The relationship between lake surface area and maximum depth

B. B. Cael[1] and D. Seekell[2,3]

1. National Oceanography Centre, Southampton, UK. cael@noc.ac.uk 2. Climate Impacts Research Centre, Umeå University, Abisko, Sweden. 3. Department of Ecology and Environmental Science, Umeå University, Umeå, Sweden. david.seekell@umu.se

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To whom correspondence should be addressed: cael@noc.ac.uk and david.seekell@umu.se.

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Abstract: Maximum depth varies among lakes from <1 to 1741 meters, but attempts to explain this variation have achieved little predictive power. In this paper, we describe the probability distribution of maximum depths based on recent developments in the theory of fractal Brownian motions. The theoretical distribution is right-tailed and adequately captures variations in maximum depth in a dataset of 8,164 lakes (maximum depths 0.1 to 135 meters) from the northeastern United States. Maximum depth increases with surface area, but with substantial random variation - the 95% prediction interval spans more than an order of magnitude for lakes with any specific surface area. Our results explain the observed variability in lake maximum depths, capture the link between topographic characteristics and lake bathymetry, and provide a means to upscale maximum-depth-dependent processes, which we illustrate by upscaling the diffusive flux of methane from northern lakes to the atmosphere.

Significance Statement: Many foundational ecosystem characteristics in lakes, such as thermal stratification, are constrained by maximum depth. Aquatic scientists have have struggled to both explain this variation and to predict maximum depths for lakes without detailed bathymetric surveys. The morphometry of collections of lakes has previously been shown to follow theoretical predictions that assume Earth's topography approximates a fractal Browning motion. In this study, we demonstrate how these theories relate maximum depth to surface area. Maximum depth increases with surface area, but massive lake-to-lake variation is an inherent characteristic of this pattern. This is because the maximum depth is a single random displacement on a topographic profile. The probability distribution of maximum depths can be used to upscale the patterns and processes for many lakes at the regional or global scale. We demonstrate this for diffusive flux of methane from temperate lakes to the atmosphere - a process that correlates strongly with maximum depth but not surface area and cannot be up-scaled without the results presented in this paper.

Introduction

Maximum depth varies among lakes between ∼0.1 and 1741 meters [27, 13]. This variation engenders patterns of diverse ecosystem characteristics including mixing and thermal stratification [15], the relative sizes of littoral and pelagic habitats [22], and carbon cycling including methane evasion [16]. However, there is a paucity of bathymetric data relative to the global abundance of lakes, and empirical relationships that relate maximum depth to lake and landscape characteristics consistently fail to develop sufficient predictive power to estimate maximum depth in lakes that have not been depth sounded [26]. In particular, there is need to develop scaling relationships that relate maximum depth to lake characteristics that are easily measured across broad geographic regions, such as surface area (e.g., [2, 23]). Such relationships provide the simple rules used to generalize understanding of aquatic ecosystem patterns and processes at regional to global scales [7].

Bathymetric surveys are time consuming, and prohibitively expensive for large numbers of lakes [26, 14]. While global perspectives have come to dominate the aquatic sciences over the last twenty years, the proportion of lakes depth sounded has remained low, preventing up-scaling of empirical results from local to global scales [26, 21, 8]. This has spurned a series of empirical studies seeking to predict lake-specific maximum depth based on surface area and other easily mapped characteristic, typically some metric of vertical relief within a buffer zone around each lake (e.g., [21, 14, 27, 19, 18, 12]). Typically, these studies assume that larger lakes should be deeper than smaller lakes, and that integrative measures of topography (i.e. variance, slope) should relate to lake bathymetry. However, these might not be reliable assumptions. For example, while the deepest lakes all have large surface areas (≥ 500 km ^{2}), many lakes with large surface areas are remarkably shallow, often only a few meters deep [13]. Further, correlations between surface area and maximum depth are often weak. For example, the Pearson correlation between the logarithms of surface area and maximum depth for Canadian lakes is $r = 0.46$ [19]. Based on this correlation, the probability of a larger lake being deeper than a smaller lake is only slightly better ($p = 0.65$) than a fair coin flip ($p = 0.5$), if the two lakes are selected at random [9]. Additionally, it is not clear that topography should predict maximum depth. Specifically, many lakes have relatively sudden and catastrophic origins that may not reflect the processes shaping topography [29]. Additionally, maximum depth is essentially a single random perturbation along a combined topographic-bathymetric profile [26]. While integrative measures of topography may be able to predict integrative measures of bathymetry, there is no clear reason to believe that they should accurately predict the value of a single random displacement [26]. The consequence of this is highly uncertain predictions that preclude the application of these equations for upscaling. For example, the error ratio for maximum depth predictions for Canadian lakes is approximately two [3, 19]. This means that a lake predicted to have a maximum depth of 10 meters will have a maximum depth between 5 and 20 meters [3]. Error ratios are even higher in other studies (e.g., [12]). This magnitude of prediction error is material across the entire size spectra of lakes.

In this paper, we describe the theoretical basis for predicting maximum depth from surface area. This theory is based on the characteristics of fractional Brownian motions, which approximate key features of Earth's topography and are the basis for other lake scaling relationships including mean depth and volume (e.g., [11, 23, 2, 25]). Specifically, we describe how recent developments generalizing the arcsine laws from Brownian motions to fractional Brownian motions relate to the problem of maximum depths [4, 5]. We test goodness-of-fit of the theoretical distribution derived based on these

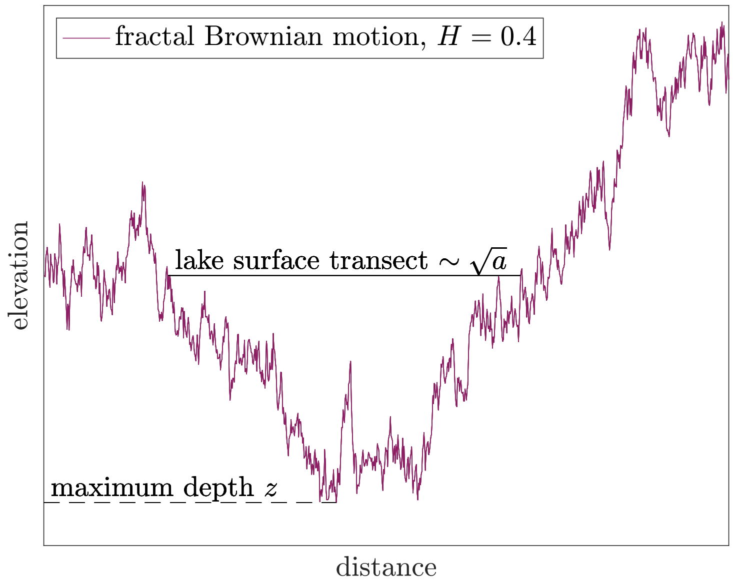

Figure 1: An example fractal Brownian motion path, with Hurst exponent $H = 0.4$. This path is taken to be an analog of a transect along a landscape; lakes fill in sections along this transect, which thus sets their maximum depth.

theoretical developments to a bathymetry database with more than 8000 lakes. Finally, demonstrate how this distribution can be applied to advance understanding of global lake characteristics by up-scaling diffusive methane flux from temperate lakes to the atmosphere, a process that has previously been shown to correlate with maximum depth but not surface area, and hence a process that is difficult to upscale without the results presented in this paper [16]. Collectively, these results both advance fundamental understanding of patterns of lake morphometry, and provide tools for upscaling depth dependent processes when seeking to place lakes within a global context.

Theory

Earth's topography is approximately scale-invariant [30]. This is self-evident for many landforms including lakes. For example, it would be difficult or impossible to determine if a lake is 1 ha or 1,000 ha on a map or image without a scale reference (e.g., a scale bar on a map or a house on an aerial image) because the key characteristics of lake geometry are similar across wide ranges of scales [25, 26, 23]. In particular, Earth's topography is self-affine, which implies different scaling in the vertical and horizontal directions [30]. This is also self-evident; when walking or driving away from a mountain range, the profile flattens more rapidly than it compresses horizontally [26]. Specifically, the Hurst exponent H captures the relationship between horizontal and vertical scales. For a surface $Z(\vec{X})$, rescaling by any coefficient b conforms, in a statistical sense, to $b^{-H}Z(b\vec{X}) = Z(\vec{X})$. H takes values in the range (0,1). When $H = \frac{1}{2}$, transects across Z have the statistical properties of a one-dimensional Brownian motion; when $H > \frac{1}{2}$ ($< \frac{1}{2}$), surfaces are smoother (rougher) than Brownian trajectories (e.g., [11]). These characteristics engender the derivation of empirically robust and theoretically sound power-law relationships between lake surface area and various aspects of lake morphometry including abundance, volume, mean depth, perimeter, and hydrological connectivity [2, 23, 25, 23]. However, to our knowledge, the relationship between maximum depth and surface area has not been described [26].

When topography approximates a Brownian motion ($H = \frac{1}{2}$), the maximum depth over a given interval is described by the third arcsine law [4]. Essentially, because the maximum depth is a single random displacement on a topographic profile, the arcsine law shows that maximum depth converges as a probability distribution based on surface area. Recently, this result has been generalized to cases where $H \neq \frac{1}{2}$ [4, 5]. This generalization allows application of the arcsine law to predict the distribution of lake maximum depth $p(z)$ based on surface area and the Hurst coefficient. Unfortunately, the mathematical form of $p(z)$ is almost comically complex. First, define the normalized maximum depth

$$y = \frac{z}{\sqrt{2}L^H}$$

and define the probability distribution in terms of y, i.e. $p(y)$ (later we recast this in terms of z as well). Doing so,

$$p(z) = \frac{1}{\sqrt{2}L^H} f(y)$$

where $\frac{1}{\sqrt{2}L^H}$ is a normalization constant that ensures the total probability integrates to one, and $f(y)$ has the form:

$$f(y) = \sqrt{\frac{2}{\pi}}\; e^{-y/2}\; y^{(1/H)-2}\; e^{(H-\frac{1}{2})(4\ln y + \mathcal{G}(y))}$$

and $\mathcal{G}(y)$ is ithe complex expression:

$$\mathcal{G}(y) = \frac{y^4}{6}\ {}_2F_2\Big(1,1;\frac{5}{2},3;\frac{y^2}{2}\Big) - 3y^2 + \pi(1-y^2)\mathrm{erfi}\Big(\frac{y}{\sqrt{2}}\Big) + \sqrt{2\pi}e^{y^2/2}y + (y^2-2)(\gamma_E + \ln(2y^2))$$

where F is the hypergeometric function, erfi is the imaginary error function, and γ_E is the Euler-Mascheroni constant [4, 5].

To date, the arcsine laws for fractal Brownian motions are only proven for the one-dimensional case. Therefore, lake areas must be recast as one-dimensional lengthscales L to empirically test the theoretical depth distribution. The lengthscale L is in a sense the length of a randomly chosen horizontal transect along a lake's bathymetry, which includes the lake's maximum depth (solid black line in Figure 1). As surface area a [m^2] is the fundamental horizontal metric for lakes, it would be preferable to cast L in terms of a; from unit considerations one necessarily must have $L \propto \sqrt{a}$, but the coefficient relating the two is not easily derived from first principles because lake surfaces often have complex shapes, and their bathymetry can also be complex, including multiple basins and/or having maximum depths far from their centers. We therefore introduce the free parameter ℓ which allows us to relate L to a according to $L = \ell\sqrt{a}$. ℓ should be no larger than what it would be for a circular lake with its maximum depth at the center, i.e. $\ell = \sqrt{2/\pi} \approx 0.8$, but is likely much smaller as lakes take myriad shapes that are often very far from circular, and their maximum depths of course do not have to be in their center [23]. To compare the theoretical distribution to observed depths, it is further useful to used a normalized maximum depth $y = z/\sqrt{2}L^H$, and define the probability distribution in terms of y, i.e. $p(y)$ (which can later be recast in terms of z). Thus in terms of maximum depth and area, normalized maximum depth is: $y = z/\sqrt{2}(\ell\sqrt{a})^H$. This allows an overall test of goodness-of-fit which is needed because of difficulty finding large numbers of lakes with identical surface areas on real landscapes.

Empirical Test

We tested goodness-of-fit of normalized maximum depths to the theoretical distribution using the publicly available LAGOS database, which includes maximum depths for $N = 8164$ lakes in the northeastern United States [20, 21] (Accessed 28 August 2021). We selected this database because it has been extensively documented and has previously been utilized for developing and testing predictive models for lake maximum depth [21]. Maximum depth spans more than three orders of magnitude among these lakes, from $z = 0.1$m to $z = 135$m while surface area spans from $a = 4$ha to $a = 989$ha. This database does not distinguish between lakes and reservoirs.

We compare the theoretical and empirical depth distributions using the Kolmogorov-Smirnov statistic $D = \max(P(y) - E(y))$, where P (E) is the theoretical (empirical) cumulative distribution function for y [28]. For a given value of H and ℓ, we calculate y for each lake, then $E(y)$ from the empirical distribution of y values, then $p(y)$ based on that H value, then $P(y)$ by integrating $p(y)$, then finally D by comparing $E(y)$ and $P(y)$. First, we set $H = 0.4$ as an estimated value for Earth's topography [17, 6], which has also been shown to capture global scale relationships between mean depth and surface area [2]. We then select ℓ by systematically varying ℓ and repeating this procedure until D is minimized. We also test what the best-fit value of H is by systematically varying both H and ℓ until D is minimized. Finally, because small lakes are often not accurately represented in lake databases, we repeat this process but only considering the upper half of the distribution, i.e. using a modified $D_{alt} = \max(P(y > \text{median}(y)) - E(y > \text{median}(y))))$ [26]. This tests the goodness-of-fit of the larger well mapped lakes to the upper tail of the theoretical distribution.

Results

The theoretical distribution $p(z)$ adequately captures the empirical distribution of maximum depth when $H = 0.4$ (Figure 2; $D = 0.033$). The distribution is right-skewed and unimodal, with few lakes having a $y \approx 0$, many lakes having a $y \approx 0.5$, and a heavy tail of relatively deep lakes having a $y \in (1, 4)$. The best-fitting ℓ value was 0.17, substantially lower than the theoretical limit of $\sqrt{2/\pi}$; this presumably reflects the extent to which these lakes are not circular and their maximum depths are not located in their centers. Furthermore, when H is left as a free parameter rather than externally set by topographic considerations, the best-fitting $H = 0.38$ is negligibly different to $H = 0.4$ (Figure A1; $D = 0.027$; ℓ also changes only slightly, from 0.17 to 0.18).

This distribution $p(y)$ can be recast as a distribution for maximum depth z for lakes of a given area a (Figure 3); for a lake with $a = 1$ha this corresponds to a median maximum depth of 3.9m and a 95% confidence interval of (0.4m, 10.5m). This wide range of predicted depths arises naturally from random topographic variations, and is consistent with the ranges of variability observed in empirical datasets (Figure 3). In this context of a particular a value, the difference between the theoretical and empirical distributions can be most easily understood by comparing the percentiles of each, which underscores

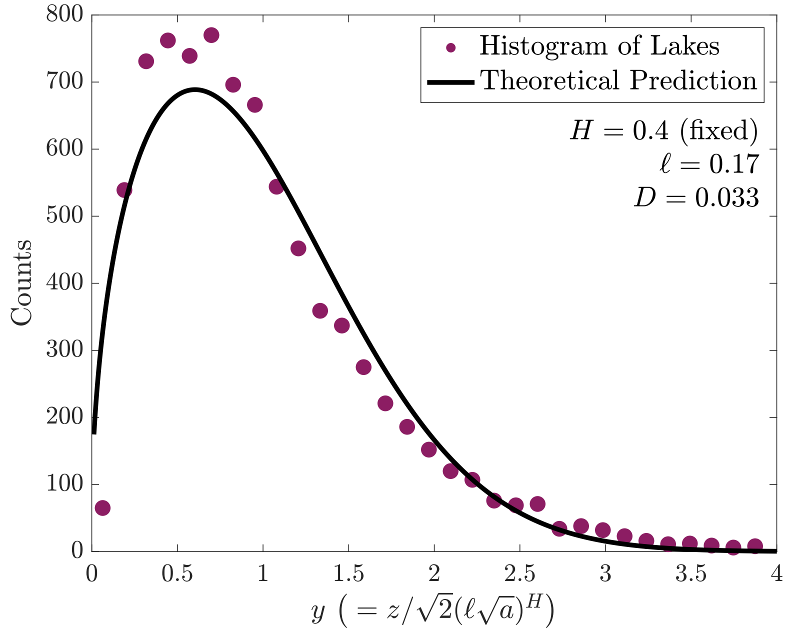

Figure 2: Theoretical probability distribution versus histogram of normalized maximum depth (y) values for 8164 lakes. The Hurst exponent is fixed at $H = 0.4$ and the area-lengthscale coefficient ℓ ($= 0.17$) is left as a free parameter. $D = 0.033$ is the Kolmogorov-Smirnov statistic.

that the correspondence is indeed quite good. If we treat the empirical and theoretical y distributions as if they were maximum depth distributions for lakes with $a = 1$ha, the 1st-91st percentiles of the theoretical distribution for maximum depth are all within 8cm of those of the empirical distribution, which is remarkable given that errors in predicting maximum depth are primarily material for shallow and medium depth lakes. Lakes in the 92-99th percentile have larger absolute deviations from their expected values (10-56cm), but as these expected values are larger these correspond to small relative deviations (5-17%).

Discussion

Scaling relationships provide simple rules for understanding hydrographic patterns at regional and global scales. Our study contributes to this understanding by describing the relationship between surface area and maximum depth. Prior work has primarily focused on developing empirical relationships with multiple linear regression or similar methods (e.g., [27, 12, 21]), whereas our study provides a rigorous theoretical perspective which is accurate and generalizable among regions. Lake area and the Hurst coefficient are the key factors controlling lake depth, with a large stochastic component. These

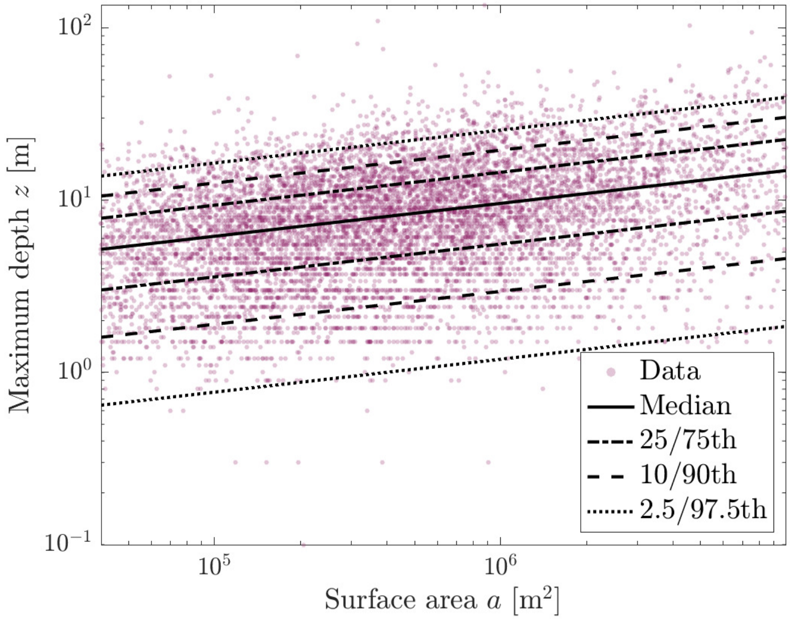

Figure 3: Lake surface areas a and maximum depths z, with percentiles of the probability distribution for z (from figure 2, for a given a) superimposed.

factors can be measured without bathymetric surveys, and hence our results can be applied to existing hydrographic and topographic data sets to estimate characteristics of lake across broad geographic extents including littoral habitat size, carbon burial, or greenhouse gas evasion [22, 16, 1].

Theoretical lake morphometry, including our study, primarily derives from the assumption that Earth's topography approximates a fractal Brownian surface [11, 2, 25]. This assumption is imperfect because 1) it implies that the landscape is a static surface whereas real landscapes are dynamic and evolve over time, 2) it implies scale-invariance whereas real landscapes are shaped by scale dependent processes, and 3) it implies that lakes are formed by flooding preexisting landscape depressions, which is often not the case [11, 29]. Within this context, linear regression analyses based on landscape characteristics essentially are trying to leverage the differences between real and random surfaces to improve maximum depth predictions. However, the low explanatory power of such analyses, and the general consistency between theoretical predictions and empirical patterns, indicates that the imperfect assumptions of the fractal Brownian motion do not materially detract from their application to lake morphometry. Hence, although somewhat stylized, the fractional Brownian motion is an effective starting point for developing predictions about the global characteristics of lakes.

In our analysis, the number of medium-depth lakes is slightly overestimated and number of small lakes slightly underestimated by the theoretical distribution, but the overall fit is remarkably good. However, whether H is set by external topographic considerations (H = 0.4) or estimated from the lake y distribution (H = 0.38), there are some discrepancies between the theoretical depth distribution and observations is not perfect. Specifically, at the lower end of the distribution, with the theory slightly overestimating the number of medium-depth lakes ($y \in (1, 2)$), underestimating the number of shallow lakes ($y \in (\frac{1}{4}, 1)$), and overestimating the number of very shallow lakes ($y < \frac{1}{4}$). The general ontongeny of lakes is decreased depth overtime due to sedimentation [22]. Empirical deviations from the theoretical depth distribution are consistent with this ontogeny, specifically very shallow lakes are rarer than expected, probably because they have transitioned from lake to wetland or terrestrial ecosystem. Additionally, the accentuated peak for shallow lakes is consistent with patterns expected due sedimentation. There are several other factors that can contribute to discrepancies between theory and empirical patterns, primarily related to the collection of bathymetric data. First, large lake databases typically contain samples strongly biased to certain lake characteristics, and the values in the data we analyzed may not be completely representative of the true maximum depth distribution [26, 22]. Second, maximum depths are often only reported to one decimal place, which can lead to significant rounding errors for shallow lakes. This is clearly visible in Figure 3, where there is a regular patterning of shallow maximum depths, but not deep maximum depths (i.e. equal spacing among points along the ordinate). Finally, very shallow lakes may be mis-classified as wetlands and therefore not included in bathymetric databases. Collectively, these factors most strongly impact small and shallow lakes, and this observation is consistent with the excellent fit between theoretical and empirical distributions for deep lakes and somewhat weaker fit across the whole depth distribution. Specifically, when we fit only the upper-half of the y distribution (i.e. using D_{alt} above; Figure A2); the goodness-of-fit was excellent ($D_{alt} = 0.012$). Advancing global limnology relies on developing robust probability distributions for lake characteristics [7]. The variety of factors causing discrepancies between the theoretical and observed distributions highlight the challenges faced in characterizing these distributions, even in ideal cases like this where theory provides clear guidance on the appropriate distribution.

Our approach to understanding the relationship between maximum depth and surface area is useful for predicting characteristics of collections of lakes, but not individual lakes. The only accurate way to measure morphometry for individual lakes remains the detailed bathymetric survey. However, many urgent questions in the aquatic sciences are framed in a global perspective where the characteristics of large populations are of specific interest [7, 8]. Our approach is well suited for application to these questions. In particular, the characteristics of small numbers of lakes are typically up-scaled based on abundance-area distributions to estimate population level characteristics [7, 24]. This poses a problem for ecosystem characteristics that are closely tied to maximum depth but not surface area. Diffusive methane flux from temperate lakes to the atmosphere is one example of these characteristics [16]. Methane is a potent greenhouse gas and estimating evasion from lakes to the atmosphere is a priority for understanding contributions of lakes to the global carbon cycle. A recent synthesis of diffusive flux measurements for temperate lakes revealed a statistically significant inverse relationship with maximum depth ($p<0.001$), but no significant relationship with surface area ($p>0.05$) [16]. We applied the empirical relationship for diffusive methane flux from [16] to the full database of lakes from 17 northeastern United States for which surface areas are available ($N = 141,265$) by randomly simulating maximum depths for these according to the distribution above (with $H = 0.4$, and $\ell = 0.17$ estimated from the 8,164 lakes from these same 17 states for which maximum depth measurements were available; Figure 2) and then calculating diffusive methane flux for each lake as a function of surface area and simulated maximum depth. We repeat this process many times to estimate uncertainty due to the randomness of simulated maximum depths. This results in an estimated 0.5 ± 0.04 Tg CH_4/year from these lakes. This is a substantially different estimate than if one just takes an average rate per unit area across all lakes, because the relationship between methane flux and maximum depth is nonlinear [16] and this average will be dominated by lakes with large surfaces, deep maximum depths, and low rates of methane flux per unit area; one may then substantially underestimate overall methane flux as a result. For instance, multiplying the average of 0.9 g CH_4/m^2/year from [16] by the total surface area of the 141,265 lakes above results in an estimate of 0.03 Tg CH_4/year. Hence, our results represent an important methodological advancement for up-scaling lake characteristics correlated with depth but not surface area. For illustrative purposes (as the parameter ℓ has been fit here to lakes from the northeastern United States rather than lakes globally), applying the relationship from [16] globally to the lake areas from the Hydrolakes database (https://hydrosheds.org) [18] via simulating maximum depths as we describe here results in a global diffusive methane flux of 27 ± 3 Tg CH_4/year. Matlab code demonstrating the application of the maximum depth distribution for up-scaling is in the supplemental materials.

Our study highlights the far reaching influence of the Hurst coefficient on global-scale lake characteristics. Specifically, the differences between horizontal and vertical scaling described by the Hurst coefficient underlie differences in characteristics between large and small lakes - small lakes are typically deeper relative to their surface area compared to large lakes which has implications for energy and carbon budgets across the lake size spectra [1]. Additionally, the Hurst coefficient is involved in most other lake scaling relationships, including for abundance, perimeter, volume, and hydrologic connectivity [11, 25, 2, 23]. Despite this, empirical measurements of the Hurst coefficient for Earth's topography and bathymetry are relatively rare and highly variable (H = 0.4-0.7)[11, 10, 31]. Developing such measurements should be an important priority for advancing global scale understanding of lakes. These measurements could explain variations in scaling relationships among regions, as well as improve the precision of predictions by reducing uncertainty in parameterization.

Appendix: alternate versions of Figure 2

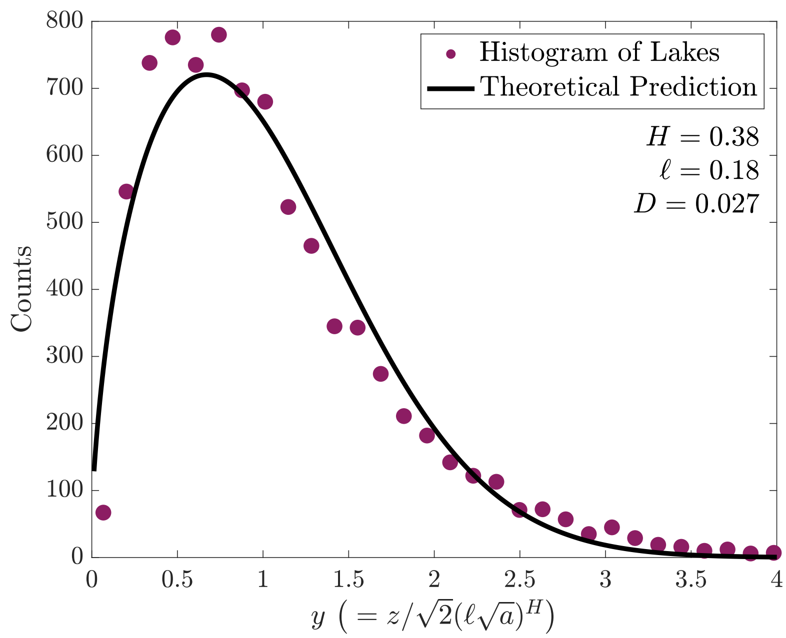

Figure A1: As in Figure 2, but with H left as a free parameter. The best-fit Hurst exponent is $H = 0.38$ (0.4 in Figure 2) and the best-fit $\ell = 0.18$ (0.17 in Figure 2); D reduces to 0.027 (0.033 in Figure 2).

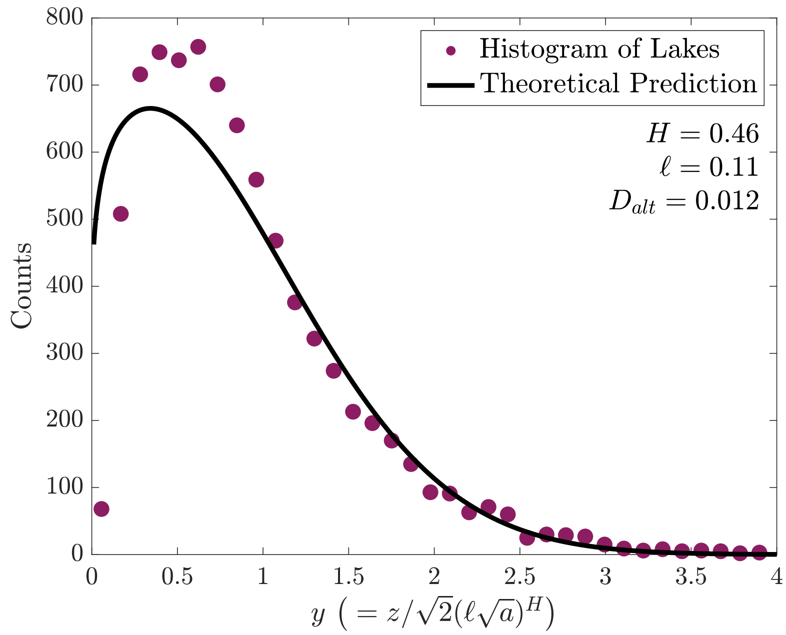

Figure A2: As in Figure 2, but with H left as a free parameter and D is only evaluated for y values above the median (i.e. D_{alt} is used). The best-fit Hurst exponent is $H = 0.46$ (0.4 in Figure 2) and the best-fit $\ell = 0.11$ (0.17 in Figure 2) while the fit to the upper half of the distribution is much improved ($D_{alt} = 0.012$ as opposed to $D = 0.033$ – albeit over the full distribution – in Figure 2).

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