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GANSim-3D for conditional geomodelling: theory and field application

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Abstract

Geomodelling of subsurface reservoirs is important for water resources, hydrocarbon exploitation, and Carbon Capture and Storage (CCS). Traditional geostatistics-based approaches cannot abstract complex geological patterns and are thus not able to simulate very realistic earth models. We present a Generative Adversarial Networks (GANs)-based 3D reservoir simulation framework, GANSim-3D, which can capture geological patterns and relationships between various conditioning data and earth models and is thus able to directly simulate multiple 3D realistic and conditional earth models of arbitrary sizes from given conditioning data. In GANSim-3D, the generator, designed to only include 3D convolutional layers, takes various 3D conditioning data and 3D random latent cubes (composed of random numbers) as inputs and produces a 3D earth model. Two types of losses, the original GANs loss and condition-based loss, are designed to train the generator progressively from shallow to deep layers to learn the geological patterns and relationships from coarse to fine resolutions. Conditioning data can include 3D sparse well facies data, 3D low-resolution probability maps, and global features like facies proportion, channel width, etc. Once trained on a training dataset where each training sample is a 3D cube of a small fixed size, the generator can be used for geomodelling of 3D reservoirs of large arbitrary sizes by directly extending the sizes of all inputs and the output of the generator proportionally. To illustrate how GANSim-3D is used for field geomodelling and also to verify GANSim-3D, a field karst cave reservoir in Tahe area of China is used as an example. The 3D well facies data and 3D probability map of caves obtained from geophysical interpretation are used as conditioning data. First, we create a training dataset consisting of facies models of $64 \times 64 \times 64$ cells with a process-mimicking simulation method to integrate field geological patterns. The training well facies data and the training probability map data are produced from the training facies models. Then, the 3D generator is successfully trained and evaluated in two synthetic cases with various metrics. Next, we apply the pretrained generator for conditional geomodelling of two field cave reservoirs of Tahe area. The first reservoir is $800\text{m} \times 800\text{m} \times 64\text{m}$ and is divided into $64 \times 64 \times 64$ cells, while the second is $4200\text{m} \times 3200\text{m} \times 96\text{m}$ and is divided into $336 \times 256 \times 96$ cells. We fix the input well facies data and cave probability maps and randomly change the input latent cubes to allow the generator to produce multiple diverse cave reservoir realizations, which prove to be consistent with the geological patterns of real Tahe cave reservoir as well as the input conditioning data. The noise in the input probability map is suppressed by the generator. Once trained, the geomodelling process is quite fast: each realization with $336 \times 256 \times 96$ cells takes 0.988 seconds using 1 GPU (V100). This study shows that GANSim-3D is robust for fast 3D conditional geomodelling of field reservoirs of arbitrary sizes.

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Key Points:

- GANSim-3D produces multiple 3D realistic conditional reservoir models of arbitrary sizes from given conditioning data.
- Application in field karst cave reservoirs shows robust performance of GANSim-3D.
- Geomodelling with GANSim-3D is quite fast: <1s for one realization with $336 \times 256 \times 96$ cells.

60

61 **1 Introduction**

62 Geomodelling, or quantitatively predicting the distribution of subsurface reservoirs, is of
63 great significance for evaluation and exploitation of underground water and energy resources as
64 well as for geological sequestration of CO₂ (CCS). Generally, various types of information (data)
65 about the subsurface are incorporated using geostatistical approaches for geomodelling. Such
66 information includes sparse well data, geophysical data, global features (e.g., facies proportion),
67 and spatial geological patterns, among which the geological patterns may be the most difficult to
68 incorporate. In traditional geostatistics-based geomodelling approaches, geological patterns can
69 be partially expressed by simple variogram functions (e.g., in Sequential Indicator Simulation
70 method; Pyrcz & Deutsch, 2014) or local multiple points statistics (MPS) (in MPS-based
71 approaches; Mariethoz & Caers, 2014). Such partial representations may not be able to
72 completely express the complicated spatial geological patterns, thus there is a lack of realism
73 (expected geological patterns) to different extents in the simulated results of these approaches,
74 e.g., facies models produced by variogram-based approaches cannot exhibit sinuous channel-like
75 shapes, while MPS-based approaches may produce discontinuous channels. In addition, due to
76 the incompleteness and imperfectness of the incorporated information (e.g., the sparse nature of
77 well data and low-resolution nature of geophysical data), uncertainty exists in geomodelling
78 results, so a number of reservoir realizations are generally produced to represent the potential
79 distribution of subsurface reservoirs.

80 Generative Adversarial Networks (GANs) (Goodfellow et al., 2014) in deep learning are
81 good at capturing complete spatial patterns (structures) of 2D images or 3D objects using a
82 generator Convolutional Neural Network (CNN), with the help of another discriminator CNN.
83 With the captured pattern knowledge, the generator CNN can map a random latent vector into a
84 realistic image or a realistic 3D object (e.g., Wu et al., 2016). Supporting Information S1 gives
85 more detail about the basic methodology of GANs.

86 In recent years, GANs have been combined with geomodelling on different aspects (e.g.,
87 Chan & Elsheikh, 2017, 2019; Dupont et al., 2018; Laloy et al., 2018; Mosser et al., 2020; Zheng
88 & Zhang, 2022; Nesvold & Mukerji, 2021; Song et al., 2021a, 2021b, 2022; Zhang et al., 2019).
89 Like any other approach, unconditional and conditional geomodelling are considered. For
90 unconditional GANs-based geomodelling, the generator CNN is first trained to abstract spatial
91 geological patterns from training facies models; then it uses the learned pattern knowledge to
92 quickly produce multiple reservoir realizations consistent with the pattern from random latent
93 vectors. Song et al. (2021a) proposed to use a progressive training approach for GANs in
94 geomodelling, going incrementally from coarse resolution to finer and finer resolutions and
95 proved that it performs better than the conventional alternative of training where the finest
96 resolution is directly produced.

97 There are two approaches to achieve conditioning in GANs-based geomodelling. Note
98 that the trained generator in the unconditional case can map a random latent vector into any
99 realistic facies model, so the key of the first conditioning approach is to search for appropriate
100 input latent vectors, with which the trained generator can produce facies models consistent with
101 expected geological patterns as well as given conditioning data. Laloy et al. (2018), Mosser et al.
102 (2020), and Nesvold & Mukerji (2021) used Markov Chain Monte Carlo optimization algorithms
103 (MCMC) to search for such appropriate latent vectors, while Dupont et al. (2018) and Zhang et
104 al. (2019) used gradient descent optimization algorithms. With these optimization-based
105 algorithms, only one optimal latent vector is found every time, and thus many optimization runs

106 are needed to produce multiple latent vectors and their corresponding multiple facies models for
107 uncertainty assessment. Chan & Elsheikh (2019) proposed to train an extra neural network
108 between a predefined distribution (e.g., Gaussian distribution) and all appropriate latent vectors
109 for given conditioning data. In this way, multiple conditional facies models can be obtained all at
110 once. However, the extra neural network is unique for the given conditioning data, and once the
111 conditioning data changes the neural network has to be trained again.

112 Song et al. (2021b, 2022) proposed another direct conditioning approach, called
113 GANSim, where the generator directly takes conditioning data and random latent vectors as
114 inputs to produce multiple conditional earth models of reservoir. The proposal of GANSim is
115 inspired by the process of experts drawing conditional geological maps by hand, where experts
116 use the geological pattern knowledge and relationship knowledge between conditioning data and
117 geological maps to finish realistic maps consistent with both expected patterns and given
118 conditioning data. The trained generator in the unconditional case only learns the pattern
119 knowledge; in GANSim, a condition-based loss function is further introduced to force the
120 generator also to learn the relationships between various conditioning data and earth models.
121 These conditioning data can include global features (e.g., facies proportion, channel sinuosity,
122 and channel width), local well facies interpretations, and probability maps of geobodies produced
123 from geophysical interpretations. With the learned geological pattern knowledge and the
124 relationship, the trained generator can directly map observed conditioning data into multiple
125 realistic conditional earth models. Compared to the previous approach, GANSim does not
126 involve optimization or extra neural network training process, and thus is more convenient and
127 faster.

128 However, there are still several aspects that need to be further improved in the current
129 GANSim research presented by Song et al. (2021b, 2022). First, the current GANSim algorithm
130 is presented in a 2D framework, where the input well data, input probability map, and the output
131 reservoir models are all set as 2D (i.e., planner sections of real 3D reservoir) and the CNNs of
132 generator and discriminator also use 2D convolutional kernels. But real subsurface reservoirs are
133 3D. How to extend GANSim into 3D space? Second, in the current GANSim framework, the
134 trained generator can only produce earth models with a fixed size equal to that on which it was
135 originally trained, e.g, if the training facies models have 64×64 cells each representing
136 $10\text{m} \times 10\text{m}$, then the trained generator also produces facies models of 64×64 cells. However, the
137 field reservoir to be predicted may be at large arbitrary sizes, different from the size of the
138 training models. Is it possible to use the generator trained on small-size facies models for
139 geomodelling of field reservoirs of large sizes? Third, Song et al. (2021b, 2022) only use
140 synthetic cases to validate the proposed GANSim. Given much more sophisticated geological
141 patterns of field reservoirs than synthetic patterns, field reservoirs are needed to verify the
142 effectiveness and efficiency of GANSim.

143 Therefore, in this study, we propose a GANSim-3D framework to address the
144 aforementioned concerns based on the GANSim research presented by Song et al. (2021b, 2022).
145 In GANSim-3D, the convolutional kernels of the generator and discriminator, the input
146 conditioning data, and the output earth models are all designed to be 3D, and the architecture
147 design of the generator is specially improved so that it can produce earth models of flexible sizes
148 after being trained on samples of small fixed size. A 3D field karst cave reservoir in Tahe area of
149 China, which proved to be very difficult for geomodelling after many years' research (Liu et al.,

150 2012; Lu et al., 2012; Li et al., 2016(a); Li et al., 2016(b)), is taken as an example to validate the
151 proposed GANSim-3D framework and showcase the field application of GANSim-3D.

152 Tahe area is located at Akekule uplift of northern Tarim Basin in western China and has
153 an area of around 60km×30km. The major reservoir is Ordovician carbonate karst caves about
154 5500 meters beneath the surface, including connected ribbon-like caves formed by paleo-
155 underground rivers, isolated caves formed by intermittent surficial water, and fault-controlled
156 caves controlled by strike slip faults and related dissolutions (Lu et al., 2020). These caves were
157 formed when the Ordovician carbonate rock with extensive faults and fractures produced in
158 earlier geomechanical stages was exposed to weathering during the middle Caledonian and
159 Hercynian periods. The Ordovician carbonate rock may experience multiple episodes of
160 weathering in some parts, where in each episode the carbonate rock is uplifted, exposed to
161 weathering, and then subsided, producing multiple layers of caves that are vertically stacked. The
162 unconformity surface (of the last weathering episode) under which the caves are formed is
163 designated as the paleo-geographic surface in this paper. In addition to the vertical multi-layer
164 feature, Tahe caves also have a very strong heterogeneity and complicated spatial patterns in
165 planar view. The diameter of the caves spans from tens of centimeters to tens of meters. Caves
166 tend to develop in the vicinity of different levels of faults, especially where multiple faults
167 intersect. There are three extension directions for the underground river caves: NNE, NNW, and
168 nearly EW. Sparsely distributed hall caves with size several times larger than normal caves and
169 cave branches with different length randomly develop in the underground river cave systems.
170 Section 3 shows more detail about the spatial structure of the underground river cave systems.

171 These caves can be fully or partially filled by clastic or chemical cement, or totally void
172 without any fill. Compared to the surrounding tight carbonate rock with porosity of less than 1%,
173 these caves, even when completely filled (porosity can be >20%), are good reservoirs for water,
174 hydrocarbons, and CO₂. During last decades, over hundreds of studies about Tahe karst cave
175 reservoir have been reported ranging from structural geology, karstification, reservoir
176 characterization, geomodelling, etc. Xu et al., (2021) present a review of these progresses.

177 Because of large fluid storage potential of Tahe cave reservoir, abundant geological data
178 and consequent analyses have been obtained, including over 3000 wells, 3D seismic data,
179 outcrop measurements, geomechanical simulation results, fault and fractures interpretation, and
180 karstification history analyses. Although the wealth of data provides a solid foundation for
181 geomodelling of the karst cave reservoir, the sophisticated geological patterns of the cave
182 reservoir and the challenges in expressing complex geological patterns using traditional
183 geostatistics-based geomodelling approaches make it difficult to produce cave reservoir models
184 consistent with expected spatial patterns. For example, Liu et al. (2012) and Lu et al. (2012)
185 utilized variogram-based approaches to simulate cave models, but the simulated caves are
186 distributed in large pieces and cannot show specific shapes. Li et al. (2016a) and Li et al. (2016b)
187 used MPS-based approaches to produce underground river caves of Tahe area; the simulated
188 caves can present a ribbon-like shape to some extent but the continuity and variability still need
189 to be improved. In this paper, we only consider the underground river cave reservoir type
190 (excluding the isolated and fault-controlled cave reservoir types) as an example for geomodelling
191 with the proposed GANSim-3D.

192 The structure of this paper is organized as follows. Section 2 describes GANSim-3D
193 geomodelling framework and specifies its designs for geomodelling of Tahe cave reservoir.
194 Section 3 constructs conceptual models of Tahe cave reservoirs by integrating field geological

195 patterns. Then in Section 4, based on the conceptual models, a training dataset (training facies
196 models, training well data, and training probability maps) is built. In Section 5, we use the
197 training dataset to train a generator based on the GANSim-3D design and evaluate the
198 generator's ability for geomodelling. Next, in Section 6 the pretrained generator is used for
199 practical uncertainty geomodelling of two field reservoirs with different sizes in the Tahe area.
200 Finally, Section 7 gives conclusions of this study.

201 **2. GANSim-3D and its designs for geomodelling of Tahe cave reservoir**

202 **2.1 GANSim-3D**

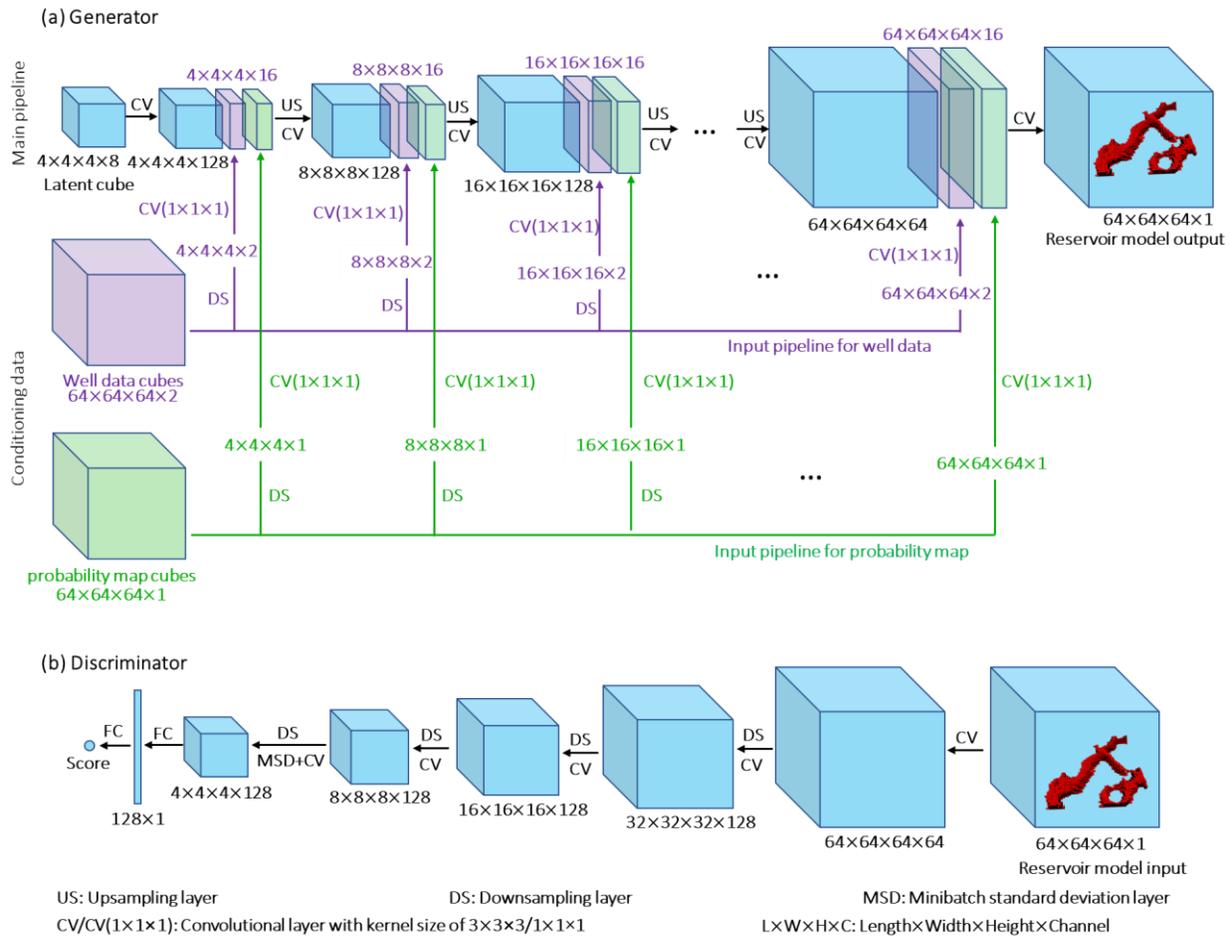
203 GANSim presented by Song et al. (2021b, 2022) can be used to produce 2D reservoir
204 models of the same size as the training data, in its original form. Supporting Information S2
205 gives detail about the basic theory of GANSim. Since real reservoirs are 3D and of arbitrary
206 sizes, in this study a 3D framework, GANSim-3D, is proposed to address the concerns of 2D and
207 fixed-size geomodelling based on the previous GANSim.

208 As illustrated in Figure S2, in the generator of GANSim, the concatenation of a random
209 latent vector and a vector of global features is mapped into a feature vector with a fully
210 connected neural network layer, and the following reshape layer further converts the feature
211 vector into multiple channels of 2D feature maps. However, in GANSim-3D, the input latent
212 vector and global feature vector are transformed into 3D concatenated cubes (i.e., random latent
213 cubes and global feature cubes), and the original fully-connected and reshape layers are replaced
214 by 3D convolutional layers. Now there are only convolutional layers in the generator which
215 enables the geomodelling of flexible sizes, which will be explained in detail in latter paragraphs.
216 In addition, to produce 3D reservoir models and condition to 3D observable data like well data
217 and probability maps resulted from 3D geophysical data, all CNN layers of the generator and the
218 discriminator are set to be 3D, i.e., the kernels of the CNN layers are transformed from 2D into
219 3D.

220 Figure 1 shows an example of the architecture design of the generator and discriminator
221 of GANSim-3D, which will be used for field geomodelling of Tahe karst cave reservoirs in
222 following sections. In this example, only well data and probability map are used as conditioning
223 data (i.e., excluding global features). There are two types of pipelines in the generator: main
224 pipeline as the backbone and pipelines for the two types of conditioning data. The main pipeline
225 of the generator is composed of 6 3D $3 \times 3 \times 3$ convolutional layers (i.e., the kernel size is $3 \times 3 \times 3$)
226 and 4 upsampling layers. The input is 8 channels of $4 \times 4 \times 4$ -latent cubes each containing 64
227 latent variables sampled from a standard Gaussian distribution, while the output is a $64 \times 64 \times 64$ -
228 karst cave facies model. The input conditioning data include $64 \times 64 \times 64$ -well facies data (a well
229 trajectory indicator and a karst cave facies indicator) and one $64 \times 64 \times 64$ -probability map of
230 karst cave. They are taken into the main pipeline of the generator through parallel input pipelines
231 of conditioning data that are composed of downsampling layers and $1 \times 1 \times 1$ convolutional layers
232 (i.e., the kernel size is $1 \times 1 \times 1$). All feature cubes converted from the input conditioning data are
233 set to have 16 channels (e.g., the feature cube with size of $4 \times 4 \times 4 \times 16$ or $8 \times 8 \times 8 \times 16$). The
234 discriminator is basically symmetrical to the main pipeline of the generator, except that two fully
235 connected layers and one minibatch standard deviation layer are included. All $3 \times 3 \times 3$ -size
236 convolutional kernels (in the main pipeline of the generator and the discriminator) have a stride
237 size of $1 \times 1 \times 1$ and are padded with zeros. The leaky rectified linear unit function (LReLU) with

238 a leaky value of 0.2 is used as the activation function in all layers of the generator and the
 239 discriminator except the last layer of the discriminator for which no activation function is used.

240



241

242 **Figure 1** Architectures of the generator and the discriminator used for geomodelling of 3D Tahe
 243 cave reservoirs. Only one data cube is shown in this figure; e.g., $4 \times 4 \times 4 \times 8$ represents 8 channels
 244 of data cube of size $4 \times 4 \times 4$, but only one such data cube is drawn. The generator includes one
 245 main pipeline and two pipelines of well data and probability map conditioning data. The input of
 246 the main pipeline includes 8 channels of $4 \times 4 \times 4$ -latent cubes, and the output is a $64 \times 64 \times 64$ -size
 247 karst cave facies model. The input well facies conditioning data include one well trajectory
 248 indicator and one cave facies indicator both of size $64 \times 64 \times 64$. The size of the input cave
 249 probability map is also $64 \times 64 \times 64$.

250

251 With only 3D CNN layers, the generator can be regarded as an amplifier that amplifies
 252 small-size input latent cubes and global feature cubes into a large-size facies model by an
 253 amplification factor, conditioned to input probability map and well facies data with the same size
 254 as the output facies model. Thus, we may first train the generator on a training dataset consisting
 255 of small-size data cubes, and then use the trained generator for geomodelling of reservoirs with
 256 large arbitrary sizes by expanding the inputs of the generator as long as the quantitative

257 relationship of the inputs sizes remains constant. For example, once trained on 64×64×64-size
 258 training data, the generator of Figure 1 can take in 20×15×5-size latent cubes and 320×240×80-
 259 size well data and probability maps to produce a 320×240×80-size cave facies model, where the
 260 size of the well data, the probability maps, and the output cave facies model is 16 times that of
 261 the latent cubes, which is the same amplification factor as the original architecture in Figure 1.
 262 This feature is very important in geosciences because, first, it is impossible to train a deep
 263 learning model to produce earth models with very large size due to the limitation of training data
 264 size and computation resources; second, practical areas of interest can be of many possible sizes.
 265 Such technique was initially proposed by Jetchev et al. (2016) and has been used in many
 266 geosciences researches (e.g. Laloy et al., 2018; Zheng & Zhang, 2022). We will use this
 267 technique for geomodelling of large-size reservoirs in Section 6.2.

268 The total loss function of GANSim-3D is constructed as the weighted sum of the original
 269 adversarial GANs loss, condition-based losses for well data, probability data, and global features:

$$270 \quad L(G, D)_{total} = \beta_1 L(G, D) + \beta_2 L(G)_w + \beta_3 L(G)_p + \beta_4 L(G)_g, \quad (1)$$

271 where, $L(G, D)_{total}$, $L(G, D)$, $L(G)_w$, $L(G)_p$, and $L(G)_g$ are the total loss, original GANs loss,
 272 and condition-based losses for the input well data, probability maps, and global features, while
 273 β_1 , β_2 , β_3 , and β_4 are predefined weights. These two types of losses, GANs loss and condition-
 274 based losses, are to enforce the generator to learn the geological pattern knowledge and the
 275 correct relationships between various input conditioning data and the output facies model. The
 276 Wasserstein loss function with gradient penalty (Gulrajani et al., 2017) is used as the original
 277 GANs loss here. For details of these condition-based losses, readers can see Supporting
 278 Information S2 or Song et al. (2021b, 2022). To better tune the weights, the four losses at the
 279 right-hand side of the equation are normalized into standard Gaussian distributions. During
 280 training, the discriminator and the generator are alternatively trained. When training the
 281 generator, $L(G, D)_{total}$ is minimized, i.e., the original GANs loss and the other three condition-
 282 based losses are all minimized; when training the discriminator, $L(G, D)_{total}$ is maximized, i.e.,
 283 only the original GANs loss is maximized.

285 In addition, a progressive training method (Karras et al., 2017) is applied, where the
 286 generator and the discriminator are progressively trained from shallow to deep neural network
 287 layers and the geological patterns and the relationships are learned from coarse to fine scales by
 288 the generator.

289 After training, when we fix the input conditioning data and randomly change the input
 290 latent cubes, the generator can produce multiple 3D facies model realizations that are both
 291 realistic and consistent with the input conditioning data. These conditional realistic earth models
 292 constitute the uncertainty space of the subsurface reservoir.

293 2.2 GANSim-3D design for geomodelling of Tahe cave reservoir

294 The proposed GANSim-3D is specified for uncertainty geomodelling of 3D field cave
 295 reservoirs in Tahe area. The architectures of the generator and the discriminator are designed as
 296 in Figure 1. Since global features are not used as conditioning data here, their corresponding
 297 condition-based loss is not included in the total loss function (Equation (1)). Three types of
 298 training data are required: 3D karst cave training facies models of size 64×64×64, 3D training
 299 well facies data of size 64×64×64, and 3D training probability map of karst cave of size

300 $64 \times 64 \times 64$. The training facies model dataset is the most important because they need to
301 incorporate the field geological patterns of the underground river cave reservoir of Tahe area. In
302 the next two sections, we first construct large conceptual models with size $655 \times 655 \times 64$ for the
303 field cave reservoirs by integrating the field geological patterns; then we crop the conceptual
304 models into the required size of $64 \times 64 \times 64$ as the training facies models; finally, the training
305 well and probability map data of the same size are obtained from the training facies models.

306 To speed up the training process, minibatch gradient descent and the Adam optimizer
307 with default parameters (Kingma & Ba, 2014) are used. The generator and discriminator are
308 alternatively trained both with a single minibatch. We use 4 GPUs (NVIDIA Tesla V100-PCIE-
309 32GB), 20 CPUs, and 160G RAM in parallel for training.

310 The layers of the generator and the discriminator are trained from shallow to deep
311 progressively. Each minibatch is set to include 32 training facies models when training the front
312 4 convolutional layers; when the last two convolutional layers are activated (i.e. when the
313 $64 \times 64 \times 64$ -size feature cubes in Figure 1 are produced), each minibatch is set to only include 16
314 training facies models to save GPU memory. The training schedule includes 5,000 training
315 iterations when only the first convolutional layer is activated, 20,000 training iterations after the
316 second and third convolutional layers are activated, 30,000 iterations when the fourth
317 convolutional layer is activated, and unlimited number of iterations after the following layers are
318 activated. The training stops when the generated cave facies models are realistic (i.e., consistent
319 with expected geological patterns), diverse, and consistent with the input conditioning data.

320 **3. Construction of conceptual models of cave reservoir of Tahe area**

321 To build the training facies model dataset of $64 \times 64 \times 64$ cells, large-size conceptual facies
322 models with $655 \times 655 \times 64$ cells of the underground river cave of Tahe area are built in this part.
323 First, we propose a process-mimicking simulation approach for underground river caves; second,
324 related parameters of Tahe cave reservoir are prepared; then, these parameters are taken into the
325 process-mimicking approach to simulate a number of conceptual models of Tahe cave reservoir;
326 finally, the geological patterns inside these conceptual models are verified. These large-size
327 conceptual models will then be cropped into training facies models of small size in next section.

328 **3.1 Process-mimicking simulation approach**

329 Conceptual models can be constructed using object-based method (e.g. Mosser et al.,
330 2020; Zhang et al., 2019), process-based method, or process-mimicking method (e.g. Pyrcz &
331 Deutsch, 2014). Process-based methods produce realistic earth models but are too expensive for
332 building a large number of 3D earth models. For example, the process-based model of Li et al.
333 (2020) takes 1-5 days to simulate a single 3D karst model of less than 0.1 million grids using a
334 hydrochemical simulation approach.

335 Researchers (e.g. Audra et al., 2010; Billi et al., 2007; Boersma et al., 2019; Klimchouk,
336 2009; Klimchouk et al., 2016) have provided insights about the formation mechanism of
337 underground river karst caves. This involves fluid flow through subsurface fracture systems
338 (occasionally including faults) eroding and scouring the surrounding rocks, often involving
339 chemical dissolution, and finally forming ribbon-like karst caves along the fracture system.
340 Following the formation mechanism, we propose a process-mimicking approach to simulate
341 underground river karst caves (see Figure 2) with the following steps.

342 (1) Select a cave simulation area. Considering all fracture sets in the area, obtain the
 343 fracture density map, the probability density function (pdf) of fracture strike, and the pdf of
 344 fracture length for each fracture set. In addition, obtain the vertical profile of the underground
 345 river cave to be simulated and the pdfs of cave width and height.

346 (2) Inside the cave simulation area, randomly select a fracture set and the center of initial
 347 fracture according to the fracture density maps.

348 (3) Based on the pdfs of strike and length of the fracture set, randomly sample the strike
 349 and length for the initial fracture. Then construct the initial fracture.

350 (4) Inside the current simulated fracture, randomly select an intersection point between
 351 current and the next fracture to be simulated. From this intersection point construct a new
 352 fracture by randomly selecting a fracture set based on the fracture density maps, and again
 353 drawing the strike and length randomly from their corresponding pdfs.

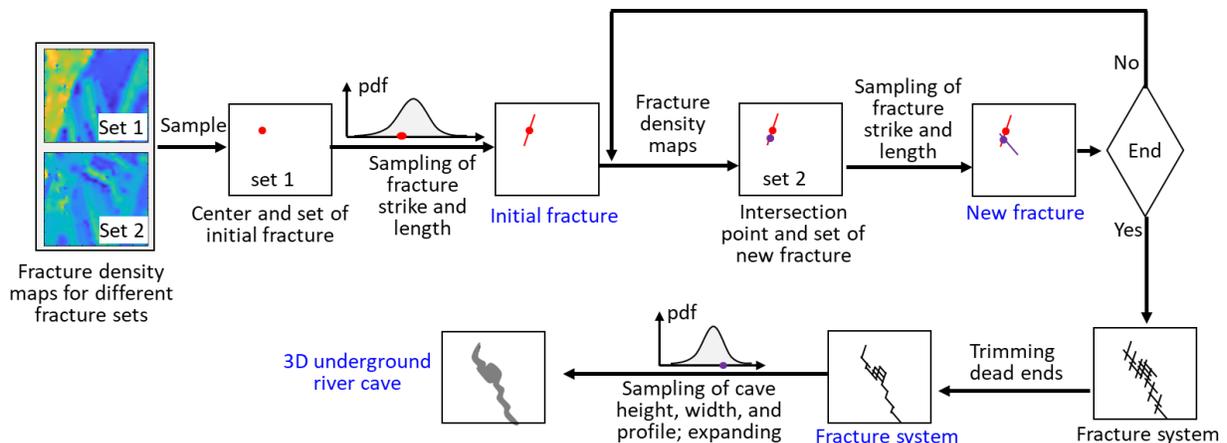
354 (5) Decide if the newly constructed fracture meets the fracture simulation stopping
 355 criteria; if yes, stop fracture simulation, otherwise repeat step (4), and (5). The stopping criteria
 356 include that, either the newly constructed fracture intersects the boundaries of the simulation area,
 357 or the number of simulated fractures reaches a predefined target value.

358 (6) Considering that underground river cave system may include several branches, for
 359 each branch, randomly select one simulated fracture as the initial fracture of this new branch.
 360 Then repeat step (4) and (5) to simulate fractures of this new branch.

361 (7) Randomly trim a fixed proportion (predefined value) of dead ends (the part between
 362 fracture end and nearest intersection point) of the simulated fracture system.

363 (8) According to pdfs of cave width and height, randomly sample width and height values
 364 for the cave system. Then based on the cave profile and the sampled cave width and height,
 365 expand the fracture system into a 3D cave system.

366



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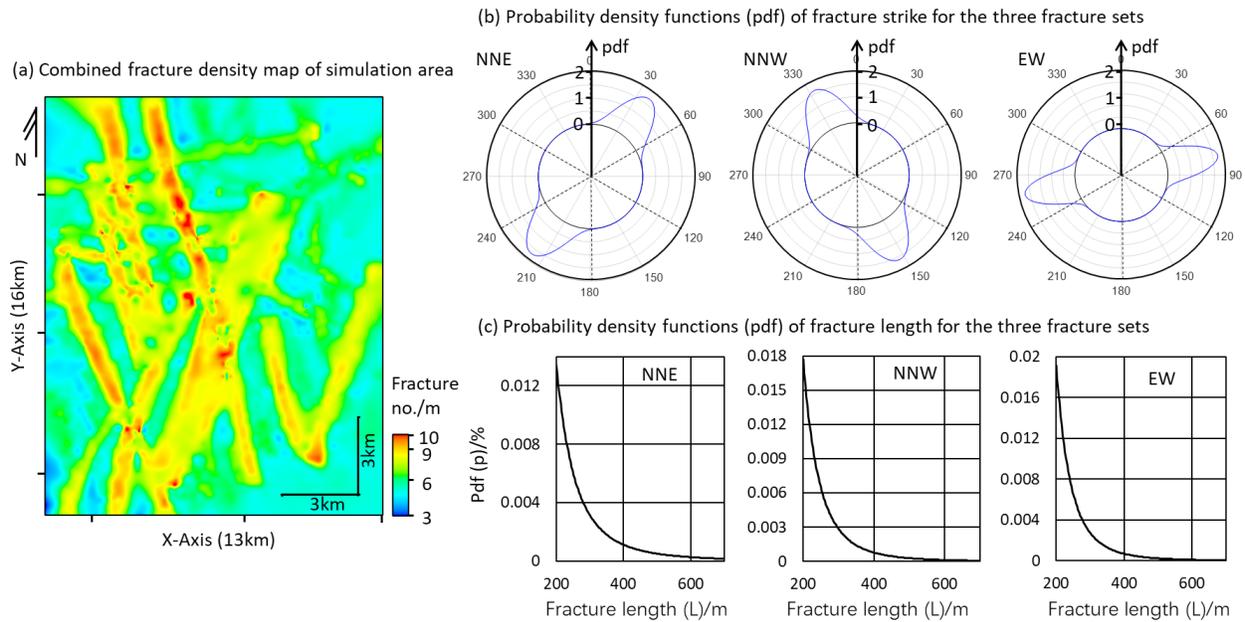
368 **Figure 2** Workflow of proposed process-mimicking approach for underground river cave

369

370 3.2 Simulation-related parameters of the Tahe cave reservoir

371 According to above workflow, we apply this process-mimicking approach to simulate
 372 conceptual facies models of underground river cave of Tahe area. There are mainly three sets of
 373 fractures with strikes of NNE, NNW, and nearly EW in Tahe cave reservoir. Based on image
 374 logs from several wells, the pdfs of fracture strike for the three sets are characterized by von
 375 Mises distributions (Berens, 2009; Fisher, 1993) as in Figure 3 (b) with the pdf parameters as
 376 given in Table 1. The pdfs of fracture length for the three sets are fit with a power law
 377 distribution based on the corrected measurement of outcrop fractures and seismically interpreted
 378 fractures presented in Méndez et al. (2020), as shown in Figure 3 (c) and Table 1. We choose a
 379 sub-area of 16km×13km inside Tahe area to simulate the underground river cave reservoir. The
 380 combined fracture density map of the three fracture sets is calculated from strain energy density
 381 distribution (obtained from Geomechanical simulation) and the relationship between the strain
 382 energy density and the fracture density estimated from core observation data. This is similar to
 383 the approach used in Feng et al. (2018). Here, the three sets of fractures are assumed to occur
 384 with the same probability at each point of the simulated fracture density map, so in the next
 385 process-mimicking simulation step the center or intersection point of a fracture is first selected
 386 according to the combined fracture density map, then one of the three fracture sets is
 387 equiprobably decided.

388



389

390 **Figure 3** Combined fracture density map, pdfs of fracture strike, and pdfs of fracture length for
 391 the three fracture groups of Tahe cave reservoir.

392

393 **Table 1** Pdfs of fracture strike and length for the three fracture groups of Tahe reservoir.

394

Strike (von Mises distribution)		Length (L)		
Average (μ)/degree	Concentration(κ)	Pdf (p)	Min/m	Max/m

NNE	38 (218)	19	$p = 2.6 \times 10^6 \times L^{-3.6}$	200	700
NNW	153 (333)	17	$p = 2.6 \times 10^6 \times L^{-4.5}$	200	700
EW	78 (258)	31	$p = 2.6 \times 10^6 \times L^{-4.8}$	200	700

395

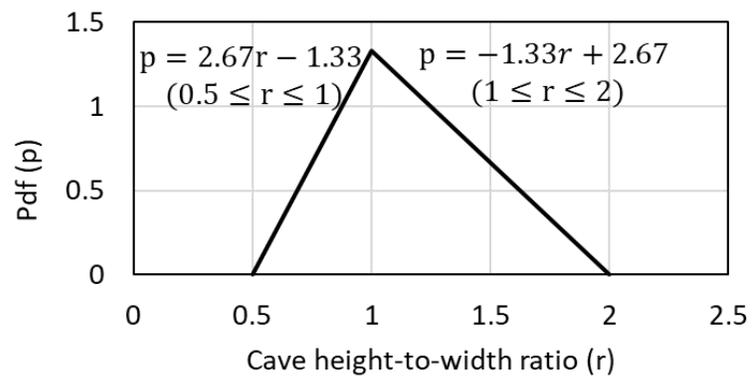
396 Based on outcrop observations of caves in the Tahe area (Figure 4 (a)), we use a half
 397 ellipse to approximate the cave profile. The size of outcrop caves (<5m) is usually much smaller
 398 than subsurface caves (several to tens meters), because large outcrop caves collapse easily. Thus,
 399 it is difficult to obtain the pdf of subsurface cave width from outcrop caves. In our case, we use
 400 the pdf of cave height-to-width ratio instead, which is obtained from the measurement of outcrop
 401 caves and the vertical sections of seismic attribute data (Figure 4 (b)). The pdf of cave height is
 402 fit from the statistics of wells drilled through underground river caves (Figure 4 (c)).

403

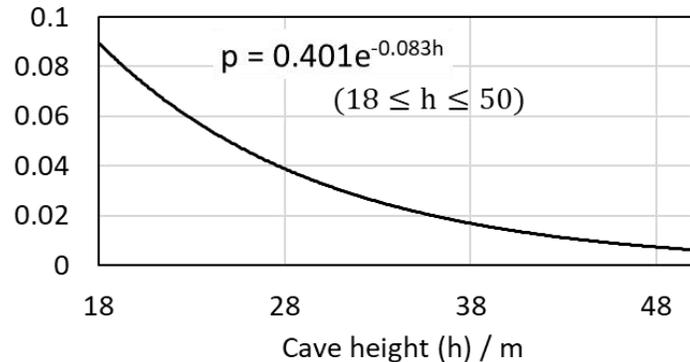
(a) Cave outcrop



(b) Pdf of cave height-to-width ratio



(c) Pdf of cave height



404

405 **Figure 4** Cave outcrop examples and pdfs of height-to-width ratio and height of underground
 406 river cave in Tahe reservoir.

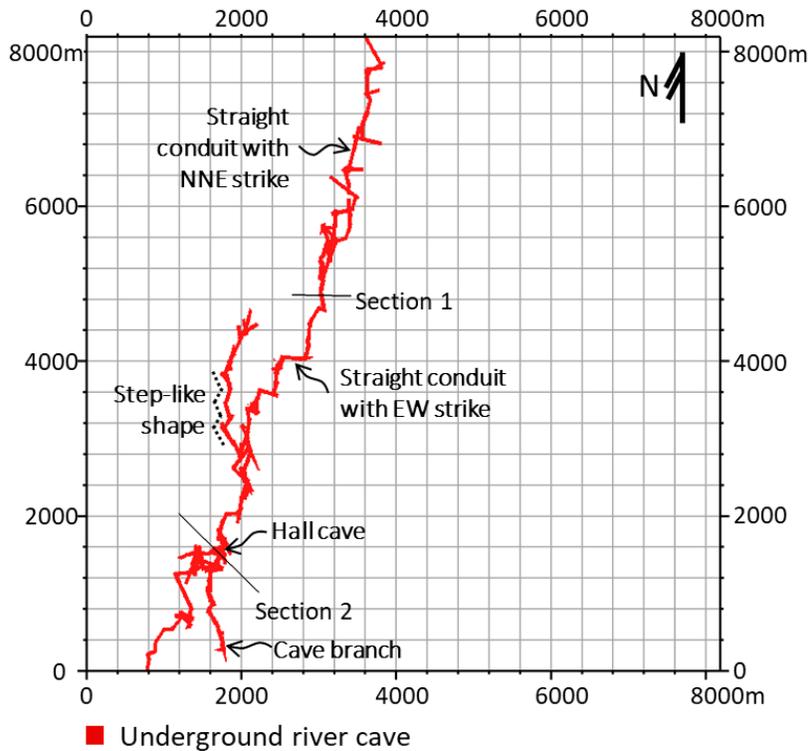
407

408 3.3 Simulation of conceptual models of Tahe area

409 To get different conceptual models of karst cave, in every simulation, we randomly crop
 410 a sub-area (8192m×8192m) from the original simulation area (Figure 3 (a); 16km×13km), and

411 use the fracture density map inside the sub-area, pdfs of fracture strike and length, and pdfs of
 412 cave height and height-to-width ratio to simulate one conceptual model, based on the proposed
 413 process-mimicking workflow. In total, 642 conceptual models of karst caves are simulated; each
 414 model includes $655 \times 655 \times 64$ cells, and each cell is 12.5m (length) \times 12.5m (width) \times 1m (height).
 415 Figure 5 and Figure 6 show a planer view and vertical sections of one randomly selected
 416 simulated model.

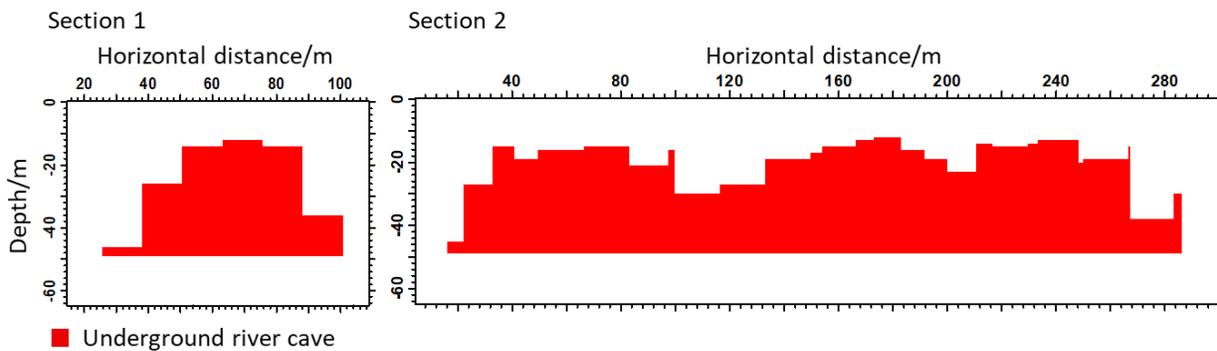
417



418

419 **Figure 5** Planer view of one randomly generated conceptual model of underground river cave
 420 reservoirs in Tahe area, simulated with the process-mimicking workflow. Only cave facies are
 421 shown.

422



423

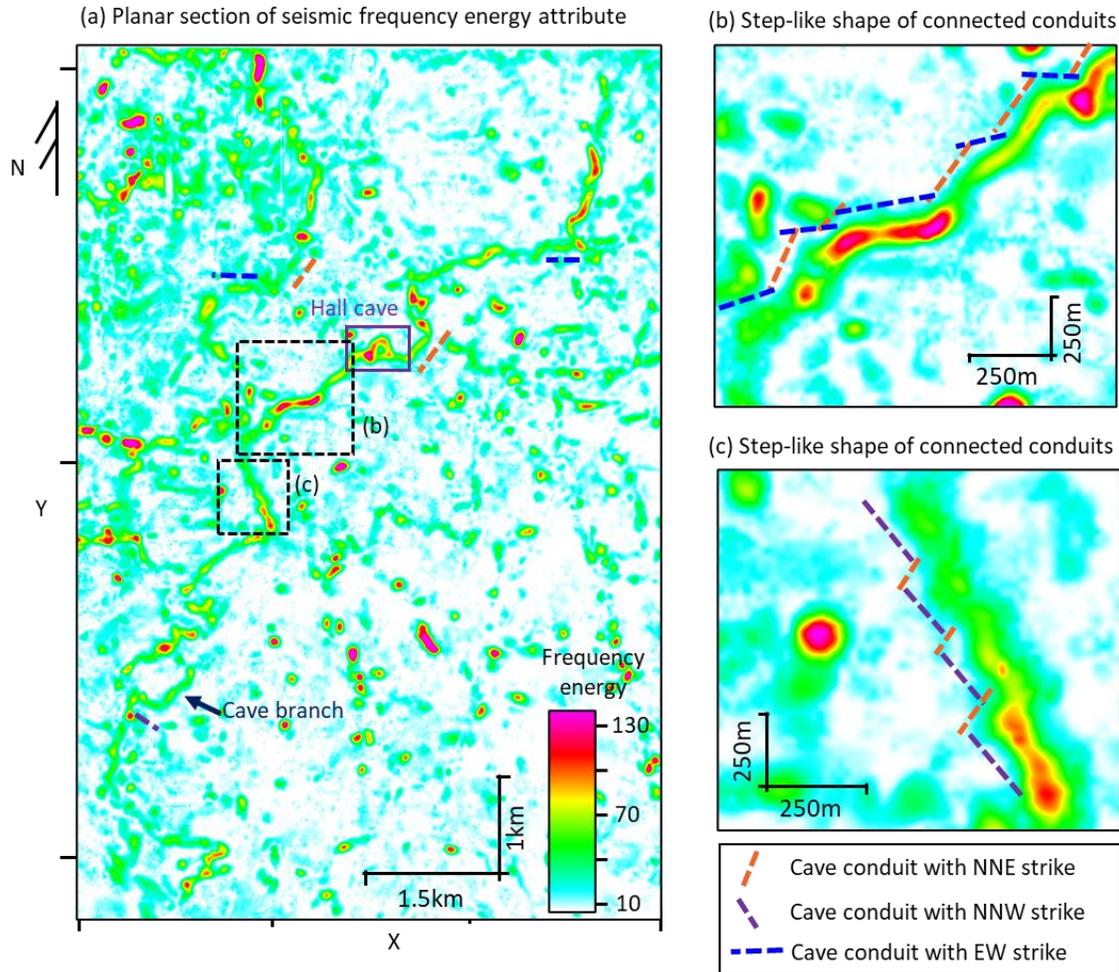
424 **Figure 6** Vertical sections of the randomly generated conceptual model shown in Figure 5. The
425 sections are marked in Figure 5. Only cave facies are shown.

426

427 3.4 Verification of geological patterns in simulated conceptual models

428 As shown in Figures 5 and 6, the simulated caves in these conceptual models are
429 connected, like ribbons. Each cave system is composed of many small straight conduits. Most of
430 these conduits have strikes of NNE, NNW, and nearly EW. A step-like shape can be observed
431 from several connected conduits. Cave branches also develop. Hall caves—cave whose width is
432 several times of normal cave width—are sparsely developed (section 2 of Figure 6). Most caves
433 have a half ellipse profile (section 1 of Figure 6). These are typical features of underground river
434 karst cave of Tahe reservoir. Figure 7 shows a planar section of the seismic frequency energy
435 attribute of a small part of Tahe cave reservoir, where one or two cave systems can be clearly
436 recognized. From the section, we can observe cave conduits with strikes of NNE, NNW, and
437 nearly EW some of which are marked by dashed lines, sparsely distributed hall caves, cave
438 branches, and step-like shape of connected conduits (Figure 7 (b) and (c)), similar to patterns
439 generated. Therefore, we can conclude that the conceptual models simulated by the process-
440 mimicking algorithm integrate the geological patterns of field cave reservoir of Tahe area.

441



442

443 **Figure 7** Planar section of seismic frequency energy attribute of a small part of Tahe cave
 444 reservoir.

445

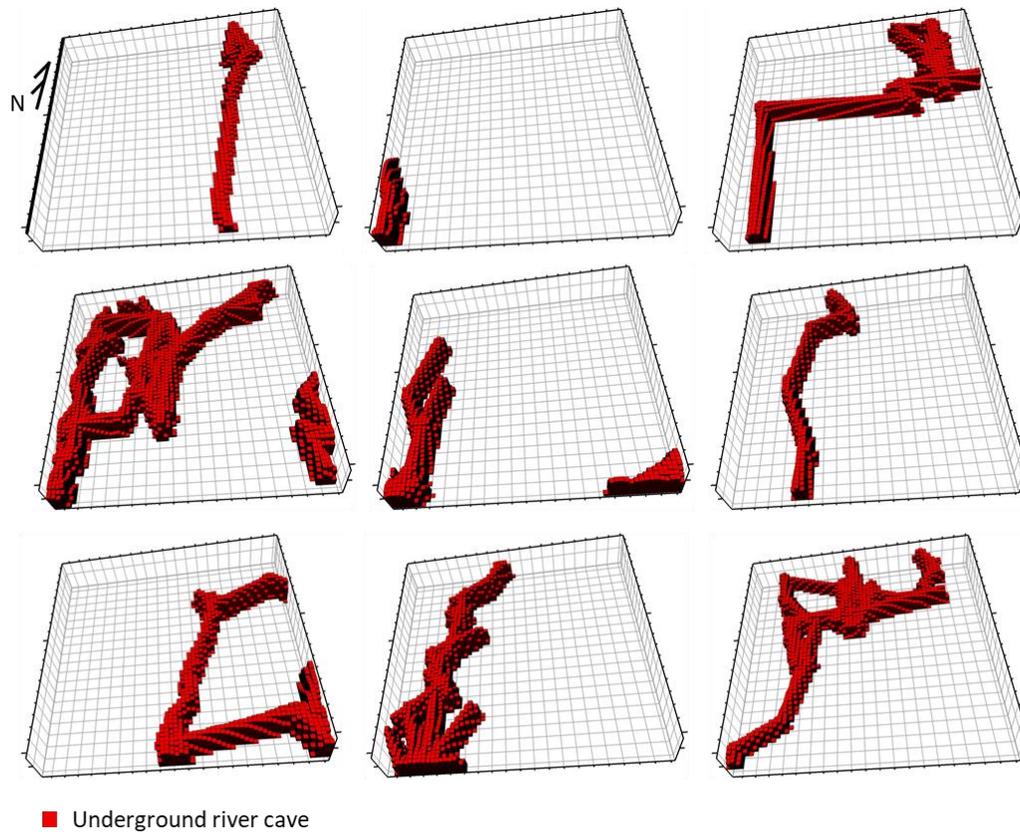
446 4. Construction of the training dataset

447 The training dataset includes training facies models ($64 \times 64 \times 64$), training well facies
 448 data ($64 \times 64 \times 64$), and training probability map of karst cave ($64 \times 64 \times 64$). We build training
 449 facies models from the simulated conceptual models of karst cave reservoir of Tahe area, by
 450 randomly cropping the large conceptual models ($655 \times 655 \times 64$) into blocks of $64 \times 64 \times 64$. The
 451 training well facies and probability maps data are obtained from these cropped training facies
 452 models.

453 4.1 Training facies models

454 After randomly cropping the 642 large conceptual models, a total of 22,695 3D training
 455 facies models ($64 \times 64 \times 64$) of underground river karst cave are obtained. Each cell represents
 456 12.5m (length) $\times 12.5\text{m}$ (width) $\times 1\text{m}$ (height). Figure 8 shows 9 random training facies models.

457



458

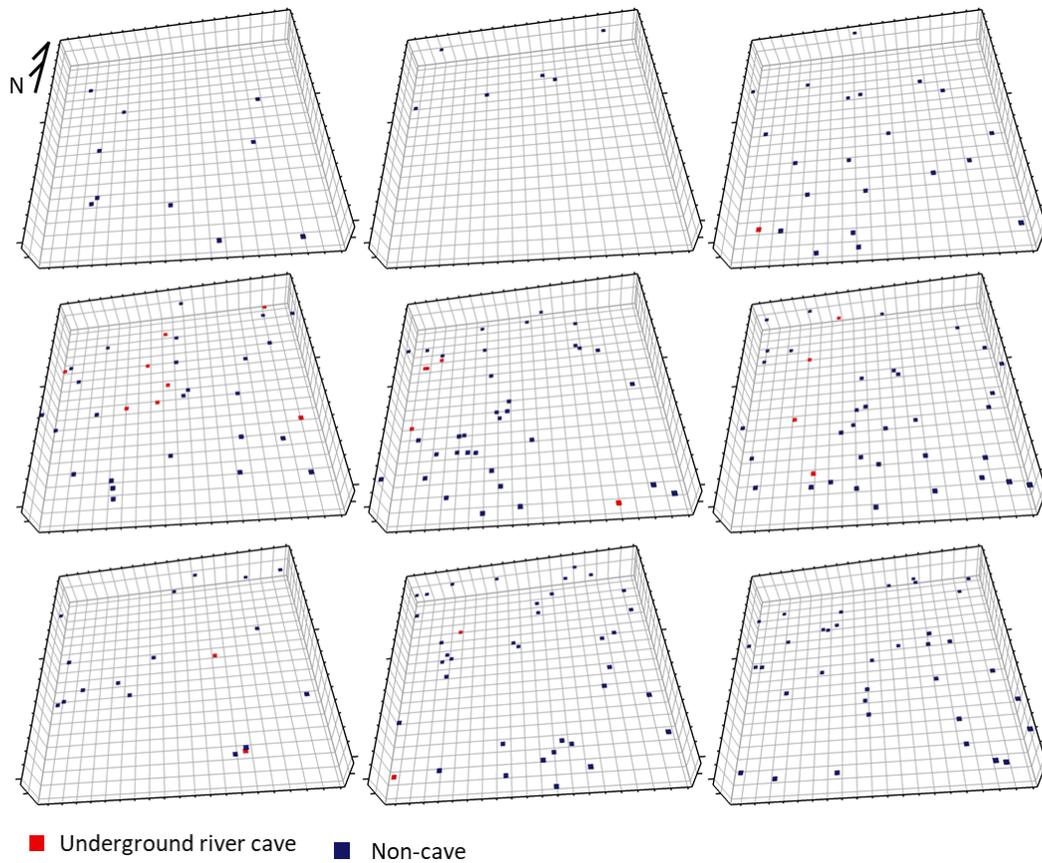
459 **Figure 8** Randomly selected 9 training models of underground river karst cave. Each model has
 460 $64 \times 64 \times 64$ cells, and each cell represents 12.5m (length) \times 12.5m (width) \times 1m (height). Only
 461 cave facies are shown.

462

463 4.2 Training well facies data

464 Theoretically, when constructing sparse training well data, all types of wells should be
 465 considered, including straight wells, inclined wells, horizontal wells, and wells not drilling
 466 through a model. In such cases, well data are distributed like strings along well trajectories. To
 467 make this process simpler, we ignore the string-like feature of well data and randomly sample
 468 sparse cells from the previously constructed training facies models to build training well data.
 469 Such non-string-like well data do not exist in practice, but they can help train the generator to
 470 condition on normal string-like well data, as is illustrated in the case studies of Section 5 and 6.
 471 From each training facies model (with $64 \times 64 \times 64$ cells), we sample 1-500 cells to form one set
 472 of training well data; the void cells between these sampled cells are set to an easily recognized
 473 numeric code for no data, e.g., -99. Finally, 22,695 sets of training well facies data (each with
 474 $64 \times 64 \times 64$ cells) are built. Figure 9 shows the 9 training well facies data sampled from the facies
 475 models in Figure 8.

476



477

478 **Figure 9** Training well facies data obtained from the 9 facies models in Figure 8. Each set of
 479 well data has $64 \times 64 \times 64$ cells, and each cell represents 12.5m (length) \times 12.5m (width) \times 1m
 480 (height).

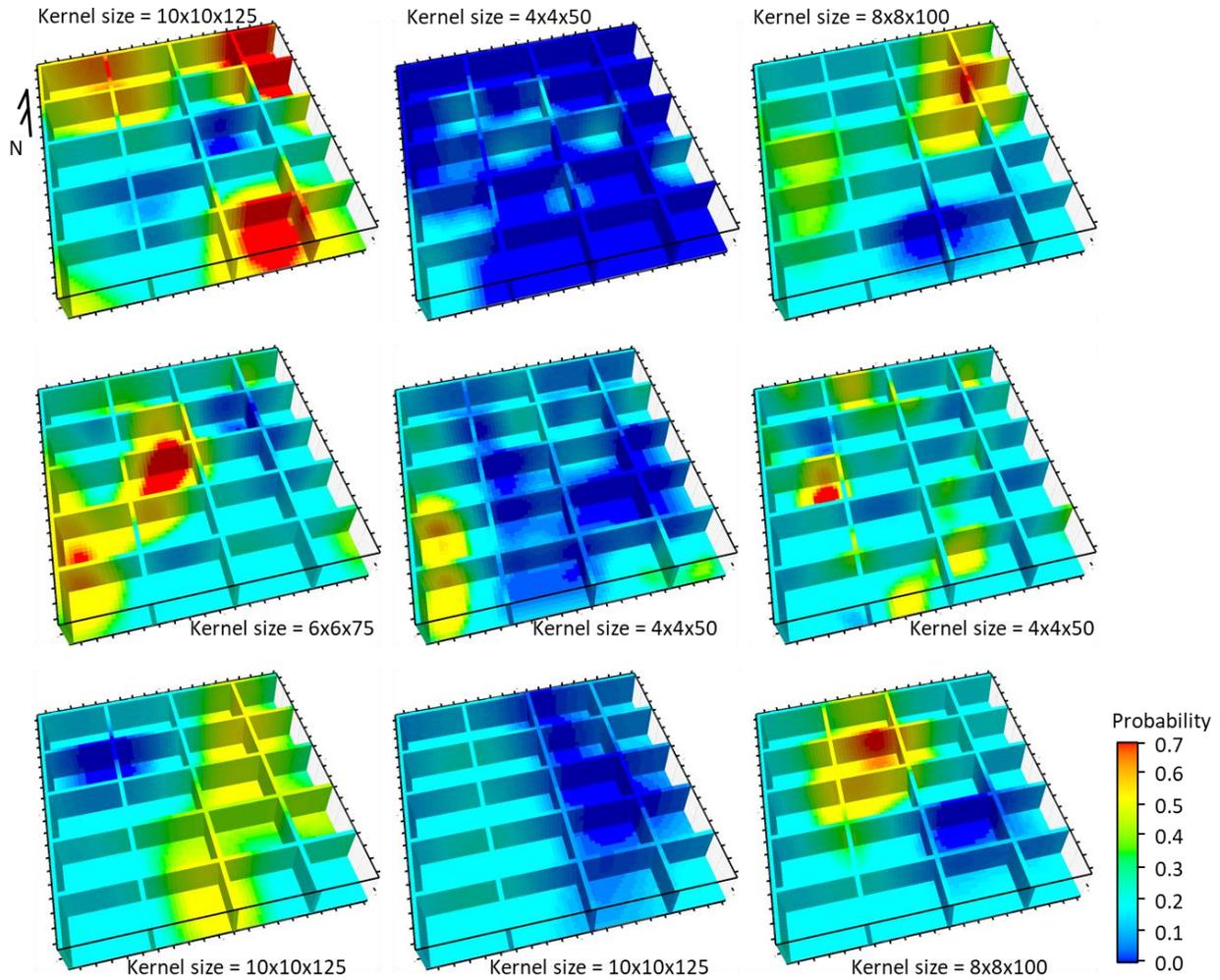
481

482 4.3 Training probability map of underground river cave

483 Training probability map data of underground river cave are also obtained from the 3D
 484 training facies models using a Gaussian smoothing approach, to mimic real geophysically
 485 interpreted probability maps. First, the facies models are preprocessed into indicator models of
 486 karst cave: the karst cave cells are set to be 1 while other cells are set to be 0. Second, these
 487 indicator models are smoothed using 3D Gaussian kernels with sizes of $4 \times 4 \times 50$, $6 \times 6 \times 75$,
 488 $8 \times 8 \times 100$, or $10 \times 10 \times 125$ to form 3D probability map of underground river caves with
 489 dimensions of $64 \times 64 \times 64$. Here, different kernel sizes are used in order to make the training
 490 probability maps similar to the practical ones (obtained from geophysical data) which may have
 491 diverse resolutions (blurriness). The vertical size of these kernels is 12.5 times of horizontal size,
 492 because the horizontal size (12.5m) of each cell is 12.5 times of the vertical size (1m). Third, 5%
 493 - 30% of Gaussian noise are randomly added into each probability map to mimic practical
 494 probability map data in which noises are inevitably introduced. Due to the added noise, the
 495 constructed training probability map data may not be strictly consistent with the training well
 496 facies data, e.g., karst cave cells in well data may correspond to low probability value in
 497 probability map data and vice versa. Such phenomena are common in practice. Finally, 22,695

498 3D probability map data are constructed. Figure 10 shows the 9 training probability maps
 499 calculated from the 9 facies models of Figure 8 with different Gaussian kernels.

500
 501



502

503 **Figure 10** Training probability maps obtained from the 9 facies models in Figure 8 using
 504 Gaussian kernels of different sizes. Each probability map has $64 \times 64 \times 64$ cells, and each cell
 505 represents $12.5\text{m (length)} \times 12.5\text{m (width)} \times 1\text{m (height)}$.

506

507 Among the 22,695 training facies models, well data, and probability maps, 20,000 are
 508 used for training GANs, while the remaining 2,695 are used as test dataset to evaluate the trained
 509 generator.

510 5. Training and evaluation of the generator

511 5.1 Training of generator

512 The weights for the three of losses (i.e., β_1 for GANs loss, β_2 for well condition-based
 513 loss, and β_3 for probability map condition-based loss in Equation (1)) are investigated through

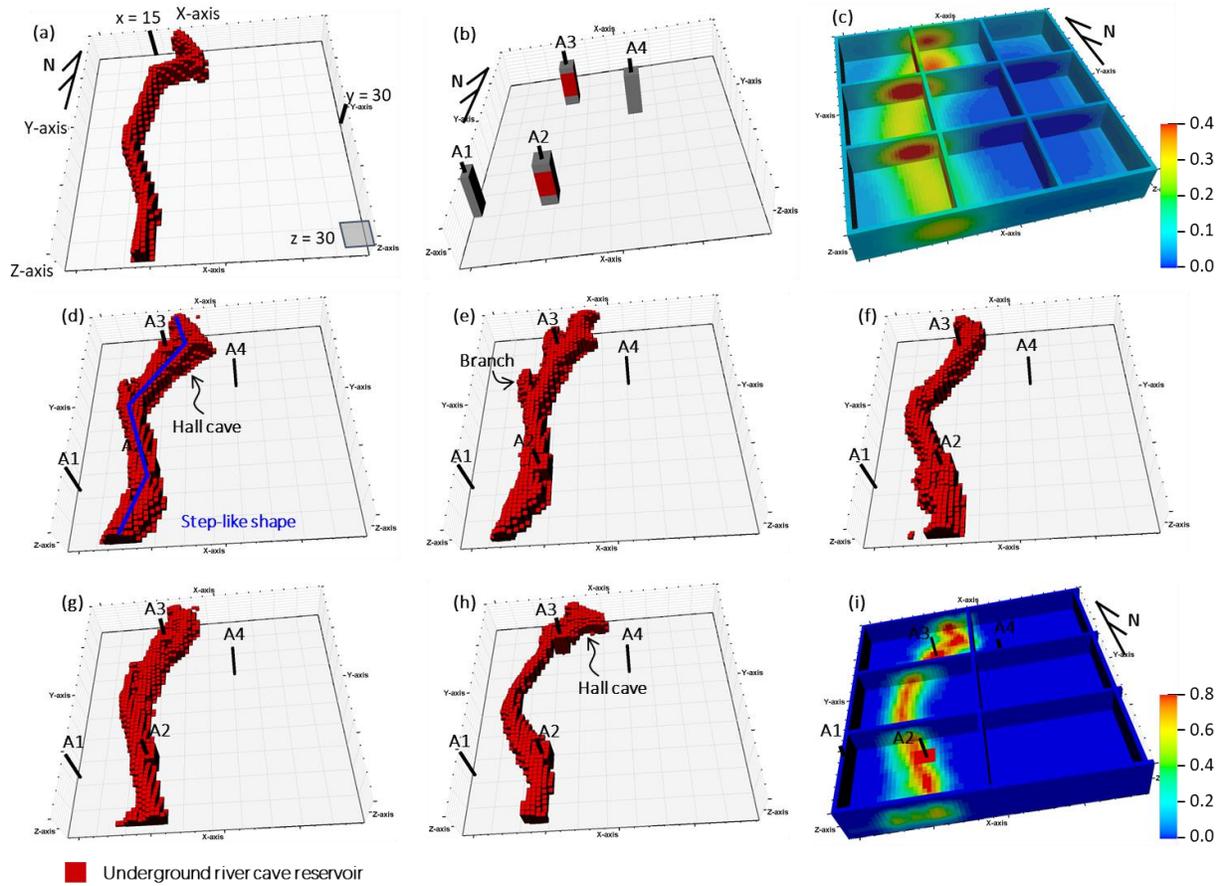
514 trial-and-error experiments as in Song et al. (2021b). Supporting Information S4 shows the
515 produced results of trained generators for various combinations of the weights. We suggest
516 setting weight β_2 and weight β_3 between 0.07 to 0.35 and 0.5 to 2.5, while setting weight β_1 as 1.
517 The weight β_2 represents a tradeoff between the conditioning of well data and the realism of the
518 earth models produced by the trained generator, while β_3 represents a tradeoff between the
519 diversity and the realism of the produced earth models. In this paper, we set β_1 , β_2 , and β_3 as 1,
520 0.35, and 0.5, respectively. With other settings as in Section 2.2, the generator was trained for 40
521 hours until the generated facies models were visually realistic, diverse, and conditioned to input
522 well and probability map data. The trained generator takes random latent cubes, 3D well facies
523 data, and 3D probability map as inputs and produces multiple 3D facies models of underground
524 river karst cave systems.

525 In Section 5.2, we first evaluate the trained generator in two synthetic cases in terms of
526 the realism, diversity, conditioning to input well facies and probability map, and prediction
527 accuracy of the simulated realizations. Then in Section 6, we use the trained generator for
528 uncertainty geomodelling of field karst cave reservoirs of Tahe area.

529 5.2 Evaluation of the pretrained generator based on synthetic cases

530 We chose two random facies models from the test dataset (not used for training) as the
531 ground truth and obtained two groups of corresponding 3D probability maps of karst cave and
532 well facies data via Gaussian smoothing (as described in Section 4.3) and random sampling. The
533 pretrained generator takes in 400 groups of random latent cubes (each group has 8 channels of
534 $4 \times 4 \times 4$ -latent cubes), the probability map, and the well data to produce 400 facies model
535 realizations for each synthetic case. Each realization has $64 \times 64 \times 64$ cells, and each cell
536 represents 12.5m (length) \times 12.5m (width) \times 1m (height). Figure 11 and Figure 12 show the
537 ground truth facies model, input 3D probability map, input well data, and 5 realizations for each
538 synthetic case. It takes 0.02 seconds to produce one realization on 1 GPU (V100).

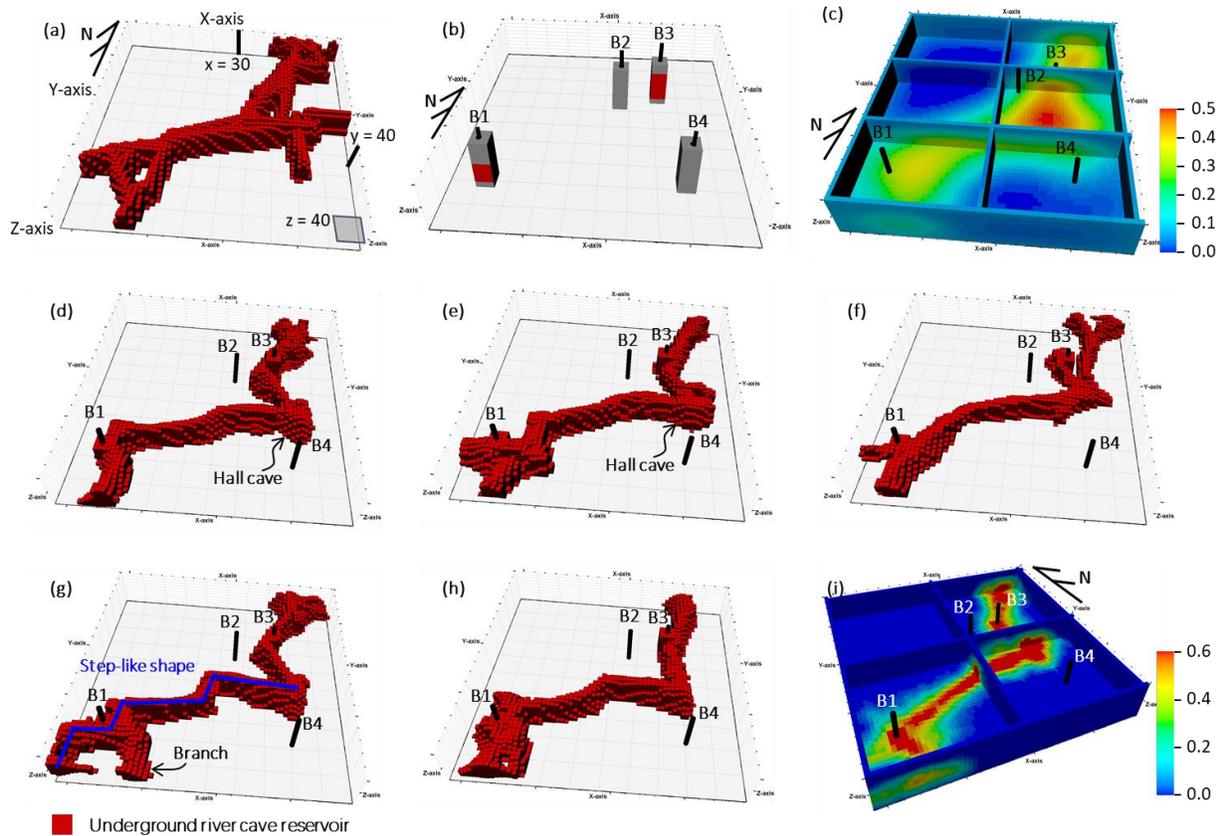
539



540

541 **Figure 11** Geomodelling results for synthetic case 1. (a) Ground truth karst cave facies model;
 542 (b) Sparse well facies data sampled from the ground truth facies model (the gray cells represent
 543 non-cave facies type); (c) Probability map of karst cave obtained from the ground truth via
 544 smoothing; (d) – (h) Random facies model realizations directly produced from the pretrained
 545 generator by taking the well data and probability map as conditioning inputs; (i) Frequency map
 546 calculated from 400 generated realizations. All subfigures include $64 \times 64 \times 64$ cells, and each cell
 547 represents 12.5m (length) \times 12.5m (width) \times 1m (height). Only karst cave cells are shown.

548



549

550 **Figure 12** Geomodelling results for synthetic case 2. (a) Ground truth karst cave facies model;
 551 (b) Sparse well facies data sampled from the ground truth facies model (the gray cells represent
 552 non-cave facies type); (c) Probability map of karst cave obtained from the ground truth via
 553 smoothing; (d) – (h) Random facies model realizations directly produced from the pretrained
 554 generator by taking the well data and probability map as conditioning inputs; (i) Frequency map
 555 calculated from 400 generated realizations. All subfigures include $64 \times 64 \times 64$ cells, and each cell
 556 represents 12.5m (length) \times 12.5m (width) \times 1m (height). Only karst cave cells are shown.

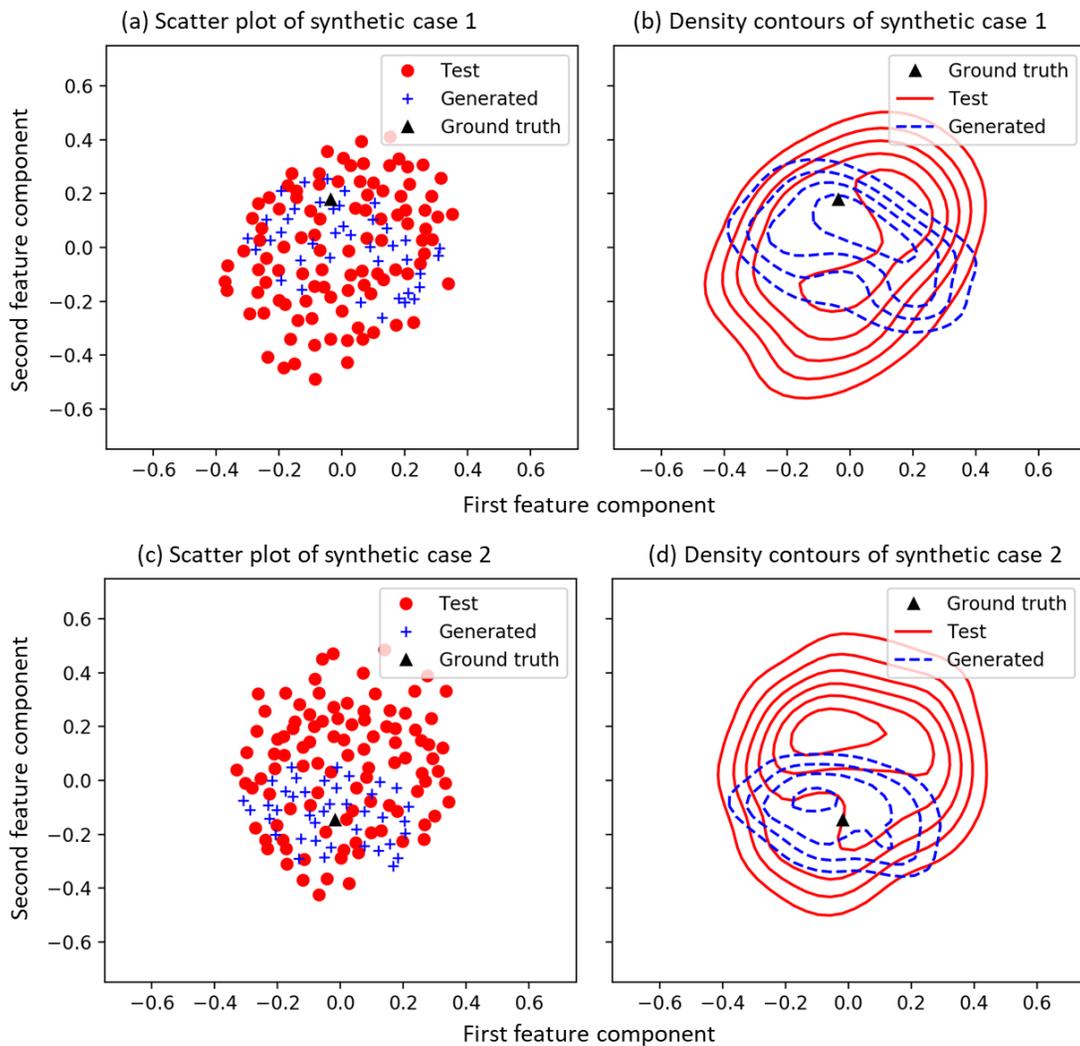
557

558 From Figure 11 and Figure 12, we can see that these generated karst cave facies model
 559 realizations are very realistic, i.e., having very similar geological patterns to the practical karst
 560 caves of Tahe area or the training/testing facies models. For example, these simulated
 561 underground river caves have a connected ribbon-like shape in planar view and a half-elliptical
 562 shape in vertical section, some cave branches develop ((d) and (e) of Figure 11, (e), (f), and (g)
 563 of Figure 12), wide hall caves are sparsely distributed ((d) and (h) of Figure 11, (d), (e), and (h)
 564 of Figure 12), most straight conduits have strikes of NNE, NNW, and nearly EW, and the step-
 565 like shape can be observed from some connected cave conduits (blue lines in (d) of Figure 11
 566 and (g) of Figure 12). These features are very typical in the actual karst caves (Figure 7 and
 567 Figure 4) and training/test facies models (Figure 8) of Tahe area.

568 To quantitatively assess the relationship of internal geological patterns between the
 569 generated realizations and the training/test facies models (representing actual karst cave
 570 geological patterns), we used Multi-Scale Sliced Wasserstein Distance combined with Multi-

571 Dimensional Scaling (MS-SWD-MDS) approach to map every facies model into a point in a
 572 reduced-dimension 2D space representing its geological pattern. Song et al. (2021a) describe this
 573 approach in detail. Supporting Information S3 also explains this approach. Figure 13 (a) and (c)
 574 show such point distributions of 100 test facies models, 40 generated realizations, and the ground
 575 truth facies model for the two synthetic cases; Figure 13 (b) and (d) show the density contours of
 576 these points, which approximate the distributions of underlying geological patterns in a 2D
 577 space. From Figure 13, we can clearly see that, first, the geological pattern distribution of the
 578 generated realizations is located inside that of the test facies models; second, the former one
 579 shrinks to cluster closely around the ground truth facies model. This proves that the generated
 580 realizations can reproduce geological patterns of practical karst caves (represented by test
 581 dataset) but is constrained due to the conditioning effect of the input probability map and well
 582 data. In addition, these realizations are very diverse, which can be seen from Figure 11, Figure
 583 12, and Figure 13.

584



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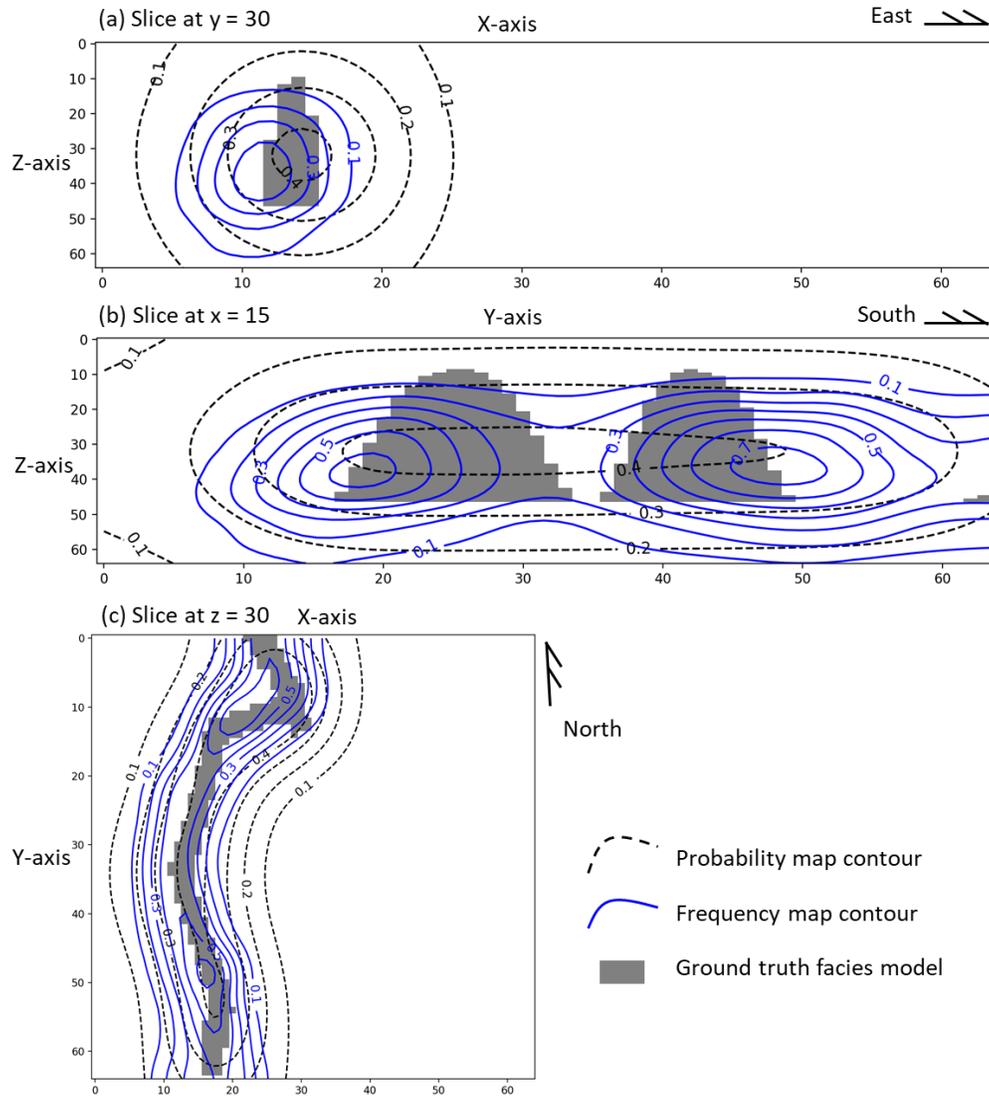
586 **Figure 13** Projection of test facies models, generated conditional facies model realizations, and
 587 the ground truth facies models in a 2D space, based on MS-SWD-MDS approach; each point

588 represents geological patterns of one facies model. (a) and (c) are the point scatter plots of the
589 two synthetic cases; (b) and (d) are the density contour maps obtained from the point scatter plots
590 of the two cases.

591

592 Based on the generated realizations, the reproduction accuracy of the input well facies
593 data was 100% for both cases. From careful visual inspection, we can see the locations and
594 shapes of the generated realizations are consistent with the input probability map very well. In
595 Figure 11 and Figure 12, we obtain frequency maps of karst cave from the 400 generated
596 realizations by calculating the proportion of simulated cave among all realizations at each cell
597 (see Equation (S2-5)). The frequency maps are quite similar with the input probability maps.
598 Figure 14 and Figure 15 compare the distributions of input probability map, frequency map, and
599 the ground truth facies models at different sections of the two synthetic cases. Apparently, the
600 generated frequency maps are more concentrated inside the input probability maps towards the
601 ground truth facies models. Especially, some features about the ground truth facies models are
602 lost in the input probability maps but are recaptured in the calculated frequency maps; for
603 example, in Figure 14 (b) and Figure 15 (a), high values of the frequency maps are concentrated
604 around the two ground truth karst caves, although the probability maps give no hint about the
605 number of the ground truth caves; in planar sections of Figure 14 (c) and Figure 15 (c), large
606 parts of the ground truth caves are outlined by the frequency maps. These discussions prove that,
607 (1) the generated realizations are consistent with the input probability maps, and (2) compared to
608 the case of geomodelling only using probability map (where the frequency map completely
609 overlaps with the probability map), the generator's prediction accuracy of caves are largely
610 increased and the uncertainty is decreased. The increased accuracy or decreased uncertainty
611 results from the integration of well data and geological patterns; the difference between the input
612 probability map and the frequency map points to the value of input well data and geological
613 patterns.

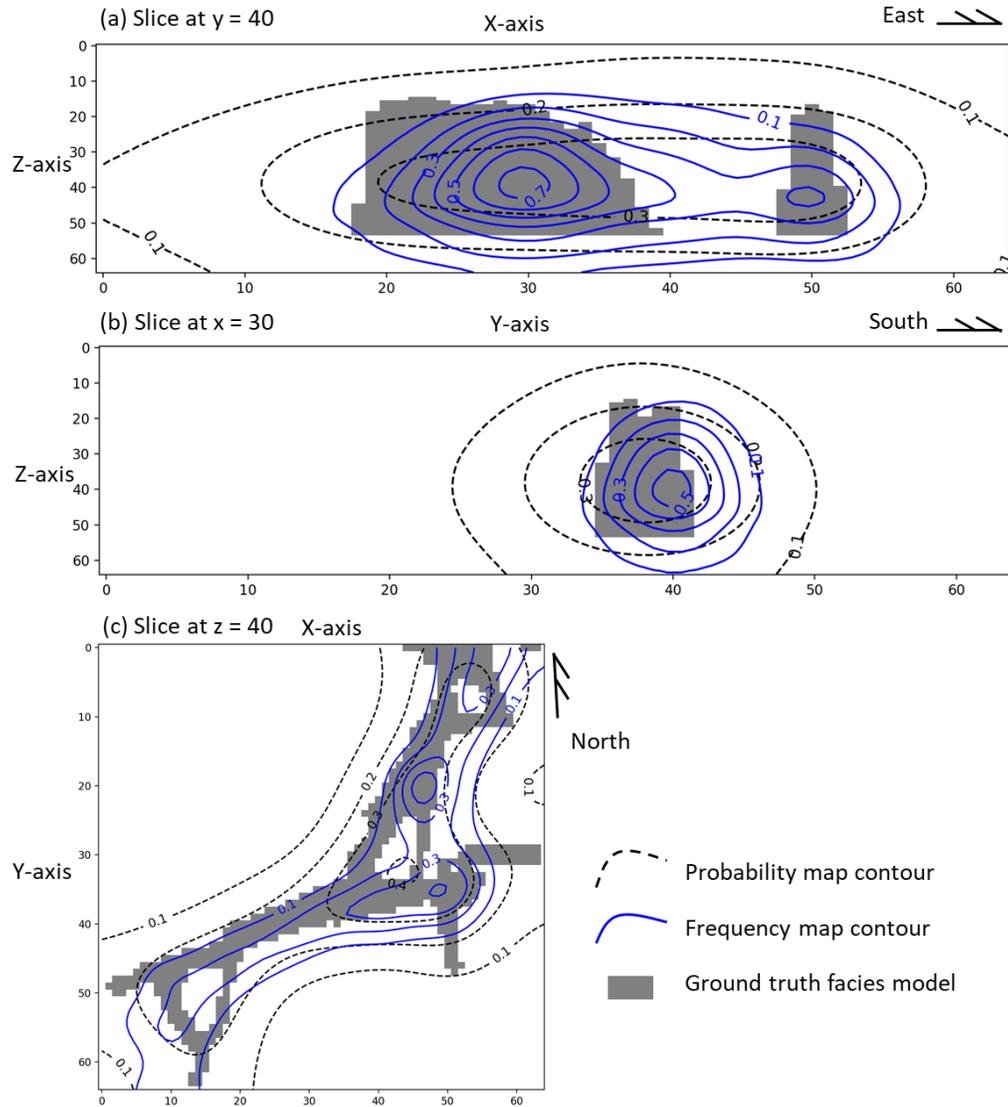
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615

616 **Figure 14** Comparison of input probability map contour, frequency map contour, and the ground
 617 truth facies model for synthetic case 1 at three different sections (i.e., $y = 30$, $x = 15$, and $z = 30$).
 618 These sections are marked in Figure 11.

619



620

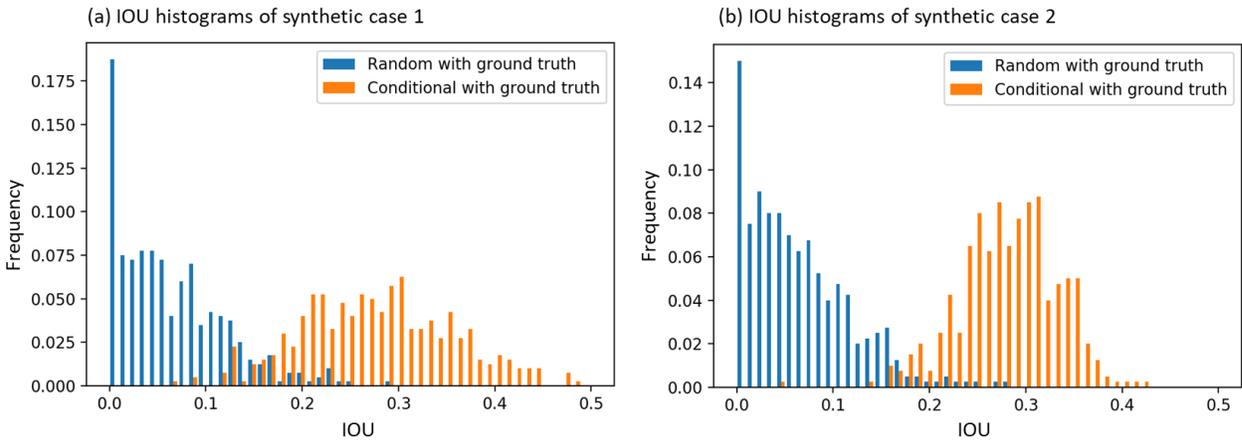
621 **Figure 15** Comparison of input probability map contour, frequency map contour, and the ground
 622 truth facies model for synthetic case 2 at three different sections (i.e., $y = 40$, $x = 30$, and $z = 40$).
 623 These sections are marked in Figure 12.

624

625 Figures 11, Figure 12, and Figure 13 show that the simulated facies model realizations
 626 are very close to the ground truth with respect to geological patterns and locations. To
 627 quantitatively assess the cave prediction accuracy of these generated realizations, we define
 628 intersection-over-union (IOU) metric as cave intersection divided by cave union between one
 629 generated (or test) facies model and the ground truth one. IOU varies from 0 to 1; the larger the
 630 IOU value is, the more accurate the generated (or test) facies model is. We calculate IOU for the
 631 400 generated realizations (IOU_g) and IOU of 400 random test facies models (IOU_t) as a
 632 comparison in both synthetic cases. Figure 16 shows the histograms of IOU_g and IOU_t for the
 633 two cases. Most IOU_g values are much larger than most IOU_t values; the maximum IOU_g values
 634 are 0.48 and 0.43 in case 1 and case 2. This means that the generated conditional facies models
 635 are much more accurate than random unconditional facies models. Figure 17 compares the

636 generated realizations with $\text{IOU} = 0.2, 0.3,$ and 0.4 and the ground truth ones for both cases. In
 637 addition, the conditioning approach for global features (Song et al., 2021b) could be used to
 638 condition the width and proportion of caves, thus further improving the accuracy of simulated
 639 realizations.

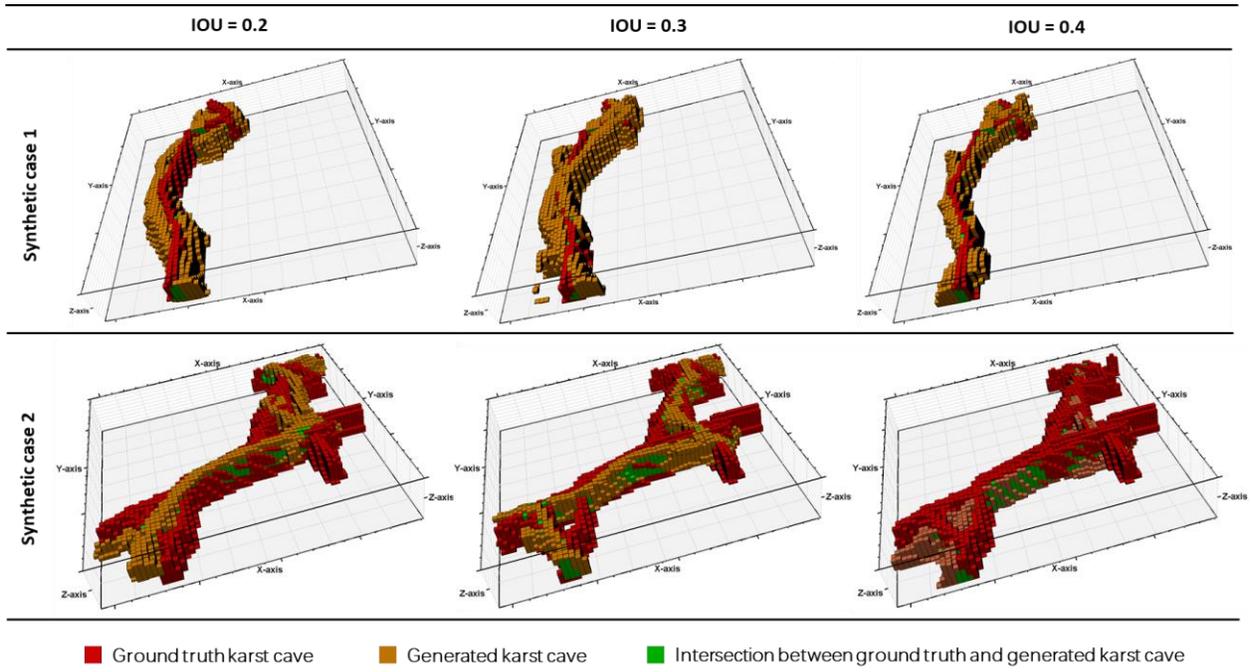
640



641

642 **Figure 16** Comparison of IOU histograms between random test facies models and produced
 643 conditional realizations for both synthetic cases.

644



645

646 **Figure 17** Comparison between the generated karst cave facies models with $\text{IOU} = 0.2, 0.3,$ and
 647 0.4 and the ground truth ones for the two synthetic cases. Only karst cave cells are shown.

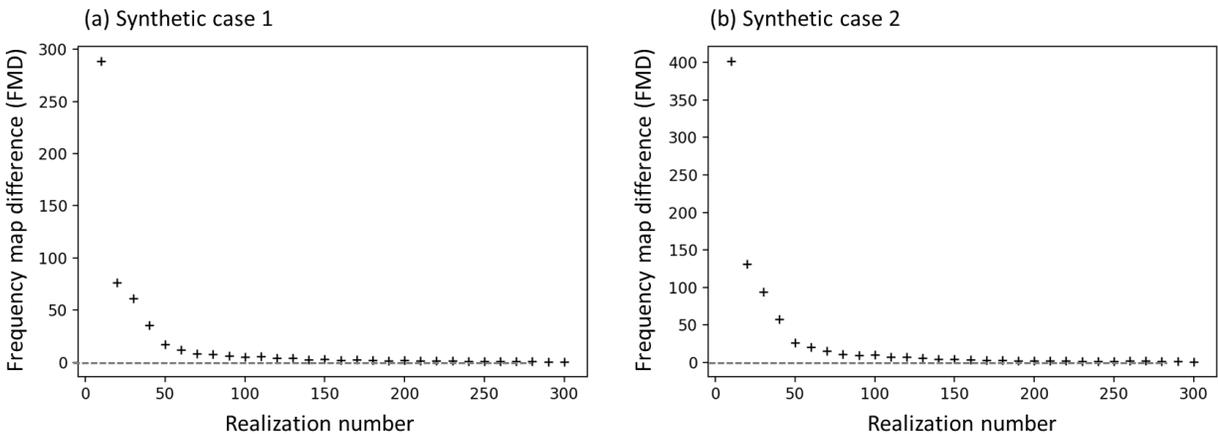
648

649 The above analyses of synthetic cases show that given observed well data and probability
 650 maps (calculated from geophysical data), the pretrained generator can quickly produce diverse
 651 realizations that are consistent with both the expected geological patterns and the input
 652 conditioning data, with acceptable accuracy. Thus, we can rely on the pretrained generator for
 653 real field case uncertainty geomodelling, i.e., to produce multiple facies model realizations to
 654 represent the uncertainty of real reservoirs. However, how many realizations should be produced?
 655 This question is addressed by examining the change in the frequency map with increasing
 656 number of realizations. The frequency map changes with increasing number of realizations
 657 generated, until for a large enough set of realizations the frequency map converges and does not
 658 change anymore. When the frequency map stabilizes, the number of realizations are taken to be
 659 enough to represent the uncertainty of the reservoir model. We define a frequency map
 660 difference (FMD) as

$$661 \quad FMD = \sum_{all\ cells} (FM_{x+10} - FM_x)^2, \quad (2)$$

662 where x is the realization number, FM_{x+10} and FM_x are the frequency map for $x + 10$ and x
 663 realizations, respectively. Thus, FMD represents the frequency map change with every 10 new
 664 additional realizations produced. Figure 18 shows the FMD change with the number of
 665 realizations for the two synthetic cases. The FMD converges to 0 when 200 realizations are
 666 produced in both cases, indicating a stable frequency map after that. Thus, we suggest producing
 667 200 facies model realizations when using the pretrained generator for practical uncertainty
 668 geomodelling.

669



670

671 **Figure 18** Change of frequency map difference (FMD) for every ten new realizations.

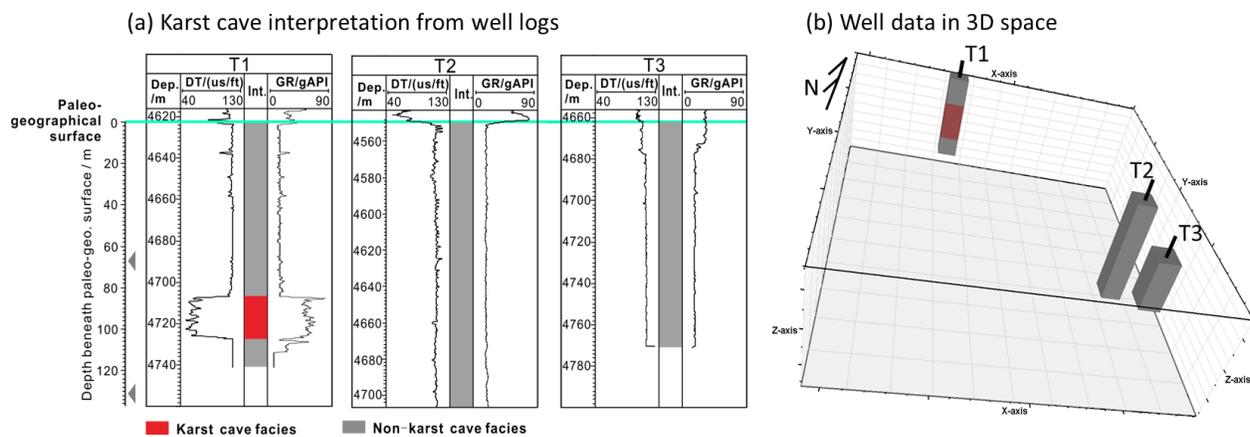
672

673 **6. Uncertainty geomodelling of field reservoirs with the pretrained generator**

674 We show two field reservoir cases from Tahe area: the first reservoir case is based on
 675 $64 \times 64 \times 64$ cells (i.e., $800\text{m} \times 800\text{m} \times 64\text{m}$) – the size at which the generator is trained, while in
 676 the second case, we apply the pretrained generator to simulate geological models of
 677 $336 \times 256 \times 96$ cells (i.e., $4200\text{m} \times 3200\text{m} \times 96\text{m}$).

678 6.1 Field case 1 based on $64 \times 64 \times 64$ cells

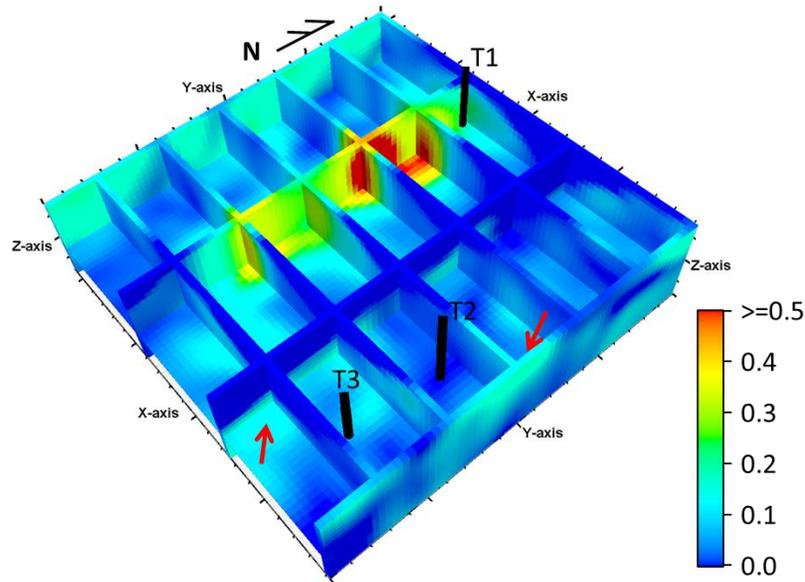
679 We choose a study area of $800\text{m} \times 800\text{m}$, with three wells, from Tahe area. The well logs
 680 are shown in Figure 19 (a), from which we can recognize a 21m-thick underground river cave
 681 interval in well T1 (i.e., 84m to 105m beneath the paleo-geographic surface below which the
 682 caves were formed) and no caves in the other two wells. The geo-space of this area between 67m
 683 to 131m beneath the paleo-geographic surface is divided into $64 \times 64 \times 64$ cells, with each cell
 684 representing 12.5m (length) \times 12.5m (width) \times 1m (height) which is the same as the training/test
 685 dataset. Figure 19 (b) shows the distribution of well data in this geo-space. The 3D probability
 686 map of underground river cave is calculated from a 3D seismic attribute (frequency energy) and
 687 the relationship between that attribute and cave occurrence probability obtained from the
 688 statistics of the 3000 wells in Tahe area. Figure 20 shows the probability map of this study area.
 689



690

691 **Figure 19** (a) Karst cave interpretation from well logs for the three wells of the study area. (b)
 692 Distribution of well facies data in the 3D $64 \times 64 \times 64$ -cell geo-space ($800\text{m} \times 800\text{m} \times 64\text{m}$) of the
 693 study area.

694



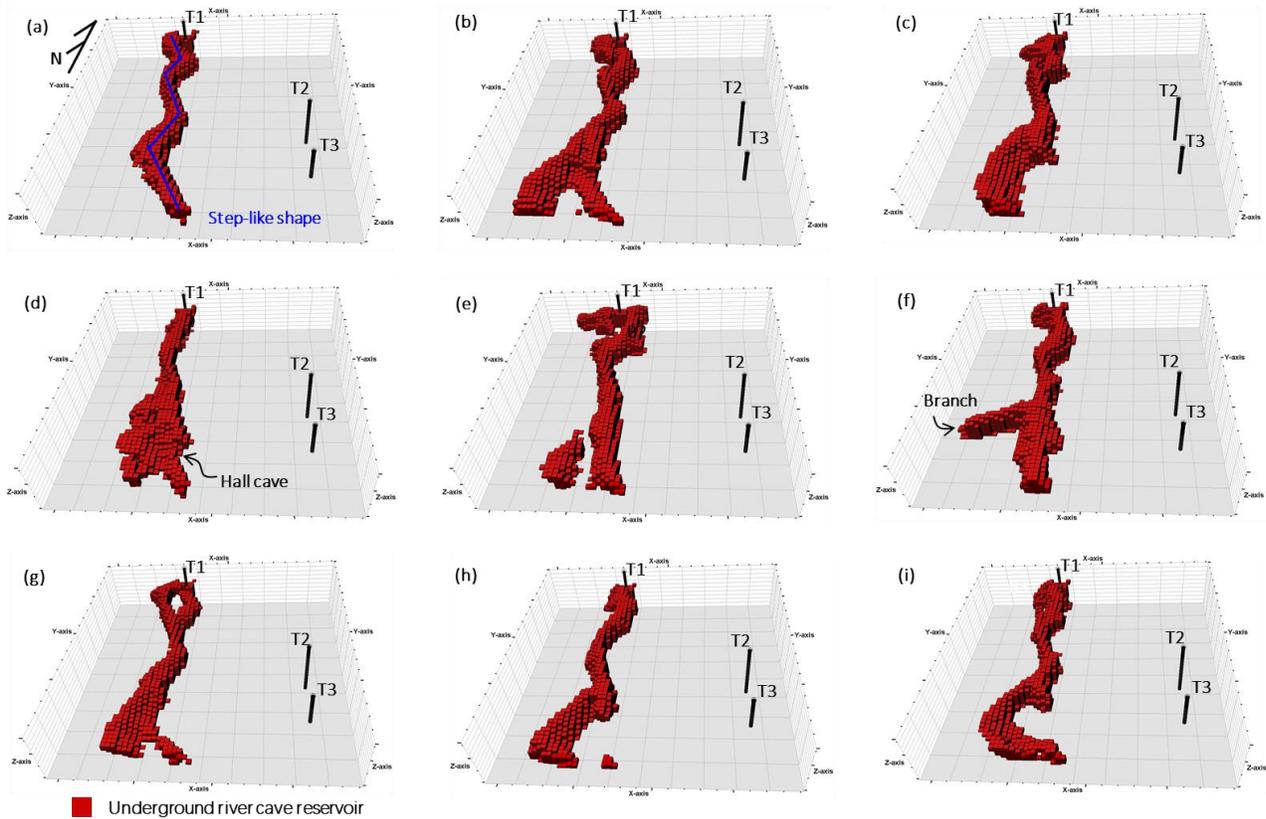
695

696 **Figure 20** 3D probability map of underground river cave for the study area. The map is
 697 calculated from the seismic frequency energy attribute and the relationship between that attribute
 698 and cave occurrence probability. Red arrows show local high probability areas away from the
 699 main trend.

700

701 We take the interpreted 3D well facies data, 3D probability map data, and 200 groups of
 702 random latent cubes (each group has 8 channels of latent cubes with size of $4 \times 4 \times 4$) into the
 703 pretrained generator to produce corresponding 200 karst cave realizations. Figure 21 shows 9 of
 704 them. Like the previous synthetic cases, the cave systems of these realizations are consistent with
 705 expected geological patterns, such as step-like shape (Figure 21 (a)), NNE, NNW, and nearly
 706 EW strikes of single conduit, sparse hall caves (Figure 21 (d)).

707



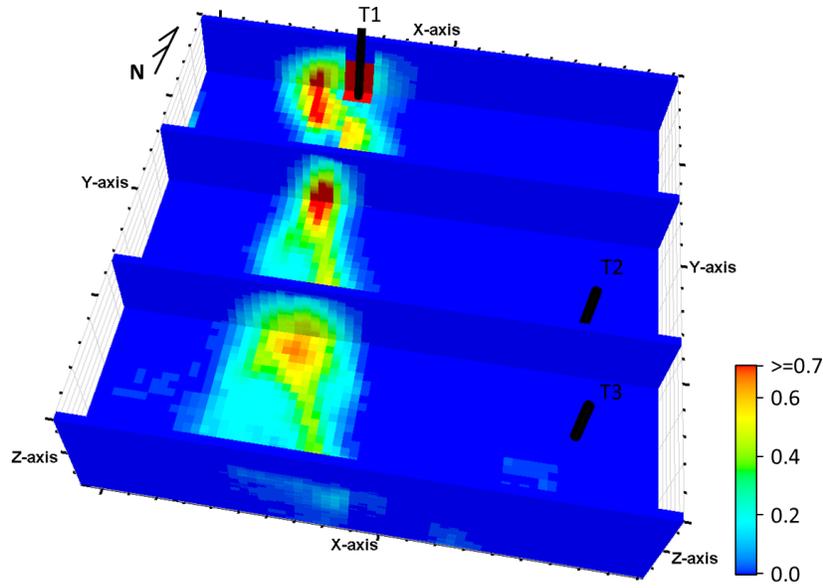
708

709 **Figure 21** Nine facies model realizations (out of 200) of underground river karst cave for the
 710 study area. Each realization has $64 \times 64 \times 64$ cells. Only underground river cells are show.

711

712 We calculate the frequency map of karst cave from the 200 simulated realizations (Figure
 713 22). By comparing Figure 21 and Figure 22 to Figure 20, we can clearly find that, first, the high
 714 value of the probability map is mainly distributed in the slightly west area with a north–south
 715 trend, and most simulated caves are also distributed in the same high-probability area and with a
 716 north–south trend; second, the high values of the frequency map are concentrated inside the high
 717 value area of the input probability map. Therefore, we can conclude that the simulated caves are
 718 consistent with the input probability map. By calculation from the 200 realizations, the
 719 reproduction accuracy of the input well data is 100%.

720

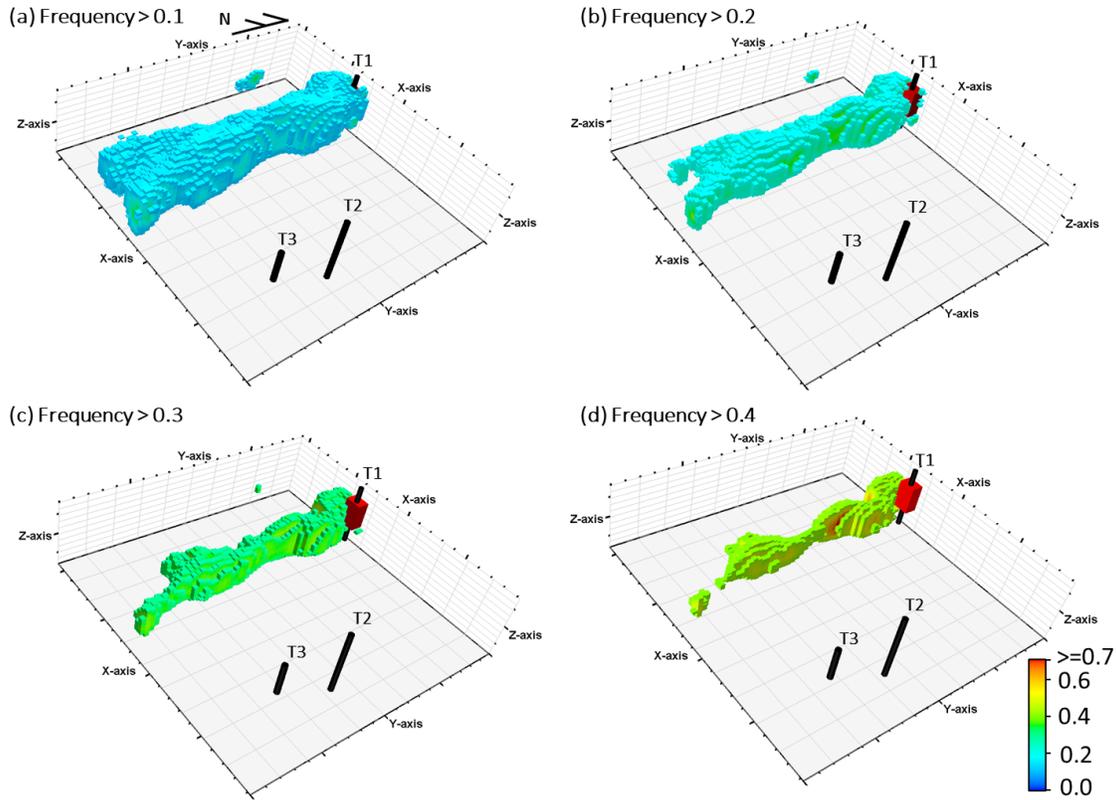


721
 722 **Figure 22** Frequency map of karst cave for the study area calculated from 200 facies models
 723 realizations.

724
 725 In the input probability map, except the major high-value area (slightly west; with value
 726 larger than 0.2), there are still other local high-value areas (red arrows in Figure 20). These local
 727 areas may result from the noise of seismic data collection, processing artifacts, interpretation
 728 uncertainties, or the isolated karst cave type that is not considered in this study. Regardless of
 729 causes, these local highs can all be regarded as noises in the background of simulating
 730 underground river caves. However, when comparing Figure 20, Figure 21, and Figure 22, we can
 731 find that these local noises have completely no effect on the distribution of simulated caves. The
 732 reason might be that the spatial shape of these local highs is inconsistent with the ribbon-like
 733 geological patterns of underground river caves learned by the generator. Thus, these local non-
 734 ribbon-like features of the input probability map are suppressed by the pretrained generator.

735 The calculated frequency map reveals the “sweet spots” of this area. Figure 23 filters the
 736 frequency map based on various thresholds. These filtered frequency maps can be used as inputs
 737 for designing well trajectories, calculating reserves of fluids in the cave reservoir, and evaluating
 738 the uncertainty of investment and revenue in a more systematic manner.

739



740

741 **Figure 23** Filtered frequency maps based on thresholds of 0.1, 0.2, 0.3, and 0.4.

742

743 The probability map obtained from seismic data involves uncertainty, which may come
 744 from the collection and processing of seismic data itself, the non-uniqueness of the relationship
 745 between seismic attribute and cave probability, error in the interpretation of well data, etc. Thus,
 746 in practice, we may not trust the seismic probability map completely. Given a relative trust level
 747 (t), varying from 0 to 1, here a compromised probability map p_t is defined to only contain t
 748 portion of the original seismic probability map information:

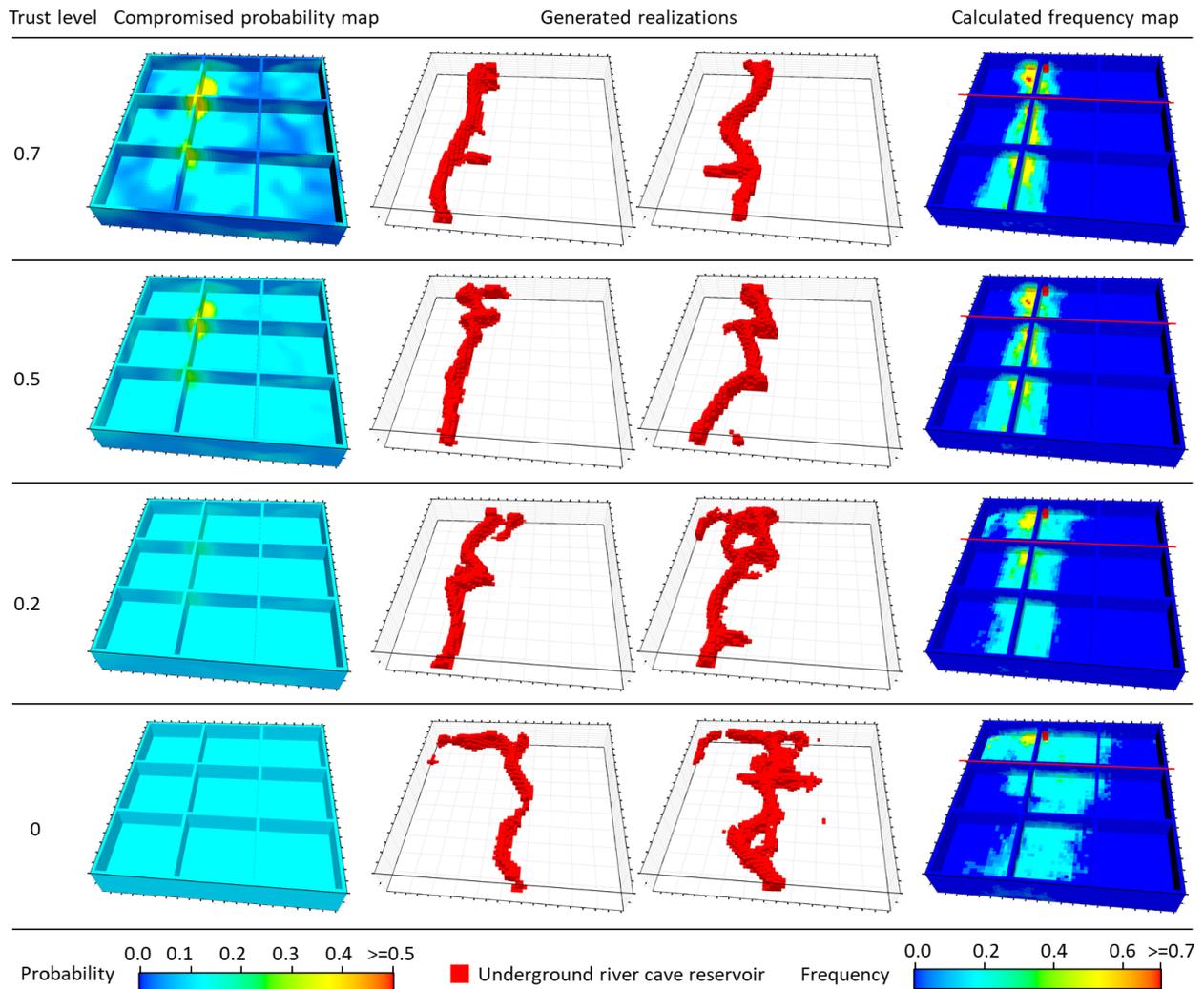
$$749 \quad p_t(x) = tp(x) + (1 - t)p_{prior}(x), \quad (3)$$

750 where x represents each cell of the geological model, $p_t(x)$ is the compromised probability map
 751 at trust level of t , $p(x)$ represents the original seismic probability map, and $p_{prior}(x)$ is the cave
 752 prior probability map prior to seismic survey. This prior probability map may come from
 753 geologic reasoning or well data interpretation. When changing the trust level from 1 to 0, the
 754 compromised probability linearly shifts from the complete seismic probability map to the
 755 complete prior probability map where no information about seismic data or its consequent
 756 probability map is kept.

757 In this case, we use the cave proportion interpreted from the three wells, 0.133, as the
 758 prior cave probability for each cell, i.e., $p_{prior}(x) = 0.133$. Then we decrease the trust level from
 759 1 to 0 (trust level = 0.7, 0.5, 0.2, and 0), and use the calculated compromised probability map at
 760 these levels and the well data for uncertainty geomodelling of caves, as shown in Figure 24. The
 761 case with trust level as 1 is just as discussed earlier in Figure 20, Figure 21, and Figure 22. It is

762 clear that as the trust level decreases to 0, the influence of the seismic probability map on the
 763 generated cave realizations and the calculated frequency map also decreases to 0, while the
 764 variability of the generated realizations gradually increases as is shown by the frequency maps
 765 (especially the vertical sections marked by red lines) of Figure 24. Note that all these generated
 766 realizations are realistic and are conditioned to the input well data with 100% accuracy.

767



768

769 **Figure 24** Compromised probability maps, generated realizations, and calculated frequency
 770 maps at various trust levels of the seismic probability map (i.e., 0.7, 0.5, 0.2, 0).

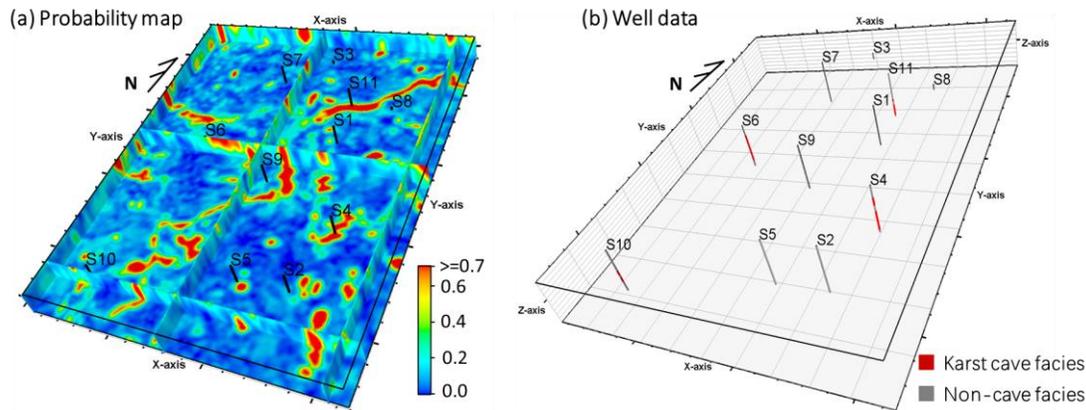
771

772 6.2 Field case 2 based on 336×256×96 cells

773 As described in Section 2.1, the generator trained on dataset of small size can be used for
 774 geomodelling of reservoirs with large arbitrary sizes by expanding the sizes of all inputs
 775 proportionally. Thus, here in case 2 we choose a field area of 4.2km×3.2km, with 11 wells, from
 776 Tahe cave reservoir. From well logs, 7 underground river karst cave intervals are recognized.
 777 The 3D probability map of underground river cave is calculated from seismic data with the same

778 method as in the previous case, i.e., by combining the seismic frequency energy attribute and the
 779 relationship between the attribute and cave occurrence probability obtained from the statistics of
 780 wells. Analyses of the well data and the probability map suggests that the underground river cave
 781 of this area should be distributed between 60m to 156m beneath the paleo-geographic surface.
 782 Thus, the geo-space inside this zone is divided into $336 \times 256 \times 96$ cells, with each cell
 783 representing 12.5m (length) \times 12.5m (width) \times 1m (height) which is the same as the training/test
 784 dataset. Figure 25 shows the probability map of karst cave and the distribution of well facies data
 785 in this $336 \times 256 \times 96$ -cell geo-space.

786



787

788 **Figure 25** 3D probability map of underground river karst cave and the distribution of well facies
 789 data in the study area. There are $336 \times 256 \times 96$ cells, with each cell representing 12.5m (length) \times
 790 12.5m (width) \times 1m (height).

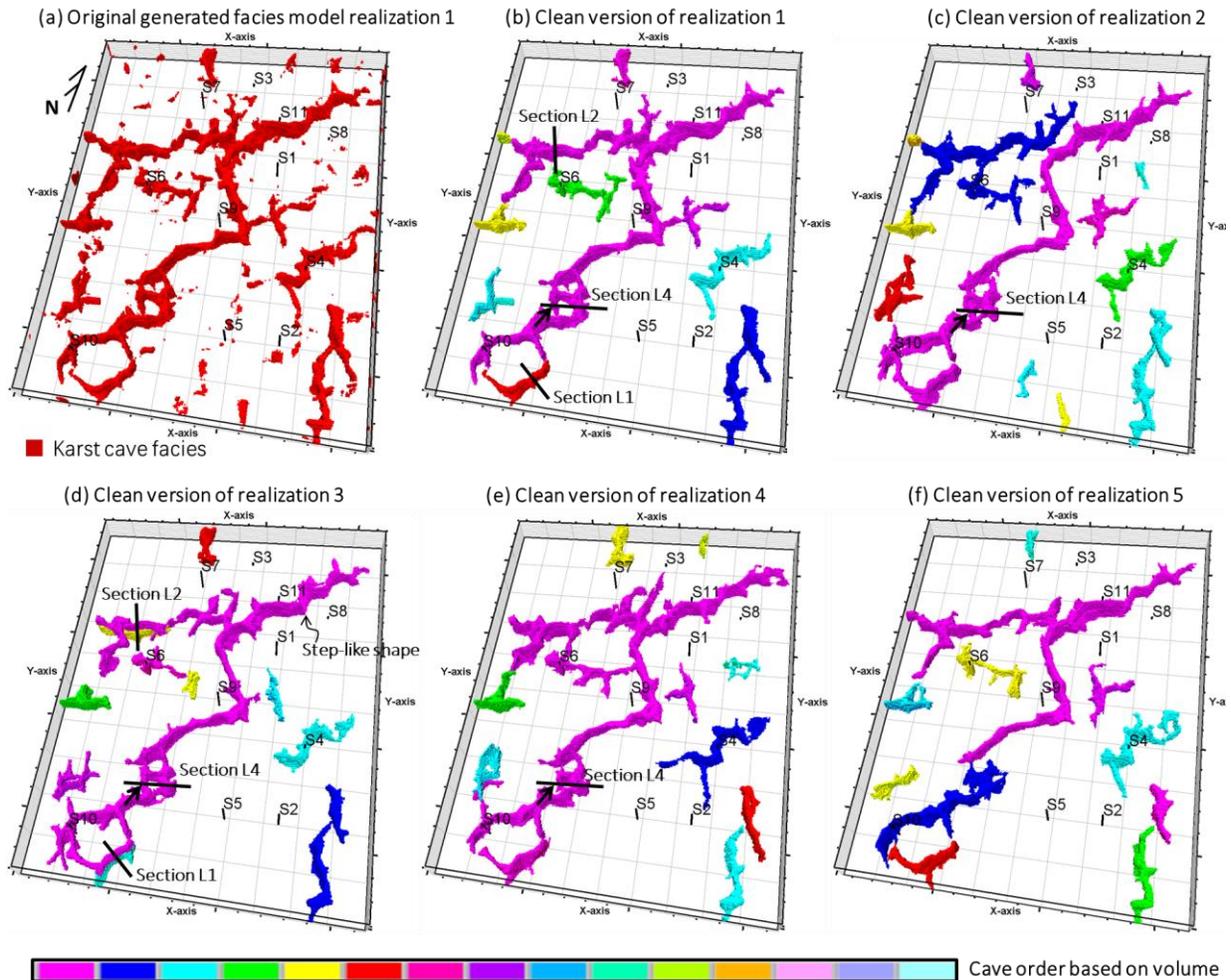
791

792 As illustrated earlier, the pretrained generator can produce facies models of any size that
 793 is 16 times of the input latent cube size. We take the 3D probability map (with size of
 794 $336 \times 256 \times 96$), the well data (with size of $336 \times 256 \times 96$), and 200 groups of random latent cubes
 795 (each group has 8 channels of random latent cubes with size of $21 \times 16 \times 6$) into the pretrained
 796 generator to produce 200 karst cave facies model realizations with $336 \times 256 \times 96$ cells. Each
 797 realization takes 0.988 seconds on average at 1 GPU (V100).

798 Figure 26 (a) shows one random realization. There are a small number of localized
 799 discrete caves with very small volume and without an apparent ribbon-like pattern in the
 800 realizations. As shown in Figure 27, most of these discrete localized features are not located at
 801 the local highs of the input probability map, thus proving the occurrence of this noise may not
 802 relate to the input probability map. One possible reason is that the generator is originally trained
 803 to produce $64 \times 64 \times 64$ -size facies models from $4 \times 4 \times 4$ -size latent cubes containing Gaussian
 804 random variables and one layer of zeros outside each latent cube for zero padding, but when the
 805 generator is applied here for $336 \times 256 \times 96$ -size facies model production, each $64 \times 64 \times 64$ -size
 806 patch of the generated facies model is calculated from $4 \times 4 \times 4$ -size latent cube patches (inside the
 807 input $21 \times 16 \times 6$ -size latent cubes) and one layer of Gaussian variables outside the latent cube
 808 patches for padding. Such a small change in the padding number may result in these small,
 809 localized noises in the generated facies models.

810 The volumes of these cave noises are generally smaller than 2000 cells (312,500m³). We
 811 remove these cave noises less than 2000 cells in post-processing. Figure 26 (b) shows the
 812 cleaned version of the realization in Figure 26 (a) after the post-processing removal of cave
 813 noise. The colors represent different simulated caves ordered by cave volume. Figure 26 (c) – (f)
 814 show the clean version of four other realizations.

815

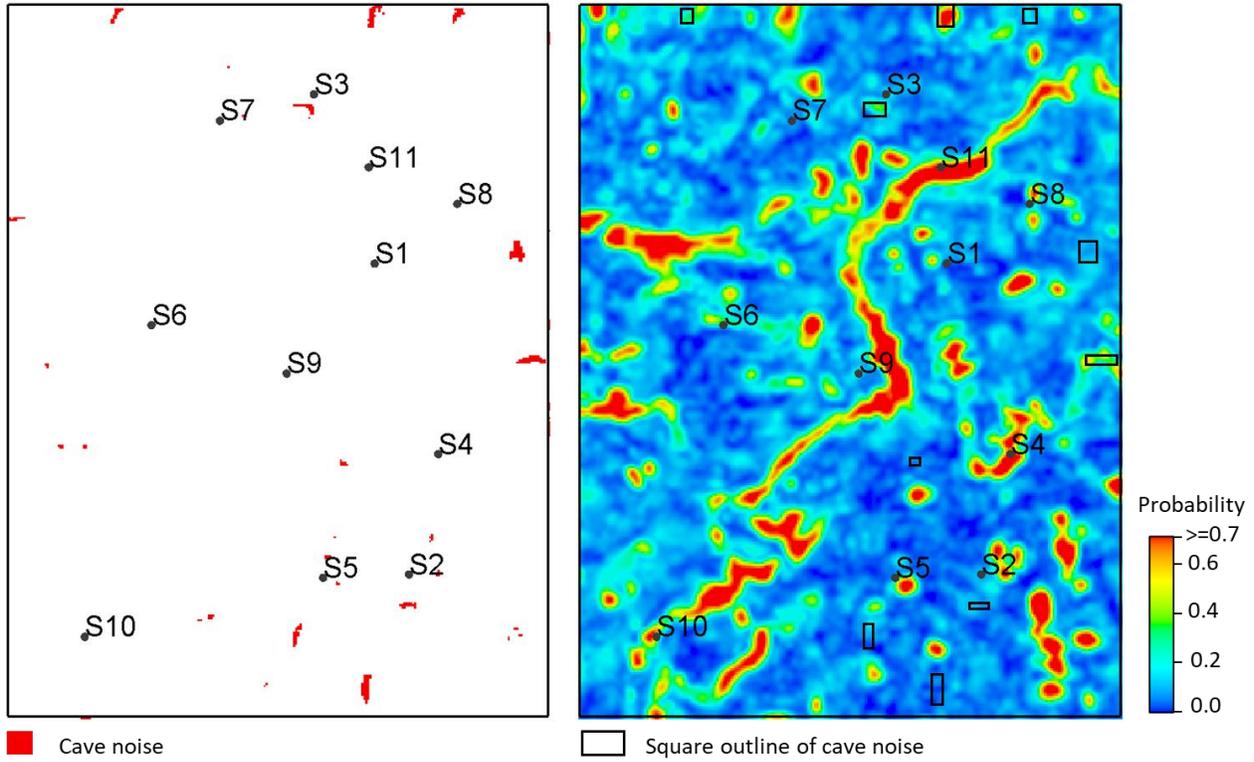


816

817 **Figure 26** Original (a) and post-processed clean version ((b) – (f)) of random realizations of
 818 underground river cave reservoir for the study area. Each realization has 336×256×96 cells, with
 819 each cell representing 12.5m (length) × 12.5m (width) × 1m (height). Only underground river
 820 cave facies are shown. The color represents the order of cave based on its volume.

821

822 (a) Planar section at 62m beneath the paleo-geographic surface



(b) Planar section at 81m beneath the paleo-geographic surface

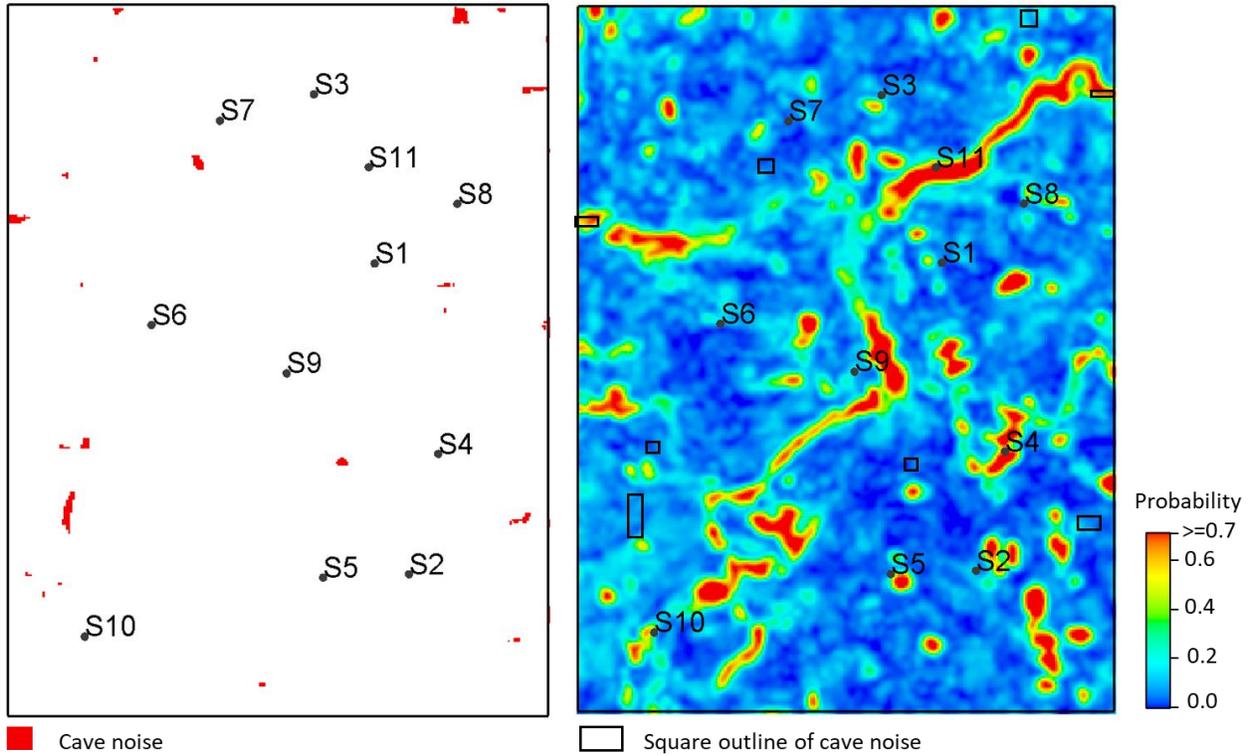


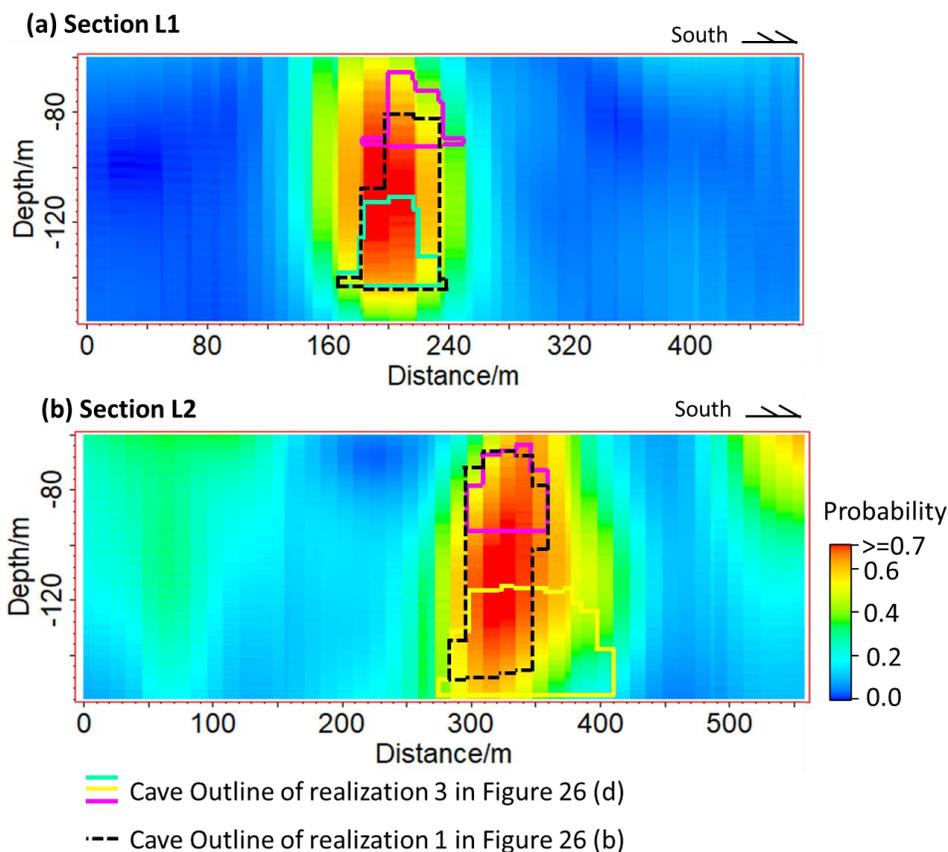
Figure 27 Cave noises (Left) of realization 1 in Figure 26 and the corresponding input probability map (Right) at different planar sections (62m and 81m beneath the paleo-geographic

828 surface). The large ribbon-like caves are not shown here. The outlines of these cave noises are
 829 overlain with the probability map.

830

831 The simulated caves in these realizations are consistent with the expected geological
 832 patterns, such as step-like cave shape (Figure 26 (d)), cave conduits with strikes of NNE, NNW,
 833 and nearly EW, sparse hall caves, etc. Especially, vertically-stacked double cave layers are
 834 simulated in some realizations, e.g., Figure 26 (d), at sections L1 and L2. Double cave layer
 835 feature does not exist in the training data yet it is common and important in the actual field. As is
 836 illustrated in Figure 28, double cave layers are simulated mainly because the high-value area of
 837 input probability map is large enough to accommodate double cave layers stacking vertically. Of
 838 course, one vertically large cave may also be simulated to replace double cave layers as shown in
 839 sections L1 and L2 in Figure 26 (b) and the black dashed lines in Figure 28. The simulated karst
 840 caves are also diverse. Note the various shapes of the largest pink karst cave in Figure 26. Figure
 841 28 also illustrates a variety of shape and location of the simulated caves: a large anomaly of high
 842 probability can give either one large cave or two vertically stacked smaller caves.

843



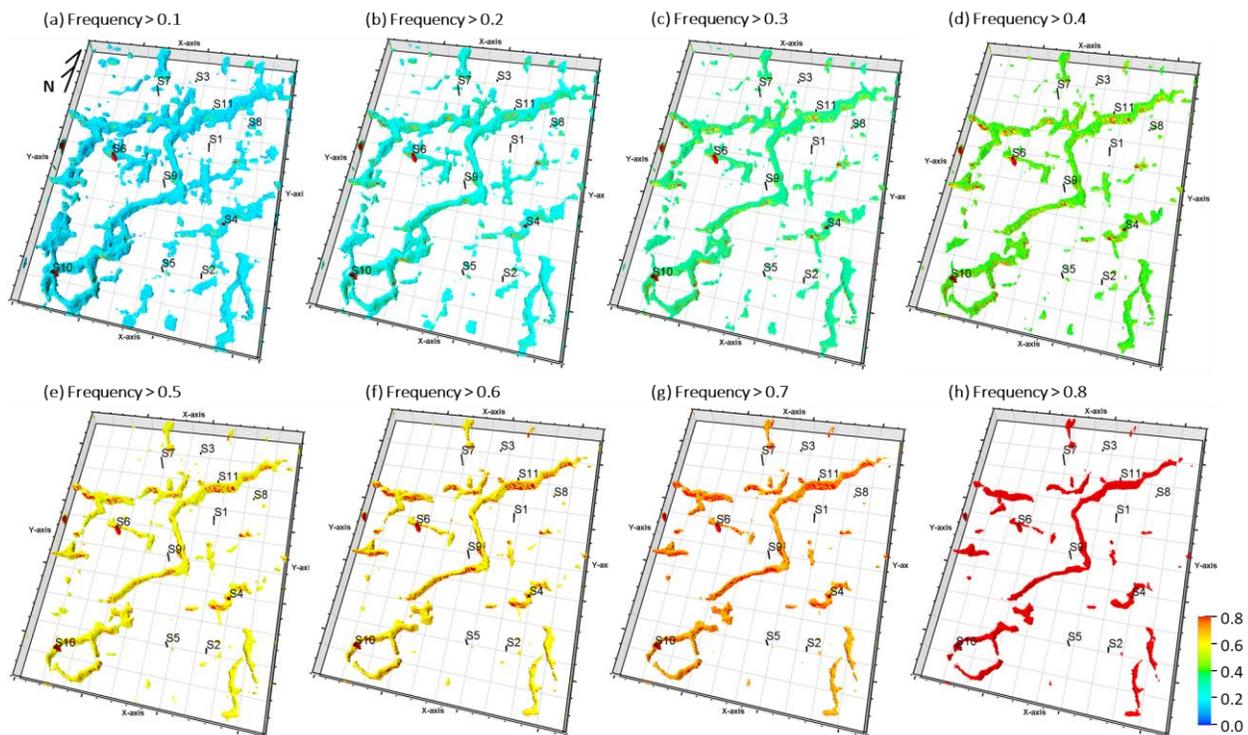
844

845 **Figure 28** Outlines of simulated caves overlap with the input probability map in two vertical
 846 sections L1 and L2. The caves are from simulated realization 1 and 3 in Figure 26 (b) and (d).
 847 The two sections are marked in the two subfigures.

848

849 Like the previous case, the shape and distribution of the simulated caves are consistent
 850 with the input probability map. We use the 200 simulated realizations before post-processing
 851 removal of cave noise to calculate a frequency map of karst caves. Figure 29 shows this
 852 frequency map filtered with various thresholds (i.e., 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, and 0.8). In
 853 Figure 30 and Figure 31, we compare the input probability map with the calculated frequency
 854 map at various horizontal and vertical sections. Most high-value areas of the frequency map are
 855 distributed inside high-value areas of the probability map, although some high frequency values
 856 are distributed at low probability value areas (e.g., section 4 and section 5 in Figure 30 and
 857 Figure 31). This is mainly because, the pretrained generator uses the learned geological pattern
 858 knowledge to connect several discrete high probability anomalies to form one connected
 859 underground river cave. The occurrence of the west cave of section L4 in Figure 26 (pointed by
 860 black arrows) is just this case. Additionally, many localized high-value anomalies of the input
 861 probability map correspond to zero value in the frequency map, such as the northwest corner in
 862 the planar section of Figure 30 and the west half of section L5 in Figure 31. These anomalies are
 863 essentially noises. The pretrained generator suppresses these noises using its learned geological
 864 pattern knowledge (e.g., the underground river cave should have ribbon-like shape). The well
 865 reproduction accuracy is 100% for both cave and non-cave facies types.

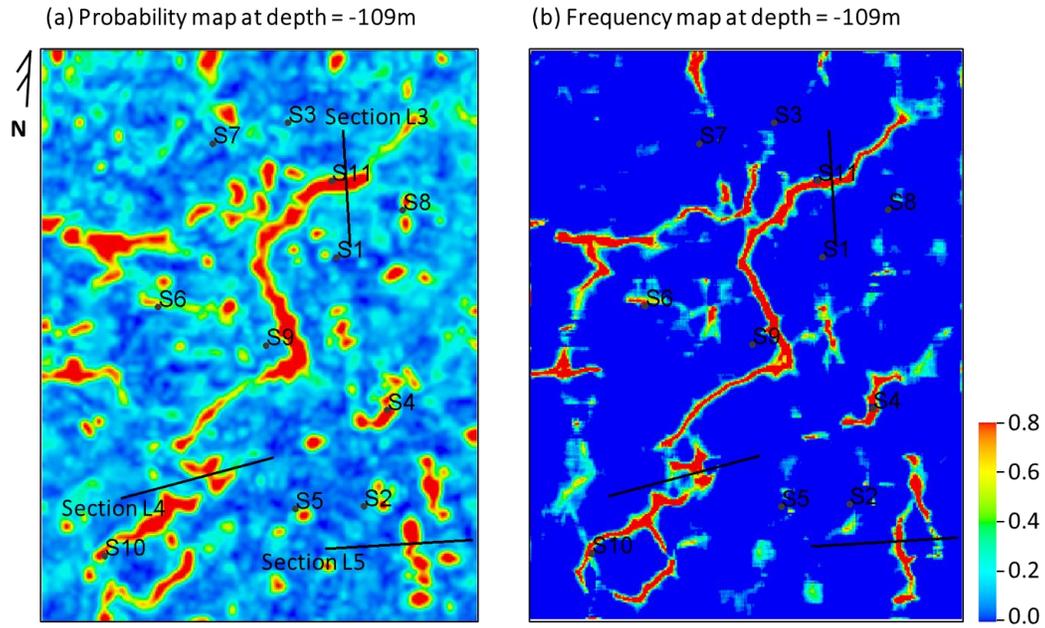
866



867

868 **Figure 29** Karst cave frequency map of simulated caves filtered with thresholds of 0.1, 0.2, 0.3,
 869 0.4, 0.5, 0.6, 0.7, and 0.8.

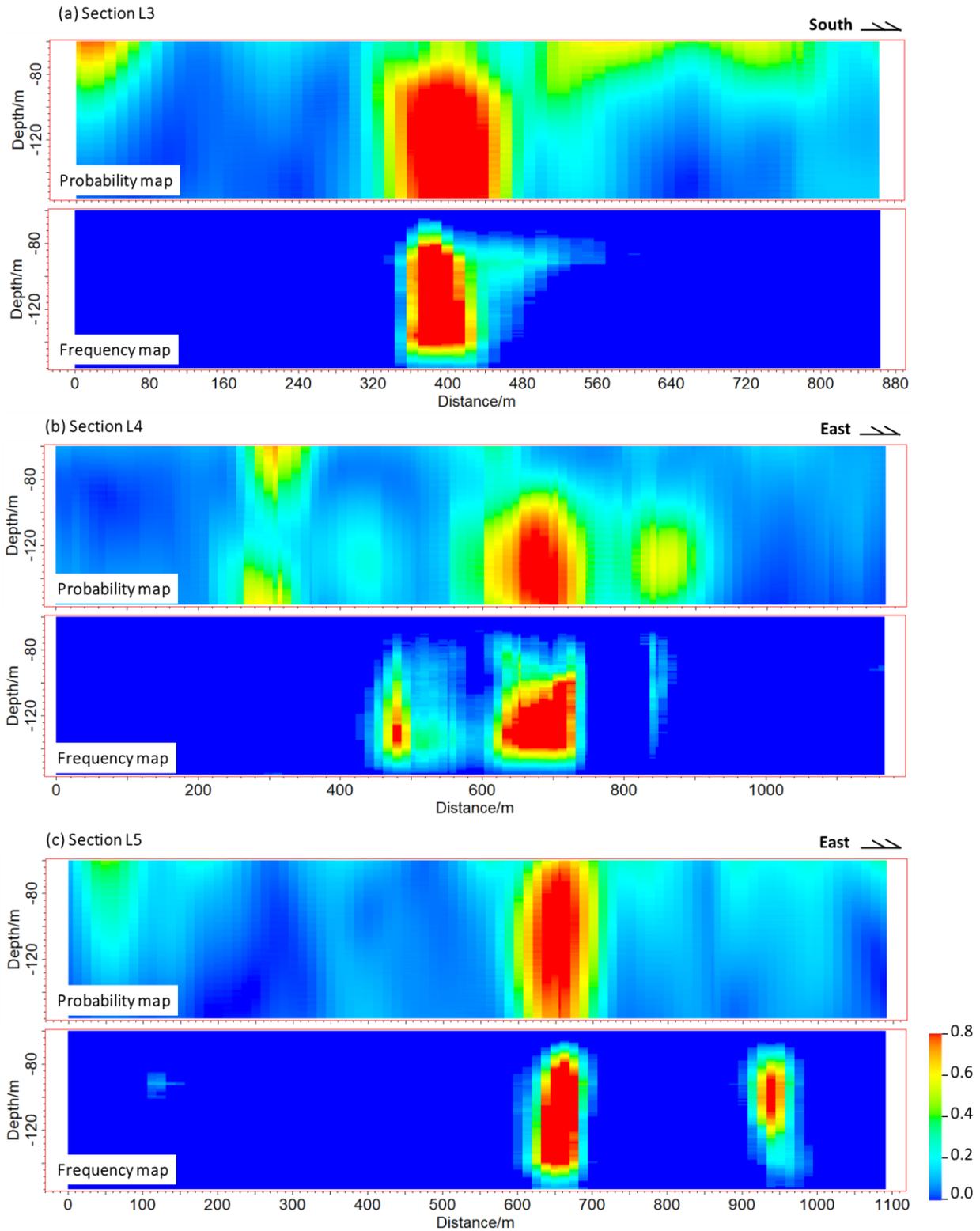
870



871

872 **Figure 30** Comparison of the input probability map and the calculated frequency map at a
873 horizontal section 109m beneath the paleo-geographic surface.

874



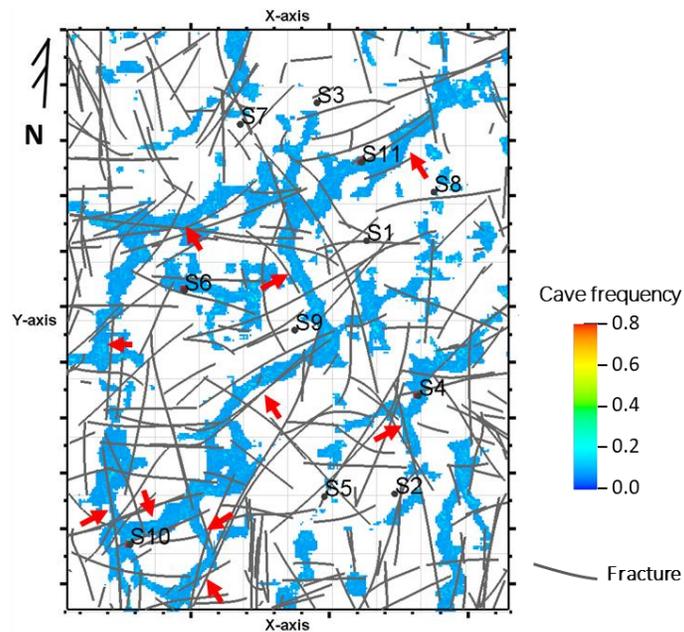
875

876 **Figure 31** Comparison of the input probability map and the calculated frequency map at various
 877 vertical sections. These sections are marked in Figure 30.

878

879 The fractures (may include some faults) of this study area are recognized from seismic
 880 data via ant tracking technique used by the commercial software Petrel. Uncertainty exists in
 881 these abstracted fractures, meaning that some of them are inaccurate, and some are not
 882 recognized. In Figure 32, the fractures are overlain with the calculated frequency map of
 883 underground river cave filtered with a threshold of 0.1. We can find most parts of the filtered
 884 frequency map coincide with the distribution of fractures, such as the areas pointed out by the
 885 red arrows in Figure 32. Figure 33 shows the relationship between the percentage of the visible
 886 cells in Figure 32 and the distance to fractures, indicating apparent inclination of generated caves
 887 to develop at the vicinity of fractures. Note there is a close genetic relationship between actual
 888 underground river caves and fractures as is discussed in Section 3.1. Thus, the overlap in Figure
 889 32 also proves the accuracy of the simulated karst caves.

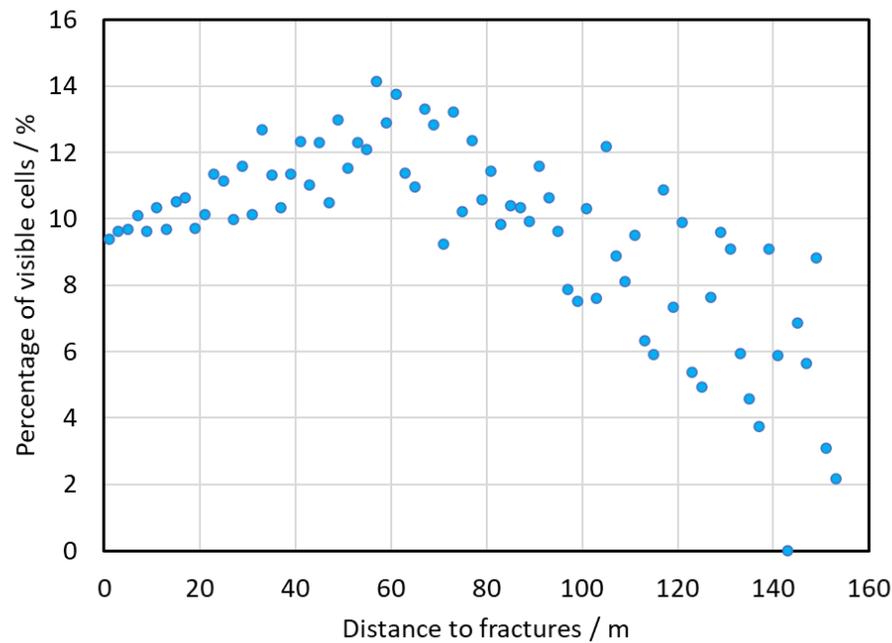
890



891

892 **Figure 32** Fractures obtained from interpreted seismic data using ant-tracking technique,
 893 overlain with the frequency map of underground river karst cave filtered with a threshold of 0.1
 894 for this study area. The red arrows point to the areas where the frequency map coincides with the
 895 fractures.

896



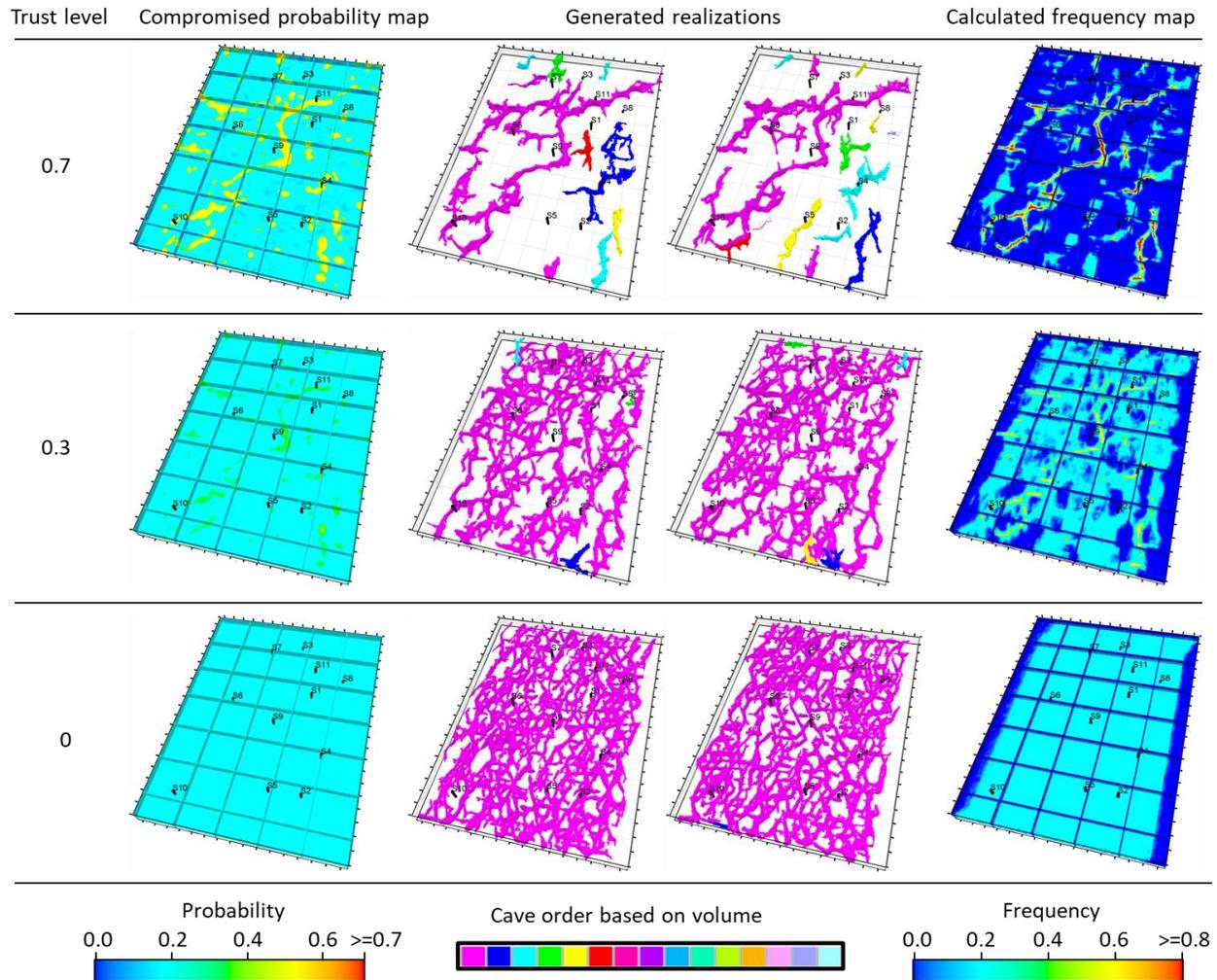
897

898 **Figure 33** Relationship between the percentage of the visible cells in Figure 32 (frequency map
 899 values larger than 0.1) and the distance to fractures.

900

901 Similar to the previous case, given decreasing trust levels for the probability map as 0.7,
 902 0.3, and 0, we calculate the compromised probability maps at the three trust levels with Equation
 903 (3). The cave proportion interpreted from wells, 0.191, is used as the prior-seismic cave
 904 probability for each cell (i.e., $p_{prior}(x) = 0.191$). Then the compromised probability maps and
 905 the original well data are used for uncertainty geomodelling. Figure 34 shows the compromised
 906 probability maps, simulated realizations, and the corresponding cave frequency maps for the
 907 three trust levels. As the trust level decreases, the original seismic probability map has less and
 908 less influence on the generated cave realizations and until no influence when the trust level
 909 equals 0. At the same time, the realizations become more and more various and finally
 910 completely consistent with the prior probability map (i.e., 0.191 for all cells) when the trust level
 911 equals 0. All generated realizations are realistic and conditioned to the input well data with 100%
 912 accuracy.

913



914

915 **Figure 34** Compromised probability maps, consequent generated realizations, and calculated
 916 frequency maps at various trust levels for the original seismic probability map (i.e., 0.7, 0.3, and
 917 0).

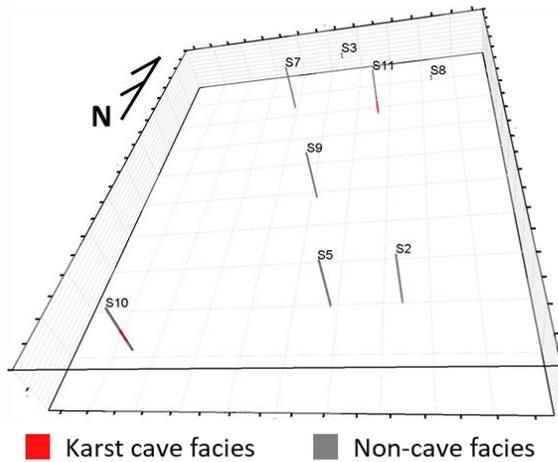
918

919 We also tried geomodelling with fewer conditioning wells. As Figure 35 (a) shows, the
 920 interpretation data of 3 wells (well S1, S4, and S6) are removed from the original input well data.
 921 Among the three removed wells, well S4 and S6 drill through underground river caves (see
 922 Figure 25). The new well data and the original seismic probability map are taken into the
 923 pretrained generator to produce 400 cave realizations. Figure 35 shows two random realizations
 924 and the cave frequency map calculated from these realizations, where the three removed wells
 925 are also shown. We can see from the generated realizations and the frequency map that, although
 926 the cave interpretation data of S4 is not taken as input, caves are produced at S4 in all
 927 realizations – with 100% cave frequency value at S4. This is due to the very strong conditioning
 928 effect of the input seismic probability map at the location of S4. For the location of well S6, the
 929 input seismic probability map has a rather weak conditioning (see Figure 25, Figure 27, and
 930 Figure 30), so no cave is produced at this location (Figure 35). Note that caves are produced at
 931 the location of S6 in all realizations when the interpretation data of S6 is used as input (Figure 26

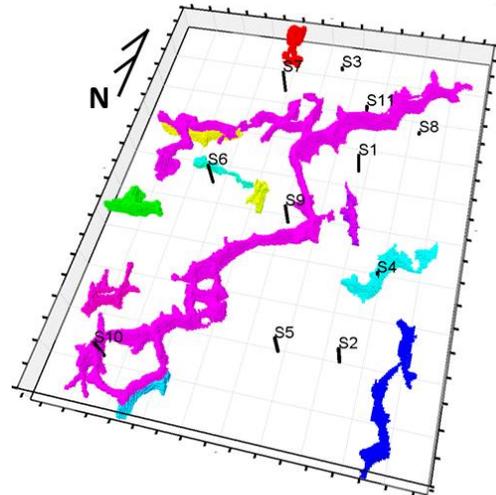
932 and Figure 29). Thus, as expected, well data improves the accuracy of the simulated earth
 933 models.

934

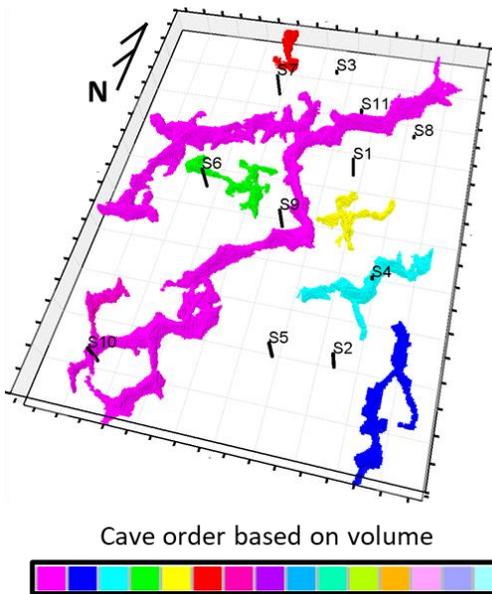
(a) Input well data (well S1, S4, and S6 are removed)



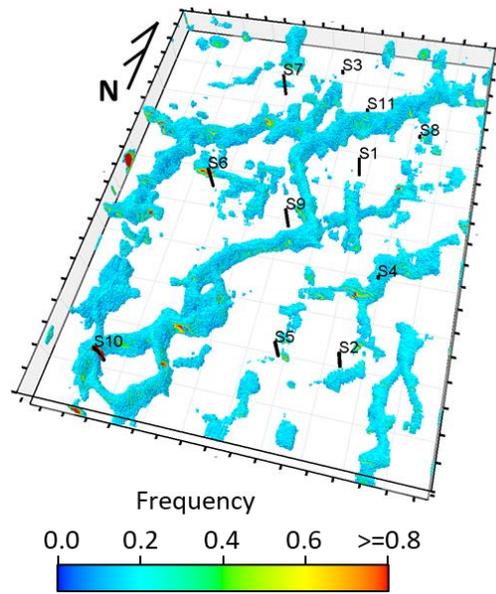
(b) Generated cave realization 1



(c) Generated cave realization 2



(d) Calculated cave frequency map (only show >0.1)



935

936 **Figure 35** New input well data after removal of the interpretation data of three wells (S1, S4, and
 937 S6), two random generated cave realizations, and the calculated cave frequency map filtered by
 938 the threshold of 0.1. The three removed wells are visualized in the realizations and the frequency
 939 map.

940

941 The above analyses shows that the generator trained at $64 \times 64 \times 64$ cells can be used to
 942 quickly produce various facies model realizations of any size, conditioned to probability map
 943 resulting from geophysical interpretation and well data. These realizations are consistent with

944 both the geological pattern and the input conditioning data. The local noises in the input
945 probability map are suppressed. Double cave layers are simulated in some realizations though
946 such a feature does not exist in the training data yet is common in the field, indicating a strong
947 generalization ability of the pretrained generator. Thus, the generator can be used for quick and
948 accurate conditional geomodelling and uncertainty assessment in other areas of Tahe cave
949 reservoir or other reservoirs with similar geological patterns. Given the uncertainty of the seismic
950 probability map itself, different trust levels for the probability map can be considered when
951 geomodelling.

952 **7. Conclusions**

953 This study proposes a 3D framework, GANSim-3D, for conditional geomodelling of
954 arbitrary sizes, based on Generative Adversarial Networks (GANs). The generator of GANSim-
955 3D only includes 3D convolutional layers, takes various 3D conditioning data and 3D random
956 latent cubes containing random numbers as inputs, and produces a 3D facies model. The
957 conditioning data can include sparse well facies data, low-resolution probability maps resulting
958 from geophysical interpretation, and global features like facies proportion, channel width, etc.
959 The original adversarial GANs loss augmented by condition-based loss is used to progressively
960 train the generator to learn geological patterns and relationships between conditioning data and
961 facies models. After training, when we fix the input conditioning data and randomly change the
962 input latent cubes, multiple realistic and conditional 3D facies model realizations are produced.
963 The trained generator can be used for geomodelling of reservoirs of large arbitrary sizes by
964 expanding the sizes of all inputs proportionally.

965 A field karst cave (underground river cave) reservoir in Tahe area of China is used as an
966 example to illustrate how GANSim-3D is used and test its field performances. We take well
967 facies and probability map resulting from geophysical interpretations as conditioning data. First,
968 642 large 3D conceptual models of Tahe cave reservoir are constructed using a newly proposed
969 process-mimicking simulation approach with parameters obtained from the field reservoir. These
970 conceptual models are unconditional and capture the geological patterns of Tahe cave reservoirs.
971 We crop these large-size conceptual models to build smaller-size 3D training facies models
972 which are then used to build 3D well facies and 3D probability maps as part of the training data.
973 Next, the generator and discriminator are trained for 40 hours, using 4 GPUs (NVIDIA Tesla
974 V100-PCIE-32GB), 20 CPUs, and 160G RAM in parallel. The trained generator is evaluated in
975 two synthetic cases with various metrics, showing excellent performances in producing realistic,
976 accurate, diverse, and conditional 3D facies models. Then, we use the generator for uncertainty
977 geomodelling of two 3D Tahe cave reservoirs of sizes $800\text{m} \times 800\text{m} \times 64\text{m}$ (divided into
978 $64 \times 64 \times 64$ cells) and $4200\text{m} \times 3200\text{m} \times 96\text{m}$ (divided into $336 \times 256 \times 96$ cells). In both cases,
979 multiple 3D realizations are quickly produced, being diverse and consistent with expected
980 geological patterns of Tahe cave reservoir and the input field conditioning data (3D well data and
981 3D probability maps). Additionally, it turns out that the generator can automatically suppress
982 localized noise patterns of the input probability map. Double cave layers are produced in the
983 second field case, which do not exist in the training facies models yet are common and important
984 in practice, indicating the robust generalization ability of the generator. Finally, various trust
985 levels for the probability map obtained from geophysical data are also considered when
986 geomodelling. Geomodelling with the trained generator is quite fast: each realization with
987 $336 \times 256 \times 96$ cells takes 0.988 seconds using 1 GPU (V100). The trained generator can be used

988 for uncertainty geomodelling of other areas of Tahe cave reservoir or other field reservoirs with
 989 similar geological patterns.

990

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 996 cluster at Stanford University. We would like to thank Stanford University and the Stanford
 997 Research Computing Center for providing computational resources and support that contributed
 998 to these research results.

999

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1117 **Supporting Information S1**1118 **Basics of GANs**

1119 GANs framework was initially proposed by Goodfellow (2014). It includes two neural
 1120 networks called generator (G) and discriminator (D), respectively. Given many observed real
 1121 samples as training dataset, the generator is designed to map a low-dimensional latent vector into
 1122 a sample, while the discriminator is designed to map the generated fake or real sample into a
 1123 value representing the probability of the input being real. The loss function in vanilla GANs
 1124 (Goodfellow et al., 2014) is

$$1125 \quad L(G_\theta, D_\varphi) = \mathbb{E}_{x_r \sim p_{data}} [\log D_\varphi(x_r)] + \mathbb{E}_{z \sim p_z} [\log (1 - D_\varphi(G_\theta(z)))] \quad (S1)$$

1126 where, L is the GANs loss, G and D are the generator and discriminator neural networks, θ and
 1127 φ are trainable parameters of G and D , p_{data} is the distribution of real samples, x_r is one of the
 1128 given real samples, z is input latent vector, p_z is the distribution of z , and \mathbb{E} is the expectation
 1129 operator. The last activation function of D is a sigmoid function. The discriminator and the
 1130 generator are alternatively trained by maximizing or minimizing this loss function, respectively.
 1131 Such alternative training process pushes the generator to learn the complete pattern knowledge
 1132 behind the given samples. The training stops until the generator produces very realistic samples
 1133 so that the discriminator cannot distinguish the fake samples from real ones. After training, the
 1134 generator is kept for practical generative applications. Aside from the above loss function
 1135 (Equation (S1)), several other forms of losses have also been proposed in recent years (Lucic et
 1136 al., 2017), among which the Wasserstein loss with gradient penalty (Gulrajani et al., 2017)
 1137 proved to have the best performances.

1138 Traditionally, all neural network layers of the generator and the discriminator are trained
 1139 concurrently, where the scale of pattern knowledge to be learned is not considered. Karras (2017)
 1140 proposed progressive GANs training approach (also called the progressive growing of GANs), in
 1141 which the layers of the generator and the discriminator are trained one by one to allow the
 1142 pattern knowledge to be learned gradually from coarse to fine scales. Progressive training
 1143 approach has proved to perform better than traditional training approach in training speed,
 1144 training stability, and quality of the results.

1145 GANs have been used for geomodelling of subsurface reservoirs with either the
 1146 traditional training approach (Chan & Elsheikh, 2017, 2019; Dupont et al., 2018; Laloy et al.,
 1147 2018; Mosser et al., 2020; Nesvold & Mukerji, 2021; Zhang et al., 2019) or the progressive
 1148 training one (Song et al., 2021a); with whichever way, the generator learns geological patterns
 1149 from given training facies models. With the learned pattern knowledge, the trained generator can
 1150 thus produce facies models consistent with the learned patterns, i.e., unconditional geomodelling.

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 1152 Lucic, M., Kurach, K., Michalski, M., Gelly, S., & Bousquet, O. (2017). Are GANs created
 1153 equal? A large-scale study. In *In Proceedings of the IEEE/CVF Conference on Computer*
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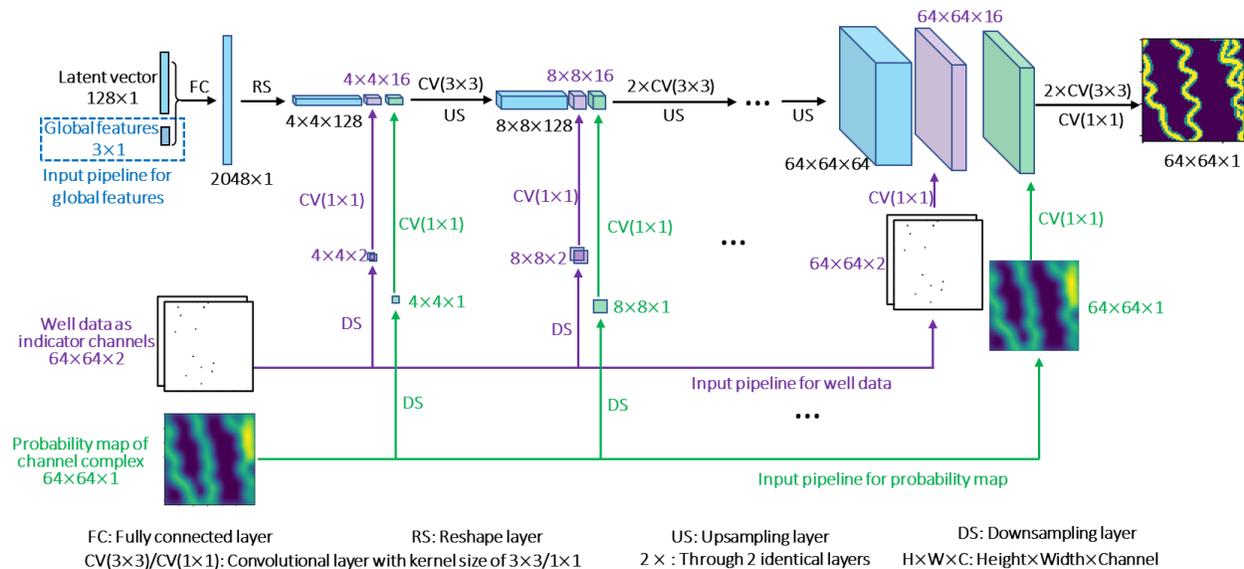
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1158 **Supporting Information S2**1159 **GANSim: a direct conditional simulation approach based on improved GANs**

1160 To directly achieve conditional geomodels, i.e., to get generated facies models consistent
 1161 with both the expected patterns and the given conditioning data (e.g., well data), first, the
 1162 generator needs to take in various conditioning data, and second, the generator needs to learn the
 1163 relationship between the input conditioning data and the output facies model in addition to the
 1164 geological pattern knowledge. Such a relationship is the key to achieve conditioning, and thus is
 1165 called conditioning ability. Song et al. (2021b, 2022) proposed the GANSim framework to
 1166 directly train such a generator, based on a progressive training method.

1167 In GANSim, three types of conditioning data are considered: non-spatial global features
 1168 of reservoirs (e.g., facies proportion and channel sinuosity), sparse well facies interpretations,
 1169 and spatially distributed probability maps of all facies calculated from geophysical data. Figure
 1170 S2 shows the input pipelines for the three types of conditioning data for an example of producing
 1171 2D facies models of sinuous channels. The conditioning data related to global features are
 1172 concatenated with the input latent vector and go through all layers of the generator. Well facies
 1173 data are first downsampled into various progressive resolutions (e.g., 8×8) and then converted
 1174 into feature cubes (e.g., the feature cube with size of $8 \times 8 \times 16$) with 1×1 convolutional layers.
 1175 These feature cubes are finally concatenated with the feature cubes at the same resolution (e.g.,
 1176 the feature cube with size of $8 \times 8 \times 128$) in the main pipeline of the generator. The input pipeline
 1177 for probability maps is the same as that of well data.

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1180 **Figure S2** Input pipeline design of generator for three types of common conditioning data (non-
 1181 spatial global features, sparse well facies data, and probability maps) in an example of producing
 1182 2D facies models of sinuous channels (modified from Song et al., 2022). Global features (e.g.,
 1183 facies proportion) are concatenated with the input latent vector; well data are first downsampled
 1184 into various resolutions and then converted into feature cubes with 1×1 convolutional layers,
 1185 which are further concatenated with other feature cubes of the same resolution in the main
 1186 pipeline; the input pipeline design for probability maps is the same as that of well data.

1187 To let the generator learn the conditioning ability, a specially designed loss function
 1188 (called condition-based loss function) is introduced in GANSim, while the original GAN loss
 1189 (Equation (S1)) is kept to guarantee the learning of geological pattern knowledge. The general
 1190 form of this condition-based loss function is

$$1191 \quad L(G)_{con} = \mathbb{E}_{z \sim p_z, con_{in} \sim p_{con}} Dist(f_{con}[G(z, con_{in})], con_{in}). \quad (S2-1)$$

1192 Here, con_{in} is the input condition (e.g., well facies data), p_{con} is the distribution of con_{in} , and
 1193 $Dist$ is some type of distance function. f_{con} is a predefined inversion function that maps the
 1194 generated facies model $G(z, con_{in})$ into the correct condition values it corresponds to. $L(G)_{con}$
 1195 essentially represents the inconsistency between the input condition con_{in} and the generated
 1196 facies model ($G(z, con_{in})$). By minimizing $L(G)_{con}$, the generator is forced to learn the
 1197 relationship between the input condition and the output facies model, i.e., conditioning ability.

1198 According to the general form (Equation (S2-1)), the condition-based loss function of
 1199 global features is specified as

$$1200 \quad L(G_\theta)_g = \mathbb{E}_{z \sim p_z, g \sim p_g} \| f_g[G(z, g)] - g \|_2, \quad (S2-2)$$

1201 where, g is input (one or multiple types of) global features, p_g is the distribution of all possible g ,
 1202 and f_g maps generated facies models into the corresponding real global features. In some cases,
 1203 f_g is hard to obtain, so an additional neural network may be trained as f_g (see Song et al., 2021b).
 1204 $Dist$ in Equation (S2-1) is specified as Euclidean L2 distance here, i.e., $\|\cdot\|_2$.

1205 The condition-based loss of well facies data is specified as

$$1206 \quad L(G_\theta)_w = \mathbb{E}_{z \sim p_z, w \sim p_w} \| I_{wloc} \odot [G(z, w)] - w \|_2, \quad (S2-3)$$

1207 where, w is input well data, p_w is the distribution of all possible w , I_{wloc} is the indicator of well
 1208 locations, and \odot is the element-wise product operator.

1209 The condition-based loss of probability map is specified as

$$1210 \quad L(G)_p = \mathbb{E}_{z_1, z_2, \dots, z_m \sim p_z, p \sim p_p} \| f_p[G(z_1, p), G(z_2, p), \dots, G(z_m, p)] - p \|_2, \quad (S2-4)$$

1211 where, z_1, z_2, \dots, z_m are random samples of the input latent vector z from its distribution p_z , p
 1212 represents input probability maps for all facies types, p_p represents the distribution of possible p ,
 1213 and f_p calculates the frequency map (in fraction) for each facies type from m generated facies
 1214 models. Parameter m is a predefined hyperparameter. For a facies type F , the value at each
 1215 location of the frequency map of this type is calculated by the sum of occurrence of that facies at
 1216 that location in the m generated models divided by m :

$$1217 \quad f_p = \frac{\sum_{i=1}^m \mathbb{I}[G(z_i, p) = C(F)]}{m}. \quad (S2-5)$$

1218 Here, $C(F)$ represents the code of facies type F , and the indicator function $\mathbb{I}(\cdot)$ equals 1 if the
 1219 condition inside the bracket is satisfied, otherwise equals 0.

1220 These three types of conditioning data are not necessarily all included depending on
 1221 specific cases of observed conditioning data. The total loss is a weighted combination of the
 1222 condition-based loss and the original GANs loss:

$$1223 \quad L(G, D)_{total} = \beta_1 L(G, D) + \beta_2 L(G)_{con}. \quad (S2-6)$$

1224 Here, β_1 and β_2 are predefined weights. When training the generator, the two types of losses are
 1225 both minimized, while only the GANs loss is maximized when training the discriminator.
 1226 GANSim uses the progressive training approach. For example, in Figure S2, the first

1227 convolutional layer (and the FC layer) at the resolution of 4×4 is initially activated and trained
1228 after taking in the 4×4 -conditioning well data and probability map; then, the following two
1229 convolutional layers at the resolution of 8×8 are further activated and trained together with the
1230 previous layers after taking in the 8×8 -conditioning data. In this way, all successively higher-
1231 resolution layers of the generator are gradually activated and trained to learn the geological
1232 patterns and conditioning ability from coarse to fine scales. After training, the generator can
1233 produce facies models consistent with both the geological patterns and the input conditioning
1234 data. Once the input latent vector changes, multiple conditional facies model realizations are
1235 produced.

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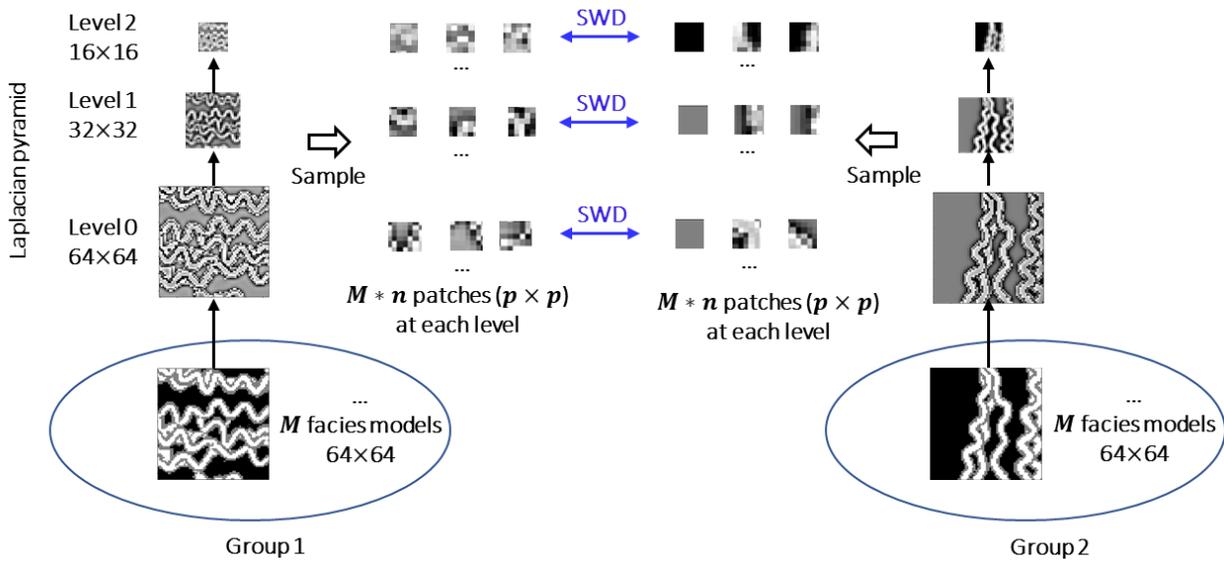
1238 **Supporting Information S3**

1239 **Multi-Scale Sliced Wasserstein Distance combined with Multi-Dimensional Scaling (MS- 1240 SWD-MDS)**

1241 MS-SWD is proposed by Karras et al., (2017) to evaluate the distance in multi-scale
1242 spatial structures between two groups of data. Here we show the calculation steps of MS-SWD
1243 with an example of two groups of facies models (64×64 , 2D). As Figure S3 shows, each group
1244 contains M facies models. First, the Laplacian pyramid representations (Burt & Adelson, 1987)
1245 of each facies model in both groups is calculated from resolution of 64×64 to 16×16 . The
1246 Laplacian pyramid representations reveal the structures of the original facies models at different
1247 scales. Second, multiple (n) small patches ($p\times p$ pixels) are randomly extracted from the
1248 Laplacian pyramid representation of each facies model at each level, to obtain $M*n$ patches from
1249 each group of facies models at each level. Third, these patches are normalized with respect to the
1250 mean and the standard deviation of each patch. Finally, the sliced Wasserstein distance (SWD),
1251 an efficient approximation to the Wasserstein distance (Rabin et al., 2012), between the patches
1252 from each group at each level is calculated. MS-SWD over different levels can be averaged as
1253 single value to represent the distance between two distributions.

1254 MDS is commonly used to project high-dimensional data into 2D or 3D space to
1255 visualize their relationship, based on certain type of distance between each pair of the data. MS-
1256 SWD is originally used to calculate the distance between two large groups of data. Song et al.,
1257 (2021a) proposed to combine MS-SWD with MDS (MS-SWD-MDS) to project the two groups
1258 of data into 2D space. In the method, each large group is divided into many small groups, and the
1259 MS-SWD is calculated for each pair of the small groups inside the two large groups. Then, all
1260 small groups are projected into 2D space using MDS, based on the calculated MS-SWD (average
1261 of MS-SWD) among these small groups. Each point in MDS represents one small group of data.
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Figure S3 Schematic illustration of how MS-SWD is calculated with an example of two groups of 2-D facies models.

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Burt, P. J., & Adelson, E. H. (1987). Readings in computer vision: issues, problems, principles, and paradigms. In M. A. Fischler & O. Firschein (Eds.) (pp. 671–679). San Francisco, CA, USA: Morgan Kaufmann Publishers Inc. Retrieved from <http://dl.acm.org/citation.cfm?id=33517.33571>

Rabin, J., Peyré, G., Delon, J., & Bernot, M. (2012). Wasserstein barycenter and its application to texture mixing. In A. M. Bruckstein, B. M. ter Haar Romeny, A. M. Bronstein, & M. M. Bronstein (Eds.), *Scale Space and Variational Methods in Computer Vision* (pp. 435–446). Berlin, Heidelberg: Springer Berlin Heidelberg.

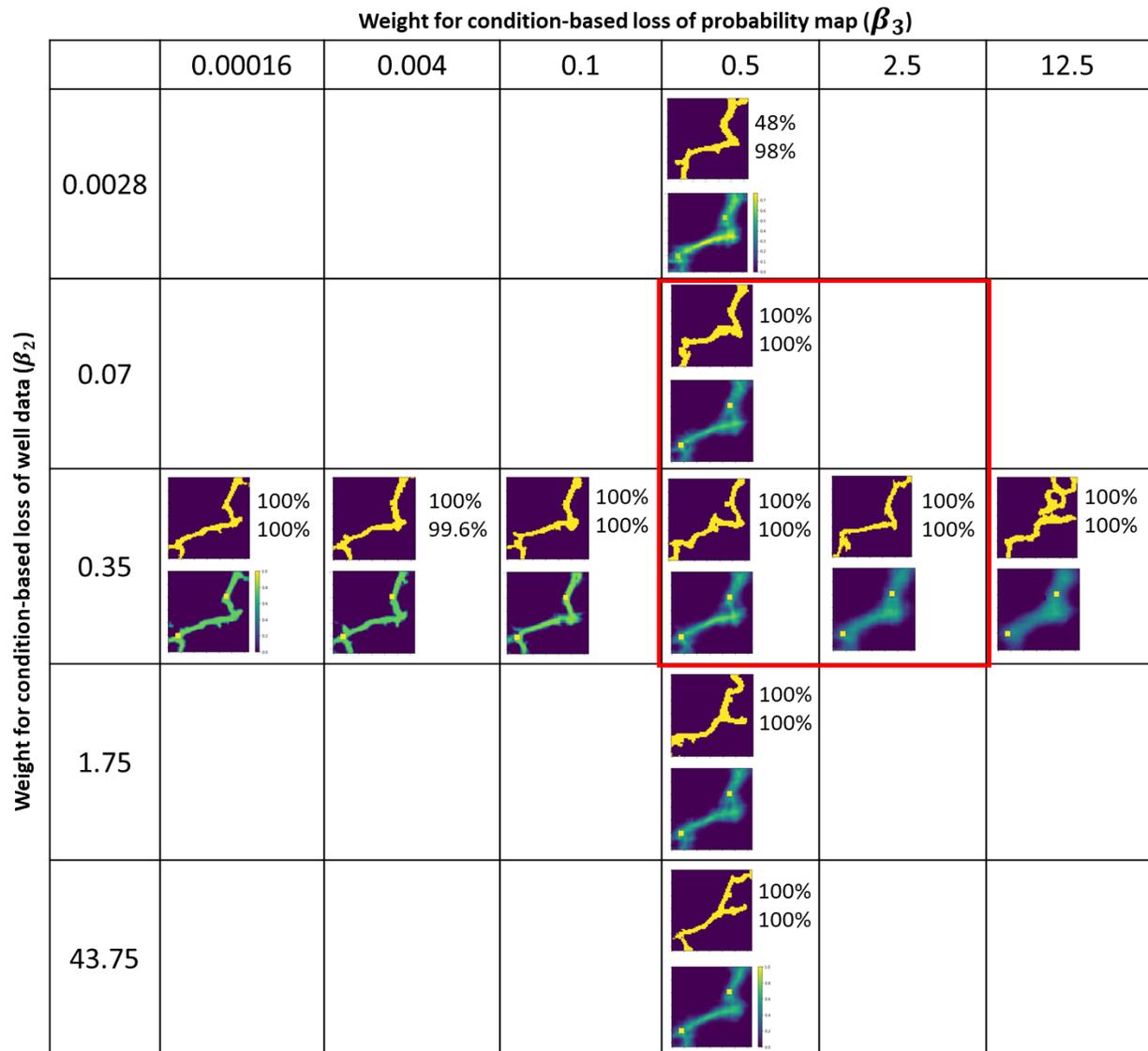
1279 Supporting Information S4

1280 Brief discussions about the weights for various types of losses

1281 Training of the GANs used in this paper involves three types of losses including the
 1282 original GANs loss and the condition-based losses for well data and probability map (Equation
 1283 (1)). We tried different combinations of the weights for these losses and then evaluated the
 1284 performances of the trained generators in terms of the conditioning effect to input well data and
 1285 probability map, diversity, and realism of the produced facies models. As shown in Figure S4,
 1286 the weight for original GANs loss (β_1) is set as 1, while the weights for well data and probability
 1287 map condition-based losses (β_2 and β_3) ranges from 0.0028 to 43.75 and 0.00016 to 12.5,
 1288 respectively. We take the well data and probability map of the synthetic case 2 (Figure 12 (b) and
 1289 (c)) into the trained generators. Figure S4 shows the produced random facies models, the
 1290 frequency maps of cave facies, and the well data reproduction accuracy values for each weight
 1291 combination.

1292 We suggest to set the weight for well condition-based loss (β_2) and the weight for
 1293 probability map condition-based loss (β_3) as 0.07 to 0.35 and 0.5 to 2.5, while the weight for the
 1294 original GANs loss is set as 1. The weight β_2 represents a tradeoff between the conditioning of
 1295 well data and the realism of the facies models produced by the trained generator, while β_3
 1296 represents a tradeoff between the diversity and the realism of the produced facies models. Note
 1297 that the process of enforcing the consistency of the produced facies models with the input
 1298 probability map (by using a larger β_3 value) is essentially a process of strengthening the diversity
 1299 of the produced facies models.

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1302 **Figure S4** Horizontal sections (32nd layer of cells from top) of the produced random facies
 1303 models (top) and frequency maps of cave (bottom) and well data reproduction accuracy values
 1304 for cave and non-cave facies types resulting from generators trained using different weight
 1305 combinations. The weight for the GANs loss is set as 1.

1306