Observations and models of dynamic topography: Current status and future directions

D. Rhodri Davies\textsuperscript{a}, Sia Ghelichkhan\textsuperscript{a}, Mark J. Hoggard\textsuperscript{a}, Andrew P. Valentine\textsuperscript{b} & Fred D. Richards\textsuperscript{c}

\textsuperscript{a}Research School of Earth Sciences, The Australian National University, Canberra, Australia.
\textsuperscript{b}Department of Earth Sciences, Durham University, Durham, UK.
\textsuperscript{c}Department of Earth Science and Engineering, Imperial College London, London, UK.


The slow creeping motion of Earth’s mantle drives transient changes in surface topography across a variety of spatial and temporal scales. Recent decades have seen substantial progress in understanding this so-called ‘dynamic topography’, with a growing number of studies highlighting its fundamental role in shaping the surface of our planet. In this review, we outline the current frontiers of geodynamical research into dynamic topography. It begins with a summary of ongoing observational, theoretical and computational efforts that aim to quantify the present-day expression of dynamic topography, including its geographical distribution and sensitivity to different components of the mantle’s flow regime. Next, observational constraints that shed light on how dynamic topography has changed over time are summarised, and compared with predictions from a range of geodynamical modelling studies, to highlight our current understanding of its evolution through the geological past. Although many model predictions can be reconciled with the available observational constraints, these comparisons demonstrate that there remain inconsistencies, particularly at shorter spatial and temporal scales. These discrepancies allow us to isolate the shortcomings of existing modelling approaches and identify pathways towards improving future reconstructions of dynamic topography through space and time. Such reconstructions are vital if we are to robustly connect the evolution of Earth’s surface environments to the processes that are occurring deep within its interior.

1 Introduction

Earth’s surface topography is a product of competition between internal and external processes; some that act to increase elevation, such as mountain-building, and others that reduce it, for example, through extension or erosion. Although much of Earth’s topography is supported by isostasy, being controlled by variations in thickness and density within the crust and lithosphere, a substantial proportion arises as a result of forces exerted by vigorous convection within the underlying mantle (e.g. Pekeris, 1935; Parsons & Daly, 1983; Davies, 1988; Hager & Richards, 1989; Braun, 2010; Hoggard et al., 2016; Eakin & Lithgow-Bertelloni, 2018; Davies et al., 2019). This so-called dynamic topography is transient, varying both spatially and temporally in response to underlying mantle flow. As such, it directly connects the evolution of surface environments to Earth’s deep interior.

Dynamic topography plays a key role in a diverse range of processes, including sea-level change and continental flooding (e.g. Mitrovica et al., 1989; Gurnis, 1993; Moucha et al., 2008). Earth’s gravitational expression (e.g. Hager & Richards, 1989; Colli et al., 2016), sedimentary stratigraphy (e.g. Burgess & Gurnis, 1995; Petersen et al., 2010; Czarnota et al., 2013), landscape and shoreline evolution (e.g. Sandford, 2007; Shephard et al., 2010; Roberts & White, 2010; Richards et al., 2010), oceanic circulation patterns (e.g. Poore et al., 2006; 2009; Parnell-Turner et al., 2015) and ice sheet stability (e.g. Austermann et al., 2015). Despite considerable advances across a number of observational, theoretical and computational studies (e.g. Mitrovica et al., 1989; Gurnis, 1993; White & Lovell, 1997; Lithgow-Bertelloni & Silver, 1998; Gurnis et al., 2006; Conrad et al., 2009; Moucha et al., 2008; Conrad & Husson, 2009; Guerri et al., 2016; Steinberger, 2016; Hoggard et al., 2016; Watkins & Conrad, 2018; Faccenna et al., 2019; Ghelichkhan et al., 2021), our understanding of dynamic topography remains incomplete, with ongoing uncertainties on the length scales and amplitude at which it manifests, the rates over which it changes, and the mechanisms driving its evolution (e.g. Rowley et al., 2013; Molnar et al., 2015; Hoggard et al., 2016; Colli et al., 2016; Yang & Gurnis, 2016; Müller et al., 2018a; Davies et al., 2019; Hoggard et al., 2021).

Several attempts have been made to constrain the spatial pattern, wavelength and amplitude of present-day dynamic topography. There are generally two ways to approach this: (i) estimation of so-called residual topography, by removal of the isostatic contribution due to sediments, ice, crust and lithosphere from the observed topography (e.g. Menard, 1973; Crough, 1983; Panasyuk & Hager, 2000; Kaban et al., 2003; Guerri et al., 2016; Hoggard et al., 2017); or (ii) estimation of the surface deflections arising from mantle flow, through predictive computational modelling (e.g. Hager et al., 1985; Ricard et al., 1993; Steinberger, 2007; Conrad & Husson, 2009; Flamant et al., 2013; Yang & Gurnis, 2016; Rubey et al., 2017; Oweis Tuna et al., 2018; Colli et al., 2018; Ghelichkhan et al., 2021). Until recently, the results obtained using these two approaches were inconsistent. Most predictive models exhibited peak amplitudes of greater than 2 km, being dominated by broad topographic highs within the Pacific and African domains, separated by a band of topographic lows extending from Antarctica, through the Americas to the Arctic region and broadening beneath the Eurasian continent. Residual topography estimates, on the other hand, showed smaller-scale structure, with key features including lows at the Australian-Antarctic Discordance (AAD) and Argentine Basin, and highs under the central and western Pacific Ocean, offshore southern Africa and the North Atlantic Ocean. Recent research has, however, broadly resolved this long-standing debate, reconciling theoretical model predic-
tions with careful measurements of residual topography from the geological record (e.g. Hoggard et al. 2016, 2017, Davies et al. 2019, Richards et al. 2020a, Valentine & Davies 2020). These studies reveal that both deep mantle flow and shallower processes involving the interaction between asthenospheric flow and lithospheric structure are fundamental to generating Earth’s dynamic surface response, with deep mantle flow primarily contributing to dynamic topography at long-wavelengths, and shallower processes controlling the shorter-wavelength components (∼10^2–10^3 km), as advocated in seminal studies by, for example, Hager & Richards (1989).

Despite this progress, there remains a fundamental gap in our understanding of dynamic topography: we do not have a clear picture of how it has evolved in space and time, even for the most recent Cenozoic history of our planet. This limitation arises partly due to increasing sparsity of observational constraints back in time, but also because transient uplift and subsidence events lead to periodic erasing and overprinting of the geological record, resulting in complex interpretational challenges (Hoggard et al. 2021). A growing number of modelling studies have attempted to fill this gap (e.g. Moucha et al. 2008, Liu et al. 2008, Spasojevic et al. 2009, Spasojevic & Gurnis 2012, Flament et al. 2013, Rubey et al. 2017, Müller et al. 2018a, Flament 2019, Ghelichkhan et al. 2021), with some now able to reproduce long-wavelength (∼10^3 km) spatial and temporal patterns of dynamic topography that are consistent with first-order geological events, such as the Cretaceous onset and Paleocene termination of large-scale marine inundation in North America (e.g. Mitrovica et al. 1989, Liu et al. 2008, Spasojevic et al. 2009), the late Tertiary rise of Africa relative to other continents (e.g. Gurnis et al. 2009, Rubey et al. 2017), the Miocene reversal of the Amazon river drainage system (e.g. Shephard et al. 2010), and post-Cretaceous tilting of Australia (e.g. DiCaprio et al. 2011, Ghelichkhan et al. 2021). Other model predictions, however, remain largely at odds with geological constraints, particularly at shorter spatial and temporal scales (e.g. Wheeler & White 2002, Sandiford & Quigley 2009, McLaren et al. 2012, Paul et al. 2014, Green et al. 2018, Gurnis et al. 2020, Ball et al. 2021). This major limitation hampers our ability to understand fundamental processes driven by time-dependent interactions between Earth’s surface and its interior. As a result, an accurate reconstruction of dynamic topography into the geological past is now regarded as a grand challenge in global geodynamical research.

Here, the aims of our contribution are twofold. Firstly, we attempt to summarise the current understanding of dynamic topography, both at the present day and into the geological past, through a combination of observational constraints and predictions from geodynamical models. Secondly, we identify important future directions and approaches that will facilitate more accurate reconstructions of dynamic topography. This review is intended to complement a recent review on observational estimates of dynamic topography through space and time by Hoggard et al. (2021), and an earlier review on observations and models of dynamic topography by Flament et al. (2013), through the integration of more recent material.

The manuscript is divided into two parts. In the first part (Section 2), we focus on present-day dynamic topography, providing a synopsis of efforts to reconcile present-day observational constraints with model predictions. This part allows us to quantify the sensitivity of dynamic topography to different components of the mantle’s flow regime for a period where the observational constraints on dynamic topography and mantle flow are more plentiful. In the second part (Section 3), we focus on dynamic topography through the geological past, summarising time-dependent observational constraints, methods and predictions from a range of geodynamical models. This synthesis allows us to highlight the strengths and weaknesses of different modelling approaches and to identify a pathway towards improving future reconstructions of dynamic topography through space and time, which we cover in Section 3.4.

2 Present-day dynamic topography

2.1 Observational estimates

Some of the earliest observational estimates of the amplitude and geographic pattern of present-day dynamic topography came from studies of oceanic bathymetry. In the late 1960s, the observed first-order pattern of increasing basement subsidence as a function of lithospheric age was successfully described using models of conductive cooling of a thermal boundary layer (e.g. Turcotte & Oxburgh 1967, McKenzie 1967). Nevertheless, localised deviations away from a classical age-depth relationship were quickly identified and mapped in many of the oceanic basins. These features became known as residual depth anomalies and were thought to be a proxy for dynamic topography (e.g. Menard 1969, 1973). Later studies further refined these maps and consensus emerged that depth anomalies could be up to ±1 km in amplitude, across typical wavelengths of 500–2500 km (e.g. Cochran & Taiwan 1977, Crough 1983, Caizeneve et al. 1988).

More recent research has demonstrated that some of the early-identified features are actually a result of isostatic topography caused by variations in the thickness and density of the sedimentary pile and crust (e.g. Colli et al. 2006). Modern residual depth analyses attempt to correct observed bathymetry for these isostatic effects, which requires detailed knowledge of local sedimentary and crustal thicknesses (e.g. Hillier & Watts 2005, Crosby et al. 2006). The most accurate measurements come from active-source marine seismic experiments, and the last decade has seen steady compilation of a global inventory of more than 2000 such measurements (e.g. Winterbourne et al. 2009, Czarnava et al. 2013, Winterbourne et al. 2014, Hoggard et al. 2017).

Careful analyses, including the propagation of uncertainties associated with the sedimentary and crustal isostatic corrections, have yielded the most reliable constraints on present-day dynamic topography to date. In well-resolved regions, such as the South Atlantic passive margins, multiple cycles residual depth variations have been observed (Winterbourne et al. 2009). These features have amplitudes of ±1 km, wavelengths of 1000–2500 km, and correlate with free-air gravity anomalies, sub-plate seismic velocity variations, and geological evidence for uplift and subsidence in the stratigraphy and geomorphology of adjacent continental shelves and their hinterlands (e.g. Al-Hajri et al. 2009, Roberts & White 2010, Hoggard et al. 2017). Crucially, these features appear to be robust, since their amplitudes are significantly larger than the associated measurement uncertainties. The oceanic residual topography database of Hoggard et al. (2017) is central to the analyses presented in this review: it has underpinned a series of studies into the spatial character of residual topography and its relationship to underlying mantle dynamics (e.g. Hoggard et al. 2016, Steinberger 2016, Yang & Gurnis 2016, Yang et al. 2017, Watkins & Conrad 2018, Steinberger et al. 2019, Davies et al. 2019, Richards et al. 2020a, Valentine & Davies 2020). Further details of its construction are described in Section 2.2.

In spite of advances in the quality, quantity and accuracy of analyses, there remain two significant challenges with
Figure 1: (a-d) – Observational estimates of present-day residual topography from the database of Hoggard et al. (2017). Spot measurements are shown in (a), with associated uncertainties in (c). High-accuracy measurements that incorporate both crustal and sedimentary corrections are shown as circles, with measurements that lack a crustal correction shown as triangles. The uncertainty associated with the latter includes an additional 0.08 km that has been determined to bring the robust, crustal-corrected data points into best agreement with the statistical model (see Valentine & Davies, 2020 for further details). Panel (b) shows 20,767 gridded measurements derived from shiptrack bathymetry, where uncertainties depicted in (d) now include an additional 0.09 km to reflect the lack of crustal corrections. (e-h) – global water-loaded residual topography determined from observational dataset. Panel (e) shows the posterior mean point-wise estimate of residual topography determined from all spot data, with (g) mapping the associated standard deviation. Note that uncertainties rapidly grow with increasing distance from measurement locations and there are large regions (including all continental interiors) where the spot data is uninformative. Panel (f) shows the mean residual topography determined from all available data, including both spot and shiptrack-derived measurements, with associated standard deviation in (h). The use of shiptrack-derived data substantially expands the area in which the model is informative. Colour scales are chosen for consistency with subsequent figures and the maximum amplitude for each map is given in the lower-right corner.

residual depth datasets. First, the use of an age-depth model curve as a baseline implies that the lithospheric structure everywhere is equivalent to that predicted by the oceanic cooling model. In reality, there will be local differences in the thermal structure of oceanic lithosphere away from this idealised case, such as thinning above mantle plumes or thickening above cold subduction zones, as indicated by seismological studies of lithospheric structure (e.g. Richards et al., 2020a). Thus, there is potentially still a component of oceanic residual depth anomalies that are supported by lithospheric isostasy, and constraining its amplitude is a focus of considerable ongoing research (e.g. Davies et al., 2019; Richards et al., 2020b).

Secondly, there is an inherent spatial bias in the high-accuracy measurements caused by the distribution of marine seismic experiments (Hoggard et al., 2017). A high density of data is found along many of the passive margins that have been the subject of extensive hydrocarbon exploration. Similarly, academics have performed many of their seismic surveys at sites of particular geological interest, such as around seamounts, igneous plateaus, volcanic hotspots, subduction zones, and other tectonic boundaries including fracture zones and microplates. The centers of the major ocean basins, where much of the more pristine, bona fide oceanic crust appears to be located, are therefore underrepresented in the database. Where they are included, the experiments often
the early days of marine seismic acquisition in the 1960s and 1970s, when surveying equipment and processing techniques were less sophisticated and precise than they are today. Improving our current understanding of Earth’s residual topography field, in particular its spectral properties, requires obtaining estimates of residual topography in these data “gaps” (e.g. Valentine & Davies 2020).

There are two avenues of research that aim to tackle this spatial bias problem. In the central ocean basins, gridded datasets derived from shiptrack bathymetry and sediment thickness maps are used to supplement the high-accuracy spot measurements (e.g. Crosby & McKenzie 2009, Hillier 2010, Winterbourne et al. 2014, Hoggard et al. 2017). Improving their accuracy necessitates returning to the original constraints used in their construction and coming up with a suitable approach to deal with the crustal correction in the absence of global grids of oceanic crustal thickness. The more substantial data gaps, however, occur in continental interiors, and obtaining residual topography estimates of sufficient accuracy in this setting is a key goal for observational geodynamics (Hoggard et al. 2021). Whilst even early residual topography studies estimated values in continental areas, the greater crustal and lithospheric thicknesses and wider range of lithologies in comparison to oceanic settings results in considerably larger uncertainties (e.g. Panasuk & Hager 2000, Guerri et al. 2015). The impact of complex and protracted tectonic histories, melt depletion events in the early Earth, and subsequent metasomatic enrichment, introduces likely density variations at the ∼1.5% level, which, in turn, produce ±1 km shifts in the isostatic corrections: this is equivalent to the amplitude of the dynamic topography signals being studied (e.g. Guerri et al. 2015, Lamb et al. 2020, Hoggard et al. 2021). Attempts are currently being made to reduce these uncertainties using additional observational constraints, such as using local seismic velocities and/or gravity to better constrain continental density structure (e.g. Brocher 2005, Davis et al. 2012, Guerri et al. 2015, Wang et al. 2022).

### 2.2 Oceanic residual topography dataset

The oceanic residual topography dataset of Hoggard et al. (2017) consists of two classes of data. The first are those from the sites of active-source seismic experiments (both reflection and wide-angle refraction surveys), and are generally referred to herein as spot measurements. The second are shiptrack measurements, which are calculated using global grids of bathymetry and sediment thickness. Each class is described in turn below.

#### 2.2.1 Spot measurements

Approximately 2,000 high-accuracy spot measurements have been obtained from seismic images of oceanic crust collected using active-source acquisition techniques. Along each of these profiles, the seabed, sediment-basement interface, and, where possible, Moho have been mapped and converted into estimates of water depth, sedimentary and crustal thickness. Careful processing allows correction for isostatic topography associated with sedimentary loading (including the effects of compaction) and crustal thickness, and age-depth cooling is accounted for using a plate model (Richards et al. 2018). A total of 1,160 residual topography observations are obtained in this manner (circles in Fig. 1a), with associated uncertainties (Fig. 1b). We consider these points to be the most robust within the database. A further 870 points (triangles in Fig. 1c) are obtained in a similar manner, but the lack of an imaged Moho prevents application of a crustal correction. These observations must therefore be assumed to be less accurate. To reflect this, Hoggard et al. (2017) increased the uncertainties reported for these data points by 0.2 km, chosen based on the median crustal correction of all the data that does image oceanic Moho, Valentine & Davies (2020) instead replaced this additional 0.2 km component by a correction, ∆ = 0.08 km, determined to ensure statistical consistency with the most robust, crustal-corrected data points. The results presented below use this latter approach.

#### 2.2.2 Shiptrack-derived measurements

The database also contains 20,767 measurements derived from global grids. Water depths are taken from the database of Smith & Sandwell (1997), using only measurements obtained along shiptracks using hull-mounted echosounder systems. The sedimentary correction is based on the sediment-thickness model of Divins (2003), infilled in locations with no data from the global synthesis of (Laske & Masters 1997). There are currently no reliable grids of oceanic crustal thickness, and so a crustal correction has not been applied. Instead, Hoggard et al. (2017) have attempted to excise anomalous areas of oceanic seafloor (fracture zones, seamounts, and large igneous plateaux) using high-resolution gravity anomalies and bathymetric data. Again, to account for the lack of a crustal correction, they increased the associated measurement uncertainties by 0.2 km, although in the analyses of Valentine & Davies (2020) that are discussed further below, this value was replaced with ∆ = 0.09 km, again determined to ensure statistical consistency with information from the most robust spot measurements. Shiptrack-derived data are illustrated in Fig. 1(c), with uncertainties in Fig. 1(b). These points clearly provide enhanced spatial coverage, but the use of global grids in their construction, which are themselves derived from disparate sources of variable quality, raises the possibility of significant unquantified systematic biases within this portion of the dataset. We therefore regard the shiptrack-derived measurements as being the least robust within our database of oceanic residual topography.

### 2.3 Global representation of observational dataset

The observation-based dataset illustrated in Fig. 1 provides information about residual topography at discrete locations. These locations are unevenly distributed around the globe, and the measurements are associated with significant uncertainty. In order to understand and analyse the patterns of residual topography and their links to underlying mantle dynamics, we need to convert this dataset into a continuous representation, quantifying our knowledge of the topography at any point on Earth’s surface. This process requires us to impose assumptions: how should we interpolate between data points and “fill in the gaps”? One starting point — adopted by Hoggard et al. (2016) and Davies et al. (2019) — is to assume that the topography can be expressed in terms of a finite set of spherical harmonic functions. Using \( h(\theta, \varphi) \) to represent the topographic height at latitude \( \theta \) and longitude \( \varphi \), we would write

\[
h(\theta, \varphi) = \sum_{l=1}^{L} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta, \varphi)
\]

where \( Y_{lm}(\theta, \varphi) \) represents a spherical harmonic of degree \( l \) and order \( m \).

\(^1\) Various definitions and normalisation conventions exist for spherical harmonics, and this has been the source of some confusion in the literature on dynamic topography. The results presented herein assume real surface spherical harmonics, as defined in Section B6 of Dahlen & Tromp (1998).
and order \( m \), \( L \) represents a pre-determined maximum spherical harmonic degree for the expansion, and the \( a_{lm} \) represent expansion coefficients. To estimate these values, one may set up and solve a regularised least-squares inversion, seeking the vector of coefficients \( a \) that minimises

\[
\phi(a) = \frac{1}{\sigma^2} \left( \sum_{l=1}^{L} \sum_{m=-l}^{l} a_{lm} y_{lm}(\theta_i, \phi_i) \right)^2 + P(a) \tag{2}
\]

where \( d_i \) represents the observation made at location \((\theta_i, \phi_i)\), and \( \sigma_i \) denotes its corresponding uncertainty. The second term, \( P(a) \), represents a regularisation function chosen to ensure that \( \phi \) has a well-defined and unique minimum; from a Bayesian viewpoint, it encapsulates our prior knowledge about the system. Both this regularisation function and the maximum spherical harmonic degree \( L \) are somewhat arbitrary choices, but each can significantly affect the characteristics of obtained results.

One common choice of regularisation function takes the form

\[
P(a) = \frac{1}{2} \sum_{l=1}^{L} \sum_{m=-l}^{l} \left[ \alpha^2 + \beta^2 l(l+1) \right] a_{lm}^2 \tag{3}
\]

where \( \alpha \) and \( \beta \) are tunable parameters governing the overall power content and roughness, respectively, of the recovered topography. Hoggard et al. (2016) used \( L = 30 \) and chose values of \( \alpha = 20 \) and \( \beta = 1 \), following a series of experiments. This formulation was generalised by Davies et al. (2019), where \( \alpha \) and \( \beta \) are instead estimated from the observational data using a hierarchical Bayesian approach developed by Valentine & Sambridge (2018); the values obtained (for the full dataset including both spot- and shiptrack-derived data) were \( \alpha = 1.25 \) and \( \beta = 1.28 \). However, this analysis relies heavily on the assumed form of \( P(a) \) in Eq. (3). As explained in more detail in Davies et al. (2019), this choice turns out to impose strong constraints on the recovered dynamic topography field and, in particular, on its spectral properties. The assumption is problematic for interpretations and comparisons, as it can be unclear whether features are mandated by the data or are simply an artefact of regularisation. To move beyond this limitation, Davies et al. (2019) instead adopted

\[
P(a) = \frac{1}{2} \sum_{l=1}^{L} \sum_{m=-l}^{l} \xi_l a_{lm}^2 \tag{4}
\]

which builds on the concept of ‘Automatic Relevance Determination’ (ARD) introduced by MacKay (1992). Again, the parameters \( \xi_l \) can be estimated from the data, allowing analysis to proceed with a minimum of external assumptions. In order to minimize the effects of spectral leakage (e.g. Trampert & Snieder 1996), the study of Davies et al. (2019) used \( L = 50 \) in their inversions, but coefficients above \( l = 30 \) were subsequently disregarded in their analyses.

A downside to the ARD approach is that it becomes unstable for small datasets, leaving Davies et al. (2019) unable to analyse the high-accuracy spot measurements in isolation. As a result, Valentine & Davies (2020) developed a new framework for analysis of the observational data, replacing the basis-function expansion of Equation (1) with an assumption that topography can be described by a Gaussian Process,

\[
h(\theta, \phi) \sim \mathcal{GP}(\mu(\theta, \phi), k(\theta, \phi; \theta', \phi')) .
\]

Here, \( \mu \) represents a ‘mean function’, quantifying our prior expectations of topography, while the covariance function, \( k \), quantifies our prior assumptions about correlations between the topography at one point \((\theta, \phi)\) and that at another, \((\theta', \phi')\). Put another way, \( k \) encodes the extent to which a point measurement can be considered to be informative on the surrounding region. Gaussian Processes extend the familiar concept of a Gaussian distribution from vector spaces to function spaces, and allow analysis of the observational data to proceed within a rigorous statistical framework; for further details, see, for example, Rasmussen & Williams (2006), or Valentine & Sambridge (2019a). As set out in Valentine & Sambridge (2020b), the Gaussian Process framework can be connected to the least-squares inversion used by Hoggard et al. (2016) and Davies et al. (2019); conceptually, it corresponds to allowing \( L \rightarrow \infty \), applying the transformation \( d_i \rightarrow d_i - \mu(\theta_i, \phi_i) \) and choosing

\[
P(a) = \frac{1}{2} a^T \mathbf{K}^{-1} a \tag{6a}
\]

with

\[
[K]_{ij} = \iint_{S^2} Y_{l_i m_i}(\theta, \phi) Y_{l_j m_j}(\theta', \phi') k(\theta, \phi; \theta', \phi') \, d\Omega \, d\Omega' \tag{6b}
\]

where \((l_i, m_i)\) represent the spherical harmonic degree and order associated with the \( i \)-th coefficient in the vector \( a \), and where integration is over the surface of the unit sphere. Specifically, Valentine & Davies (2020) assumed a constant mean function,

\[
\mu(\theta, \phi) = \mu_0 \tag{7a}
\]

and a Matern covariance function

\[
k(\theta, \phi; \theta', \phi') = \alpha^2 2^{2-\nu} \Gamma(\nu) \left( \frac{\sqrt{2\nu d(\theta, \phi; \theta', \phi')}}{\nu} \right) \sqrt{\nu \pi} \times K_\nu \left( \sqrt{2\nu \pi d(\theta, \phi; \theta', \phi')} \right) . \tag{7b}
\]

Here, \( \Gamma \) represents the Gamma function, and \( K_\nu \) a modified Bessel function of the second kind. The function \( d \) quantifies the distance between any two points on Earth’s surface: Valentine & Davies (2020) adopted an epicentral-angle distance metric

\[
d(\theta, \phi; \theta', \phi') = \arccos \left[ \sin \theta \sin \theta' + \cos \theta \cos \theta' \cos(\phi - \phi') \right] . \tag{7c}
\]

The quantities \( \mu_0, \sigma_1, \sigma_2 \) and \( \nu \) all represent hyperparameters that are tuned to match the characteristics of the data.

Both the least-squares and Gaussian Process frameworks enable the construction of a ‘posterior’ (i.e., data-informed) estimate of the dynamic topography field, which we denote \( \tilde{h}(\theta, \phi) \). Following Hoggard et al. (2016), we define the power at degree \( l \) within this field as

\[
p_l = \frac{1}{\pi} \iint \tilde{h}(\theta, \phi) Y_{lm}(\theta, \phi) \, d\Omega . \tag{8}
\]

If knowledge of \( \tilde{h} \) is Gaussian — as is the case for both Gaussian Process and Bayesian least squares results — then our knowledge of \( p_l \) is represented by a generalised \( \chi^2 \) distribution. This formulation has some counter-intuitive features: in particular, the most-probable power spectrum is greater than the power spectrum of the mean (i.e. most-probable) dynamic topography field.

The primary advantage of the Gaussian Process approach is its mathematical elegance: the only assumptions made are (i) that the topography is representable using a Gaussian Process, and (ii) that the mean and covariance functions take certain forms. These choices have straightforward physical interpretations, allowing principled choices to be made and any impact on results to be assessed. Beyond this, analysis proceeds without further assumption or approximation.
Figure 2: Comparisons of power spectra for observed residual topography with synthetic predictions of dynamic topography. In all panels, the spectrum obtained using the observational constraints is shown in grey, both for the spot measurements only (a,c,e,g) and for the combined dataset of spot and shiptrack measurements (b,d,f,h). Solid lines represent the mean spherical harmonic coefficients; coloured bands represent the ranges spanned by the central 50% and 99% of spectra computed for 100,000 randomly-generated models consistent with the data (i.e., samples from the posterior distribution). In each panel, these observational constraints are compared to a model prediction of dynamic topography. Panels (a) and (b) are from the model that neglects mantle structure shallower than 250 km depth, sampled at either only spot or combined spot and shiptrack locations, respectively. Panels (c) and (d) are the same for the simulation that includes shallow structure: the observational data is more compatible with our simulation that includes shallow mantle structure. In panels (e-f), we show results from a model where no filtering of degree 2 density anomalies has been performed in the mid mantle (1000–2000 km depth range) and in panels (g-h) seismic structure is converted into physical structure assuming a pyrolite composition, with no chemically anomalous material at the base of LLVPs (purely thermal – T, as opposed to thermo-chemical – TC). In both latter cases, power is dramatically over-predicted at long wavelengths.

The Gaussian Process model of residual topography from Valentine & Davies (2020), derived from spot measurements only, is mapped in Fig. 1(e), with associated uncertainties in Fig. 1(g). The spatial pattern is dominated by broad topographic highs within the Pacific, African and North Atlantic regions, separated by a band of topographic lows that extend from Antarctica, through the Americas to the Arctic, broadening beneath the Eurasian continent and extending south of Australia. Unsurprisingly, given the sparse and uneven data coverage, this model has large uncertainties in many regions of the globe — particularly continental interiors, the Pacific Ocean and the Southern Ocean. This effect must be considered when interpreting the residual topography map.

A repeated analysis, from Valentine & Davies (2020), where the dataset has been extended to include the additional measurements derived from shiptrack bathymetry, is shown in Fig. 1(f). It is remarkably consistent with that determined from the spot data alone, despite a significant expansion in spatial coverage of the dataset. Given the ten-fold increase in the number of measurements used to construct...
the model, finer scale detail is now evident that was not visible using the spot measurements alone. As expected, the data is informative throughout the oceans with uncertainties markedly reduced, but the lack of onshore data results in significant uncertainties within continental interiors, as noted in Section 2.2.

These residual topography models are next expressed in terms of spherical harmonics, with the resulting power spectra shown in Fig. 2. To represent the (non-Gaussian) uncertainties associated with these spectra, Valentine & Davies (2020) generated power spectra for 100,000 random samples from the posterior residual topography models, and Fig. 2 depicts the ranges spanned by the central 95% and 50% of samples. In general, both datasets tell a similar story: the spectrum of residual topography is relatively flat, peaking at degree-2 (wavelength ∼ 1000 km), with steady declining power at shorter length scales. Based on the spot data only, the most-probable model has degree-2 power 0.54 km\(^{-2}\), with power likely in the range 0.47–0.70 km\(^{-2}\) (central 50% of data), although the data could support power up to ∼ 1.12 km\(^{-2}\). By degree-10 (wavelength ∼ 4000 km) and degree-20 (∼ 2000 km), power is likely in the ranges 0.13–0.18 km\(^{-2}\) and 0.04–0.06 km\(^{-2}\), respectively. In general, the additional information available when the dataset is expanded to include shiptrack-derived estimates enables a modest reduction in the spectral uncertainty, but does not substantially alter the most-probable power. These results imply that degrees 1–3 account for about 86% of the total power (up to degree 30) in the residual topography field (this drops marginally to about 84% for spot and shiptrack data).

In summary, the analyses of Valentine & Davies (2020), reviewed above: (i) express a preference for 0.54 km\(^{-2}\) of residual topography power at long-wavelength (l = 2), likely in the range of 0.47–0.70 km\(^{-2}\), with peak amplitudes of 0.74 ± 0.10 km at degrees 1–3; (ii) indicate that spectral power decreases by over an order of magnitude from l = 2 to l = 30; and (iii) demonstrate the robustness of the low-amplitude, short-wavelength (l = 15 – 30) residual topography component.

2.4 Predictions from simulations of mantle flow

To determine how the observed dynamic topography power spectrum might be related to underlying mantle dynamics, we can investigate predictions of surface deflections that are obtained from simulations of mantle flow. A number of different modelling approaches are available, as reviewed by Flament et al. (2013) and further discussed in Section 3.2 below. These include time-dependent forward and adjoint simulations of global mantle flow that evolve towards the present day (e.g. Moucha et al. 2008; Spasojevic & Gurnis 2012; Flament et al. 2013; Rubey et al. 2017; Müller et al. 2018a; Flament 2019; Ghelechkhian et al. 2021), focused studies that examine the sensitivity of topographic predictions to various controlling parameters (e.g. Gurnis 1993; Arnould et al. 2018), regional studies that aim to connect dynamic topography to specific aspects of the geological record (e.g. Mitrovica & Jarvis 1989; Mitrovica et al. 1999; Burgess & Gurnis 1999; Petersen et al. 2010; Schellart & Spakman 2015; Uicoak et al. 2021), and global models of instantaneous mantle flow that consider the present-day state of the mantle only (e.g. Ricard et al. 1993; Panasyuk & Hager 2000; Karban et al. 2003; Steiberg 2007; Conrad & Huse 2009; Steinberger 2016; Yang & Gurnis 2016). Due to a comparatively low computational cost and the multitude of observational constraints that are available at the present day, in particular seismological and geodetic observations constraining the mantle’s current thermo-chemical structure, recent analyses of present-day dynamic topography have tended to focus on instantaneous flow models (e.g. Hager & O’Connell 1979; Richards & Hager 1984; Ricard et al. 1993; Lithgow-Bertelloni & Silver 1998; Steiberg 2007; Conrad & Husson 2009; Steinberger 2016; Davies et al. 2019; Richards et al. 2020b). In these calculations, flow velocities and pressures are calculated in relation to prescribed density and viscosity fields, using either a semi-analytical approach (e.g. Richards & Hager 1984) or discretised numerical approaches (e.g. Davies et al. 2019).

2.4.1 Modelling approach and end-member cases

Assuming incompressibility and the Boussinesq approximation, the governing equations for instantaneous mantle flow are the Stokes and continuity equations, expressed in non-dimensional form as:

\[ \nabla \cdot \left[ \mu \left( \nabla \vec{u} + (\nabla \vec{u})^T \right) \right] - \nabla p = g \delta \rho \hat{k} \quad (9) \]

\[ \nabla \cdot \vec{u} = 0 \quad (10) \]

where \( \vec{u} \) is the velocity, \( p \) the pressure, \( \mu \) the viscosity, \( \rho \) the density, \( g \) gravitational acceleration, and \( \hat{k} \) the unit vector in the direction opposite to gravity. These equations describe the balance between driving (buoyancy) and resisting (viscous) forces at any instant in time and include no time-dependence. For this reason, models that only incorporate Equations (9) and (10) are termed instantaneous: the velocity and pressure fields are computed for a prescribed density and viscosity structure. Dynamic topography can subsequently be computed from predicted radial stresses, \( \tau_r \), at the surface via: \( h = \tau_r / (\delta \rho_{\text{ext}} \cdot g) \), where \( \delta \rho_{\text{ext}} \) is the density contrast between uppermost mantle density and air (continents: 3300 kg m\(^{-3}\)) or water (oceans: 2300 kg m\(^{-3}\)).

In the results that follow, we solve these governing equations inside a spherical shell, using a modified version of Fluidity (e.g. Davies et al. 2011; Kramer et al. 2012; Davies et al. 2016), recently validated against an extensive set of analytical solutions introduced by Kramer et al. (2021) and further benchmarked against published results from alternative spherical shell mantle convection codes (e.g. Zhong et al. 2008; Tackley 2008; Davies et al. 2013). In our simulations, the inner radius corresponds to the CMB and the outer radius to Earth’s surface, with free-slip mechanical boundary conditions specified at both boundaries. Models employ a fixed icosahedral mesh with a lateral resolution of ∼ 50 km at the surface. This mesh is extruded in the radial direction, with radial spacing increasing linearly from 10 km at the surface to 100 km at the CMB.

We focus on two end-member simulations of global mantle flow. In the first, lateral variations in density and viscosity are ignored above 250 km depth, allowing us to quantify the first-order topographic expression of deeper mantle flow. In the second, we account for the effects of shallow mantle flow and its interaction with the lithosphere by incorporating variations in density and viscosity for the entire convecting mantle and lithosphere. In both cases, when converting normal stresses to dynamic topography, we assume a global water-load at the surface. These simulations are similar to those of Davies et al. (2019), albeit with some important updates: (i) our tomographically derived density field has been modified to exploit a robust thermodynamic approach for converting between seismic and physical structure; and (ii) our radial viscosity profile has also been updated for consistency with the aforementioned conversion and to ensure that our model predictions are compatible with observations of Earth’s geoid (discussed further below).
2.4.2 Physical properties: Density and viscosity

To determine the present-day mantle density field, we adopt the hybrid approach described in detail in [Richards et al., 2022]. Above 300 km depth, density is computed using the thermomechanically self-consistent parameterisation described in [Richards et al., 2020b], in which a series of independent constraints on upper mantle temperature, seismic attenuation and viscosity are used to calibrate an experimentally derived parameterisation of anelasticity for a particular seismic tomographic model [Yamanauchi & Takenaka, 2016]. In this case, we use the upper mantle V$_{SV}$ model of SL2013v [Schaeffer & Lebedev, 2013], augmented with regional higher resolution V$_{SV}$ models that employ the same tomographic procedure: SL2013NA for North America [Schaeffer & Lebedev, 2014]; AF2019 for Africa [Celli et al., 2020a]; and SA2019 for South America and the South Atlantic Ocean [Celli et al., 2020b], as outlined in Hoggard et al. [2020]. Importantly, density variations related to the cooling of oceanic lithosphere are removed. This process involves subdividing the principal oceanic basins into 2 million year bins, extracting density profiles from each location within a given bin, and stacking them to yield globally averaged age- and depth-dependent density changes. The difference between these mean values and those of PREM [Dziewonski & Anderson, 1981] is subtracted from the oceanic density field according to local lithospheric age, ensuring that computed topography does not incorporate isostatic seafloor subsidence.

Below 400 km depth, the paucity of independent constraints on the covariation of V$_S$, temperature, attenuation and viscosity forces us to adopt an alternative methodology. Over this depth range, we use Perple_X [Connolly, 2005, 2009] alongside the thermodynamic database of [Stixrude & Lithgow-Bertelloni, 2011] to determine equilibrium phase assemblages throughout the mantle as a function of composition, temperature and pressure, and their associated anharmonic V$_S$ and density values. Temperature-dependent discontinuities in these properties that are caused by phase transitions are smoothed, since the anomalies they generate are too short-wavelength to be captured by most seismic tomographic models. In addition, to account for anelasticity, anharmonic V$_S$ is corrected using the approach of [Matas & Bukowinski, 2007], as outlined in the supplements of [Richards et al., 2022]. This procedure involves generating an ensemble of seismic quality factor (Q) profiles from the full range of possible upper and lower mantle anelastic parameters (e.g., activation energies, activation volumes and frequency dependence factors), discarding those that fall outside of independently constrained bounds, and then selecting the profile and associated parameters that produce the median value of average lower mantle Q. The pressure- and temperature-dependent V$_S$ reduction associated with these values is then subtracted from the Perple_X-derived anharmonic V$_S$ predictions. Having made these adjustments, tomographically inferred V$_S$ can be converted to temperature and corresponding density for a prescribed mantle composition. Given their ability to simultaneously satisfy a range of geodynamic, geodetic and seismological constraints, we adopt the TX2011 seismic tomographic model [Grand, 2002] and the compositional model of [Richards et al., 2022], in which the mantle is assumed to be pyrolytic [Workman & Hart, 2005], aside from a ~200 km-thick basal layer within Large Low Velocity Provinces (LLVPs) comprising a mechanical mixture of iron-enriched Hadean basalt [Tolstikhin & Hofmann, 2005] and pyrolite (~ 50% basalt; ~ 50% pyrolite). LLVPs are delineated using the $\delta V_S$ = 0.65 velocity anomaly contour.

Between 300 km and 400 km depth, where the sensitivity of the surface wave-dominated upper mantle model tends to zero, density anomalies are calculated using both methodologies independently. We then take their depth-weighted average in order to ensure a smooth transition in between the two parameterisations. The resulting models differ from those of [Richards et al., 2022] only within the 1000 km–2000 km depth range, where instead of zeroing out buoyancy anomalies to combat inferred tomographic smoothing artefacts [Davaille & Romanowicz, 2020], a smooth high-pass filter of the form,

$$f(l) = -\left(\frac{l - l_{\min}}{l_{\max} - l_{\min}}\right)^4 + 2\left(\frac{l - l_{\min}}{l_{\max} - l_{\min}}\right)^2,$$

is convolved with the spherical harmonic coefficients of the density field, assuming a minimum degree, $l_{\min} = 1$, and a maximum degree, $l_{\max} = 8$. This procedure allows reliably imaged smaller-scale features to be retained, such as slabs and larger mantle plumes, while the long-wavelength, low-velocity ‘halo’ around LLVPs is muted. As in [Richards et al., 2022], we find that these adjustments are required for our models to generate predicted $l = 2$ geoid and dynamic topography amplitudes that are in reasonable agreement with the observed ratios, but also incorporate realistic convective structures from the mid-mantle.

An estimate of mantle temperature is also derived from tomography using the aforementioned approach. In contrast to the density field, however, age-dependent oceanic cooling trends are now retained. This temperature field controls the viscosity distribution in our models, which varies with depth and temperature, following the relation

$$\mu_v = \exp\left[E_0(0.5 - T^*)\right].$$

Here, $T^*$ is the non-dimensional temperature, $E_0 = 13.81$ controls the sensitivity of viscosity to this temperature, whilst $\mu_v$ varies with depth. The latter is set to ensure a mean radial viscosity profile that closely matches that of the purely depth-dependent model S10 from [Steinberger et al., 2010], which has been constructed to be compatible with observations of Earth’s geoid, heat flux, post-glacial rebound, and CMB ellipticity. The resulting density and viscosity profiles are displayed in Fig. 3. Relative to S10, our mean viscosity profile incorporates a lower viscosity asthenosphere between 100 and 410 km depth, marginally higher viscosities between 660 and 1000 km depth, and a more muted reduction in viscosity below 2500 km depth towards the CMB.

2.4.3 Synthetic predictions of dynamic topography

Predicted dynamic topography from these simulations is shown in Fig. 4. The model without shallow structure shares many characteristics with published models (e.g., Steinberger, 2007; Conrad & Hüssn, 2009; Spasojevic & Gurnis, 2012; Flament et al., 2013; Rubey et al., 2017; Flament, 2019), displaying long-wavelength topographic highs within the Pacific Ocean, southern and eastern Africa, and the North Atlantic Ocean, with lows extending across Central and South America, Europe, northern Africa and southeast Asia. The simulation that also incorporates shallow structure, on the other hand, exhibits shorter wavelength features. Nonetheless, the Pacific domain is generally associated with a topographic high, albeit with a broad low off the west coast of South America and more localised lows in the northeast Pacific Ocean. Large topographic highs are visible in the western US, the margins of Antarctica, southern and eastern Africa and adjacent to Iceland, with major lows focused along the AAD, the South Atlantic, the northwest Atlantic.
basin and southeast Asia. These predictions closely resemble those of Steinberger (2016), Davies et al. (2019), and Richards et al. (2020b), who also incorporate shallow mantle and lithospheric structure, albeit with some spatial variability.

Following Davies et al. (2019), these predictive models are next sampled at the locations of spot and shiptrack oceanic residual topography estimates, thus enabling fully-consistent comparisons with the observational constraints. The Gaussian Process approach is subsequently used to perform an independent inversion for each model. The resulting power spectra are displayed in Fig. 2. For the model that neglects shallow structure (Fig. 2a–b), spectral power displays a clear peak at $l = 2$ and rapid drop off at higher $l$. This is inconsistent with the observational constraints, which show significant power at shorter-wavelengths. For the model that incorporates shallow structure, spectral power also peaks at $l = 2$ (Fig. 2c–d), but it does not drop off as strongly at higher $l$. We note that the mean model predicts marginally more power ($\sim 0.75$ km$^2$) than the observational constraints ($\sim 0.54$ km$^2$) at $l = 2$, but, in general character, the spectrum is consistent with the observational constraints, with the range of plausible models overlapping at all $l$. This result represents an improvement on the fit between model prediction and observational data when compared to Davies et al. (2019), who, despite reproducing the general characteristics of the observational spectrum, generally over-predicted power across all spherical harmonic degrees. This improvement is principally driven by our revised approach for converting between seismic and physical structure in both the upper Richards et al. (2020b) and lower Richards et al. (2022) mantle.

2.4.4 Comparisons with the observed geoid

The shape of Earth’s geoid, an equipotential surface of the gravity field that has been well constrained for decades through satellite geodesy (e.g. Kaulla 1967), depends not only on internal density anomalies, but also the boundary deflections that they produce at Earth’s surface and CMB (e.g. Hager 1984 Hager et al. 1985). Model predictions that satisfy observational constraints on dynamic topography must therefore also be compatible with the observed geoid (e.g. Richard et al. 1993 Flament 2019 Richards et al. 2022). To facilitate such a comparison, we have computed the synthetic non-hydrostatic geoid for our simulations using an analytical solution to the Poisson equation in spherical coordinates. In addition to the direct gravitational contribution of mantle density anomalies, contributions from dynamically induced boundary undulations (i.e., surface and CMB dynamic topography) are superimposed (e.g. Zhong et al. 2008). Whilst our modelling approach allows us to capture the effects of lateral viscosity variations, we note that, in contrast to previous geoid studies using the semi-analytical propagator matrix technique (e.g. Richards & Hager 1984 Ricard et al. 1993 Steinberger 2016 Richards et al. 2022), our formulation does not account for self-gravitation.

Maps of the observed and predicted geoid, filtered up to spherical harmonic degree 50, are presented in Fig. 4a,b,d,f.h). The observed non-hydrostatic geoid height anomaly varies between $-121$ and $+107$ metres, being dominated by highs in the African and Pacific regions that connect beneath South America, separated by a band of lows that runs through Asia, the Indian Ocean and the poles. These spatial patterns are generally well recovered by our model predictions: the model that neglects shallow structure yields a correlation coefficient of 56.1, whilst the model that incorporates shallow structure yields a marginally improved correlation coefficient of 60.0. It is clear from Fig. 4 however, that geoid amplitudes predicted by our synthetic models are slightly lower than the observations: for the model neglecting shallow structure, amplitudes range between $-73$ and $+94$ metres, whilst they range between $-89$ and $+93$ metres in the model incorporating shallow structure. This amplitude reduction is reflected in the geoid variance reduction (VR) diagnostics, which are $\sim 31.5$ and $\sim 36.0$, respectively. We note that the similarity in predicted geoid anomalies for these synthetic models is expected given that it is dominated by long-wavelength structure in the deep mantle (e.g. Hager 1984 Hager et al. 1985 Colli et al. 2016), which remains consistent between both simulations.

Our geoid VR diagnostics are lower than those of Steinberger (2010), who recorded values of $\sim 70\%$ for many cases, and Richards et al. (2022), who recorded a value of $\sim 59\%$ for the density model used herein. It should be noted, however, that these results are not directly comparable: as outlined above, our approach allows us to resolve the effects of lateral viscosity variations but neglects self-gravitation, whereas the semi-analytical approach utilised by Steinberger (2010) and Richards et al. (2022) neglects lateral viscosity variations but accounts for self-gravitation. Although we cannot directly
quantify the effects of self-gravitation here, it is known to impact dynamic topography and the geoid at long-wavelengths (e.g. Zhong et al. 2008). Its omission, therefore, likely explains part of the deterioration in geoid VR relative to previous studies. On the other hand, we are able to explore the significance of lateral viscosity variations. To do so, we have examined a simulation where viscosity varies as a function of depth only, using the depth-averaged viscosity of our model with shallow structure (Fig. 3b). The geoid VR is improved by this test, driving our results closer to previous studies. This interesting result does not imply that the mantle’s viscosity varies with depth only: it is well known that lateral viscosity variations are fundamental in dictating the planform of mantle convection, being required to reproduce imaged slab morphologies (e.g. Garel et al. 2014; Goes et al. 2017) and key characteristics of mantle plumes (e.g. Griffiths & Campbell 1991; Davies 1999; Ito et al. 2003). Rather, since we have chosen to calibrate our simulations such that the depth-averaged viscosity closely matches the S10 radial profile from Steinberger et al. 2010, this result suggests that

Figure 4: Observed versus predicted dynamic topography (a,c,e,g) and non-hydrostatic geoid anomalies (b,d,f,h), using the reference geoid height anomaly from Pail et al. (2010) corrected for hydrostatic effects as in Chambat et al. (2010). Panels (c, e, g) display predicted water-loaded dynamic topography from instantaneous flow models. In (c), density and viscosity heterogeneity is neglected at depths shallower than 250 km, whereas in panel (e) it is included. The model shown in panel (g) incorporates shallow density heterogeneity but includes a pressure-dependent viscosity only, corresponding to the depth-average of the model shown in (e). Minimum and maximum values for each map are given in the lower-right corner. For geoid maps, the correlation coefficient (CC) and variance reduction (VR), both in %, between predicted and observed geoid is also shown in the upper-right corner (see Richards et al. 2022 for details of CC and VR calculations).
future efforts to constrain mantle viscosity from observations including mantle heat flux, post-glacial rebound and CMB ellipticity, should carefully consider the role of lateral viscosity variations, alongside self-gravitation, to determine how these factors combine to influence the predicted geoid.

The results presented here, alongside those in Section 2.4.3, demonstrate that models incorporating shallow mantle structure can simultaneously satisfy observational constraints on both residual topography and the geoid (noting the shortcomings outlined above), whereas the model that neglects shallow structure only provides a satisfactory fit to the geoid. Taken together, these comparisons imply: (i) a dominant role for deep-mantle flow in dictating Earth’s dynamic topography and geoid expression at long wavelength; and (ii) an important role for shallow processes, involving the interaction between asthenospheric flow and lithospheric structure, in dictating the shorter-wavelength components of dynamic topography and the general character of the power spectrum.

2.5 Summary of present-day dynamic topography

Observational estimates of dynamic topography, alongside measurements of the geoid, provide a fundamental constraint on the structure and dynamics of Earth’s mantle. Whilst the long-wavelength harmonics of the geoid have been well constrained for decades using satellite geodesy (e.g. Kaula 1967), observational constraints on dynamic topography are still being developed. A recently compiled global database of residual topography measurements within the world’s oceans provides the most robust constraint on present-day dynamic topography that is currently available (Hoggard et al. 2017). Nevertheless, these measurements are made at discrete, unevenly-distributed locations on Earth’s surface and, hence, it is challenging to make inferences concerning global properties.

Building on a series of studies (e.g. Steinberger 2016; Hoggard et al. 2016; Yang & Gurnis 2016; Hoggard et al. 2017; Yang et al. 2017; Steinberger et al. 2019; Davies et al. 2019), Valentine & Davies (2020) used a novel approach based upon the statistical theory of Gaussian processes (Valentine & Sambridge 2020a) to infer the spatial pattern, wavelength, and amplitude of global residual topography. Results, which have been reviewed herein, indicate that the associated spherical harmonic power spectrum peaks at \( l = 2 \), with power likely in the range \( 0.47 - 0.70 \text{km}^2 \) for a global water load. This value decreases by over an order of magnitude at \( l = 30 \), with around 86% of the total power concentrated in degrees 1–3. Comparisons between this spectrum and updated synthetic predictions from instantaneous models of mantle flow demonstrate that the long-wavelength components of Earth’s residual topography are principally driven by deep mantle flow, but shallow processes involving the interaction between asthenospheric flow and lithospheric structure are also crucial in explaining the general form of the power spectrum. This relationship is to be expected based on the sensitivity kernels that illustrate how effective density anomalies at different depths and spherical harmonic degrees are at creating topography (e.g. Hager 1984; Hager & Richards 1985; Colli et al. 2016; Steinberger 2016; Hoggard et al. 2016).

Although we are now at the stage where model predictions of dynamic topography and observational constraints on residual topography arc broadly consistent with one another, until recently, this was not the case. This review demonstrates that reconciliation of model predictions and observational constraints, in a manner that is also compatible with the geoid, requires a carefully constructed dataset of residual topography (Hoggard et al. 2017), robust methods for inferring global characteristics from this data (Davies et al. 2019; Valentine & Davies 2020), a carefully calibrated approach for converting from seismic velocities to physical structure (Richards et al. 2020; 2022), and computational modelling tools that are capable of using the resulting mantle structure as input (Davies et al. 2019).

Some aspects, however, require further work. For example, at present, there is no modelling framework available for computing mantle flow whilst simultaneously accounting for the full (non-linear) impact of self-gravitation. As demonstrated in Section 2.4.3, this shortcoming affects our ability to directly compare synthetic predictions with observations of Earth’s geoid and the development of such a capability is an important avenue for future research. Furthermore, although the range of plausible models for our simulation incorporating shallow structure overlaps with the observational estimates of dynamic topography at all degrees, our flow models generally predict too much power at \( l = 2 \). This issue is consistent with the findings of Steinberger et al. (2019), although our amplitudes are only \( \sim 10\% \) too large, rather than a factor of two, as is visually demonstrated in Fig. 5 where we plot the \( l = 1–3 \) components of both the residual topography measurements and our flow model predictions (sampled at spot and shiptrack locations and spectrally analysed in a self-consistent manner). Although the peak amplitude of positive dynamic topography is in reasonable agreement between observations and predictions (a difference of only 0.06 km), the difference in peak negative values is larger, being -0.68 km for observations versus -0.80 km for predictions. Inspection of the spatial maps reveals that it is in the region of the southeast Asia subduction zones that predicted drawdown is currently too large.

Nevertheless, the principal reason for our improved ability to match long-wavelength dynamic topography is our filtering out of degree 2 density anomalies in the mid mantle (1000–2000 km depth range). Richards et al. (2022) demonstrate that it is \( l = 2 \) structure at these depths that erroneously inflates dynamic topography amplitudes in mantle flow simulations, and also breaks the observed ratio between geoid and long-wavelength dynamic topography. This issue is further highlighted in Fig. 2(e–h), where we plot predicted dynamic topography power spectra for simulations similar to the case with shallow structure examined herein, but with no filtering of degree 2 density anomalies in the mid mantle. The first case assumes a thermo-chemical LLSPV with a consistent composition to the cases examined thus far (Fig. 2(f)), whilst the second assumes a pyrolitic composition with density anomalies purely driven by temperature variations (Fig. 2(h)). In both scenarios, power is dramatically over-predicted at long wavelengths, particularly at \( l = 2 \). Given the long ray paths for body waves, low density of crossing rays, and broad sensitivity of normal modes to velocity structures at these depths, it is perhaps of no surprise that there appears to be substantial smearing of deep LLSPV mantle structure upwards to shallower depths, and also a smoothing of higher frequency structure, such as plumes and slabs, into lower spherical harmonic degrees, as recently suggested by Davaille & Romanovics (2020), and demonstrated in synthetic models using the SNOwTS resolution operator (Ritsma et al. 2007; 2011) by Davies et al. (2012) and Jones et al. (2020). We acknowledge that our current approach to dealing with this problem is somewhat rudimentary and keenly await further tomographic studies that might investigate these potential resolution issues.
We emphasise that fine details of the predicted dynamic topography in our simulations and their associated power spectra are sensitive to several model parameters. These include: (i) the depth- and lateral-dependence of mantle viscosity, which remain poorly constrained (e.g. Rudolph et al. 2015; Marquardt & Miyagi 2015; Lau et al. 2017; Ballmer et al. 2017) and may influence coupling between upper and lower mantle, the transmission of stress across the asthenosphere to the lithosphere, and through the lithosphere to the surface; (ii) the seismic tomography model used as a basis for defining the mantle’s density and thermal structure—although tomographic models now show broad similarity in the distribution of heterogeneity at a large-scale (Lekic et al. 2011), they differ in amplitude and in the distribution of smaller-scale heterogeneity; and (iii) the lack of phase transitions in our models, which will influence how density anomalies at various depths couple to the surface and CMB (e.g. Thoraval & Richards 1997; Steinberger 2007), impacting both dynamic topography and geoid predictions. Nonetheless, our models are based upon a reasonable set of parameters that allow us to illustrate the likely roles of shallow and deep mantle flow in generating Earth’s surface response. These insights gained from analysis of present-day datasets are of fundamental importance if we are to accurately reconstruct the evolution of dynamic topography through the geological past.

3 Dynamic topography into the geological past
3.1 Observational constraints

Investigating the temporal evolution of dynamic topography requires identification and dating of geological features that can track changes in elevation through time. The accuracy of each individual constraint depends upon its indicative range of formation (i.e. the range of elevations that it could have formed at), and by far the most robust markers tend to be generated in the vicinity of sea level since, in this setting, a variety of starkly contrasting geological environments operate over a narrow paleo-elevation band. For example, many marine organisms only grow in the shallow waters of the photic zone, hence uplift and subsidence of features such as coral reefs can be used to place tight bounds on the amplitude and rates of evolving dynamic topography (e.g. D’Agostino et al. 2010; Czarnota et al. 2013; Austermann et al. 2017; Stephenson et al. 2019). Other proxies for former sea levels that have been used to reconstruct vertical motions include estuarine and deltaic sediments (e.g. lignites and coals), paleosols, fluvial base levels in the vicinity of coastlines, transitions between erosional and depositional environments within basin stratigraphy (including unconformities and the geometries of clinoform packages), and other uplifted marine rocks. Away from sea level, techniques involving paleoaltimetry (e.g. clumped-isotope analysis), biological tiering (e.g. leaf palynology), sedimentary deposits from ocean currents, and the carbonate compensation depth (CCD) have also been exploited, but suffer from larger uncertainties due to the wider indicative range associated with these paleoelevation proxies. For an in-depth review of these approaches and many others, we refer the reader to the recent review paper of Hoggard et al. (2021).

Whilst the most robust constraints on the pattern of dynamic topography at the present day are obtained in the oceanic realm (Section 2), it remains a relatively poor place to attempt reconstruction of changes in dynamic topography through time for two important reasons. First, in spite of possible interactions with the CCD (e.g. Campbell et al. 2018), a transient, kilometer-scale uplift and subsidence event of 6 km-deep abyssal ocean floor is less likely to have an impact on the geological record than if that cycle were to occur on a continental shelf and therefore cross the boundary between depositional and erosional settings (e.g. Hartley et al. 2011). Secondly, the geological record in the ocean basins extends to only ∼ 200 Ma before subduction destroys it, whereas continental features such as cratonic basins can survive for billions of years and, therefore, potentially store signals associated with dynamic topography for much longer temporal durations (e.g. Crosby et al. 2010; Young et al. 2021). Thus, existing observational estimates for changes in dynamic topography are generally focused on continental interiors and their surrounding margins. It is not possible to cover all of these studies in sufficient detail here, so we instead restrict ourselves to summarising those related to Cenozoic vertical motions in North America, Australia, and Africa in order to permit later comparison with numerical simulations of mantle flow in Section 3.3.

North America: The geological record of Mexico, the United States and Canada contains substantial evidence for transient vertical motions throughout the Cenozoic Era. One of the most iconic examples of large-scale marine incursions comes from the western US, where a phase of Cretaceous drawdown lead to the development of an interior seaway that was up to ∼ 1400 km wide and resulted in more than 3 km of sedimentary deposition in its deeper western portions (e.g. Sloss 1963; Bond 1978). There is a general absence of large-scale tectonic features in the Upper Cretaceous stratigraphy that might indicate a flexural or extensional basin formation mechanism (e.g. thrust or normal faults, horsts and grabens).
Combined with the timing and widespread nature of the de-pocentre, this observation lead several early studies to link formation of the seaway to underlying mantle flow, in particular subsidence above the downgoing Farallon slab (e.g. Bond, 1978a; Cross & Pilger, 1978; Mitchell et al., 1989; Burgess et al., 1997). The fact that this marine stratigraphy is now exposed at elevations of up to ~2 km above sea level attests to subsequent dynamic rebound of the seaway following westward passage of the region over the downgoing slab. Several studies have focussed on reconstructing the rates and timing of this uplift (see Fernandes et al. 2019 for a comprehensive review). Evidence that has been exploited includes the age and elevation of uplifted marine and coastal rocks, paleoaltimetry estimates from clumped-isotope analysis, the incision and retreat of river profiles draining the region, estimates of sedimentary flux into surrounding basins, and landscape denudation measurements made from techniques including apatite (U-Th)/He and fission track analysis (e.g. Robinson Roberts & Kirschbaum, 1995; Flowers et al., 2007; Huntington et al., 2010; Roberts et al., 2012b; Stephenson et al., 2014; Winn et al., 2017). While the total amplitude of cumulative uplift is well constrained, there has been considerable debate concerning its timing. Some studies suggest that most of the present-day topography had already formed by Eocene times, whilst many others argue for multiple stages of uplift, including a substantial phase of Late Neogene growth that continues today. This latter phase coincides with extensive intraplate volcanism throughout the Colorado Plateau and Wyoming, and has been linked to incursion of warm asthenospheric material (as manifest by slow sub-plate seismic velocities) and arrival of the Yellowstone plume (Klieging et al., 2018). Elsewhere in North America, there has been sufficient evidence for similar Cenozoic uplift (or incision) events throughout central Mexico, the Appalachian mountains, the Rockies, and in parts of the Canadian shield (e.g. Chamberlain et al., 2012; Ault et al., 2013; Stephenson et al., 2014; McKeon et al., 2014; Rovere et al., 2015). In contrast, further to the east on the Atlantic passive margin, an anomalously thick sequence of shallow-water sediments appears to indicate a widespread phase of Neogene subsidence (e.g. Steckler & Watts, 1978; Cloetingh et al., 1990). Recent stratigraphic analyses estimate that 0.3–1.0 km of water-loaded drawdown has occurred over the last 20 Myr, centered stratigraphic analyses estimate that 0.3–1.0 km of water-loaded drawdown has occurred over the last 20 Myr, centered on the Baltimore Canyon Trough (Morris et al., 2020). Since the rifting event responsible for forming the passive margin that likely continued into Neogene times (Wellman 1987; Fernandes & Roberts, 2021; Holdgate et al., 2008; Czarnota et al., 2014; Engel et al., 2020; Ball et al., 2021). Nevertheless, multiple different datasets including uplifted marine rocks, speleothem records, palynology, river profile modelling, sedimentary flux into fringing basins, and the age and morphology of basaltic volcanism indicate significant Cenozoic uplift of the margin that likely continued into Neogene times (Wellman 1987; Fernandes & Roberts, 2021; Holdgate et al., 2008; Czarnota et al., 2014; Engel et al., 2020; Ball et al., 2021).

Africa: It has been known since the earliest continental-scale studies of marine incursion that Africa has undergone several cycles of epeirogenic motions during Cretaceous and Cenozoic times (Bond, 1978a, 1979; Sahagian, 1988). Today, African topography is dominated by a large, high-elevation plateau in the south that steadily dies away northwards, while the entire continent is punctuated by a series of shorter wavelength domes and intermediate basins that have been termed ‘basin-and-swell’ morphology (Burke et al., 2008). Once again, the timing and rates of uplift are heavily debated, but the classical view that much of the topography had existed since passive margin formation in the Jurassic and Early Cretaceous periods is now understood to be largely incorrect. Uplifted marine and coastal rocks dating from the Late Cretaceous through to the Eocene can be found at present-day elevations of more than 500 m in parts of north and east Africa (Bond, 1978a; Sahagian, 1988; Fernandes & Roberts, 2021). Several prominent planation surfaces, often exhibiting well-developed chemical weathering profiles and sometimes blanket ed in younger sedimentary and volcanic rocks, have been dated and mapped throughout the continent and attest to a multistage uplift history since 40 Ma (e.g. Sembroni et al., 2016; Chardon et al., 2016; Guilloucheau et al., 2018; Baby et al., 2018; Friedrich et al., 2018). The erosion and supply of clastic material to surrounding continental margins exhibits a similarly pulsed history – for example two prominent phases of erosion appear to have occurred in the Zambezi catchment from 34–24 Ma and 10 Ma to recent (Walford et al., 2005). Further south, the Limpopo delta shows an increase in clastic flux from 23 Ma onwards that is likely due to Miocene uplift of the northeastern part of the South African Plateau (Said et al., 2015). Further evidence for Cenozoic unroofing of parts of the continental interior comes from thermochronological studies of denudation. For example, (U–Th)/He thermochronology on granitic basement rocks and kimberlites across southern Africa indicates that the plateau planation surface likely formed in response to mid-to-late Cretaceous incision events that propagated from west to east across the continent, but generally limit Cenozoic unroofing to less than 1 km (Flowers & Schloen, 2010; Stanley et al., 2019). Nevertheless, there are several additional strands of evidence pointing to substantial Neogene uplift, including a large angular unconformity on the Angolan continental shelf (Ali-Haji et al., 2009), Quaternary marine terraces that have been uplifted in Kenya, Angola, and Madagascar (Odada, 1996; Guiraud et al., 2010; Stephenson et al., 2019), and uplift histories determined from inversion of longitudinal river profiles (Roberts & White, 2010; Roberts et al., 2012a; Wilson et al., 2014; Paul et al., 2015; O’Malley et al., 2021). Inter-regional Cenozoic hiatus surfaces exhibit a strong expansion of total unconformable area

Australia: Analogous to the situation in North America, much of the central and eastern portions of present-day Australia were flooded by an inland sea in mid-Cretaceous times (Frakes et al., 1987). Today, this marine stratigraphy is exposed onshore (primarily within the Eromanga Basin), and this transient subsidence-rebound event has been linked to a similar story of a downwelling slab in the upper mantle (Gurnis et al., 1998). On a broad scale, Neogene vertical motions in Australia have been dominated by uplift in the south and subsidence in the north. Geological evidence for this behaviour includes the relative width of continental shelves (narrow in the south, wide in the north), the stratigraphic architecture of Neogene sediments (offlap in the south, onlap in the north), and a series of paleoshorelines that are uplifted and exposed onshore in the south, but drowned in the north (Sandiford, 2007). Stratigraphic analysis and backstripping of carbonate clineiforms from the Northwest Shelf indicate that a water-loaded subsidence event of up to 600 m has occurred since ~10 Ma (Czarnota et al., 2013). The general picture of down-to-the-north Cenozoic tilting is, however, punctuated by some shorter wavelength uplift and subsidence events within the continent. In particular, there are several lines of evidence pointing to localised uplift of the Eastern Highlands by up to 2 km sometime since Jurassic/Cretaceous times, although there remains significant controversy over the age and relative rates of this vertical motion (e.g. Jones & Vevers, 1982; Bishop, 1988; Holdgate et al., 2008).
for the base of Miocene outcrops, suggesting that the preceding period likely involved substantial uplift across much of southern Africa (Carena et al., 2019; Hayek et al., 2020). There was also a Neogene flare up in intraplate volcanism at many of the topographic swells across northern Africa (Bali et al., 2019). Taken together, the Cenozoic history of epeirogeny in Africa appears to involve a long-wavelength, potentially early uplift of much of southern Africa, which is punctuated by renewed uplift and sporadic volcanism of several shorter wavelength domes throughout the African and Arabian continents in Neogene times.

### 3.2 Computational approaches for dynamic topography reconstructions

As is evident from the instantaneous flow models presented in Section 2.3, our ability to predict dynamic topography into the geological past requires knowledge of the thermo-chemical state of the mantle at previous points in Earth’s history. Direct seismological and geodetic constraints on mantle structure, however, are limited to the present day, which represents a major obstacle for time-dependent computational models that aim to reconstruct the evolution of dynamic topography in space and time.

As explained above, the coupled Stokes and continuity equations (Eqs. [3] and [10]) are instantaneous. For time-dependent simulations of mantle dynamics, these equations should be augmented with additional conservation laws describing the temporal evolution of mantle heterogeneity. These laws appear in form of energy and, in the case of flow with multiple chemical compositions, compositional equations. Assuming isochronal flow, the energy equation, expressed once again in its non-dimensional form under the Boussinesq approximation, is:

\[
\frac{\partial T}{\partial t} + u \cdot \nabla T = \kappa \Delta T + \phi,
\]

where \(T\) is temperature, \(t\) time, \(\kappa\) thermal diffusivity and \(\phi\) a heat source term. Equation (13) is coupled to the Stokes equation through an equation of state that relates density variations to temperature. Equations [3], [10] and [13] can be solved for velocity, pressure and temperature, given appropriate boundary conditions at the surface and CMB, and an initial condition for the mantle’s thermo-chemical structure.

The latter is of fundamental importance given that the spatial and temporal evolution of mantle convection is uniquely determined by its initial condition. The lack of direct constraints on mantle structure in the past – specifically, this unknown initial condition – is the main difficulty of inferring mantle flow and associated dynamic topography, into the past. As illustrated in the schematic of Fig. 3.(a), the strong sensitivity of the mantle’s evolutionary pathway to this initial condition means that without knowing it, we cannot accurately reconstruct the mantle’s evolution from forward models, even for the most recent Mesozoic and Cenozoic history of our planet (e.g. Bunge et al., 2003).

The initial condition problem can be at least partially overcome through data assimilation (e.g. Hager & O’Connell, 1979; Bunge et al., 2002). As stated by Bunge et al. (2003), simply put, the purpose of data assimilation in a mantle convection model is to use all available information to determine, as accurately as possible, the state of mantle flow. From an algorithmic point of view, data assimilation is often implemented through sequential filtering (e.g. Wunsch, 1996); here, the model is run forward in time and, whenever an instant is reached where observations are available, the model is ‘updated’ or ‘corrected’. The amplitude of the correction can be determined in an optimal sense using schemes such as the Kalman filter (e.g. Wunsch, 1996; Bocher et al., 2018), with the model subsequently restarted from the updated state, and the process repeated until all available information has been assimilated.

**Mantle circulation models**, where plate tectonic reconstructions are integrated chronologically, provide an end-member class of data assimilation approach. In these models, reconstructed plate velocities (e.g. Stampfl & Borel, 2004; Section 2.1; Muller et al., 2016a), often processed via the open-source Gplates software (e.g. Gurnis et al., 2012; Muller et al., 2018a), are either prescribed through a kinematic boundary condition (e.g. Bunge et al., 1998; Davies et al., 2012; Rubey et al., 2015), or through an approach that tries to better capture the expected thermal structure, thickness, dip angle and upper mantle descent rate of slabs (e.g. Bower et al., 2015; Hassan et al., 2016; Flament et al., 2017; Bocher et al., 2016). Nonetheless, the approach has been demonstrated to facilitate forward modelling simulations where the predicted distribution of present-day heterogeneity closely mimics that of Earth’s mantle, as imaged through seismic tomography (e.g. Bunge et al., 2002; McNamara & Zhong, 2005; Schubert et al., 2009; Davies et al., 2012; Barry et al., 2017; Flament et al., 2017; Flament, 2019), demonstrating the central role of plate tectonics in organising underlying mantle flow. As will be discussed further in Section 3.3 when the surface deflections from these simulations are examined through time, they are able to reproduce many of the long-wavelength observational constraints on dynamic topography through the Cenozoic Era, although shorter wavelength variations are not captured. The consistency with observational constraints also deteriorates further back in time (e.g. Flament et al., 2013; Rubey et al., 2017; Muller et al., 2018a), due to the impact of the prescribed (arbitrary) initial condition, although recent work to extend plate tectonic reconstructions further into the geological past (e.g. Young et al., 2019; Merdith et al., 2021) offers a potential avenue for reducing the dependence on the unknown initial condition (e.g. Zhang et al., 2010; Davies et al., 2012; Colli et al., 2015; Flament, 2019; Cao et al., 2021).

To circumvent the key shortcomings of mantle circulations models, Bocher et al. (2016) developed a novel *sequential data assimilation* approach, based upon a suboptimal Kalman filter, which integrates a time series of surface observations chronologically into a mantle convection model until all observations are taken into account. Whenever an observation is available, the analysis evaluates the most likely state of the mantle at this time, considering a prior guess. By running multiple simulations simultaneously to produce multiple evolutionary states, this approach is able to account for observational uncertainties, including those in plate tectonic reconstructions, whilst also allowing surface velocities to evolve more consistently with the underlying force-balance. In contrast to the mantle circulation approach described above, surface velocities are only assimilated at specific points in time, and these observational constraints are weighted with those of the convection simulations at that time, rather than being fully prescribed. The approach of Bocher et al. (2016) was extended by Bocher et al. (2018) using a more advanced
Figure 6: Illustration of different procedures available for estimating mantle structure into the geological past: (a) forward modelling prediction, where an unknown initial condition is estimated at $t_0$, with prediction error, measured as the distance between the predicted and true state, growing in time; (b) sequential data assimilation – having estimated an initial condition at $t_0$, the forward model is run until $t_1$. An analysis is subsequently undertaken from the resulting prediction and the available observation, and a new prediction computed until the next observation at $t_2$. The process is repeated until $t_3$. The information flow diagram depicts how information is carried from both the past and present, using current data; (c)(d) variational data assimilation, via an adjoint model, capable of carrying information explicitly backward in time. In (c) observational data that constrains present-day mantle structure (e.g., images from seismic tomography) are used to optimise the unknown initial condition: with limited data, the predicted initial condition has large uncertainty. This can be reduced, and the evolutionary pathway more tightly constrained, if data from different points in space and time can be brought to bear on the problem, as illustrated in (d). Here, all available observations between $t_2$ and $t_0$ contribute to the analysis. We note that, in reality, this data is scattered in space and time. The true (unknown) signal is represented by the solid blue line. Observations (blue stars), predictions (green circles) and analyses (red squares) are surrounded by ellipsoids of a size proportional to the estimated uncertainty. Modified from Carrassi et al. (2018).

sequential data assimilation method, built around an ensemble Kalman filter (EnKF) (e.g. Evensen 1994, Burgers et al. 1998). At each point in time, the EnKF approximates the probability density function of the state of the system, via a finite ensemble of states. As noted by Bocher et al. (2018), this method is able to exploit information across the ensemble more accurately and, accordingly, is able to provide a more formal estimate of uncertainty. This avenue of research is highly promising and will likely open up new opportunities for geodynamical research, particularly in the context of improving our treatment of uncertainties in plate tectonic reconstructions. To date, however, due to the computational cost of performing numerous global 3-D spherical simulations, the approach has not been applied to 3-D spherical simulations exploiting real-Earth data at realistic convective vigour, which is of fundamental importance when reconstructing the spatial and temporal evolution of mantle flow and its impact at Earth’s surface (e.g. Davies & Davies 2009, Nerlich et al. 2016).

An appealing aspect of sequential filtering and the approach advocated by Bocher et al. (2016, 2018) is that updates are continuously applied to the convection model, with each new observation used to correct the latest model state and, as a result, the consistency between model predictions and observations tends to improve towards the present day as more and more observations are assimilated. There is, however, a fundamental drawback: because of the sequential nature of the assimilation, each observation is used only once and influences the model only at later times. As illustrated in Fig. 6(b), information propagates from the past to the future, but not back into the past. This limitation is a major disadvantage in mantle convection studies, where our knowledge of the mantle at the present day is more detailed than at any earlier time. It is therefore necessary to explore a formulation capable of carrying information explicitly backward in time or, more precisely, one that allows estimation of a time-dependent model of mantle evolution that best fits all available constraints. We term this process inverse geodynamics, where present-day and time-dependent observations are propagated backwards through geological history.

The main challenge in backward-in-time propagation of information is the unconditional instability of diffusive pro-
cesses when one reverses time-stepping. The most computationally straightforward approach for addressing this issue comes from the so-called backward advection method. Here, backward-in-time modelling of mantle flow is achieved by reversing the direction of gravity in the Stokes equation, with thermal diffusion either ignored or set to a small value. Whilst the latter ensures numerical stability, it causes diffusion to act in the wrong direction during backwards-in-time integration. Using present-day mantle heterogeneity, as inferred from seismic observations, Steinberger & O’Connell (1995) simulated mantle flow backwards in time from the present day to examine the advection of plumes. A similar approach was used by Conrad & Gurnis (2003) to examine uplift of the African continent over the Cenozoic Era and Moucha et al. (2008) to investigate the stability of continental platforms. Ignoring thermal diffusion, of course, leads to unrealistic dynamics in regions of the mantle dominated by diffusion, such as the thermal boundary layers at Earth’s surface and CMB. Given that the analyses of present-day dynamic topography presented in Section 2.4 demonstrate a strong sensitivity to shallow mantle structure and flow, the backward advection approach is therefore of limited use in reconstructing the evolution of dynamic topography into the geological past.

The shortcomings of backward advection can be partially overcome using the quasi-reversibility method (QRV), originally proposed by Lattès & Lions (1969). Beginning from a starting guess, an initial condition is reconstructed using a backward energy equation that contains an additional regularisation term designed to stabilise diffusion during the back-in-time component of the simulation. The resulting estimate of the initial condition is subsequently run forward in time using the correct energy equation, and the misfit between the final model state and present-day structure is used to update the starting guess. Through a number of iterations (minimum of five; Glišović & Forte, 2019), it is anticipated that the initial condition will improve sufficiently such that it evolves along a pathway that produces a final model state as close as possible to the known present-day state. The original QVR method was introduced within a geodynamical context by Ismail-Zadeh et al. (2007) and has been updated and used to simulate the evolution of mantle flow and its impact at Earth’s surface by, for example Glišović & Forte (2014, 2016) and Faccenna et al. (2019). These studies demonstrate that the approach provides a substantial improvement on the results of backward advection, offering a means to reconstruct key aspects of the mantle’s 3-D structure and dynamics over the Cenozoic Era that can include the effects of thermal diffusion. Nevertheless, synthetic tests highlight concerns with the accuracy of the QVR approach (Glišović & Forte, 2014), in particular for regions of the mantle that are dominated by diffusion (Ismail-Zadeh et al., 2007; Ismail-Zadeh & Tackley, 2010). This shortcoming is likely a result of introducing the aforementioned regularisation term, which will impact diffusive heat transfer during the back-in-time step and ultimately limit the retrievability of the unknown initial condition. As an aside, the QVR studies of Glišović & Forte (2014) and Glišović & Forte (2016) also use a treatment of surface velocity boundary conditions originally developed by Forte & Peletier (1991) – rather than prescribing plate velocities, plate geometries are used to partition underlying forces in such a way that surface velocities are determined using a combination of free-slip and no-slip boundary conditions.

Given the significance of thermal diffusion in dictating structure and dynamics in the shallow mantle, an optimal approach for geodynamical inversion will robustly account for this process. A formal inverse approach, based upon the so-called adjoint method, provides a means to accomplish this. The method provides a way to globally correct a model with respect to observations that are distributed in space and time using formal adjoint equations that provide sensitivity information, in conjunction with a forward code (e.g. Tarantola, 1987; Talagrand, 1997). Adjoint equations have been derived for incompressible (Bunge et al., 2003) Ismail-Zadeh et al. (2004), compressible (Ghelichkhan & Bunge, 2016) and thermo-chemical (Ghelichkhan & Bunge, 2018) mantle flow. The method has been used to compute an optimal initial condition for mantle flow that provides a present-day state that is consistent, within observational and epistemic errors, with seismological observations of the mantle (e.g. Bunge et al. 2003; Liu & Gurnis, 2008; Liu et al. 2008; Colli et al. 2018; Price & Davies, 2018; Ghelichkhan et al., 2021). In contrast to sequential assimilation methods, the adjoint approach propagates information and uncertainties both forwards and backwards in time (Fig. 4). Furthermore, with the diffusion term in the adjoint energy equation having the opposite sign to that of the forward energy equation, it is unconditionally stable to backward-in-time integration (Bunge et al., 2003).

Despite their promise, the uptake of adjoint methods within the geodynamics community has, so far, been limited. This situation is principally due to: (i) the challenge of developing and implementing adjoint models for non-linear, time-dependent problems, which is notoriously difficult (e.g. Farrell et al., 2013) – to date, global studies have not utilised the full set of compressible adjoint equations applicable to simulations with strongly varying viscosities, which is essential if we are to resolve the role of lithospheric structure and uppermost mantle flow in controlling dynamic topography; and (ii) the computational expense of running this class of models – they require a sequence of forward and adjoint iterations in order to minimise the misfit between model prediction and observation. Nonetheless, owing to increasing computational resources and algorithmic development, this area of computational geodynamics is rapidly evolving and is set to fundamentally change our understanding of the mantle’s thermo-chemical evolution. It is therefore our view that this approach provides the most promising avenue for accurate reconstructions of dynamic topography through space and time.

3.3 Time-dependent global predictions of dynamic topography

The first global predictions of dynamic topography through space and time were developed by Gurnis (1993), using a semi-analytical approach from Richards & Hager (1984), combined with constraints on the reconstructed location of trenches through time from Scotese & Golonka (1992), this study revealed that a large fraction of Phanerozoic continental flooding could be attributed to dynamic subsidence in the vicinity of active subduction zones. Lithgow-Bertelloni & Gurnis (1997) built on this approach to predict dynamic topography at different stages of the Cenozoic Era, reproducing observed flooding trends in the North American, Australian and Indonesian regions. Such simulations also formed a key component of the analyses of Lithgow-Bertelloni & Stiller (1998), which linked dynamic uplift of the African continent to underlying deep mantle structure. Although the amplitudes of uplift and subsidence predicted in these simulations is greater than those inferred from stratigraphic constraints, they were influential in demonstrating that, when constrained by reconstructions of trench locations, models of global mantle flow could be used to predict changes in
long-wavelength dynamic topography at Earth’s surface and, hence, the marine inundation of continents.

Building on these early studies, Bunge et al. (1998, 2002) developed the first global mantle circulation models, using improved constraints on late Mesozoic and Cenozoic plate motion and subduction history (119 Ma to present) from Lithgow-Bertelloni & Richards (1998). These prescribed tectonic reconstructions organise underlying mantle flow, generating a planform of mantle heterogeneity in which the location and depth of slabs is consistent with those imaged through magnetic anomalies (e.g. North Atlantic) and earthquakes, thereby directly connecting ancient ocean floors to structures within Earth’s interior (Bunge & Grand 2000). Nonetheless, owing to the limited temporal extent of the plate motion reconstructions incorporated, these simulations were unable to reproduce salient features of imaged structure in the deep mantle, including LLVPs. It should be noted that time-dependent predictions of dynamic topography from these simulations were not analysed by Bunge et al. (1998, 2002). However, given the strong sensitivity of long-wavelength dynamic topography to deep mantle structure, it seems unlikely that predictions of dynamic topography would match available observational constraints.

Over the intervening years, there has been a major effort to improve global plate tectonic reconstructions, extending them further into deep time and quantifying their uncertainties by incorporating an ever-growing body of observational and mathematical constraints (e.g. Stampfli & Borel 2002; Müller et al. 2008; Seton et al. 2012; Gurnis et al. 2012; Vérard et al. 2015; Müller et al. 2016b; 2019; Young et al. 2019; Ketley et al. 2019; Merdith et al. 2021). More recent mantle circulation models, driven by these enhanced reconstructions, predict a present-day upper-mantle planform that is dominated by strong downwellings in regions of plate convergence. In the mid-mantle, cold downwellings are prominent beneath North America and southeast Asia, whilst remnants of older subduction are visible above the CMB. These downwellings modulate the location of hot material such that it becomes concentrated beneath Africa and the Pacific Ocean, closely matching the general structure of deep mantle LLVPs imaged with seismic tomography (e.g. Schubert et al. 2009; Davies et al. 2012; Flament 2019).

We emphasise that the resemblance between model predictions and tectono-magnetical constraints would deteriorate if these simulations incorporated unreasonable estimates of the mantle’s material properties. Of fundamental importance are: (i) a realistic convective vigour, which controls the length- and time-scales at which convection operates (e.g. Davies & Davies 2009; Nerlich et al. 2016); (ii) an increase in viscosity with depth, which exerts a primary control on the sinking velocity of slabs (e.g. Bunge et al. 1996, 2002; Rubey et al. 2017); (iii) a temperature-dependent viscosity, which helps to maintain the coherence of the lithosphere and the morphology of slabs during their descent (e.g. Davies et al. 2012; Flament 2019); and (iv) compressibility, which modulates the relative temperature and buoyancy of downwelling and upwelling features (e.g. Bunge 2003; Meng & Zhong 2008; Davies et al. 2012; Flament 2019). As confidence has grown that these models capture the distribution of mantle heterogeneity through space and time with sufficient accuracy, they have been used to reveal the role of mantle convection in driving long-wavelength dynamic topography in a number of regions, including North America (Flament et al. 2013), the Arctic region (Shephard et al. 2014), the South Atlantic Ocean (Flament et al. 2015), southeast Asia (Zahirovic et al. 2016), eastern Australia (Müller et al. 2016a), northern Africa (Barnett-Moore et al. 2017) and eastern China (Cao et al. 2018). Globally, such models have been able to reproduce some aspects of long-wavelength subsidence patterns extracted from well data (Rubey et al. 2017), whilst also providing an excellent match with observations of continental flooding throughout the Cenozoic Era (Müller et al. 2018a), reinforcing conclusions from the seminal study of Gurnis (1993).

It is now generally accepted that mantle circulation models robustly predict the long-wavelength topographic expression of the current plate of mantle convection and those components of mantle flow driven by the mantle’s upper thermal boundary layer; Davies (1999). This element includes subsidence in the vicinity of active and recent subduction zones, rebound as the slab descends and its influence on the surface waves, and broad uplift associated with passive upwelling away from regions of subduction. This class of model has, however, had limited success in explaining dynamic topography changes at smaller spatial and temporal scales (e.g. Rubey et al. 2017; Müller et al. 2018a). Careful studies by, for example, Osei Tutu et al. (2018), Coltice et al. (2018) and Arnold et al. (2018), demonstrate that these components of dynamic topography are particularly sensitive to rheological assumptions such as yielding in the shallow crust and lithosphere. Such aspects have, thus far, been neglected in global mantle circulation models owing to algorithmic and computational limitations. Furthermore, our analyses of present-day dynamic topography demonstrate that these components will be sensitive to lithospheric structure and shallow mantle flow, neither of which are well resolved by the current generation of mantle circulation models. Finally, despite some simulations generating primary upwelling mantle plumes in locations that are not inconsistent with expectations from the observational record (e.g. Davies et al. 2015a; Hassan et al. 2016), mantle circulation models have been less successful in resolving the topographic consequences of the plume mode of mantle convection (i.e. those components of mantle flow driven by the lower thermal boundary layer; Davies 1999).

The key reason for this discrepancy is that this class of models does not exploit any direct observational constraints on short- and intermediate-scale upwelling flow, including constraints on present-day mantle structure from seismological and geodetic observations. As noted in Section 5.2, overcoming this limitation requires an inverse approach.

Using a backward advection scheme, alongside constraints on present-day mantle structure from seismic tomography, Conrad & Gurnis (2003) were able to connect Cenozoic uplift of the African continent to upwelling flow associated with the underlying deep mantle LLVP, corroborating the conclusions of Lithgow-Bertelloni & Silver (1998). By exploiting a similar approach, alongside additional constraints from convection-related surface observables, Moucha et al. (2008) concluded that there is likely to be no such thing as a stable continental platform, which has major implications for our understanding of long-term global sea-level change. Although several issues associated with the backward advection approach have been documented in Section 5.2, the first order conclusions of these studies have stood the test of time. This finding is likely related to the fact that these studies focussed on the analysis of long-wavelength features over short temporal durations, where the impacts of erroneous diffusion are less apparent. As noted in Section 5.2, some of the shortcomings of backward advection are overcome by the QRV method, and this is evidenced by the fact that global models by Glijovic & Forte (2016) are amongst the first to produce upwelling plume dynamics that can be closely reconciled with the surface vol-
cyclic record (Glišović & Forte 2017, 2019). The potential for such an approach to capture the topographic consequences of the plume mode of mantle convection is demonstrated by Faccenna et al. (2019), where a long-lived topographic gradient, sustained by upwelling flow beneath Ethiopia and downwelling flow beneath the Levant Sea and northern Egypt, has been invoked to explain the persistence and stability of the Nile river for ~30 Myr. Nonetheless, given concerns with the accuracy of the QRV approach in regions of the mantle dominated by diffusion (e.g. Ismail-Zadeh et al. 2007; Ismail-Zadeh & Tackley 2010), it is important for future studies to verify these predictions using more formal adjoint schemes as their usage, efficiency and applicability continues to develop.

The first application of the adjoint approach for global mantle convection was by Liu et al. (2008) and Spasojevic et al. (2009), who focussed on reconstructing subduction of the Farallon slab and its impact at Earth’s surface over the past 100 Myr. In doing so, they were able to directly link the formation and subsequent disappearance of the Cretaceous interior seaway to underlying mantle flow, reconciling model predictions with many of the observational constraints outlined in Section 3.1 and confirming the hypotheses of Bond (1978b); Cross & Pijler (1978); Mitrovica et al. (1989) and Burgess et al. (1997). An important finding from these studies was the sensitivity of the scale and temporal duration of subsidence to the mantle’s viscosity structure: models deemed consistent with stratigraphic observations could therefore be used to constrain the depth dependence of mantle viscosity. The simulations of Liu et al. (2008) were further analysed by Shephard et al. (2010), to reveal the key role of dynamic topography in driving Miocene reversal of the Amazon River in South America. Together, these studies were influential in demonstrating the power of adjoint methods for reconstructing the spatial and temporal evolution of dynamic topography. Nevertheless, they suffered from two important shortcomings: (i) a ‘stress-guide’ was required to recover the subduction pathway of the Farallon slab, likely owing to the neglect of complex rheologies and interactions that control the evolution of flat-slab subduction (e.g. Garel et al. 2014); (ii) the adjoint approach was only applied to the energy equation (Liu & Gurnis 2008) with no coupling to the Stokes and continuity equations.

The latter shortcoming was overcome by Colli et al. (2018), who used the complete adjoint equations for compressible convection to analyse the evolution of dynamic topography in the South Atlantic and African regions. It is reassuring that at long-wavelengths, predictions of dynamic topography from Colli et al. (2018) are generally compatible with earlier generations of mantle circulation models, although they also capture intermediate wavelength features associated with the plume mode of mantle convection, including the ‘basin and swell’ morphology of the African continent. Comparing predictions more closely, exemplar mantle circulation models by Rubey et al. (2017) predict dynamic support beneath southern Africa that gradually develops through the Cretaceous period. The models of Colli et al. (2018), however, predict a multi-stage uplift history, including a substantial post-30 Ma uplift phase, which appears to be more consistent with many of the observational constraints that argue for significant Neogene rejuvenation of African topography (Section 3.1).

The study of Ghelichkhan et al. (2021) was the first to undertake systematic comparisons between adjoint models and observational constraints at the global scale. Ghelichkhan et al. (2021) examined a suite of eight high-resolution adjoint simulations, comprising different combinations of depth-dependent viscosity profiles and estimates of present-day mantle heterogeneity. Shallow mantle structure (<300 km) is constrained by the SL2013sv surface-wave tomography model (Schaefier & Lebedev 2013), which introduces substantially higher resolution features that have previously been absent from most time-dependent studies, while the deeper mantle makes use of two different whole-mantle models (Simmons et al. 2015; French & Romanowicz 2015). The dynamic topography predictions from one such simulation, RMµ3-SL, are next described. The model assumes a present-day deeper mantle structure based on the SEMUCB-WM1 model of French & Romanowicz (2015) and employs the 5-layer radial viscosity profile illustrated in Fig. 3(b) (see Ghelichkhan et al. (2021) for further details).

To allow for straightforward comparison with the observational constraints summarised in Section 3.1, Fig. 7 displays the reconstructed dynamic topography from 50 Ma to the present day in the three continental regions of North America, Australia and Africa. In the interior of the North American continent (Fig. 7 top row), the model predicts a dynamic topography low to be already present during early Eocene times that is driven by the downgoing Farallon slab. Much of this negative dynamic topography disappears over the following 20 Myr due to a rebound induced by a combination of the westward drift of North America and continued descent of the slab into the deeper mantle. In addition, the model predicts positive dynamic topography along the eastern margin of North America during the early Eocene, which transitions into negative dynamic topography at the present day. These predictions are broadly consistent with the observed Cenozoic rebound of the Cretaceous interior seaway and also Neogene subsidence recorded in the Baltimore Canyon Trough. Consistent with Liu et al. (2008) and Spasojevic et al. (2009), Ghelichkhan et al. (2021) find a strong sensitivity of dynamic topography predictions to the radial viscosity structure, influencing both the scale and rates at which dynamic topography changes. As will be demonstrated below, they also find a sensitivity to the tomographic model used as a constraint on present-day mantle density. For North America, however, all eight models consistently predict a dynamic topography low at 50 Ma over the interior of the continent. This general agreement between models is likely related to robust imaging of fast seismic velocity anomalies associated with the Farallon slab in the mantle beneath this region.

Changing focus to Australia (Fig. 7 middle row), the model predicts regional Cenozoic uplift for eastern and southern Australia. This uplift, driven by motion of the Australian plate over the margin of the Pacific LLVP (as suggested by Muller et al. 2016a) has long-wavelength features that are consistent with observations of regional Cenozoic uplift for the continent’s central and eastern regions (e.g. Czarnota et al. 2014). It appears after an initial negative signal over much of the continent, which migrates southwards to lie at the AAD at the present day, being driven by the passage of Australia over a relic slab from the Gondwanaland-Pacific convergent margin (Gurnis et al. 1998). The model predicts subsidence in the northern part of the continent in the late Neogene (shown at 10 Ma), consistent with a range of stratigraphic and geomorphological observations that point towards northward tilting of the continent (e.g. Sandford 2007; DiCaprio et al. 2009; Czarnota et al. 2013). Importantly, the Australian region illustrates the sensitivity of adjoint models to their tomographic input structure – the models of Simmons et al. (2015) and French & Romanowicz (2015) that are used by Ghelichkhan et al. (2021) are less consistent in this region, particularly in the deep mantle, which impacts on regional topographic predictions. This discrep-
Figure 7: Air-loaded dynamic topography predictions from the RM-µ1-SL adjoint simulation of Ghelichkhan et al. (2021) in three regions: top row = North America, middle row = Australia; bottom row = Africa. Reconstructed coastlines are obtained by back-rotation of present-day coastlines using the plate reconstruction model of Young et al. (2019). See text for further details.

Figure 7: Air-loaded dynamic topography predictions from the RM-µ1-SL adjoint simulation of Ghelichkhan et al. (2021) in three regions: top row = North America, middle row = Australia; bottom row = Africa. Reconstructed coastlines are obtained by back-rotation of present-day coastlines using the plate reconstruction model of Young et al. (2019). See text for further details.

Recent observations of dynamic topography at different spatial and temporal scales; and (iii) a means to supplement the observational record where no markers of dynamic topography have been identified.

It is our view that adjoint methods provide the most promising avenue for reconstructing the spatial and temporal evolution of dynamic topography at the global scale. The reasoning behind this, from an algorithmic perspective, has
been outlined in Section 3.2 the approach offers a robust mechanism for exploiting diverse observational constraints on the mantle’s present-day structure and temporal evolution to guide dynamical simulations that are underpinned by fundamental physical principles. The power of adjoint schemes has been illustrated in Section 3.3 whilst existing studies have been forced to make some physical simplifications that impact results, they clearly highlight the transformative nature of this approach and its potential to fundamentally improve our understanding of the evolution of dynamic topography across a wide-range of spatial and temporal scales.

Nonetheless, the uptake of these schemes within the geodynamics community has been limited, with only a small number of global studies using the approach (Lu et al. 2008; Spasojevic et al. 2009; Shephard et al. 2010; Coll et al. 2015; Ghelichkhan et al. 2021). As noted in Section 3.2 this is principally due to the computational expense of running this class of model, alongside the formidable practical challenges of deriving and implementing adjoints for these problems. In Section 3.3, we introduced a limitation of the models of Ghelichkhan et al. (2021): a rheological approximation that fails to incorporate lateral viscosity variations and yielding in the uppermost mantle. However, extending these models to incorporate more realistic visco-plastic rheologies is a non-trivial task: the non-linear coupling introduces a number of additional terms in the forward and adjoint equations, the derivation, subsequent implementation and validation of which is challenging and time-consuming. Given these bottle-necks, thus far, global studies have not utilised the full set of compressible adjoint equations relevant for simulations with non-linear rheologies (e.g. Li et al. 2017). As a result, they are unable to capture a fundamental aspect of uppermost mantle dynamics and its contribution to dynamic topography. Based upon our analyses of present-day dynamic topography (Section 2), we hypothesise that this shallow mantle contribution is key to explaining the shorter scale spatial and temporal evolution of dynamic topography, thus explaining why existing models cannot currently be reconciled with all of the available observational constraints.

It is encouraging, therefore, that recent developments in computational science promise to open up this class of problem to geodynamical research. High-level finite element software frameworks, such as FEniCS (e.g. Logg et al. 2012; Alnæs et al. 2014) and Firedrake (e.g. Rathgeber et al. 2016), allow a user to specify the discrete formulation of a forward model at a high level of mathematical abstraction. State-of-the-art compilers subsequently generate low-level code from these abstractions, incorporating sophisticated performance optimisations that few users would have the resources to code by hand. The combination of these packages with tools that automatically derive a fully consistent adjoint of the discrete equations (e.g. Farrell et al. 2013), offers a radical new approach for the development and use of adjoint-based research software infrastructure, providing a straightforward mechanism for incorporating different approximations of the governing equations. Demonstrating the applicability of this approach for geodynamical simulation is an ongoing area of research, although early results are promising (Davies et al. 2022).

Of course, these next-generation tools can only do so much: their success at facilitating reconstructions of dynamic topography in space and time will ultimately be determined by the available observational constraints. Key to successful inverse geodynamic models are tight constraints on the present-day state of the system. Thus far, all global adjoint studies have utilised present-day constraints on mantle structure from seismic tomography models (e.g. Schaeffer & Lebedev 2013; Simmons et al. 2015; French & Romanowicz 2015). As illustrated in Section 3.3 and more completely in the study of Ghelichkhan et al. (2021), predictions of dynamic topography remain strongly sensitive to differences between these tomographic models. The importance of ongoing improvements to tomographic models (e.g. Taklic et al. 2015; Mouniari et al. 2021) and a robust estimate of their uncertainty (e.g. Zaroli 2019), therefore, cannot be overstated. It is also crucial to exploit up-to-date thermodynamic and experimental constraints on density and phase proportions, as well as anelastic corrections, when converting between seismic and physical structure (e.g. Stixrude & Lithgow-Bertelloni 2005; Connolly 2009; Stixrude & Lithgow-Bertelloni 2011; Cline et al. 2018; Richards et al. 2020), tuned to other interdisciplinary geodetic, geodynamic and seismic considerations (e.g. Ghelichkhan et al. 2018; Richards et al. 2020a,b), and to also account for the uneven resolution inherent to tomographic models (e.g. Bunge & Davies 2001; Risense et al. 2007). Ongoing progress in this area will be fundamental to a successful adjoint model of mantle evolution. Finally, we note that further insight into present-day mantle structure for future adjoint models can also be derived by combining seismological observations with the complementary constraints from other geodynamical observables, such as the geoid, as advocated by, for example, Moucha et al. (2009) and Giliovic & Forte (2016).

Whilst knowledge of the mantle’s present-day structure is vital, it provides only limited constraints on the mantle’s evolution into the geological past: other observations that extend into the time domain are needed to constrain the time dependence of the system (Fig. 6b). A key constraint comes from the velocity of plates at Earth’s surface, and the location and nature of plate boundaries (e.g. Coll et al. 2015; Müller et al. 2019). Revealing how tectonic plates and their boundaries have evolved over geological time provides a geographical context that is a prerequisite for understanding how Earth’s surface interacts with its interior. As noted in Section 3.3, there have been dramatic improvements in plate tectonic reconstructions over recent years, including extending them into deep time, accounting for lithospheric deformation, and optimising absolute plate motions in the mantle reference frame (e.g. Müller et al. 2019; Tetley et al. 2019; Meredith et al. 2021). Further developments, including a more robust quantification of uncertainties potentially through the approach of Bocher et al. (2018), will underpin future efforts to reconstruct the evolution of dynamic topography.

A shortcoming of the observational datasets discussed thus far is that they provide only limited constraints on the deep mantle’s thermochemical evolution. Amongst other features of the geological record, geochemical observations from volcanic rocks could potentially help to fill this gap. The principal challenge is to decode these datasets to infer key characteristics of their mantle source (e.g. Niu et al. 2011; Davies et al. 2015b; Jones et al. 2017), since the geochemistry of any melt depends upon source composition, depth, and temperature of melting (e.g. McKenzie & Bickle 1988). Volcanic rocks therefore have the potential to provide pointwise constraints on mantle composition and thermal state at specific locations in space and time (e.g. McNab et al. 2018; Klockling et al. 2018; Ball et al. 2019; 2021), and successfully incorporating these data into future simulations is likely to prove fruitful.

This is an exciting time in geodynamics. With an ever-growing body of high-quality observational constraints on the mantle’s thermochemical structure and evolution, alongside
the development of cutting-edge computational algorithms that provide a framework for fully exploiting these datasets, the stage is set to illuminate the history of our planet’s interior in unprecedented detail. Doing so will underpin progress across many areas of geoscience, opening up new possibilities to explore connections between Earth’s surface and its interior across a previously unimaginable range of spatial and temporal scales.

References


Acknowledgments: All authors acknowledge support from the Australian Research Data Commons (ARDC: [https://ardc.edu.au/]) under the `node platform grant: PL031), ANSscope: Geoscience Australia and the National Computational Infrastructure (NCI), DRD and APV acknowledge support from the Australian Research Council (ARC) (DP200100053). MJH acknowledges support from the Australian Government’s Exploring for the Future program and an ARC Discovery Early Career Research Award (DE220105139). APV acknowledges an ARC
Discovery Early Career Research Award (DE180100040). Numerical simulations were undertaken at the NCI National Facility in Canberra, Australia, which is supported by the Australian Commonwealth Government. The authors are grateful to David Al-Attar, Jacky Austermann, Hans-Peter Bunge, Karol Czarnota, Caroline Eakin, Nicolas Flament, Saskia Goes, Stephan Kramer, Jerry Mitrovica, Dietmar Müller, Nicholas Rawlinson, Malcolm Sambridge, Cian Wilson, Jeff Winterbourne and many other colleagues for insightful discussions and advice over many years of research on this topic.

**Data availability:** The specific present-day residual topography dataset used here, which was originally constructed by [Hoggard et al. (2017)](https://github.com/drhodrid/Davies_etal_NGeo_2019_Datasets), may be obtained from [https://github.com/drhodrid/Davies_etal_NGeo_2019_Datasets](https://github.com/drhodrid/Davies_etal_NGeo_2019_Datasets). Gaussian Process code to analyse the present-day residual topography dataset may be obtained from [https://github.com/valentineap/DynamicTopographyGP](https://github.com/valentineap/DynamicTopographyGP) (doi: 10.5281/zenodo.3895317). Model predictions of dynamic topography and geoid height anomalies may be obtained from [https://github.com/drhodrid/Davies_etal_2022_observations_models_dynamic_topography](https://github.com/drhodrid/Davies_etal_2022_observations_models_dynamic_topography).

**Competing interests:** The authors declare no competing financial interests.