The size-distribution of Earth's lakes and ponds: limits to power-law behavior

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This paper is a non-peer reviewed preprint submitted to EarthArXiv. This paper is in review at *Frontiers in Environmental Science*.

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ABSTRACT

Global-scale characterizations of Earth's lakes and ponds assume their surface areas are power-law distributed across the full size range. However, empirical power-laws only hold across finite ranges of scales. In this paper, we synthesize evidence for upper and lower limits to power-law behavior in lake size-distributions. We find support for the power-law assumption in general. We also find strong evidence for a lower limit to this power-law behavior, although the specific value for this limit is highly variable (0.001 - 1 km^2), corresponding to orders of magnitude differences of the total number of lakes and ponds. The exact mechanisms that break the power-law at this limit are unknown, but we are able rule out mapping errors as a first-order factor. There is no evidence for an upper limit to power-law behavior at the global scale. There is inconsistent evidence for an upper limit at regional-scales. Explaining variations in these limits stands to improve the accuracy of global lake characterizations and shed light on the specific mechanism responsible for forming and breaking lake power-law distributions.

Keywords: Limnology, Lake Size Distribution, Caspian Sea, Pond Abundance, Power Law

1 INTRODUCTION

From first principles, it is expected that lake areas should be power-law distributed Russ (1994); Seekell et al. (2013); Cael et al. (2015); Cael and Seekell (2016). This is because the areas enclosed by the contours formed on a level-set of a self-affine fractal surface are power-law distributed Mandelbrot (1983); Russ (1994); Seekell et al. (2013), and Earth, whose lake shorelines are analogs to these contours, has an approximately self-affine fractal topography Mandelbrot (1983); Russ (1994); Seekell et al. (2013). However, empirical power-laws hold across a finite range of scales and frequently exhibit exponential truncation. Such limits to power-law behavior are poorly explored for lakes, and estimates of global lake characteristics typically assume power-law behavior across the full size-distribution Downing (2009, 2010); Seekell and Pace (2011); Seekell et al. (2013); Cael and Seekell (2016). This is a critical assumption because ponds and small lakes are omitted from maps Benson and MacKenzie (1995); Seekell (2018). (n.b. here we do not distinguish lakes from ponds by a cut-off size; 'lake' refers to both lakes and ponds). The abundance and area of these small lakes is therefore estimated by extrapolating a power-law fit of large, well mapped lakes Meybeck (1995); Birkett and Mason (1995); Ryanzhin (2005); Downing et al. (2006);

Downing (2009, 2010); Messsager et al. (2016); Lazzarino et al. (2009); Minns et al. (2008); Chumchal et al. (2016); Seekell (2018); Tamrazyan (1974); Pace and Prairie (2005); Kastowski et al. (2011). This extrapolation will massively overestimate the abundance of lakes if there are limits to power-law behavior not apparent when examining small numbers of large lakes in isolation Seekell and Pace (2011); McDonald et al. (2012); Seekell et al. (2013); Cael and Seekell (2016). These errors are propagated through subsequent analyses. For example, global estimates of photosynthesis in lakes are 45% higher when based on a full power-law distribution compared to a plausible alternate with lower limit to power-law behavior Lewis (2011).

Few studies have actually tested the goodness-of-fit of lake sizes to the power-law distribution Seekell and Pace (2011); Seekell et al. (2013). Such tests often indicate deviation from a power-law, but because these tests are applied across the full size range, it is unresolved if this deviation is due to the complete absence of power-law behavior or because power-law behavior is limited to certain scales Seekell and Pace (2011); Seekell et al. (2013). Lower-limits are likely because the approximate scale-invariance of topography that engenders power-law distributions breaks down at small-scales where there are strong imprints of scale-dependent geological processes Cael and Seekell (2016). Upper-limits to power-law distributions are also possible and have been hypothesized as components of some regional lake-size distributions where large lakes are unable to form within a finite area Hamilton et al. (1992); Cael et al. (2015). Identifying limits to power-law behavior stands to improve the accuracy of global lake characterizations, and also stands to shed light onto the specific mechanisms responsible for shaping lake size-distributions Cael and Seekell (2016).

In this paper, we synthesize the general evidence for a power-law lake size-distribution, as well as evidence for upper and lower limits to power-law behavior. We also contribute new empirical analyses aimed at resolving uncertainties relative to upper and lower limits. We argue that the development of alternate hypotheses for the form of the lake size-distribution and its generating processes, compared through the application of rigorous statistical analyses, is important for the advancement of global limnology.

2 EVIDENCE FOR THE POWER-LAW DISTRIBUTION

Empirical power-laws are ubiquitous in scientific reports, but the strength of evidence supporting these power-laws is often weak due lack of statistical support, lack of a generative mechanism, or both Stumpf and Porter (2012). Additionally, few empirical power-laws contribute genuinely new insights Stumpf and Porter (2012). In this context, the power-law distribution of lakes has a moderate level of support.

Widespread acceptance that lake areas may be power-law distributed is credited with precipitating a paradigm shift whereby environmental scientists recognized that there are many more lakes than previously believed, particularly small lakes, and that therefore lakes have a much greater contribution to the global system than previously believed Downing et al. (2006); Downing (2009, 2010, 2014). Historically, lakes were studied individually or as small groups in close proximity. The adoption of a power-law size-distribution is credited with providing a means of generalizing such studies to the global scale, ensuring the relevance of the entire discipline of limnology during a time of immense focus on global change Downing (2009, 2010, 2014). Hence, there is strong evidence that this power-law has led to transformation scientific insights, regardless of the actual level of statistical and mechanistic support. Despite its transformational impact, there is only moderate and incomplete evidence that lakes actually exhibit a power-law distribution. While there is substantial graphical support for a power-law distribution, this is a relatively weak form of evidence Mandelbrot (1963); Perline (2005); Seekell and Pace (2011) (Supplementary Material §1).

In general, the dynamic processes creating power-laws are poorly understood for lakes when compared to other landforms Mandelbrot (1983); Seekell et al. (2013, 2021). One conceptual model used to explain patterns of lake size and abundance is that depressions are randomly located on the landscape, with hypothetical flooding to outlets sills used to identify the location of lakes (e.g., Mandelbrot (1983); Cael et al. (2015); Cael and Seekell (2016); Goodchild (1988); Downing and Duarte (2009); Seekell et al. (2013); Bhang et al. (2019); Mandelbrot (1995)). Connected depressions represent lakes on river networks, with overlapping regions merging to become multi-basin lakes Cael and Seekell (2016); Goodchild (1988); Downing and Duarte (2009). This has been presented as analogous to the processes that give rise to power-laws in percolation theory or in level-set theory for self-similar surfaces Goodchild (1988); Seekell et al. (2013); Cael et al. (2015); Cael and Seekell (2016); Mandelbrot (1995); Blaudeck et al. (2006). There is inconsistent evidence from these models for a generating mechanism. The primary merit of these models is their relative simplicity. Additionally, they typically provide dual criteria for evaluating data, the power-law form and a specific exponent, which is a much stronger test than examining functional form alone Goodchild (1988); Seekell et al. (2013); Cael and Seekell (2016). There have been mixed results when confronting these models with data. In one case, the global lake distribution for lakes $\geq O(1\text{km}^2)$, had the power exponent equal to that expected from percolation theory to four decimal places Messsager et al. (2016). In another case, lakes at the mean landscape elevation had approximately the same exponent as predicted by level set theory based on independent measures of the landscape's fractal dimension Seekell et al. (2013). However, in other cases, lakes have failed to exhibit power-law behavior or failed to produce the expected power exponent, even under idealized conditions with simulated data Goodchild (1988); Cael and Seekell (2016); Bhang et al. (2019). The reasons for this variable performance have not been studied.

A common feature of these generating models is the assumption of a static, scale-invariant topography, but the actually geologic and hydrological processes responsible for lake formation and disappearance are scale-dependent and evolve over time Cael and Seekell (2016); Englund et al. (2013). The extent to which these simple models capture the collective behavior of these scale-dependent processes is not clear because few studies have sought to directly test theoretical frameworks Mandelbrot (1995); Goodchild (1988); Cael and Seekell (2016); Cael et al. (2015); Seekell et al. (2013). Additionally, it is difficult to discern what theoretical framework might be optimal, both because different frameworks predict similar results, and because the relevant dynamics often occur at temporal scales exceeding observational records Downing (2010).

3 EVIDENCE FOR A LOWER LIMIT

By definition, all power-law distributions have a positive lower limit Vidondo et al. (1997); Clauset et al. (2009); Newman (2005). If lakes are power-law distributed across their entire size spectra, this lower limit will be equal to the smallest sized water body that can be perceived by humans to be a lake (this can be any size down to a single molecule of water) Vidondo et al. (1997); Downing et al. (2006); Clauset et al. (2009); Newman (2005). Empirical size spectra are typically power-law distributed across part but not the full range of scales Clauset et al. (2009); Newman (2005). In this case there is a lower limit to power-law behavior which is greater than the smallest perceptible lake. This lower limit is visible on rank-size plots as a downward defection from a straight line among small lakes Seekell and Pace (2011); Cael and Seekell (2016); Newman (2005).

It has long been hypothesized that such deviations reflected the omission of small lakes in regional and global lake data sets Benson and MacKenzie (1995); Downing et al. (2006); Muster et al. (2013). While there is no doubt that mapping omission occurs and can contribute to this pattern, it is unlikely to be the primary cause because the defection typically begins with lakes that are much larger than the minimum reliably mapped size (i.e. the deflection is visible for lakes that are reliably mapped) Benson and MacKenzie (1995); Seekell (2018); Seekell and Pace (2011); Cael and Seekell (2016); Muster et al. (2013). For example, in one global study, the deflection began two orders higher than the minimum reliably mapped lake size Cael and Seekell (2016). We examined pond size data from the United Kingdom Centre for Ecology and Hydrology which completed a survey of randomly selected swaths of Britain, measuring all water bodies in those swaths down to 25m^2, totaling just over 1,000 small lakes. Departure from power-law behavior is clearly visible in these data despite the specific focus on counting very small ponds (Figure 1). Using the maximum likelihood estimation and bootstrap methodology of Clauset et al. Clauset et al. (2009), we estimated that this break point is $800 \pm 400\text{m}^2$. The upper tail appears power-law distributed with an exponent $\tau = 2.04 \pm 0.09$ highly consistent with the theoretical expectation from percolation theory $187/91 = 2.054$ Cael and Seekell (2016). Another study specifically focused on very small lakes and ponds (< 1000 m^2) in the Swedish Arctic found that their areas lognormally distributed, which is consistent with an approximate power-law fit for larger lakes and the downward deflection for small lakes found on rank-size plots Seekell and Pace (2011); Rocher-Ros et al. (2017). Collectively, these pieces of evidence comprise strong evidence of a lower limit to power-law behavior that is greater than the smallest perceptible lake size and not caused by mapping errors.

There are several factors that could be responsible for the lower limit to power-law behavior. Earth's topography is approximately scale-invariant at large scales, but the signature of scale-dependent geological processes can become strong at some smaller scales Dodds and Rothman (2000). This loss of invariance could be reflected in the lake size distribution Cael and Seekell (2016). Another factor could be size-dependent lake formation and destruction processes. For example, a study of lakes in northern Sweden found a power-law lake distribution on young landscapes, but not on older landscapes where small lakes were less abundant than predicted by the power-law distribution, presumably due to the cumulative effects of sedimentation during the longer landscape history Englund et al. (2013). The specific mechanisms that cause deviation from the power-law distribution, including those that cause loss of topography invariance, have not been enumerated but can include by geological processes (e.g., erosion, sedimentation) and human activities (e.g., urbanization, agriculture) that both form and destroy lakes Cael and Seekell (2016); Steele and Heffernan (2014, 2017); Hayes (2016). Quantitative assessments of the lower limit to power-law behavior are rare and current evidence does not allow discrimination among these processes. Visual evidence suggests that lower bounds may vary between 0.001 km^2 and 1 km^2, depending on the geographic region and scale of the analysis (cf. Downing et al. (2006); Cael and Seekell (2016); Seekell and Pace (2011)). Such a wide uncertainty in the value of the lower cutoff has tremendous implications for the total number of lakes (Supplementary Material §2). The quantification of these patterns, especially relative other landscape characteristics, is the first step to identifying the factors shaping lake size distributions.

4 EVIDENCE FOR AN UPPER LIMIT

By definition, the upper tails of power-law size-distributions extend infinitely Vidondo et al. (1997); Clauset et al. (2009); Newman (2005). Strictly speaking, this is not possible for empirical power-laws because of Earth's finite surface area Goodchild (1988); Hamilton et al. (1992); Barton and Pointe (1997). This can create an upper limit for power-law behavior, beyond which large lakes are scarcer than predicted by the power-law. On a rank-size plot, this appears as downward deflection by largest lakes relative to a straight line Hamilton et al. (1992); Cael et al. (2015); Barton and Pointe (1997). Graphical analyses of global-scale lake data by independent research groups using independent data sets have revealed no

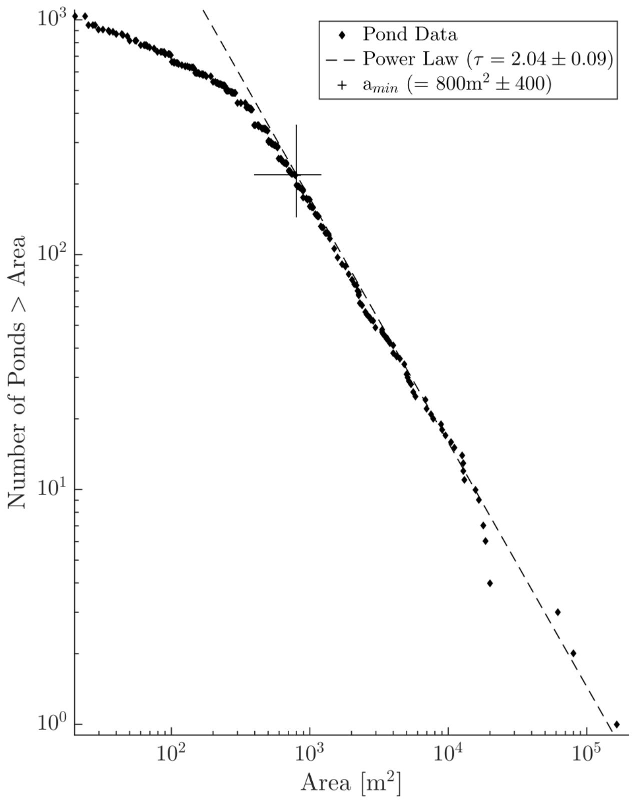

Figure 1. Number of ponds greater than a given area versus that area for the CEH dataset, fit by a power law. Diamonds correspond to empirical distribution; dashed line corresponds to a power-law fit with an exponent of 2.04 and a minimum area of 800m^2. Power-law parameters and uncertainties calculated according to Clauset et al (2009). The predicted exponent from percolation theory is 187/91 = 2.054.

evidence of such an upper bound impacting the lake size distribution (i.e. the power-law fit is visually good for all large lakes) Meybeck (1995); Barton and Pointe (1997); Downing et al. (2006); Cael and Seekell (2016). However, there is graphical evidence for such boundaries in some smaller scale studies. For example one study reported a power-law distribution for very small ponds (1 - 1000 cm^2) on a tidal mud flat, but not for larger ponds ($>$ 1000 cm^2) Cael et al. (2015). An upper bound was also reported for Amazonian floodplain lakes Hamilton et al. (1992). Graphical evaluation of lake size-distributions in the major eco-regions of the United States also suggests deviation from a power-law for large lakes McDonald et al. (2012). However, there is little statistical evidence for an upper limit to power-law behavior at the regional-scale because tests for these patterns are rarely applied. These are a major need because graphical evaluations using rank-size plots can be difficult to interpret and are sometimes misleading Perline (2005); Seekell and Pace (2011). Similar to for lower limits, quantifying the scales of power-law behavior is the first step to shedding light on the factors responsible for upper limits.

A common question related to upper limits is whether the Caspian Sea (374,000 km^2) is a lake or a sea. This is relevant to the discussion of upper limits because the Caspian is sometimes excluded from analyses of the global lake size-distribution on the basis that it is an outlier Lehner and Döll (2004); Messsager et al. (2016). Additionally, there is great societal interest in this question. From 1991 to 2018, there was an international dispute relative to the status of the Caspian Sea, which had significant legal consequences for the distribution and extraction of hydrocarbon resources Pietkiewicz (2021); Zimnitskaya and von Geldern

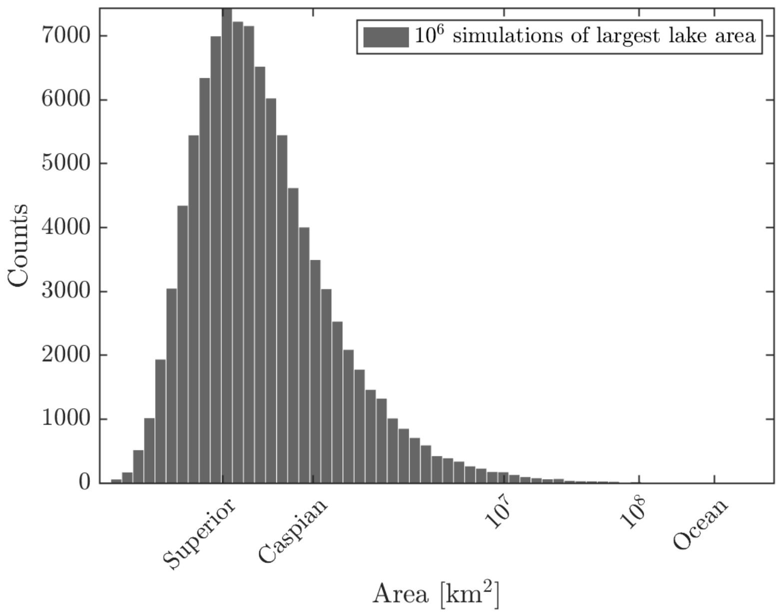

Figure 2. Histogram of 1,000,000 simulated values for the area of Earth's largest lake, based on the total number of Earth's lakes $>$1km^2 and the power-law exponent of Earth's lakes (excluding the Caspian Sea and Lake Superior). X-axis shows the area of the Caspian Sea and Lake Superior.

(2011). A quick search of many social media networks will reveal substantial and persistent interest in the question from the general public (our favorite is the reviews on Google Maps). In a sense this question relates to whether or not an upper-limit should be imposed on lake size-distributions.

From a limnological perspective, the Caspian Sea is a lake and there is no scientific debate about this status Dumont (1998). We have conducted an empirical analysis to settle the question of whether or not it is an outlier that should be considered differently from other lakes. Specifically, we used the lake size-distribution to develop expectations for the area of Earth's largest lake by taking many times the largest of N random samples from a power-law distribution with a minimum size of 1 km^2 and the same exponent as the lake size distribution, where N is the number of lakes on Earth with areas $\geq$1km^2. We did this 1 million times, and compared our simulated estimates with the areas of the Caspian Sea. We found that the Caspian Sea is between the 78th-87th percentile of the distribution depending on the estimates for the relevant parameters (area of Caspian Sea, N, and τ)(Figure 2). In other words, the Caspian Sea is well within the range of sizes expected for Earth's largest lake, based on the scaling characteristics of its other lakes. Based on this analysis, the Caspian Sea would have to be 4.3x larger to be considered an outlier. For comparison, Earth's oceans are larger than 99.99% of these simulated largest lake areas. Hence there is a distinct different between lakes and the ocean not seen when comparing lakes to the Caspian Sea. Collectively, these analyses indicate that there is no need to create an artificial upper-limit by excluding the Caspian Sea from lake size analyses.

5 DISCUSSION

Application of the power-law size-distribution has had a transformational effect on the understanding of Earth's lakes, but empirical evidence that lakes actually exhibit power-law behavior is still incomplete. Specifically, there has been an over-reliance on rank-size plots that are difficult to interpret when only

only large lakes are accurately mapped Mandelbrot (1963); Seekell and Pace (2011); Muster et al. (2013). There is a major need for the application of statistical goodness-of-fit tests both to support the general application of the power-law distribution and identify any limits to power-law behavior. These tests have societal relevance because the power-law size distribution is the basis for most estimates of the global contributions of lakes to the carbon cycle, including some of those cited in IPCC reports (e.g., Tranvik et al. (2009); Bastviken et al. (2011)); they also have relevance for ecology, biogeochemistry, and even the study of other planets (Supplementary Material §3). Inclusion of values based on a power law assumption in high profile science-policy interfaces (e.g., Ciais et al. (2013)) engenders a responsibility to ensure accurate characterization, but rigorous evaluations are never completed prior to extrapolation and even cursory evaluations are rare (e.g., Lehner and Döll (2004); Meybeck (1995); Telmer and Costa (2007); Ryaanzhin (2010, 2015); Downing et al. (2006); Chumchal et al. (2016); Raymond et al. (2013); Lazzarino et al. (2009); Minns et al. (2008); Kastowski et al. (2011)). Application of goodness-of-fit tests is a simple and pragmatic way to fulfill this responsibility.

A limitation to the study of lake size-distributions is a lack of plausible alternate hypotheses. Perhaps without exception, every study that has evaluated lakes size distributions has done so based on the premise it should be power-law distributed (e.g., Mandelbrot (1983, 1995); Hamilton et al. (1992); Meybeck (1995); Lehner and Döll (2004); Downing et al. (2006); Messsager et al. (2016); Benson and MacKenzie (1995); Cael et al. (2015); Cael and Seekell (2016); Seekell et al. (2013); Lazzarino et al. (2009); Minns et al. (2008); Chumchal et al. (2016); Kastowski et al. (2011)). As a consequence, contorted reasoning is sometimes used to confirm a power-law distribution when graphical evidence clearly indicates another distribution (e.g., Chumchal et al. (2016)). This lack of alternate hypotheses is probably why there has been few substantive changes to the understanding of lake size distributions over the last 50 years Platt (1964). The development of plausible alternate hypotheses and generating mechanisms would promote a thorough understanding of the factors shaping lake size-distributions by forcing the consideration and rejection of alternate patterns and mechanisms, a process needed to engender robust results Chamberlin (1965); Platt (1964).

There is a growing number of regional and global lake databases based on either map compilations or remote sensing (e.g., Lehner and Döll (2004); Messsager et al. (2016); Feng et al. (2019); aand T. Kutser et al. (2014); anad J. O. Sexton et al. (2015); Pekel et al. (2016); Rocher-Ros et al. (2017)). In the future, these data sets may reduce the need to extrapolate small lake abundance from the distribution of large lakes aand T. Kutser et al. (2014). This is a positive development given the magnitude of potential errors (e.g. a factor of 10, 100, or 1000 over-/undercounting of total lakes and ponds) that can be caused by these extrapolations Seekell and Pace (2011); Seekell et al. (2013); aand T. Kutser et al. (2014). These developments do not diminish the need for a thorough characterization of lake size distributions. First and foremost, current global data sets do not have sufficient resolution to accurately resolve small lakes aand T. Kutser et al. (2014); anad J. O. Sexton et al. (2015); Pekel et al. (2016); Downing (2010). There is still a need for improved extrapolation to estimate the abundance of these small systems. Second, improved data sets do not, on their own, resolve the fundamental questions related to the origins of lake size distributions. The lake size-distribution is a fundamental constraint to global patterns of lake characteristics, it it requires both improve data sets and rigorous characterizations of these data to shed light on the factors constraining global patterns of lake characteristics. Hence, these new data sets do not supplant the analysis of lake size distributions, but are an important complement to them in advancing the fundamental understanding of lakes at the global scale.

6 CONCLUSION

Widespread acceptance of a power-law size-distribution for lakes precipitated a paradigm shift from local to global understanding of lakes. Despite this, there is still incomplete evidence for the power-law distribution. In particular, there is evidence for a lower-limit to power-law behavior. There is no evidence of an upper-limit to power-law behavior at the global-scale, but such limits may exist regionally. The factors determining the scales exhibiting power-law behavior are poorly studied. Resolving these uncertainties involves the application of rigorous statistics and the development of new alternate hypotheses. Quantifying scales of power-law behavior by identifying these limits is the first step to understanding the ultimate constraints on global-scale patterns of lake properties.

CONFLICT OF INTEREST STATEMENT

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

AUTHOR CONTRIBUTIONS

Cael and Seekell conceived the study. Cael performed analyses. Biggs provided data. Cael and Seekell wrote the paper with input from Biggs.

FUNDING

This paper is based on research supported by the Knut and Alice Wallenberg Foundation and Umeå University

DATA AVAILABILITY STATEMENT

Data will be given a Zenodo DOI before publication should this manuscript be accepted.

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