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Robust Probabilities of Detection and Quantification Uncertainty for Aerial Methane Detection: Examples for Three Airborne Technologies

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Abstract

Thorough understanding of probabilities of detection (POD) and quantification uncertainties is fundamentally important when applying aerial measurement technologies as part of alternative means of



emission limitation (AMEL) or alternate fugitive emissions management programs (Alt-FEMP), as part of monitoring, reporting, and verification (MRV) efforts, and in surveys designed to support measurement-based emissions inventories and mitigation tracking. This paper presents a robust framework for deriving continuous probability of detection functions and quantification uncertainty models for aerial measurement techniques based on controlled release data. Using extensive fully- and semi-blinded controlled release experiments to test Bridger Photonics Inc.'s Gas Mapping LiDAR (GML)TM, as well as available release data for Kairos LeakSurveyorTM and NASA/JPL AVIRIS-NG technologies, robust POD functions are derived that enable calculation of detection probability for any given source rate, wind speed, and flight altitude. Uncertainty models are separately developed that independently address measurement bias, bias variability, and measurement precision, allowing for a distribution of the true source rate to be directly calculated from the source rate estimated by the technology. Derived results demonstrate the potential of all three technologies in methane detection and mitigation, and the developed methodology can be readily applied to characterize other techniques or update POD and uncertainty models following future controlled release experiments. Finally, the analyzed results also demonstrate the importance of using controlled release data from a range of sites and times to avoid underestimating measurement uncertainties.

35 **Highlights**

- 36 • Generalized method presented to derive aerial methane detection sensitivity
- 37 • Generalized error model also developed to derive quantification uncertainty
- 38 • Continuous probability of detection functions derived for three aerial technologies
- 39 • Results give detection probability for any source, wind, and flight altitude
- 40 • Enables use of aerial data in MRV, AMEL/Alt-FEMP, and measurement-based inventories

41 **1 Introduction**

42 Methane is a potent yet short-lived greenhouse gas and rapid reductions in methane emissions from energy,
43 waste, and agriculture sectors are an essential part of the pathway to limiting global temperature rise (Arias
44 et al., 2021; CCAC, 2021; IPCC, 2018). However, successful mitigation of emissions is contingent on the
45 ability to reliably detect both known and unknown sources of methane. Moreover, development of
46 trustworthy emission inventories and tracking progress toward mitigation targets requires accurate
47 measurements within defined uncertainties. This challenge is at the heart of emerging monitoring, reporting
48 and verification (MRV) efforts (European Commission, 2021) and the associated verification role of the
49 United Nations International Methane Emissions Observatory (IMEO).

50 In recent years, a range of potential detection and/or measurement technologies have been explored
51 with promise to significantly reduce time and labour costs to find and measure sources of methane,
52 especially for applications in the oil and gas sector (Bell et al., 2020; Fox et al., 2019; Kemp and Ravikumar,
53 2021; Rashid et al., 2020; Ravikumar et al., 2019; Schwietzke et al., 2019). Of particular interest are
54 airplane-mounted technologies which are increasingly used in large-scale field campaigns with success
55 (Chen et al., 2022; Cusworth et al., 2021; Tyner and Johnson, 2021) and gaining acceptance in alternate
56 fugitive emissions management programs (Alt-FEMP) replacing or supplementing optical gas imaging
57 (OGI) surveys using hand-held infrared cameras (AER, 2021; Bridger Photonics, 2022; InvestableUniverse,
58 2021; Kairos Aerospace, 2022a). With sensitivities >100-1000 times better than current satellite systems,
59 airplane-mounted sensors have emerged as a key tool for mitigating methane, well-suited to the challenging
60 “verification” component of MRV and capable of being used to create measurement-based inventories.
61 However, successful application of these technologies and interpretation of collected data requires a
62 thorough understanding of the probability of detecting unknown sources under different conditions and
63 uncertainty in quantifying emissions from detected sources. To date, only limited controlled release studies
64 have appeared in the literature (Bell et al., 2020; Johnson et al., 2021; Ravikumar et al., 2019; Sherwin et
65 al., 2021; Thorpe et al., 2016) and robust methodologies to meet these requirements have not been
66 developed.

67 This paper has four main objectives. First, a novel generalized approach to deriving continuous
68 probability of detection (POD) functions is presented that significantly improves upon existing formulations
69 in the literature that are often non-physical. Generalized POD functions are essential for understanding
70 what is or is not captured in field measurements and modelling applicability and mitigation potential of
71 technologies in programs like FEAST (Fugitive Emissions Abatement Simulation Toolkit; Kemp et al.,
72 2016). Second, a statistical error model is presented to derive quantification uncertainties in aerial-
73 estimated source rates. Together with robust POD data, quantification uncertainties are essential for
74 defensibly applying airborne measurements for MRV and ultimately for using aerial data in measurement-
75 based inventories. Third, using extensive controlled release experiments completed to evaluate Bridger
76 photonics' gas mapping LiDAR (GML) system (Bridger Photonics, 2021) as an initial case study, a
77 continuous POD function and quantification uncertainty model are derived. Finally, using available
78 published controlled release data, the methods are extended to also estimate robust POD and quantification
79 uncertainty of Kairos LeakSurveyor™ (Kairos Aerospace, 2022b) and POD of NASA's Jet Propulsion
80 Laboratory's Next-Generation Airborne Visible/Infrared Imaging Spectrometer (AVIRIS-NG) platform
81 (Thorpe et al., 2016).

82 **2 Methodology**

83 **2.1 Methane Detection Technologies**

84 **2.1.1 Bridger Photonics Gas Mapping LiDAR™**

85 Bridger Photonics Gas Mapping LiDAR (GML) uses an airplane-mounted scanning laser, camera, and
86 Global Navigation Satellite System – Inertial Navigation System (GNSS-INS) to detect methane sources
87 and produce quantitative geo-located imagery of associated plumes (Bridger Photonics, 2021; Hunter and
88 Thorpe, 2017; Johnson et al., 2021; Kreitinge and Thorpe, 2018). Originally developed through the
89 Advanced Research Project Agency – Energy (ARPA-E) MONITOR program (ARPA-E, 2018), the
90 technology uses wavelength modulation spectroscopy at 1651 nm to measure path-integrated methane
91 concentrations between the aircraft and the ground, which acts as a topographic backscatterer. Forward
92 and backward looking measurements as the plane flies give information on the detected plume height,
93 typically within 2 m accuracy (Johnson et al., 2021). At typical target altitudes between 168 and 230 m
94 above ground level (AGL), the sensor's 31° field-of-view results in an approximately 94–130 m wide
95 measurement swath on the ground and resolves plumes with ~1–2 m spatial resolution. Source emission
96 rates are estimated by a proprietary method that combines information about the spatial concentration of
97 methane in the detected plume, the height of the plume above ground level, the horizontal wind speed at
98 the time of detection (Bridger Photonics typically uses interpolated hourly meteorological station data
99 provided by Meteoblue ([meteoblue.com](https://www.meteoblue.com))), and the assumed vertical profile of wind speed. Preliminary

100 analysis of blinded controlled releases by Johnson et al., (2021) suggests that 1σ quantification uncertainties
101 of ± 31 – 68% can be expected for sources near the sensitivity limit. However, uncertainties at higher release
102 rates and over a broader range of conditions are not well-described in the literature and a robust
103 understanding of these uncertainties is an important goal of this paper.

104 2.1.2 Kairos Aerospace LeakSurveyor™

105 Kairos Aerospace’s LeakSurveyor is an airplane-mounted methane imaging system that combines an
106 infrared imaging spectrometer, global positioning system (GPS) and inertial monitoring unit (IMU), and
107 optical camera to detect methane plumes (Berman et al., 2021; Branson et al., 2021; Schwietzke et al.,
108 2019). Path integrated methane concentrations are measured via absorption of reflected sunlight from the
109 ground in spectral regions where there is no interference from other common hydrocarbons (Berman et al.,
110 2021). For the targeted flight altitude of 900 m AGL, each measurement swath is approximately 800 m
111 wide with a spatial resolution of ~ 3 m (Sherwin et al., 2021). As summarized in (Berman et al., 2021;
112 Sherwin et al., 2021), quantification is via a proprietary algorithm that calculates pixel-level methane
113 column density between the airplane and the ground, sums these estimates within a core-plume region with
114 distinguishable methane enhancements from background, divides by the length of this core plume region,
115 and multiplies by an estimated wind speed. Compared to Bridger Photonics’ *active* GML sensor, the
116 *passive* LeakSurveyor from Kairos Aerospace trades potential advantages of larger measurement swath
117 permitting greater facility coverage per airplane pass with the disadvantages of lower spatial resolution and
118 higher minimum detection limits as well as potentially greater sensitivity to environmental lighting
119 conditions.

120 Because in-situ wind speed is not generally available for aircraft-detected sources and database wind
121 speed can be highly uncertain, Kairos Aerospace typically provides source rate estimates on a wind-
122 normalized basis – i.e., in units of emission rate per wind speed (Branson et al., 2021). Kairos’ in-house
123 (Berman et al., 2021) and third-party (Sherwin et al., 2021) assessments of the LeakSurveyor technology
124 have estimated detection sensitivities in these units of approximately 8.2 (at a 50% POD) and
125 5-15 (kg/h)/(m/s) ("partial detection range"), respectively. Quantification bias was also assessed by Kairos
126 Aerospace on wind-normalized source rate-basis and found to be approximately -2% (Branson et al., 2021);
127 precision errors were not analyzed. In their controlled release study, Sherwin et al., (2021) independently
128 evaluated quantification error in emission rate (non-normalized units of kg/h) by multiplying
129 LeakSurveyor-reported wind-normalized source rate data by wind speed estimated from four different
130 sources. The parity slope of estimated-to-controlled source rates ranged from 0.88 to $1.45\times$, representing
131 a bias on the order of -12 to $+45\%$ depending on the source of wind speed data. Precision errors were
132 estimated using the residuals of linear fits to controlled release data and were on the order of 30-42% (1σ).

133 2.1.3 *NASA JPL's Next-Generation Airborne Visible/Infrared Imaging Spectrometer*

134 The next-generation airborne visible/infrared imaging spectrometer (AVIRIS-NG; Hamlin et al., 2011) is
135 an improvement on the original AVIRIS instrument (Green et al., 1998) developed by the U.S. National
136 Aeronautics and Space Administration's (NASA) Jet Propulsion Laboratory (JPL). The AVIRIS-NG
137 instrument is a push-broom imaging spectrometer with approximately 5 nm spectral resolution over the
138 visible and near-infrared spectra (380 to 2510 nm). At flights altitudes relevant for methane point source
139 detection (~400 to 3800 m AGL), the 34° field-of-view provides swath widths of approximately 250 to
140 2500 m with spatial resolutions of 0.4 to 3.6 m. Methane columns are retrievable using differential optical
141 absorption spectroscopy (e.g., Thompson et al., 2015) or matched filter methods (e.g., Foote et al., 2020)
142 permitting downstream processing to identify methane plumes.

143 Although the development of AVIRIS(-NG) was not specifically motivated by methane detection,
144 AVIRIS-NG has been successfully used to detect, map, and monitor large-scale methane emitters.
145 Methane-relevant studies have targeted measurements at an array of assorted facility types (Duren et al.,
146 2019; Guha et al., 2020) with some focusing on oil and gas facilities (Cusworth et al., 2021; Frankenberg
147 et al., 2016; Thorpe et al., 2020) solid waste facilities (Cusworth et al., 2020; Krautwurst et al., 2017), and
148 arctic permafrost (Elder et al., 2020). In 2013, Thorpe et al. (2016) mounted the AVIRIS-NG instrument
149 on a Twin Otter aircraft during controlled release experiments to evaluate methane retrieval algorithms and
150 assess detection sensitivity as a function of wind speed and aircraft altitude; the accuracy of methane source
151 rate estimation using AVIRIS-NG has not been evaluated at the time of writing.

152 2.2 *Controlled Releases – Bridger GML*

153 For this study, controlled methane releases were completed during two separate field campaigns in 2020
154 and 2021 at oil production sites near Lloydminster, Saskatchewan to assess Bridger Photonics' GML
155 technology. These releases were completed as part of broader measurement surveys across western Canada
156 and included both semi-blinded and fully blinded experiments to assess quantification accuracy as well as
157 detection sensitivity under varying conditions. First, working collaboratively with Bridger Photonics and
158 the contracted airplane operator, high-flowrate *semi*-blinded controlled releases were completed to derive
159 GML quantification uncertainties when measuring methane sources emitting between 1 and 66 kg/h,
160 consistent with 96% of sources found in a recent survey of oil and gas infrastructure in BC, Canada (Tyner
161 and Johnson, 2021). Releases were made from a set of four inactive oil and gas facilities conveniently
162 arranged in a line approximately 375 m apart (refer to supplemental information (SI) for additional detail).
163 Over several days during each campaign, the plane flew laps over the test facilities while flow rates at each
164 site were independently varied between each lap at predetermined random flow rates (including zero
165 releases) that were not shared with Bridger Photonics.

166 Second, following the same approach used in (Johnson et al., 2021), additional low-flowrate controlled
 167 releases (0.4–5.2 kg/h) plus zero-releases were performed from active sites included in parallel contracted
 168 surveys of oil and gas infrastructure in the region. In collaboration with industry operators, methane was
 169 released at random rates near the expected sensitivity limit of the GML technology to test its ability to
 170 correctly detect unknown sources at unknown locations. These tests were fully blinded in that they were
 171 conducted without informing Bridger Photonics that the experiments were taking place.

172 At each release location time-resolved wind speed at 3 m above ground level was measured at 1 Hz
 173 using an ultrasonic wind sensor (Anemoment, TriSonica mini) with a rated accuracy of ± 0.2 m/s over the
 174 relevant range of 0–10 m/s. As in (Johnson et al., 2021), after initial results were first obtained from
 175 Bridger, these in-situ wind speed data were subsequently provided to Bridger Photonics Inc., who
 176 reprocessed their results and also returned a single-valued wind speed at plume height for each flight pass
 177 and detected source. Methane from compressed cylinders (PraxAir, >99% purity) was released through
 178 Bronkhorst thermal mass flow controllers (various models, rated accuracy of $\pm 0.1\%$ of full scale or $\pm 0.5\%$
 179 of reading). For the larger flow rates, a custom-built heated regulator and liquid-gas heat exchanger system
 180 were used to overcome Joule-Thomson cooling of the gas and ensure temperatures were near ambient as it
 181 entered the flow controllers and was subsequently released to atmosphere. At each release location, GPS-
 182 synchronized data loggers were used to record methane release rate and wind speed data that could
 183 subsequently be matched with time-stamped data provided by Bridger. This was especially important in
 184 confirming missed detections during the fully blinded releases from within sites included in the parallel
 185 surveys of oil and gas infrastructure. Table 1 summarizes the collected controlled release data.

186 **Table 1: Summary of Controlled Release Experiments to test Bridger Photonics’ GML completed**
 187 **as part of the present study.**

| Release Set | Year | Count |
|---|--------------------|--------------------|
| High-flowrate (1–66 kg/h), semi-blinded releases from a fixed set of inactive facilities ^a | 2020 | 122 (+16 zeros) |
| | 2021 | 162 (+13 zeros) |
| Low-flowrate (0.4–5.2 kg/h), fully blinded releases from within active sites included in parallel oil & gas sector surveys ^b | 2020 | 67 (38 Misses) |
| | 2021 | 115 (24 Misses) |
| Total | 495 total releases | |

^a All non-zero semi-blinded releases were detected.

^b Representative scene noise was provided with the standard data product for small-volume releases in 2020 and 2021

188

189 **2.3 Available Controlled Release Data for Kairos' LeakSurveyor**

190 Using the new methodology presented below, a robust POD function and uncertainty model were also
191 developed for Kairos' LeakSurveyor using published controlled release data from Sherwin et al. (2021)
192 augmented with internal controlled release data obtained from Kairos similar to (Chen et al., 2022).
193 Sherwin et al. (2021) completed 234 semi-blinded controlled release tests of Kairos' LeakSurveyor from a
194 single facility located in San Joaquin County, California over four days spanning October 8 to 15, 2019.
195 These included 210 non-zero controlled releases between 18 to 1,025 kg/h. Three data points were
196 discarded following Sherwin et al.'s (2021) initial quality control and the remaining 207 releases were used
197 to assess detection sensitivity in the present work – 40 of these 207 releases were purposely performed at
198 low flowrates near the lower limit of the flowmeter (<50 kg/h). Of the original 210 releases, 149 were
199 considered for the present assessment of quantification error, corresponding to the subset of release data
200 with a successful detection, a controlled rate > 50 kg/h, and no quality control concerns. Wind speeds were
201 measured in situ at 8 ft (~2.43 m) above ground level using two instruments: a cup-based wind meter and
202 a two-dimensional ultrasonic anemometer (on the latter three days only). Sherwin et al. (2021)'s analysis
203 also evaluated quantification error for the practical scenario where in-situ wind speed data are not available,
204 testing accuracy when using minute-resolved data from the commercial Dark Sky database (Apple Inc.,
205 2022) and hourly data from the public High-Resolution Rapid Refresh (HRRR) database (NOAA, 2020).
206 The LeakSurveyor sensor was used to detect and, where possible, quantify the controlled releases and was
207 flown at a nominal altitude of 900 m AGL throughout the study. For the present analysis of POD, in-situ
208 wind speed from the ultrasonic anemometer is favoured when available due to its improved accuracy over
209 the cup-based meter; for the measurement day where only data from the cup-meter were available, these
210 data are corrected based on a linear fit with available ultrasonic data. Sherwin et al. (2021) chose the one-
211 minute gust as the representative measured wind speed (corresponding to the maximum speed during the
212 minute prior to the aircraft overpass) in the main text of their analysis, which matches the wind speed
213 preferred in Kairos' quantification as further discussed below. By contrast, the present analysis uses the
214 one-minute *averaged* wind speed prior to the aircraft overpass as it is likely to be more indicative of
215 convective dispersion of the plume prior to detection and is the relevant windspeed to consider when
216 planning a survey or modelling expected performance in simulators like FEAST. To standardize wind
217 speeds against the present controlled releases, all available wind data were scaled to a 3-m height AGL
218 using a logarithmic profile with a specified zero-displacement plane, d , of 0.066 m and a surface roughness,
219 z_0 , of 0.01 m representative of the graded areas around oil and gas areas as used in Bridger's algorithm
220 (Johnson et al., 2021).

221 Additional data from internal controlled release studies were provided by Kairos' to augment the
222 present analysis of detection probabilities and quantification error. These confidential data include
223 controlled source rate, estimated wind-normalized source rate, measured wind speed, and one-minute gust
224 wind speed from the Dark Sky database for 375 additional non-zero releases. Within these data a total of
225 45 releases were missed and 296 releases were automatically detected by Kairos' algorithm; the remaining
226 34 were tagged as partial detects, which required human interpretation to identify a plume. When combined
227 with the publicly available controlled release data of Sherwin et al. (2021) (which are treated as automated
228 detects since the available data did not distinguish partial detects), there were a total of 485 detects, 34
229 partial detects, and 63 missed detections. Additional analysis in the SI shows the effects of treating these
230 data sets separately. For the quantification uncertainty modelling, partial detections were not quantified,
231 and there were total of 149–495 available source measurements depending on which wind data source was
232 considered.

233 ***2.4 Published Controlled Release Data for NASA JPL's AVIRIS-NG***

234 A POD function for the AVIRIS-NG sensor was derived using the controlled release data reported by
235 Thorpe et al. (2016). These experiments were originally designed to evaluate the ability of AVIRIS-NG in
236 detecting methane point sources and the available data do not include separate source rate estimates from
237 the plane. A total of 143 controlled releases were completed over seven days in June 2013 from three
238 separate sites within the Rocky Mountain Oilfield Testing Center in Wyoming, U.S.A. Thorpe et al. (2016)
239 measured wind speeds at 8–9 m AGL using a 3D ultrasonic wind anemometer. For the present analysis of
240 detection probability, reported wind speeds (averaged over the minute preceding a detection) were scaled
241 from an average height of 8.5 m AGL to 3 m AGL using the same logarithmic profile and parameters noted
242 above; the resulting wind speeds at 3 m spanned 0.66–7.5 m/s. Controlled release rates ranged from 2.2 to
243 96 kg/h and flight altitudes were between 430 to 3800 m AGL. For each release, the methane plume was
244 flagged as either detected (automatic detection by algorithm, $N = 94$), partially detected (requiring human
245 interpretation, $N = 25$), or missed ($N = 24$).

246 **3 Statistical Analysis**

247 ***3.1 Generalized Approach to Deriving Robust Probability of Detection Functions***

248 For a specified remote detection technology, the probability of detection (POD) function represents the
249 likelihood of successfully detecting an emitter at some source rate for a given set of conditions during a
250 single measurement observation. Although different technologies may be affected by additional
251 parameters, in general, detectability of a given source (at rate Q) depends on the wind field that drives
252 plume dispersion, the spatial resolution of the measurement, and the effective signal-to-noise ratio (SNR)

253 of the measurement system. For simplicity, the effects of wind can be parameterized by the measured 3-m
 254 wind speed (u_3). For Bridger’s GML technology, the measured 3-m wind speed is averaged according to
 255 their algorithm and sourced directly from Bridger’s reported results using the in-situ wind data at 1 Hz. For
 256 the imaging spectrometers, measured wind speed is averaged over one minute prior to the aircraft overpass
 257 to be consistent with (Sherwin et al., 2021). For a fixed set of sensor optics, the ground-level spatial
 258 resolution is defined by the altitude of the measurement system above ground level (\tilde{h} [m]). The effective
 259 SNR for a given measurement is itself a function of Q and u_3 (which affect the observed path-integrated
 260 concentration of the plume), \tilde{h} (which affects signal strength through the inverse square law), the spectral
 261 albedo of the ground (which affects the strength of the return signal), and potentially other parameters
 262 specific to the technology. Although additional SNR data may or may not be readily available for a given
 263 technology as further considered below, it is initially considered in this general analysis as a representative
 264 scene noise in units of column density (\tilde{n} [ppm-m]). Using these parameters, a POD function can be derived
 265 that depends on $\mathbf{x} = [Q, u_3, \tilde{h}, \tilde{n}]^T$. Technically, the plume height (\tilde{z}_p) is also a relevant parameter since
 266 plume dispersion is height-dependent; however, since this is undefined for a failed detection, it is
 267 necessarily ignored in the derivation of a POD function.

268 A broad range of potential POD functions were evaluated using binary regression on the collected
 269 controlled release data. The objective of binary regression is to model a *discrete* binary dependent variable,
 270 here D representing a successful (1) or failed (0, “missed”) detection, which follows a Bernoulli
 271 distribution. The parameter of the Bernoulli distribution is the probability of detection, i.e.,

$$D \sim \text{Bernoulli}(\text{POD}(\mathbf{x})) \quad (1)$$

272 The distribution parameter $\text{POD}(\mathbf{x})$ is modelled via a chosen predictor function ($g(\mathbf{x}; \boldsymbol{\phi})$), with
 273 variables \mathbf{x} and coefficients $\boldsymbol{\phi}$, and a *continuous* inverse link function ($F(g(\mathbf{x}; \boldsymbol{\phi}); \boldsymbol{\theta})$), with coefficients
 274 $\boldsymbol{\theta}$:

$$\text{POD}(\mathbf{x}) \equiv F(g(\mathbf{x}; \boldsymbol{\phi}), \boldsymbol{\theta}) \quad (2)$$

275 For a candidate pair of predictor and inverse link functions, $\boldsymbol{\phi}$ and $\boldsymbol{\theta}$ are obtained by maximum likelihood
 276 estimation (MLE) of the Bernoulli distribution using controlled release data. This can be found via
 277 optimization to minimize ℓ , the negative logarithm of the likelihood function, where for the Bernoulli
 278 distribution:

$$\ell(\boldsymbol{\phi}, \boldsymbol{\theta}) = \sum_i -(D_i \ln F_i + (1 - D_i) \ln(1 - F_i)) \quad (3)$$

279 and $F_i = F(g(\mathbf{x}_i; \boldsymbol{\phi}), \boldsymbol{\theta})$ for each controlled release data point, i .

280 For a fixed probability of detection (p), the POD function may be inverted to define contours of constant
 281 sensitivity for the measurement technique. In the present case, this permits calculation of a critical source
 282 rate at some detection probability, as a function of the remaining parameters in \mathbf{x} – i.e., $Q_p(u_3, \tilde{h}, \tilde{n}; p)$. A
 283 linear prediction model is often used in binary regression, such that $g(\mathbf{x}) = \boldsymbol{\phi}^T \mathbf{x}$, which is coupled with a
 284 logistic inverse link function (logistic regression) or a normal cumulative distribution function (CDF; probit
 285 model). However, in the present application, this approach produces lines of constant detection probability
 286 that converge to zero at zero wind ($u_3 = Q = 0$) for a fixed aircraft altitude and scene noise. This implies
 287 that an infinitesimally small emitter could be detected as wind reduces towards zero, which is non-physical
 288 for a noise-laden system. To avoid this, we allow candidate predictor functions to be nonlinear, while
 289 remaining monotonic with each element in \mathbf{x} and non-negative (consistent with the definition of each
 290 element). Candidate predictor functions are also required to provide a non-negative output that increases
 291 with the likelihood of detection. The inverse link function maps the output of predictor function to the
 292 range $[0, 1]$ as required.

293 **3.2 Source Quantification Uncertainty**

294 To interpret estimated source rate data, it is critical that measurement uncertainties are thoroughly
 295 understood. This section presents the method by which controlled release data can be used to derive
 296 predictive estimates for the true source rate (Q) given an estimated source rate (\tilde{Q}). Mathematically, the
 297 objective is to derive the conditional probability of Q given \tilde{Q} – i.e., $\pi(Q|\tilde{Q})$. This challenge was
 298 approached by parsing observed errors during controlled release experiments into bias and precision
 299 components.

300 Consider hypothetical multiple detections/measurements of a single, steady-state source observed on a
 301 single, specific date. It can be assumed that, *on average*, there will be some error in the estimated value of
 302 the source rate, which represents *bias* in the measurement of this source on the specific date. A bias-
 303 correction procedure that accounts for this average error in \tilde{Q} can be developed using a *bias-corrected*
 304 estimate of the source rate ($\hat{Q} = f_B(\tilde{Q})$), which may be assumed to follow a conditional distribution
 305 $\pi(\hat{Q}|\tilde{Q})$. A *precision distribution* that accounts for precision error of the bias-corrected estimate can be

306 similarly defined, $\pi(Q|\hat{Q})$. The desired distribution of the true source rate given the estimated source rate
 307 can then be computed from these distributions via:

$$\pi(Q|\tilde{Q}) = \int_{\hat{Q}} \pi(Q|\hat{Q}) \pi(\hat{Q}|\tilde{Q}) d\hat{Q} \quad (4)$$

308 where the integration is performed over all possible values of \hat{Q} . For convenience, Eq. (4) can be re-written
 309 in terms of probabilistic correction parameters κ_Q and λ_Q where $\kappa_Q = \hat{Q}/f_B(\tilde{Q})$ is a bias-correction
 310 parameter and $\lambda_Q = Q/\hat{Q}$ is a precision-correction parameter. Letting the probability distributions of these
 311 correction parameters be $\pi_{\kappa_Q}(\kappa_Q)$ and $\pi_{\lambda_Q}(\lambda_Q)$, respectively, substitution into Eq. (4) gives:

$$\pi(Q|\tilde{Q}) = \int_{\hat{Q}} \pi_{\lambda_Q}\left(\frac{Q}{\hat{Q}}\right) \pi_{\kappa_Q}\left(\frac{\hat{Q}}{f_B(\tilde{Q})}\right) \frac{1}{\hat{Q} f_B(\tilde{Q})} d\hat{Q} \quad (5)$$

312 Since bias-correction accounts for the average error in \tilde{Q} , the parameters of the precision-correction
 313 distribution (π_{λ_Q}) must be chosen to yield a unit mean. Likewise, the parameters of the bias-correction
 314 distribution (π_{κ_Q}) can be constrained to have a unit mean while optimizing for the coefficients of the bias-
 315 correction function (f_B).

316 There is one simplifying limiting case for the conditional distribution shown in Eq. (5) that is necessary
 317 if controlled release data are constrained to a small set of sites and/or measurement days. In this case,
 318 measurement error must be assumed to be *independent* of time and location, implying that the required
 319 bias-correction is non-probabilistic. With this assumption, Eq. (4) simplifies to:

$$\pi(Q|\tilde{Q}) = \pi_{\lambda_Q}\left(\frac{Q}{f_B(\tilde{Q})}\right) \frac{1}{f_B(\tilde{Q})} \quad (6)$$

320 The conditional probability distributions in Eq. (6) were computed via MLE using controlled release data
 321 for Bridger's GML and Kairos' LeakSurveyor technology. This approach optimizes the parameters for π_{λ_Q}
 322 (constrained to yield a unit mean) and the coefficients of the bias-correction function, f_B .

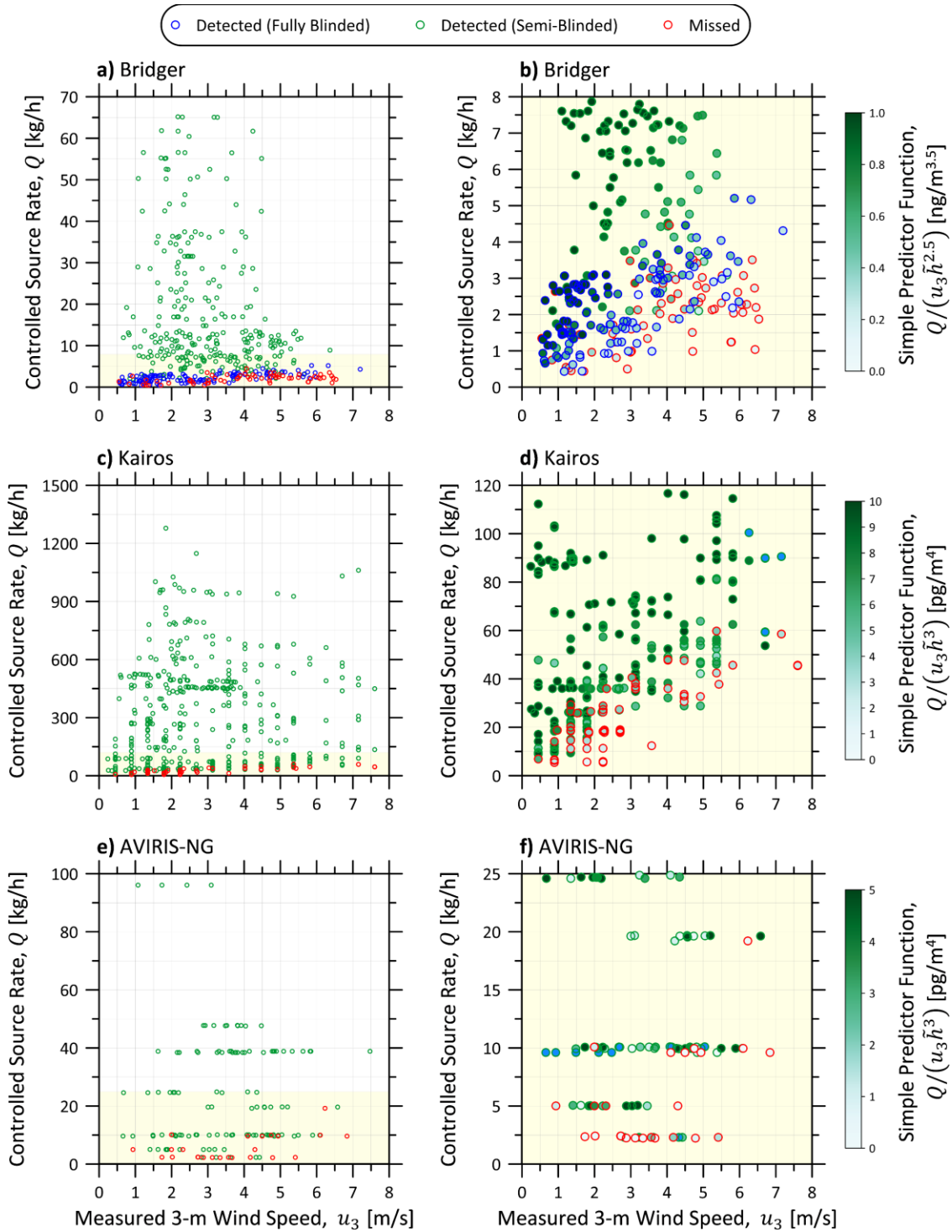
323 Myriad other parameters could influence error in source rates estimated from aerial measurements.
 324 These include the time-history of the turbulent wind field over the site as well as parameters impacting the
 325 quality of the measurement signal (e.g., aircraft altitude/orientation and surface albedo). In the most general

326 sense, the desired probability distribution(s) should be conditioned on these additional parameters.
327 However, error caused by these parameters are likely to be highly site-, source-, and time-dependent, such
328 that these confounding variables are inherently considered if extensive controlled release data for multiple
329 sites over multiple days are available and Eq. (5) can be used to model quantification error. Conversely,
330 since Eq. (6) assumes errors are independent of site, source, and time, this latter model can be expected to
331 underestimate variance in the quantification error. This is further explored in Section 4.2.1 below.

332 **4 Results**

333 ***4.1 Probability of Detection***

334 Starting first with the detailed case-study of Bridger's GML, Figure 1a plots the 466 non-zero controlled
335 releases obtained during the 2020 and 2021 campaigns as a function of measured 3-m wind speed.
336 Successful detections of fully and semi-blinded releases are identified in blue and green, respectively, and
337 misses in red. There were no false positives during the 29 zero controlled releases. Over this range of wind
338 speeds between 0.5 and 7.2 m/s, all sources greater than ~4.5 kg/h were detected. Figure 1b shows a
339 magnified view of the same data for source rates less than 8 kg/h, which highlights the probabilistic nature
340 of detection success.



341
342 **Figure 1: Available controlled release data for (a,b) Bridger Photonics GML, (c,d) Kairos LeakSurveyor, and**
343 **(e,f) AVIRIS-NG. Successful detections are outlined in blue (fully blinded data) or green (semi-blinded data)**
344 **and missed detections are outlined in red. Righthand panels (b, d, and f) show a zoomed subset of lower**
345 **release rate data from the corresponding left panels, where the data points are also shaded according to each**
346 **technique’s simple predictor function (described in the main text) as outlined on the right of each panel.**

347 Expectedly, successful detection appears more likely at higher source rates and lower wind speeds –
 348 i.e., detection probability is correlated with the wind-normalized source rate as in previous studies (e.g.,
 349 Sherwin et al., 2021). This is anticipated by the simplified Gaussian plume dispersion model (Hanna et al.,
 350 1982), where the wind-normalized source rate is proportional to the plume column density along the vertical
 351 axis (i.e., the observable “signal” for an airborne measurement). However, detection is also affected by the
 352 strength of the return signal at the optics which is proportional to \tilde{h}^{-2} (inverse square law) and the spatial
 353 resolution of the imagery, which for Bridger’s scanning laser and GML optics is approximately proportional
 354 to $\tilde{h}^{-0.5}$. Including these effects, while still ignoring the effect of instrument noise for the time-being,
 355 provides an informative, non-parametric, *simple* predictor function for Bridger’s GML, $g(\mathbf{x}; \Phi) \approx \frac{Q}{u_3 \tilde{h}^{2.5}}$.
 356 This function is used to colour the data in Figure 1b, scaled to units of ng/m^{3.5}. Visually, the colour gradient
 357 in the data from the top-left (high detection probability) to the bottom right (low detection probability)
 358 suggests strong correlation of this simple predictor with detectability.

359 Similar data are shown for the Kairos LeakSurveyor (Figure 1c-d) and AVIRIS-NG (Figure 1e-f)
 360 instruments. In contrast to Bridger’s GML with actively scanning optics, the detection sensitivity of these
 361 passive imaging spectrometers is expectedly lower, such that some emissions likely to be detected by
 362 Bridger may be missed by Kairos’ LeakSurveyor or AVIRIS-NG. Additionally, for these imaging optics
 363 that can be approximated with a pinhole model, spatial resolution at the ground/plume is linear with aircraft
 364 altitude, such that the equivalent *simple* prediction function these techniques should be $g(\mathbf{x}; \Phi) \approx \frac{Q}{u_3 \tilde{h}^3}$,
 365 indicating a greater sensitivity to aircraft altitude than Bridger’s GML. Figure 1d and f show the controlled
 366 release data according to this latter predictor function in units of pg/m⁴ – recall however, that available
 367 Kairos data were acquired at the single targeted altitude of 900 m AGL. Interestingly, in contrast to
 368 Bridger’s GML, the gradient in this colouring scheme is less pronounced for AVIRIS-NG, indicating that
 369 detection sensitivity is not well-captured by the simple predictor model.

370 Although potentially useful, the simple predictor functions $g(\mathbf{x}; \Phi) \approx \frac{Q}{u_3 \tilde{h}^{2.5}}$ and $g(\mathbf{x}; \Phi) \approx \frac{Q}{u_3 \tilde{h}^3}$ in
 371 Figure 1b, d, and f are sub-optimal since, in addition to being non-parametric and approximate, this
 372 formulation forces contours of constant POD to be linear and converge at the origin in the Q - u_3 domain.
 373 Thus, for a fixed aircraft altitude, this formulation results in the same non-physical POD at low wind speeds
 374 as the linear predictor model. To avoid this issue and to generalize the predictor model, the present analysis
 375 considers an optimizable model of the form:

$$g(\mathbf{x}; \boldsymbol{\phi}) = \phi_7 \frac{(Q_{[\text{kg/h}]} - \phi_1)^{\phi_3}}{\tilde{n}_{[\text{ppm}\cdot\text{m}]}^{\phi_4} \left(\frac{h_{[\text{m}]}}{1000}\right)^{\phi_5} (u_{3[\text{m/s}]} - \phi_2)^{\phi_6}} \quad (7)$$

376 where representative scene noise (\tilde{n}) has been introduced for completeness and may be optionally
 377 considered via optimization of coefficient ϕ_4 and units of each variable have been explicitly stated in square
 378 brackets. Choosing $\phi_1 > 0$ and/or $\phi_2 < 0$ ensures a physically reasonable POD at zero-wind, unlike the
 379 linear prediction model and the simple, non-parametric predictor functions described above and used to
 380 colour data in Figure 1b, d, and f. Similarly, non-negative exponents ϕ_{3-6} allow for deviation from
 381 linearity or, in the case of ϕ_5 for aircraft altitude, from the expected value of 2.5 (GML) or 3.0
 382 (LeakSurveyor and AVIRIS-NG). Importantly, the generalized predictor model of Eq. (7) is non-negative
 383 and monotonically increases with source rate and decreases with scene noise, aircraft altitude, and 3-m
 384 wind speed. This means that candidate inverse link functions can take the form of the cumulative
 385 distribution function (CDF) of any distribution with non-negative support (e.g., lognormal, Fréchet, etc.).

386 As further detailed in the SI (see especially Table S1), the optimization considered a broad range of
 387 possible inverse link functions while independently testing the importance of each variable in Eq. (7).
 388 Considering first the subset of controlled release measurements where scene noise data were available, in
 389 all instances the optimization showed that including either scene noise *or* aircraft altitude in the model, i.e.,
 390 permitting ϕ_4 *or* ϕ_5 to be non-zero, was strongly statistically justified. By contrast, including both
 391 parameters was either not justified or only marginally justified ($\Delta AICc < \sqrt{10}$, see SI); that is, classed as
 392 “not worth more than a bare mention” (Kass and Raftery, 1995; Snipes and Taylor, 2014). Thus, given that
 393 aircraft altitude is a trivial parameter to quantify (and in the present case available for Bridger’s GML as a
 394 standard output), the remainder of the POD derivation ignores scene noise, forcing $\phi_4 = 0$ and optimizing
 395 for the exponent on aircraft altitude, ϕ_5 .

396 Subsequent optimization was performed using all available controlled release data plotted in Figure 1a
 397 for Bridger’s GML (N = 466), Figure 1c for Kairos’ LeakSurveyor (N = 207), and Figure 1f for AVIRIS-
 398 NG (N = 139). As an example, the best-fitting model for the GML data had the following optimized
 399 predictor function:

$$g(Q, u_3, \tilde{h}) = \frac{0.1518 Q_{[\text{kg/h}]}^{1.072}}{\left(\frac{\tilde{h}_{[\text{m}]}}{1000}\right)^{2.440} (u_{3[\text{m/s}]} + 2.139)^{1.692}} \quad (8)$$

400 and employed a Fréchet CDF for the inverse link function:

$$F(g) = \exp(-0.3719g^{-2.530}) \quad (9)$$

401 Combined, these give the probability of detection for any specific source rate, wind speed, and altitude
402 using Bridger’s GML. Importantly, the generalized approach used to produce this detailed model can be
403 readily extended to any other technology for which sufficient controlled release data are available. Using
404 published controlled release data for Kairos’ LeakSurveyor (Sherwin et al., 2021) and AVIRIS-NG (Thorpe
405 et al., 2016), POD functions were derived for each of these technologies using the developed method. The
406 composite POD functions joining the predictor and inverse link are summarized for each technology in
407 Table 2; optimized coefficients for the predictor functions are available in the SI. For both Kairos’
408 LeakSurveyor and AVIRIS-NG cases, representative instrument noise data for the controlled releases were
409 not available (hence, $\phi_4 = 0$). Additionally, for Kairos’ LeakSurveyor, aircraft altitude was constant
410 during controlled release experiments so the optimized exponent on aircraft altitude (ϕ_5) was also
411 necessarily ignored. Coefficients of the optimized predictor and inverse link functions for each
412 measurement technology are summarized in Table 2.

413 **Table 2: Derived POD functions for GML, LeakSurveyor, and AVIRIS-NG, combining optimized**
 414 **predictor and inverse link functions. Detailed equations of the predictor and inverse link functions**
 415 **for each technology are summarized in Table S3 of the SI.**

| Technology | Optimized Probability of Detection (POD) Function |
|---|--|
| Bridger Photonics Inc. Gas-mapping LiDAR (GML) | $\text{POD} = \exp \left(- \left(\frac{0.2244 Q_{[\text{kg/h}]}^{1.072}}{\left(\frac{\tilde{h}_{[\text{m}]}}{1000} \right)^{2.440} (u_3[\text{m/s}] + 2.139)^{1.692}} \right)^{-2.530} \right)$ |
| Kairos Aerospace LeakSurveyor | $\text{POD} = 1 - \left(1 + \left(\frac{(5.266 \times 10^{-3}) Q_{[\text{kg/h}]}^{2.054}}{(u_3[\text{m/s}] + 0.9296)^{2.056}} \right)^{1.303} \right)^{-3.906}$ |
| Including Partial Detections | $\text{POD} = 1 - \left(1 + \left(\frac{(2.116 \times 10^{-3}) Q_{[\text{kg/h}]}^{2.027}}{(u_3[\text{m/s}] + 0.07006)^{1.527}} \right)^{1.303} \right)^{-3.906}$ |
| NASA JPL AVIRIS-NG | $\text{POD} = 1 - \left(1 + \left(\frac{(5.613 \times 10^9) Q_{[\text{kg/h}]}^{2.431}}{\left(\frac{\tilde{h}_{[\text{m}]}}{1000} \right)^{1.684} (u_3[\text{m/s}] + 36.44)^{7.643}} \right)^{1.303} \right)^{-3.906}$ |
| Including Partial Detections | $\text{POD} = \exp \left(- \left(\frac{(3.141 \times 10^6) Q_{[\text{kg/h}]}^{1.101}}{\left(\frac{\tilde{h}_{[\text{m}]}}{1000} \right)^{0.7343} (u_3[\text{m/s}] + 37.11)^{4.420}} \right)^{-2.530} \right)$ |

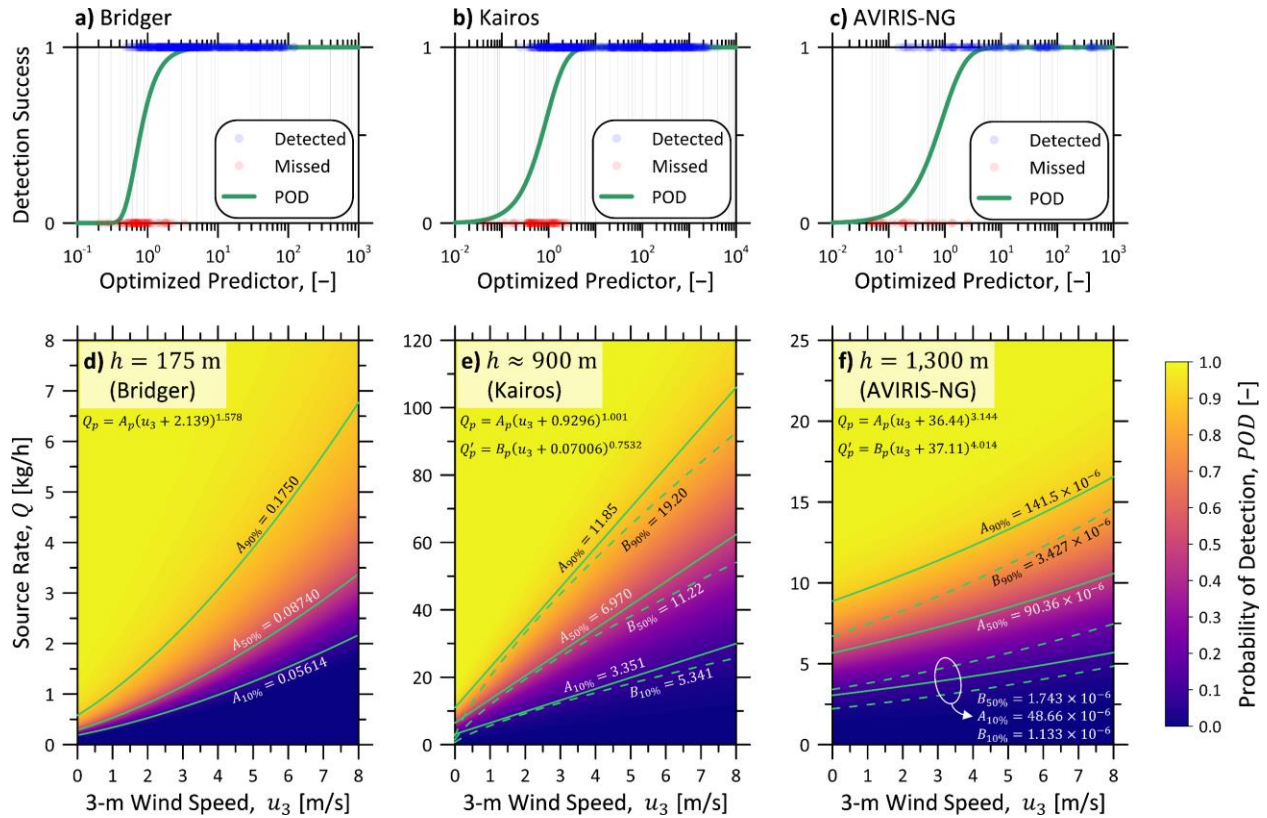
^a Aircraft altitude during controlled release experiments of Kairos' LeakSurveyor did not deviate from the targeted aircraft altitude of 900 m AGL (approximately 3000 ft), so aircraft altitude is necessarily ignored in the stated POD function. The optimized model can theoretically be extended to other altitudes by forcing the exponent on aircraft altitude to its expected value of 3.0 and updating other coefficients as necessary. Note however that there are no public data to support model accuracy at other altitudes and this extrapolation should be performed with caution given the observed deviation of AVIRIS-NG's optimized predictor function from the same expected value of 3.0.

416

417 Optimization of the generalized predictor function in Eq. (8) using the controlled release data for
 418 Bridger's GML technology identified an optimal exponent on aircraft altitude (ϕ_5) of 2.440, quite close to
 419 the theoretical/expected value of 2.5. By contrast, optimization of the AVIRIS-NG controlled release data
 420 yielded an optimal exponent on aircraft altitude of 0.7373–1.684, which is lower than the expected value
 421 of 3.0 assuming simple pinhole optics. Given this deviation and noting that aircraft altitude was not varied
 422 from the targeted level in the controlled release studies of Kairos' Leak Surveyor, one should use caution
 423 if seeking to extrapolate from the presented POD function for Kairos to other altitudes.

424 Figure 2a-c plots detection success against the value of the optimized predictor function for the
 425 controlled releases of Bridger's GML, Kairos' LeakSurveyor, and AVIRIS-NG, respectively. The

426 optimized inverse link function is overlaid in each plot. Figure 2d-f combines the optimized predictor and
 427 inverse link to display the POD function within the Q - u_3 domain at typical aircraft altitudes for Bridger's
 428 GML (175 m), Kairos' LeakSurveyor (900 m), and AVIRIS-NG (1300 m, Thorpe et al., 2016),
 429 respectively. Contours at probabilities of detection of 10, 50, and 90% – and the associated functions, Q_p –
 430 are also plotted as solid green lines. The dashed green lines (and associated functions, Q'_p) show the POD
 431 if partial (human-identified) detections are included in the analyses of Kairos' LeakSurveyor and AVIRIS-
 432 NG and treated equally as algorithmic detections.



433
 434 **Figure 2: Robustly derived probability of detection (POD) functions for Bridger's GML technology, Kairos'**
 435 **LeakSurveyor technology, and AVIRIS-NG. a-c) detection success against optimized predictor function**
 436 **values for all available controlled release data for each instrument alongside the corresponding optimized**
 437 **inverse link function (green line). d-f) calculated probability of detection as a function of source rate and 3-m**
 438 **wind speed at typical flight altitudes for each instrument. Contours for probabilities of detection of 10, 50,**
 439 **and 90% and their associated functions (Q_p) are overlaid in each plot as solid green lines. For comparison,**
 440 **POD contours if partial detections are included are also plotted with their associated functions (Q'_p) as dashed**
 441 **green lines. Table 2 provides general equations for POD as a function of source rate, wind speed, and altitude**
 442 **(where relevant) for all cases in this figure.**

443 The POD functions plotted in Figure 2d-f and summarized in Table 2 provide continuous detection
 444 probabilities on a *measurement-specific* basis for any given wind speed, source rate, and altitude. These
 445 functions have not existed to date and are precisely what is required for realistic analysis using emissions

446 abatement simulators like FEAST (Kemp et al., 2016) and modelling efforts supporting alt-FEMP
447 applications. In FEAST for example, detection sensitivity has to date been treated as a binary variable with
448 successful detection assumed if an instrument's sensitivity is exceeded by the maximum plume
449 concentration estimated from Gaussian plume dispersion theory. This approach inherently ignores the
450 continuous nature of detection probability and assumes idealized plume dispersion that is not supported by
451 the data. The continuous POD functions developed in this work identify non-linear sensitivities to source
452 rate size and measurement conditions and can be readily implemented within FEAST and other models to
453 probabilistically assess detection success. Similarly, robust POD data are vital for objective analysis of
454 missed detections in situations where multiple measurements are made over the same facility.

455 As expected and noting the different scales in Figure 2d-f, the detection sensitivities of Bridger's active
456 sensor are much lower than either of the passive sensors. Considering typical altitudes of 175, 900, 1300 m
457 for each technology respectively, at a common reference wind speed of 3 m/s, Bridger's GML can be
458 expected to detect a 1.2 kg/h source at 50% probability, Kairos a 26/27 kg/h source, and AVIRIS-NG a
459 4.7/7.3 kg/h source (the latter two lower/upper values depending on whether partial, human-reviewed
460 detections are considered as detections or not). At fixed altitudes, the optimized POD functions for all three
461 technologies provide physically realistic non-zero intercepts at zero wind speed. These contours contrast
462 with assumed detection sensitivities or partial detection ranges with non-physical zero-intercepts based on
463 wind-normalized source rates for Kairos' LeakSurveyor (Berman et al., 2021; Chen et al., 2022; Sherwin
464 et al., 2021) as well as the assumed linear model of Johnson et al. (2021) for Bridger's GML. Figure S2 of
465 the SI compares the newly derived continuous POD functions with these previously published detection
466 sensitivities for each technology. There is a slight improvement in the detection sensitivity of Bridger's
467 GML over that estimated from limited tests in the 2019 data of (Johnson et al., 2021). Detection sensitivities
468 are of similar magnitude for Kairos' LeakSurveyor as in Sherwin et al.'s (2021) and Berman et al.'s (2021)
469 analyses. Likewise, the present approach overlaps significantly with Thorpe et al.'s (2016) stated partial
470 detection range, however the new result improves upon this by parameterizing the POD with wind speed
471 and altitude aircraft.

472 The optimized POD function for Kairos' LeakSurveyor is approximately linear with wind speed. While
473 this result is justified by goodness-of-fit statistics, subjective inclusion/exclusion of data can yield
474 significantly different results. Using this technology as an example and referring to Figure S3 in the SI,
475 POD contours are super-linear if Sherwin et al.'s (2021) data are considered alone (Figure S3a) but, by
476 contrast, become sub-linear if only Kairos' internal data are considered (Figure S3b). Only when combining
477 these unique data sets does the optimized POD function yield contours that are approximately linear (Figure
478 S3c). This sensitivity to data inclusion is likely due to the scarcity of data near the sensitivity limit in

479 Sherwin et al.'s (2021) experiments (see Figure S3a in the SI). For instance, one-minute-averaged 3-m
 480 wind speeds during Sherwin et al.'s (2021) experiments did not exceed 5.5 m/s as compared to maximum
 481 wind speeds of 7.4 m/s in the Bridger GML and >8.0 m/s in the AVIRIS-NG controlled release data.
 482 Moreover, due to instrumentation constraints noted by Sherwin et al. (2021), releases near the sensitivity
 483 limit were occasionally held constant during consecutive (up to 16) flight passes, letting the variable wind
 484 perturb detectability of the plume. Consequently, the available controlled release data tend to be clustered
 485 in the Q - u_3 domain, such that a POD function for Kairos' LeakSurveyor derived from Sherwin et al.'s
 486 (2021) data alone should not be extrapolated. Nevertheless, the observed sensitivity of the optimized POD
 487 function to the contributing datasets supports the continued acquisition (and public sharing) of controlled
 488 release data for these technologies.

489 As presented, the derived POD assumes accurate knowledge of aircraft altitude and 3-m wind speed.
 490 This is the appropriate form when trying to understand what might be detectable in a range of field study
 491 scenarios and/or modelling of alternate fugitive emissions management programs (Alt-FEMP) or
 492 alternative means of emission limitation (AMEL) proposals. However, when interpreting data from a
 493 specific field campaign, accurate in-situ wind data are generally not available and database/modelled wind
 494 speed must instead be used to infer the POD. This scenario necessarily requires an error model for the wind
 495 speed. Such a model is likely to be highly dependent on time and location as well as the source of the wind
 496 speed estimate and ideally should be derived from data relevant to any particular measurement campaign.
 497 However, if a wind error model of the form $\pi(u_3|\tilde{u}_3)$ exists (i.e., a conditional distribution of the true 3-m
 498 wind speed given the available estimate), then the POD can be readily quantified considering bias/precision
 499 in the estimated 3-m wind speed via:

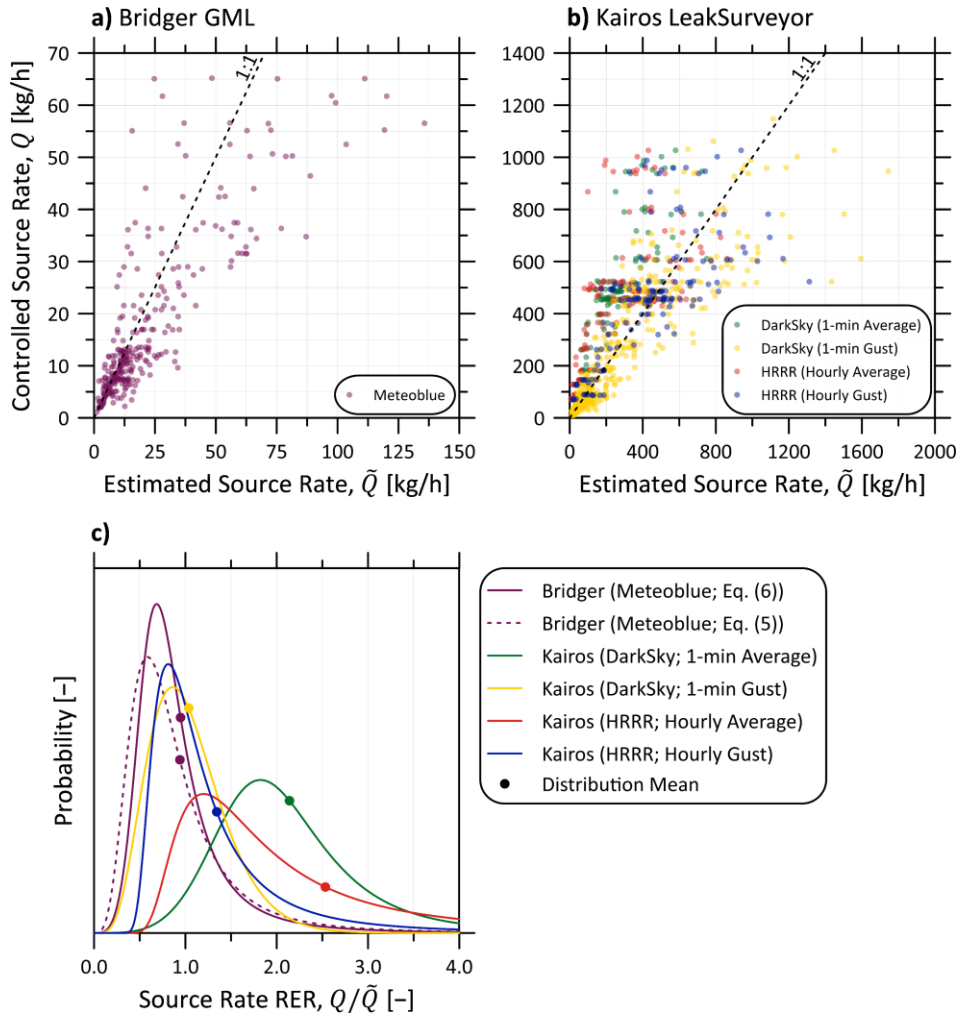
$$\text{POD}(Q, \tilde{u}_3, \tilde{h}) = \int_0^{\infty} \text{POD}(Q, u_3, \tilde{h})\pi(u_3|\tilde{u}_3)du_3 \quad (10)$$

500 To enable this type of analysis, wind speed error distributions, $\pi(u_3|\tilde{u}_3)$, were derived using available wind
 501 data from the controlled release trials of Bridger's GML and Kairos' LeakSurveyor. The resulting
 502 distributions are summarized in Table S5 of the SI and can be used with the optimized POD functions in
 503 Table 2 to compute probabilities of detection given *estimated* wind speed via Eq. (10).

504 **4.2 Measurement Uncertainty**

505 Figure 3 compares the known (Q) and estimated (\tilde{Q}) source rates across the controlled release studies of
 506 Bridger's GML and Kairos' LeakSurveyor technologies. Estimated source rates for Bridger's GML
 507 technology were taken directly from their reported results; all 284 non-zero, semi-blinded, high-flowrate

508 releases in the 2020 and 2021 campaigns are shown in Figure 3a alongside a 1:1 parity line. Figure 3b plots
509 similar data from Sherwin et al.'s (2021) controlled release experiments of Kairos' LeakSurveyor. The 149
510 data points in this latter case correspond to all controlled releases greater than 50 kg/h and without any
511 identified quality control concerns. Source rates were computed from Kairos' estimated wind-normalized
512 source rates and multiplied by modelled wind speed at 3-m height above ground. Four datasets are shown
513 in Figure 3b corresponding to wind data from Dark Sky – one-minute average (green) and gust (yellow) –
514 and HRRR – one-hour average (red) and gust (blue). Figure 3c plots the resulting probability distributions
515 for the relative error ratio ($RER = Q/\tilde{Q}$) from the data in Figure 3a and b according to Eq. (6), which
516 ignores potential site-to-site and day-to-day variability in measurement accuracy; means of each
517 distribution, representing overall measurement biases, are identified by points. Bridger's GML estimates
518 using Meteoblue wind data and Kairos' LeakSurveyor estimates using one-minute *gust* data from Dark Sky
519 show minimal bias errors, with relative error ratios of 0.95 and 1.04, respectively. By contrast, bias errors
520 can be large (1.34) when using one-hour gust wind data from HRRR and prohibitively large using one-
521 minute *average* Dark Sky or one-hour *average* HRRR data (2.14 and 2.53, respectively) with Kairos'
522 LeakSurveyor technology. Table 3 summarizes key statistics (mean, median, and 95% equal tail
523 distributions) for each of these distributions shown in Figure 3c and Table S4 of the SI provides detailed
524 equations for the conditional probability distribution, $\pi(Q|\tilde{Q})$, for each combination of technology and
525 wind speed data source. These distributions are the essential inputs for Monte Carlo methods enabling
526 comprehensive uncertainty analysis in large measurement campaigns (e.g., Tyner and Johnson, 2021),
527 which may include multi-pass measurements of single sources/facilities, and specifically include
528 aggregation of detected sources to develop measurement-based inventories.



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Figure 3: Summary of controlled release data and quantification error analysis for a) Bridger’s GML technology using Meteoblue wind data (purple) and b) Kairos’ LeakSurveyor technology, computed using Dark Sky one-minute average (green) and gust (yellow) and HRRR hourly average (red) and gust (blue) wind data. (c) Resulting distributions of the source rate relative error ratio (RER) for each technique and wind source via fitting of Eq. (6) in addition to the source rate RER for Bridger’s GML technology using Eq. (5). Distribution means, representing quantification bias error are identified for each distribution by a point.

536 **Table 3: Statistics of the relative error ratio ($RER = Q/\tilde{Q}$) for Bridger’s GML and Kairos’**
 537 **LeakSurveyor technologies; source data corresponds to the high-flowrate (1-66 kg/h) controlled**
 538 **releases from the present study and all valid controlled releases > 50 kg/h from Sherwin et al.**
 539 **(2021). RER statistics (mean, median, and 95% equal tail confidence interval (CI)) are shown for**
 540 **each technique and, for Kairos’ LeakSurveyor, when using different sources of wind speed data.**

| Instrument | Wind Source | Wind Statistic | Mean (Bias) | Median | 95% Equal Tail CI |
|---------------------|-------------|----------------|-------------|--------|-------------------|
| Bridger GML | Meteoblue | Proprietary | 0.95 | 0.81 | 0.35 – 2.41 |
| Kairos LeakSurveyor | Dark Sky | 1-min Gust | 1.04 | 0.98 | 0.38 – 2.02 |
| | | 1-min Average | 2.14 | 1.99 | 0.93 – 4.27 |
| | HRRR | 1-hr Gust | 1.34 | 1.06 | 0.56 – 3.81 |
| | | 1-hr Average | 2.53 | 1.75 | 0.77 – 8.82 |
| | In Situ | 1-min Gust | 0.98 | 0.91 | 0.43 – 1.92 |
| | | 1-min Average | 1.39 | 1.28 | 0.58 – 2.85 |

541

542 The improved quantification accuracy when using *gust* instead of *average* wind speeds to estimate
 543 source rate with Kairos’ LeakSurveyor is somewhat counterintuitive since average wind speed is more
 544 indicative of the history of plume propagation prior to any observation. This seemingly anomalous
 545 observation could be a result of the coarse spatiotemporal resolution in database/modelled winds, which
 546 might tend to underestimate averages of non-negative and right-skewed wind speeds. However, this is
 547 much more likely related to how Kairos’ wind-normalized source rate is estimated based on a defined “core”
 548 of the plume. Specifically, Kairos estimates wind-normalized source rate by dividing the total observed
 549 excess methane mass in the “core” of the plume by the length of this plume “core” in the direction of the
 550 wind; to then estimate source rate, this parameter is multiplied by the estimated wind speed. This is
 551 equivalent to averaging the flux of methane through control surfaces orthogonal to and spanning the length
 552 of the plume core. This approach is only valid in the case of infinite sensitivity where excess methane at
 553 the edges of the plume is fully resolved. In practice though, finite sensitivity implies that the excess mass
 554 of methane in the plume is inherently underestimated, and this effect is accentuated by constraining the
 555 analysis to an arbitrary plume core. To overcome this underestimation of plume mass, an upward correction
 556 to wind speed would be necessary. This same result has been identified for satellite-based methane
 557 detection methods – particularly the cross-sectional flux (CSF) method (e.g. Varon et al., 2018 and
 558 references therein), which is similar to Kairos’ approach. Robust analyses of this quantification method in
 559 the context of satellite remote sensing confirms that database/modelled average wind speeds must be
 560 calibrated/corrected to accurately recover known source rates. The calibration correction has been found
 561 to be sensor noise- and plume-dependent and studies have estimated it to range from +30 to +75% for
 562 satellite instruments (Jervis et al., 2021; Varon et al., 2020). Recognizing that database/modelled wind

563 speeds are inherently biased and uncertain, it is possible and perhaps likely that the upward correction used
564 to estimate gust wind speed from an average wind speed tends to mimic this required calibration correction.

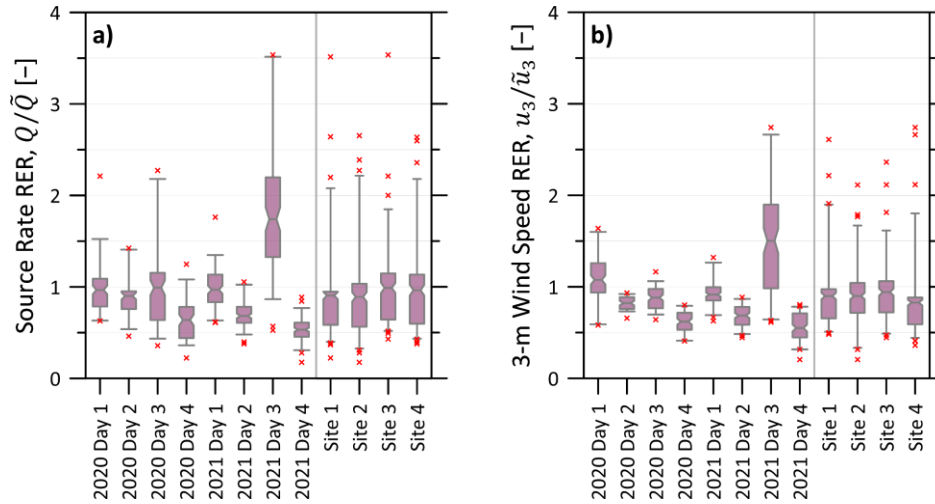
565 To explore this further, Table 3 shows RER statistics for Kairos' LeakSurveyor using in-situ wind speed
566 data from Sherwin et al. (2021) and Kairos' internal controlled release studies. One-minute-averaging of
567 in-situ wind speed tends to underestimate the true source rate (RER of 1.39), corresponding to a +39%
568 calibration correction needed to minimize bias; this is consistent with published corrections needed for
569 satellite imagery using the CSF method (Jervis et al., 2021; Varon et al., 2020). However, the in-situ, one-
570 minute gust wind speed compensates for this underestimation (RER of 0.98). Thus, in this specific
571 example, if wind-normalized source rate is derived using Kairos' plume "core", then the one-minute gust
572 wind speed empirically minimizes bias.

573 4.2.1 *Spatiotemporal Variability of Measurement Bias*

574 Use of the simpler error model shown in Eq. (6) assumes that site-to-site and day-to-day bias in
575 measurement error for a given technique is negligible. While this is a necessary assumption if controlled
576 release data are limited to few locations/days, it is also uncertain. For example, drift in optical components
577 and general atmospheric conditions may influence day-to-day variability in quantification accuracy, while
578 localized conditions such as wind direction/turbulence and ground albedo can affect site-to-site variability.
579 To glean insight into this bias variability, an additional analysis was performed using the present controlled
580 release data for Bridger's GML technology, which includes releases from four oil and gas sites recorded
581 over multiple days in two field campaigns one year apart. Figure 4a presents a box-whisker diagram for
582 the relative error ratio (RER) of the Bridger GML-estimated source rate, which takes the 284 controlled
583 releases and computes statistics for data aggregated by measurement day (eight days spanning 2020 and
584 2021) and site (four locations). In these diagrams the central bar represents the interquartile range (25th to
585 75th percentile), the gray bars extend to the 90% equal tail confidence interval (CI), and the red crosses
586 indicate extreme data outside the 90% CI. The central bars are notched at the mean value of the aggregated
587 data, which represents bias for a specific measurement day or location. Measurement bias (quantified as
588 the mean source rate RER at a particular site or on a particular measurement day) varied moderately on a
589 site-by-site basis, from 0.89 to 0.99, and significantly on a day-to-day basis, from 0.53 to 1.74. This implies
590 that bias on any one day and/or at any one site can be significant; however, available data also imply that,
591 on average, bias across multiple days/sites is not statistically different than unity at a 5% significance level.

592 Figure 4b provides insight into the source of bias variability by showing the same box-whisker diagrams
593 for the RER in modelled 3-m wind speed from Meteoblue (used in Bridger's quantification) vs. the actual
594 measured wind speed. As evidenced by these figures, day-to-day bias errors in estimated source rate

595 correlate highly with the errors in the modelled 3-m wind speed ($\rho = 0.974$), implying that source rate bias
 596 on a day-to-day basis is driven by error in the windspeed. By contrast, source rate and wind speed bias are
 597 negligibly correlated on a site-by-site basis ($\rho = 0.048$). This implies that site-specific sources of bias like
 598 surface albedo and site infrastructure that affects wind speed error are likely unimportant relative to day-
 599 to-day variability in wind speed error.



600
 601 **Figure 4: Box-whisker diagrams for the relative error ratio (RER) of source rate using Bridger's**
 602 **estimates with Meteoblue wind data, accumulated by measurement day (a) and site (b). The central**
 603 **bars of the box-whisker diagrams are notched at the mean error (i.e., bias) and span the**
 604 **interquartile range; whiskers correspond to the 90% equal tail confidence interval (CI) and red**
 605 **crosses mark extreme data outside the 90% CI. Day-to-day variability is significant with bias**
 606 **errors ranging from 0.53 to 1.74.**

607 Thus, while Eq. (6) is the only practical error model when constrained by limited controlled released data,
 608 Eq. (5) is preferred to avoid underestimation of uncertainties given the potential significance of day-to-day
 609 variability in measurement bias. The difference between these approaches is demonstrated in Figure 3c,
 610 where the present controlled release data for Bridger's GML from four sites over eight unique days in two
 611 different years is sufficient to model uncertainties via either Eq. (5) or (6). Use of Eq. (5) in place of the
 612 simplified Eq. (6) yielded no meaningful effect on the average bias, which changed less than 1%. However,
 613 as shown in the figure, and expected given the proper consideration of bias variability, Eq. (5) estimates
 614 higher dispersion in source rate RER (15% increase in standard deviation) than Eq. (6). This increased
 615 variability when considering day- and site-dependent bias is moderate but not insignificant, and implies an
 616 underestimation of quantification uncertainty if controlled release data are limited to a small number of
 617 locations and/or measurement days and Eq. (6) is used for quantification error analysis. Based on these
 618 results, it is highly recommended that future controlled release studies be completed from a range of unique
 619 locations and over as many different days as feasible.

620 5 Conclusions

621 Generalized models to characterize probabilities of detection and quantification error were developed and
622 applied to three aerial methane-detection technologies: Bridger Photonics Inc.'s Gas-Mapping LiDAR
623 (GML), Kairos Aerospace's LeakSurveyor, and the (U.S.) National Aeronautic and Space Administration's
624 Jet Propulsion Laboratory's Next-Generation Airborne Visible/Infrared Imaging Spectrometer (AVIRIS-
625 NG). Leveraging binary regression with a generalized predictor function, this new method improves upon
626 existing techniques in the literature by enabling derivation of continuous and physically realizable POD
627 functions that are variable on methane source rate, ambient wind speed, and aircraft altitude (where
628 available). POD functions optimized to available controlled release data identified technology-specific
629 detection sensitivities that vary with wind speed and altitude. At typical/target aircraft altitudes and a
630 representative average wind speed of 3 m/s, Bridger's GML, Kairos' LeakSurveyor, and AVIRIS-NG were
631 predicted to identify methane emissions of 1.2, 27, and 6.0 kg/h with 50% probability, respectively.

632 Using a subset of controlled release data for Bridger's GML and Kairos' LeakSurveyor that included
633 source rate estimates for comparison with ground truth controlled source rates, quantification uncertainties
634 were separately characterized, including analysis of effects of using four optional database sources of wind
635 speed for Kairos' LeakSurveyor. The developed statistical model permits analysis of measurement bias,
636 *variability* in measurement bias (where data permitted), and measurement precision, where the latter two
637 were treated as probabilistic variables. Using the Meteoblue wind speed data product, the source rate
638 relative error ratio (RER – i.e., controlled over estimated source rate) for Bridger's GML averaged to 0.95
639 with a 95% confidence interval of 0.35–2.41. The analysis of Kairos' LeakSurveyor identify that source
640 rate RER was highly sensitive to the wind speed data source and statistic (i.e., gust vs. average wind speed)
641 and gust wind speed provided significantly less-biased results. One-minute gust wind speed from the Dark
642 Sky database and one-hour gust wind speed from the High-Resolution Rapid Refresh database yielded mean
643 source rate RERs of 1.04 and 1.34 with 95% confidence intervals of 0.38–2.02 and 0.56–3.81, respectively.
644 Data from the present controlled release study of Bridger's GML demonstrated that day-to-day variability
645 in measurement bias was strongly correlated with wind speed error and appreciably increased the dispersion
646 of the source rate RER. These results identify the need to target an assortment of different measurement
647 locations and maximize measurement days during future controlled release studies.

648 Ultimately, the described methods – successfully applied to three example technologies – yield the robustly
649 derived continuous POD function and probabilistic quantification error model that are needed to properly
650 simulate emissions abatement/reduction and support methane monitoring, reporting, and verification via
651 aircraft-based remote sensing. Moreover, the developed generalized methods are readily extensible to

652 analysis of other remote sensing techniques or can be used to update POD and uncertainty models as further
653 controlled release data become publicly available.

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658 **Declaration of Competing Interest**

659 The authors have no competing interests to declare.

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672 **Supplementary Information**

673 Supplementary information to this article can be found online.

674 **References**

- 675 AER, 2021. Alternative Fugitive Emission Management Program Approvals [WWW Document].
676 URL [https://www.aer.ca/protecting-what-matters/holding-industry-accountable/industry-](https://www.aer.ca/protecting-what-matters/holding-industry-accountable/industry-performance/methane-performance/alternative-fugitive-emission-management-program-approvals)
677 [performance/methane-performance/alternative-fugitive-emission-management-program-](https://www.aer.ca/protecting-what-matters/holding-industry-accountable/industry-performance/methane-performance/alternative-fugitive-emission-management-program-approvals)
678 [approvals](https://www.aer.ca/protecting-what-matters/holding-industry-accountable/industry-performance/methane-performance/alternative-fugitive-emission-management-program-approvals) (accessed 2.15.21).
- 679 Apple Inc., 2022. Dark Sky API [WWW Document]. URL <https://darksky.net/dev> (accessed
680 6.9.22).
- 681 Arias, P.A., Bellouin, N., Coppola, E., Jones, R.G., Krinner, G., Marotzke, J., Naik, V., Palmer,
682 M.D., Plattner, G.-K., Rogelj, J., Rojas, M., Sillmann, J., Storelvmo, T., Thorne, P.W.,

683 Trewin, B., Rao, K.A., Adhikary, B., Allan, R.P., Armour, K., Bala, G., Barimalala, R.,
684 Berger, S., Canadell, J.G., Cassou, C., Cherchi, A., Collins, W., Collins, W.D., Connors, S.L.,
685 Corti, S., Cruz, F., Dentener, F.J., Dereczynski, C., Luca, A. Di, Niang, A.D., Doblus-Reyes,
686 F.J., Dosio, A., Douville, H., Engelbrecht, F., Eyring, V., Fischer, E., Forster, P., Fox-
687 Kemper, B., Fuglestvedt, J.S., Fyfe, J.C., Gillett, N.P., Goldfarb, L., Gorodetskaya, I.,
688 Gutierrez, J.M., Hamdi, R., Hawkins, E., Hewitt, H.T., Hope, P., Islam, A.S., Jones, C.,
689 Kaufman, D.S., Kopp, R.E., Kosaka, Y., Kossin, J., Krakovska, S., Lee, J.-Y., Li, J.,
690 Mauritsen, T., Maycock, T.K., Meinshausen, M., Min, S.-K., Monteiro, P.M.S., Ngo-Duc, T.,
691 Otto, F., Pinto, I., Pirani, A., Raghavan, K., Ranasinghe, R., Ruane, A.C., Ruiz, L., Sallée, J.-
692 B., Samset, B.H., Sathyendranath, S., Seneviratne, S.I., Sörensson, A.A., Szopa, S.,
693 Takayabu, I., A.-M.Tréguier, Hurk, B. van den, R.Vautard, Schuckmann, K. von, Zaehle, S.,
694 Zhang, X., Zickfeld, K., 2021. Technical Summary, in: Masson-Delmott, V., Zhai, P., Pirani,
695 A., Connors, S.L., Péan, C., Berger, S., Caud, N., Chen, Y., Goldfarb, L., Gomis, M.I., Huang,
696 M., Leitzell, K., Lonnoy, E., Matthews, J.B.R., Maycock, T.K., Waterfield, T., Yelekçi, O.,
697 Yu, R., Zhoue, B. (Eds.), *Climate Change 2021: The Physical Science Basis. Contribution of*
698 *Working Group I to the Sixth Assessment Report of the Intergovernmental Panel on Climate*
699 *Change.* Cambridge University Press, pp. 33–144.
700 <https://doi.org/10.1017/9781009157896.002>

701 ARPA-E, 2018. Impact Sheet - Bridger Photonics (MONITOR) [WWW Document]. URL
702 <https://arpa-e.energy.gov/impact-sheet/bridger-photonics-monitor> (accessed 5.23.22).

703 Bell, C., Vaughn, T.L., Zimmerle, D.J., 2020. Evaluation of next generation emission
704 measurement technologies under repeatable test protocols. *Elem Sci Anth* 8, 32.
705 <https://doi.org/10.1525/elementa.426>

706 Berman, E., S.F., W., B., E., Jones, B.B., 2021. Kairos Aerospace Technical White Paper: Methane
707 Detection (Version 1F). Kairos Aerospace. <https://doi.org/10.17605/OSF.IO/HZG52>

708 Branson, K., Jones, B.B., Berman, E.S.F., 2021. Methane Emissions Quantification (No. Version
709 2). Kairos Aerospace, Mountain View, CA.

710 Bridger Photonics, 2022. Gas Mapping LiDAR gains regulatory approval for ALARM use in New
711 Mexico [WWW Document]. URL [https://www.bridgerphotonics.com/blog/gas-mapping-](https://www.bridgerphotonics.com/blog/gas-mapping-lidar-gains-regulatory-approval-alarm-use-new-mexico)
712 [lidar-gains-regulatory-approval-alarm-use-new-mexico](https://www.bridgerphotonics.com/blog/gas-mapping-lidar-gains-regulatory-approval-alarm-use-new-mexico) (accessed 5.23.22).

713 Bridger Photonics, 2021. Gas Mapping LiDAR™ [WWW Document]. URL
714 <https://www.bridgerphotonics.com/gas-mapping-lidar> (accessed 5.31.21).

715 CCAC, 2021. Global Methane Pledge [WWW Document]. URL
716 <https://www.globalmethanepledge.org/> (accessed 4.28.22).

717 Chen, Y., Sherwin, E.D., Berman, E.S.F., Jones, B.B., Gordon, M.P., Wetherley, E.B., Kort, E.A.,
718 Brandt, A.R., 2022. Quantifying Regional Methane Emissions in the New Mexico Permian
719 Basin with a Comprehensive Aerial Survey, *Environmental Science & Technology*.
720 <https://doi.org/10.1021/acs.est.1c06458>

721 Cusworth, D.H., Duren, R.M., Thorpe, A.K., Olson-Duvall, W., Heckler, J., Chapman, J.W.,
722 Eastwood, M.L., Helmlinger, M.C., Green, R.O., Asner, G.P., Dennison, P.E., Miller, C.E.,
723 2021. Intermittency of Large Methane Emitters in the Permian Basin. *Environ. Sci. Technol.*
724 *Lett.* 8, 567–573. <https://doi.org/10.1021/acs.estlett.1c00173>

725 Cusworth, D.H., Duren, R.M., Thorpe, A.K., Tseng, E., Thompson, D., Guha, A., Newman, S.,
726 Foster, K.T., Miller, C.E., 2020. Using remote sensing to detect, validate, and quantify

727 methane emissions from California solid waste operations. *Environ. Res. Lett.* 15.
728 <https://doi.org/10.1088/1748-9326/ab7b99>

729 Duren, R.M., Thorpe, A.K., Foster, K.T., Rafiq, T., Hopkins, F.M., Yadav, V., Bue, B.D.,
730 Thompson, D.R., Conley, S., Colombi, N.K., Frankenberg, C., McCubbin, I.B., Eastwood,
731 M.L., Falk, M., Herner, J.D., Croes, B.E., Green, R.O., Miller, C.E., 2019. California's
732 methane super-emitters. *Nature* 575, 180–184. <https://doi.org/10.1038/s41586-019-1720-3>

733 Elder, C.D., Thompson, D.R., Thorpe, A.K., Hanke, P., Walter Anthony, K.M., Miller, C.E., 2020.
734 Airborne Mapping Reveals Emergent Power Law of Arctic Methane Emissions. *Geophys.*
735 *Res. Lett.* 47. <https://doi.org/10.1029/2019GL085707>

736 European Commission, 2021. Proposal for a Regulation of the European Parliament and of the
737 Council on methane emissions reduction in the energy sector and amending Regulation (EU)
738 2019/942. European Commission, Brussels.

739 Foote, M.D., Dennison, P.E., Thorpe, A.K., Thompson, D.R., Jongaramrungruang, S.,
740 Frankenberg, C., Joshi, S.C., 2020. Fast and Accurate Retrieval of Methane Concentration
741 From Imaging Spectrometer Data Using Sparsity Prior. *IEEE Trans. Geosci. Remote Sens.*
742 58, 6480–6492. <https://doi.org/10.1109/TGRS.2020.2976888>

743 Fox, T.A., Barchyn, T.E., Risk, D., Ravikumar, A.P., Hugenholtz, C.H., 2019. A review of close-
744 range and screening technologies for mitigating fugitive methane emissions in upstream oil
745 and gas. *Environ. Res. Lett.* 14, 053002. <https://doi.org/10.1088/1748-9326/ab0cc3>

746 Frankenberg, C., Thorpe, A.K., Thompson, D.R., Hulley, G., Kort, E.A., Vance, N., Borchardt, J.,
747 Krings, T., Gerilowski, K., Sweeney, C., Conley, S.A., Bue, B.D., Aubrey, A.D., Hook, S.,
748 Green, R.O., 2016. Airborne methane remote measurements reveal heavytail flux distribution
749 in Four Corners region. *Proc. Natl. Acad. Sci. U. S. A.* 113, 9734–9739.
750 <https://doi.org/10.1073/pnas.1605617113>

751 Green, R.O., Eastwood, M.L., Sarture, C.M., Chrien, T.G., Aronsson, M., Chippendale, B.J.,
752 Faust, J.A., Pavri, B.E., Chovit, C.J., Solis, M., Olah, M.R., Williams, O., 1998. Imaging
753 Spectroscopy and the Airborne Visible/Infrared Imaging Spectrometer (AVIRIS). *Remote*
754 *Sens. Environ.* 65, 227–248. [https://doi.org/10.1016/S0034-4257\(98\)00064-9](https://doi.org/10.1016/S0034-4257(98)00064-9)

755 Guha, A., Newman, S., Fairley, D., Dinh, T.M., Duca, L., Conley, S.C., Smith, M.L., Thorpe,
756 A.K., Duren, R.M., Cusworth, D.H., Foster, K.T., Fischer, M.L., Jeong, S., Yesiller, N.,
757 Hanson, J.L., Martien, P.T., 2020. Assessment of Regional Methane Emission Inventories
758 through Airborne Quantification in the San Francisco Bay Area. *Environ. Sci. Technol.* 54,
759 9254–9264. <https://doi.org/10.1021/acs.est.0c01212>

760 Hamlin, L., Green, R.O., Mouroulis, P., Eastwood, M., Wilson, D., Dudik, M., Paine, C., 2011.
761 Imaging spectrometer science measurements for Terrestrial Ecology: AVIRIS and new
762 developments, in: 2011 Aerospace Conference. IEEE, pp. 1–7.
763 <https://doi.org/10.1109/AERO.2011.5747395>

764 Hanna, S.R., Briggs, G.A., Hosker Jr., R.P., 1982. Handbook on atmospheric diffusion.

765 Hunter, D., Thorpe, M.J., 2017. Gas Mapping LiDAR Aerial Verification Program Final Report.
766 Alberta Upstream Petroleum Research Fund Project 17-ARPC-03, Petroleum Technology
767 Alliance of Canada (PTAC).

768 InvestableUniverse, 2021. Breakthrough For LiDAR Tech In Oil And Gas Industry After Exxon
769 Filing [WWW Document]. URL <https://investableuniverse.com/2021/04/09/bridger-photonics-lidar-methane-emissions-exxon-mobil/> (accessed 5.23.22).

770

771 IPCC, 2018. Summary for Policymakers, in: Masson-Delmotte, V., Zhai, P., Pörtner, H.-O.,
772 Roberts, D., Skea, J., Shukla, P.R., Pirani, A., Moufouma-Okia, W., Péan, C., Pidcock, R.,
773 Connors, S., Matthews, J.B.R., Chen, Y., Zhou, X., Gomis, M.I., Lonnoy, E., Maycock, T.,
774 Tignor, M., Waterfield, T. (Eds.), *Global Warming of 1.5°C. An IPCC Special Report on the*
775 *Impacts of Global Warming of 1.5°C above Pre-Industrial Levels. Intergovernmental Panel*
776 *on Climate Change.*

777 Jervis, D., McKeever, J., Durak, B.O.A., Sloan, J.J., Gains, D., Varon, D.J., Ramier, A., Strupler,
778 M., Tarrant, E., 2021. The GHGSat-D imaging spectrometer. *Atmos. Meas. Tech.* 14, 2127–
779 2140. <https://doi.org/10.5194/amt-14-2127-2021>

780 Johnson, M.R., Tyner, D.R., Szekeres, A.J., 2021. Blinded evaluation of airborne methane source
781 detection using Bridger Photonics LiDAR. *Remote Sens. Environ.* 259, 112418.
782 <https://doi.org/10.1016/j.rse.2021.112418>

783 Kairos Aerospace, 2022a. New Mexico Authority Approves Kairos Aerospace Technology in
784 Methane Emissions Mitigation Efforts [WWW Document]. URL
785 [https://kairosaerospace.com/new-mexico-authority-approves-kairos-aerospace-technology-](https://kairosaerospace.com/new-mexico-authority-approves-kairos-aerospace-technology-in-methane-emissions-mitigation-efforts/)
786 [in-methane-emissions-mitigation-efforts/](https://kairosaerospace.com/new-mexico-authority-approves-kairos-aerospace-technology-in-methane-emissions-mitigation-efforts/) (accessed 6.2.22).

787 Kairos Aerospace, 2022b. Methane Detection from a Unique Perspective [WWW Document].
788 URL <https://kairosaerospace.com/methane-detection/> (accessed 5.29.22).

789 Kass, R.E., Raftery, A.E., 1995. Bayes Factors. *J. Am. Stat. Assoc.* 90, 773–795.

790 Kemp, C.E., Ravikumar, A.P., 2021. New Technologies Can Cost Effectively Reduce Oil and Gas
791 Methane Emissions, but Policies Will Require Careful Design to Establish Mitigation
792 Equivalence. *Environ. Sci. Technol.* 55, 9140–9149. <https://doi.org/10.1021/acs.est.1c03071>

793 Kemp, C.E., Ravikumar, A.P., Brandt, A.R., 2016. Comparing Natural Gas Leakage Detection
794 Technologies Using an Open-Source “virtual Gas Field” Simulator. *Environ. Sci. Technol.*
795 50, 4546–4553. <https://doi.org/10.1021/acs.est.5b06068>

796 Krautwurst, S., Gerilowski, K., Jonsson, H.H., Thompson, D.R., Kolyer, R.W., Iraci, L.T., Thorpe,
797 A.K., Horstjann, M., Eastwood, M., Leifer, I., Vigil, S.A., Krings, T., Borchardt, J., Buchwitz,
798 M., Fladeland, M.M., Burrows, J.P., Bovensmann, H., 2017. Methane emissions from a
799 Californian landfill, determined from airborne remote sensing and in situ measurements.
800 *Atmos. Meas. Tech.* 10, 3429–3452. <https://doi.org/10.5194/amt-10-3429-2017>

801 Kreitinge, A.T., Thorpe, M.J., 2018. High-Sensitivity Gas-Mapping 3D Imager and Method of
802 Operation. 9970756 B2.

803 NOAA, 2020. High-Resolution Rapid Refresh (HRRR) Model [WWW Document]. URL
804 <https://rapidrefresh.noaa.gov/> (accessed 6.9.22).

805 Rashid, K., Speck, A., Osedach, T.P., Perroni, D. V., Pomerantz, A.E., 2020. Optimized inspection
806 of upstream oil and gas methane emissions using airborne LiDAR surveillance. *Appl. Energy*
807 275, 115327. <https://doi.org/10.1016/j.apenergy.2020.115327>

808 Ravikumar, A.P., Sreedhara, S., Wang, J., Englander, J.G., Roda-Stuart, D., Bell, C.S., Zimmerle,
809 D.J., Lyon, D.R., Mogstad, I., Ratner, B., Brandt, A.R., 2019. Single-blind inter-comparison
810 of methane detection technologies – results from the Stanford/EDF Mobile Monitoring
811 Challenge. *Elem Sci Anth* 7, 37. <https://doi.org/10.1525/elementa.373>

812 Schwietzke, S., Harrison, M., Lauderdale, T., Branson, K., Conley, S.A., George, F.C., Jordan, D.,
813 Jersey, G.R., Zhang, C., Mairs, H.L., Pétron, G., Schnell, R.C., 2019. Aerially guided leak
814 detection and repair: A pilot field study for evaluating the potential of methane emission

815 detection and cost-effectiveness. *J. Air Waste Manag. Assoc.* 69, 71–88.
816 <https://doi.org/10.1080/10962247.2018.1515123>

817 Sherwin, E.D., Chen, Y., Ravikumar, A.P., Brandt, A.R., 2021. Single-blind test of airplane-based
818 hyperspectral methane detection via controlled releases. *Elem. Sci. Anthr.* 9.
819 <https://doi.org/10.1525/elementa.2021.00063>

820 Snipes, M., Taylor, D.C., 2014. Model selection and Akaike Information Criteria: An example
821 from wine ratings and prices. *Wine Econ. Policy* 3, 3–9.
822 <https://doi.org/10.1016/j.wep.2014.03.001>

823 Thompson, D.R., Leifer, I., Bovensmann, H., Eastwood, M., Fladeland, M., Frankenberg, C.,
824 Gerilowski, K., Green, R.O., Kratwurst, S., Krings, T., Luna, B., Thorpe, A.K., 2015. Real-
825 time remote detection and measurement for airborne imaging spectroscopy: a case study with
826 methane. *Atmos. Meas. Tech.* 8, 4383–4397. <https://doi.org/10.5194/amt-8-4383-2015>

827 Thorpe, A.K., Duren, R.M., Conley, S., Prasad, K.R., Bue, B.D., Yadav, V., Foster, K.T., Rafiq,
828 T., Hopkins, F.M., Smith, M.L., Fischer, M.L., Thompson, D.R., Frankenberg, C.,
829 McCubbin, I.B., Eastwood, M.L., Green, R.O., Miller, C.E., 2020. Methane emissions from
830 underground gas storage in California. *Environ. Res. Lett.* 15, 045005.
831 <https://doi.org/10.1088/1748-9326/ab751d>

832 Thorpe, A.K., Frankenberg, C., Aubrey, A.D., Roberts, D.A., Nottrott, A.A., Rahn, T.A., Sauer,
833 J.A., Dubey, M.K., Costigan, K.R., Arata, C., Steffke, A.M., Hills, S., Haselwimmer, C.,
834 Charlesworth, D., Funk, C.C., Green, R.O., Lundeen, S.R., Boardman, J.W., Eastwood, M.L.,
835 Sarture, C.M., Nolte, S.H., Mccubbin, I.B., Thompson, D.R., McFadden, J.P., 2016. Mapping
836 methane concentrations from a controlled release experiment using the next generation
837 airborne visible/infrared imaging spectrometer (AVIRIS-NG). *Remote Sens. Environ.* 179,
838 104–115. <https://doi.org/10.1016/j.rse.2016.03.032>

839 Tyner, D.R., Johnson, M.R., 2021. Where the Methane Is—Insights from Novel Airborne LiDAR
840 Measurements Combined with Ground Survey Data. *Environ. Sci. Technol.* 55, 9773–9783.
841 <https://doi.org/10.1021/acs.est.1c01572>

842 Varon, D.J., Jacob, D.J., Jervis, D., McKeever, J., 2020. Quantifying Time-Averaged Methane
843 Emissions from Individual Coal Mine Vents with GHGSat-D Satellite Observations. *Environ.*
844 *Sci. Technol.* 54, 10246–10253. <https://doi.org/10.1021/acs.est.0c01213>

845 Varon, D.J., Jacob, D.J., McKeever, J., Jervis, D., Durak, B.O.A., Xia, Y., Huang, Y., 2018.
846 Quantifying methane point sources from fine-scale satellite observations of atmospheric
847 methane plumes. *Atmos. Meas. Tech.* 11, 5673–5686. [https://doi.org/10.5194/amt-11-5673-](https://doi.org/10.5194/amt-11-5673-2018)
848 2018

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1 **Supplemental Information**

2
3 **Robust Probabilities of Detection and Quantification Uncertainty**
4 **for Aerial Methane Detection: Examples for Three Airborne**
5 **Technologies**

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12 **S1 Facilities for High-Flowrate Controlled Release Experiments of Bridger**
13 **Photonics Inc.’s Gas-Mapping LiDAR**

14 High-flowrate, controlled release experiments were completed to support the present analysis of
15 quantification error for Bridger Photonic Inc.’s Gas-Mapping LiDAR (GML) technology.
16 Expanding on the description in the main text, these were performed at four inactive oil and gas
17 facilities (approximate GPS coordinates: 53.12°N, 109.65°W) located in Western Saskatchewan,
18 Canada and shown in Figure S1. These previously oil-producing facilities were approximately
19 30 km southeast of the town of Lloydminster and each less than 100 m in size in the north-south
20 direction such that they were easily captured in a single swath of Bridger’s GML, mounted on an
21 aircraft flying approximately west/eastward. The facilities were sufficiently spaced
22 (approximately 350, 390, and 410 m apart from west to east) to avoid overlap of controlled release
23 plumes granting Bridger the opportunity to measure four controlled releases in quick succession
24 (<30 s at typical flight speeds). The aircraft looped over the facilities approximately every 4
25 minutes, permitting the ground teams to adjust controlled release rate (semi-blindly) between
26 passes and ensuring detectable plumes from the previous pass were sufficiently dispersed and the
27 new plumes had time to establish.



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Figure S1: Four inactive, previously oil-producing, facilities used for high-flowrate controlled releases to study Bridger Photonics Inc.'s Gas-Mapping LiDAR technology. The four facilities were approximately 375 m apart in the west-east direction and less than 100 m in size in the north-south direction, permitting measurement of four controlled releases in rapid succession.

35 S2 Model Optimization and Selection

36 This section describes the model optimization and selection procedure for the derived probability
37 of detection (POD) functions ($POD(\mathbf{x})$) and quantification error distributions ($\pi(Q|\tilde{Q})$).

38 The methodology to derive POD functions for a given measurement technology combines a
39 predictor function ($g(\mathbf{x}; \boldsymbol{\phi})$), which is variable on measured parameters and conditions (\mathbf{x}) and
40 parameterized by $\boldsymbol{\phi}$, and an inverse link function ($F(g; \boldsymbol{\theta})$), which is variable on the predictor
41 function output and is parameterized by $\boldsymbol{\theta}$. As discussed in the manuscript, a generalized predictor
42 function of seven optional coefficients was used (Eq. (7)) in the manuscript and repeated below
43 for convenience) and candidate inverse link functions included cumulative distribution functions
44 (CDFs) of probability distributions with non-negative support. To avoid over-determination of
45 this optimization problem, the coefficients of the candidate inverse link functions ($\boldsymbol{\theta}$) were
46 constrained such that the distribution represented by the candidate CDF had a unit mean and unit
47 variance. Eight probability distributions were considered for the inverse link function including
48 the Burr Type XII, Fréchet, Gamma, and Log-logistic distributions. The candidate model that
49 minimizes the corrected Akaike Information Criterion (AICc; Akaike, 1974) was deemed optimal.

$$g(\mathbf{x}; \boldsymbol{\phi}) = \phi_7 \frac{(Q_{[\text{kg/h}]} - \phi_1)^{\phi_3}}{\tilde{n}_{[\text{ppm}\cdot\text{m}]}^{\phi_4} \left(\frac{h_{[\text{m}]}}{1000}\right)^{\phi_5} (u_{3[\text{m/s}]} - \phi_2)^{\phi_6}} \quad (7)$$

50 Referring to the predictor function (Eq. (7)) and discussion in the manuscript, coefficients
51 could be optionally fixed to ignore the effect of, for example, scene noise or aircraft altitude on

52 detection probability. With the present controlled release data for Bridger’s GML, for which a
53 subset included scene noise data, the importance of each coefficient was studied. Firstly, if
54 coefficients ϕ_1 and ϕ_2 were optimized (such that $\phi_1 \geq 0$ and $\phi_2 \leq 0$), the optimum was obtained
55 when ϕ_1 was approximately zero; this was consistent across technologies, where the optimized
56 $\phi_1 = 0$ throughout. After this observation, the relative importance of scene noise and aircraft
57 altitude was assessed by optionally fixing ϕ_4 and/or ϕ_5 to zero and optimizing the POD for the
58 subset of Bridger GML data where scene noise was available (N = 178). The marginal benefit of
59 including an additional non-fixed coefficient (i.e., ϕ_4 and/or ϕ_5) was assessed using the AICc.
60 This parameter, technically a “Bayes Factor” (e.g., Snipes and Taylor, 2014), is used to quantify
61 the relative goodness of models and is a function of the optimized value of the negative log-
62 likelihood function (i.e., the objective function of the optimization) and with a penalty on number
63 of optimized variables. The difference in the AICc (ΔAICc) between an *initial model* and an
64 *alternative model* is indicative of the statistical justification for the latter over the former. This
65 result is typically interpreted using Kass and Raftery’s (1995) classification, where ΔAICc in (0,
66 $10^{0.5}$) implies that the difference between models is “not worth more than a bare mention” and
67 ΔAICc in ($10^{0.5}$, 10^1) and (10^1 , 10^2) imply that there is “substantial” and “strong” justification for
68 the alternative model over the initial model. As summarized in Table S1 and discussed in the
69 manuscript, the consideration of scene noise *or* aircraft altitude is strongly justified ($\Delta\text{AICc} \approx$
70 $10^{1.25}$). By contrast, the marginal benefit of including noise or aircraft altitude if the other is
71 already in the initial model is much weaker, cussing the “not worth more than a bare mention”
72 classification.

73 **Table S1: Summary of goodness-of-fit statistics for models that incrementally include consideration of**
74 **representative scene noise or aircraft altitude.**

| Initial Model: | | Alternative Model: | | Average ΔAICc of Eight Candidate Models |
|----------------|----------|--------------------|----------|---|
| Noise | Altitude | Noise | Altitude | |
| No | No | No | Yes | $10^{1.25}$ |
| No | No | Yes | No | $10^{1.25}$ |
| No | Yes | Yes | Yes | $10^{0.51}$ |
| Yes | No | Yes | Yes | $10^{0.50}$ |

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76 Coefficients (ϕ) of the optimized predictor function ($g(\mathbf{x}; \phi)$) are shown in Table S2 for each
77 technology: Bridger Photonics Inc.’s Gas-Mapping LiDAR (GML), Kairos Aerospace’s

78 LeakSurveyor, and AVIRIS-NG (Next-Generation Airborne Visible/Infrared Imaging
79 Spectrometer) from the (U.S.) National Aeronautics and Space Administration’s (NASA’s) Jet
80 Propulsion Laboratory (JPL). Table S3 additionally provides the complete equations for these
81 predictor functions as well as the inverse link functions ($F(g; \theta)$); the composition of these
82 equations yields the POD function ($POD(x)$) for each technique, which are detailed in the final
83 column of the table.

84 **Table S2: Optimized coefficients of the predictor function ($g(x; \phi)$) for each measurement technology.**

| Function/ Coefficient | | Bridger GML | Kairos LeakSurveyor | | NASA JPL AVIRIS-NG | |
|-------------------------------|----------|----------------|----------------------------------|-------------------------------|----------------------------------|-------------------------------|
| | | | without partial detections | with partial detections | without partial detections | with partial detections |
| Predictor function (g) | ϕ_1 | 0 | 0 | 0 | 0 | 0 |
| | ϕ_2 | 1.072 | 2.054 | 2.027 | 2.431 | 1.101 |
| | ϕ_3 | -2.139 | -0.929 | -0.07006 | -36.44 | -37.11 |
| | ϕ_4 | 0 | — | — | — | — |
| | ϕ_5 | 2.440 | — | — | 0.1.684 | 0.7343 |
| | ϕ_6 | 1.692 | 2.056 | 1.527 | 7.643 | 4.420 |
| | ϕ_7 | 0.1518 | 13.35×10^{-3} | 5.363×10^{-3} | 14.23×10^9 | 2.125×10^6 |

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86 Modelling of the quantification error distributions was performed via Eq. (5) and (6) in the
87 manuscript depending on the assumed (in)dependence of measurement bias with measurement
88 date and location. Regardless of the assumption, model optimization and selection were performed
89 using the AICc, like the model optimization/selection for POD. Table S4 summarizes the results
90 for the quantification error analysis of Bridger’s GML and Kairos’ LeakSurveyor. The optimized
91 bias-correction functions ($\hat{Q} = f_B(\tilde{Q})$) and bias- and precision-distributions (π_{κ_Q} and π_{λ_Q} as
92 needed, where $\kappa_Q = Q/\tilde{Q}$ and $\lambda_Q = Q/\hat{Q}$) are shown for each of the technologies and wind data
93 sources discussed in the manuscript. These results are combined in the last column of the table to
94 yield the conditional distribution for quantification error, $\pi(Q|\tilde{Q})$. Table S5 summarizes the same
95 analysis for wind speed error distributions, $\pi(u_3|\tilde{u}_3)$ to support calculations via Eq. (10) in the main
96 text.

Table S3: Detailed equations for the predictor, inverse link, and composite POD functions for each measurement technology.

| Instrument | Predictor Function $g(Q, u_3, \tilde{h})$ | Inverse Link Function $F(g)$ | Detailed Equation for POD(Q, u_3, \tilde{h}) |
|--|---|--|---|
| Bridger Photonics Inc. GML | $\frac{0.1518 Q_{[\text{kg/h}]}^{1.072}}{\left(\frac{\tilde{h}_{[\text{m}]}}{1000}\right)^{2.440} (u_{3[\text{m/s}]} + 2.139)^{1.692}}$ | Fréchet CDF: $\exp(-0.3719g^{-2.530})$ | $\exp\left(-\left(\frac{0.2244 Q_{[\text{kg/h}]}^{1.072}}{\left(\frac{\tilde{h}_{[\text{m}]}}{1000}\right)^{2.440} (u_{3[\text{m/s}]} + 2.139)^{1.692}}\right)^{-2.530}\right)$ |
| Kairos Aerospace LeakSurveyor ^a | $\frac{(13.35 \times 10^{-3}) Q_{[\text{kg/h}]}^{2.054}}{(u_{3[\text{m/s}]} + 0.9296)^{2.056}}$ | Burr Type XII CDF: $1 - (1 + 0.2976g^{1.303})^{-3.906}$ | $1 - \left(1 + \left(\frac{(5.266 \times 10^{-3}) Q_{[\text{kg/h}]}^{2.054}}{(u_{3[\text{m/s}]} + 0.9296)^{2.056}}\right)^{1.303}\right)^{-3.906}$ |
| Including Partial Detections | $\frac{(5.363 \times 10^{-3}) Q_{[\text{kg/h}]}^{2.027}}{(u_{3[\text{m/s}]} + 0.07006)^{1.527}}$ | Burr Type XII CDF: $1 - (1 + 0.2976g^{1.303})^{-3.906}$ | $1 - \left(1 + \left(\frac{(2.116 \times 10^{-3}) Q_{[\text{kg/h}]}^{2.027}}{(u_{3[\text{m/s}]} + 0.07006)^{1.527}}\right)^{1.303}\right)^{-3.906}$ |
| AVIRIS-NG | $\frac{(14.23 \times 10^9) Q_{[\text{kg/h}]}^{2.431}}{\left(\frac{\tilde{h}_{[\text{m}]}}{1000}\right)^{1.684} (u_{3[\text{m/s}]} + 36.44)^{7.643}}$ | Burr Type XII CDF: $1 - (1 + 0.2976g^{1.303})^{-3.906}$ | $1 - \left(1 + \left(\frac{(5.613 \times 10^9) Q_{[\text{kg/h}]}^{2.431}}{\left(\frac{\tilde{h}_{[\text{m}]}}{1000}\right)^{1.684} (u_{3[\text{m/s}]} + 36.44)^{7.643}}\right)^{1.303}\right)^{-3.906}$ |
| Including Partial Detections | $\frac{(2.125 \times 10^6) Q_{[\text{kg/h}]}^{1.101}}{\left(\frac{\tilde{h}_{[\text{m}]}}{1000}\right)^{0.7343} (u_{3[\text{m/s}]} + 37.11)^{4.420}}$ | Fréchet CDF: $\exp(-0.3719g^{-2.530})$ | $\exp\left(-\left(\frac{(3.141 \times 10^6) Q_{[\text{kg/h}]}^{1.101}}{\left(\frac{\tilde{h}_{[\text{m}]}}{1000}\right)^{0.7343} (u_{3[\text{m/s}]} + 37.11)^{4.420}}\right)^{-2.530}\right)$ |

^a Result for Kairos' LeakSurveyor technology at fixed aircraft altitude of 900 m.

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Table S4: Optimized bias-correction functions and precision distributions for estimated source rate using Bridger’s GML and Kairos’ LeakSurveyor technologies with various wind speed data. Optimized bias-correction functions were proportional models and precision-correction distributions took the form of various non-negative probability distributions with unit mean. Detailed equations for the resulting quantification error distribution ($\pi(Q|\bar{Q})$) are also provided.

| Instrument | Wind Source | Bias-correction Function $f_B(\bar{Q}) = d\bar{Q}$ | Bias- and Precision-Correction Distributions ^a π_{κ_Q} and π_{λ_Q} | Detailed Equation for $\pi(Q \bar{Q})$ |
|---|---------------------------------------|---|--|--|
| Bridger Photonics Inc. GML | Meteoblue – Proprietary averaging | $d = 0.948$ | $\lambda_Q \sim \text{Burr} \left(\frac{0.743}{\alpha}, \frac{4.55}{c}, \frac{0.659}{k} \right)$ | $\frac{\frac{kc}{\alpha} \left(\frac{Q}{d\alpha\bar{Q}} \right)^{c-1}}{d\bar{Q} \left(1 + \left(\frac{Q}{d\alpha\bar{Q}} \right)^c \right)^{k+1}}$ |
| Bridger Photonics Inc. GML ^b | Meteoblue – Proprietary averaging | $d = 0.939$ | $\kappa_Q \sim \text{LL} \left(\frac{0.944}{\alpha}, \frac{5.36}{\beta} \right)$ $\lambda_Q \sim \text{Burr} \left(\frac{0.752}{\alpha}, \frac{4.53}{c}, \frac{0.674}{k} \right)$ | $\pi(Q \bar{Q}) = \int_{\bar{Q}} \pi_{\lambda_Q} \left(\frac{Q}{\hat{Q}} \right) \pi_{\kappa_Q} \left(\frac{\hat{Q}}{f_B(\bar{Q})} \right) \frac{1}{\bar{Q} f_B(\bar{Q})} d\hat{Q}$ |
| Kairos Aerospace LeakSurveyor | Dark Sky – 1-minute average | $d = 2.14$ | $\lambda_Q \sim \text{LL} \left(\frac{0.930}{\alpha}, \frac{4.79}{\beta} \right)$ | $\frac{\left(\frac{\beta}{\alpha} \right) \left(\frac{Q}{d\alpha\bar{Q}} \right)^{\beta-1}}{d\bar{Q} \left(1 + \left(\frac{Q}{d\alpha\bar{Q}} \right)^\beta \right)^2}$ |
| | Dark Sky – 1-minute gust ^c | $d = 1.04$ | $\lambda_Q \sim \Gamma \left(\frac{5.93}{\alpha}, \frac{0.169}{\beta} \right)$ | $\frac{(d\bar{Q}\beta)^{-\alpha}}{\Gamma(\alpha)} Q^{\alpha-1} \exp \left(-\frac{Q}{d\bar{Q}\beta} \right)$ |
| | HRRR – 1-hour average | $d = 2.53$ | $\lambda_Q \sim \text{GEV} \left(\frac{0.489}{k}, \frac{0.282}{\sigma}, \frac{0.576}{\mu} \right)$ | $\frac{1}{d\sigma\bar{Q}} \left(1 + k \frac{\left(\frac{Q}{d\bar{Q}} - \mu \right)}{\sigma} \right)^{-\frac{k+1}{k}} \exp \left(- \left(1 + k \frac{\left(\frac{Q}{d\bar{Q}} - \mu \right)}{\sigma} \right)^{-\frac{1}{k}} \right)$ |
| | HRRR – 1-hour gust | $d = 1.34$ | $\lambda_Q \sim \text{GEV} \left(\frac{0.386}{k}, \frac{0.265}{\sigma}, \frac{0.686}{\mu} \right)$ | |

^a Legend: *Burr* = Burr Type XII distribution; Γ = Gamma distribution. *GEV* = Generalized extreme value distribution; *LL* = Log-logistic distribution.

^b Quantification error distribution fit to Bridger GML data assuming time- and location-dependent measurement bias (i.e., fit using Eq. (5) in the manuscript).

^c Includes an additional 296 data from controlled release studies completed by Kairos.

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Table S5: Optimized bias-correction functions and precision distributions for various 3-m wind speed data sources relevant to Bridger’s GML and Kairos’ LeakSurveyor technologies. Optimized bias-correction functions were proportional models and precision-correction distributions took the form of various non-negative probability distributions with unit mean. The detailed equations for the resulting quantification error distribution $\pi(u_3|\tilde{u}_3)$ are provided for use with Eq. (10) in the main text if seeking to derive POD functions that use modelled wind rather than in situ measured/actual wind.

| Instrument | Wind Source | Bias-correction Function $f_B(\tilde{u}_3) = d\tilde{u}_3$ | Bias- and Precision-Correction Distributions ^a $\pi_{\kappa_{u_3}}$ and $\pi_{\lambda_{u_3}}$ | Detailed Equation for $\pi(u_3 \tilde{u}_3)$ |
|---|---------------------------------------|---|--|---|
| Bridger Photonics Inc. GML | Meteoblue – Proprietary averaging | $d = 0.906$ | $\lambda_{u_3} \sim LL\left(\frac{0.903}{\alpha}, \frac{4.05}{\beta}\right)$ | $\frac{\left(\frac{\beta}{\alpha}\right)\left(\frac{u_3}{d\alpha\tilde{u}_3}\right)^{\beta-1}}{d\tilde{u}_3\left(1 + \left(\frac{u_3}{d\alpha\tilde{u}_3}\right)^\beta\right)^2}$ |
| Bridger Photonics Inc. GML ^b | Meteoblue – Proprietary averaging | $d = 0.939$ | $\kappa_{u_3} \sim IG\left(\frac{1}{\mu}, \frac{10.6}{\xi}\right)$ $\lambda_{u_3} \sim LL\left(\frac{0.904}{\alpha}, \frac{4.08}{\beta}\right)$ | $\pi(u \tilde{u}_3) = \int_{\tilde{u}_3} \pi_{\lambda_{u_3}}\left(\frac{u_3}{\hat{u}_3}\right) \pi_{\kappa_{u_3}}\left(\frac{\hat{u}_3}{f_B(\tilde{u}_3)}\right) \frac{1}{\hat{u}_3 f_B(\tilde{u}_3)} d\hat{u}_3$ |
| Kairos Aerospace LeakSurveyor | Dark Sky – 1-minute average | $d = 1.83$ | $\lambda_{u_3} \sim LL\left(\frac{0.951}{\alpha}, \frac{5.77}{\beta}\right)$ | $\frac{\left(\frac{\beta}{\alpha}\right)\left(\frac{u_3}{d\alpha\tilde{u}_3}\right)^{\beta-1}}{d\tilde{u}_3\left(1 + \left(\frac{u_3}{d\alpha\tilde{u}_3}\right)^\beta\right)^2}$ |
| | Dark Sky – 1-minute gust ^c | $d = 0.780$ | $\lambda_{u_3} \sim W\left(\frac{1.11}{\xi}, \frac{3.61}{k}\right)$ | $\frac{k}{d\xi\tilde{u}_3}\left(\frac{u_3}{d\xi\tilde{u}_3}\right)^{k-1} \exp\left(-\left(\frac{u_3}{d\xi\tilde{u}_3}\right)^k\right)$ |
| | HRRR – 1-hour average | $d = 1.92$ | $\lambda_{u_3} \sim IG\left(\frac{1}{\mu}, \frac{2.80}{\xi}\right)$ | $\frac{1}{d\tilde{u}_3} \sqrt{\frac{\xi}{2\pi}} \left(\frac{u_3}{d\tilde{u}_3}\right)^{-3} \exp\left(-\frac{d\xi\tilde{u}_3\left(\frac{u_3}{d\tilde{u}_3} - 1\right)^2}{2u_3}\right)$ |
| | HRRR – 1-hour gust | $d = 1.05$ | $\lambda_{u_3} \sim LL\left(\frac{0.908}{\alpha}, \frac{4.18}{\beta}\right)$ | $\frac{\left(\frac{\beta}{\alpha}\right)\left(\frac{u_3}{d\alpha\tilde{u}_3}\right)^{\beta-1}}{d\tilde{u}_3\left(1 + \left(\frac{u_3}{d\alpha\tilde{u}_3}\right)^\beta\right)^2}$ |

^a Legend: *IG* = Inverse Gaussian distribution; *LL* = Log-logistic distribution; *W* = Weibull distribution.

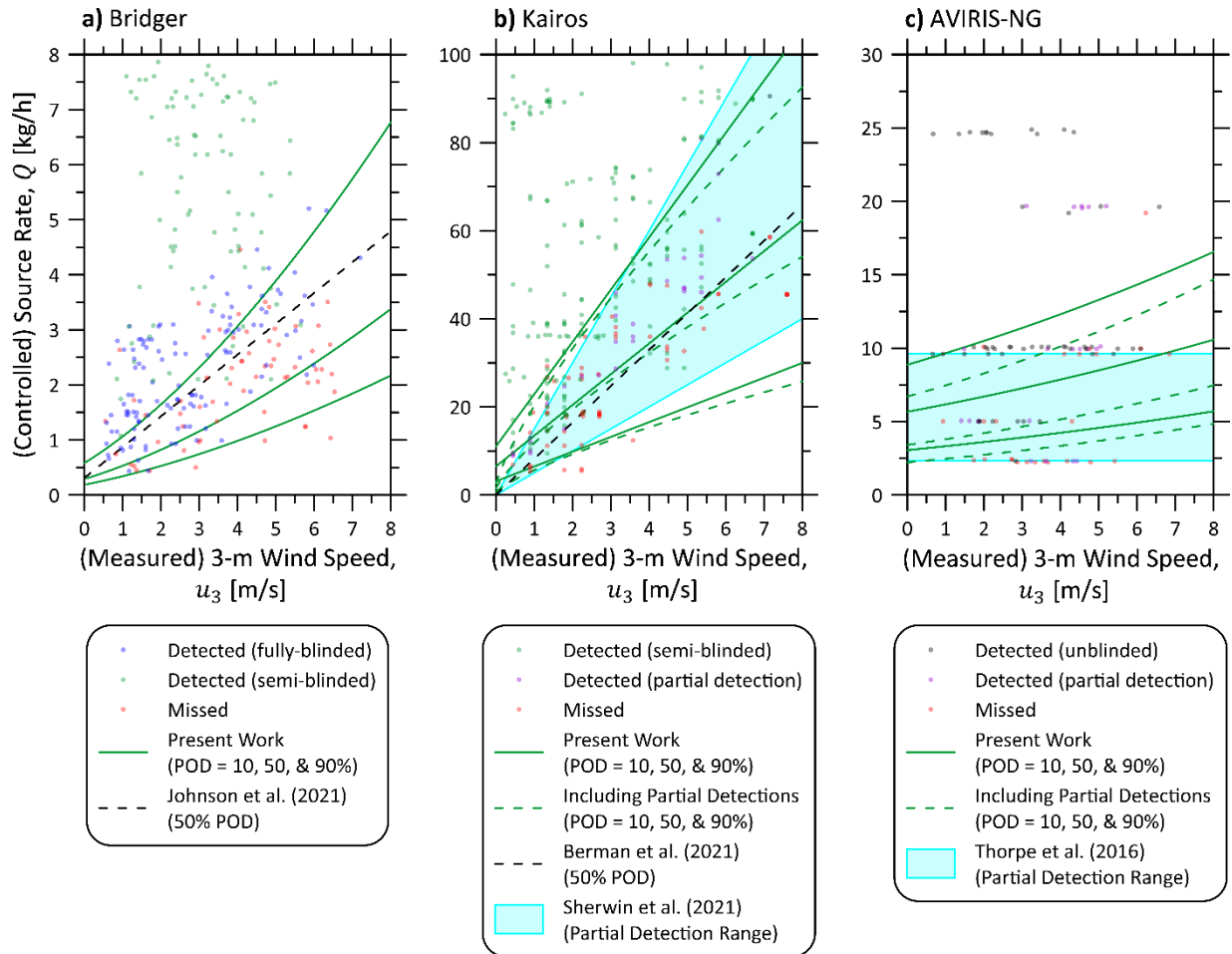
^b Quantification error distribution fit to Bridger GML data assuming time- and location-dependent measurement bias (i.e., fit using Eq. (5) in the manuscript).

^c Includes an additional 296 data from controlled release studies completed by Kairos.

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111 **S3 Additional Detail of Data used in POD Derivations**

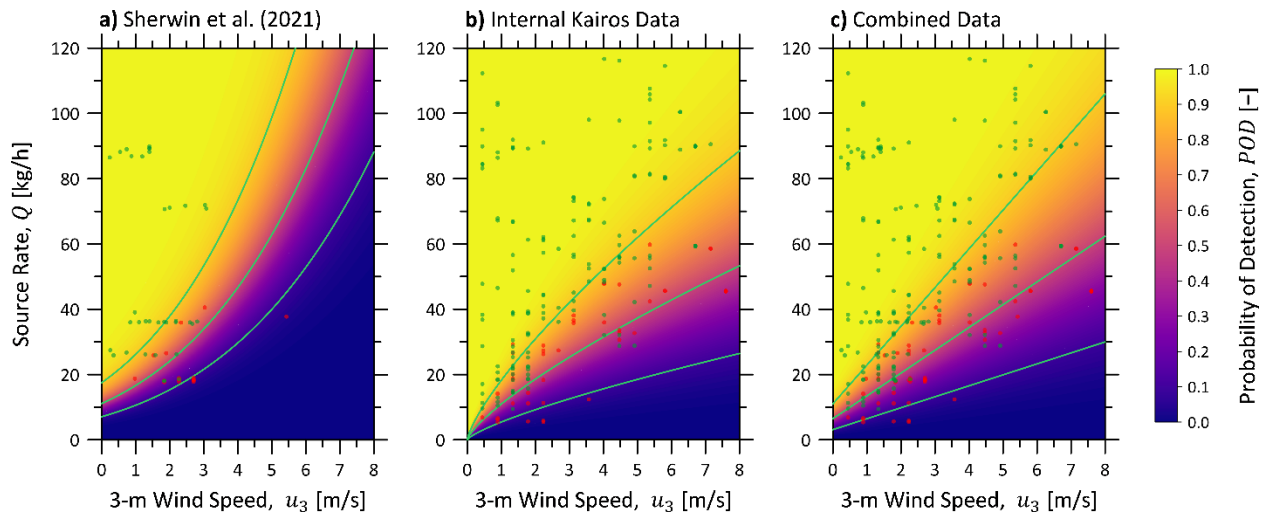
112 The following figures support the discussion in the manuscript. Figure S2 plots available
 113 controlled release data for each measurement technology, the POD contours from the derived POD
 114 functions (replicated from the manuscript), and previously published detection sensitivities. Note
 115 that for each plot there are additional large-scale release data beyond the limits of the y-axis.



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 117 **Figure S2: Comparison of probabilities of detections for Bridger's GML (a), Kairos' LeakSurveyor (b), and**
 118 **NASA JPL's AVIRIS-NG (c) technologies. Each figure plots probability contours using the present**
 119 **methodology (at 10, 50, and 90% POD) alongside available controlled release data which are coloured**
 120 **according to detection (black = unblinded, blue = fully blinded, green = semi-blinded) and miss (red). Each**
 121 **figure also identifies previously published estimates of detection sensitivity: 50% POD for Bridger's GML**
 122 **(Johnson et al., 2021) and Kairos' LeakSurveyor (Berman et al., 2021), and partial detection ranges for**
 123 **Kairos' LeakSurveyor (Sherwin et al., 2021) and NASA JPL's AVIRIS-NG (Thorpe et al., 2016).**

124 Figure S3 provides the optimized POD function using the methodology in the present work and
 125 controlled release data for Kairos' LeakSurveyor. Subplots a) through c) show significant

126 differences in the optimized POD when using Sherwin et al.'s (2021) data alone (a), Kairos'
 127 internal controlled release data alone (b), and the combination of these data sources (c).



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 129 **Figure S3: Optimized POD functions for Kairos' LeakSurveyor technology, ignoring partial detections using**
 130 **different data sets: a) the data of Sherwin et al. (2021) alone, b) the confidential data from Kairos' internal**
 131 **studies alone, and c) the combination of these data sources.**

132 **S4 References**

133 Akaike, H., 1974. A new look at the statistical model identification. *IEEE Transactions on Automatic*
 134 *Control* 19, 716–723. <https://doi.org/10.1109/TAC.1974.1100705>

135 Berman, E., S.F., W., B., E., Jones, B.B., 2021. Kairos Aerospace Technical White Paper: Methane
 136 Detection (Version 1F). Kairos Aerospace. <https://doi.org/10.17605/OSF.IO/HZG52>

137 Johnson, M.R., Tyner, D.R., Szekeres, A.J., 2021. Blinded evaluation of airborne methane source detection
 138 using Bridger Photonics LiDAR. *Remote Sensing of Environment* 259, 112418.
 139 <https://doi.org/10.1016/j.rse.2021.112418>

140 Kass, R.E., Raftery, A.E., 1995. Bayes Factors. *J Am Stat Assoc* 90, 773–795.

141 Sherwin, E.D., Chen, Y., Ravikumar, A.P., Brandt, A.R., 2021. Single-blind test of airplane-based
 142 hyperspectral methane detection via controlled releases. *Elementa: Science of the Anthropocene* 9.
 143 <https://doi.org/10.1525/elementa.2021.00063>

144 Snipes, M., Taylor, D.C., 2014. Model selection and Akaike Information Criteria: An example from wine
 145 ratings and prices. *Wine Economics and Policy* 3, 3–9. <https://doi.org/10.1016/j.wep.2014.03.001>

146 Thorpe, A.K., Frankenberg, C., Aubrey, A.D., Roberts, D.A., Nottrott, A.A., Rahn, T.A., Sauer, J.A.,
 147 Dubey, M.K., Costigan, K.R., Arata, C., Steffke, A.M., Hills, S., Haselwimmer, C., Charlesworth, D.,
 148 Funk, C.C., Green, R.O., Lundeen, S.R., Boardman, J.W., Eastwood, M.L., Sarture, C.M., Nolte, S.H.,
 149 Mccubbin, I.B., Thompson, D.R., McFadden, J.P., 2016. Mapping methane concentrations from a
 150 controlled release experiment using the next generation airborne visible/infrared imaging
 151 spectrometer (AVIRIS-NG). *Remote Sensing of Environment* 179, 104–115.
 152 <https://doi.org/10.1016/j.rse.2016.03.032>

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