

1 **Streamflow depletion estimation for conjunctive water management in a**
2 **heavily-stressed aquifer using analytical depletion functions**

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17
18 **Keywords:** analytical models, MODFLOW, water management, decision support, streamflow
19 depletion

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21 **Key Points:**

- 22 ● We compared streamflow depletion estimates from analytical depletion functions to a
23 numerical model in a heavily-stressed aquifer.
24 ● Analytical depletion functions had similar estimates of streamflow depletion with lower
25 data and computational costs than numerical models.
26 ● Analytical depletion functions are a potential tool for decision making in settings where
27 numerical models are not available.
28

29 **Abstract**

30 Groundwater pumping can lead to reductions in streamflow (‘streamflow depletion’) and
31 estimating streamflow depletion is critical for conjunctive groundwater-surface water
32 management. Streamflow depletion can be quantified using either analytical models, which have
33 low data requirements but many simplifying assumptions, or numerical models, which represent
34 physical processes more realistically but have high data, effort, and expertise requirements.
35 Analytical depletion functions are a recently-developed tool that address some of the limitations
36 of analytical models, but to date have only been evaluated under relatively simple conditions.
37 Here, we evaluate eight different analytical depletion functions across a range of groundwater
38 abstraction, physiographic, and hydrostratigraphic conditions via comparison to the Republican
39 River Compact Administration groundwater model, a calibrated MODFLOW numerical model
40 used for conjunctive water management in a heavily-stressed portion of the High Plains Aquifer
41 (USA). We find mostly strong agreement between the analytical depletion functions and the
42 MODFLOW model, though analytical depletion functions underestimate depletion for wells
43 close to surface water features in high transmissivity environments. Compared to previous work,
44 there is little variability among the eight analytical depletion functions, indicating that function
45 formulation plays a minor role in this setting. Analytical depletion function performance is
46 strongly influenced by hydrostratigraphic parameters (storativity and transmissivity) but
47 performance is insensitive to pumping rate, confirming a key assumption of analytical models.
48 Overall, analytical depletion functions provide comparable estimates of streamflow depletion to
49 numerical models at a fraction of the time and data requirements. Accurate hydrostratigraphic
50 data are essential to estimating streamflow depletion regardless of modeling approach.

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52 **Plain Language Summary**

53 Estimating the impacts of groundwater pumping on streamflow (‘streamflow depletion’) is
54 challenging but essential for effectively managing water resources. In this study, we test a low-
55 cost, low-effort approach (called an ‘analytical depletion function’) to estimate streamflow
56 depletion by comparing it to a more complex tool that is currently used for water management in
57 a heavily-irrigated setting in the central US. We find that there is general agreement between the
58 analytical depletion function and the more complex approach. We also test whether analytical
59 depletion function performance is better or worse for different conditions, and find that
60 performance is similar regardless of pumping rate but very sensitive to properties of the
61 subsurface. Overall, our results indicate that analytical depletion functions could be useful tools
62 for estimating streamflow depletion where more complex approaches are unavailable, but having
63 accurate data about the subsurface is essential.

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65 **1 Introduction**

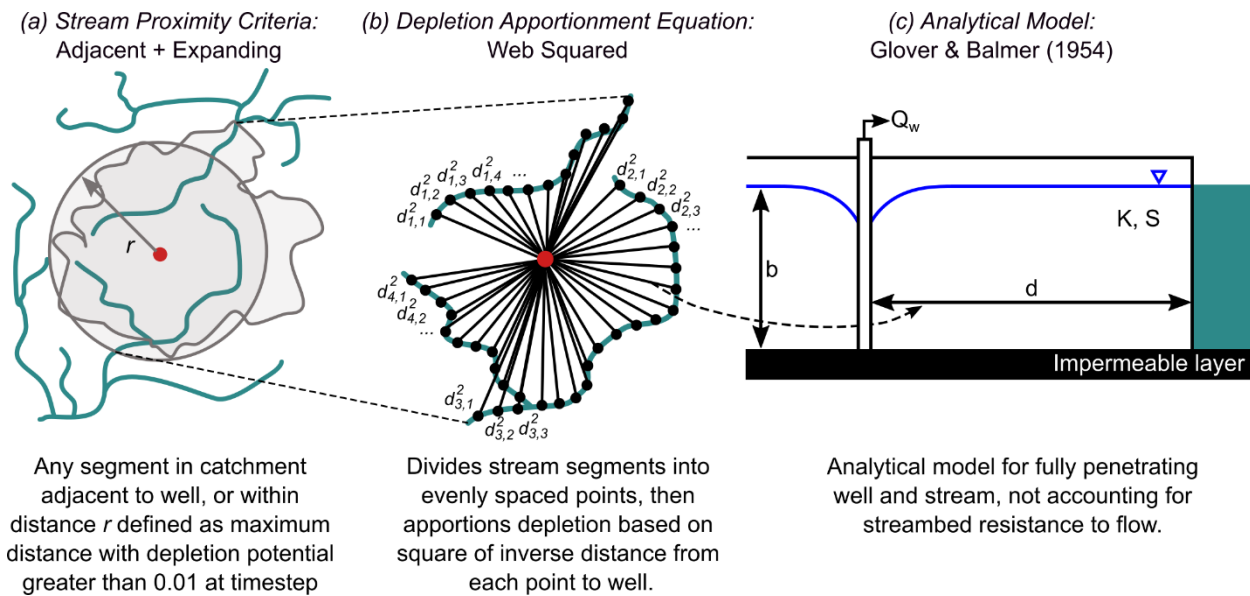
66 Groundwater is an essential contributor to streamflow around the world (Beck et al.
67 2013), providing a relatively cool and stable supply of water particularly during dry periods.

68 Groundwater inflow to streams ('baseflow') is essential for a number of aquatic and
69 groundwater-dependent ecosystems (Rohde et al. 2017; Larsen and Woelfle-Erskine 2018).
70 However, groundwater abstractions can lead to reductions in streamflow ('streamflow
71 depletion') via the capture of discharge, which includes interception of water which otherwise
72 would have discharged into a stream or, in extreme cases, induced infiltration from a previously
73 gaining stream (Bredehoeft et al. 1982; Bredehoeft 2002; Barlow et al. 2018).

74 Streamflow depletion cannot be directly observed and is often difficult to estimate due to
75 complex groundwater flow systems, lag times between pumping and streamflow reductions, and
76 natural variability in streamflow resulting from other processes such as weather, water control
77 structures such as dams, and surface water withdrawals (Barlow and Leake 2012). For this
78 reason, conjunctive groundwater-surface water management typically relies on numerical models
79 (e.g. MODFLOW), which are physics-based simulations of groundwater flow processes
80 (McDonald and Harbaugh 1988; Fienen et al. 2018). However, there are large time, effort, and
81 computational costs associated with numerical models (Fienen et al. 2015, 2016). They are,
82 therefore, not available in most settings.

83 In locations where numerical models are not available, analytical models are often used
84 instead (Hamilton and Seelbach 2011; Huang et al. 2018). Analytical models are simpler
85 representations of stream-aquifer interactions, but contain many limiting assumptions such as
86 one (or occasionally two) streams, homogeneous subsurface conditions, and simplified stream
87 and aquifer geometry. While analytical models have been proposed that account for some of
88 these assumptions (Butler et al. 2007; Yeh et al. 2008; Zlotnik and Tartakovsky 2008; Singh
89 2009), there are still many real-world environments which violate the core assumptions of
90 analytical models including settings where there are multiple and/or highly sinuous streams.

91 Recently, analytical depletion functions (Figure 1) have been proposed as a potential
92 extension of existing analytical models which empirically address some of these limitations for
93 use in complex, real-world settings (Zipper et al. 2019b). An analytical depletion function
94 consists of (i) *stream proximity criteria*, which identify the streams that may be affected by a
95 well; (ii) a *depletion apportionment equation*, which calculates how depletion from a single well
96 should be allocated to multiple stream segments meeting the stream proximity criteria; and (iii)
97 an *analytical model*, which estimates depletion in each stream segment meeting the proximity
98 criteria.



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Figure 1. Components of an analytical depletion function. Modified from Zipper et al. (2019a) under CC-BY 3.0 license.

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Analytical depletion functions have only been subjected to limited testing in a small number of study sites. During the development of the State of Michigan's Water Withdrawal Assessment Tool, Reeves et al. (2009) compared nine different depletion apportionment equations for a small watershed in Michigan and found that an inverse distance-based approach best matched output from a MODFLOW numerical model. Subsequently, Zipper et al. (2018a) evaluated five depletion apportionment equations for several real-world stream networks in British Columbia, finding that a new inverse distance-based approach which considers stream geometry (called web squared; Figure 1b) performed the best under steady-state conditions. Zipper et al. (2019b) introduced the concept of stream proximity criteria (Figure 1a) and performed a sensitivity analysis to 50 combinations of stream proximity criteria, depletion apportionment equation, and analytical model, finding that analytical depletion functions were able to accurately estimate the distribution and magnitude of streams affected by a well. However, Zipper et al. (2019b) compared analytical depletion functions to an archetypal numerical model with several simplifications including a homogeneous subsurface and stream properties. Li et al. (2020) conducted the first comparison of analytical depletion functions to a calibrated numerical model, but this was in unstressed conditions with only a single well pumping at any given time and did not evaluate different analytical depletion function formulations.

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As a result, it remains unknown whether analytical depletion functions are suitable tools for heavily-stressed aquifers with significant ongoing pumping, particularly since evidence indicates that cumulative impacts of multiple wells may not be linearly additive (Ahlfeld et al. 2016). Further, the degree to which well and hydrostratigraphic characteristics influence the performance of analytical depletion functions has not been previously evaluated. Thus, the ability of analytical depletion functions to predict streamflow depletion in complex,

126 heterogeneous, and highly-stressed real-world settings where cumulative impacts of multiple
127 wells are occurring simultaneously remains unknown. In this study, we address this knowledge
128 gap by comparing a suite of analytical depletion functions to a complex, calibrated groundwater
129 model of the Republican River Basin (USA) which is currently used for conjunctive water
130 management (RRCA 2003). Specifically, we ask:

- 131 (1) Do analytical depletion functions estimates of streamflow depletion in a complex, highly-
132 stressed, unconfined aquifer agree with an existing calibrated numerical model?
- 133 (2) How does agreement between analytical depletion functions and the numerical model
134 vary as a function of formulation, hydrogeological properties, stream properties,
135 landscape attributes, and time of year?

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137 **2 Methods**

138 *2.1 Republican River Compact Administration groundwater model*

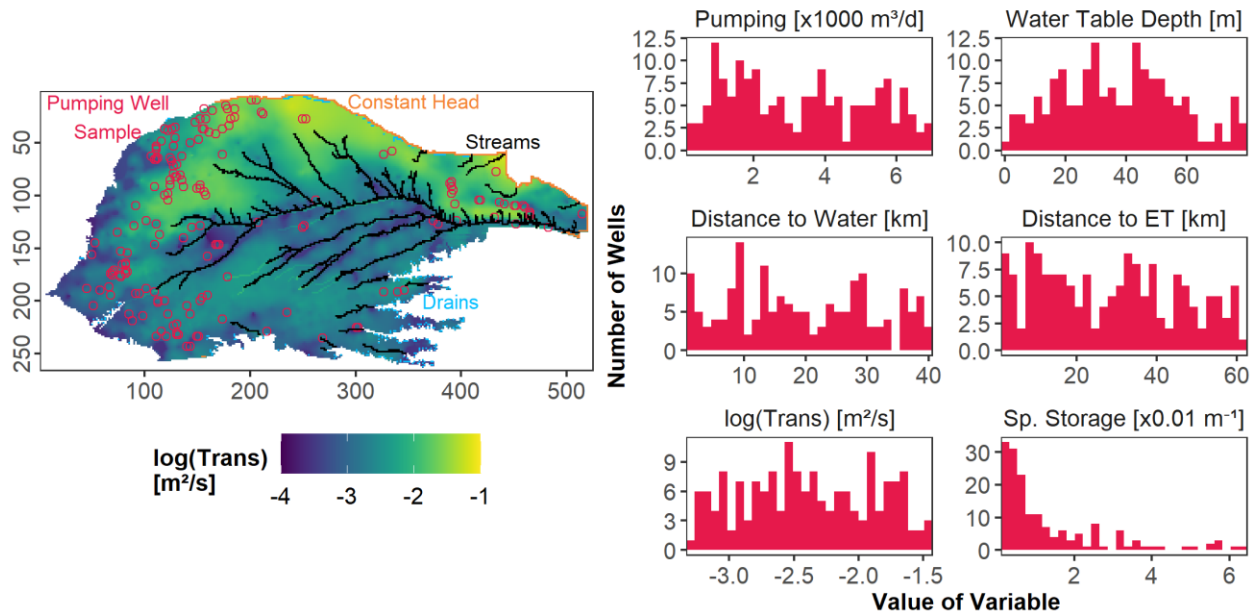
139 The Republican River Watershed drains approximately 64,500 km² (24,900 mi²) of the
140 US High Plains, flowing through Colorado, Nebraska, and Kansas (RRCA 2003). There is
141 significant irrigated agriculture within the watershed (Deines et al. 2017, 2019) and, as a result,
142 the surface and groundwater resources in the Republican River Watershed and surrounding High
143 Plains Aquifer are heavily stressed (Haacker et al. 2016; Butler et al. 2018). To allocate and
144 manage the water of the Republican River, the three states entered into the Republican River
145 Compact in 1942 (Khan and Brown 2019). Following a US Supreme Court decision in 2002,
146 representatives from Kansas, Nebraska, Colorado, the US Bureau of Reclamation, and the US
147 Geological Survey collaboratively constructed the Republican River Compact Administration
148 (RRCA) groundwater model as a tool for quantifying streamflow depletion caused by
149 groundwater pumping. The model is updated annually to guide water allocations among the three
150 states. The model is described in detail in RRCA (2003) and all model files are available on the
151 Republican River Compact Administration website (<https://www.republicanrivercompact.org/>).

152 The RRCA model spans an area larger than the Republican River watershed and is
153 bounded by the Platte River in the north and outcrops of the High Plains Aquifer on the east,
154 west, and south. The model is constructed using MODFLOW-2000 (Harbaugh et al. 2000)
155 covering a total active domain of 77,868 km² (30,065 mi²), which is discretized into 2.6 km² (1
156 mi²) grid cells. The model is updated annually with each year's estimated pumping data
157 submitted by each state. Here, we use version 12p, which is the version originally released that
158 spans the period 1918-2000 at a monthly timestep. While the High Plains Aquifer in this region
159 is unconfined, the RRCA model adopts an assumption of constant transmissivity to improve
160 model stability. In MODFLOW-2000, constant transmissivity in transient simulations requires
161 that the aquifer is parameterized as confined and, as a result, requires specific storage as model
162 input rather than specific yield. To appropriately represent unconfined aquifer storage properties
163 with a confined parameterization, the RRCA calculates specific storage for each grid cell as the
164 estimated specific yield divided by saturated thickness in each grid cell (RRCA 2005).

165 The model represents surface water features using a combination of three MODFLOW
166 packages (Figure 2; Figure S1): the stream package (STR), which is used for the Republican
167 River and tributaries and allows stream cells to dry in response to pumping; constant head
168 boundaries (CHB), which are used for the Platte River at the north edge and the eastern edge of
169 the model; and the drain package (DRN), which represent springs and are primarily along the
170 southeastern portion of the domain. Direct uptake of groundwater by phreatophytes is
171 represented using the evapotranspiration (EVT) package, and primarily occurs in cells along
172 stream channels with shallow groundwater. There are a total of 9372 pumping wells in the
173 domain, each of which has a monthly pumping schedule which primarily occurs during the June-
174 September growing season (Figure S2; Figure S3).

175 The model was calibrated via comparison to groundwater levels (350,233 records from
176 10,835 locations) and baseflow (65 records) for the historical pumped period. Hydraulic
177 conductivity and precipitation recharge rates were the primary calibration parameters. Since the
178 calibration period included the expansion of pumping across the watershed, calibrating to
179 baseflow across the historic period provides confidence that the model is able to simulate the
180 response of groundwater-surface water interactions in response to groundwater pumping.
181 Complete calibration results are available at the RRCA website for baseflow
182 (<https://www.republicanrivercompact.org/v12p/html/ch07.html>) and groundwater levels
183 (<https://www.republicanrivercompact.org/v12p/html/ch08.html>). Since baseflow is the more
184 relevant calibration target for our investigation, we included several calibration scatterplots in the
185 supplemental information (Figure S4, S5, S6). There tends to be more variability in performance
186 in smaller tributaries compared to the Republican River main stem. During model development,
187 fit between observations and predictions was evaluated by experts from the model development
188 team, which was made up of representatives from each of the affected states and the federal
189 government, and deemed to be acceptable for water allocation decisions related to streamflow
190 depletion (RRCA, 2003).

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 193 Figure 2. (left) Map showing Republican River Compact Administration MODFLOW model domain,
 194 hydrostratigraphic properties, and selected pumping wells for pumping experiment. (right) Properties of
 195 wells sampled for pumping experiment, which are described in detail in Section 2.3.1 of the text.

196 *2.2 Analytical depletion functions*

197 Analytical depletion functions are described in detail in Zipper et al. (2019b), so only a
 198 brief overview is presented here. Analytical depletion functions consist of three components
 199 (Figure 1):

- 200 (i) stream proximity criteria, which identify the stream segments that may be depleted by a
 201 well as a subset of the total stream network;
- 202 (ii) a depletion apportionment equation, which calculates the fraction (f_i) of the well's total
 203 depletion that is sourced from each stream segment (i) meeting the stream proximity
 204 criteria. For each well, f_i of all stream segments must sum to 1.0; and
- 205 (iii) an analytical model, which calculates the streamflow depletion for each stream segment
 206 without considering other stream segments (Qa_i). The calculation of Qa_i follows the
 207 typical use of analytical models which assume infinite stream length.

208 For that well, the estimated volumetric streamflow depletion in each segment, Qs_i , is then
 209 calculated as, $Qs_i = f_i * Qa_i$.

210 The inclusion of stream proximity criteria and depletion apportionment equations in
 211 analytical depletion functions are intended to empirically account for two major limitations of
 212 analytical models: typically analytical models only include one or a limited number of streams,
 213 and do not address stream sinuosity. Depletion apportionment equations that subdivide stream
 214 segments into multiple points, such as the web and web squared approaches developed by Zipper
 215 et al. (2018a), approximate the integral of streamflow depletion along a stream segment of finite
 216 length, which addresses the problematic analytical assumption of infinite stream length (Kollet et
 217 al., 2002). However, analytical depletion functions still include many of the assumptions of

218 analytical models, including that pumping does not change in recharge and therefore all pumped
219 water is sourced from either groundwater depletion or streamflow depletion. In settings where
220 recharge is unaffected by pumping, the spatial distribution of recharge does not influence the
221 streamflow depletion (Barlow & Leake, 2012; Bredehoeft et al. 1982; Bredehoeft 2002;
222 Feinstein et al., 2016).

223 Numerous options exist for stream proximity criteria, depletion apportionment equations,
224 and analytical models. These components were systematically evaluated in Zipper et al. (2019b)
225 via comparison to an uncalibrated numerical model of the Navarro River Watershed (California,
226 USA). Zipper et al. (2019b) found that analytical depletion function performance was most
227 sensitive to the choice of depletion apportionment equation, secondarily sensitive to the choice of
228 stream proximity criteria, and relatively insensitive to the choice of analytical model.

229 In this study, we will compare a subset of 8 of the 50 combinations evaluated by Zipper
230 et al. (2019b). Since Zipper et al. (2019b) found the greatest sensitivity of model performance to
231 the choice of depletion apportionment equation, we compared four unique but related depletion
232 apportionment equations here: web and web squared, which were the two best-performing
233 equations for the Navarro River Watershed, CA (Zipper et al. 2019b); and inverse distance and
234 inverse distance squared, the former of which was the best-performing equation for the
235 Kalamazoo Valley, MI (Reeves et al. 2009). All four depletion apportionment equations can be
236 expressed as:

$$f_i = \frac{\sum_{p=1,P_i} \frac{1}{d_{i,p}^w}}{\sum_{j=1,n} \left(\sum_{p=1,P_j} \frac{1}{d_{j,p}^w} \right)} \quad \{\text{Eq. 1}\}$$

237 The result, f_i is the fraction of total depletion occurring in stream segment i . Required inputs
238 include d , the distance from the well to the stream segment; w , a weighting factor that is equal to
239 1 for inverse distance and web and equal to 2 for inverse distance squared and web squared; n is
240 the total number of affected stream segments identified by the stream proximity criteria
241 (turquoise lines in Figure 1b); and P is the number of points each stream segment is divided into
242 (black dots in Figure 1b). The inverse distance and inverse distance squared methods are
243 simplified formulations of Eq. 1 where only the closest point to the well on each stream segment
244 is used and therefore $P = 1$.

245 Since stream proximity criteria were of secondary importance, we compared two stream
246 proximity criteria: adjacent catchments only, which was used by Reeves et al. (2009), and
247 adjacent+expanding, which Zipper et al. (2019b) found worked best in the Navarro River
248 Watershed. The adjacent+expanding methods includes both adjacent catchments to the well and
249 any catchments in which the estimated streamflow depletion would be greater than or equal to
250 1% of the pumping rate at a given timestep. As a result, the number of stream segments meeting
251 the stream proximity criteria increases over time.

252 We tested a single, relatively simple Glover and Balmer (1954) analytical model (herein
253 referred to as the Glover model; Eq. 2):

$$Qa = Qw * \operatorname{erfc} \left(\sqrt{\frac{Sd^2}{4Tt}} \right) \quad \{\text{Eq. 2}\}$$

254 In Eq. 2, Qw is the pumping rate of the well; S is the storativity, typically specific yield or
 255 specific storage; T is the transmissivity; and t is the time since pumping began. To account for
 256 monthly variation in Qw , we used standard superposition techniques to turn the wells on and off
 257 (Jenkins 1968). The Glover model contains numerous simplifying assumptions which are
 258 violated in the RRCA domain, including a stream fully penetrating to the bottom of the aquifer, a
 259 single linear stream, an aquifer of infinite extent extending away from the stream, and no
 260 changes in transmissivity in response to pumping. Since Zipper et al. (2019b) found minimal
 261 sensitivity to the choice of analytical model and the RRCA MODFLOW model represents
 262 surface water using a mixture of packages, some of which do not include streambed conductance
 263 as an input which is required for more complex analytical models such as Hunt (1999), we did
 264 not test multiple analytical models in this study. However, many analytical models exist with
 265 different formulations (reviewed in Huang et al. 2018) and in other hydrogeological settings,
 266 different analytical models may be appropriate.

267

268 *2.3 Analytical depletion function performance evaluation*

269 *2.3.1 Selecting pumping well sample*

270 The goal of our study is to examine the performance of analytical depletion functions
 271 relative to a MODFLOW model over a range of hydrogeological and physiographic
 272 characteristics. Therefore, we selected a subset of 166 wells (of 9372 total pumping wells in the
 273 domain) based on the following characteristics: (i) the mean pumping rate, (ii) pre-development
 274 water table depth from MODFLOW steady-state output, (iii) the transmissivity of the
 275 MODFLOW cell containing the well (log-transformed), (iv) the storativity of the MODFLOW
 276 cell containing the well, (v) the distance from the well to the closest surface water feature (active
 277 STR, DRN, or CHB grid cell), and (vi) the distance from the well to the closest cell with
 278 potential phreatophytic ET (active EVT cell). The distribution of these properties among our
 279 experimental sample is shown in Figure 2 and the distribution among all wells is shown in Figure
 280 S7. To span each of these characteristics as uniformly as possible, we used the Latin Hypercube
 281 Sampling method (McKay et al. 1979) to randomly sample 250 parameter combinations
 282 spanning these six characteristics (Zipper et al. 2018b). We then selected the pumping well that
 283 had the shortest euclidean distance in parameter space to each parameter sample, resulting in a
 284 total of 166 unique pumping wells in our final evaluation because some wells were closest to
 285 multiple samples. Several of the selected wells are spatially close to each other because they are
 286 in locations that have relatively rare parameter conditions within the multi-dimensional
 287 parameter space, such as high specific storage (Figure S7), and therefore multiple nearby wells
 288 were selected to effectively sample a wide range of parameter combinations.

289

290 2.3.2 Calculating streamflow depletion in MODFLOW model

291 To calculate the streamflow depletion associated with each of these pumping wells in the
292 MODFLOW model, we first ran a baseline RRCA simulation to calculate the stream-aquifer flux
293 under baseline conditions for each cell containing a surface water feature. The cell-resolution
294 stream-aquifer fluxes were then aggregated to net stream-aquifer fluxes for each stream segment,
295 which were defined (for the STR package) or manually delineated based on stream network
296 geometry (for the DRN and CHB packages). Since many of the DRN cells were discontinuous
297 (Figure 2) and represent springs or seeps rather than stream channels, each DRN segment was
298 delineated as a cluster of nearby DRN cells. To evaluate the sensitivity of the calculated
299 performance metrics to the inclusion of these features, we performed analytical depletion
300 function calculations both including and excluding DRN cells. We then turned off each of the
301 selected wells one-at-a-time for 166 unique numerical experiments (which we refer to as
302 ‘pumping simulations’ herein) and calculated the net stream-aquifer flux for each stream
303 segments in each pumping simulation. Turning off each well one-at-a-time and comparing to the
304 baseline simulation isolates the amount of streamflow depletion caused by that specific well.
305 Quantitatively, the decrease in stream-aquifer flux in the baseline simulation relative to a
306 pumping simulation is equal to the streamflow depletion caused by that pumping well (and
307 potential numerical model error), and therefore we can calculate streamflow depletion in each
308 segment and at each stress period for each of our 166 pumping wells. We automated
309 MODFLOW runs using the FloPy package for Python (Bakker et al. 2016, 2018).

310 Since changing groundwater model boundary conditions (such as pumping rates) can
311 impact model convergence and stability, we screened MODFLOW output for anomalous results
312 prior to comparison. We used two approaches to screen for anomalous results in all MODFLOW
313 simulations, each of which corresponds to a single pumping well. First, we identified additive
314 outliers in the timeseries of the difference in overall mass balance error between the pumping
315 simulation and the baseline simulation (Chen and Liu 1993; López-de-Lacalle 2019). Second, we
316 identified any stress period where MODFLOW estimated negative streamflow depletion either in
317 an individual segment or summed across all segments exceeding 1% of the maximum pumping
318 rate for that well. For both of these comparisons, we identified the stress periods at which
319 anomalous MODFLOW results occurred and limited our comparison to the time period between
320 the onset of pumping and the stress period prior to the first anomalous MODFLOW mass
321 balance change (Figure S8). Mass balance outliers indicating convergence issues were identified
322 for 31 of the 166 pumping experiments tested, occurring as early as the 499th stress period and
323 as late as the final (996th) stress period and stress periods with anomalous results from these 31
324 pumping simulations were removed from analysis as described above. For all other pumping
325 experiments, our comparison was conducted between the onset of pumping and the end of the
326 model simulation period (December 2000).

327

328 2.3.3 Calculating streamflow depletion with analytical depletion functions

329 Input for the analytical depletion functions (the formulations of which are described in
330 Section 2.1) was extracted directly from the RRCA MODFLOW model, described in Section
331 2.1. Using consistent parameters between the two approaches was intended to focus our
332 comparison on the impact of the simplifications of analytical depletion functions relative to the
333 numerical model on predicted streamflow depletion, since we do not do a direct comparison to
334 field observations of streamflow depletion. The pumping rate (Q_w) was extracted from the
335 MODFLOW WEL package (Figure S3), with each grid cell representing a unique pumping well.
336 The well-stream geometry (d) was extracted as the distance from each well to each grid cell with
337 a STR, CHB, and DRN cell, which were grouped into the same segments used by the
338 MODFLOW model which are described in Section 2.3.2. Due to the discretization of the
339 MODFLOW model, the point spacing for the discretization of segments in the web and web
340 squared depletion apportionment equations was 2.6 km^2 (1 mi^2). Effective transmissivity (T) and
341 storativity (S) were calculated as the average T and S for all MODFLOW cells intersected by a
342 straight line between the well and each stream segment. For S , we used specific yield, rather than
343 specific storage, as input to the analytical depletion functions because the use of specific storage
344 in the RRCA model is an artifact of model design, as described in Section 2.1. However, as an
345 experiment to test the importance of the storativity approach, we also ran a set of analytical
346 depletion function calculations using specific storage from the MODFLOW model. The output
347 from the analytical depletion functions is the estimated streamflow depletion in each stream
348 segment caused by each well throughout the entire RRCA simulation period.

349

350 2.3.4 Comparison between analytical depletion functions and MODFLOW

351 To systematically assess different aspects of analytical depletion function performance,
352 we calculated four fit metrics which are described below. To assess the drivers of analytical
353 depletion function performance, we conducted a regional sensitivity analysis for each of the fit
354 metrics in response to each of the well and landscape characteristics used to define the well
355 sample (Figure 2; Section 2.3.1). The regional sensitivity analysis is meant to identify conditions
356 under which the performance of analytical depletion functions exceeds a defined performance
357 threshold (Spear and Hornberger 1980; Wagener et al. 2001; Pianosi et al. 2016), which we set
358 separately for each of the four fit metrics:

- 359 1. *Spatial distribution of primary impact*: The percentage of pumping simulations in which
360 the stream segment most affected by groundwater pumping matched between the
361 analytical depletion functions and MODFLOW model. For regional sensitivity analysis, a
362 threshold value of 50% was used to separate good (match $\geq 50\%$) from poor (match $<$
363 50%) performance.
- 364 2. *Magnitude of primary impact*: The mean absolute difference (MAD) between the
365 volumetric depletion (Q_s) in each stream segment predicted by the analytical depletion
366 function and MODFLOW model, normalized to the range of predicted Q_s values from

367 MODFLOW. The normalization allows for comparison across a range of depletion
368 conditions - for instance, a difference in predicted Q_s of $100 \text{ m}^3 \text{ d}^{-1}$ is more problematic
369 when actual Q_s is $200 \text{ m}^3 \text{ d}^{-1}$ than when actual Q_s is $5000 \text{ m}^3 \text{ d}^{-1}$. For regional sensitivity
370 analysis, a threshold value of 0.25 was used to separate good (normalized $\text{MAD} \leq 0.25$)
371 from poor (normalized $\text{MAD} > 0.25$) performance. Note that we use the term MAD,
372 rather than Mean Absolute Error, because differences between the MODFLOW model
373 and analytical depletion functions may be caused by errors in either of the two
374 approaches.

- 375 3. *Spatial distribution of overall impacts:* The Kling-Gupta Efficiency (KGE) between the
376 volumetric depletion (Q_s) in each stream segment predicted by the analytical depletion
377 function and MODFLOW model. KGE is a hydrological performance indicator which
378 accounts for differences in correlation, variability, and bias (Gupta et al. 2009; Kling et
379 al. 2012). Similar to the Nash-Sutcliffe Efficiency, a KGE value of 1.0 indicates perfect
380 agreement and lower values indicate worse agreement. As a benchmark, $\text{KGE} > -0.41$
381 indicates better agreement than simply using the mean (Knoben et al. 2019). For regional
382 sensitivity analysis, a threshold value of -0.41 was used to separate good ($\text{KGE} \geq -0.41$)
383 from poor ($\text{KGE} < -0.41$) performance.
- 384 4. *Magnitude of overall impacts:* The bias between the total streamflow depletion across all
385 segments for an individual well simulated by the analytical depletion function and the
386 MODFLOW model, normalized to the range of predicted total streamflow depletion from
387 MODFLOW. For regional sensitivity analysis, a threshold value of 75% was used to
388 separate good (absolute bias $\leq 75\%$) from poor (absolute bias $> 75\%$) performance.
389

390 This study focused on a comparison between the analytical depletion functions and the
391 RRCA MODFLOW model because of the lack of large-scale streamflow depletion estimates. At
392 the scale of an individual stream segment, analytical approaches can be evaluated via controlled
393 field experiments (Hunt et al., 2001; Kollet & Zlotnik, 2003). However, at large spatial scales
394 streamflow depletion is obscured by interannual variability in weather, lag times between
395 pumping and depletion, and other management activities such as surface water withdrawals
396 (Gleeson & Richter, 2018), and therefore calibrated numerical models are the preferred approach
397 to quantify streamflow depletion (Barlow & Leake, 2012). Despite the limitations of the RRCA
398 model (described in Section 2.1), the RRCA model was calibrated via comparison to historical
399 baseflow data and therefore represents the best available estimates of streamflow depletion, and
400 has been widely used for previous scientific studies (de Graaf et al., 2019; Hu et al., 2015, 2017;
401 Ou et al., 2018). However, we acknowledge the potential for error in both the RRCA models and
402 analytical depletion functions, and therefore our study focuses on agreement and differences
403 between the two approaches.
404

405 **3 Results and Discussion**

406 *3.1 Overall performance*

407 Overall, we find a strong agreement between the analytical depletion functions and
408 MODFLOW predictions of streamflow depletion. Variability among analytical depletion
409 function formulations is explored in the following section. The best-performing analytical
410 depletion function combined the adjacent+expanding stream proximity criteria, the web squared
411 depletion apportionment equation, and the Glover analytical model, which agrees with the results
412 from a previous comparison in the Navarro River Watershed (Zipper et al. 2019b).

413 There is strong agreement between this analytical depletion function and the MODFLOW
414 model across our four performance criteria (Table 1). Over the final 20 years of the simulation
415 period, the most-affected stream segment is identified correctly for 53.9% of pumping wells, the
416 MAD of predicted depletion in the most-affected segment is 0.048 of the range in predicted
417 depletion, the KGE of predicted depletion across all segments is 0.779, and the bias for predicted
418 total streamflow depletion is 0.4%. Analytical depletion function is significantly better than
419 using a standalone analytical model (without stream proximity criteria or depletion
420 apportionment equations) for all metrics except the identification of the most affected stream
421 segment (Table 1), highlighting the ability of analytical depletion functions to improve
422 predictions for real-world settings compared to a standalone analytical model.

423 Identification of the most-affected segment is substantially lower than previous work,
424 which was generally >70% correct in the Navarro River Watershed (Zipper et al. 2019b). Results
425 from the standalone analytical model, which always used the stream segment closest to each
426 well, had the same skill in identifying the most-affected segment (53.9%) which indicates that
427 for 46.1% of wells the most-affected stream segment was not the closest stream segment to the
428 well. This may be driven by the fact that well-stream distances and numerical model
429 discretization are an order of magnitude larger in this domain compared to the Navarro River
430 watershed, and therefore subsurface controls on flow such as spatial heterogeneity in T and S
431 exert a stronger control over the distribution of pumping impacts.

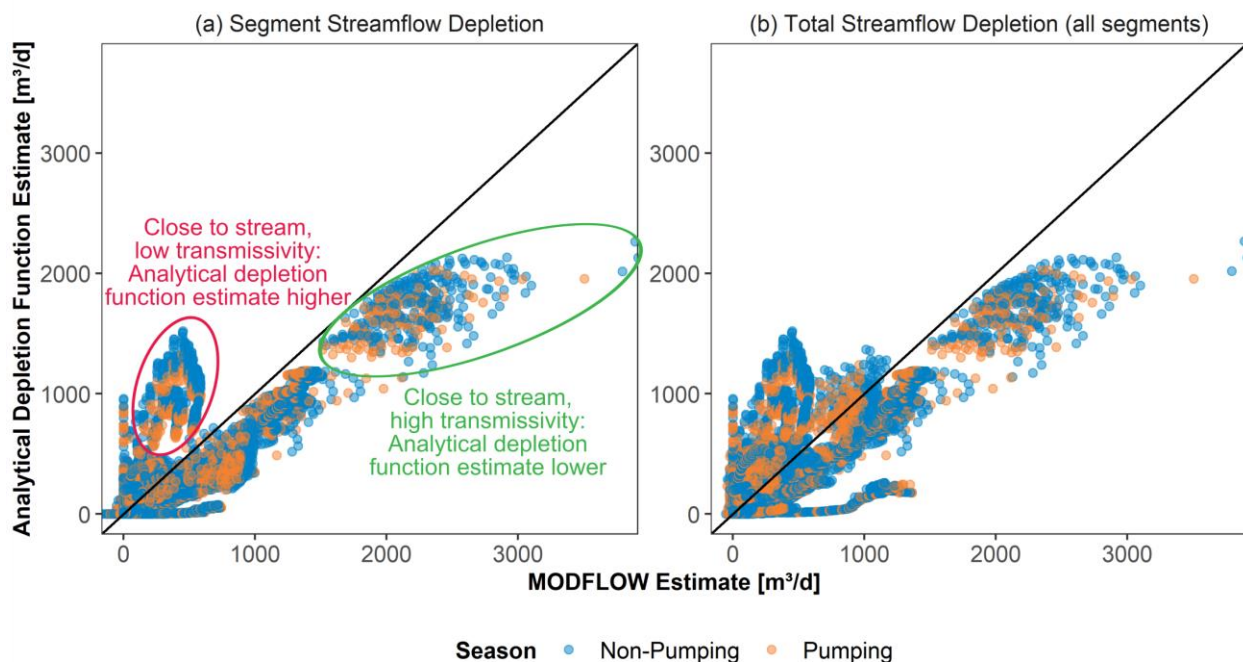
432 Despite a negligible overall bias, this analytical depletion function tends to have a higher
433 estimate than MODFLOW for both segment-resolution and total streamflow depletion at low
434 magnitudes and a lower estimate at high magnitudes (Figure 3). The largest differences between
435 MODFLOW and analytical depletion function estimates tend to be driven by a relatively small
436 number of wells which are located near the edge of the domain, which is consistent for both
437 segment-resolution and total streamflow depletion. The cluster of points in which the analytical
438 depletion function produces substantially higher estimates of depletion compared to MODFLOW
439 are associated with two wells on the southeastern margin of the domain that have several
440 extreme characteristics relative to the overall well sample. These wells are the minimum possible
441 distance from surface water cells given the model discretization (one grid cell, or 1.6 km) yet
442 relatively far from evapotranspiration cells (>45 km, or 85th percentile among well sample), and
443 also have very low transmissivity ($\sim 50 \text{ m}^2 \text{ d}^{-1}$, or 6th percentile). In contrast, the points in which

444 analytical depletion functions have the largest underestimate relative to MODFLOW are
445 associated with two wells on the northern edge of the domain that are also extremely close to a
446 stream (two grid cells, or 3.2 km) but have a very high transmissivity ($\sim 1900 \text{ m}^2 \text{ d}^{-1}$, or 95th
447 percentile).

448 The primary differences between segment-resolution streamflow depletion (Figure 3a)
449 and total streamflow depletion caused by a well (Figure 3b) occur at low magnitudes of
450 depletion. As discussed above, all points with high magnitudes of depletion tend to be very close
451 to a surface water feature of some sort and therefore depletion is dominated by a single segment.
452 In contrast, wells causing low levels of depletion tend to be far from stream segments and
453 therefore depletion is distributed throughout more segments, but remains low even when the
454 depletion is added together across all segments.

455 While there is variability in bias among analytical depletion functions (Table 1), none of
456 the analytical depletion function formulations predict substantially higher depletion than the
457 best-performing analytical depletion function (Figure S9 and Figure S10), indicating that the
458 underestimate in depletion at high magnitudes (Figure 3) may be a persistent problem for
459 analytical depletion functions in this setting. This finding is in contrast to the typical assumption
460 that analytical approaches provide conservative estimates of streamflow depletion (Sophocleous
461 et al. 1995; Rathfelder 2016) and may be problematic from a water management perspective
462 because underestimating the depletion from the wells with the largest impacts could lead to an
463 overallocation of water resources (Zipper et al. 2018a). As a result, analytical depletion functions
464 should not be considered a “worst-case” estimate of depletion, but rather a minimally-biased
465 estimate which may overestimate or underestimate depletion relative to the MODFLOW model.

466
467



468

469 Figure 3. Comparison between analytical depletion function and MODFLOW predictions of (a) segment-
 470 scale streamflow depletion, (b) total streamflow depletion summed across all segments for a given well.
 471 Circles on depletion plot indicate ‘higher estimate’ and ‘lower estimate’ points discussed in Section 3.1
 472 text.

473
 474 Table 1. Analytical depletion function performance for different fit metrics, calculated for each month of
 475 simulations and averaged over the final 20 years of the simulations. All of the analytical depletion
 476 functions shown in this table used specific yield for the storage parameter and included DRN features.

Stream Proximity Criteria	Depletion Apportionment Equation	Spatial distribution of primary impact [% most-affected correct]	Magnitude of primary impact [MAD segment streamflow depletion, normalized]	Spatial distribution of overall impacts [KGE, segment streamflow depletion]	Magnitude of overall impacts [% bias, total streamflow depletion]
Adjacent	Inverse Distance	50.7	0.056	0.701	-8.0
Adjacent	Inverse Distance Squared	50.8	0.054	0.695	15.1
Adjacent	Web	52.4*	0.045*	0.699	-20.9
Adjacent	Web Squared	53.9*	0.048	0.767*	4.2
Adjacent + Expanding	Inverse Distance	50.0	0.057	0.697	-18.0
Adjacent + Expanding	Inverse Distance Squared	50.1	0.053	0.737	9.2
Adjacent + Expanding	Web	52.5*	0.047	0.671	-26.5
Adjacent + Expanding	Web Squared	53.9*	0.048	0.779*	0.4*
Analytical Only	Analytical Only	53.9*	0.06	0.555	32.5

477 *Bold and starred values in each column indicate analytical depletion functions which are not significantly different
 478 from the best-performing function for that metric ($p > 0.05$ using Tukey’s honest significant difference test).
 479

480 3.2 Performance response to analytical depletion function formulation and input data

481 Analytical depletion formulation had relatively little impact on most of the model
 482 performance metrics. Comparing among stream proximity criteria, there is little difference
 483 between the Adjacent and Adjacent + Expanding stream proximity criteria (Table 1), likely due
 484 to the large size of the domain and relatively sparse stream network compared with Zipper et al.
 485 (2019b). Comparing among depletion apportionment equations, the inverse distance and inverse
 486 distance squared depletion apportionment equations do not perform the best for any of the fit
 487 metrics evaluated (Table 1) indicating that considering stream network geometry with the web

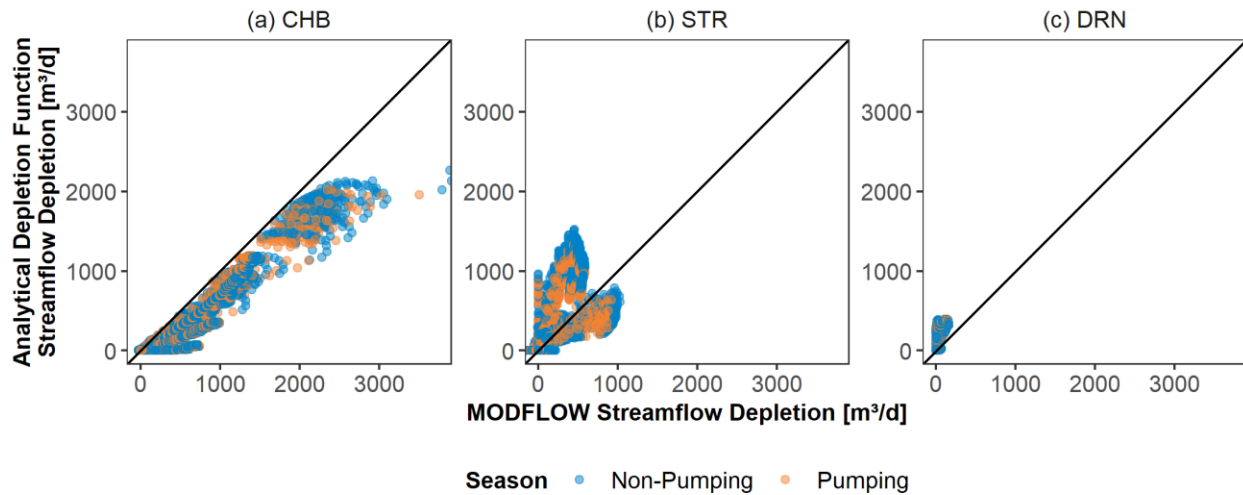
488 and web squared improves performance, though differences between approaches are only a few
489 percentage points. Notably, the web squared depletion apportionment equation performed the
490 best at the spatial distribution of the overall impacts (regardless of stream proximity criteria
491 used), demonstrating its effectiveness at identifying streamflow depletion across a stream
492 network. The superior overall performance of the web squared depletion apportionment equation
493 relative to the web equation is due to a negative bias for wells with high amounts of total
494 streamflow depletion (Table 1; Figure S10). This is caused by the increased weight given to
495 near-well stream segments in the web squared approach (Zipper et al. 2018a).

496 Both the magnitude of predicted depletion and the relationship between MODFLOW and
497 analytical depletion functions varied as a function of the boundary condition used in the
498 MODFLOW model. In general, the highest levels of depletion tended to be predicted for the
499 constant head boundary, which runs along the north side of the model domain (Figure 2a), and
500 analytical depletion function estimates of depletion were consistently lower than MODFLOW
501 (Figure 4a). In contrast, predicted depletion from the stream package tended to be more evenly
502 distributed with a mixture of overestimates and underestimates relative to the MODFLOW
503 model (Figure 4b). Depletion from drain features (which are diffuse boundaries representing
504 springs in this model; Figure 2a) was small, but analytical depletion functions had consistently
505 higher estimates than MODFLOW (Figure 4c).

506 The differences in predicted depletion among these boundary conditions are likely driven
507 by a combination of hydrostratigraphy and MODFLOW model design. First, the constant head
508 boundaries are found along the north side of the model domain and this region has the highest
509 estimated transmissivity (Figure 2a) due to more conductive sediments and a greater saturated
510 thickness (RRCA 2003). These higher conductivity materials, along with the close proximity of
511 some wells to the stream (discussed in Section 3.1), explains why the largest depletion estimates
512 are found for the northern part of the domain along the constant head boundary. Second,
513 MODFLOW uses a streambed conductance term to simulate potential low-conductivity
514 streambed materials for stream and drain features but not for constant head boundaries. This
515 streambed layer is not represented in the Glover analytical model we use in our analytical
516 depletion functions. Use of an analytical model that includes streambed conductance, such as
517 Hunt (1999), may improve agreement for stream and drain boundary conditions, but would cause
518 further underestimation of depletion in constant head boundaries.

519 Since analytical models are not traditionally designed for use in diffuse discharge features
520 such as the springs represented by the drain package, we also compare analytical depletion
521 function performance with and without drain features (Figure S11). Removing drains had
522 relatively small impacts on model performance, particularly at high levels of depletion. Overall,
523 removing DRNs meant that estimates of depletion in DRN segments went to 0, and as a result
524 estimates of depletion in some other boundary conditions increased (Figure S11). Removing
525 DRN features from the analytical depletion functions had mixed impacts on our four
526 performance metrics. When DRNs were excluded from analytical depletion function
527 calculations, the identification of the most affected segments degraded (42.2%, compared to

528 53.9% when DRNs were included) and total streamflow depletion had a negative bias of -8.4%
529 (compared to 0.4% with DRNs). However, normalized MAD of depletion for the most affected
530 segment improved to 0.043 without DRNs (compared to 0.048 with DRNs) and KGE for
531 depletion of all stream segments rose to 0.811 without DRNs (compared to 0.779 with DRNs).
532

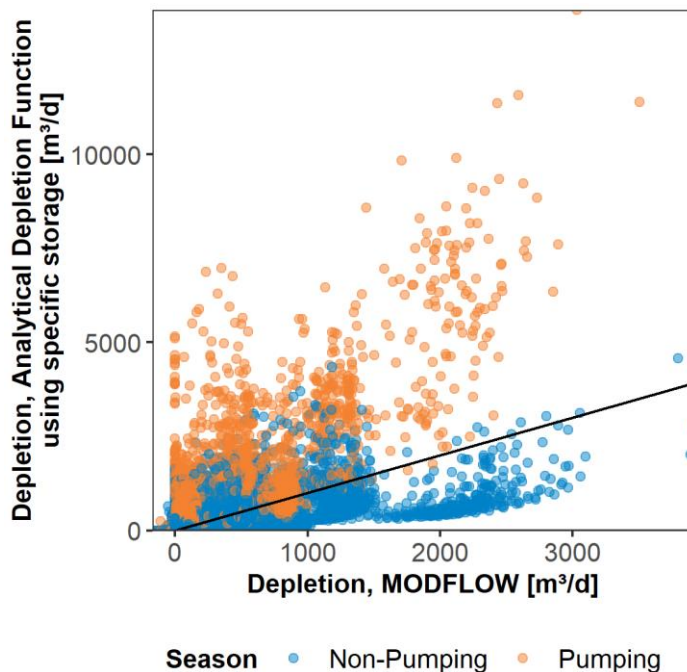


533
534 Figure 4. Comparison between analytical depletion function and MODFLOW predictions of segment-
535 scale streamflow depletion for (a) constant head boundary package, (b) stream package, and (c) drain
536 package in MODFLOW.

537 The storage parameter used in analytical depletion functions has a substantial impact on
538 depletion predictions. While specific yield is typically used to represent storage in unconfined
539 aquifers, the RRCA MODFLOW model uses a specific yield-based approximation of specific
540 storage since the model assumes a constant transmissivity (details in Section 2.1). Recent work
541 found that accurate estimates of specific yield are critical to obtain accurate estimates of
542 groundwater depletion in the High Plains Aquifer, including parts of the RRCA domain (Butler
543 et al. 2020). To assess the importance of using appropriate storage parameterizations in the
544 analytical depletion functions, we compared our best-performing analytical depletion function in
545 Table 1 (adjacent + expanding stream proximity criteria and web squared depletion
546 apportionment equation) with both specific yield (as in Table 1) and specific storage as input.
547 Using specific storage instead of specific yield led to substantially worse analytical depletion
548 function performance for three of our performance metrics. Normalized MAD of depletion for
549 the most affected segment rose to 0.130 with specific storage (compared to 0.048 with specific
550 yield), KGE for depletion of all stream segments declined to -3.40 with specific storage
551 (compared to 0.779 with specific yield), and percent bias for total streamflow depletion increased
552 to 431% with specific storage (compared to 0.4% with specific yield).

553 The degradation in performance when analytical depletion functions use specific storage
554 as input is primarily driven by a systematic overestimation of depletion relative to the
555 MODFLOW model during the pumping season (Figure 5). This occurs because specific storage
556 values in the RRCA MODFLOW model were defined by dividing specific storage by the

557 saturated thickness, causing them to be 1-2 orders of magnitude lower than specific yield values.
 558 The lower specific storage values cause the streamflow depletion to occur much more quickly
 559 after the onset of pumping, and confirm the inappropriateness of using specific storage as input
 560 for analytical depletion functions in unconfined settings where specific yield is more appropriate.
 561 Combined, our analysis indicates that data collection efforts should prioritize high-accuracy
 562 estimates of transmissivity and storativity to improve accuracy of both streamflow depletion and
 563 groundwater depletion predictions, since hydraulic diffusivity (the ratio of storativity to
 564 transmissivity) is fundamental to the timing and magnitude of streamflow depletion (Barlow &
 565 Leake, 2012). In areas with high-quality water use and water level data, emerging data-driven
 566 approaches may be a valuable tool for improving storativity estimates (Whittemore et al. 2016;
 567 Butler et al. 2016).
 568

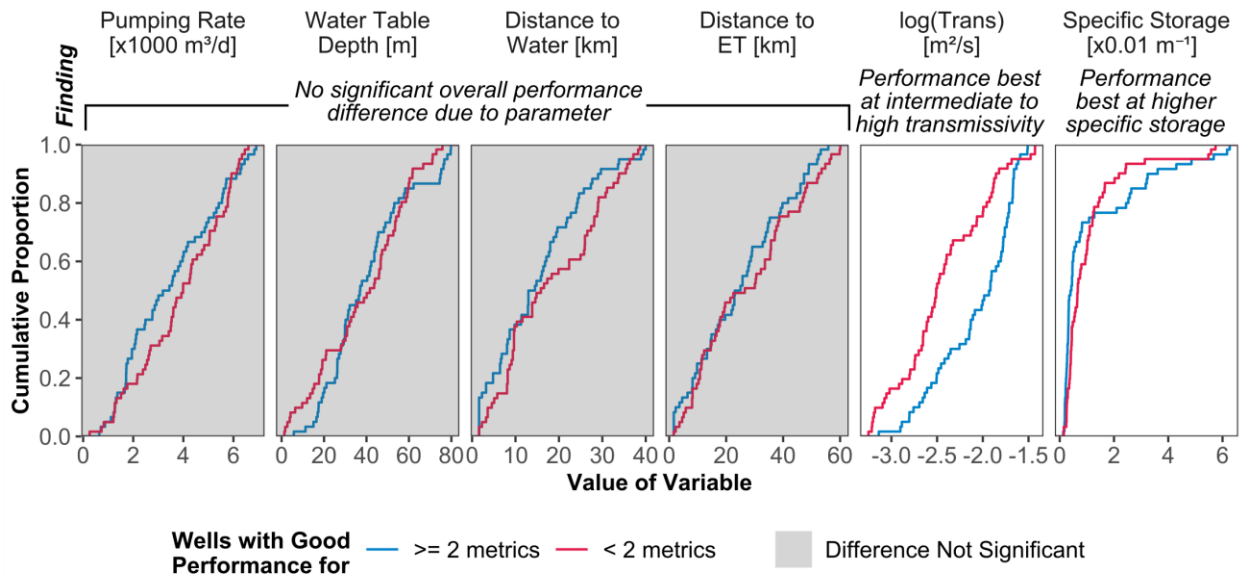


569
 570 Figure 5. Comparison of predicted depletion from analytical depletion function using specific storage to
 571 MODFLOW estimates.

572 *3.3 Well and landscape drivers of performance variability*

573 Using the threshold defined in Section 2.3.4 for each performance metric, we found that
 574 13 wells had good performance for all four metrics, 13 wells had good performance for three
 575 metrics, 33 wells had good performance for two metrics, 38 wells had good performance for a
 576 single metric, and 23 wells had good performance for no metrics. To investigate the relative
 577 importance of each parameter with comparable sample sizes, we compared wells with good
 578 performance in at least two fit metrics to wells with good performance in less than two fit
 579 metrics (Figure 6). Differences in the empirical cumulative distribution functions between the
 580 two samples for a given parameter indicates a potentially significant impact of that parameter on

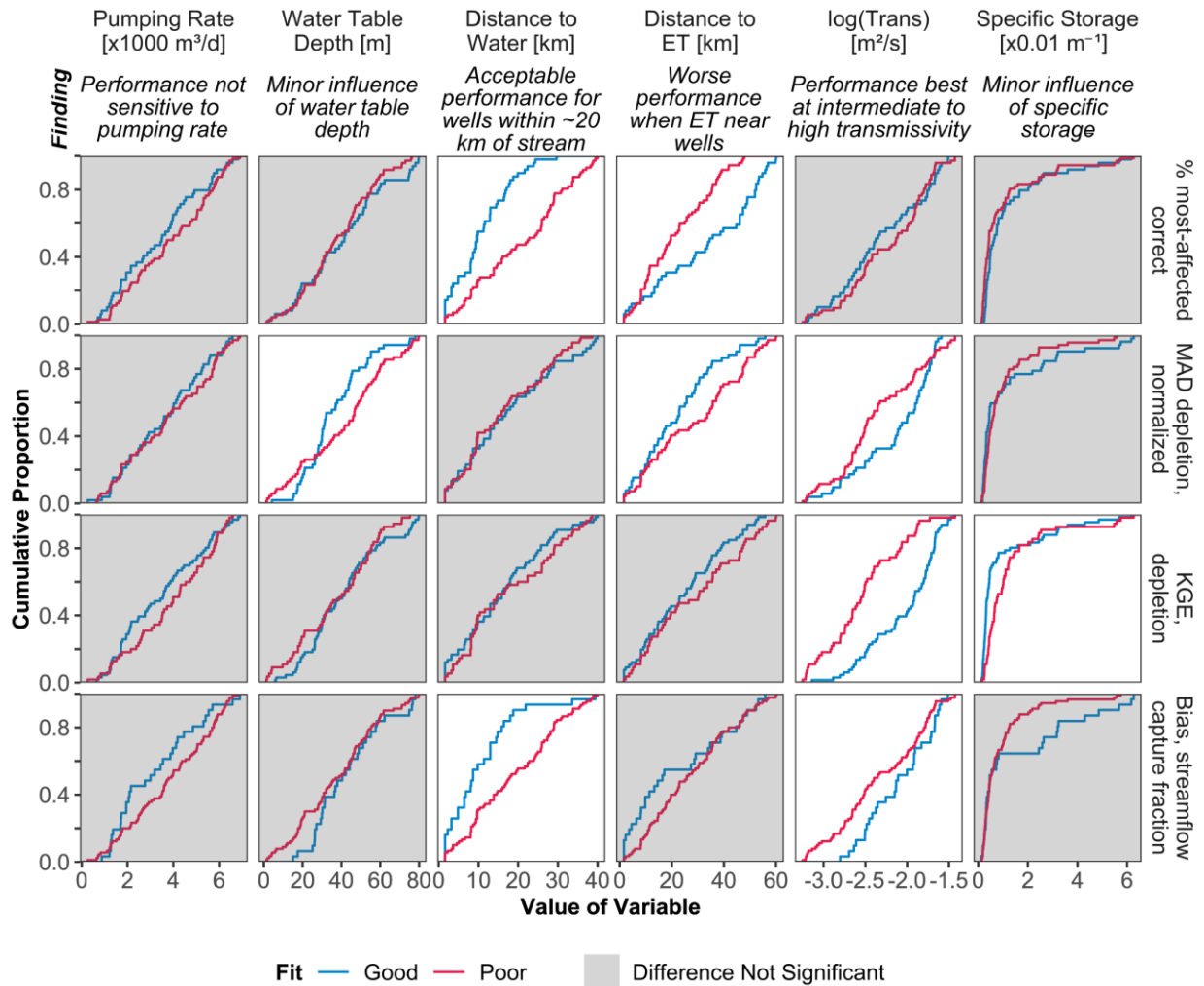
581 overall analytical depletion function performance (Wagener et al. 2001; Pianosi et al. 2016).
 582 Comparing across all fit metrics concurrently, the only two parameters that led to significant
 583 differences between these two samples were the hydrostratigraphic properties of transmissivity
 584 and specific storage. Wells tended to have better performance at intermediate to high values of
 585 transmissivity, agreeing with the observed drivers of over- and underestimated depletion (Figure
 586 3). Specific storage has a strongly skewed relationship in our domain, but performance tended to
 587 be better when specific storage was higher.
 588



589
 590 Figure 6. Regional sensitivity analysis results, expressed as empirical cumulative distribution functions
 591 for wells with good performance in at least two fit metrics and wells with good performance in less than 2
 592 fit metrics. In shaded panels, distributions of the two samples are expected to be drawn from the same
 593 distribution (two-sample Kolmogorov-Smirnov test $p > 0.05$).

594 Investigating individual fit metrics, the regional sensitivity analysis found a significant
 595 impact of all of the well and landscape properties except pumping rate on at least one of the fit
 596 metrics, but no well or landscape property had a significant impact on all fit metrics (Figure 7).
 597 For pumping rate, none of the fit metrics differed significantly between wells with a good and
 598 poor fit, which supports the long-held assumption that streamflow response to pumping is
 599 independent of abstraction rate (Theis 1941; Glover and Balmer 1954; Hunt 1999).
 600 Transmissivity affected the most fit metrics, with intermediate transmissivity values associated
 601 with improved prediction of depletion (MAD in the most-affected segment and KGE in all
 602 segments) and bias. The difference between the ‘good’ and ‘poor’ wells at intermediate values of
 603 transmissivity further supports the observation that performance degrades in extremely high and
 604 low transmissivity settings near streams (Figure 3), and that variability in hydraulic conductivity
 605 is an important control over analytical depletion function accuracy (Sophocleous et al. 1995; Li
 606 et al. 2016). Unlike the overall assessment (Figure 6), looking at individual fit metrics revealed
 607 only a minor sensitivity to specific storage for one fit metric, the KGE of segment-resolution

608 streamflow depletion, where good performing wells had a lower specific storage (Figure 7). The
 609 specific storage distribution across the domain is strongly skewed (Figure 2b), and the
 610 differences indicate that assessing the relative impact and importance of a given parameter
 611 depends on the aspect of model performance being considered.
 612



613
 614 Figure 7. Regional sensitivity analysis results, expressed as empirical cumulative distribution functions
 615 for well characteristics for each performance criteria. In shaded panels, distributions of 'good' and 'poor'
 616 groups are expected to be drawn from the same distribution (two-sample Kolmogorov-Smirnov test $p >$
 617 0.05).

618 There was an apparent threshold-type response as distance to the closest surface water
 619 feature increased; the identification of the most-affected segment and the total streamflow
 620 depletion bias both degraded significantly at distances greater than ~20 km. For distance to cells
 621 with ET, which is an alternate source of capture in the MODFLOW model that is not considered
 622 by the analytical depletion functions, wells that performed poorly for identifying the most-
 623 affected segment and MAD of depletion were primarily concentrated at shorter distances, while
 624 the MAD of depletion estimates improved at further distances to ET. Since phreatophytic ET is

625 primarily concentrated along stream channels in the MODFLOW model (RRCA 2003), this
626 indicates a spatial interplay between streamflow and ET capture sources which merits future
627 investigation (Condon and Maxwell 2019). The water table depth had only a minor influence on
628 the fit metrics, with better depletion predictions at intermediate water table depths (~20-50 m). It
629 is important to note that we were assessing fit between the analytical depletion functions and the
630 MODFLOW model, not agreement with field observations (which are not available). As a result,
631 the division of fit into ‘Good’ and ‘Poor’ categories may be driven by errors in the MODFLOW
632 model instead of or in addition to errors in the analytical depletion functions, and potential errors
633 in the MODFLOW model may also vary in response to parameters evaluated here such as well-
634 stream distance.

635 While we did not use the distance to the edge of the model domain as one of the variables
636 guiding our well sample selection, we conducted a *post hoc* analysis to evaluate whether it had a
637 significant impact on performance. We found that analytical depletion functions more
638 successfully identify the most-affected segment and have an acceptable bias for wells that were
639 closer to no-flow boundaries (Figure S12). This response is very similar to the observed
640 influence of distance to the closest surface water feature (Figure 6) and we were not able to
641 isolate the impacts of these no-flow boundaries because they are often co-located with or near
642 surface water features (Figure 2). In aquifers of limited lateral extent, analytical models for
643 bounded aquifers (Huang et al., 2018) may be useful methods to integrate into analytical
644 depletion functions, but would need additional testing.

645 646 *3.4 Synthesis with previous analytical depletion function evaluations*

647 This work extends previous evaluations of analytical depletion functions by comparing
648 their output to a calibrated numerical model in a highly stressed aquifer, a setting where
649 analytical depletion functions have not previously been tested, and by systematically assessing
650 the influence of well and hydrostratigraphic characteristics on results. Synthesizing across
651 studies, we find general agreement that the adjacent + expanding stream proximity criteria and
652 the web squared depletion apportionment equation produce the best agreement with numerical
653 model output (Zipper et al. 2018a, 2019b). We also found performance was best when wells are
654 close to streams, a finding that is consistent with previous work in coastal California (Zipper et
655 al. 2019b) but in contrast to a study in British Columbia that found better agreement for wells
656 further from streams (Li et al. 2020). We also extend this previous work by testing performance
657 across 166 wells with a variety of pumping rates and demonstrated that performance is
658 insensitive to pumping rate, indicating that analytical depletion functions are likely to be equally
659 useful regardless of the magnitude of groundwater abstractions.

660 This study and previous work raise several key questions for further evaluation. First, we
661 identify a potential spatial performance-related interaction between the distance from the well to
662 the closest stream and closest ET cell (Figure 7). Additional testing is necessary to determine the
663 conditions in which phreatophytic ET confounds analytical depletion function estimates of
664 streamflow depletion (Condon and Maxwell 2019). Second, this and previous evaluations have

665 focused on evaluation of a single well in isolation. While the current study investigates
666 performance in the context of a heavily-stressed aquifer with many pumping wells, we isolated
667 effects of an individual well by turning wells on/off one-at-a-time. While it is widely assumed
668 that the output from analytical models is additive, work in the Republican River Watershed has
669 shown this may not be the case (Schneider et al. 2017). For application of analytical depletion
670 functions in heavily-stressed aquifers, systematic testing of cumulative impacts by evaluating the
671 impacts of multiple wells concurrently is critical. Finally, recent field investigations found that
672 analytical model performance in an urban setting varied as a function of stream stage (Flores et
673 al. 2020), and will be important to test analytical depletion functions in a variety of stream stage
674 conditions.

675 Given the low computational and data requirements of analytical depletion functions
676 relative to numerical models, they may be a particularly valuable tool for applications requiring
677 many streamflow depletion estimates under different conditions such as decision support tools,
678 time series analysis, and simulation-optimization management modeling. Huggins et al. (2018)
679 developed a workflow for integrating depletion apportionment equations and analytical models
680 into existing web-based water decision support tools. Further, analytical depletion functions
681 could be used to improve parameterization of pumping impacts in time series analysis
682 approaches, which typically require a head response function that is often based on analytical
683 methods (Bakker & Schaars, 2019; Obergfell et al., 2019; Shapoori et al., 2015). Finally,
684 simulation-optimization models require the ability to test many different management
685 approaches (Wagner, 1995; Singh, 2014). While this can be accomplished in relatively small
686 domains using numerical models (Fienen et al., 2018), analytical depletion functions may
687 complement other approaches such as metamodeling (Fienen et al., 2015) to provide estimates of
688 streamflow depletion under diverse scenarios and identify optimal management solutions. For all
689 of these potential applications, however, care should be taken to ensure that uncertainty and
690 limitations of analytical approaches are appropriately considered, quantified, and shared with
691 relevant stakeholders, so that decision-makers can determine whether the accuracy is sufficient
692 for their needs.

693

694 **4 Conclusions**

695 Reliable estimates of streamflow depletion are critical for effective conjunctive management of
696 groundwater and surface water resources. This study is the first systematic evaluation of
697 analytical depletion functions for use in a heavily-stressed unconfined aquifer, and assesses how
698 agreement between the numerical and analytical model varies as a function of well and
699 hydrostratigraphic characteristics. We found that analytical depletion functions can produce
700 similar estimates of streamflow depletion to an existing numerical model during both the
701 pumping and non-pumping seasons, though they tend to over- or underestimate depletion relative
702 to MODFLOW for wells very close to surface water features. Comparing among eight different
703 analytical depletion functions, we found relatively little sensitivity to analytical depletion
704 function formulation, but a strong response to the input storage parameter, indicating the critical

705 importance of reliable parameter estimates. Among the analytical depletion functions, the one
706 that performs most similarly to the numerical model included time-varying stream proximity
707 criteria and a depletion apportionment equation that accounted for stream network geometry,
708 which is consistent with previous studies. The analytical depletion function and numerical model
709 are most similar for wells within ~20 km of a stream and intermediate values of transmissivity,
710 with no sensitivity to pumping rate. These results do not suggest that numerical models should be
711 replaced or superseded by analytical depletion functions, but rather that analytical depletion
712 functions are a useful low-cost, low-effort approach to obtain comparable estimates of
713 streamflow depletion in settings where calibrated numerical models are not available.

714

715 **Acknowledgments and Data**

716 Data and code are available at <https://github.com/samzipper/RRCA-analytical> during the review
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724

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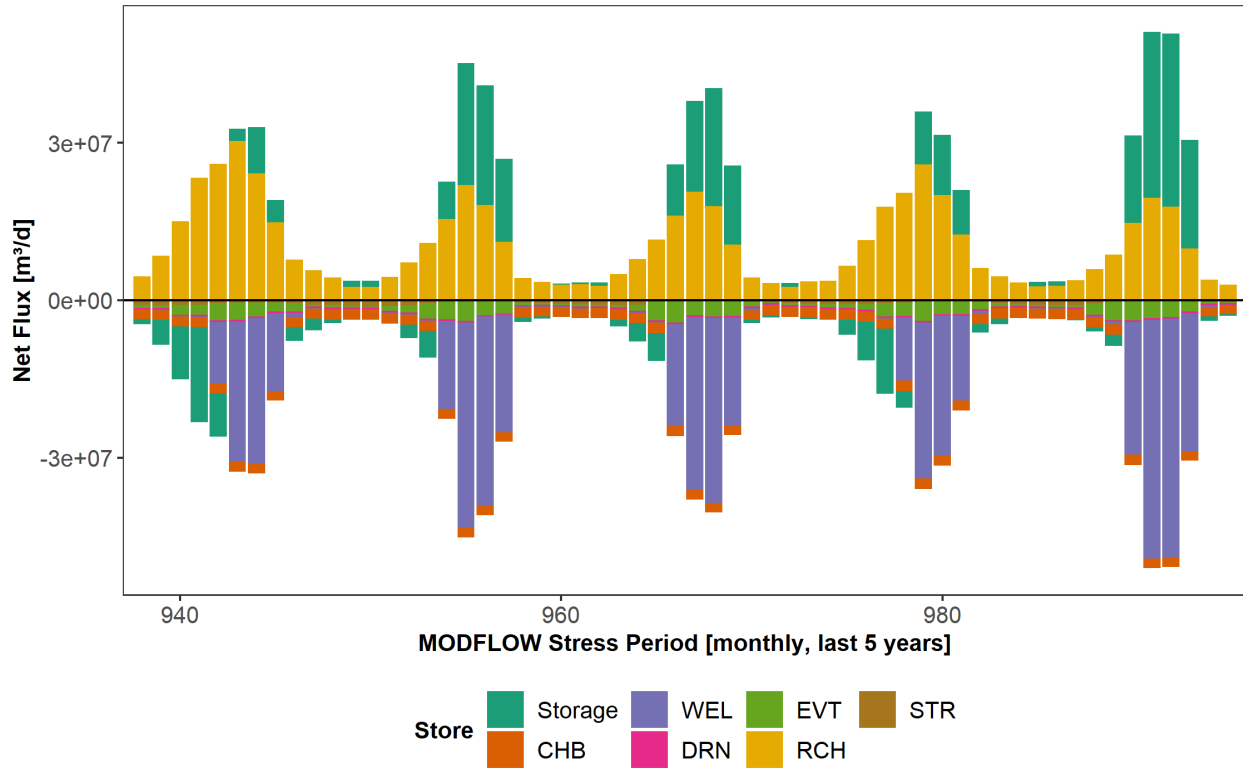
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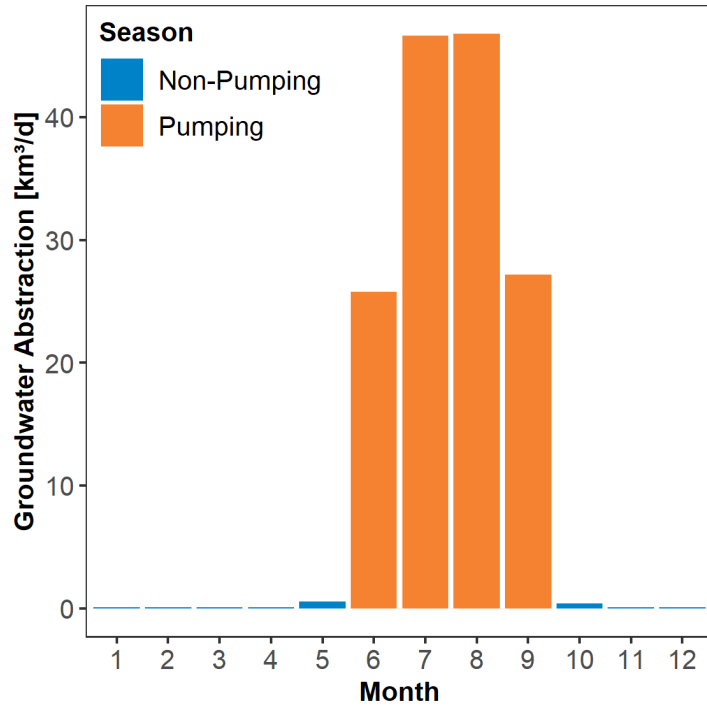
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923 **Supplemental Information**



924 **Figure S1.** Net water budget for baseline RRCA MODFLOW model over last 5 years of
925 simulation. The black line which is indistinguishable from 0 is the net error (inflow - outflows) at
926 each timestep.
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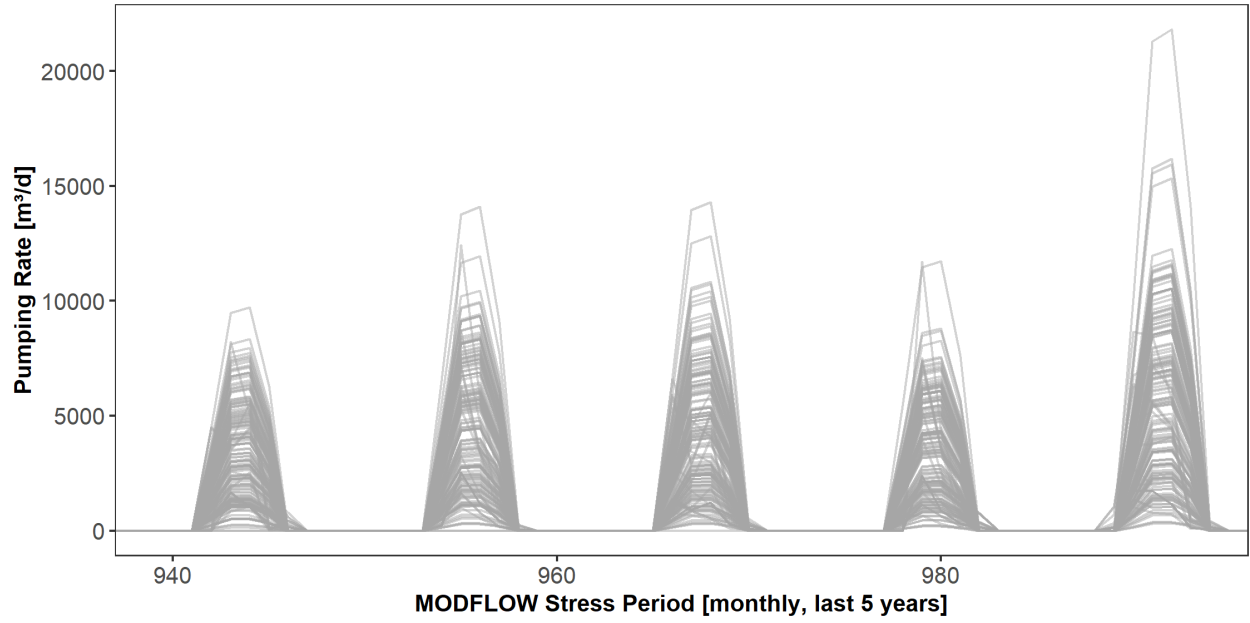
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Figure S2. Monthly total pumping by all wells within MODFLOW model domain for the final year of simulation, showing division of year into Pumping Season (June-September) and Non-Pumping Season (all other months).

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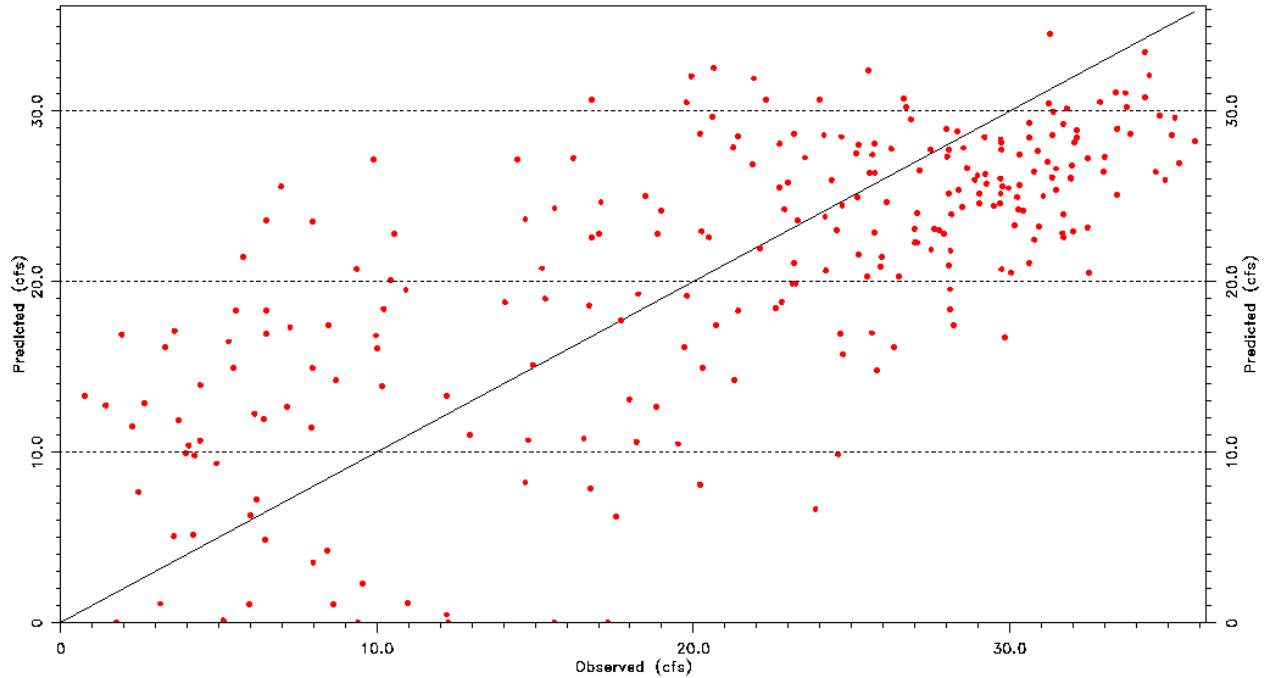
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Figure S3. Monthly pumping schedule for each of the 166 wells included in well sample for last 5 years of simulation.



South Fork Republican River Near Idalia, CO

Republican River Settlement Model Version 12p



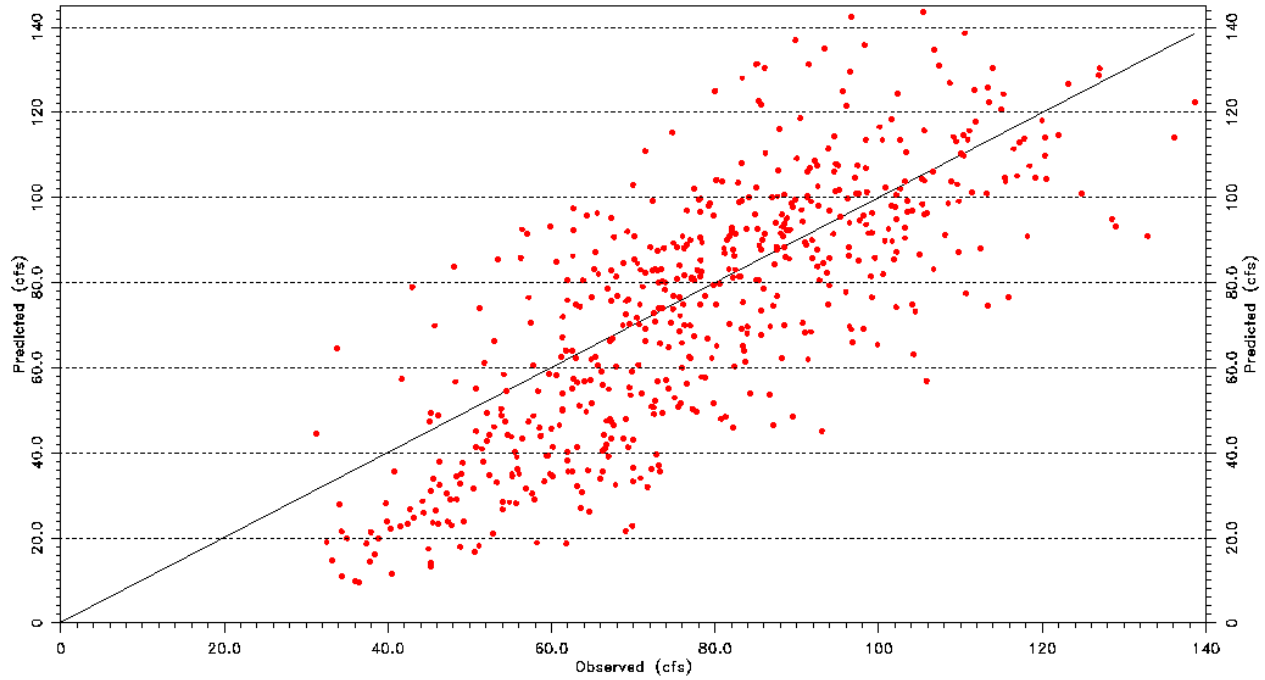
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Figure S4. RRCA model calibration from the South Fork Republican River near Idalia CO, which is the furthest point upstream for which calibration results are available. Source: <https://www.republicanrivercompact.org/v12p/html/bf/05c.html>



Republican River at Benkelman, NE

Republican River Settlement Model Version 12p



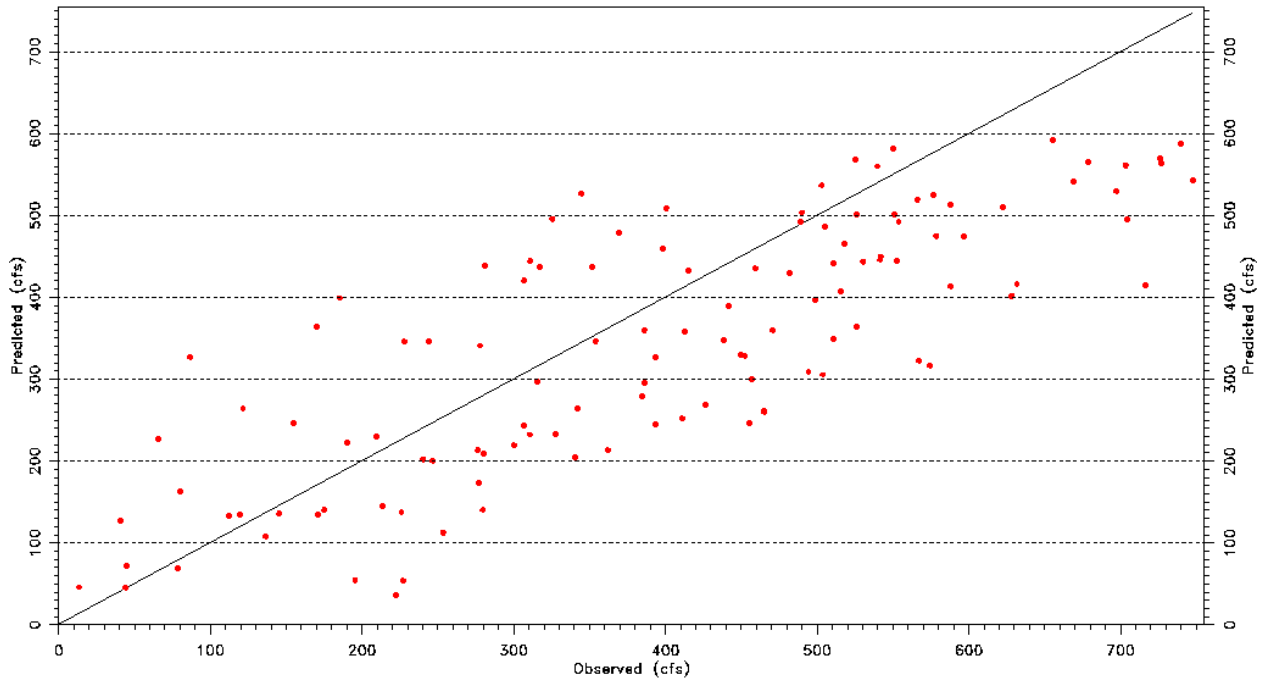
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Figure S5. RRCA model calibration from Republican River at Benkelman NE, which is between Idalia CO and Hardy NE. Source: <https://www.republicanrivercompact.org/v12p/html/bf/45c.html>



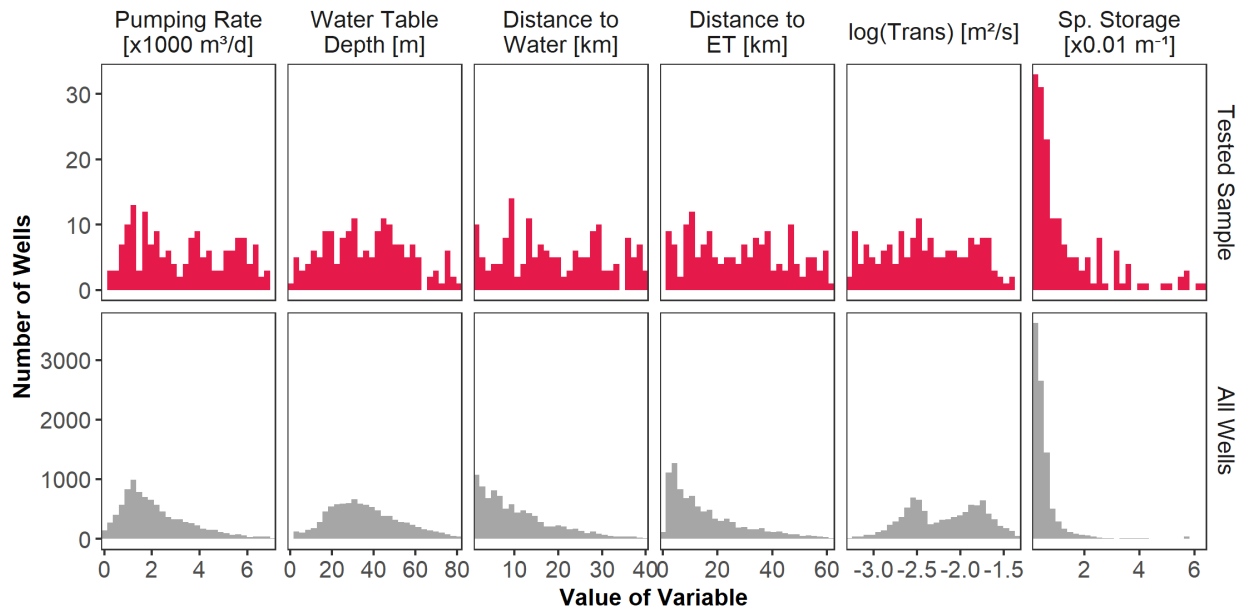
Republican River near Hardy, NE

Republican River Settlement Model Version 12p



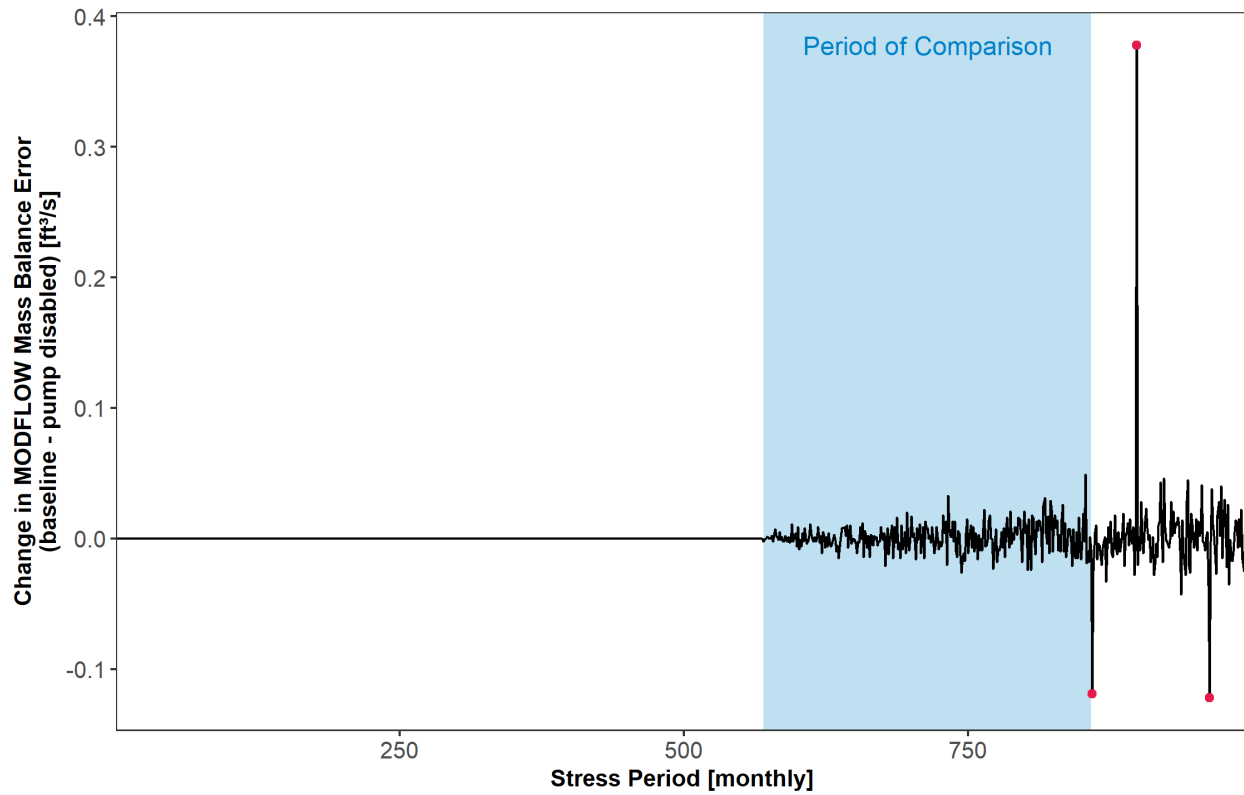
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Figure S6. RRCA predicted baseflow vs. observed baseflow from the Republican River near Hardy NE, which is the furthest downstream point for which calibration results are available. Source: <https://www.republicanrivercompact.org/v12p/html/bf/48c.html>



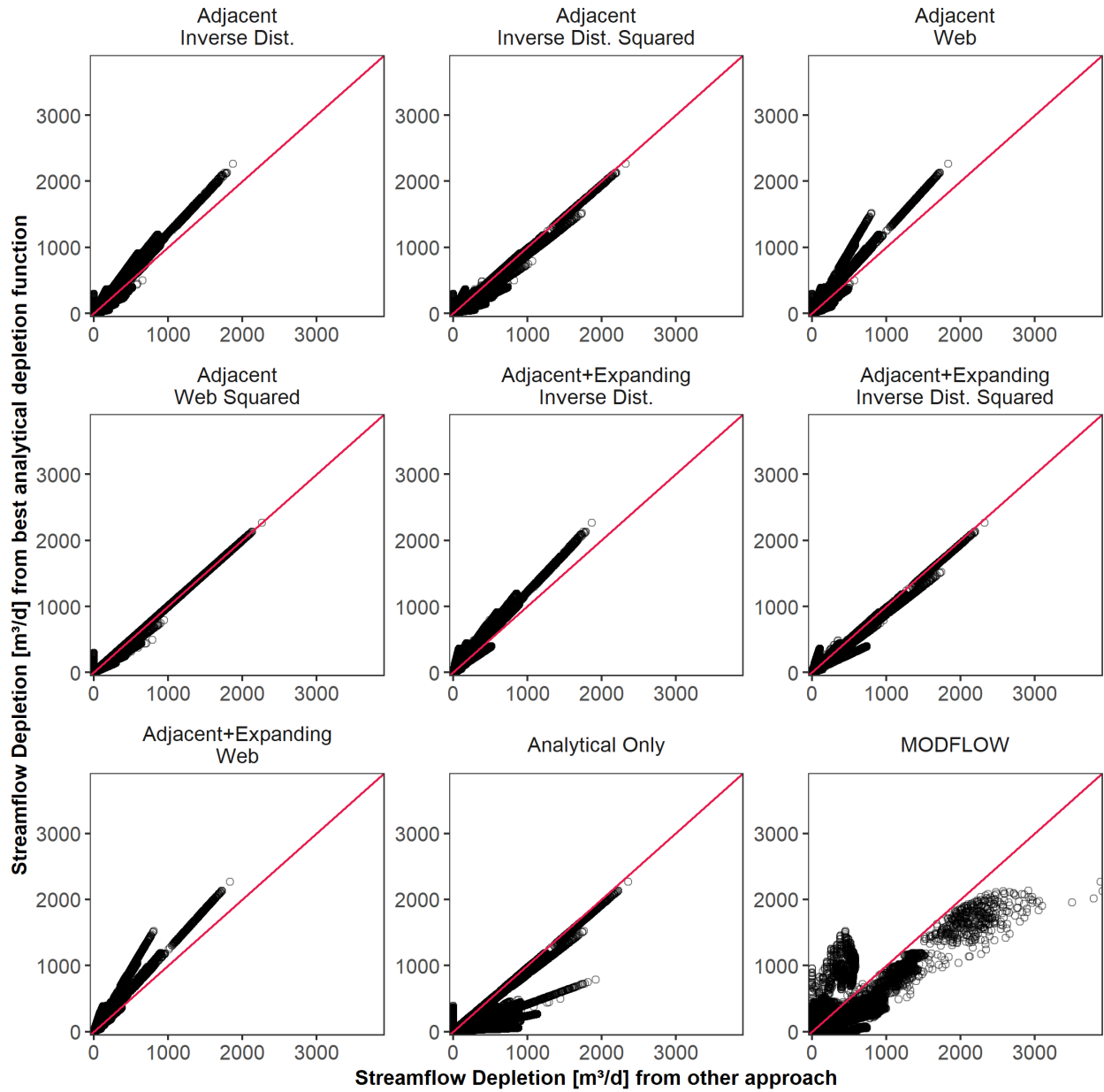
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Figure S7. (top row) Properties of wells selected for model experiments, identical to Figure 2b.
 (bottom row) Properties of all wells in model domain.



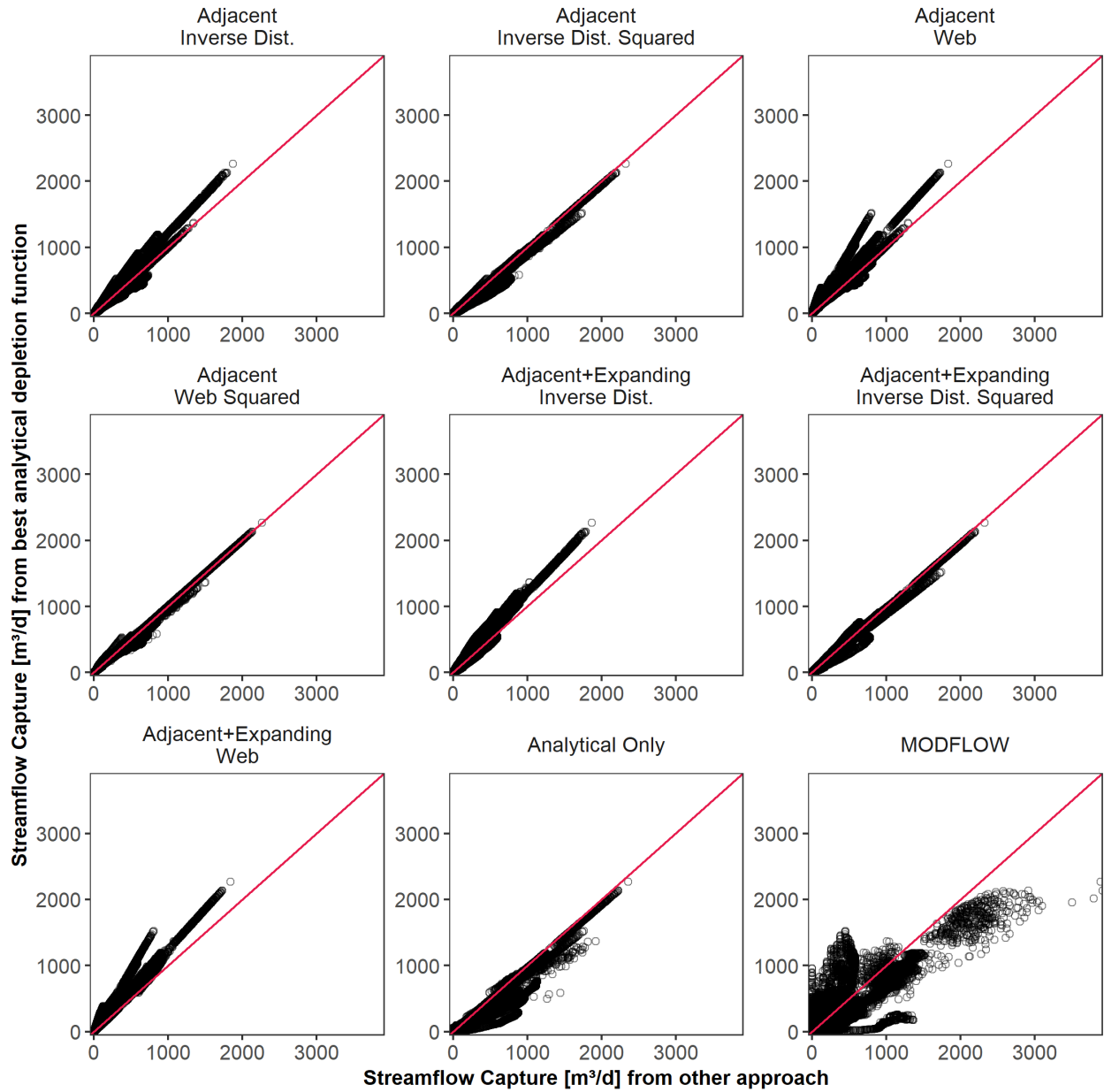
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959 **Figure S8.** Demonstration of selection of period of comparison for a well, shown as shaded blue
960 background. The period of comparison begins at the onset of pumping and ends at the timestep
961 prior to the first detected outlier. The red dots indicate outliers in the timeseries of mass balance
962 change.

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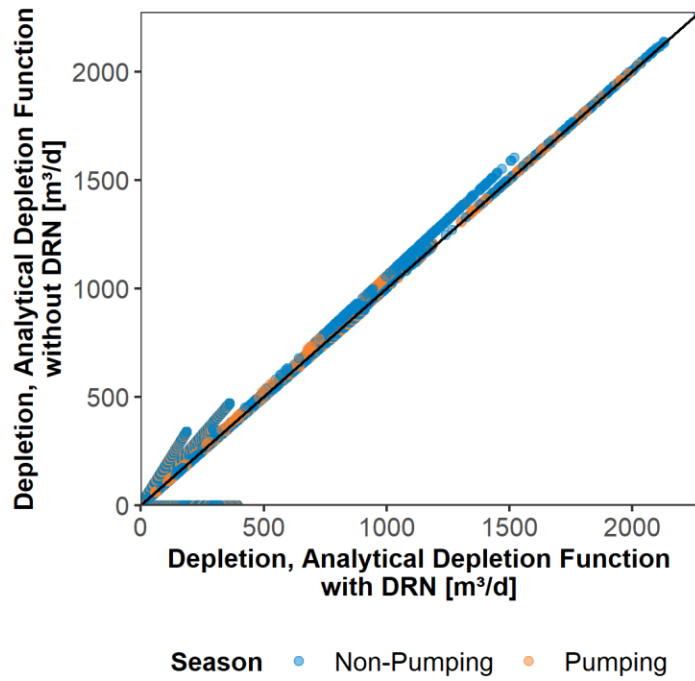
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 965 **Figure S9.** Comparison between segment streamflow depletion predicted by the best-performing
 966 analytical depletion function, which uses the adjacent+expanding stream proximity criteria and
 967 web squared depletion apportionment equation (Table 1), and all other analytical depletion
 968 functions, analytical model only, and MODFLOW model. Each point shows the streamflow
 969 depletion at a single timestep for the response of one stream segment to a single well. Red lines
 970 show 1:1 relationship.

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 973 **Figure S10.** Comparison between total streamflow depletion predicted by the best-performing
 974 analytical depletion function, which uses the adjacent+expanding stream proximity criteria and
 975 web squared depletion apportionment equation (Table 1), and all other analytical depletion
 976 functions, analytical model only, and MODFLOW model. Each point shows the total streamflow
 977 depletion summer across all segments at a single timestep for a single well. Red lines show 1:1
 978 relationship.

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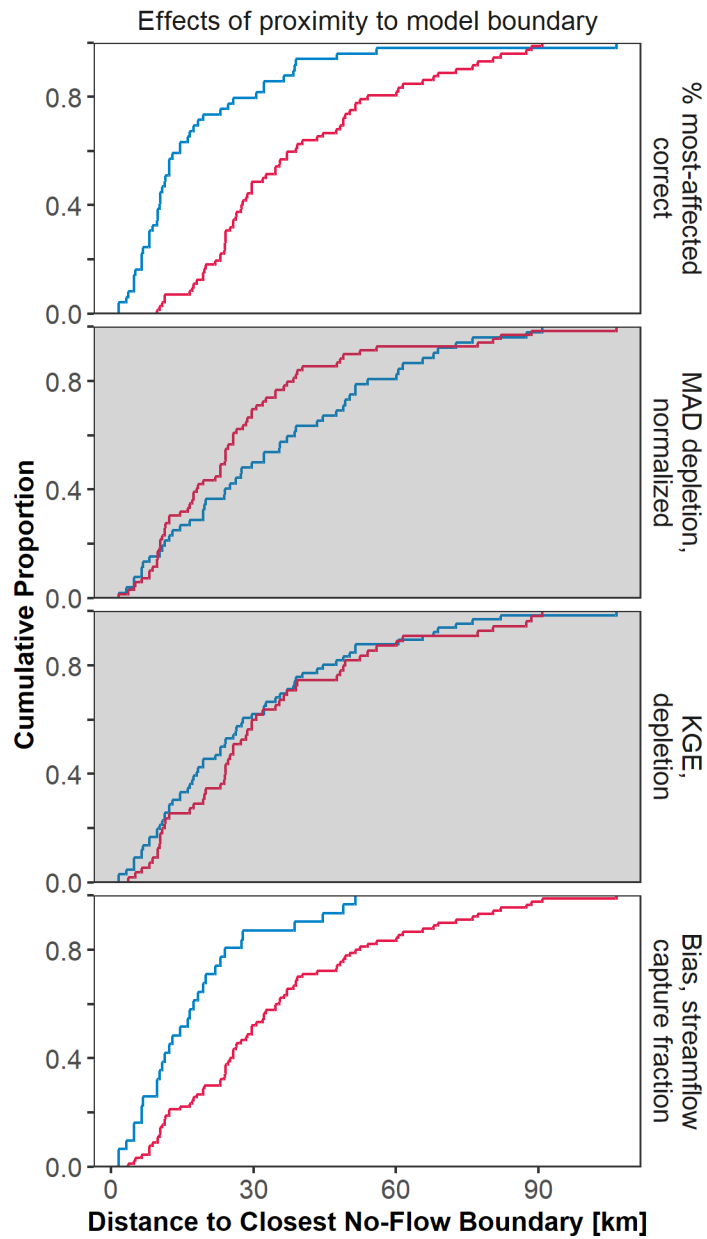


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981 **Figure S11.** Depletion predicted by analytical depletion function with and without DRN

982 features.

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Fit — Good — Poor Difference Not Significant

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Figure S12. Regional sensitivity analysis of performance as a function of distance to the closest no-flow cell. As in Figure 6, each plot is an empirical cumulative distribution functions for all wells with respect to each performance criteria. In shaded panels, distributions of ‘good’ and ‘poor’ groups are expected to be drawn from the same distribution (two-sample Kolmogorov-Smirnov test $p > 0.05$).