

1 **The sulfur solubility minimum and maximum in silicate melt**

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Abstract

The behaviour of sulfur in magmas is complex because it dissolves as both sulfide (S^{2-}) and sulfate (S^{6+}) in silicate melt. An interesting aspect in the behaviour of sulfur is the solubility minima (SS^{\min}) and maxima (SS^{\max}) with varying oxygen fugacity (f_{O_2}). We use a simple ternary model (silicate– S_2 – O_2) to explore the varying f_{O_2} paths where these phenomena occur. Both SS^{\min} and SS^{\max} occur when S^{2-} and S^{6+} are present in the silicate melt in similar quantities due to the differing solubility mechanism of these species. At constant T , a minimum in dissolved total S content (w^m_{ST}) in vapour-saturated silicate melt occurs along paths of increasing f_{O_2} and either constant f_{S_2} or P ; for paths on which w^m_{ST} is held constant with increasing f_{O_2} , the SS^{\min} is expressed as a maximum in P . However, the SS^{\min} is not encountered during closed-system depressurisation in the simple system we modelled. The SS^{\max} occurs when the silicate melt is multiply-saturated with vapour, sulfide melt, and anhydrite. The SS^{\min} and SS^{\max} influence processes throughout the magmatic system, such as mantle melting, magma mixing and degassing, and SO_2 emissions; and calculations of the pressures of vapour-saturation, f_{O_2} , and SO_2 emissions using melt inclusions.

Supplementary material: Additional information and data used to create the figures are included with the submission of this manuscript. The code used to generate the data is available at <https://github.com/eryhughes/SSminmax>.

31 It is widely accepted that there is a minimum in the solubility of sulfur (abbreviated to the SS^{\min} ,
32 for the “sulfur solubility minimum”) in silicate melts (i.e., in the concentration of dissolved S in a
33 silicate melt coexisting with an S-bearing vapour) as a function of oxygen fugacity (f_{O_2}) when the
34 speciation of S in the silicate melt changes from being sulfide (S^{2-}) to sulfate (SO_4^{2-} or abbreviated
35 as S^{6+}) dominated. This is based on experimental studies of the concentration of S in vapour-
36 saturated silicate melts spanning a range in f_{O_2} (e.g., Fincham and Richardson 1954; Katsura and
37 Nagashima 1974; Carroll and Rutherford 1985; Backnaes and Deubener 2011; Lesne *et al.* 2015;
38 Matjuschkin *et al.* 2016; Nash *et al.* 2019). Such a minimum in S solubility has implications for
39 magmatic and volcanic processes. For example, any process where f_{O_2} progressively changes and
40 becomes closer to the f_{O_2} of SS^{\min} (e.g., mixing, progressive reduction or oxidation, degassing etc.)
41 will result in a decrease in the S-solubility. This minimum has been used as evidence of the
42 presence of additional, low solubility, potentially unquenchable, S-bearing species in silicate melts
43 that could be important for metal transport in arc systems (Matjuschkin *et al.* 2016). Also,
44 understanding the thermodynamic basis for this feature is critical for calculating the pressure of
45 vapour-saturation of S-bearing magmas using the volatile concentrations of quenched glasses (e.g.,
46 Lesne *et al.*, 2015).

47 Despite its potential importance, the existence of an SS^{\min} with varying f_{O_2} has been somewhat
48 mischaracterised in the literature. As emphasized by O’Neill (2020), this is at least in part due to
49 a lack of clarity regarding the independent variables and the path followed by sulfur fugacity (f_{S_2})
50 with increasing f_{O_2} for specific natural or experimental processes (e.g., Moretti *et al.*, 2003) and
51 the number and identity of additional S-bearing phase(s) with which the silicate melt is saturated
52 (e.g., Jugo *et al.*, 2005). In this paper, we explore the conditions and paths for which an SS^{\min}
53 occurs and some of the implications for magmatic and volcanic processes. We also expand upon
54 the work of Jugo (2009) regarding a S solubility *maximum* (referred to as an SS^{\max}) for silicate
55 melts that are multiply saturated with sulfide melt + anhydrite \pm vapour. We note that although
56 silicate melt and vapour in most natural systems contain H, C, halogens, metals, etc., in addition
57 to S and O; here, we limit ourselves to a simple system in which S and O are the only volatile
58 components in the silicate melt (although such a system may be appropriate of Jupiter’s moon Io:
59 e.g., Zolotov and Fegley, 2000). By limiting our treatment to this simple end-member system, the
60 factors leading to an SS^{\min} or an SS^{\max} can be more easily isolated and understood.

61 **Thermodynamic modelling**

62 Equilibria between silicate melt, vapour, sulfide melt, and anhydrite

63 Sulfur occurs in several phases in magmatic systems, including dissolved S-bearing species in
64 silicate melt, gaseous species in vapour, immiscible sulfide melts, and various sulfate phases (e.g.,
65 reviews by Parat *et al.*, 2011; Wallace and Edmonds, 2011). In addition to silicate melt and vapour,
66 we consider pure Fe-sulfide melt (FeS) and anhydrite ($CaSO_4$) (Figure 1a and b). The silicate melt
67 end member can be compositionally simple (e.g., SiO_2 , $CaMgSi_2O_6$, $NaAlSi_3O_8$) or complex (e.g.,
68 a natural basalt), provided it is fixed in composition. In our calculations, the silicate component is
69 taken to be a S-free Hawaiian basaltic melt in which all the Fe is present as Fe^{2+} (composition
70 given in the Supplementary Material). We assume that the silicate melt is insoluble in the vapour.

71 The vapour is assumed to be constrained to the S–O binary subsystem and to contain only three
72 species (S_2 , O_2 , and SO_2 ; Figure 1b), hence

73
$$x^v_{S_2} + x^v_{O_2} + x^v_{SO_2} = 1, \quad (1)$$

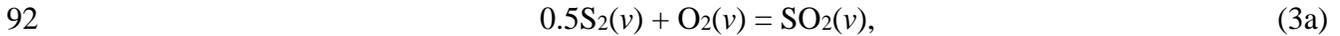
74 where x^v_i is the mole fraction of species i in the vapour. Other species are present in an S–O vapour
 75 (e.g., SO_3 , SO , S polymers, etc.) and could be added to our treatment. However, the three species
 76 in equation (1) are the most significant (e.g., Oppenheimer *et al.* 2011; Renggli *et al.* 2017; Henley
 77 and Seward 2018; Henley and Fischer 2021) and are sufficient to illustrate the salient points.

78 Sulfide (S^{2-}) and sulfate (S^{6+}) are assumed to be the only significant S-bearing species dissolved
 79 in natural silicate melts based on XANES measurements and solubility experiments (e.g., Fincham
 80 and Richardson 1954; Paris *et al.* 2001; Métrich *et al.* 2009; Wilke *et al.* 2011). Intermediate S-
 81 bearing species have been observed or inferred (e.g., S^{4+} , S_3^{2-} , S^0 , molecular SO_2 : Clemente *et al.*
 82 2004; Métrich and Wallace 2009; Burgisser *et al.* 2015; Lesne *et al.* 2015; Matjuschkin *et al.* 2016;
 83 Colin *et al.* 2020), but these species are not thought to be significant in natural (especially in Fe-
 84 bearing) silicate melts. Hence, the silicate melt is assumed to contain only two S-bearing species
 85 (S^{2-} and S^{6+} ; Figure 1b), such that

86
$$w^m_{S^{2-}} + w^m_{S^{6+}} = w^m_{S_T}, \quad (2)$$

87 where w^m_i is the weight fraction of species i in the silicate melt, and S_T refers to total dissolved S
 88 content.

89 Three independent reactions control the coexisting compositions of silicate melt and vapour in
 90 this system. The first reaction describes a homogeneous equilibrium that governs the speciation of
 91 the vapour (v):

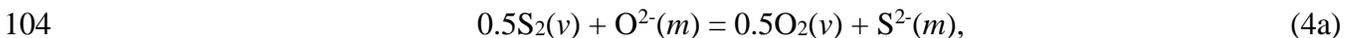


93 which is governed at equilibrium by

94
$$K_3(T) = \frac{f_{SO_2}}{(f_{S_2})^{0.5} f_{O_2}}, \quad (3b)$$

95 where K_3 is the equilibrium constant for reaction (3a) and f_i is the fugacity of species i in the
 96 vapour. If the treatment were to include other S ± O-bearing vapour species (such as SO_3 , SO , etc.;
 97 see above), a statement of homogeneous equilibrium would have to be added for each additional
 98 vapour species.

99 The second reaction describes a heterogeneous equilibrium between silicate melt (m) and
 100 vapour that governs the dissolution of sulfur from the vapour as S^{2-} in the silicate melt (e.g.,
 101 Fincham and Richardson 1954; Moretti and Ottonello 2005, 2003; Moretti and Papale 2004;
 102 Gaillard and Scaillet 2009, 2014; Baker and Moretti 2011; Gaillard *et al.* 2011, 2013, 2015;
 103 Baumgartner *et al.* 2017; Moretti 2021), described by the following three relations:

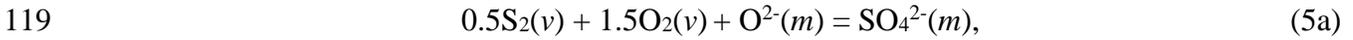


105
$$K_4(P, T) = \frac{a^m_{S^{2-}}}{a^m_{O^{2-}}} \left(\frac{f_{O_2}}{f_{S_2}} \right)^{0.5} \approx \frac{x^m_{S^{2-}}}{x^m_{O^{2-}}} \left(\frac{f_{O_2}}{f_{S_2}} \right)^{0.5}, \text{ and} \quad (4b)$$

106
$$C_{S_{2-}} = w_{S_{2-}}^m \left(\frac{f_{O_2}}{f_{S_2}} \right)^{0.5}; \quad (4c)$$

107 where a^m_i , x^m_i , and w^m_i are the activity, mole fraction, and weight fraction, respectively, in the
 108 silicate melt of the i^{th} ion (either S^{2-} or oxide $[O^{2-}]$, in this case); and $C_{S_{2-}}$ is referred to as the sulfide
 109 capacity (e.g., Fincham and Richardson 1954; O'Neill 2020). The final term of equation (4b)
 110 makes the approximation that a^m_i can be replaced by x^m_i , and this approximation is adopted
 111 throughout. Given this approximation, $C_{S_{2-}}$ is simply related to K_4 , the equilibrium constant for
 112 reaction (4a), but by convention it is defined in terms of w^m_i rather than x^m_i . Finally, we assume
 113 throughout that $x^m_{O_{2-}}$ (i.e., the O^{2-} that is part of the silicate melt that can be replaced by S^{2-}) can
 114 be approximated as constant (i.e., $x^m_{S_{2-}} \ll x^m_{O_{2-}}$).

115 A third reaction describes an additional heterogeneous equilibrium between silicate melt and
 116 vapour that governs the dissolution of sulfur from the vapour as SO_4^{2-} in the silicate melt (e.g.,
 117 Fincham and Richardson 1954; Moretti and Ottonello 2003, 2005; Moretti and Papale 2004; Baker
 118 and Moretti 2011; Moretti 2021):



120
$$K_5(P, T) = \frac{a_{S_{6+}}^m}{a_{O_{2-}}^m (f_{S_2} f_{O_2^3})^{0.5}} \approx \frac{x_{S_{6+}}^m}{x_{O_{2-}}^m (f_{S_2} f_{O_2^3})^{0.5}}, \text{ and} \quad (5b)$$

121
$$C_{S_{6+}} = w_{S_{6+}}^m (f_{S_2} f_{O_2^3})^{-0.5}; \quad (5c)$$

122 where $a^m_{S_{6+}}$ and $x^m_{S_{6+}}$ are the activity and mole fraction, respectively, of sulfate dissolved in the
 123 silicate melt; and $C_{S_{6+}}$ is referred to as the sulfate capacity (Fincham and Richardson 1954). Again,
 124 $C_{S_{6+}}$ is simply related to K_5 , the equilibrium constant for reaction (5a), but using weight fraction
 125 instead of mole fraction, and assuming $x^m_{O_{2-}}$ is constant.

126 An alternative heterogeneous equilibrium between silicate melt and vapour can be used instead
 127 of either reaction (4a) or (5a) to describe the conversion of S^{2-} to S^{6+} in the silicate melt (e.g.,
 128 Wallace and Carmichael 1994; Matthews *et al.* 1999; Métrich *et al.* 2009; Jugo *et al.* 2010;
 129 Baumgartner *et al.* 2017):



131
$$K_6(P, T) = \frac{a_{S_{6+}}^m}{a_{S_{2-}}^m (f_{O_2})^2} \approx \frac{x_{S_{6+}}^m}{x_{S_{2-}}^m (f_{O_2})^2}, \text{ and} \quad (6b)$$

132
$$\frac{x_{S_{6+}}^m}{x_{S_{2-}}^m} = \left[\frac{S^{6+}}{S^{2-}} \right]^m = \frac{w_{S_{6+}}^m}{w_{S_{2-}}^m} = \frac{C_{S_{6+}}}{C_{S_{2-}}} f_{O_2}^2. \quad (6c)$$

133 Reaction (6a) can be obtained by subtracting reaction (4a) from reaction (5a) and rearranging.
 134 Reaction (6a) is useful as it emphasizes that the oxidation state of S dissolved in the silicate melt
 135 (i.e., $[S^{6+}/S^{2-}]^m$ or $[S^{6+}/S_T]^m$) at a given T and P is controlled only by f_{O_2} , $C_{S_{2-}}$, and $C_{S_{6+}}$. However,
 136 $C_{S_{2-}}$ and $C_{S_{6+}}$ are highly dependent on the composition of the silicate melt (e.g., O'Neill and
 137 Mavrogenes 2002, 2019; Moretti and Ottonello 2005; Nash *et al.* 2019; O'Neill 2020; Boulliung
 138 and Wood 2021; Moretti 2021). Additionally, they will depend on T and P (because K_4 and K_5

139 must depend on T and P unless the standard state enthalpy and volume changes of the reactions
 140 are zero) and are likely to depend on the speciation of other multivalent elements in the silicate
 141 melt (e.g., $[\text{Fe}^{3+}/\text{Fe}^{2+}]^m$). Although such factors could lead indirectly to variations in $[\text{S}^{6+}/\text{S}^{2-}]^m$ at
 142 constant f_{O_2} , these are expected to be minor effects for most of the examples considered here.
 143 Therefore, we assume $C_{\text{S}^{2-}}$ and $C_{\text{S}^{6+}}$ depend only on T and the composition of the silicate
 144 component (and depend only on total Fe not $[\text{Fe}^{3+}/\text{Fe}^{2+}]^m$), all of which are constant in all our
 145 calculations.

146 Given equation (1), if an S–O vapour is present in the system (i.e., if the silicate melt is vapour-
 147 saturated), the sum of the partial pressures (p_i) of the species in the vapour equals the total pressure
 148 of the system (P):

$$149 \quad P = p_{\text{O}_2} + p_{\text{S}_2} + p_{\text{SO}_2}, \quad (7)$$

150 where the partial pressures are related to fugacity and mole fraction in the vapour through fugacity
 151 coefficients (γ_i):

$$152 \quad f_i = \gamma_i p_i = \gamma_i x_i^v P. \quad (8)$$

153 When the silicate melt is saturated with sulfide melt, the chemical potential of FeS in the
 154 silicate melt (μ_{FeS}^m) and the sulfide melt are equal. The “sulfide content at sulfide-saturation” (S^{2-} -
 155 CSS) is the dissolved S^{2-} concentration in the silicate melt in equilibrium with sulfide melt (e.g.,
 156 Shima and Naldrett 1975; O’Neill and Mavrogenes 2002; Smythe *et al.* 2017; O’Neill 2020):

$$157 \quad w_{\text{S}^{2-}}^m = w_{\text{S}^{2-}\text{-CSS}}^m. \quad (9)$$

158 The total S content of a silicate melt that is saturated with sulfide melt ($\text{S}_{\text{T}}\text{CSS}$) is then given by
 159 equation (9) in combination with equations (2) and (6c):

$$160 \quad w_{\text{S}_{\text{T}}}^m = w_{\text{S}_{\text{T}}\text{CSS}}^m = (1 + (C_{\text{S}^{6+}}/C_{\text{S}^{2-}})(f_{\text{O}_2})^2)w_{\text{S}^{2-}\text{-CSS}}^m. \quad (10)$$

161 Alternatively, when the silicate melt is saturated with anhydrite, the chemical potential of
 162 CaSO_4 in the silicate melt ($\mu_{\text{CaSO}_4}^m$) and anhydrite are equal. The “sulfate content at anhydrite-
 163 saturation” ($\text{S}^{6+}\text{-CAS}$) is the dissolved S^{6+} concentration in the silicate melt in equilibrium with
 164 anhydrite (e.g., Baker and Moretti 2011; Chowdhury and Dasgupta 2019; Zajacz and Tsay 2019):

$$165 \quad w_{\text{S}^{6+}}^m = w_{\text{S}^{6+}\text{-CAS}}^m. \quad (11)$$

166 The total S content of a silicate melt that is saturated with anhydrite ($\text{S}_{\text{T}}\text{CAS}$) is then given by
 167 equation (11) in combination with equations (2) and (6c):

$$168 \quad w_{\text{S}_{\text{T}}}^m = w_{\text{S}_{\text{T}}\text{CAS}}^m = (1 + (C_{\text{S}^{2-}}/C_{\text{S}^{6+}})(f_{\text{O}_2})^{-2})w_{\text{S}^{6+}\text{-CAS}}^m. \quad (12)$$

169 The importance of the phase rule in our treatment of S-solubility in silicate melt

170 In its simplest form, the phase rule relates the number of components (c) and phases (φ) in a system
 171 to the variance (or the degrees of freedom, F) of the assemblage:

172
$$F = c + 2 - \varphi. \quad (13)$$

173 For most of the calculations presented here, our system has three components (silicate, S₂, and O₂;
 174 $c = 3$) and two phases (silicate melt + vapour; $\varphi = 2$) (Figure 1a). This silicate melt + vapour
 175 assemblage is thus trivariant ($\varphi = 2, F = 3$); if only silicate melt were present, the system would
 176 be quadrivariant ($\varphi = 1, F = 4$). Therefore, for the silicate melt + vapour assemblage, if any three
 177 linearly independent intensive variables are chosen as independent variables, the state of the
 178 system is fully defined, and all other intensive variables are dependent. Consequently, the values
 179 of three independent variables *must* be given to completely specify the state of the system. The
 180 values of the other intensive variables can be calculated given the values chosen for the
 181 independent variables and knowledge of the thermochemistry of the silicate melt and vapour
 182 phases. Again, for vapour-undersaturated silicate melt, only one phase is present and four
 183 independent intensive variables are needed to define fully the state of the system.

184 The important point here is a restatement of the cautionary note in O’Neill (2020) about the
 185 SS^{\min} when f_{O_2} is an independent variable: i.e., assuming temperature (T) is constant, the variation
 186 of w^m_{ST} is *not* uniquely defined if only f_{O_2} is independently varied. The behaviour of a third variable
 187 must also be specified for the state of the silicate melt + vapour assemblage to be defined at each
 188 point on a path of varying f_{O_2} . Only then can the variation in w^m_{ST} (including the nature of any
 189 minimum or maximum) as a function of f_{O_2} be uniquely characterized.

190 There are a variety of intensive variables that could be chosen as independent or dependent
 191 variables in our model system. For example, temperature (T); pressure (P); the dissolved S²⁻, S⁶⁺,
 192 and S_T content of the silicate melt, specified as x^m_i or w^m_i (given equation (2), only two of these
 193 three quantities can be chosen as independent variables); the bulk composition of the silicate melt,
 194 vapour, or the system as a whole, given as the mole or weight fractions of S, O, and/or silicate in
 195 the silicate melt, vapour, or system; or O₂, S₂, or SO₂ in the vapour (in either case these three
 196 variables must add up to one, so only two can be specified independently); the chemical potentials
 197 of all but one of the vapour species (μ^v_i , or equivalently fugacities, f_i , where $i = O_2, S_2, \text{ or } SO_2$);
 198 the chemical potential of FeS and/or CaSO₄ in the silicate melt (μ^m_i) or in the coexisting sulfide
 199 melt and/or anhydrite if they are present; or the oxidation state of Fe and/or S in the silicate melt,
 200 (either $[Fe^{3+}/Fe^{2+}]^m$ or $[Fe^{3+}/FeT]^m$ and/or $[S^{6+}/S^{2-}]^m$ or $[S^{6+}/S_T]^m$), which is equivalent to specifying
 201 f_{O_2} .

202 The composition of the silicate melt and vapour phases are described by our idealised three-
 203 component system. Although the compositions of the sulfide melt and anhydrite phases could be
 204 incorporated into our treatment, their compositions fall outside of the plane of our chosen three-
 205 component system. This means the composition of the silicate melt does not vary if sulfide melt
 206 and/or anhydrite phases are saturated in our calculations (i.e., Fe and Ca concentrations in the
 207 silicate melt are constant). Despite this, we can still model the effects of sulfide melt- and
 208 anhydrite-saturation on the properties of the silicate melt and vapour phases using μ^m_{FeS} or $\mu^m_{CaSO_4}$
 209 as described in “Silicate melt + (vapour and/or sulfide melt and/or anhydrite)Silicate melt + vapour
 210 + (sulfide melt and/or anhydrite)”.

211 We use this conceptual background to explore trends in, and interrelationships among, various
 212 choices of independent and dependent variables, focussing on the implications for the SS^{\min} and
 213 the SS^{\max} . We model the silicate melt as a Hawaiian basalt; its composition and details of our
 214 choices of thermodynamic parameters for vapour and silicate melt can be found in the

215 Supplementary Material. For all of our calculations, T and the composition of the silicate
 216 component are held constant. Since we assume that C_{S2-} and C_{S6+} depend only on T and the
 217 composition of the silicate component, both of these parameters are the same in all calculations
 218 presented here. In particular, C_{S2-} and C_{S6+} are independent of P and $[Fe^{3+}/Fe_T]^m$, and therefore f_{O2} .
 219 It is important to emphasize that the results based on our chosen parameters only describe the
 220 representative behaviour of the particular Hawaiian melt composition given in the Supplementary
 221 Material. Although we are confident that the trends and insights derived from this choice are
 222 robust, the exact behaviour depends strongly on the chosen values of C_{S2-} and C_{S6+} . Therefore, the
 223 specific values of various variables – including the precise values of f_{O2} where shifts in behaviour
 224 are predicted to occur – will likely vary strongly with T , composition of the silicate component,
 225 and the C_{S2-} and C_{S6+} parameterisation used (e.g., O’Neill and Mavrogenes 2002, 2019; Moretti
 226 and Ottonello 2005; Nash *et al.* 2019; O’Neill 2020; Boulliung and Wood 2021; Moretti 2021).

227 Independent variables of T , f_{O2} , f_{S2} , $w^{m_{ST}}$, P , μ_{FeS} , and μ_{CaSO4}

228
 229 In this section we calculate the state of the system by choosing T (1200°C) and a value of f_{O2} (-5
 230 < ΔFMQ < +5, where FMQ is the Fayalite-Magnetite-Quartz buffer; FM β Q in Frost, 1991) as two
 231 of the independent variables. The other independent variables considered are f_{S2} , $w^{m_{ST}}$, P , μ_{FeS} , and
 232 μ_{CaSO4} . For any given state of the system (i.e., where the values of all intensive variables are
 233 defined), the results will be the same if any three variables are chosen as independent. Therefore,
 234 the following figures convey the same results, but with different variables as the axes and contours.
 235 For most of the P range shown in the following figures, $\gamma_i \sim 1$, hence $f_i \sim p_i$ using equation (8) (e.g.,
 236 for $\log_{10}[P, \text{bar}] < 3$, $\gamma_i < 1.25$ for all species). “Silicate melt + vapour” describes the two-phase
 237 silicate melt + vapour assemblages, whilst “Silicate melt + vapour + (sulfide melt and/or
 238 anhydrite)” and “Silicate melt + (vapour and/or sulfide melt and/or anhydrite)” describe the silicate
 239 melt + (vapour and/or sulfide melt and/or anhydrite) assemblages. If the S content of the silicate
 240 melt is less than $w^{m_{ST}}$ at vapour- and/or sulfide melt- and/or anhydrite-saturation, silicate melt is
 241 the only stable phase present.

242 Silicate melt + vapour

243 For silicate melt + vapour assemblages, $\varphi = 2$ and $F = 3$; therefore, in addition to T and f_{O2} , only
 244 one other independent variable is needed to specify fully the state of the system. This third
 245 independent variable will be referred to as the “ Y ” variable: we first choose $Y = f_{S2}$, then $Y = w^{m_{ST}}$,
 246 and finally $Y = P$. For each of these choices of the independent variables, we solve the system of
 247 equations (3b), (7), (8), and two out of (4c), (5c), and (6c).

248 *S speciation in the silicate melt and vapour*

249 Figure 2 shows how the compositions of the silicate melt and vapour change with varying f_{O2} at
 250 constant T and either constant f_{S2} (Figure 2a–b), constant $w^{m_{ST}}$ (Figure 2c–d), or constant P (Figure
 251 2e–f) given that the silicate melt is vapour-saturated. As f_{O2} changes, the speciation of sulfur in the
 252 silicate melt ($[S^{6+}/S_T]^m$) and in the vapour ($x^{v_{SO2}}$) change. This leads to changes with f_{O2} in the
 253 dominant species (or multiple species) in the silicate melt and vapour, which is indicated by the
 254 vertical background colour bands in Figure 2. For the vapour, S_2 is the dominant species at low f_{O2}
 255 ($x^{v_{SO2}} < 0.1$, purple); SO_2 is the dominant species at high f_{O2} ($x^{v_{SO2}} > 0.9$, turquoise-green-yellow);

256 and in between there is a transition from dominantly S_2 to SO_2 ($0.1 \leq x^{v_{SO_2}} \leq 0.9$, blue). At
 257 sufficiently high f_{O_2} , O_2 becomes more abundant than S_2 in the vapour (i.e., to the right of the point
 258 labelled α in the yellow band in Figure 2b, d, and f), but both are much less abundant than SO_2 in
 259 the range shown. For the silicate melt, S^{2-} is the dominant S-bearing species at low f_{O_2} ($[S^{6+}/S_T]^m$
 260 < 0.1 , purple-blue-turquoise); S^{6+} is dominant at high f_{O_2} ($[S^{6+}/S_T]^m > 0.9$, yellow); and in between
 261 it transitions from dominantly S^{2-} to S^{6+} ($0.1 \leq [S^{6+}/S_T]^m \leq 0.9$, green).

262 At constant T , the effects of varying the third independent variable ($Y = f_{S_2}$, $w^{m_{ST}}$, or P) in
 263 addition to f_{O_2} on the dependent variables (which are referred to as “Z” variables) are shown using
 264 contour plots in Figure 3–Figure 5. The coloured regions from Figure 2 showing the dominant
 265 silicate melt and vapour species and their changes are also shown in Figure 3–Figure 5 using the
 266 same colour scheme. These regions in Figure 3–Figure 5 are separated by black curves, where
 267 dashed curves indicate isopleths of vapour speciation ($x^{v_{SO_2}} = 0.1$ and 0.9) and solid curves indicate
 268 isopleths of silicate melt speciation ($[S^{6+}/S_T]^m = 0.1$ and 0.9). Comparison of panels (d) in Figure
 269 3–Figure 5 shows that the topology of the silicate melt and vapour speciation (indicated by the
 270 coloured fields) is similar for all three of these choices of Y .

271 A key point of Figure 3–Figure 5d is that regardless of the choice of the Y variable, at low f_{O_2}
 272 ($\Delta FMQ \lesssim +0.7$) nearly all dissolved S is present as S^{2-} (purple-blue-turquoise). Under these
 273 conditions $w^{m_{ST}} \cong w^{m_{S_2}}$, and $w^{m_{ST}}$ is controlled by equation (4c), giving $w^{m_{S_2}}$ as a simple function
 274 of f_{O_2} and f_{S_2} . Likewise, at sufficiently high f_{O_2} ($\Delta FMQ \gtrsim +1.7$), nearly all S is dissolved as S^{6+}
 275 (yellow). Here $w^{m_{ST}} \cong w^{m_{S_2}}$, and $w^{m_{ST}}$ is instead controlled by equation (5c), which gives $w^{m_{S_2}}$
 276 as a simple function of f_{O_2} and f_{S_2} . At intermediate f_{O_2} ($+0.7 \lesssim \Delta FMQ \lesssim +1.7$), the silicate melt
 277 transitions from being dominated by S^{2-} to S^{6+} (green). This region is narrow because the S
 278 speciation in the silicate melt is defined by equation (6c), which depends on $(f_{O_2})^2$ (i.e., the
 279 difference in $\log_{10}[f_{O_2}]$ between $[S^{6+}/S_T]^m = 0.1$ and 0.9 is $\log_{10}[9] \sim 0.95$). The silicate melt
 280 isopleths (the black dashed curves defining the edges of the green region) are sub-vertical (i.e., not
 281 precisely vertical) due to the P -dependence of f_{O_2} on the FMQ buffer. When the value of Y is large,
 282 P is large, which leads to p_i deviating from f_i , causing the silicate melt isopleths to deviate from
 283 vertical (i.e., to curve to lower f_{O_2}). They would, however, be vertical if plotted against $\log_{10}[f_{O_2}]$
 284 without normalisation to a buffer, given that C_{S_2} and C_{S_2} are assumed to be P -independent.

285 Unlike the speciation of S dissolved in the silicate melt, which depends only on f_{O_2} via equation
 286 (6c), vapour speciation (i.e., $x^{v_{SO_2}}$) depends on f_{S_2} in addition to f_{O_2} from equations (3b) and (8).
 287 Therefore, the boundaries separating where the vapour phase is dominated by S_2 (purple), both
 288 S_2+SO_2 (blue), and SO_2 (turquoise-green-yellow) depend on f_{O_2} (see the vapour isopleths in Figure
 289 3–Figure 5). However, the slopes of the boundaries depend on the choice of the Y variable and its
 290 value, as well as the silicate melt speciation (e.g., see the change in slope of $x^{v_{SO_2}} = 0.9$ in Figure
 291 4 where it enters the green region with increasing f_{O_2}). The change in vapour speciation from
 292 dominantly S_2 to SO_2 occurs over a wider range of f_{O_2} than the silicate melt speciation (compare
 293 the widths of the blue and green bands in Figure 2 and Figure 3–Figure 5d) because f_{S_2} (and $x^{v_{SO_2}}$
 294 via equation (8)) depends on $(f_{O_2})^1$ from equation (3b), in contrast to $[S^{6+}/S_2]^m$, which depends on
 295 $(f_{O_2})^2$ from equation (6c).

296 Combining changes in silicate melt and vapour speciation, there are three main regions plus
 297 three transitional regions in $\log_{10}[f_{O_2}]-\log_{10}[Y]$ space (Figure 3–Figure 5): at low f_{O_2} and high Y ,
 298 the vapour is S_2 -dominated and the silicate melt is S^{2-} -dominated (purple); at higher f_{O_2} and lower
 299 Y , the vapour contains both S_2 and SO_2 in similar concentrations, whereas the silicate melt is still

300 S²⁻-dominated (blue); at higher f_{O_2} and lower Y , the vapour is now SO₂-dominated and the silicate
 301 melt is still S²⁻-dominated (turquoise); at higher f_{O_2} and most Y values shown, the vapour remains
 302 SO₂-dominated but the silicate melt contains both S²⁻ and S⁶⁺ in similar concentrations (green); at
 303 high f_{O_2} and all Y values shown, the vapour is still SO₂-dominated but the silicate melt is S⁶⁺-
 304 dominated (yellow); and there is also a small region where the vapour contains both S₂ and SO₂,
 305 and the silicate melt both S²⁻ and S⁶⁺, in similar concentrations (light-turquoise region labelled [S²⁻-
 306 +S⁶⁺]^{*m*}+ [S₂+SO₂]^{*v*} in Figure 3–Figure 5: it is not intersected in Figure 2). At higher Y values than
 307 shown in Figure 3–Figure 5, there are three additional regions: at intermediate f_{O_2} , the vapour is
 308 S₂-dominated and the silicate melt contains both S²⁻ and S⁶⁺ in similar concentrations; at high f_{O_2}
 309 but intermediate Y , the vapour contains S₂ and SO₂ in similar concentrations and the silicate melt
 310 is S⁶⁺-dominated; and at higher Y , the vapour is S₂-dominated and the silicate melt is still S⁶⁺-
 311 dominated (a sketch of this topology is shown in the Supplementary Material).

312 *Regions with a single dominant species in the silicate melt and vapour*

313 For all three of our choices of a constant independent Y variable, curves (Figure 2) and contours
 314 (Figure 3–Figure 5) of the dependent variables (Z) shown in the figures have constant slopes when
 315 there is a single dominant species in the silicate melt *and* a single dominant species in the vapour.
 316 For example, when the silicate melt is S²⁻-dominated (purple-blue-turquoise),

$$317 \quad w_{ST}^m \cong w_{S^{2-}}^m, \quad (14a)$$

318 and when S⁶⁺-dominated (yellow):

$$319 \quad w_{ST}^m \cong w_{S^{6+}}^m. \quad (14b)$$

320 Similarly, when the vapour is S₂-dominated (purple),

$$321 \quad P \cong p_{S_2} \cong f_{S_2}, \quad (15a)$$

322 and when SO₂-dominated (turquoise-green-yellow):

$$323 \quad P \cong p_{SO_2} \cong f_{SO_2}. \quad (15b)$$

324 When T is constant and there is only one dominant species in the silicate melt and one dominant
 325 species in the vapour

$$326 \quad Z \propto (f_{O_2})^a (Y)^b, \quad (16)$$

327 for all choices of Z and Y given here (see Table 1). Therefore, contours of constant Z in $\log_{10}(Y)$ -
 328 $\log_{10}(f_{O_2})$ plots have slopes (σ) $\sim -a/b$ (Figure 3–Figure 5) and, when Y is constant, curves of
 329 different Z have slopes (ζ) $\sim a$ (Figure 2). Hence, the values of the slopes of the curves and contours
 330 of Z depend on the specific choice of the Y variable. Note that ζ and σ are approximately, rather
 331 than exactly, equal to these values because: (1) the x-axis is $\log_{10}(f_{O_2})$ relative to FMQ rather than
 332 strictly $\log_{10}(f_{O_2})$; (2) partial pressure rather than fugacity is sometimes plotted, which are related
 333 through equation (8); and (3) although one species is dominant, the concentrations of the other
 334 species are not zero. At constant T , the relationships between f_{O_2} , Y (f_{S_2} , w_{ST}^m , and P), and Z (f_{S_2} ,

335 f_{SO_2} , P , w^{mS_2} , w^{mS_6} , and w^{mST}) when the silicate melt is S^{2-} -dominated and the vapour is S_2 -
 336 dominated (purple), the silicate melt is S^{2-} -dominated but the vapour is SO_2 -dominated (turquoise),
 337 and the silicate melt is S^{6+} -dominated and the vapour is SO_2 -dominated (yellow) can be derived
 338 by variously substituting equations (14) and (15) into equations (3)–(8) and rearranging into the
 339 form of equation (16). These relationships are derived in the Supplementary Material; summarised
 340 (including ζ and σ values) in Table 1; and labelled in Figure 2–Figure 5.

341 *Regions with mixed speciation in the silicate melt or vapour*

342 Changes in the slopes of curves (ζ , Figure 2) and contours (σ , Figure 3–Figure 5) occur for
 343 dependent variables (Z) when their slopes in $\log_{10}(Y)$ - $\log_{10}(f_{\text{O}_2})$ space differ in regions dominated
 344 by different species (i.e., S_2 (purple) vs. SO_2 (turquoise) in the vapour and/or S^{2-} (turquoise) vs.
 345 S^{6+} (yellow) in the silicate melt). The regions in which the slopes transition from one essentially
 346 constant value to another are where there is mixed speciation in the vapour (i.e., both S_2 and SO_2
 347 in similar concentrations: blue) or in the silicate melt (i.e., both S^{2-} and S^{6+} in similar
 348 concentrations: green).

350 If the log-log slope of a Z variable with respect to f_{O_2} has the same sign in the low- and high-
 351 f_{O_2} regions on either side of one of the mixed species regions, the slope simply gets steeper or
 352 shallower along a constant Y path (Figure 2–Figure 5). For example, with increasing f_{O_2} and $Y =$
 353 constant $w^{\text{mST}} = 400$ ppm, the slope in Figure 2d of $\log_{10}(P)$ vs. $\log_{10}(f_{\text{O}_2})$ is $\zeta \cong +1.0$ where the
 354 silicate melt is S^{2-} -dominated and the vapour is S_2 -dominated (in the purple region). This is less
 355 than the value of $\zeta \cong +1.5$ where the vapour is SO_2 -dominated (in the turquoise region), but both
 356 slopes are positive (Table 1). For the contour plot in Figure 4c, this is shown as σ decreasing from
 357 -0.5 to -1.5 , but both being negative. This is because when the silicate melt is S^{2-} -dominated, the
 358 log-log slope for $Z = p_{\text{SO}_2}$ is steeper than for $Z = p_{\text{S}_2}$ ($\zeta \cong +1.5$ vs. $+1.0$ or $\sigma \cong -1.5$ vs. -0.5 in the
 359 purple-blue-turquoise regions in Figure 2d and Figure 4a–c). Therefore, the curves cross when p_{S_2}
 360 $= p_{\text{SO}_2}$ (at the point labelled β in the blue region of Figure 2d). Hence, along this path P is essentially
 361 equal to p_{S_2} when S_2 dominates the vapour ($\zeta \cong +1.0$ or $\sigma \cong -0.5$, purple), steepens continuously in
 362 the region where the concentration of SO_2 in the vapour increases (blue), and is then well
 363 approximated by p_{SO_2} when SO_2 dominates the vapour ($\zeta \cong +1.5$ or $\sigma \cong -1.5$, turquoise).

364 If the log-log slope of a Z variable with respect to f_{O_2} has opposite signs on the two sides of a
 365 mixed speciation region, a maximum (positive to negative slope with increasing f_{O_2}) or minimum
 366 (negative to positive slope with increasing f_{O_2}) in Z occurs (Figure 2–Figure 5). For example, for
 367 $Y = \text{constant } f_{\text{S}_2}$, there is a minimum in $Z = w^{\text{mST}}$ at $[\text{S}^{6+}/\text{S}_T]^m = 0.5$ (this corresponds to a maximum
 368 in $Z = \log_{10}[w^{\text{mST}}]$ contours), where the silicate melt changes from being S^{2-} - to S^{6+} -dominated
 369 (i.e., the green region in Figure 2a and Figure 3c). This is an example of an SS^{min} on a specific
 370 path of increasing f_{O_2} (i.e., f_{S_2} and T are constant). However, there are paths with monotonically
 371 increasing f_{O_2} and variable f_{S_2} for which a minimum is not encountered (e.g., all linear paths on
 372 Figure 3c with $\sigma \geq +1$ or $\sigma \leq -3$). There is also an SS^{min} when $Y = \text{constant } P$, as shown by the white
 373 S_T curve in Figure 2e and by the maxima in the $\log_{10}[w^{\text{mST}}]$ contours in Figure 5c. As for the
 374 constant f_{S_2} -path, this minimum also occurs at $[\text{S}^{6+}/\text{S}_T]^m = 0.5$ (i.e., in the green region). Similarly,
 375 there are paths with monotonically increasing f_{O_2} and variable P that do not encounter the
 376 minimum (e.g., all $\log_{10}(f_{\text{O}_2})$ - $\log_{10}(P)$ paths on Figure 5c with $\sigma \geq +1.5$ and $\sigma \leq -0.5$).

377 These SS^{\min} occur because curves and contours of w^m_{ST} have opposite slopes at lower- f_{O_2} where
 378 S^{2-} dominates the silicate melt compared to at higher- f_{O_2} where S^{6+} dominates (Figure 2a and e,
 379 Figure 3c, and Figure 5c). This occurs because O_2 in the vapour is on the product side of reaction
 380 (4a) when sulfur dissolves dominantly as S^{2-} in the silicate melt, but on the reactant side when
 381 sulfur dissolves dominantly as S^{6+} in the silicate melt in reaction (5a). This is true whether the
 382 reactions are written for dissolving SO_2 or S_2 from the vapour into the silicate melt. Thus, for any
 383 path of constant f_{S_2} or P (i.e., horizontal slices in Figure 3c or Figure 5c, respectively), w^m_{ST} reaches
 384 a minimum at $[S^{6+}/S_T]^m = 0.5$.

385 When $Y = w^m_{ST}$ and is held constant as f_{O_2} increases, the SS^{\min} manifests itself as a *maximum*
 386 in $Z = P$ in Figure 2d (and corresponding minima in $Z = P$ contours in the green region in Figure
 387 4c). This is because at the SS^{\min} , a higher P is required to maintain the same S content in the
 388 vapour-saturated silicate melt. Since SO_2 is the dominant vapour species on both sides of the
 389 maximum in P (i.e., $P \cong p_{SO_2}$), there is also a maximum in p_{SO_2} at essentially the same point (and
 390 corresponding minima in $Z = f_{SO_2}$ contours in Figure 4b). The change in sign of the slope of p_{SO_2}
 391 at the maximum with increasing f_{O_2} (e.g., from $\zeta \sim +1.5$ to -0.5 in Figure 2d) is due to the crossover
 392 of S^{2-} to S^{6+} as the dominant species in the silicate melt (see “Regions with a single dominant
 393 species in the silicate melt and vapour”). Note that $Z = p_{S_2}$ also has a maximum in Figure 2d (and
 394 corresponding minima in $Z = f_{S_2}$ contours in Figure 4a) with increasing f_{O_2} in the same vicinity as
 395 the maxima in P and p_{SO_2} . This is also because of the change in silicate melt speciation: however,
 396 since $p_{S_2} \ll p_{SO_2}$, the maximum in p_{S_2} has negligible influence on the maximum in P .

397 The maxima (and corresponding minima in the equivalent contours) in p_{SO_2} , p_{S_2} , and P all
 398 occur where the silicate melt speciation is mixed. The maxima in p_{SO_2} and P essentially coincide
 399 because $P \cong p_{SO_2}$, but the p_{S_2} maximum is at a different f_{O_2} , and none (unlike the w^m_{ST} minima)
 400 occur exactly at $[S^{6+}/S_T]^m = 0.5$ (e.g., compare Figure 2c and d). When the silicate melt has mixed
 401 speciation, equation (2) is needed rather than the approximations of equations (14a) and (14b).
 402 Substituting equations (4c) and (5c) into equation (2), and converting to partial pressure using
 403 equation (8) gives:

$$404 \quad f_{S_2} = \gamma_{S_2} p_{S_2} = (w^m_{ST}/(C_{S_2} \cdot (f_{O_2})^{-0.5} + C_{S_6+} \cdot (f_{O_2})^{1.5}))^2, \quad (17a)$$

405 and substituting equation (3b) gives:

$$406 \quad f_{SO_2} = \gamma_{SO_2} p_{SO_2} = K_3 w^m_{ST} / ((C_{S_2} \cdot (f_{O_2})^{0.5} + C_{S_6+} \cdot (f_{O_2})^{2.5})). \quad (17b)$$

407 The f_{O_2} values at the maximum for p_{S_2} and p_{SO_2} are given by differentiating equations (17a) and
 408 (17b) with respect to f_{O_2} and solving for f_{O_2} when this equals zero. As the relationships between
 409 p_{S_2} and p_{SO_2} with f_{O_2} are different, the f_{O_2} values of the maxima are also different.

410 We refer to the f_{O_2} where the w^m_{ST} minimum or P maximum occurs as the $SS^{\min} f_{O_2}$, but it can
 411 *only* be defined for silicate melt + vapour at a given T and for a choice of $Y = f_{S_2}$, w^m_{ST} , or P . The
 412 SS^{\min} always occurs when both S^{2-} and S^{6+} are dissolved in the silicate melt in similar
 413 concentrations, although it only occurs at $[S^{6+}/S_T]^m = 0.5$ when $Y = f_{S_2}$ or w^m_{ST} . Moreover, the
 414 minimum is not symmetric with changing f_{O_2} (see Figure 2a, d, and e; and Figure 3–Figure 5c)
 415 because the stoichiometry of the reactions for sulfur dissolution as S^{2-} and S^{6+} differ. The different
 416 manifestations of the SS^{\min} are related but not identical, reflecting the different f_{S_2} – f_{O_2} paths for
 417 different choices of Y . For example, there is a P maximum when $Y = w^m_{ST}$ (Figure 2d and Figure

418 4c), but no P maximum when $Y = f_{S2}$ (Figure 2b and Figure 3b) or P (Figure 2f, by definition).
419 Similarly, there is a w_{ST}^m minimum when $Y = f_{S2}$ (Figure 2a and Figure 3c) and P (Figure 2e and
420 Figure 5c), but not when $Y = w_{ST}^m$ (Figure 2c, again by definition). This highlights the importance
421 of the choice of the third independent variable for silicate melt + vapour in understanding the
422 occurrence of the SS^{\min} , which is required to specify the path followed by the system with changing
423 f_{O2} .

424 Silicate melt + vapour + (sulfide melt and/or anhydrite)

425 For Y values below the curves indicating sulfide melt-saturation (the grey solid curves labelled
426 “*sulf^m*”) or anhydrite-saturation (the grey dashed curves labelled “*anh*”) in Figure 3–Figure 5 (and
427 shown as vertical grey lines in Figure 2), the two-phase silicate melt + vapour assemblage is stable.
428 At Y values above these curves, the silicate melt is vapour-undersaturated but instead saturated
429 with sulfide melt and/or anhydrite, which will be discussed in “Silicate melt + (vapour and/or
430 sulfide melt and/or anhydrite)”. Therefore, the calculations involving silicate melt + vapour in this
431 region describe metastable equilibria.

432 This section describes the behaviour on the grey curves in Figure 3–Figure 5, where
433 silicate melt + vapour in the model ternary system are stable and saturated with sulfide melt and/or
434 anhydrite. In our model ternary system, there are still only two phases (silicate melt + vapour) and
435 therefore $F = 3$. Hence, given T and f_{O2} , the curve for sulfide melt-saturation is defined by a
436 particular value of $Y = \mu_{FeS}^m$ that is equal to that of FeS in the saturating sulfide melt. In practice
437 we apply the formulation of O’Neill (2020) to calculate S^2 -CSS for the silicate melt, which is
438 conceptually equivalent to holding μ_{FeS}^m equal to a constant chemical potential of sulfide melt at
439 the relevant conditions. In our calculations, we assume the sulfide melt is pure FeS (i.e., $\mu_{FeS} =$
440 μ_{FeS}°), although this could be modified. Similarly, the curve for anhydrite-saturation is defined by
441 $Y = \mu_{CaSO4}^m$. Again, although S^{6+} CAS is calculated based on Chowdhury and Dasgupta (2019), this
442 is conceptually equivalent to holding μ_{CaSO4}^m equal to a constant chemical potential of pure
443 anhydrite at the relevant conditions. The stable saturation condition (i.e., sulfide melt- or anhydrite-
444 saturation) is the one with the lowest w_{ST}^m .

445 If the silicate melt + vapour assemblage is saturated with either sulfide melt or anhydrite, P
446 and w_{ST}^m are no longer independent variables. Therefore, having chosen values of T , f_{O2} , and μ_{FeS}^m
447 or μ_{CaSO4}^m (which confines the state of the system to the *sulf^m* or *anh* curves), both P and w_{ST}^m are
448 fixed, as can be visualized in Figure 3–Figure 5c. Moreover, when the silicate melt + vapour
449 assemblage is saturated with both sulfide melt and anhydrite at a fixed T , three independent
450 variables are specified, so no other parameters can be chosen independently. This is demonstrated
451 graphically in Figure 3c and Figure 5c by the grey star, which for a given T has fixed values of P ,
452 f_{O2} , and w_{ST}^m at the unique intersection of the grey solid and dashed curves. We can thus describe
453 quantitatively with our model a constant- T , increasing- f_{O2} path for an assemblage containing
454 silicate melt + vapour + (sulfide melt and/or anhydrite). The variations in the values of the Z
455 variables along this path can be read from the contours crossed by the grey curves in Figure 3–
456 Figure 5 and Figure 6a and b show the variation in w_{ST}^m and P on this path.

457 At low f_{O2} ($\Delta FMQ \lesssim +0.6$), the independent variables are T , f_{O2} , and μ_{FeS} : the silicate melt is
458 S^{2-} -dominated and saturated with vapour and sulfide melt (i.e., the solid grey *sulf^m* curves in the
459 purple-blue-turquoise regions of Figure 3–Figure 5 and Figure 6a–b). Therefore, w_{ST}^m is controlled
460 by S^2 -CSS from equations (14a) and (9), hence:

461 $w^{m_{ST}} \cong w^{m_{S2-}} = w^{m_{S2-CSS}}.$ (18)

462 S²⁻CSS is sub-parallel to the log₁₀[w^{m_{ST}}] ~ 3 contour, although there is a small increase in w^{m_{S2-CSS}}
 463 with increasing f_{O2} because: (1) there is always some S present in the silicate melt as S⁶⁺; and (2)
 464 w^{m_{S2-CSS}} depends on P and [Fe³⁺/Fe_T]^m (O'Neill 2020), both of which increase with increasing f_{O2}.
 465 Nevertheless, w^{m_{ST}} is nearly constant and therefore for all practical purposes P (and the p_i's)
 466 behave as for the case when T-f_{O2}-w^{m_{ST}} were independent variables (Figure 2c–d and Figure 4).

467 At intermediate f_{O2} (+0.6 ≲ ΔFMQ < +1.5), the conditions remain the same (Y = μ_{FeS}; the
 468 silicate melt is saturated with vapour and sulfide melt, and w^{m_{S2-}} = w^{m_{S2-CSS}}), but the silicate melt
 469 contains significant quantities of both S²⁻ and S²⁺ (i.e., the solid grey curves in the green region of
 470 Figure 3–Figure 5 and Figure 6a–b). As the solubility of S⁶⁺ increases, w^{m_{ST}} increases based on
 471 equation (10) (e.g., in Figure 4c where the nearly horizontal grey *sulf^m* curve turns steeply upward
 472 as it enters the green region with increasing f_{O2}). With increasing f_{O2}, w^{m_{ST}} increases until the
 473 silicate melt reaches anhydrite-saturation when w^{m_{S6+}} = w^{m_{S6+CAS}} from equation (11). At this point,
 474 the silicate melt is saturated with vapour + sulfide melt + anhydrite, and the independent variables
 475 are T, μ_{FeS}, and μ_{CaSO4}. It is important to emphasize that for this choice of independent variables,
 476 at a given T, the vapour-saturated silicate melt defines a unique point in Figure 3 and Figure 5 (the
 477 grey star at the intersection of the solid and dashed grey curves) and fixes the values of all other
 478 intensive parameters. Therefore, combining equations (2), (9), and (11):

479 $w^{m_{ST}} = w^{m_{S2-CSS}} + w^{m_{S6+CAS}},$ (19a)

480 and substituting equations (9) and (11) into (6c):

481 $w^{m_{S6+CAS}}/w^{m_{S2-CSS}} = [C_{S6+}/C_{S2-}](f_{O2})^2.$ (19b)

482 This corresponds to the maximum w^{m_{ST}} at ΔFMQ ≅ +1.5 (i.e., the star on the grey curves in Figure
 483 3c, Figure 5c, and Figure 6a: this multiply saturated point is not shown in Figure 4 because it
 484 occurs above the w^{m_{ST}} range of the figure).

485 At higher f_{O2} levels than those described in the previous paragraph (i.e., ΔFMQ > +1.5), the
 486 vapour-saturated silicate melt is no longer saturated with sulfide melt. However, it is still saturated
 487 with anhydrite (independent variables = T, f_{O2}, and μ_{CaSO4}) and thus follows the dashed grey *anh*
 488 curves. The silicate melt is S⁶⁺-dominated and w^{m_{S6+}} is fixed at S⁶⁺CAS (which is taken to be a
 489 constant); hence, w^{m_{S2-}} is given by equation (6c) rather than S²⁻CSS. Therefore, the amount of S²⁻
 490 in the silicate melt decreases whilst w^{m_{S6+}} remains constant. Hence, w^{m_{ST}} initially decreases with
 491 increasing f_{O2} based on equation (12), resulting in the maximum at the star (this is most visible in
 492 Figure 6a). At sufficiently high f_{O2} (ΔFMQ > +2.0), w^{m_{ST}} is essentially constant at S⁶⁺CAS from
 493 equations (14b) and (11)

494 $w^{m_{ST}} \cong w^{m_{S6+}} = w^{m_{S6+CAS}},$ (20)

495 and independent of P and f_{O2} given the parameterisation we have used. Hence, w^{m_{S6+CAS}} is
 496 essentially parallel to the log₁₀[w^{m_{ST}}] ≈ 4 contour over most of the yellow regions in Figure 3c and
 497 Figure 5c, except for a small (but effectively undetectable) deviation because some S²⁻ is always
 498 present, which decreases with increasing f_{O2}.

499 At low f_{O_2} when f_{O_2} is increased, $w^{m_{S_{6+}}}$ increases while $w^{m_{S_{2-}}}$ is essentially constant at $w^{m_{S_{2-CSS}}}$.
 500 Conversely, at high f_{O_2} when f_{O_2} is decreased, $w^{m_{ST}}$ increases because $w^{m_{S_{2-}}}$ increases while $w^{m_{S_{6+}}}$
 501 is constant at $w^{m_{S_{6+CAS}}}$. Thus, there is a maximum in $w^{m_{ST}}$ when the silicate melt is saturated with
 502 vapour + sulfide melt + anhydrite (i.e., the star in Figure 3c, Figure 5c, and Figure 6c). Whilst the
 503 system is at this particular T - P - f_{O_2} value, $w^{m_{ST}}$ is fixed and any excess sulfur in the system above
 504 this value would form more vapour, sulfide melt, and/or anhydrite: hence, it is a *global SS^{max}*. A
 505 maximum in P also occurs at this f_{O_2} (Figure 6b) because a higher P is required to keep the total S
 506 content dissolved in the silicate melt as described in “Regions with mixed speciation in the silicate
 507 melt or vapour”. Although this maximum occurs in the vicinity of the SS^{min} (described in “Regions
 508 with mixed speciation in the silicate melt or vapour”), it follows a different path that results in an
 509 SS^{max} .

510 Silicate melt + (vapour and/or sulfide melt and/or anhydrite)

511 For Y values above the grey curves in Figure 3–Figure 5, the stable phase assemblage is no longer
 512 silicate melt + vapour. The contours for vapour-saturated silicate melt shown beyond these curves
 513 are metastable in these regions. Since in our treatment the silicate melt is confined to the model
 514 ternary system and it is vapour-undersaturated, the silicate melt is the only stable phase. Therefore,
 515 $\varphi = 1$ and $F = 4$, meaning four independent variables are needed to specify the state of the system.
 516 Although silicate melt is the only stable phase in the model ternary system under these conditions,
 517 the silicate melt can be saturated with sulfide melt and/or anhydrite (which are not in the ternary
 518 system) if the values of μ_{FeS} and/or μ_{CaSO_4} in the silicate melt correspond to the values of these
 519 phases. In this section, we choose T , f_{O_2} , P and either μ_{FeS} if the silicate melt is sulfide melt-
 520 saturated or μ_{CaSO_4} if it is anhydrite-saturated. If the silicate melt is both sulfide melt- and anhydrite-
 521 saturated, only two of the three other variables (T , f_{O_2} , and P) can be independent. For given values
 522 of T , f_{O_2} , and P (and parameterizations of S^2 -CSS, S^{6+} -CAS, CS_{2-} , and CS_{6+}), $w^{m_{ST}}$ at sulfide melt-
 523 saturation can be calculated using equation (10) and at anhydrite-saturation using equation (12).
 524 As before, the stable saturation condition (i.e., sulfide melt- or anhydrite-saturation) is the one with
 525 the lowest $w^{m_{ST}}$.

526 Figure 4f shows in P - f_{O_2} space (at $T = 1200^\circ\text{C}$) the phase(s) with which the silicate melt on the
 527 model ternary plane is saturated: the dark and light grey regions are for sulfide melt- and anhydrite-
 528 saturation, respectively, and the coloured regions are for vapour-saturation as described in “Silicate
 529 melt + vapour”. Where these regions intersect, the silicate melt is saturated with two of the three
 530 other phases and these curves define fully constrained paths in P - f_{O_2} space at constant T . The
 531 behaviour of $w^{m_{ST}}$ and P with increasing f_{O_2} for silicate melt saturated with vapour + sulfide melt
 532 or vapour + anhydrite are described in “Silicate melt + vapour + (sulfide melt and/or anhydrite)”.
 533 With increasing P , for silicate melt saturated with sulfide melt + anhydrite (without vapour), $w^{m_{ST}}$
 534 decreases and f_{O_2} is almost constant. The three curves (silicate melt + vapour + sulfide melt; silicate
 535 melt + vapour + anhydrite; and silicate melt + sulfide melt + anhydrite) meet at a point (the star),
 536 where silicate melt is saturated with vapour, sulfide melt, and anhydrite), corresponding to a global
 537 maximum in $w^{m_{ST}}$ for a given T (described in “Silicate melt + vapour + (sulfide melt and/or
 538 anhydrite)”).

539 We now explore how $w^{m_{ST}}$ changes along isobaric paths of increasing f_{O_2} , where P is above or
 540 below $P = 1543$ bar where silicate melt is multiply-saturated with vapour, sulfide melt, and
 541 anhydrite (i.e., the grey star in Figure 5g and Figure 6c and e). At $P > 1543$ bar, the silicate melt

542 is vapour-undersaturated but the trend of $w^{m_{ST}}$ with increasing f_{O_2} is similar to the case where
 543 vapour-saturation occurs in addition to sulfide melt- and/or anhydrite-saturation as described in
 544 “Silicate melt + vapour + (sulfide melt and/or anhydrite)” (e.g., compare Figure 6a and c). The
 545 maximum without vapour but sulfide melt + anhydrite saturation is lower than with vapour +
 546 sulfide melt + anhydrite, as the latter is a global maximum at a fixed T . The maximum S content
 547 at a given P occurs at a unique f_{O_2} (depending on the composition of the silicate component) at the
 548 near vertical curve defining saturation of the silicate melt with both sulfide melt and anhydrite.
 549 Both μ_{FeS} and μ_{CaSO_4} are specified in addition to P at this point, and therefore f_{O_2} is fixed for this
 550 set of three independent variables (in addition to T).

551 At $P < 1543$ bar, the stable isobaric path crosses into the stable vapour-saturated field at
 552 intermediate f_{O_2} (e.g., at the grey solid curve in Figure 5g, described in “Silicate melt + vapour +
 553 (sulfide melt and/or anhydrite)”). The behaviour (and calculations – equation (10)) at low f_{O_2}
 554 ($\Delta FMQ \lesssim +0.8$), when the silicate melt is only saturated with sulfide melt, is the same as at higher
 555 P (i.e., compare the black solid curves in Figure 6c and e: these two curves are nearly identical
 556 since the dependence of $w^{m_{S_2-CSS}}$ on P is minor; O’Neill, 2020). A local maximum ($\Delta FMQ \cong +0.8$)
 557 occurs when the silicate melt is saturated with both sulfide melt and vapour (i.e., labelled “ δ ” at
 558 the solid grey vertical line in Figure 6e). When f_{O_2} is further increased beyond this point ($+0.8 \lesssim$
 559 $\Delta FMQ \lesssim +3.6$), the silicate melt is only vapour-saturated, and $w^{m_{ST}}$ displays the SS^{\min} as described
 560 in “Regions with mixed speciation in the silicate melt or vapour” (i.e., labelled “ ϵ ” on the dotted
 561 black curve in Figure 6e). When $w^{m_{S_6+}} = w^{m_{S_6+CAS}}$ ($\Delta FMQ \cong +3.6$), the silicate melt is saturated
 562 with vapour and anhydrite (e.g., the grey dashed vertical line in Figure 6e). There is a small
 563 maximum at (anhydrite + vapour)-saturation (labelled “ ζ ”), but it cannot be distinguished in Figure
 564 6e because for the isobar chosen it is at sufficiently high f_{O_2} that there is essentially only S^{6+} (i.e.,
 565 very little S^{2-}) dissolved in the silicate melt at this point. Therefore, although $w^{m_{S_2-}}$ decreases with
 566 increasing f_{O_2} beyond this point, it is already so low that the magnitude of the decrease in $w^{m_{ST}}$ is
 567 insignificant. At f_{O_2} values above this point ($\Delta FMQ \gtrsim +3.6$), the silicate melt is only saturated with
 568 anhydrite (i.e., the dashed black curve in Figure 6e) and behaves as described previously under
 569 these conditions. For a given P , along all isobaric paths of increasing f_{O_2} when vapour-saturation
 570 is encountered (i.e., $P < 1543$ bars), there is an SS^{\max} when the silicate melt is saturated with sulfide
 571 melt + vapour (δ , which gets larger with increasing P); an SS^{\min} when the silicate melt is only
 572 vapour-saturated (ϵ); and another SS^{\max} when the silicate melt is saturated with anhydrite + vapour
 573 (ζ).

574 **Isothermal, decompression-induced degassing**

575 The examples developed in “Independent variables of T , f_{O_2} , f_{S_2} , $w^{m_{ST}}$, P , μ_{FeS} , and μ_{CaSO_4} ” and
 576 Figure 2–Figure 5 do not apply to the f_{O_2} – f_{S_2} paths followed by magmas as they degas on
 577 decompression (or due to crystallisation) under equilibrium conditions because f_{O_2} is not an
 578 independent variable in this process. We have evaluated whether the SS^{\min} has any impact on
 579 closed- and open-system degassing during isothermal decompression using our simple model
 580 ternary system. Inclusion of other volatiles (e.g., H and C) would add complexity to the system
 581 and are important for getting the details correct for simulation of degassing in nature (e.g., Moretti
 582 *et al.* 2003; Moretti and Papale 2004; Burgisser and Scaillet 2007; Gaillard and Scaillet 2009,
 583 2014; Wallace and Edmonds 2011; Gaillard *et al.* 2011, 2015; Burgisser *et al.* 2015; Iacovino

584 2015; Liggins *et al.* 2020). However, their exclusion here allows us to isolate the behaviour of
585 sulfur during degassing.

586 We model closed-system degassing by decreasing P at constant T and constant bulk
587 composition for the two-phase system silicate melt + vapour; hence, the independent variables are
588 T , P , and bulk composition. A closed-system requires the bulk composition of the system (i.e.,
589 silicate melt \pm vapour) to be constant at all P . The bulk composition comprises the proportions of
590 the silicate melt component, total S_2 , and O_2 (the latter is the O_2 in excess of that in the model
591 silicate component). For open-system degassing, any vapour present at each P step (1 bar
592 increments) is removed from the system. Hence, the bulk composition of the system changes
593 during depressurisation and the silicate melt composition becomes the bulk composition of the
594 system for the subsequent P step. We do not allow sulfide melt or anhydrite to be present; hence,
595 in some regions the silicate melt + vapour assemblage encountered during degassing is metastable
596 (to the left of the solid grey curve in Figure 7a). Results of closed- and open-system degassing
597 calculations are essentially indistinguishable (Figure 8), hence we only discuss closed-system
598 degassing calculations.

599 For a particular bulk composition, when $P > P^{v_{\text{sat}}}$ (pressure of vapour-saturation; above the
600 solid black curve in Figure 7), the system is vapour-undersaturated and only silicate melt is present.
601 At $P = P^{v_{\text{sat}}}$ (on the solid black curve), the system is still 100 % silicate melt, but the silicate melt
602 is vapour-saturated (i.e., equation (7) is satisfied using equations (3b), (4c), (5c), (8), and the bulk
603 composition of the system). When $P < P^{v_{\text{sat}}}$ (below the solid black curve), the silicate melt is
604 supersaturated with respect to vapour, such that vapour exsolves from the silicate melt and the two
605 phases coexist under equilibrium conditions. The 5000 ppm contour is the locus of $P^{v_{\text{sat}}}$ (i.e., the
606 P at which degassing begins; Figure 7c) for a silicate melt corresponding to the bulk composition
607 of the system, where the white diamond indicates $^{SS_{\text{min}}}f_{O_2}$ (i.e., $\Delta\text{FMQ}+1.30$ for these parameters,
608 at the maximum of the black curve).

609 If the silicate melt starts off more reduced than $^{SS_{\text{min}}}f_{O_2}$, f_{O_2} decreases with decreasing P during
610 closed-system degassing (e.g., the yellow curves in Figure 7d and g, and Figure 8b). This reflects
611 that although the silicate melt is S^{2-} -dominated, the vapour is S_2 -dominated and/or SO_2 -dominated
612 (Figure 7f), so O_2 is consumed during degassing to convert S^{2-} dissolved in the silicate melt into
613 the more oxidised S_2 and SO_2 vapour species (see reaction (4a) and the reaction generated by
614 subtracting (3a) from (4a)). Therefore, reduction of the system (i.e., a decrease in f_{O_2}) must occur
615 for degassing under these conditions, although this effect is buffered by changes in $[\text{Fe}^{3+}/\text{Fe}_T]^m$,
616 which are included in our calculations. However, if the silicate melt starts off more oxidised than
617 $^{SS_{\text{min}}}f_{O_2}$, f_{O_2} increases with decreasing P during closed-system degassing (e.g., the turquoise curve
618 in Figure 7d and g, and Figure 8b). This reflects that the silicate melt is S^{6+} -dominated but the
619 vapour is SO_2 -dominated (Figure 7f), so O_2 is generated during degassing to convert SO_4^{2-}
620 dissolved in the silicate melt into the more reduced SO_2 vapour species (see the reaction generated
621 by subtracting (3a) from (5a)). Hence, oxidation of the system (i.e., an increase in f_{O_2}) must occur
622 for degassing under these conditions. Close inspection of Figure 7d demonstrates that for a narrow
623 range of bulk compositions that start degassing close to, but slightly more reducing than, $^{SS_{\text{min}}}f_{O_2}$,
624 f_{O_2} initially increases and then decreases at lower P . In contrast to bulk compositions that are
625 displaced in f_{O_2} from $^{SS_{\text{min}}}f_{O_2}$ by more than ~ 0.1 log units, there is minimal change in f_{O_2} during
626 depressurisation for bulk compositions that start close to $^{SS_{\text{min}}}f_{O_2}$ (e.g., the green curves in Figure
627 7d and g, and Figure 8b).

628 At constant bulk composition when $P < P^{v_{\text{sat}}}$, $w^{m_{\text{ST}}}$ decreases monotonically with decreasing P
629 to 1 bar (Figure 7c and Figure 8c). Hence, the SS^{min} is not encountered in our simple system during
630 closed-system, depressurisation-induced degassing because changes in f_{O_2} do not cause the silicate
631 melt to cross the SS^{min} . However, the shapes of the $w^{m_{\text{ST}}}$ with decreasing P are different at the
632 SS^{min} (roughly linear) compared to either side (concave down). With decreasing P , f_{S_2} (Figure 7a
633 and Figure 8a) and f_{SO_2} (Figure 7b) always decrease; the rate of decrease depends on the bulk
634 composition of the system. The rate of change is controlled by the trade-off between decreasing
635 $w^{m_{\text{ST}}}$ and decreasing or increasing f_{O_2} to the left and right of SS^{min} , respectively, through equations
636 (17a) and (17b).

637 **Implications of SS^{min} and SS^{max} for magmatic and volcanic processes**

638 The SS^{min} for silicate melt + vapour and the SS^{max} for silicate melt \pm vapour + sulfide melt +
639 anhydrite occur in a similar f_{O_2} range since both features depend on the silicate melt having mixed
640 S speciation (“Independent variables of T , f_{O_2} , f_{S_2} , $w^{m_{\text{ST}}}$, P , μ_{FeS} , and μ_{CaSO_4} ”). The SS^{min} manifests
641 when T (held constant) and f_{O_2} are independent variables, and f_{S_2} , $w^{m_{\text{ST}}}$, or P are the third
642 independent variable and held constant or varied in particular ways (see “Regions with mixed
643 speciation in the silicate melt or vapour”; Figure 2–Figure 5). The SS^{min} reflects the prominent
644 “valleys” – centered where the silicate melt has mixed S speciation – in the isothermal surfaces
645 showing $w^{m_{\text{ST}}}$ as a function of f_{S_2} or P vs. f_{O_2} (Figure 3c and Figure 5c). An alternative expression
646 of the SS^{min} is the prominent “ridge” in the topography of the isothermal P surface plotted as a
647 function of $w^{m_{\text{ST}}}$ vs. f_{O_2} (Figure 4c). However, there are simple linear paths that cross the valley
648 (or ridge) with increasing f_{O_2} in Figure 3–Figure 5c for which there is no SS^{min} . There are also
649 geological important paths (including closed- or open-system depressurisation; “Isothermal,
650 decompression-induced degassing”) that nearly parallel the valley floor (or ridge crest) and
651 therefore do not encounter an SS^{min} .

652 Our model calculations place the SS^{min} for this Hawaiian basalt composition at 1200 °C at
653 $SS^{\text{min}}f_{\text{O}_2} \cong \Delta\text{FMQ}+1.2$. For comparison, this is slightly more oxidised than Hawaiian basalts
654 ($\Delta\text{FMQ}-0.5$ to $+1.0$; Moussallam *et al.* 2016; Brounce *et al.* 2017; Lerner *et al.* 2021) and more
655 oxidised than mid-ocean ridge basalts (MORB), but within the range measured in arc and ocean
656 island basalts (OIB) (e.g., Cottrell *et al.*, 2021). The position of $SS^{\text{min}}f_{\text{O}_2}$ will depend on T ,
657 composition of the silicate component, and additional volatile components (e.g., H_2O , CO_2). To
658 explore how the effects described here vary with T and melt composition, a robust parameterisation
659 of Cs_{6+} is required to model S solubility accurately and therefore to understand fully the importance
660 of the SS^{min} and SS^{max} in natural magmas. However, numerous experimental and modelling studies
661 have demonstrated the SS^{min} at the f_{O_2} range relevant to abundant terrestrial magma types (e.g.,
662 Fincham and Richardson 1954; Katsura and Nagashima 1974; Carroll and Rutherford 1985;
663 Moretti *et al.* 2003; Clemente *et al.* 2004; Backnaes and Deubener 2011; Lesne *et al.* 2015;
664 Matjuschkin *et al.* 2016; Nash *et al.* 2019). When these experimental results are combined with
665 our model results, this phenomenon is likely to play a role in magmatic and volcanic processes
666 affecting terrestrial magmas. We explore the influence of the SS^{min} and SS^{max} in experiments
667 (“Solubility experiments”) and in natural processes and describe its possible application as a tool
668 for constraining magmatic f_{O_2} (“Using $w^{m_{\text{ST}}}$ as an oxybarometer”). These potential impacts on
669 natural magmatic systems are summarised schematically in Figure 1c. We highlight natural
670 processes throughout the magmatic and volcanic system, starting deep with mantle melting

671 (“Mantle melting”); up through the crust where magmas undergo mixing (“Magma mixing and
672 crustal assimilation”) and degassing (“Magma ascent and degassing”); and finally to volcanic
673 emissions into the atmosphere (“Volcanic emissions”).

674 Solubility experiments

675 The SS^{\min} and SS^{\max} described in “Independent variables of T , f_{O_2} , f_{S_2} , w^m_{ST} , P , μ_{FeS} , and μ_{CaSO_4} ”
676 have been observed in experiments that can externally control P , T , f_{O_2} , and f_{S_2} . For example, the
677 SS^{\min} has been observed using one-atmosphere gas-mixing experiments at constant T and P with
678 varying f_{O_2} (e.g., Fincham and Richardson, 1954; Katsura and Nagashima, 1974). In these
679 experiments, f_{S_2} was *not* constant as the volume percentage of SO_2 in the gas prior to heating in
680 the gas mixing furnace was constant instead, which was mixed with other gas species to attain the
681 required f_{O_2} . Despite the more complex gas speciation of such experiments, and that the systems
682 were not simple ternaries, the geometry of w^m_{ST} vs. f_{O_2} are reproduced by our modelling (e.g.,
683 compare Figure 2a in Fincham and Richardson (1954) with our Figure 2e). Consistent with our
684 analysis (“Regions with mixed speciation in the silicate melt or vapour”), Fincham and Richardson
685 (1954) attributed the first change in slope in their Figure 2a to a change in vapour speciation and
686 the second to change in silicate melt speciation. At constant T and P , a different f_{S_2} - f_{O_2} path can be
687 followed in one-atmosphere gas-mixing experiments using the bulk composition of the input gas
688 to control f_{S_2} as well as f_{O_2} (e.g., O’Neill and Mavrogenes 2002; Nash *et al.* 2019). O’Neill and
689 Mavrogenes (2002) did not observe an SS^{\min} because the f_{O_2} was always lower than the expected
690 value of $SS^{\min} f_{O_2}$, but Nash *et al.* (2019) did observe an SS^{\min} (although not the change in slope due
691 to changes in vapour speciation).

692 Piston cylinder experiments by Matjuschkin *et al.* (2016) produced an SS^{\min} in (sulfide melt +
693 vapour)-saturated silicate melts at constant P and T , in which f_{O_2} was controlled using solid-state
694 buffers (their Figure 8b). Their experiments were always saturated with sulfide melt (except one
695 set at very high f_{O_2} that were anhydrite-saturated), whereas the minimum described in “Silicate
696 melt + (vapour and/or sulfide melt and/or anhydrite)” (ϵ in Figure 6e) is only vapour-saturated
697 (i.e., it is *not* sulfide melt-saturated). Matjuschkin *et al.* (2016) attributed their SS^{\min} to the presence
698 of an additional S-bearing melt species with intermediate charge between 2- and 6+ that was not
699 quenchable. Alternatively, their minimum could be influenced by a decrease in S^2 -CSS reflecting
700 compositional changes in the silicate melt due to crystallisation as f_{O_2} varied.

701 A variety of studies using high-pressure, high-temperature apparatus at constant T and P and
702 varying f_{O_2} have observed an increase in w^m_{ST} as the system transitioned from sulfide melt- to
703 anhydrite-saturation (e.g., Carroll and Rutherford 1985, 1987; Jugo *et al.* 2004; Beermann *et al.*
704 2011; Botcharnikov *et al.* 2011). However, these studies suggested a plateau rather than a
705 maximum, likely due to the narrow f_{O_2} range of the SS^{\max} . However, Jugo (2009) modelled the
706 data from Carroll and Rutherford (1985, 1987) and Jugo *et al.* (2004) and produced an SS^{\max}
707 similar to what we have described in “Silicate melt + (vapour and/or sulfide melt and/or
708 anhydrite)” (although they did not consider vapour-saturation as an additional constraint). It is
709 encouraging that our calculated trends match Figure 1 from Jugo (2009), despite using different
710 sets of equations, solubility mechanisms, and thermochemical parameters.

711 Mantle melting

712 The SS^{\max} we have described defines the maximum S content for silicate melts generated by partial
713 mantle melting (i.e., in the presence of sulfide melt + anhydrite \pm vapour at a given T). Once either
714 sulfide melt or anhydrite is exhausted, the S content of the partial melt decreases to w^m_{STCAS} or
715 w^m_{STCSS} , respectively, and then decreases further by dilution when both phases are exhausted (e.g.,
716 Chowdhury and Dasgupta, 2019). Chowdhury and Dasgupta (2019) explored the S content of
717 silicate melts generated by mantle melting when the mantle contains either sulfide or anhydrite.
718 They found that the S content of most arc magmas could be generated by mantle melting in the
719 presence of sulfide when S^{2-} is the dominant silicate melt species. For some arc magmas, they
720 found this was not the case, but sulfide should still be present in the mantle source during melting.
721 They attributed these higher S contents to the presence of sulfate in the silicate melt: this can be
722 visualised in Figure 6c by partial mantle melting producing silicate melts between the dashed green
723 vertical lines. For two of their arc magmas, the S concentrations were too high for even anhydrite-
724 saturated melting, which Chowdhury and Dasgupta (2019) suggested requires an additional S
725 source, such as crustal assimilation of sulfate. Alternatively, our modelling suggests these S-rich
726 magmas could be generated by melting of mantle sources containing both sulfide and anhydrite
727 (or with only one of these phases but at an f_{O_2} close to SS^{\max}), since this would result in w^m_{ST}
728 of the silicate partial melt higher than $S^{6+}CAS$. However, a thorough assessment requires knowledge
729 of the T and melt composition dependence of $C_{S^{6+}}$ to model mantle melting. The presence of mixed
730 S speciation in the melt also means melting from sulfide- or anhydrite-bearing mantle can generate
731 a wide range of S concentrations in the silicate melt (e.g., Jugo, 2009).

732 Magma mixing and crustal assimilation

733 In nature, approximately constant P paths could be important when mixing reduced and oxidised
734 S-bearing silicate melts. For example, mixing vapour-undersaturated silicate melts from either side
735 of $SS^{\min}f_{O_2}$ would generate a vapour-saturated silicate melt if the combined S content is greater than
736 w^m_{ST} at the intermediate f_{O_2} , which can be visualised using Figure 7g. At $P = 550$ bar (black dashed
737 horizontal line), both a relatively reduced (white circle at 3.26 wt% O_2) and oxidised (white square
738 at 4.23 wt% O_2) silicate melt containing 5000 ppm w^m_{ST} would be vapour-undersaturated (Figure
739 7c). Mixing these silicate melts isobarically results in a new bulk composition between the two
740 end members in Figure 7 that depends on the mixing proportions. If the mixed silicate melt has a
741 bulk composition inside the vapour-saturation curve for 5000 ppm S_T (i.e., the black curve;
742 between $+1.0 \lesssim \Delta FMQ \lesssim +1.7$ for the example shown in Figure 7g), the silicate melt will become
743 vapour-saturated (Figure 7f). The mixed magma will then degas until w^m_{ST} decreases to the contour
744 at that point (Figure 7c) and the amount of vapour degassed will depend on the mixing proportions
745 (Figure 7e). This simple analysis assumes the oxidized and reduced silicate melts are the same in
746 composition (other than in total O_2). If they differ in major element chemistry or T , there would be
747 added complexity because $C_{S^{2-}}$, $C_{S^{6+}}$, and T will change depending on the proportions of the two
748 silicate melts in the mixture (and the possibility of crystallization of the mixture). These factors
749 would also influence the solubility of S as a function of the mixing proportions of the two silicate
750 melts. This would be particularly important if a reduced silicate melt assimilated oxidised country
751 rock or an oxidised silicate melt assimilated reduced country rock (e.g., Tomkins *et al.* 2012;
752 Iacono-Marziano *et al.* 2017). Nevertheless, encountering SS^{\min} during mixing could be relevant
753 to eruptive dynamics and volcanic SO_2 contributions to the atmosphere by rapidly producing a
754 large amount of vapour (e.g., Figure 7e), potentially driving eruption (e.g., Kress 1997; Di Muro

755 *et al.* 2008). It could also lead to deeper degassing, wringing out S-rich gas at higher P than would
756 occur if the magmas in the mixture had erupted without mixing (see “Magma ascent and
757 degassing”).

758 Magma ascent and degassing

759 For vapour-saturated magma ascent to the surface, P and bulk composition are two key
760 independent variables. Our calculations for the model ternary system show that f_{O_2} paths during
761 isothermal depressurisation of a representative Hawaiian basalt are unlikely to cross $SS^{\min}f_{O_2}$
762 (Figure 7d and Figure 8b; “Isothermal, decompression-induced degassing”). Thus, in this system,
763 w^m_{ST} decreases monotonically with decreasing P for a closed- (and open-) system once vapour-
764 saturation is reached (Figure 7c and Figure 8c). In this simple system at low P , this is expected
765 because the partial molar volumes of the gaseous S-bearing species are higher than those of the
766 dissolved silicate melt species. Hence, the progressively more degassed state is the more stable
767 one with decreasing P (i.e., a stable isothermal system requires that volume increases as P
768 decreases so that the bulk isothermal compressibility is always positive). We have not evaluated
769 the possible effects of changes in T or silicate melt composition during degassing (e.g., due to
770 crystallization resulting from heat loss and/or from an increase in the liquidus on degassing). These
771 changes would result in changes in C_{S_2} - and/or C_{S_6+} and therefore the value of $SS^{\min}f_{O_2}$ at a particular
772 P could vary during degassing, possibly resulting in SS^{\min} influencing w^m_{ST} during degassing.

773 Degassing of C- and H-bearing species from a silicate melt can cause f_{O_2} to increase (e.g., Sato
774 and Wright 1966; Sato 1978; Mathez 1984; Candela 1986; Holloway 2004; Burgisser and Scaillet
775 2007; Brounce *et al.* 2017; Métrich 2021). The magnitude of this f_{O_2} increase is greater when the
776 initial f_{O_2} of the magma is lower and depends on the relative solubilities of oxidised and reduced
777 C- and H-bearing silicate melt species (e.g., Gaillard *et al.* 2015). Therefore, it may be possible to
778 have a vapour-saturated silicate melt that starts S^{2-} -dominated but oxidises sufficiently due to early
779 CO_2 and/or H_2O degassing to drive the system across SS^{\min} during depressurisation. If so, this
780 could be manifested by initially decreasing, followed by increasing, w^m_{ST} with progressive closed-
781 system, depressurisation-induced degassing. It is also possible that a maximum in w^m_{ST} could occur
782 during degassing, although this would not be due to changes in f_{O_2} or the SS^{\min} . For example, the
783 loss of CO_2 and H_2O prior to significant S degassing could cause w^m_{ST} to initially increase (i.e.,
784 because the total mass of the silicate melt is decreasing while the mass of dissolved S is nearly
785 constant); then, when S begins to degas, w^m_{ST} would decrease.

786 Progressive reduction or oxidation of magmas during degassing is expected based on previous
787 modelling efforts and has been observed in natural samples (e.g., Anderson and Wright 1972;
788 Candela 1986; Carmichael and Ghiorso 1986; Burgisser and Scaillet 2007; Métrich *et al.* 2009;
789 Gaillard *et al.* 2011, 2015; Kelley and Cottrell 2012; Moussallam *et al.* 2016, 2014; Brounce *et al.*
790 2017). Our degassing calculations demonstrate that the f_{O_2} after extensive S-degassing will not
791 represent the initial silicate melt unless the silicate melt began degassing near SS^{\min} (Figure 7d).
792 For example, when f_{O_2} differs from that of $SS^{\min}f_{O_2}$ (i.e., at the diamond) by more than ~ 0.1 log unit
793 (either positively or negatively), the f_{O_2} after nearly complete degassing (i.e., $P = 1$ bar) has
794 increased or decreased by more than 0.5 log units (and up to > 1 log unit) relative to the initial
795 (i.e., undegassed) f_{O_2} of the silicate melt (Figure 7d). Therefore, as has been highlighted previously,
796 using the f_{O_2} of volcanic gases (or from the Fe^{3+}/Fe_T or S^{6+}/S_T of silicate glasses) as a proxy for
797 the f_{O_2} of the initial silicate melt (and potentially of the mantle), should be approached with caution
798 (e.g., Anderson and Wright 1972; Carmichael and Ghiorso 1986; Burgisser and Scaillet 2007;

799 Métrich *et al.* 2009; Gaillard *et al.* 2015). We emphasize, however, that the direction of the change
800 in f_{O_2} on degassing differs for more oxidized and reduced magmas relative to $^{SS^{min}}f_{O_2}$ (Figure 7d
801 and Figure 8b).

802 The unlikelihood of crossing the SS^{min} during closed- or open-system degassing does not mean
803 that SS^{min} is unimportant during degassing. Decompression paths to the left and right of SS^{min}
804 begin to degas at lower P than one that passes right through the maximum (Figure 7c). The shapes
805 of the contours in Figure 7c are simple expressions of the SS^{min} described in “Silicate melt +
806 vapour” for the case $Y = w^{m}_{ST}$ (Figure 2d and Figure 4c). The maximum in P at the SS^{min} is occurs
807 because a higher P is required to keep a given concentration of S dissolved in the silicate melt
808 (5000 ppm in Figure 7c) at the SS^{min} than when the S solubility is higher on either side. Based on
809 our modelling, a Hawaiian basaltic melt with 5000 ppm S_T reaches vapour-saturation at 666 bars
810 at SS^{min} (the green curve in Figure 8). This is 164 bars higher than when the melt is initially 0.5
811 log units higher in f_{O_2} (the turquoise curve) and 363 bars higher than when the melt is initially 0.6
812 log units lower in f_{O_2} (the yellow curve). Thus, SS^{min} can exert significant control on the P at which
813 S degassing begins as a function of f_{O_2} (Figure 7). Therefore, even for S-rich magmas, S will degas
814 only at shallow depths if the silicate melt has an initial f_{O_2} sufficiently lower than (e.g., MORB) or
815 higher than $^{SS^{min}}f_{O_2}$. However, silicate melts with initial f_{O_2} close to $^{SS^{min}}f_{O_2}$ (e.g., many arc and
816 OIB magmas) would – all other things being equal – begin to degas S deeper than more oxidized
817 or reduced magmas. Therefore, the SS^{min} could be a contributing mechanism – in addition to S
818 partitioning into the H₂O-rich vapour exsolved from H₂O-rich magmas (e.g., Wallace and
819 Edmonds 2011; Zajacz *et al.* 2012; Edmonds and Mather 2017; Edmonds and Woods 2018) – for
820 the deep degassing of S in arc and some ocean island settings.

821 The SS^{min} also has implications for calculating the $P^{v_{sat}}$ using the volatile concentrations of
822 glasses (e.g., in pillow rims and glassy melt inclusions) (e.g., Anderson *et al.* 1989; Blundy and
823 Cashman 2008). Such calculated pressures are often used to constrain the architecture of magmatic
824 systems and link volcanic products to eruptive vents in submarine systems. These calculations are
825 based on generalisations of equation (7) that include other gaseous species for which partial
826 pressures can be determined (e.g., CO₂ and H₂O). This exercise is comparable to when T , f_{O_2} , and
827 $Y = w^{m}_{ST}$ are taken as independent variables in a silicate melt + vapour assemblage and the
828 dependent variable $Z = P$ is calculated (e.g., Figure 4c). The effect of dissolved S is currently not
829 included in such calculations, which include only H₂O and CO₂ (e.g., Newman and Lowenstern
830 2002; Papale *et al.* 2006; Iacono-Marziano *et al.* 2012; Ghiorso and Gualda 2015; Allison *et al.*
831 2019; Iacovino *et al.* 2021). Also, the effect f_{O_2} on volatile speciation in the silicate melt and vapour
832 is mostly ignored (but see Scaillet and Pichavant 2004; Wetzal *et al.* 2015).

833 MORB magmas have $S^{6+}/S_T < 0.05$ (e.g., Métrich *et al.* 2009; Jugo *et al.* 2010; Labidi *et al.*
834 2012) and therefore including S for such melts would have a negligible effect on calculated $P^{v_{sat}}$
835 because p_{S_2} and p_{SO_2} would be low (e.g., Figure 2d). However, OIB and arc volcanic glasses can
836 have $[S^{6+}/S_T]^m$ up to 1 (e.g., Jugo *et al.* 2010; de Moor *et al.* 2013; Labidi *et al.* 2015; Muth and
837 Wallace 2021), and therefore the effects of the SS^{min} on the contribution of partial pressures from
838 S-bearing species to the total P at vapour-saturation (e.g., S₂ and SO₂ in the S–O system, but
839 potentially H₂S and OCS as well in C–O–H–S system) could be non-negligible (e.g., in the green
840 band of Figure 2d). Melt inclusions from arc and OIB magmas reach $w^{m}_{ST} \sim 5000$ ppm (Wallace
841 2005) and are ΔFMQ -3 to +3 (Cottrell *et al.* 2021), which could lead to underestimating $P^{v_{sat}}$ by
842 up to 650 bars for melts at $\Delta FMQ = +1$ to +2 (Figure 9). The exact value of the underestimate
843 would depend on the f_{O_2} , $C_{S^{6+}}$, and C_{S_2} , and hence T and silicate melt composition, and including

844 the H and C species (e.g., Burgisser *et al.* 2015; Lesne *et al.* 2015; Hughes *et al.* 2021). However,
845 the potential magnitude of this SS^{\min} -related effect implied by our calculations is robust. Therefore,
846 in such cases neglecting S could result in calculated P^v_{sat} values that significantly underestimate
847 the P^v_{sat} the glass is recording.

848 We note that the maximum in P^v_{sat} at SS^{\min} based on our calculations contrasts with the
849 modelling of Lesne *et al.* (2015), who predict a *minimum* in P^v_{sat} when both reduced and oxidised
850 S species are present in silicate melts. The difference could be due to: (1) different assumptions
851 regarding the speciation of oxidised S in the silicate melt (i.e., as SO_2 rather than SO_4^{2-} ; see
852 “Equilibria between silicate melt, vapour, sulfide melt, and anhydrite” for discussion of potential
853 S-bearing species in the silicate melt); and (2) the effect of H on the SS^{\min} , which adds H_2S species
854 to the silicate melt and vapour.

855 Volcanic emissions

856 The El Chichón 1982 and Pinatubo 1991 eruptions released the largest quantities of SO_2 during
857 explosive events recorded by satellites, and most of this SO_2 was sourced from a coexisting vapour
858 present prior to eruption (e.g., Wallace and Gerlach 1994; Krueger *et al.* 1995; Gerlach *et al.* 1996;
859 Keppler 1999; Bluth *et al.* 2015). Additionally, both magmas contain anhydrite and
860 pyrrhotite/sulfide in the erupted products (e.g., Luhr *et al.* 1984; Bernard *et al.* 1991; Luhr 2008a).
861 We infer from this observation that both these magmas were stored at SS^{\max} prior to eruption (i.e.,
862 at the condition represented by the grey star in Figure 5g). Independent f_{O_2} estimates from these
863 magmas are within the range for experimental constraints for the coexistence of sulfide melt and
864 anhydrite (e.g., Luhr *et al.* 1984; Rye *et al.* 1984; Carroll and Rutherford 1987; Evans and Scaillet
865 1997; Luhr 2008b). Older eruptions have released even greater quantities of SO_2 (e.g., Figure 5 in
866 Vidal *et al.* 2016) and these magmas may have also been stored at SS^{\max} . For example, eruptive
867 products from the Samalas 1257 eruption contain sulfides and vapour, and anhydrite has been
868 observed as microcrystals on the walls of fluid inclusions (Vidal *et al.* 2016) (although anhydrite
869 dissolves in water and is therefore rarely observed in volcanic products; Luhr *et al.* 1984). These
870 results from older eruptions are consistent with the SS^{\max} being connected to unusually large
871 releases of SO_2 during explosive eruptions. If this is correct, it suggests a connection between the
872 unique f_{O_2} at which the SS^{\max} occurs and these events.

873 The “petrologic method” is often used to estimate the volatiles emissions from volcanic
874 eruptions by subtracting the volatile concentration of the degassed matrix glass from that in melt
875 inclusions (the latter is assumed to represent undegassed silicate melt) (e.g., Devine *et al.* 1984;
876 Thordarson *et al.* 1996; Wallace 2001; Sharma *et al.* 2004). The total SO_2 emitted during eruption
877 estimated in this way are often low relative to those measured using other techniques (e.g., satellite-
878 based techniques; e.g., Stoiber and Jepsen 1973; Rose *et al.* 1982; Andres *et al.* 1991; Wallace
879 2001; Shinohara 2008). This “excess sulfur” problem reflects the large amounts of SO_2 -rich vapour
880 often present during magma storage (only minor amounts of SO_2 are thought to be released from
881 the break-down of sulfide melt and anhydrite during degassing) (e.g., Anderson 1975; Luhr *et al.*
882 1984; Andres *et al.* 1991; Wallace and Gerlach 1994; Gerlach and McGee 1994; Gerlach *et al.*
883 1994, 1996; Giggenbach 1996; Keppler 1999; Wallace 2001; Scaillet and Pichavant 2003; Scaillet
884 *et al.* 2003; Sharma *et al.* 2004; Shinohara 2008). Large amounts of S-rich vapour could be
885 generated due to the SS^{\min} (and to some extents SS^{\max}). Therefore, using glass-only melt inclusions
886 trapped at SS^{\min} or SS^{\max} will result in the largest discrepancies with other methods due to the
887 additional S in other phases. Melt inclusions that co-entrap these additional phases cannot be used

888 because the proportions of the different phases present in the inclusion are unlikely to represent
889 the bulk system. Additionally, as melt inclusions evolve as a closed-system to S post-entrapment,
890 SS^{\min} and SS^{\max} may be encountered causing additional phases to form within the inclusion (e.g.,
891 vapour bubbles, sulfide blebs). Measuring only the silicate glass within these melt inclusion results
892 in a greater underestimate of SO_2 emissions because the S contribution from the silicate melt is
893 additionally underestimated (e.g., Venugopal *et al.* 2020).

894 Using w^m_{ST} as an oxybarometer

895 For a silicate melt with a given value of w^m_{ST} and T , it is possible to place constraints on its f_{O_2}
896 using the calculations we have presented based on the presence or absence of vapour, sulfide melt,
897 and/or anhydrite as saturating phases (Figure 6; e.g., the empirical approach of Beermann *et al.*,
898 2011). Suppose it is known that the silicate melt is vapour-saturated. If it is also saturated with
899 sulfide melt it must fall on the solid grey *sulf^m* curve in Figure 6a: so for a known w^m_{ST} , the f_{O_2} can
900 be read directly from the *sulf^m* curve (e.g., α and β for 3000 and 13000 ppm w^m_{ST} , respectively, in
901 Figure 6a). Likewise, if the silicate melt is known to be anhydrite-saturated it must fall on the
902 dashed grey *anh* curve in Figure 5a: the implied f_{O_2} for a given value of w^m_{ST} can be read from the
903 *anh* curve (e.g., γ for 13000 ppm w^m_{ST} in Figure 6a). If the silicate melt is known to be saturated
904 with both sulfide melt and anhydrite, the f_{O_2} is constrained to the SS^{\max} value at the grey star.

905 Suppose, however, that we do not know whether the vapour-saturated silicate melt is sulfide
906 melt and/or anhydrite saturated (or we know that it is not). Then the f_{O_2} can be constrained to be
907 between the *sulf^m* and *anh* curves in Figure 6a for a given value of w^m_{ST} (e.g., at 13000 ppm w^m_{ST} ,
908 the f_{O_2} must be between β and γ on Figure 6a). When nearly all of the dissolved S is S^2 (i.e., in the
909 purple-blue-turquoise region), the *sulf^m* curve plateaus at $\sim w^m_{S_2-CSS}$, so for values of $w^m_{ST} < w^m_{S_2-}$
910 css , no constraint can be placed on a lower bound to f_{O_2} using this approach. Likewise, the *anh*
911 curve plateaus at $\sim w^m_{S_6+CAS}$, and thus no constraint on an upper bound on f_{O_2} can be determined
912 by this approach if $w^m_{ST} < w^m_{S_6+CAS}$. If the silicate melt is not thought to be vapour-saturated, but
913 P is known independently, a figure like Figure 6c at the relevant P can be used instead. Although
914 the presence of other volatiles (e.g., C and H) will modify the results from the simple S–O system,
915 the principles are the same.

916 The technique described using Figure 6a is most sensitive when measured S concentrations are
917 high and the S speciation in the silicate melt is mixed ($\Delta FMQ \sim +1$ to $+2$); e.g., to obtain both
918 lower and upper bounds on f_{O_2} in the example described requires $w^m_{ST} > w^m_{S_6+CAS} \sim 13000$ ppm
919 (i.e., the white dotted line in Figure 6a). Although this is much higher than the typical total
920 dissolved S contents of common magmas, $S^{6+}CAS$ decreases significantly with increasing
921 dissolved H_2O and decreasing T : e.g., $S^{6+}CAS = \sim 3000$ ppm at 5 wt% H_2O and 1200 °C for a
922 basaltic melt (Chowdhury and Dasgupta 2019). Melt inclusions from arcs and ocean islands can
923 be S-rich (up to ~ 5000 ppm S_T), hydrous (up to ~ 6 wt% H_2O for arcs and ~ 3 wt% H_2O for ocean
924 islands), and relatively oxidised (up to $+3$ ΔFMQ) (e.g., Wallace 2005; Moussallam *et al.* 2019;
925 Cottrell *et al.* 2021) and may therefore provide useful f_{O_2} estimates based on this technique.
926 However, this will require accurate knowledge of C_{S_2} , C_{S_6+} , S^2-CSS , and $S^{6+}CSS$ at the relevant
927 conditions (especially the effects of T and silicate melt composition – including the influence of
928 H_2O), as these parameters strongly influence the f_{O_2} of the transition from S^{2-} - to S^{6+} -dominated
929 silicate melt (e.g., O'Neill and Mavrogenes 2002, 2019; Li and Ripley 2005; Moretti and Ottonello

930 2005; Baker and Moretti 2011; Chowdhury and Dasgupta 2019; Nash *et al.* 2019; Zajacz and Tsay
931 2019; O'Neill 2020; Boulliang and Wood 2021).

932 Conclusions

933
934 Depending on the choice of independent variables, vapour-saturated silicate melts can with
935 increasing f_{O_2} encounter a “sulfur solubility minimum” (SS^{\min}) when both S^{2-} and S^{6+} are dissolved
936 in the silicate melt in similar concentrations (Figure 2–Figure 5). This occurs because O_2 is on
937 different sides of the reactions describing S^{2-} and S^{6+} dissolution in the silicate melt from S_2 - and/or
938 SO_2 -dominant vapour (e.g., reactions (4a) and (5a)). Examples of choices of independent variables
939 and the paths they follow that exhibit a minimum in the dissolved total S content (w^m_{ST}) in vapour-
940 saturated silicate melt include paths of increasing f_{O_2} for which T and either f_{S_2} or P are held
941 constant (Figure 3c and Figure 5c). For paths on which w^m_{ST} is held constant with increasing f_{O_2}
942 and constant T , the SS^{\min} is expressed a maximum in P (Figure 4c). However, not all choices of
943 independent variables or paths defined by changes in these variables display the SS^{\min} . An
944 important geological example of this is that despite changing f_{O_2} , w^m_{ST} decreases monotonically
945 (i.e., no minimum in w^m_{ST} is encountered) during isothermal, closed-system, decompression-
946 induced degassing in a system in which the vapour contains only S- and O-bearing species (Figure
947 7c).

948 There is also a maximum in w^m_{ST} (SS^{\max}) of silicate melts that are multiply-saturated with
949 sulfide melt and anhydrite at fixed T , and a global maximum if vapour is also present. Like the
950 SS^{\min} , this SS^{\max} also occurs at an f_{O_2} value at which both S^{2-} and S^{6+} are dissolved in the silicate
951 melt in similar concentrations (Figure 5g). This maximum can be explained by the constraints of
952 sulfide melt- and anhydrite-saturation leading to simultaneous maximization of the concentrations
953 of both the S^{2-} and S^{6+} species in the silicate melt. Additionally, P and f_{O_2} are dependent variables
954 at SS^{\max} once the independent conditions of constant T , sulfide melt-saturation (which sets the
955 value of μ_{FeS}), and anhydrite-saturation (which sets μ_{CaSO_4}) are imposed. A maximum in w^m_{ST} is
956 also encountered at constant T with increasing f_{O_2} when a vapour-undersaturated silicate melt is
957 both sulfide melt- and anhydrite-saturated (Figure 6c).

958 These SS^{\min} and SS^{\max} features can play important roles over the entire magmatic and volcanic
959 system, extending from the mantle to eruption (Figure 1c). However, their influences depends on
960 the independent variables governing the system at each point during the process, the paths followed
961 by these variables, and their effects on dependent parameters. For example, these features can
962 influence the maximum S concentration in mantle melts; the volatile release from magma mixing
963 and crustal assimilation; the depth that significant amounts of S begin to degas from silicate melt;
964 the f_{O_2} of erupted magma and emissions of volcanic gases; and the amounts of SO_2 released to the
965 atmosphere during explosive eruptions. Additionally, they may also impact some of the tools used
966 to infer intensive and extensive variables of these systems. Excluding the effects of the SS^{\min} (and
967 SS^{\max}) can cause significant underestimations in calculations of P^v_{sat} , as well as SO_2 emissions
968 using the petrologic method. SS^{\max} also provides the possibility of constraining f_{O_2} for S-bearing
969 magmas based on limits set by the f_{O_2} dependence of w^m_{ST} of silicate melts saturated with vapour,
970 sulfide melt, and/or anhydrite.

971 We emphasize that our approach has been to use a simplified ternary system (silicate– O_2 – S_2)
972 to model thermodynamically the coexistence of silicate melt + (vapour and/or sulfide melt and/or

973 anhydrite). This choice allows us to isolate and analyse the interplay of key variables in a system
974 in which all the volatile species in the vapour are on the S-O join. Therefore, we have not included
975 the effects of other volatile components that typically make up most of the gas phase. Although
976 we are confident that the patterns and behaviour expressed in this simple system can be generalized
977 to natural systems, an important next step will be to include other components and species in the
978 silicate melt and vapour. Our approach can be readily expanded to model such complex natural
979 systems, particularly when H and C are present. Principally, this involves including additional
980 species and homogeneous equilibria to the vapour (e.g., H₂, H₂O, CO, CO₂, CH₄, H₂S, OCS, etc.)
981 and solubility reactions for the species that dissolve in the silicate melt (e.g., OH⁻, H₂O, H₂, CO₃²⁻
982, CO₂, CO, CH₄, H₂S, SH⁻ etc.). Full generalization of our results to natural systems will require
983 exploring the effects of variations in composition on solubility and speciation (e.g., the
984 composition of the silicate melt, including H₂O and [Fe³⁺/Fe_T]^m) and the *P* and *T* dependence of
985 the equilibrium constants for heterogeneous silicate melt-vapour equilibria. This is currently of
986 particular importance for the sulfate capacity (C_{S6+}), which controls the behaviour of dissolved
987 sulfate and is expected to be strongly influenced by *T* and melt composition (e.g., Moretti and
988 Ottonello 2005; Nash *et al.* 2019; O'Neill and Mavrogenes 2019; Boulliang and Wood 2021;
989 Moretti 2021). Although such an expanded treatment will be important and useful in detailed
990 modelling of natural systems, the complexity already present in our simple system – for a single
991 Hawaiian basaltic melt composition in which only S ± O-bearing species are included – highlights
992 in our view the importance of adding complexity (especially in terms of silicate melt and vapour
993 composition) incrementally to such end member systems for understanding the behaviour of sulfur
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1001 **Conflict of interest**

1002 The authors declare no known conflicts of interest associated with this publication.

1003 **Author contributions**

1004 ECH, EMS, and LS conceived the project idea. ECH wrote the code with the help of PL. ECH ran
1005 the analyses. ECH and EMS led manuscript production, with further contribution from LS and PL.

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1359

1360 **Tables**

1361 *Table 1* Summary of the relationships [$Z \propto (f_{O_2})^a(Y)^b$] and slopes between independent (Y) and dependent (Z) variables
 1362 when T and f_{O_2} are always independent variables for silicate melt + vapour.

Z	Y											
	f_{S_2}				w_{ST}^m				P			
	a	b	ζ	σ	a	b	ζ	σ	a	b	ζ	σ
Silicate melt $\cong S^{2-}$ and vapour $\cong S_2$ (purple)												
f_{S_2}	0.0	+1.0	0.0	0.0	+1.0	+2.0	+1.0	-0.5	0.0	+1.0	0.0	0.0
f_{SO_2}	+1.0	+0.5	+1.0	-2.0	+1.5	+1.0	+1.5	-1.5	+1.0	+0.5	+1.0	-2.0
$P (\cong p_{S_2} \cong f_{S_2})$	0.0	+1.0	0.0	0.0	+1.0	+2.0	+1.0	-0.5	0.0	+1.0	0.0	0.0
$w_{S_2-}^m$	-0.5	+0.5	-0.5	+1.0	0.0	+1.0	0.0	0.0	-0.5	+0.5	-0.5	+1.0
$w_{S_6+}^m$	+1.5	+0.5	+1.5	-3.0	+2.0	+2.0	+2.0	-1.0	+1.5	+0.5	+1.5	-3.0
$w_{ST}^m (\cong w_{S_2-}^m)$	-0.5	+0.5	-0.5	+1.0	0.0	+1.0	0.0	0.0	-0.5	+0.5	-0.5	+1.0
Silicate melt $\cong S^{2-}$ and vapour $\cong SO_2$ (turquoise)												
f_{S_2}	0.0	+1.0	0.0	0.0	+1.0	+2.0	+1.0	-0.5	-2.0	+2.0	-2.0	+1.0
f_{SO_2}	+1.0	+0.5	+1.0	-2.0	+1.5	+1.0	+1.5	-1.5	0.0	+1.0	0.0	0.0
$P (\cong p_{SO_2} \cong f_{SO_2})$	+1.0	+0.5	+1.0	-2.0	+1.5	+1.0	+1.5	-1.5	0.0	+1.0	0.0	0.0
$w_{S_2-}^m$	-0.5	+0.5	-0.5	+1.0	0.0	+1.0	0.0	0.0	-1.5	+1.0	-1.5	+1.5
$w_{S_6+}^m$	+1.5	+0.5	+1.5	-3.0	+2.0	+2.0	+2.0	-1.0	+0.5	+1.0	+0.5	-0.5
$w_{ST}^m (\cong w_{S_2-}^m)$	-0.5	+0.5	-0.5	+1.0	0.0	+1.0	0.0	0.0	-1.5	+1.0	-1.5	+1.5
Silicate melt $\cong S^{6+}$ and vapour $\cong SO_2$ (yellow)												
f_{S_2}	0.0	+1.0	0.0	0.0	-3.0	+2.0	-3.0	+1.5	-2.0	+2.0	-2.0	+1.0
f_{SO_2}	+1.0	+0.5	+1.0	-2.0	-0.5	+1.0	-0.5	+0.5	0.0	+1.0	0.0	0.0
$P (\cong p_{SO_2} \cong f_{SO_2})$	+1.0	+0.5	+1.0	-2.0	-0.5	+1.0	-0.5	+0.5	0.0	+1.0	0.0	0.0
$w_{S_2-}^m$	-0.5	+0.5	-0.5	+1.0	-2.0	+1.0	-2.0	-2.0	-1.5	+1.0	-1.5	+1.5
$w_{S_6+}^m$	+1.5	+0.5	+1.5	-3.0	0.0	+1.0	0.0	0.0	+0.5	+1.0	+0.5	-0.5
$w_{ST}^m (\cong w_{S_6+}^m)$	+1.5	+0.5	+1.5	-3.0	0.0	+1.0	0.0	0.0	+0.5	+1.0	+0.5	-0.5

1363 *Notes:* Derivations are in the Supplementary Material. The slopes of $\log_{10}(Y)$ vs. $\log_{10}(Z)$ are given as ζ when Y is
 1364 constant (slopes of Z -curves in Figure 2) and σ when Y varies (slopes of Z -contours in Figure 3–Figure 5). $p_i \cong f_i$
 1365 because $\gamma_i \sim 1$. **Bold** indicates the independent variable (Y). For example, when the silicate melt is S^{2-} -dominated and
 1366 the vapour is S_2 -dominated in the purple region (first group of f_{S_2} - f_{SO_2} - P - $w_{S_2-}^m$ - $w_{S_6+}^m$ - w_{ST}^m horizontally) and $Y = f_{S_2}$
 1367 (first group of a - b - ζ - σ vertically), the slope of $w_{S_6+}^m$ when f_{S_2} is constant (ζ) is +1.5 (Figure 2a) and the slope of the
 1368 contours (σ) of $Z = w_{ST}^m$ are $-(-0.5)/(0.5) = +1.0$ (Figure 3c).

1369 **Figure Captions**

1370 *Figure 1* Schematic figures of sulfur in magmatic and volcanic systems. In all panels, silicate melt
 1371 is shown in pink, vapour in blue, sulfide melt in yellow, and anhydrite in grey. (a) Ternary diagram
 1372 of the idealised silicate-S₂-O₂ system used in this study, which contains silicate melt and vapour.
 1373 The vapour is constrained to the S₂-O₂ binary, which includes S₂, O₂, and SO₂ as species. An
 1374 example tie-line between the composition of the silicate melt and vapour (including the
 1375 composition of the bulk system) is shown by the black line. Sulfide melt and anhydrite lie outside
 1376 this ternary. (b) Schematic illustration of the interplay between the different species in the silicate
 1377 melt, which contains S²⁻, SO₄²⁻, FeO, and FeO_{1.5}; vapour, which contains S₂, O₂, and SO₂; sulfide
 1378 melt, which contains S²⁻; and anhydrite, which contains SO₄²⁻. Phases and species that dominate
 1379 under reducing conditions are to the left, whilst those that dominate under oxidising conditions are
 1380 to the right. Two-way arrows indicate species that can interact within, and between different,
 1381 phases. (c) Illustration showing the involvement of sulfur in different magmatic and volcanic
 1382 processes. In detail, the process and techniques influenced by the sulfur solubility minimum (SS^{min},
 1383 left) and maximum (SS^{max}, right) are highlighted from “Implications of SSmin and SSmax for
 1384 magmatic and volcanic processes”. SS^{min}, where silicate melt + vapour are stable, effects the gas
 1385 release from magma mixing and crustal assimilation (“Magma mixing and crustal assimilation”),
 1386 degassing depth (“Magma ascent and degassing”), and the *f*_{O₂} of surface gas emissions (“Magma
 1387 ascent and degassing”), as well as causing *P*^{v_{sat}} to be underestimated (“Magma ascent and
 1388 degassing”) and affecting the reliability of the petrologic method (“Volcanic emissions”). SS^{max},
 1389 where silicate melt + vapour + sulfide melt + anhydrite are stable, sets the maximum S content of
 1390 partial mantle melts (“Mantle melting”) and is a common feature of large SO₂ emissions from
 1391 explosive eruptions (“Volcanic emissions”). It also effects the reliability of the petrologic method
 1392 (“Volcanic emissions”) and can be utilised as an oxybarometer from *w*^{mST} measurements (“Using
 1393 *w*^{mST} as an oxybarometer”). *Abbreviations*: *sil*^m, silicate melt; *v*, vapour; *sulf*^m, sulfide melt; *anh*,
 1394 anhydrite; *P*^{v_{sat}}, pressure of vapour-saturation; *f*_{O₂}, oxygen fugacity; *w*^{mST}, total dissolved S content
 1395 of the silicate melt.

1396 *Figure 2* Different dependent variables (*Z*) versus oxygen fugacity (*f*_{O₂}) relative to the Fayalite-
 1397 Magnetite-Quartz (FMQ) buffer (shown as ΔFMQ) for silicate melt + vapour: (a, c, e)
 1398 concentration of dissolved S-bearing species in the silicate melt (*w*^{m_i}), where *i* = total sulfur (S_T,
 1399 white solid), sulfide (S²⁻, black dot), or sulfate (S⁶⁺, black dash); and (b, d, f) pressure (*P*, white
 1400 solid) or partial pressure (*p_i*), where *i* = sulfur (S₂, black dot), sulfur dioxide (SO₂, black dash), or
 1401 oxygen (O₂, black dot dash). In each column a different independent variable (*Y*) is fixed (indicated
 1402 by the horizontal line in either the top or bottom row): (a–b) *f*_{S₂} = 10⁻⁴ bar; (c–d) *w*^{mST} = 400 ppm;
 1403 and (e–f) *P* = 1 bar. The background colour indicates the S speciation of the silicate melt and
 1404 vapour, where boundaries are defined at [S⁶⁺/S_T]^m = 0.1 and 0.9 ([S²⁻]^m where [S⁶⁺/S_T]^m < 0.1, [S²⁻
 1405 +S⁶⁺]^m where 0.1 ≤ [S⁶⁺/S_T]^m ≤ 0.9, and [S⁶⁺]^m where [S⁶⁺/S_T]^m > 0.9) and *x*^{v_{SO₂}} = 0.1 or 0.9 ([S₂]^v
 1406 where *x*^{v_{SO₂}} < 0.1, [S₂+SO₂]^v where 0.1 ≤ *x*^{v_{SO₂}} ≤ 0.9, or [SO₂]^v where *x*^{v_{SO₂}} > 0.9): purple = [S²⁻
 1407]^m+ [S₂]^v; blue = [S²⁻]^m+ [S₂+SO₂]^v; turquoise = [S²⁻]^m+ [SO₂]^v; green = [S²⁻+S⁶⁺]^m+ [SO₂]^v; and
 1408 yellow = [S⁶⁺]^m+ [SO₂]^v. The vertical grey lines show where sulfide melt (solid) or anhydrite
 1409 (dashed) would saturate. Slopes (*ζ*) are labelled in boxes outlined in black. *α* (in panels b, d, and f)
 1410 indicates when *p*_{O₂} = *p*_{S₂}, and *β* in panel (d) indicates a change in slope of *P* from *ζ* = +1 to +1.5
 1411 with increasing *f*_{O₂}. Note that to the right of *α* in (b) that *γ*_{S₂} = 2.14 at *P* = 4507 bar, causing *p*_{S₂} to
 1412 deviate noticeably from *f*_{S₂}, just visible as the slight downturn in the black dotted curve at high *f*_{O₂}.
 1413 *Abbreviations*: *sil*^m, silicate melt; *v*, vapour; *sulf*^m, sulfide melt; and *anh*, anhydrite.

1414 *Figure 3* Sulfur fugacity ($Y = fs_2$) versus oxygen fugacity (f_{O_2}) for silicate melt + vapour at $T =$
1415 1200 °C. Contours of Z are shown for: (a) sulfur dioxide fugacity (fs_{O_2}); (b) pressure (P), and (c)
1416 total S content of the silicate melt ($w^{m_{ST}}$). (d) Sulfur speciation of the vapour (S_2 and/or SO_2) and
1417 silicate melt (S^{2-} and/or S^{6+}), where boundaries are defined for the vapour at $x^v_{SO_2} = 0.1$ or 0.9
1418 (black dashed curves) and for the silicate melt at $[S^{6+}/S_T]^m = 0.1$ and 0.9 (black solid curves):
1419 purple = $[S^{2-}]^m + [S_2]^v$; blue = $[S^{2-}]^m + [S_2 + SO_2]^v$; turquoise = $[S^{2-}]^m + [SO_2]^v$; green = $[S^{2-}$
1420 $+ S^{6+}]^m + [SO_2]^v$; yellow = $[S^{6+}]^m + [SO_2]^v$, light turquoise with black arrow = $[S^{2-} + S^{6+}]^m + [S_2 + SO_2]^v$
1421 (as in Figure 2, although the light turquoise section was not previously intersected). (e) Paths
1422 followed in f_{O_2} - fs_2 space for different processes (overlain on contours of $Z = w^{m_{ST}}$): dark turquoise
1423 is constant $fs_2 = 10^{-4}$ bar (Figure 2a–b); purple is constant $w^{m_{ST}} = 400$ ppm (Figure 2c–d); blue is
1424 constant $P = 1$ bar (Figure 2e–f); yellow, green, and light turquoise are constant bulk $O_2 = 3.26,$
1425 $3.87,$ and 4.23 wt%, respectively (Figure 8); and grey is constant μ_{FeS} and/or μ_{CaSO_4} equal to sulfide
1426 melt- and anhydrite-saturation, respectively (Figure 6a–b). In the white region at very low and very
1427 high f_{O_2} and high fs_2 , $w^{m_{ST}} > 10$ wt% and hence calculations are terminated. The value of the slopes
1428 of the curves and contours (σ) are labelled in boxes with a thick black outline. The grey curve
1429 indicates the boundary above which silicate melt + vapour is metastable, which is solid when
1430 sulfide melt would saturate, dashed when anhydrite would saturate, and the grey star indicates the
1431 silicate melt is multiply-saturated with vapour, sulfide melt, and anhydrite. The f_{O_2} for different
1432 tectonic settings from Cottrell et al. (2021) is shown by horizontal blue bars (extent of the bar shows
1433 the range and black vertical line shows the mode) for mid-ocean ridge (light blue), ocean island
1434 (intermediate blue), and arc (dark blue) basalts. *Abbreviations:* sil^m , silicate melt; v , vapour; $sulf^m$,
1435 sulfide melt; anh , anhydrite; MORB, mid-ocean ridge basalts; OIB, ocean island basalts; and Arc,
1436 arc basalts.

1437 *Figure 4* Total S content of the silicate melt ($Y = w^{m_{ST}}$) versus oxygen fugacity (f_{O_2}) for silicate
1438 melt + vapour at $T = 1200$ °C. Contours of Z are shown for: (a) sulfur fugacity (fs_2); (b) sulfur
1439 dioxide fugacity (fs_{O_2}); and (c) pressure (P). (d) Sulfur speciation of the silicate melt and vapour.
1440 (e) Paths followed in f_{O_2} - fs_2 space for different processes (overlain on contours of $Z = P$). See
1441 Figure 3 for details of the coloured regions, different curves, annotations, abbreviations, and f_{O_2}
1442 ranges. *Note:* anhydrite is not stable at these $w^{m_{ST}}$ and therefore anh does not appear.

1443 *Figure 5* Pressure ($Y = P$) versus oxygen fugacity (f_{O_2}) at $T = 1200$ °C for (a–e) silicate melt +
1444 vapour and (f–g) silicate melt + (vapour and/or sulfide melt and/or anhydrite) (indicated using *).
1445 Contours of Z are shown for: (a) sulfur fugacity (fs_2); (b) sulfur dioxide fugacity (fs_{O_2}); and (c, g)
1446 total S content of the silicate melt ($w^{m_{ST}}$). (d, f) Sulfur speciation of the silicate melt and vapour.
1447 In the vapour-undersaturated region of (f): dark grey indicates S^{2-} -dominated silicate melt + sulfide
1448 melt; medium grey indicated silicate melt (both S^{2-} and S^{6+} in similar concentrations) and sulfide
1449 melt; and light grey indicates S^{6+} -dominated silicate melt + anhydrite. (e, g) Paths followed in f_{O_2} -
1450 fs_2 space for different processes (overlain on contours of $Z = w^{m_{ST}}$). In the white region at high f_{O_2}
1451 and low P , calculations are unfeasible as $x^v_{O_2} > 1$. See Figure 3 for details of the coloured regions,
1452 different curves, annotations, abbreviations, and f_{O_2} ranges. *Note:* the blue horizontal dotted and
1453 dashed lines in (g) are the $P = 200$ and 2000 bar, respectively, slices shown in Figure 6c–f.

1454 *Figure 6* (a, c, e) Total S content of the silicate melt ($w^{m_{ST}}$) and (b, d, f) pressure (P) versus oxygen
1455 fugacity (f_{O_2}) against at $T = 1200$ °C. When the silicate melt is vapour-saturated, the background
1456 is coloured (no vapour is present in the white regions) and the silicate melt and vapour speciation
1457 are shown by the background colour (see Figure 3 for details). Grey or black solid curves indicate
1458 sulfide melt-saturation and dashed curves indicate anhydrite-saturation. (a–b) Silicate melt +

1459 vapour + (sulfide melt and/or anhydrite) with independent variables of T , f_{O_2} , and μ_{FeS} and/or
 1460 μ_{CaSO_4} . At the grey star, both sulfide melt and anhydrite are saturated; hence, both μ_{FeS} and μ_{CaSO_4}
 1461 are specified and f_{O_2} is no longer independent. The white horizontal lines and annotations (α , β ,
 1462 and γ) are for the melt inclusion example containing 400 (solid), 3000 (dash), and 13000 (dot) ppm
 1463 S_T described in “Using $wmST$ as an oxybarometer”. (c–d) Vapour-undersaturated silicate melt +
 1464 (sulfide melt and/or anhydrite) with independent variables of T , f_{O_2} , P , and μ_{FeS} and/or μ_{CaSO_4} ; when
 1465 both μ_{FeS} and μ_{CaSO_4} are specified, f_{O_2} or P is no longer independent. Silicate melt speciation is
 1466 indicated by the green vertical lines ($[S^{6+}/S_T]^m = 0.1$ or 0.9): $[S^{2-}]^m$ at low f_{O_2} , $[S^{2-}+S^{6+}]^m$ at
 1467 intermediate f_{O_2} , and $[S^{6+}]^m$ at high f_{O_2} . At the vertical grey line, both sulfide melt and anhydrite
 1468 are saturated. (e–f) Silicate melt + (vapour and/or sulfide melt and/or anhydrite) with independent
 1469 variables of T , f_{O_2} , P , and μ_{FeS} and/or μ_{CaSO_4} ; when both μ_{FeS} and μ_{CaSO_4} are specified, f_{O_2} or P is no
 1470 longer independent. The green vertical line indicates the boundary between $[S^{2-}]^m$ (lower f_{O_2}) and
 1471 $[S^{2-}+S^{6+}]^m$ (higher f_{O_2}) when vapour is *not* present ($[S^{6+}/S_T]^m = 0.1$). At the solid grey vertical line,
 1472 sulfide melt + vapour are saturated; along the dotted segment, vapour is saturated; and at the dashed
 1473 grey vertical line, vapour + anhydrite are saturated. Annotations (δ , ϵ , and ζ) indicate maxima and
 1474 minima described in “Silicate melt + (vapour and/or sulfide melt and/or anhydrite)”. f_{O_2} ranges are
 1475 shown as described in Figure 3. *Abbreviations*: sil^m , silicate melt; v , vapour; $sulf^m$, sulfide melt;
 1476 anh , anhydrite; MORB, mid-ocean ridge basalts; OIB, ocean island basalts; and Arc, arc basalts.

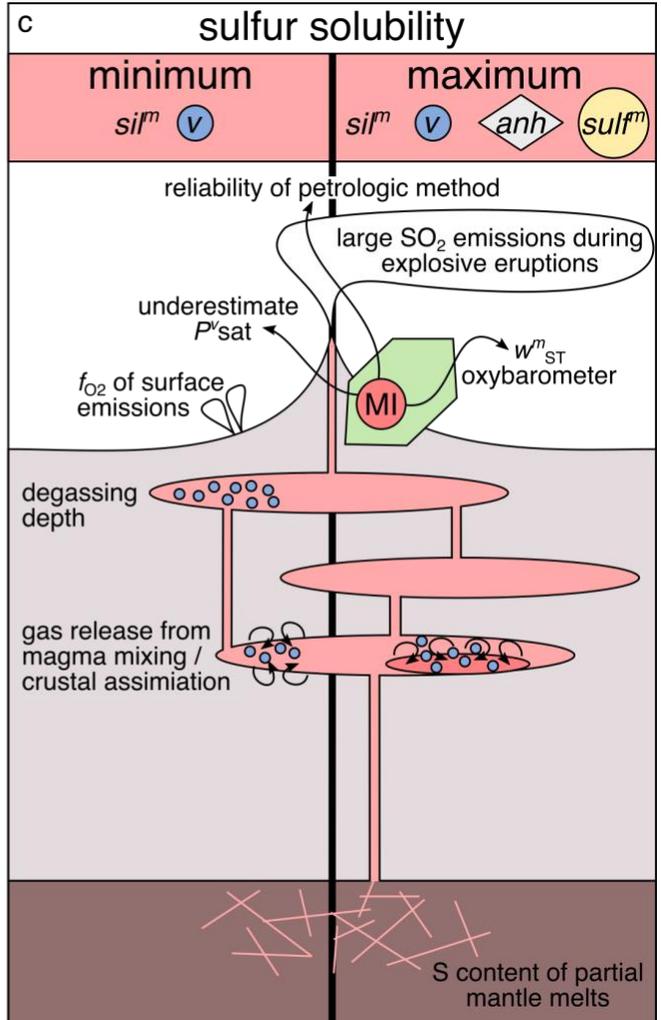
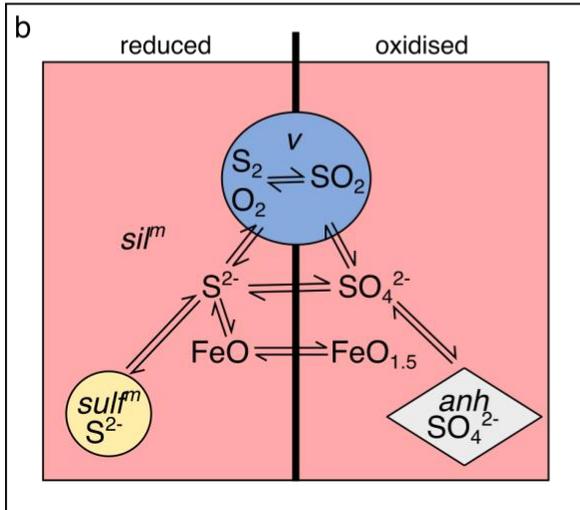
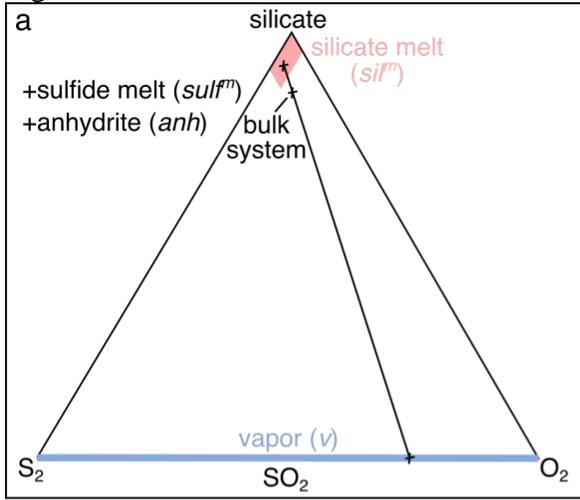
1477 *Figure 7* Closed-system degassing: pressure (P) versus bulk O_2 content of the system for silicate
 1478 melt + vapour at $T = 1200$ °C with a bulk S content of 5000 ppm. Contours of Z are shown for: (a)
 1479 sulfur fugacity (f_{S_2}); (b) sulfur dioxide fugacity (f_{SO_2}); (c) total S content dissolved in the silicate
 1480 melt (w^{mST}); (d) oxygen fugacity (f_{O_2}); (e) total vapour content of the system (w^v_T) – this includes
 1481 both S and O in the vapour and hence can exceed 0.5 wt%. (f) Sulfur speciation of the silicate melt
 1482 and vapour. (g) Paths followed in P -bulk O_2 space for closed-system degassing and the white circle
 1483 and square connected by a black-long dashed-horizontal line is the mixing example described in
 1484 “Magma mixing and crustal assimilation” (overlain on contours of $Z = w^{mST}$). The solid black
 1485 curve is equal to the 5000 ppm w^{mST} contour and $P^{v_{sat}}$, and $f_{O_2}^{SSmin}$ is indicated by a white diamond.
 1486 The bulk O_2 ranges shown below each figure correspond to the f_{O_2} by tectonic setting bars in other
 1487 figures at $P^{v_{sat}}$. See Figure 3 for details of the coloured regions, different curves, annotations, and
 1488 abbreviations. *Note*: anhydrite is not stable at these conditions and therefore anh does not appear.

1489 *Figure 8* Closed (solid) and open (black dash) -system depressurisation paths for silicate melt +
 1490 vapour at $T = 1200$ °C: (a) sulfur fugacity (f_{S_2}); (b) oxygen fugacity (f_{O_2}); and (c) total S content
 1491 dissolved in the silicate melt (w^{mST}). Three initial bulk compositions are used, as shown on Figure
 1492 7g: 3.26 (yellow), 3.87 (green), and 4.23 (turquoise) wt% O_2 .

1493 *Figure 9* Pressure of vapour-saturation ($P^{v_{sat}}$) versus oxygen fugacity (f_{O_2}) for a Hawaiian basaltic
 1494 melt + vapour at 1200 °C containing different w^{mST} : 400 (solid), 2000 (dash), and 5000 (dot) ppm.
 1495 The background colour indicates the speciation of the silicate melt and vapour and the bars indicate
 1496 f_{O_2} of different tectonic settings (see Figure 3 for details).

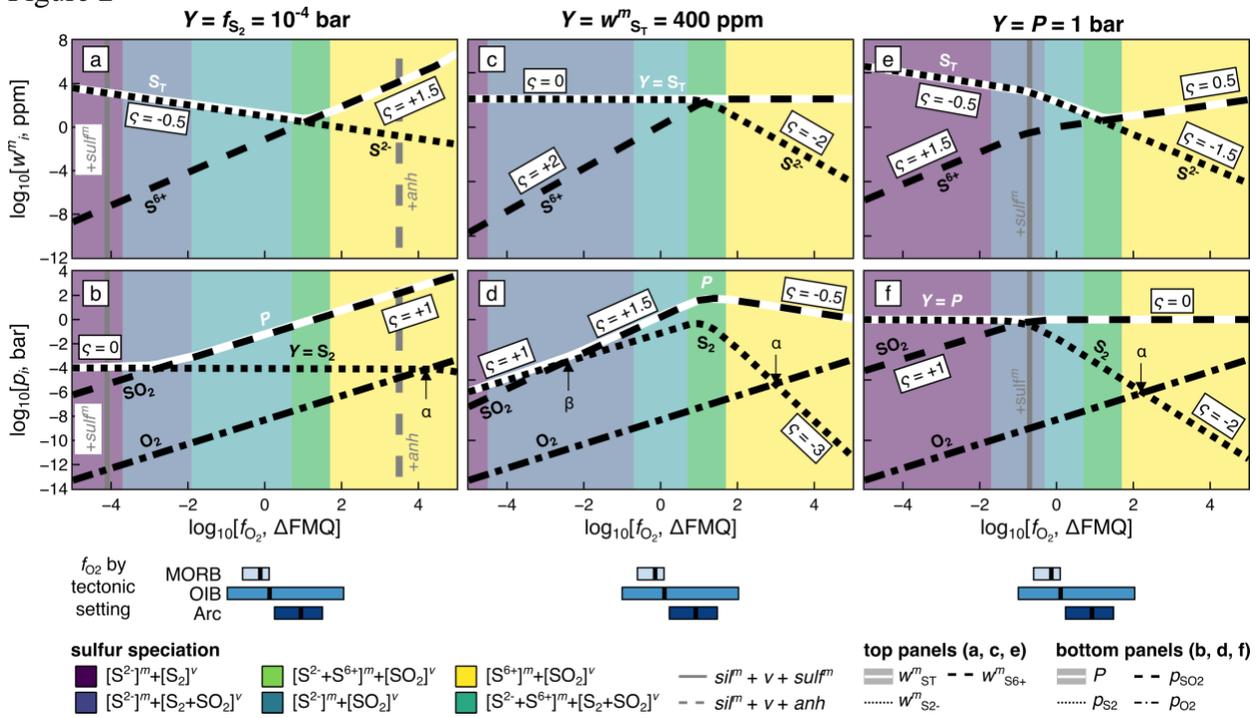
1497
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1499 **Figures**
 1500
 1501 Figure 1



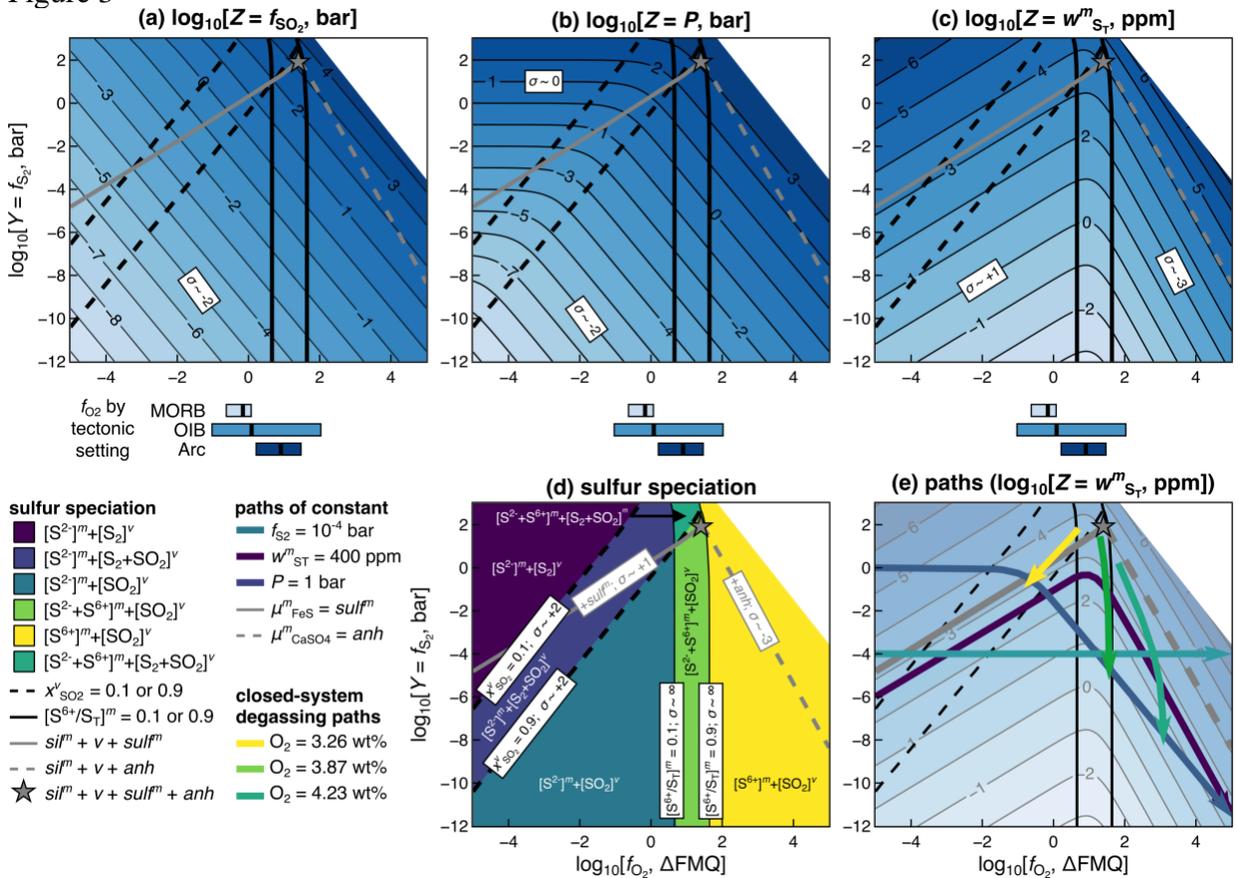
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1503 Figure 2

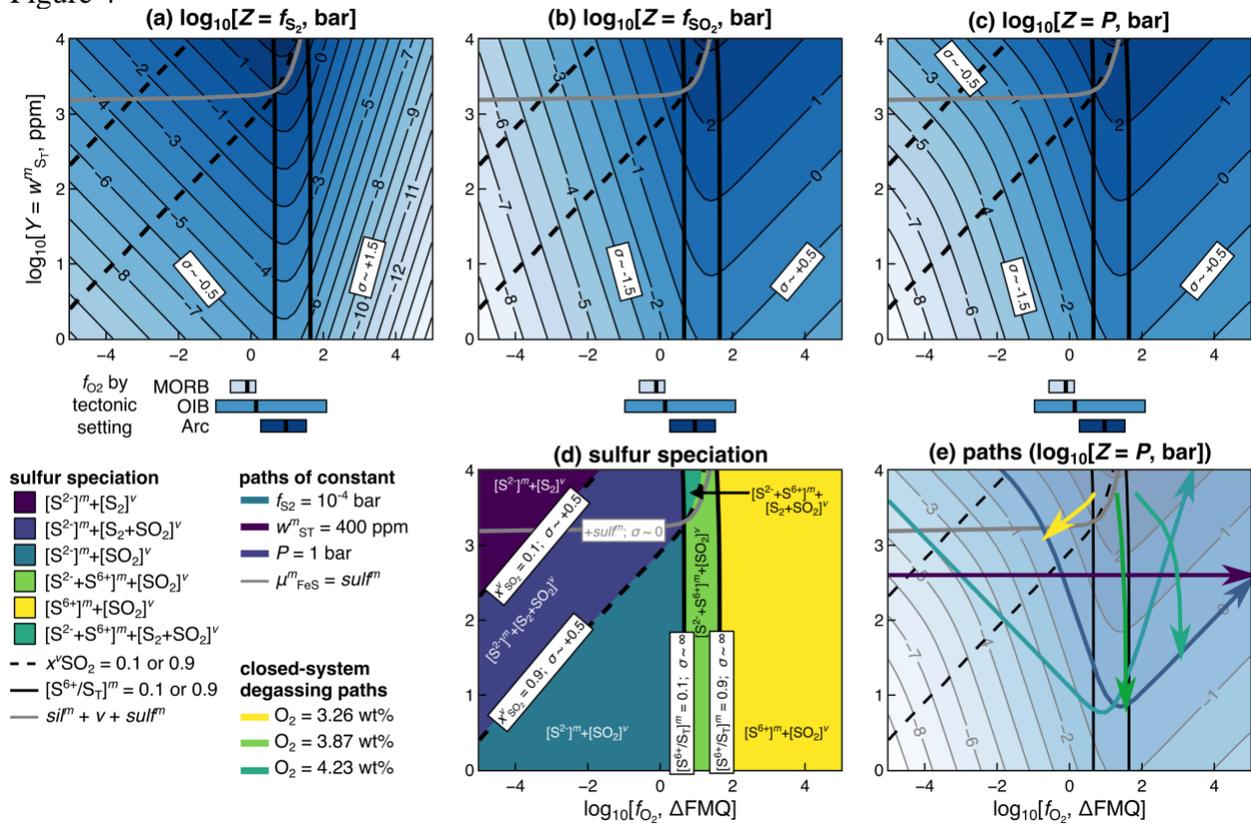


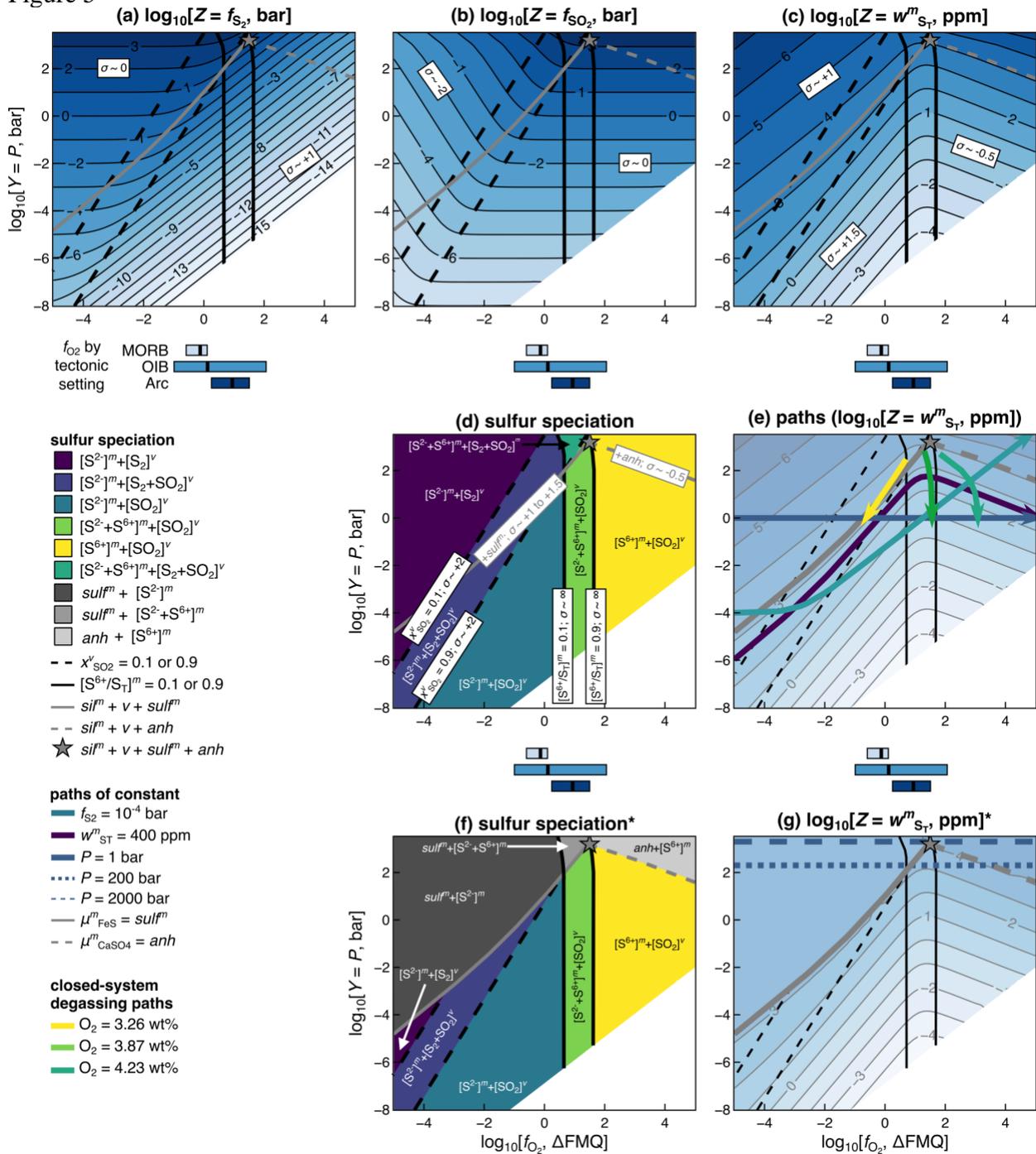
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1505 Figure 3

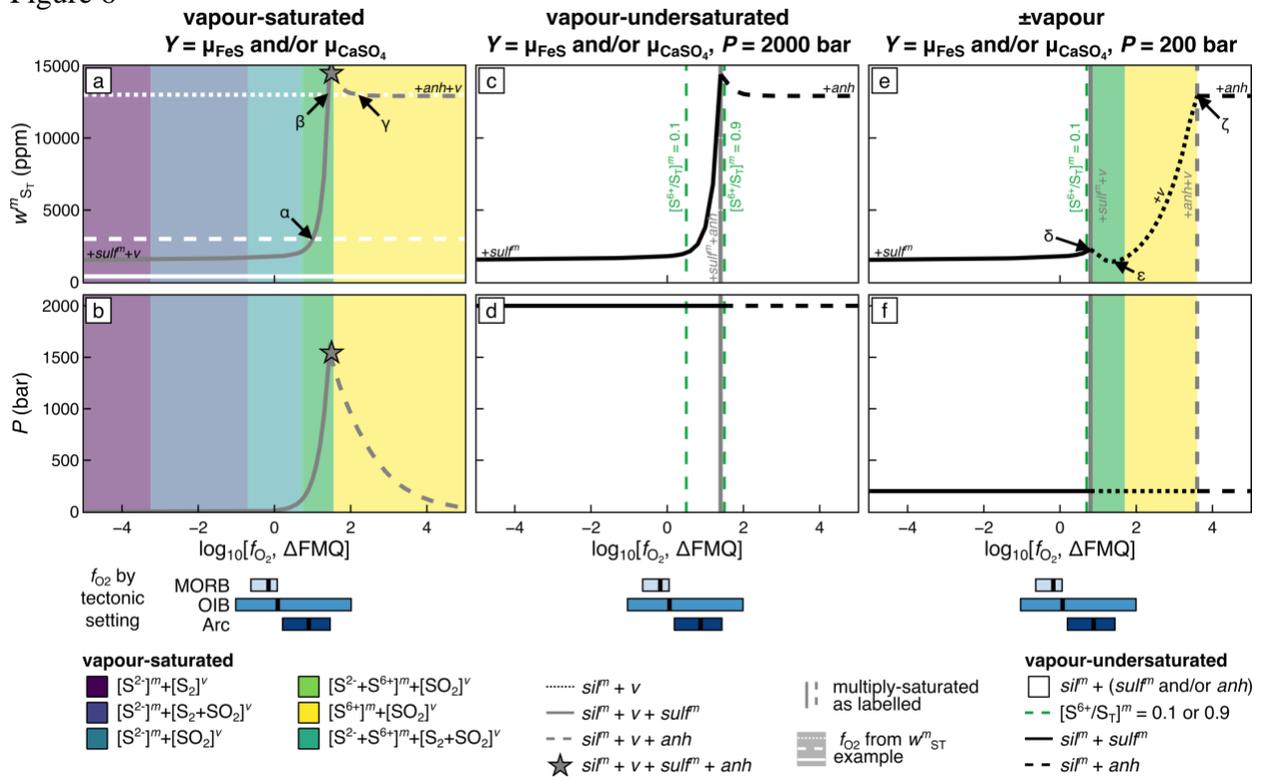


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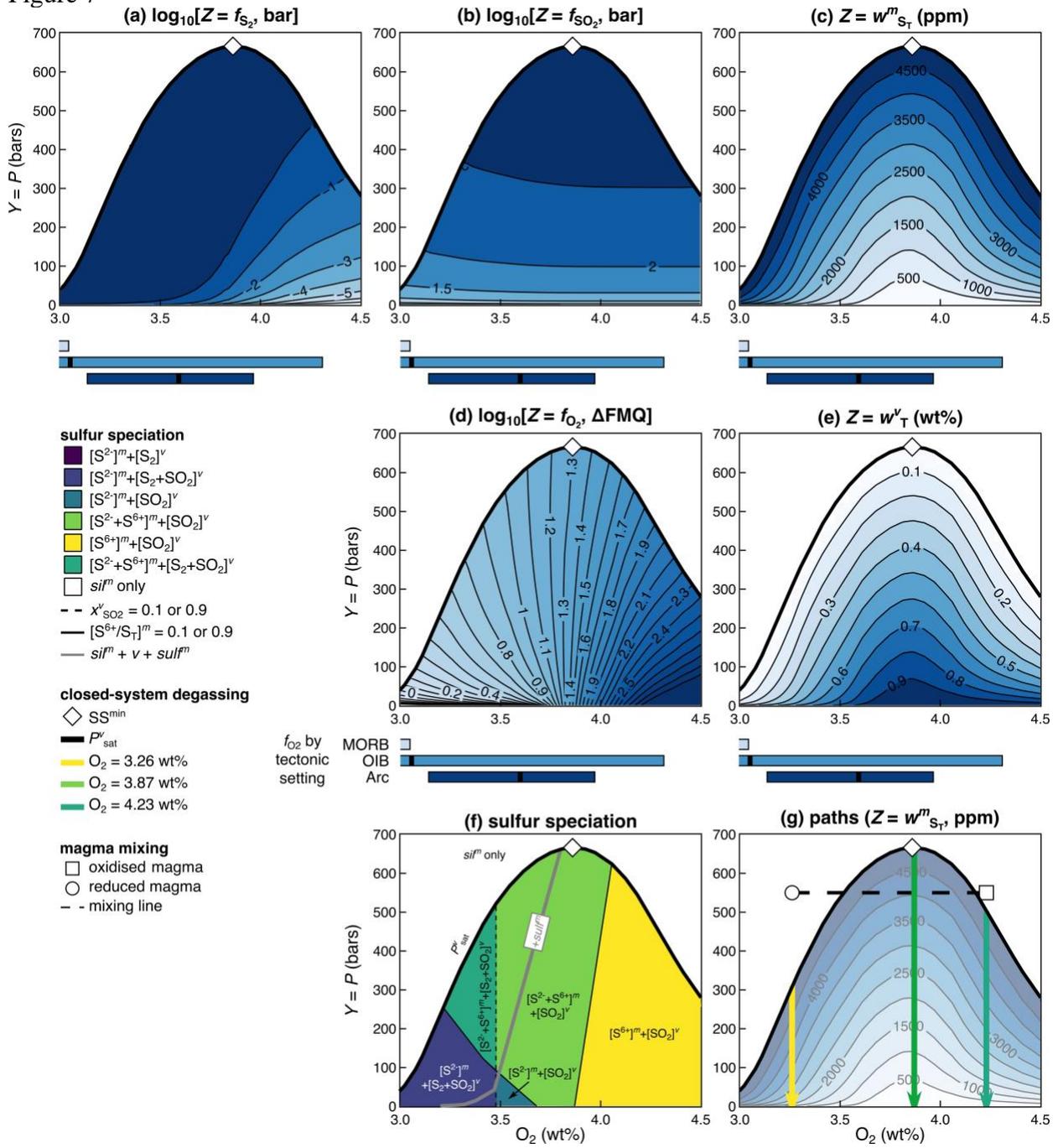




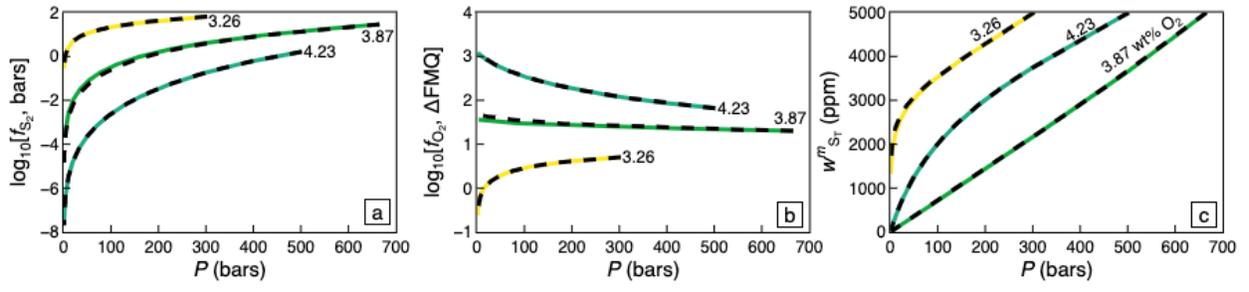
1512 Figure 6



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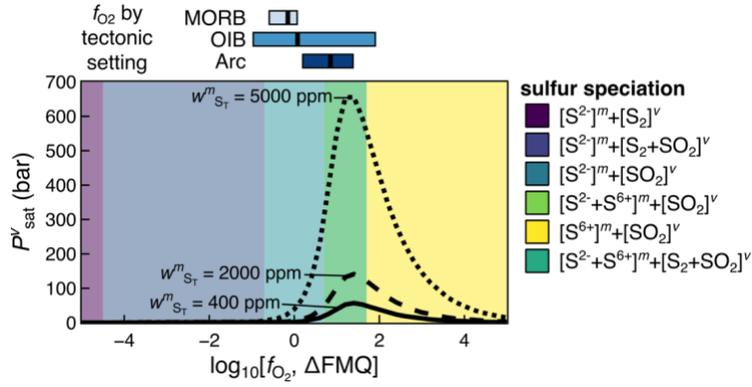


1516 Figure 8



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1518 Figure 9



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