Supplement: Meltwater generation in ice stream shear margins: case study in Antarctic ice streams

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COMPARISON TO NUMERICS FOR DIFFERENT SHEAR HEATING RATE

Figure S1 shows comparison to numerics for $\mathtt{Br} = 6$. The comparison shows good agreement with the analytical estimates presented in this study and the numerics for a different Brinkman number, and therefore a different temperate zone thickness ($\sim 38\%$ of the ice thickness).

DEPENDENCE OF FLUX ON δ

Figure S2 shows results for varying δ. The parameter δ varies for many orders of magnitude and does not significantly affect porosity or meltwater flux. The difference in flux between orders of magnitude variations in δ is < 0.05 in nondimensional flux.

EFFECT OF Θ

Here, we compute ice temperature using the model derived in Meyer and Minchew (2018), which makes the implicit assumption that all of the work done during ice deformation is dissipated as heat. However, this assumption has not yet been evaluated and this may overestimate the rate of heating in shear margins, as some fraction of that work is actually stored in the ice microstructure. We defined a parameter Θ to represent the fraction of work done during deformation that is dissipated as heat, in which $\mathtt{Br} = \mathtt{Br}(\Theta)$.

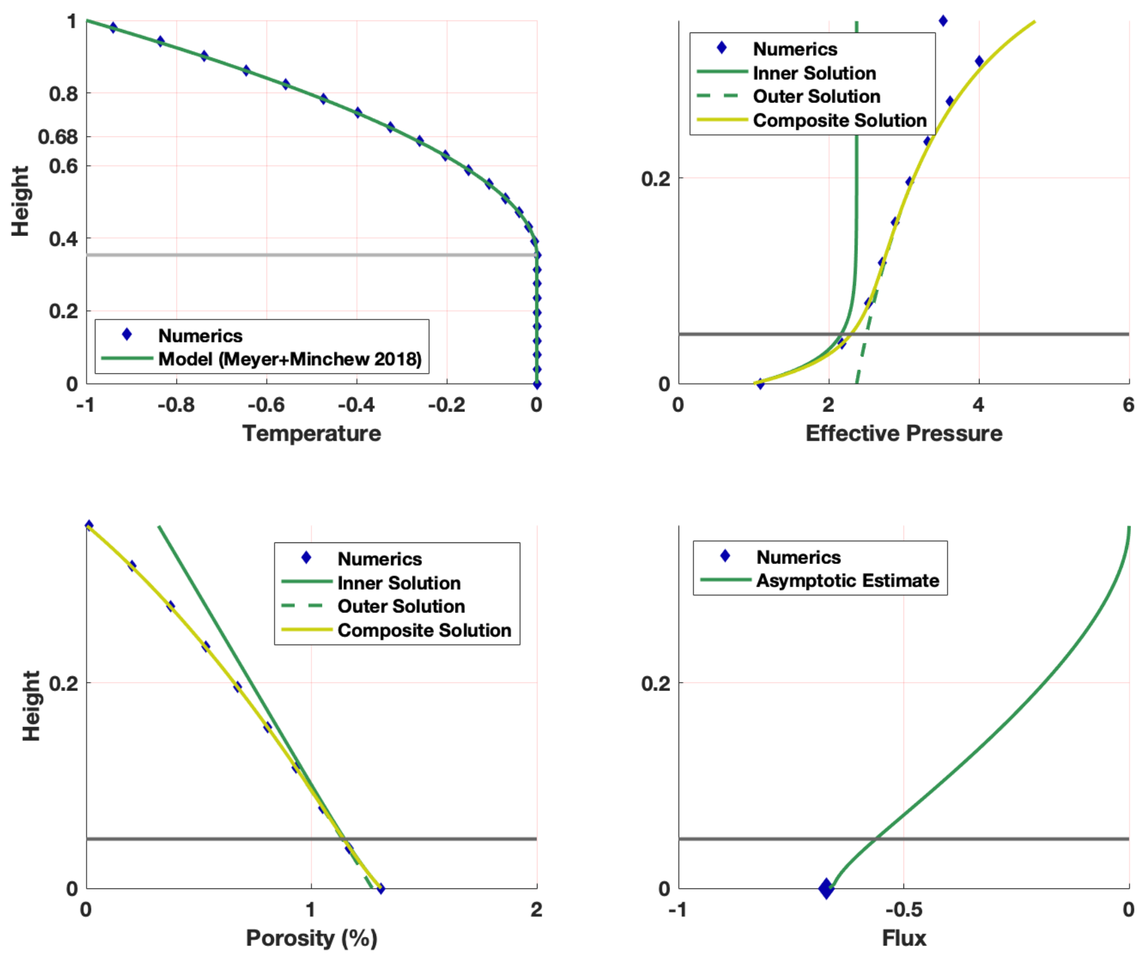

Fig. S1. Ice temperature, effective pressure, porosity, and meltwater flux, compared to numerics. In comparison to numerics, we let $H = 200$, $\texttt{Pe} = -1.1115$, $\kappa = 0.4416$, $\texttt{Br} = 6$, $\delta = 0.0023$, $\alpha = 2$, $\Delta T = 1$, $N_0 = 1$.

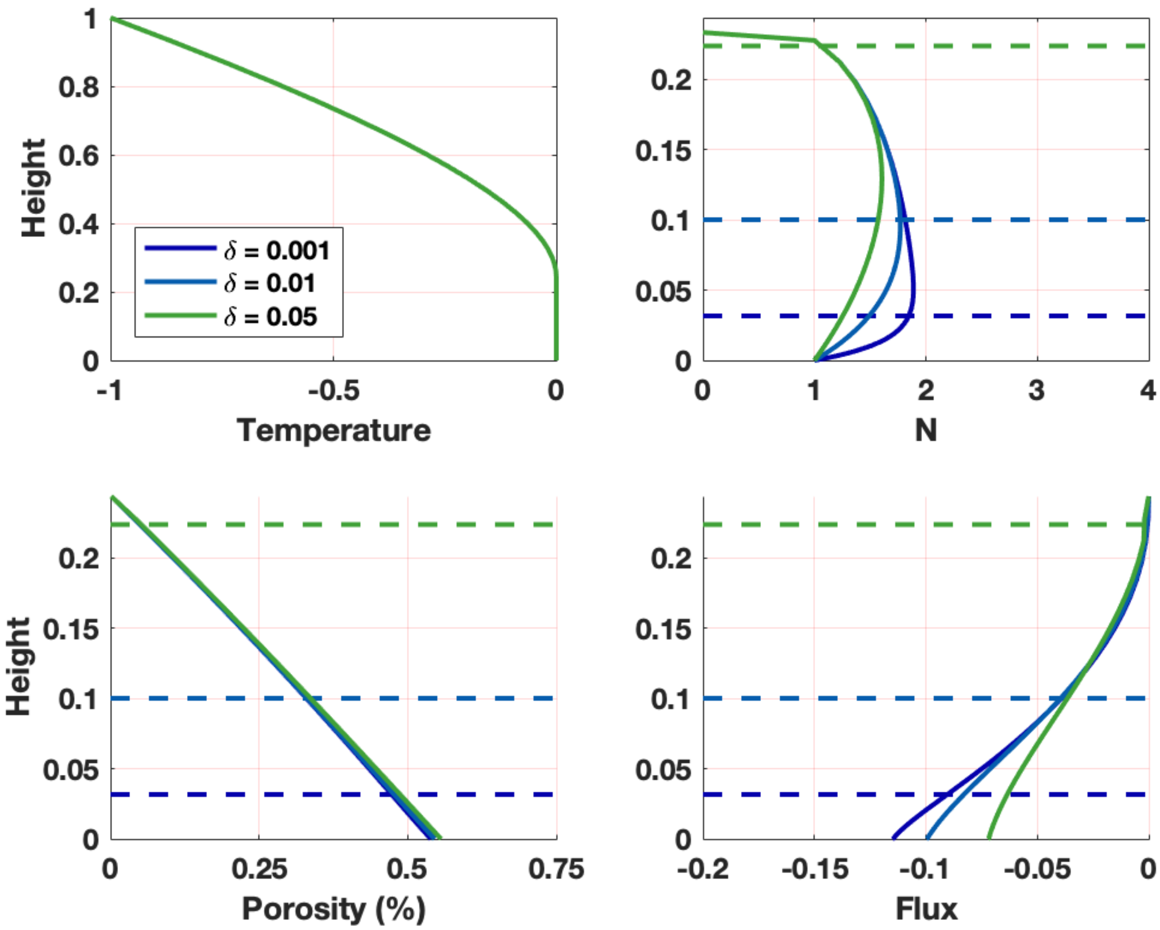

Fig. S2. Ice temperature, effective pressure, porosity, and meltwater flux, computed with varying δ. The other parameters are $h = 1000$ m, $\mathtt{Pe} = -2.5$, $\kappa = 0.52$, $\mathtt{Br} = 6$, $\alpha = 2.5$, $\Delta T = 25$ K, $N_0 = 1$.

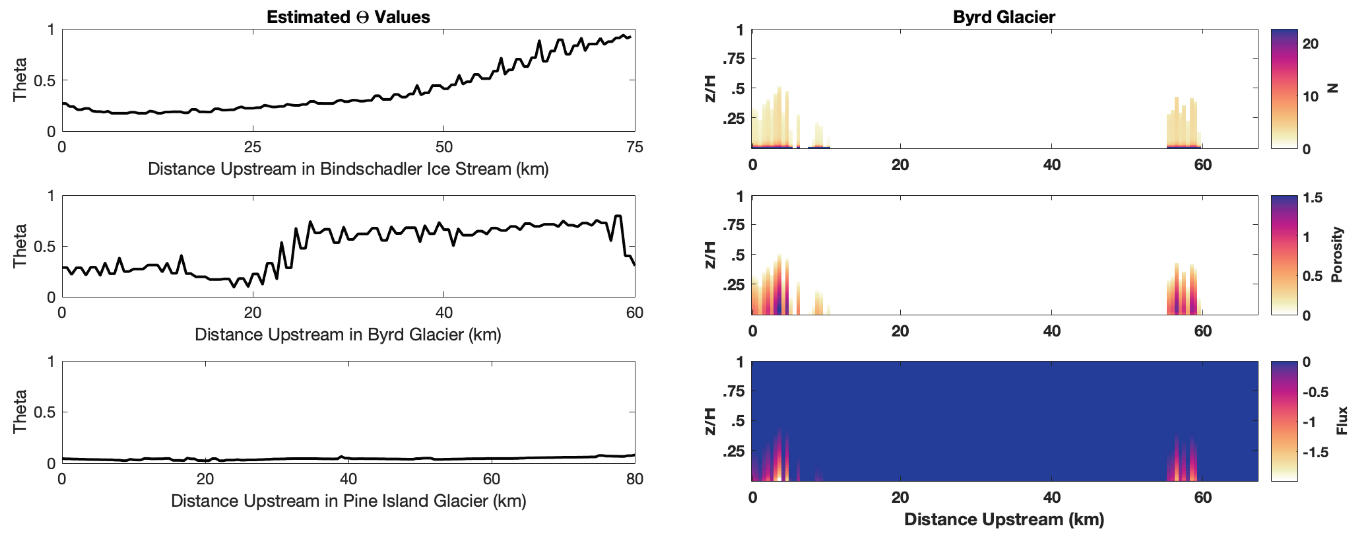

Fig. S3. Effect of considering the partitioning of deformational work into stored energy and dissipated heat: (a) Estimates of Θ, the fraction of deformational work that is dissipated as heat, in Bindschadler, Byrd, and Pine Island Glacier margins. (b) Estimates of effective pressure, porosity, and meltwater flux in Byrd Glacier accounting for the fraction of deformational work that is dissipated as heat.

Here, we show how the results presented in the main text change when incorporating Θ and calculating ice temperature assuming that only a fraction Θ of mechanical work is dissipated as heat.

Figure S3a shows estimated Θ values for Bindschadler Ice Stream's southern margin, Byrd Glacier's northern margin, and Pine Island Glacier's southern margin. In general, the value of Θ decreases closer to the margins, as Θ decreases with increasing strain rate. The value of Θ is therefore particularly low in Pine Island Glacier, which has rapidly deforming margins.

For both Bindschadler's southern margin and Pine Island Glacier's southern margin, incorporating Θ results in no estimated temperate zone, and therefore no meltwater flux at the bed. Byrd Glacier, on the other hand, still retains a smaller temperate zone due to the lower ice thickness in its margin. However, there is much less temperate ice and therefore the meltwater flux at the bed (in nondimensional terms) is approximately an order of magnitude less. More detail on the effect of Θ on ice temperature, ice softness, and the presence of temperate ice is left for future work and, notably, more work needs to be done to verify the energetics that occurs when ice deforms rapidly in order to correctly estimate meltwater flux out of temperate zones. For now, we stick with the canonical assumption that $\Theta = 1$, meaning all work done to deform the ice is dissipated as heat.

EFFECT OF TEMPERATURE-VARYING A

In this study, we use a constant value for the ice softness parameter A, in which we define A to be the flow-rate parameter that corresponds to ice at its melting point. However, typically A is represented as a function of ice temperature T by

$$A = A_0 \exp\left[-\frac{Q}{RT}\right] \tag{1}$$

where Q is the activation energy for creep, R is the ideal gas constant, and A_0 is a constant prefactor. This parameter then enters the equation for ice flow as

$$\dot{\epsilon} = A\tau^n \tag{2}$$

where $\dot{\epsilon}$ is effective strain-rate, τ is effective deviatoric stress, and n is the stress exponent. In Figure S4, we show results for using a coupling between ice temperature and A. Including the temperature-dependent A results in larger and more extensive temperate ice zones in all regions studied, suggesting that this would produce larger meltwater fluxes than those presented in this study.

REFERENCES

Meyer CR and Minchew BM (2018) Temperate ice in the shear margins of the Antarctic Ice Sheet: Controlling processes and preliminary locations. *Earth and Planetary Science Letters*, **498**, 17–26 (doi: 10.1016/j.epsl.2018.06.028)

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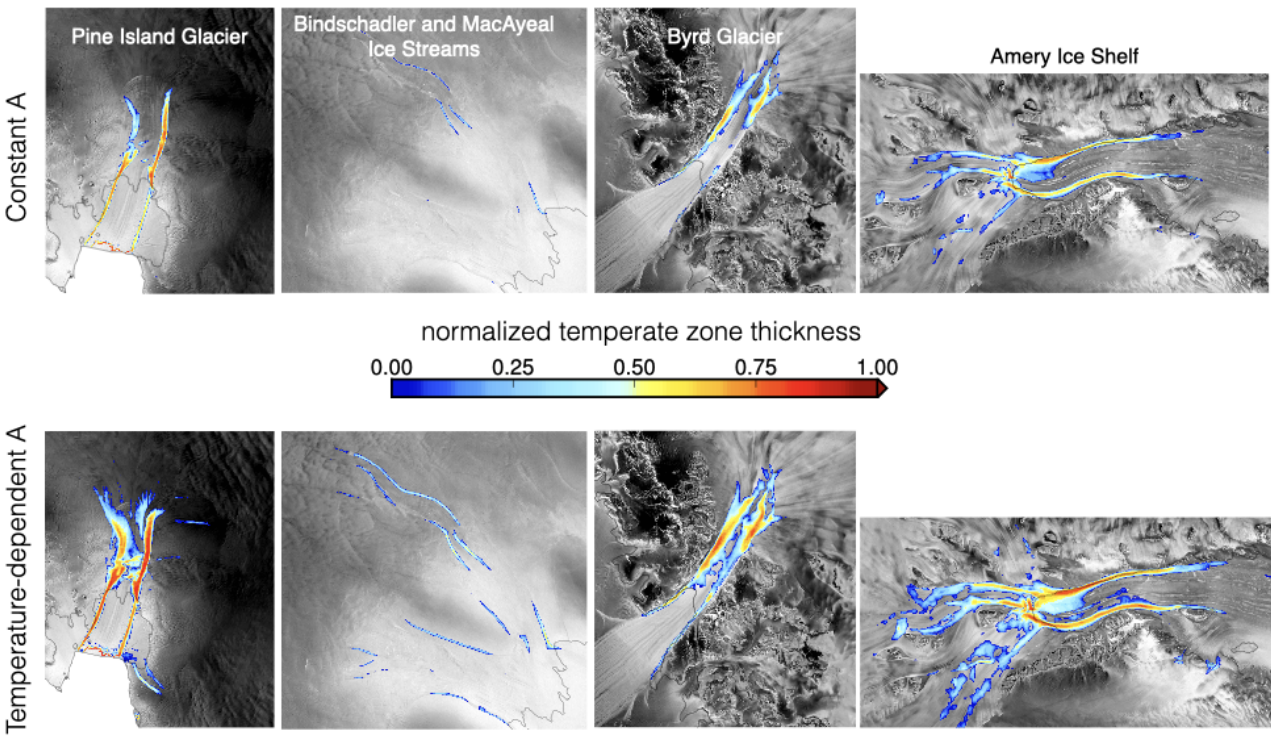

Fig. S4. Thickness of temperate ice zone, computed from the model presented in Meyer and Minchew (2018), for (top row) constant ice softness parameter A, and (bottom row) temperature-dependent ice softness parameter A.