

1 **Evaluating essential features of proppant transport at engineering scales**
2 **combining field measurements with machine learning algorithms**

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17 **Abstract**

18 The characterization of the proppant transport at a field-engineering scale is still challenging due to
19 the lack of direct subsurface measurements. Features that control the proppant transport may link the
20 experimental and numerical observations to the practical operations at a field scale. To improve the
21 numerical and laboratory simulations, we propose a machine-learning-based workflow to evaluate the
22 essential features of proppant transport and their corresponding calculations. The proppant flow in
23 fractures is estimated by applying the GRU and SVM algorithms to the measurements obtained from
24 shale gas fracturing operations. Over 430,000 groups of fracturing data are collected and pre-
25 processed by the proppant transport models to calculate key features, including settlement, stratified
26 flow and inception of settled particles. The features are then fed into machine learning algorithms for
27 pressure prediction. The root mean squared error (RMSE) is used as the criterion for ranking selected
28 features *via* the control variate method. Our result shows that the stratified-flow feature (fracture-
29 level) possesses better interpretations for the proppant transport, in which the Bi-power model helps to
30 produce the best predictions. The settlement and inception features (particle-level) perform better in
31 cases that the pressure fluctuates significantly, indicating that more complex fractures may have been
32 generated. Moreover, our analyses on the remaining errors in the pressure-ascending cases suggest
33 that (1) an introduction of the alternate-injection process, and (2) the improved calculation of proppant
34 transport in complex fracture networks and highly-filled fractures will be beneficial to both
35 experimental observations and field applications.

36 **Keywords:** proppant transport; feature evaluation; machine learning; pressure variation; field scales

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37 **1 Introduction**

38 Hydraulic fracturing has become an important technique to enhance hydrocarbon recovery from
39 unconventional gas resources, aiming to meet the growing demand for clean energy globally. To
40 avoid fracture closure after the dissipation of the hydraulic injecting pressure, proppant injections are
41 essential in the hydraulic fracturing process, and their effectiveness plays an important role in
42 enhancing the stimulated reservoir volume (Barree & Conway, 1994; L. Fan, Thompson, & Robinson,
43 2010; Nassir, Settari, & Wan, 2014). To inject thousands of tons of proppant particles down into the
44 induced fractures, a deep well in a shale gas reservoir (>4000 m) is usually operated under a wellhead
45 pressure exceeding 100 MPa (B. Hou, Chang, Fu, Muhadasi, & Chen, 2019; Mao, Zhang, Chun, &
46 Wu, 2021). Therefore, how to inject proppant particles under safe operating pressures is very
47 challenging, especially with the application of low-viscosity slickwater (Liang, Sayed, Al-Muntasheri,
48 Chang, & Li, 2016). Proppant transport is, therefore, an essential research topic in hydraulic fracturing
49 engineering (Economides & Nolte, 1989).

50 The behaviours of the proppant settlement, stratified flow and inception of settled particles in low-
51 viscosity fracturing fluid have been characterized numerically based on experimental tests (Gadde,
52 Liu, Norman, Bonnacaze, & Sharma, 2004; Wei, Babadagli, Huang, Hou, & Li, 2020; Zhao et al.,
53 2019), which are the essential features of proppant transport. The recent trend of the proppant
54 transport research is to bring in more realistic subsurface scenarios by replacing the single smooth-
55 panel fracture with artificial-coarse fracture networks (Manchanda, Zheng, Hirose, & Sharma, 2020;
56 Raki Sahai & Moghanloo, 2019; Tong & Mohanty, 2016). However, it is still challenging to simulate
57 and characterize the realistic morphology (scale, tortuosity, branches, et. al.) of fractures numerically
58 (Dahi-Taleghani & Olson, 2011). Moreover, the direct subsurface measurements and observations
59 during fracturing operations are still limited. Many numerical models for calculating the proppant
60 transport, therefore, are usually verified by laboratory experiments (Mack, Sun, & Khadilkar, 2014;
61 Patankar et al., 2002; Raki Sahai & Moghanloo, 2019). However, the approaches for examining the
62 numerical observations at a field scale are still in demand. The quantitative and qualitative links
63 between indoor research and field practice may rely on exploring the key property features that

64 control the proppant transports (Cai, Guo, Li, & Yang, 2017). As machine learning techniques are
65 widely employed to provide new insights into engineering problems (Ben et al., 2020; Hu, Khan,
66 Zhang, & Tian, 2020), it is efficient to use data-driven approaches to better understand how the key
67 features being tested in the lab influence the real operations at the engineering level, which may, in
68 turn, promote the numerical and experimental simulations.

69 In this study, we examine and compare essential features of proppant transport and their
70 corresponding calculations by using a new workflow, where we introduce machine learning (ML)
71 algorithms, including Support Vector Regression (Al-Anazi & Gates, 2010) and Gated Recurrent
72 Units (Sun, Battula, Hruby, & Hossaini, 2020). The ML method can process the field measurements
73 directly without in-depth characteristics of the realistic fracture morphology, which may build a
74 bridge between the fundamental research and field applications. Based on the data-driven approach,
75 our study is aimed to 1) propose a new workflow to estimate the proppant transport at field
76 engineering scales; and 2) better understand the essential features that control the proppant transport,
77 which is valuable for both field engineering and basic research work.

78 **2 Methodology**

79 The field measurements of the shale gas fracturing treatments are collected and carefully pre-
80 processed (splitting, trimming, and denoising) for training. The proppant transport features,
81 specifically the velocity ratio and the height of the flowing layers within fractures, are initially
82 calculated by several popular proppant transport models. The calculation outputs consisting of the
83 features relevant to the proppant flow, as long as the other subsurface measurements, are then fed into
84 the machine learning algorithms to predict downhole pressure. The predictions are further analyzed
85 using the control variate method and error analyses to evaluate the proppant transport features and
86 their corresponding calculations.

87 **2.1 Data collection and preprocessing**

88 55 stages, including over 430,000 groups, of fracturing measurements (in second) are collected
89 from 10 shale gas wells, which are selected from 5 different platforms in the Sichuan basin, China
90 (Table 1). The field measurements include the geological data (vertical and well depths), clustering

91 data (stage length, cluster number and perforation number), and fracturing data (fluid and proppant
 92 types, pump rate, proppant concentration and wellhead pressure). Five of the ten wells are set as the
 93 training well ($A_1 - E_1$), of which 50 stages of fracturing data are pre-processed for training the
 94 machine learning models. Five testing stages are selected from the remaining five wells ($A_2 - E_2$),
 95 defined as testing wells. To constrain the effect of large spatial variation in geological uncertainty and
 96 formation properties on the predictions, each training well has its own testing well that is selected
 97 from the same platform. For instance, both Well A_1 and Well A_2 (neighbouring wells) are from
 98 Platform A, and so forth, as shown in Table 1. This is one of our strategies to eliminate interference
 99 factors of pressure variation and promote the influence of proppant transport.

100

Table 1 Division of training and testing datasets.

	Platform A		Platform B		Platform C		Platform D		Platform E	
Well No.	A_1		B_1		C_1		D_1		E_1	
Training Dataset / Stages	10	/	10	/	10	/	10	/	10	/
Well No.	A_2		B_2		C_2		D_2		E_2	
Testing Dataset / Stages	/	1	/	1	/	1	/	1	/	1

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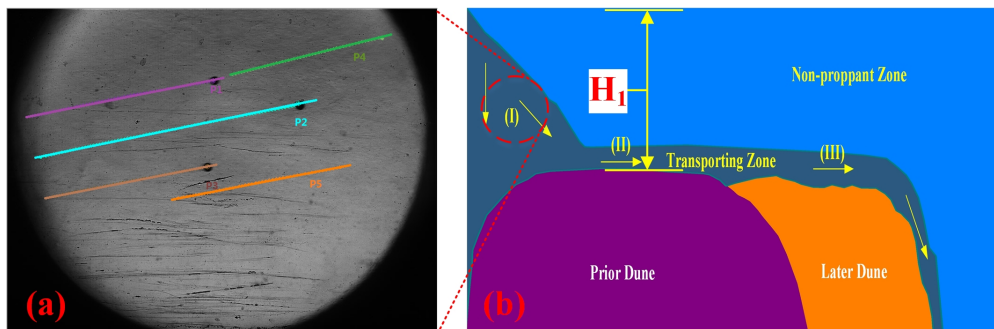
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The other strategy during the data preprocessing is to convert the wellhead pressure into the downhole pressure after the perforation hole (Appendix A), defined as the DPP. The conversion can rule out the potential effects of hydrostatic pressure and friction variations, leaving the proppant transport to control the fracture pressure fluctuation (Dontsov & Peirce, 2014; Willingham, Tan, & Norman, 1993). Other denoising methods involve trimming the pressure at the beginning (when the fracture is created) and the end (pump-off) of the fracturing operation, repeating predictions and averaging the errors obtained from all the platforms ($A - E$), as long as applying two different machine learning algorithms.

109 **2.2 Features for proppant transport**

110



111 **Fig. 1. Proppant transport features at (a) particle-level (particle settlement and inception); (b)**
 112 **fracture-level (H_l – the height of the flowing layer) (L. Hou, Jiang, Liu, et al., 2017; Patankar et al., 2002).**

113 In general, we divide the proppant transport features into two categories by their scales – particle
 114 level (Fig.1 a) and fracture level (Fig.1 b), including the particle settling velocity (Gadde et al., 2004;
 115 Mack et al., 2014; McCabe, Smith, & Harriott, 1993; Richardson & Zaki, 1954; Yew & Weng, 2014),
 116 the critical velocity to restart the settled proppant (also used as the critical turning velocity in complex
 117 fractures) (Cao, Pender, & Meng, 2006; L. Hou, Jiang, Li, Zeng, & Cheng, 2017; L. Hou, Jiang, Liu,
 118 et al., 2017; Rakshit Sahai, Miskimins, & Olson, 2014), the flowing layer height (H_l) (L. Hou et al.,
 119 2019; Novotny, 1977; Patankar et al., 2002; Jing Wang, Joseph, Patankar, Conway, & Barree, 2003)
 120 and the equilibrium dune level (EDL – the dune height divided by fracture height) (Alotaibi &
 121 Miskimins, 2019). Based on the field pumping schedules, those features are further calculated by
 122 employing the Velocity, Settling, Bi-power, and EDL models to yield a group of independent
 123 variables, which is one of the inputs for ML models (Table 2). Details about the equations and their
 124 applications can be found in Appendix B.

125 **Table 2 Summary of calculations, features and control variate method for data processing.**

	Inputs		Output (Dependent variable)	Error analysis	Notes	
	Independent Variable (Selected features)	Control variable (field records)				
Pressure conversion	/	/	DPP (Reference)	/	Appendix A	
Original data	/		DPP (without independent variable)		For comparisons /	
Velocity model	$v_{turning} / v_f$ $v_{settling} / v_f$	$\mu, Q, C \text{ \& } d$	DPP (with independent variable)	RMSE	Derived from forces acting on particles	
Settling model	H_l				Derived from particle settling	Appendix B
Bi-power model	H_l				Derived from particle and fluid Reynold's numbers	
EDL model	EDL				Empirical model	

126 The representative features, particle settlement ($v_{settling} / v_f$), inception ($v_{turning} / v_f$) and stratified-flow
 127 (H_l and EDL) behaviours, are selected and tabulated in Table 2. The particle settlement and inception
 128 are grouped because they are decomposition features of particle movements in vertical and horizontal
 129 directions. The selected features in Table 2 control the proppant transport in the low-viscosity fluid,

130 based on which more comprehensive models coupling fracture propagation, fluid leak-off, etc. are
131 derived (Barboza, Chen, & Li, 2021; Isah, Hiba, Al-Azani, Aljawad, & Mahmoud, 2021). Besides, the
132 calculations (Appendix B) for the selected features are analytical, which is more calculational
133 effective to pre-processing our datasets (over 430,000 groups of measurements) compared with
134 numerical solutions. Furthermore, the models in Table 2 are mainly derived from observations of
135 experimental simulations (Appendix B). By evaluating the calculation outputs at field-practical scales,
136 the experimental techniques may be improved in the aspects of equipment, parameters, methodology,
137 measurements, etc.

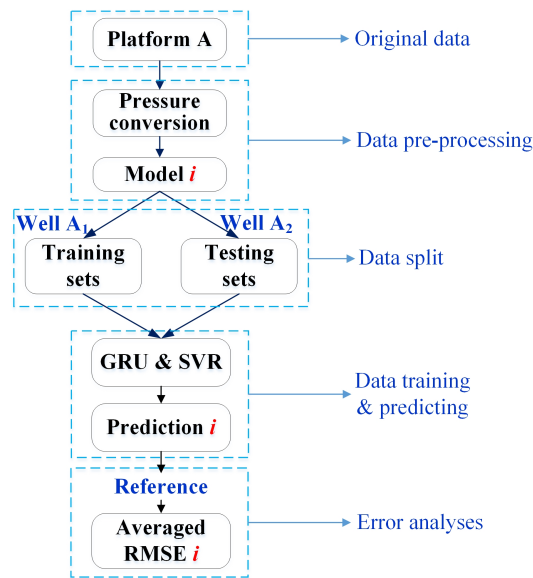
138 During the calculation, the fracture width is the only unknown parameter that is presumably set to
139 a value of $100 \times d_{max}$ (d_{max} is the largest diameter of injected proppant) referring to the result of slant
140 core drilling through a stimulated shale reservoir (Elliott & Gale, 2018). For an alternate pumping
141 schedule (injecting pure fluid and slurry alternatively), the results of the velocity model are discrete
142 and are all treated as zeros as pure fluid is injected. As shown in Table 2, the independent variables
143 and the control variables (field measurements: fluid type – μ , pump rate – Q , proppant concentration –
144 C and proppant type – d) are jointly fed into the ML models for the prediction (dependent variable:
145 downhole pressure after perforation – DPP) and error analyses (the Root Mean Square Error –
146 RMSE). The non-numeric variables (fluid and proppant types) are replaced with the values of fluid
147 viscosity and averaged proppant diameters, respectively. For comparison purposes, the original field
148 measurements alone are directly used to train the ML models to predict the DPP, defined as the
149 unprocessed DPP (Table 2).

150 **2.3 Machine learning models and workflow**

151 To constrain the high variance and boost the prediction performance, two different machine
152 learning algorithms are applied for training and predicting. The Support Vector Regression (SVR)
153 model, with a Radial Basis Function (RBF) kernel, is capable of both linear and non-linear regression
154 (Al-Anazi & Gates, 2010), being of memory efficiency, and performing well in various petroleum
155 engineering applications (Goel, Saurabh, Patil-Shinde, & Tambe, 2017; Guo et al., 2018).
156 Furthermore, we apply Gated Recurrent Units (GRU) to the same datasets. The GRU is a deep
157 learning algorithm designed for extracting information from time-sequence data. In GRU models, the

158 current state and prediction can be influenced by the preceding state and will affect the following
 159 prediction at the next time step as well, making the GRU models appropriate for handling continuous
 160 hydraulic fracturing data (Sun et al., 2020; Jinjiang Wang, Yan, Li, Gao, & Zhao, 2019). According to
 161 previous modelling experience (Cho et al., 2014; D. Fan et al., 2021), a three-layer (including the
 162 output layer) GRU model is constructed with the activation function of ReLU. The dropout (0.2)
 163 layers are applied to avoid the overfitting of the model (Gal & Ghahramani, 2015). The Adam
 164 optimizer is used in the model with a learning rate starting at 0.0005 (Kingma & Ba, 2014).

165 Using Platform A as an example, our workflow for the data processing is shown in Fig. 2. Model i
 166 represents one of the four proppant transport models given in Table 2. The reference is the DPP
 167 converted directly from the surface pressure records, and Prediction i is the predicted DPP based on
 168 Model i . The pressure prediction is made for each platform (A to E), and for each prediction, a new
 169 GRU model and SVR model are created and trained respectively. Eventually, the prediction errors for
 170 each platform are averaged to evaluate the performance of the selected features and the corresponding
 171 models.



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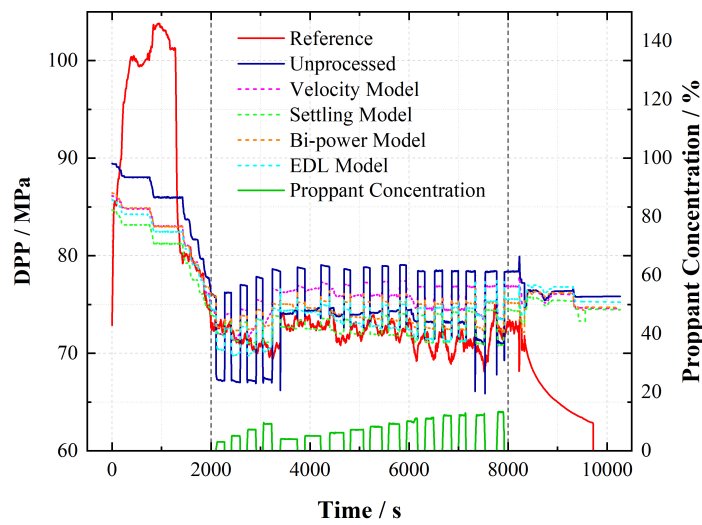
173 **Fig. 2. Schematics of the data processing workflow using Platform A as an example.**

174 **3 Results**

175 **3.1 Evaluation of proppant transport features at engineering scales**

176 The pressure predictions by GRU for Well A₂ at Platform A are plotted in Fig. 3, using as an
 177 example of the evaluation results. In general, the predicted DPP curves corresponding to the proppant

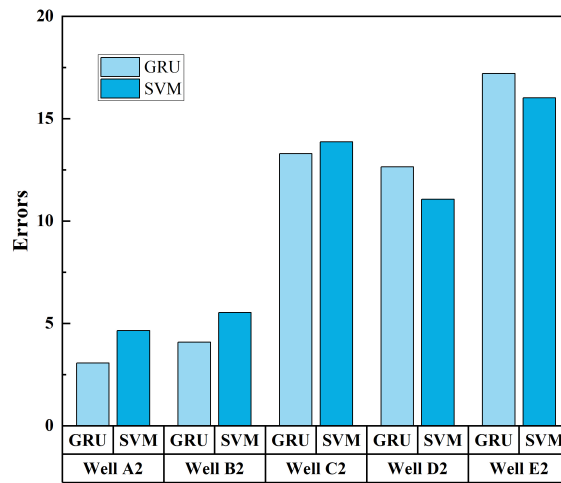
178 transport models match the reference DPP curve much better than the unprocessed curve, indicating
 179 that the introduction of the proppant transport features improves the prediction accuracy and reduces
 180 the variance (Fig. 3). The proppant concentration during the fracturing operation is also presented
 181 (green solid line in Fig. 3), demonstrating that the proppant-injection-induced pressure fluctuations
 182 influence the variation in pressure predominantly. To denoise the pressure variation induced by the
 183 injection, the pressure data ranging between the time of 2000 s (the period of the fracture initiation
 184 and propagation) and the time of 8000 s (the period of the fracture closure and fluid diffusion after
 185 pump-off) are only used for error analyses (the region between two vertical grey dash lines in Fig. 3).



186
 187 **Fig. 3. Comparisons of DPP between the Reference and Predictions based on Platform A using the GRU**
 188 **model. The Reference curve (red solid line) is obtained by pressure conversion. The Unprocessed curve**
 189 **(blue solid line) is the prediction based on original injection parameters, referring to Table 2. The dashed**
 190 **lines are the predictions by GRU models corresponding to different proppant transport models. The**
 191 **green solid line is the proppant concentration.**

192 Four more ML predictions are carried out for Platforms B – E. The errors based on different
 193 proppant transport models are averaged to compare the performances of the GRU and SVM
 194 algorithms, as shown in Fig. 4. Generally, the two algorithms exert close performances according to
 195 the generated errors. The GRU-based workflow produces smaller errors for Wells A₂, B₂ and C₂.
 196 Therefore, the predictions based on the GRU model are selected for further investigations. Besides,
 197 the errors for Wells A₂ and B₂ are significantly smaller than the rest of the cases. We divided the cases

198 into two groups in the later analyses – the small error group (Wells A₂ and B₂) and the large error
 199 group (Wells C₂, D₂, and E₂).



200

201 **Fig. 4 The average errors produced by GRU and SVM algorithms based on Wells A₂ – E₂. Each error**
 202 **bar represents the averaged errors based on different proppant transport models.**

203 The detailed errors between the reference and the ML prediction for each testing well (A₂ – E₂)
 204 based on the GRU algorithm are summarized in Table 3. By comparing the averaged RMSE, we find
 205 that the DPP predictions are enhanced by introducing the Velocity, Settling, and Bi-power models, in
 206 which the stratified-flow feature performs better than the settlement and inception features. The Bi-
 207 power model helps yield the best DPP predictions, followed by the Settling model. The introduction
 208 of the EDL model promotes low RMSEs for Wells A₂, B₂, and C₂, whereas leads to large prediction
 209 errors for Wells D₂ and E₂. The performance of the Velocity model is probably limited by the
 210 simplification employed under the pure-fluid condition. However, exceptions are observed for Wells
 211 D₂ and E₂, where the Velocity model helps to produce smaller errors than the Settling model does.

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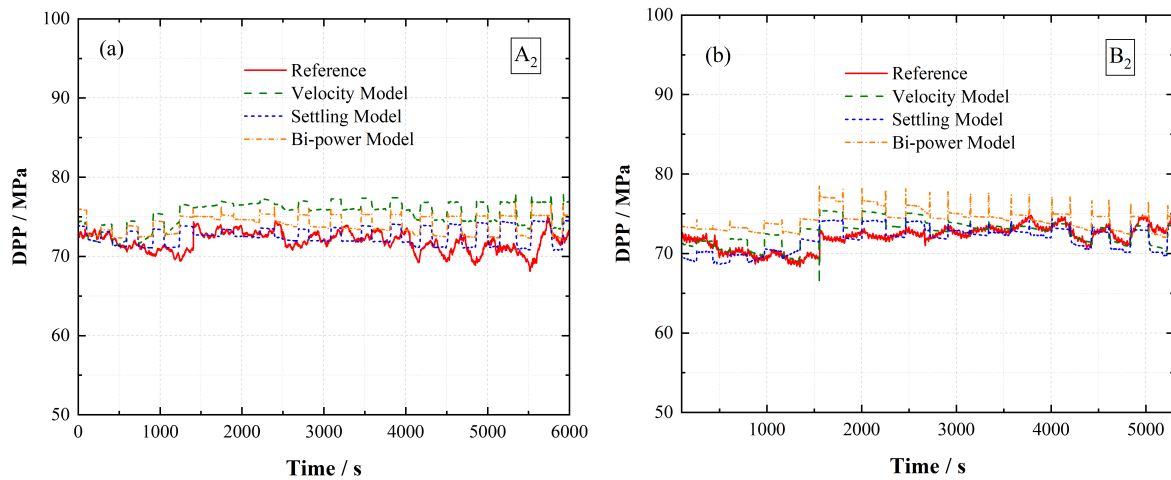
Table 3 Summary of the RMSE based on each proppant transport model.

	Well A ₂	Well B ₂	Well C ₂	Well D ₂	Well E ₂	Averaged RMSE
Algorithms	GRU	GRU	GRU	GRU	GRU	
Unprocessed	4.61	4.39	14.2	8.43	15.92	9.51
Velocity model	4.09	4.08	13.37	6.98	14.32	8.57
Settling model	1.72	4.02	12.71	9.53	14.55	8.51
Bi-power model	2.59	4.25	12.65	8.35	11.9	7.95
EDL model	2.31	3.65	13.53	29.92	29.32	15.75

213 3.2 Error variance in different cases

214 According to the results in Table 3, we also plot the DPP curves predicted by the GRU algorithm
215 for further investigation, as shown in Figs. 5 and 6. Each dash curve corresponds to a different model
216 used to calculate the input features, including the Velocity (particle-level feature), Settling and Bi-
217 power (fracture-level feature) models. As the different calculations are applied for the pure-fluid and
218 slurry injections (Appendix B) when pre-processing the input data, the predicted curves derived from
219 the alternative injection schedule are relatively discrete. The predictions, therefore, fluctuate around
220 the reference at a frequency following the oftenness of the injection switching.

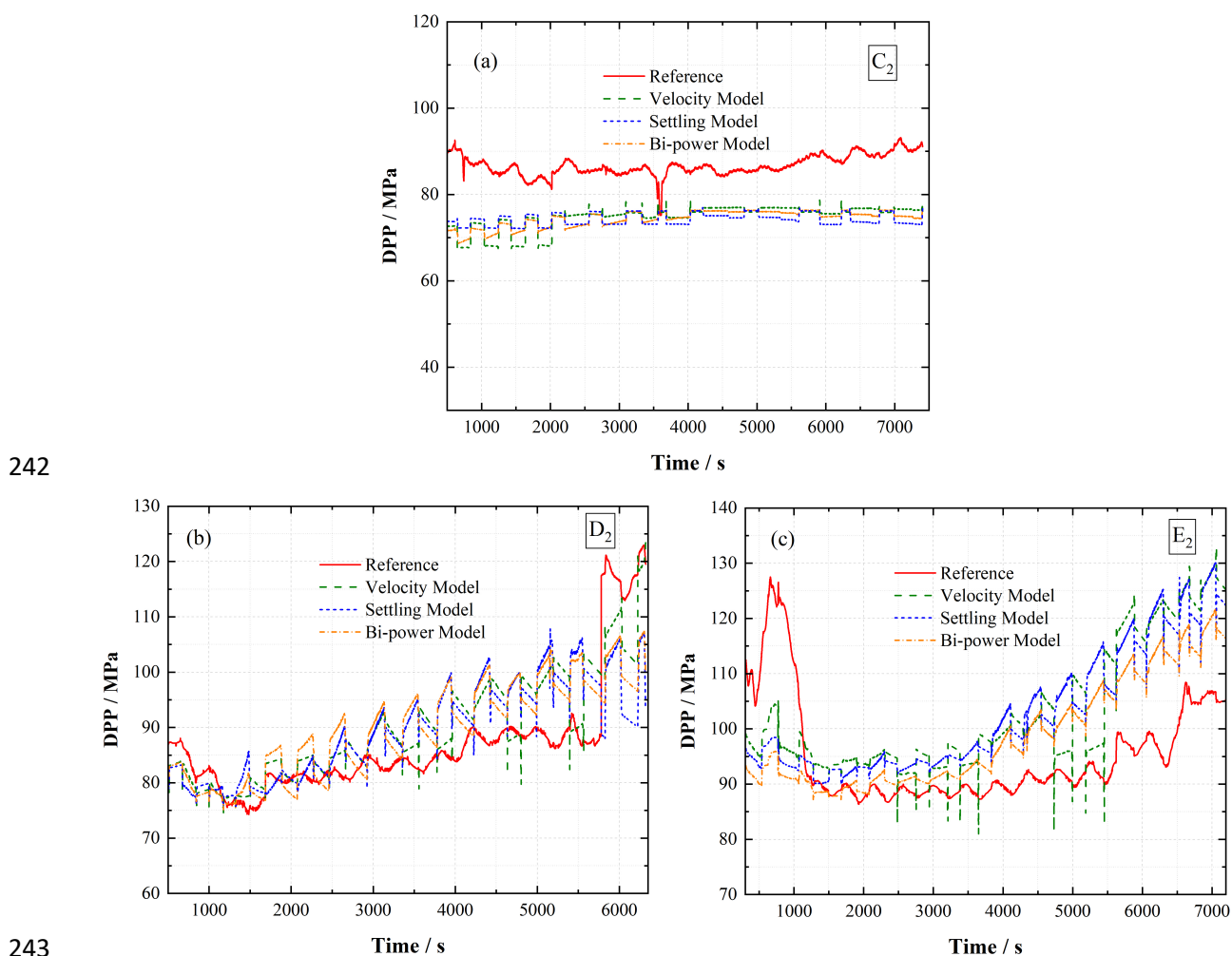
221 For the small error group (Fig. 5), a relatively flat trend of DPP along with a constant pumping rate
222 can be found throughout the injection treatment in Wells A₂ and B₂. The effect of proppant transport
223 on pressure variation is moderate, suggesting that the fracture volume may be sufficient for the
224 current proppant injection rate. Besides, the predicted pressure curve is unsmooth and exerts vertical
225 climbing and jumping between slugs (the alternate from pure fracturing fluid into proppant slurry).



226
227 **Fig. 5. Comparisons of the pressure evolution based on (a) Well A₂ and (b) Well B₂ using the GRU**
228 **algorithm. The Reference curve (red solid line) is obtained by pressure conversion. The dashed lines are**
229 **the proppant-transport-model-based predictions.**

230 In contrast, an ascending pressure trend (red solid curves in Fig. 6) can be found in the large error
231 group (Wells C₂, D₂ and E₂). Compared with the proppant concentrations in Wells A₂ and B₂ (~20%),
232 the proppant concentrations for Well C₂, D₂, and E₂ are all under 10%, indicating that their DPPs are
233 relatively sensitive to the proppant transport. In Well C₂, D₂, and E₂, the proppant particles are likely
234 driven into fractures possessing insufficient volume, where the continuously injected proppant may

235 accumulate, and then block the flowing pass, resulting in a gradual increase in flowing friction,
 236 reflected by the ascending operation pressure. Besides, the performance of the Velocity model is
 237 unexpected and even better than the Settling and Bi-power models, as shown in Figs. 6 (b) and (c).
 238 Integrating the severe fluctuations of fracturing pressure into account, the underground fracture in Fig.
 239 6 may be more complex than that in Fig. 5. The proppant may be transported in fracture networks.
 240 Therefore, the Velocity model, calculating the critical condition of proppant turning from the main
 241 fracture into the minor fracture, produces better predictions.



243
 244
 245 **Fig. 6. Comparisons of the pressure evolution based on (a) well C₂, (b) well D₂ and (c) well E₂ using**
 246 **the GRU algorithm. The Reference curve (red solid line) is obtained by pressure conversion. The dashed**
 247 **lines are the proppant-transport-model-based predictions.**

248 **4 Discussion**

249 Our comparison among pressure predictions derived from different proppant transport features
250 exhibits that the stratified-flow feature (H_I , calculated by the Bi-power and Settling models) can
251 improve the pressure prediction considerably. The unstable predictions produced by the EDL model
252 may be attributed to the limited application range of the empirical equations used in the model
253 (Appendix B). The Velocity model characterizes the particle flowing feature during the slurry
254 injection but fails to take into account the scenario when the pure fluid is injected, which results in a
255 restrained improvement in the pressure prediction.

256 However, relatively large prediction errors exist for cases of Wells C₂, D₂, and E₂ (Fig. 6), which
257 are non-negligible and likely attributed to the following factors:

258 (i) Effect of the injection alternation – The proppant transport models are featured by taking into
259 account the accumulation of the proppant dune being in an equilibrium state. Hence, the prediction
260 curves in Figs 5 and 6 are relatively discrete under an alternate injection schedule. However, the time
261 interval (around 3–5 mins) between the injecting alternation may be insufficient to allow the proppant
262 dune to reach the equilibrium state (Yew & Weng, 2014), thus resulting in the discrete pattern of the
263 predicted curve and vertical pressure variations between slugs. It is likely that the transition state of
264 the proppant dune between two alternating injections influences the pressure substantially, also
265 contributing to the errors for the cases with pressure-ascending trends (Fig. 6). This feature describing
266 the transition state, however, is not reflected by any model we evaluate in this study.

267 (ii) Fracture propagation during proppant injection – The introduction of the velocity feature
268 (describing the critical flow condition that drives the proppant to turn into branching fractures)
269 enhances the prediction performance for Wells D₂ and E₂ (Fig. 6), implying that more complex
270 fracture networks may be generated. The amplitudes of the pressure fluctuations shown in Fig. 6 is
271 broadly larger than those observed from Fig. 5, which may be attributed to the development of
272 branching or minor fractures. The random fracture propagation may cause unexpected pressure
273 variation and thus extra prediction errors (Fig. 6).

274 (iii) Proppant transport in highly-filled fractures – According to the discrepancies between
275 reference and predicted DDP curves, the largest errors emerge at the beginning and end of proppant
276 injections (Fig. 6). Initially, the fracture is underdeveloped with limited volumes. At the end of
277 operations, a large volume of proppant has been injected into the fractures. The similarity of these two
278 conditions is that the fracture is highly filled due to the relative volumes of fractures and proppant.
279 However, few relevant research works can be found during our literature review. The highly-filled-
280 fracture operating condition may be critical for pressure-sensitive cases and the sand screen-out, thus
281 deserving more studies.

282 Therefore, we suggest investigating further (1) the evolution of proppant dune based on a staged
283 pumping schedule, (2) a better assessment of proppant transport in fracture networks and highly-filled
284 fracture.

285 **5 Conclusions**

286 In this study, we propose a machine-learning-based (GRU and SVM) workflow to process field
287 measurements collected from shale gas fracturing to assess the essential proppant transport features
288 and their corresponding calculations at field-practical scales. The new workflow, where the fracturing
289 pressure is processed and predicted to estimate the proppant transport indirectly, paves a path to
290 potentially establish a link between laboratory work and engineering practices. The feature analysis
291 improves the awareness of underground proppant transport in engineering scales, which may provide
292 a complement to numerical and experimental simulations. The main conclusions are generalized as
293 follows:

294 (1) The Bi-power model exerts the best interpretation of proppant stratified-flow feature and
295 predictions of fracturing pressures, followed by the Settling model (stratified-flow feature) and
296 Velocity model (settlement and inception features). The performance of the EDL model is unstable
297 and produces larger errors. The introduction of the essential features enhances the pressure predictions
298 for the cases where relatively flat trends of pressure evolution (under constant pump rates) are present,
299 implying that the proppant is likely injected into a sufficient volume of fracture, comparable to the
300 conditions simulated by the lab research.

301 (2) For the cases, where an ascending trend of pressure is shown throughout the proppant
302 injections, all features and calculations bring in relatively large prediction errors. However, the
303 Velocity model (characterizing the critical flow velocity that drives proppant to turn into branching
304 fractures) helps to yield less prediction error in these cases, indicating that the proppant may be
305 transported into un-fully-developed fracture networks. The underground fracture may be more
306 complex in the pressure-ascending cases according to the more severely fluctuated pressure that may
307 be induced by the generations of branching or minor fractures.

308 (3) The existing errors in the pressure-ascending cases can be improved by enhancing the accuracy
309 of feature calculating models, where the alternate injection schedule and the random propagation of
310 fracture are still missing. Based on the feature tests at field scales, we suggest that the evolution of
311 proppant dune during an alternate pumping schedule may play a critical role in pressure evolutions.
312 Moreover, the proppant transport in fracture networks and highly-filled fractures should be defined
313 more accurately for the pressure-sensitive operations to mitigate the operating risk and improve the
314 proppant injection.

315

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317

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320

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435

Appendix A – Calculations used for pressure conversion

The original wellhead pressure (or surface pressure) is converted into downhole pressure to remove noises of flowing resistance and hydrostatic pressure variation caused by changes of pump rate, fluid type and proppant type and concentration.

$$P_{downhole} = P_{wellhead} + P_{statics} - P_{pipeloss} - P_{perforation} \quad (1)$$

The hydrostatic pressure ($P_{statics}$) is calculated by the vertical well depth (h_v)

$$P_{statics} = \rho_s g h_v \quad (2)$$

The friction loss of the wellbore ($P_{pipeloss}$) is estimated by the Darcy-Weisbach equation

$$P_{pipeloss} = 2f \frac{\rho_s v_s^2 L}{h_v} \quad f = 0.046 \left(\frac{\rho_s v_s h_v}{\mu_s} \right)^{-0.2} \quad (3)$$

where L is the wellbore length from its wellhead to the fracturing stage, m; v_s is the flowing rate of slurry in the wellbore, m/s; μ_s is the slurry viscosity, Pa·s. The slurry viscosity is calculated by (Dontsov & Peirce, 2014)

$$\mu_s = \mu_f \left[\frac{5}{2} C_m A^{-1} + \left(0.32 + \frac{0.38}{1 + 5 \times 10^{-5} A^{-2}} \right) A^{-2} \right] \quad A = \frac{C_m}{C} - 1 \quad (4)$$

where C_m is the maximum proppant concentration and is assigned a value of 0.585.

The pressure drop through the perforation hole is estimated based on the hydraulic and perforation parameters (Willingham, Tan, & Norman, 1993)

$$P_{perforation} = \frac{2.233 \times 10^{-4} Q^2 \rho_s}{n^2 d_h^4 C_p^2} \quad (5)$$

where d_h is the diameter of the perforation hole, m; C_p is the coefficient of discharge and is 0.6–0.95 for slurry; n is the number of the opening perforation hole. According to the mini-fracturing test, around half of the designed perforation holes will be opened.

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466

467

Appendix B – Summary of feature calculations

468

469 1 Velocity model

470 The slickwater, widely used for massive hydraulic fracturing, could only suspend the proppant for
471 minutes during the fracturing operation (Yew & Weng, 2014). Therefore, the proppant settling
472 velocity, one of the most fundamental issues, is given by (Mack, Sun, & Khadilkar, 2014; McCabe,
473 Smith, & Harriott, 1993)

$$474 \quad v_{\text{settling}} = \left[\frac{0.072g(\rho_p - \rho_f)d^{1.6}}{\rho_f^{0.4}\mu_f^{0.6}} \right]^{0.71} \quad (1)$$

475 where v_{settling} is the proppant settling velocity, m/s; ρ_p and ρ_f are densities of proppant and
476 fracturing fluid, respectively, kg/m³; μ_f is the fluid viscosity, Pa·s; d is the averaged diameter of
477 proppant, m.

478 Settling in a fracture, proppant is slowed down by fracture walls and interactions between
479 particles, which can be modified by (Gadde, Liu, Norman, Bonnecaze, & Sharma, 2004; Richardson
480 & Zaki, 1954)

$$481 \quad \begin{aligned} v_{\text{settling}}^{\cdot} &= v_{\text{settling}} (2.37C^2 - 3.08C + 1) \\ v_{\text{settling}}^* &= v_{\text{settling}} \left[0.563\left(\frac{d}{w}\right)^2 - 1.563\frac{d}{w} + 1 \right] \end{aligned} \quad (2)$$

482 where C is the volume fraction of proppant; w is the fracture width, m.

483 For a complex fracture network, the dragging of the carrying fluid is one of the most important
484 motivations to drive the proppant turn into branch fractures (Sahai, Miskimins, & Olson, 2014). A
485 minimum flowing rate of the slurry (v_{turning}) is required for proppant turning, and is estimated by the
486 proppant restarting condition (Cao, Pender, & Meng, 2006; Hou, Jiang, Li, Zeng, & Cheng, 2017)

$$487 \quad v_{\text{turning}} = \frac{0.05(\rho_p - \rho_f)gdw}{8\mu_f} \quad (3)$$

488 The particle movements could reflect the proppant transporting by evaluating the ratio between
489 particle and fluid velocities (Hou, Jiang, Liu, et al., 2017). The velocity model is defined as

$$490 \quad \begin{cases} v_{\text{settling}} / v_f \\ v_{\text{turning}} / v_f \end{cases} \quad (4)$$

491 **2 Settling model**

492 The proppant is tending to form an equilibrium dune in low-viscosity fluids under constant
 493 injection conditions (Hou et al., 2019), as shown in Fig. 1 (b). The height of the flowing layer above
 494 the dune is a core parameter that evaluates the proppant transport, and could be estimated by the
 495 settling model expressed as (Novotny, 1977)

$$496 \quad H_1 = \frac{16.67Q}{wv_{eq}} \quad (5)$$

497 where H_1 is the height of the flowing layer, m; Q is the pump rate, m³/s; v_{eq} is the flowing rate
 498 when the particle settling and restarting reach equilibrium, and is calculated by

$$499 \quad \begin{cases} v_{eq} = \left(\frac{(v_w)_{eq}}{0.2}\right)^{0.143} \frac{(2w\rho_f/\mu_f)^{0.143}}{(\rho_f/\rho_{SC})^{0.571}} \\ (v_w)_{eq} = 18.5v_{Settling} \left(\frac{v_{Settling} d \rho_f}{\mu_f} \sqrt{\frac{2w}{d}}\right)^{-0.5} \\ \rho_s = \frac{\rho_f + \rho_p C(1-\phi)}{1 + C(1-\phi)} \end{cases} \quad (6)$$

500 where ρ_s is the density of the slurry, m³/s; ϕ is the porosity of the proppant dune.

501 **3 Bi-power model**

502 The Bi-power law correlations are proposed to directly calculate the height of the flowing layer
 503 (Wang, Joseph, Patankar, Conway, & Barree, 2003), which is defined as

$$504 \quad \frac{H_1}{w} = [-0.00023 \ln(R_G) + 0.00292] R_f^{1.2-0.00126\lambda^{-0.428}[15.2-\ln(R_G)]} R_p^{[-0.0172 \ln(R_G)-0.12]} \quad (7)$$

505 where R_f , R_p , R_G and λ are calculated by

$$506 \quad \begin{cases} R_f = \frac{\rho_f Q_f}{w\mu_f} & R_p = \frac{\rho_p Q_p}{w\mu_f} \\ R_G = \frac{\rho_f(\rho_p - \rho_f)gd^3}{\mu_f^2} & \lambda = \frac{\mu_f / \rho_f}{w^{1.5} \sqrt{g}} \end{cases} \quad (8)$$

507 where Q_f is the pump rate of fracturing fluid, m³/s; Q_p is the pump rate of proppant, m³/s.

508 There is a special condition when pure fluid is injected to push the injected proppant deeply into
 509 the fracture. The pure fluid may rebalance the proppant dune, which could be calculated by (Patankar
 510 et al., 2002)

511
$$H_1 = \left(\frac{Q_f \rho_f w^{0.0937}}{2053.4 \mu_f} \right)^{\frac{1}{1.0937}} \quad (9)$$

512 **4 EDL model**

513 A similar empirical power-law formula for dune height has been derived based on a series of sand-
 514 carrying experiments (Alotaibi & Miskimins, 2019). The equilibrium dune level (*EDL*) is defined as
 515 the dune height divided by fracture height, which is proposed as

516
$$EDL = -0.003496d^{-0.3277} \left[\frac{v_{eq}^{(0.7749d^{-0.1955})}}{C^{(0.6684d+0.04398)}} \right] + 0.9901d^{-0.02667} \quad (10)$$

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518

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