Lithology, pore-filling media, and pore closure depth beneath InSight on Mars inferred from shear wave velocities

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Key Points:
• Mars’ upper crust (0-8 km) beneath InSight comprises fractured gas-filled rocks and weakly cemented sediments
• Mars’ deeper crust (8-20 km) beneath InSight could comprise fractured basalts or more-felsic igneous rocks with 0-23% porosity
• No seismically detectable cryosphere exists in the crust and pores in the deeper crust host gas, liquid water, or up to 2% cement

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Abstract

We quantify the volume and distribution of water, cement, sediments, and fractured rocks within the Martian crust beneath NASA’s InSight (Interior Exploration using Seismic Investigations, Geodesy, and Heat Transport mission) lander by using rock physics models to interpret shear wave velocities ($V_s$) measured from InSight data. The models assume that Mars’ crust comprises sediments and fractured rocks whose pores and fractures host variable combinations of gas, liquid water, and mineral cements. Measured $V_s$ in the upper crust (0-8 km) can be explained by layers of minimally (< 2%) cemented sediments and gas-filled fractured basalts. Measured $V_s$ in the deeper crust (8-20 km) can be explained by fractured basalts or more felsic igneous rocks (modeled here as 100% plagioclase feldspar) that is unfractured or has up to 23% porosity. Open pores in the deeper crust could host liquid water, liquid water, and up to 2% cement. Modeled $V_s$ are too low for a seismically detectable ice-saturated cryosphere in the upper crust and temperatures are too high to freeze liquid water in the deeper crust. Notably, with $V_s$ alone, we are unable to distinguish between liquid water and gas within the pores.

Plain Language Summary

Liquid water may have existed on Mars as oceans, rivers, or ground water. Surface water was likely lost to space, buried as liquid water and ice, and/or incorporated in subsurface minerals and mineral cements. The InSight lander on Mars has a seismometer whose measurements can be used to estimate the velocity of seismic shear waves. Seismic velocities change based on rock type and the material that fills the pores within rocks (e.g., liquid water, gas, or ice and other mineral cements). We show that the measured seismic velocities in the upper (0-8 km) crust can be explained by layers of gas-filled basalts and minimally (2%) cemented sediments rather than ice-filled sediment or basalt. Measured seismic velocities in the deeper (8-20 km) crust can be explained by fractured basalt. More feldspar-rich rocks could explain the velocities in the deeper crust and they could be unfractured or have up to 23% porosity. Fractures within the deeper crust could host liquid water, gas, and up to a couple percent of mineral cements.

1 Introduction

Quantifying the volume and distribution of Mars’ subsurface lithologies, mineral cements, and liquid water are critical to unraveling the planet’s geologic evolution (Carr & Head, 2003; Di Achille & Hynek, 2010; Carr & Head, 2019; Scheller et al., 2021). Mars’ crust comprises igneous and sedimentary rocks that are lithified and fractured to varying degrees (Tanaka et al., 2014; Golombek et al., 2018; Pan et al., 2020). Two open questions are (1) what is the depth where pores close entirely within the Martian crust and (2) what percentages of existing pores host water as liquid or ice, or that was incorporated into other mineral cements.

Gravity and heat flow models provide constraints on Mars’ subsurface porosity and pore closure depth (Clifford, 1993; Clifford et al., 2010; Goossens et al., 2017; Gyalay et al., 2020; Wieczorek et al., 2022). Goossens et al. (2017) used gravity data to infer that Mars’ average bulk density in the upper 20 km is 2,582±209 kg/m$^3$. From this bulk density, a porosity of 0.10 to 0.23 in the upper 20 km can be obtained. Wieczorek et al. (2022) later integrated gravity and shear wave velocity data to hypothesize that a lower density (higher porosity) layer extends to 8-11 or 20-23 km below the surface and the pores close entirely beneath one of these depths. Gyalay et al. (2020) used heat flow models to argue that pore collapse via thermally-activated viscous creep should occur between 12 km and 23 km below the surface. The transition from open to closed pores should occur over 1 km (Gyalay et al., 2020). Gyalay et al. (2020)’s and Wieczorek et al. (2022)’s
proposed pore closure depth overlaps with two possible seismic discontinuities (8±3 and 20±5 km) (Figure 1).

Surface exposures alongside heat and fluid flow models provide constraints on the presence, volume, and distribution of water within the Martian crust. Rover and satellite images showing sediment structures and stratigraphy characteristic of ancient delta, marine, and fluvial depositional environments alongside direct and remotely inferred observations of ice and liquid water at the polar regions evidence past and current water on Mars (Carr, 1987; Baker, 2006; Orosei et al., 2018; Nazari-Sharabian et al., 2020). Evidence for past subsurface liquid water also includes Hesperian and Amazonian-aged outflow channels, whose discharges were sometimes a few orders of magnitude greater than Earth's largest floods (Colaprete & Jakosky, 1998; Carr & Head, 2002; Burr et al., 2002; Manga, 2004; Bibring et al., 2005; Clifford et al., 2010; Di Achille & Hynek, 2010; Rodriguez et al., 2015; Weiss & Head, 2017; Voigt & Hamilton, 2018). Mars’ past surface water in rivers, lakes, and possible oceans may have been lost to space or infiltrated the ground (Colaprete & Jakosky, 1998; Bibring et al., 2005; Di Achille & Hynek, 2010). Liquid water may have percolated through the pores of rock layers, whose permeability may have been increased by impacts (Clifford, 1997; Wang et al., 2005). Heat flow models suggest that a 0-9 km and 10-22 km thick regional cryosphere could exist at Mars’ equator and poles, respectively (Clifford et al., 2010). Atmospheric carbon dioxide may have dissolved in water, then precipitated as carbonate cement (up to 2%) (Boynton et al., 2009; Halevy & Schrag, 2009; Adam et al., 2013). Thus, mapping Mars’ subsurface ice and other mineral cements, liquid water, and lithology may help constrain (1) the volume of water buried versus lost to space (Jakosky, 2021), (2) the planet’s water budget and cycle through time (Clifford & Parker, 2001), (3) the fates of past surface water (Citron et al., 2018), (4) the volume of water sequestered by minerals, and (5) the lithology of Martian subsurface layers in the past and present (Mustard, 2019; Scheller et al., 2021; Wernicke & Jakosky, 2021).

Rock physics models and shear wave velocities \( V_s \) derived from seismograms collected by the seismometer on the InSight (Interior Exploration using Seismic Investigations, Geodesy, and Heat Transport mission) lander provide opportunities to explore Mars’ subsurface mechanical properties further. \( V_s \) is sensitive to several rock and sediment properties, including mineralogy, fracture density, porosity, and ice and other mineral cements (Mindlin, 2021; Dvorkin & Nur, 1996; Jenkins et al., 2005; Waite et al., 2009). The Martian crust beneath InSight has at least two seismically detectable km-scale layers (Knappmeyer-Endrum et al., 2021). \( V_s \) are 1.7-2.1 km/s in the upper crust (i.e., between 0 km and ~8-11 km) and 2-3.4 km/s in the deeper crust (i.e., between ~8-11 km and 20 km) (Figure 1). Interpretations using self-consistent fractured-media rock physics models (Te Wu, 1966; Berryman, 1980) indicate that \( V_s \) within the upper 8-11 km is lower than expected for a cryosphere (Manga & Wright, 2021; Wright et al., 2022). \( V_s \) between 11 km and 20 km may be consistent with basalts whose fractures are 1-5% filled with calcite cement (Manga & Wright, 2021). Thus, Mars’ subsurface is likely a mix of sediments (i.e., layers with unconsolidated grains) and fractured layers of consolidated sediments or igneous rocks.

Our study uses granular and self-consistent fractured-media rock physics models to infer the volume and distribution of liquid water, ice and other mineral cements, and lithology from InSight-measured \( V_s \). We infer that (1) the upper crust beneath InSight comprises layers of fractured gas-filled basalts and weakly cemented sediments, (2) the deeper crust could be fractured basalts or more felsic igneous rocks that are either unfractured or has up to 23% porosity, (3) the pores of fractured rocks in the deeper crust could host liquid water, gas, or 2% cement and 98% liquid water or gas, and (4) no seismically detected ice-saturated cryosphere layer exists beneath InSight.
Inversion results from computing P-to-S and S-to-P receiver functions for two marsquakes.

Figure 1. InSight derived shear-wave velocities ($V_s$) (Knapmeyer-Endrun et al., 2021). The brown and purple lines show $V_s$ from P-to-S receiver function analyses for four and two marsquakes, respectively.

2 Methods

We compare modeled and measured $V_s$ to infer Mars’ subsurface mechanical properties, constraining model uncertainties with Monte Carlo simulations and sensitivity analyses. We use granular and fractured-media rock physics models to model $V_s$ in the upper crust; we use only the fractured-media models for the deeper crust because we do not expect sediment layers in the deeper crust.

We calculate $V_s$ from

$$V_s = \sqrt{\frac{\mu_e}{\rho}},$$

where $\mu_e$ and $\rho$ are the effective shear modulus and bulk density, respectively. Rock physics models, described next, provide estimates for $\mu_e$. We estimate bulk density $\rho$ using

$$\rho = \sum_i \phi_i \rho_i$$

where $\rho_i$ and $\phi_i$ are densities and volume fractions of the $i^{th}$ constituents, respectively.

2.1 Modelling $V_s$ for sediments

We estimate $\mu_e$ for cementless sediments using the Hertz-Mindlin rock physics model (Mindlin, 2021). The cementation model (Dvorkin & Nur, 1996) provides $\mu_e$ for sediments with cements (e.g., ice and calcite) deposited at grain contacts or that surround
grains in contact. The models’ equations can be found in Supporting Information Method S1, Mindlin (2021), Dvorkin and Nur (1996), and Mavko et al. (2020). Model input parameters are mineral Poisson’s ratio $\nu_m$, mineral bulk moduli $\kappa_m$, mineral shear moduli $\mu_m$, cement fraction $c_f$, volume fraction of rough versus smooth grain contacts $f$ (smooth grain contacts allow elastic micro-scale slip during seismic wave propagation and rough grain contacts do not), porosity $\phi$, effective stress $P$, and coordination number $c_n$ (average number of grains contacting each other). We calculate mineral Poisson’s ratio $\nu_m$ using

$$\nu_m = \frac{3\kappa_m - 2\mu_m}{6\kappa_m + 2\mu_m},$$

where $\mu_m$ and $\kappa_m$ are mineral shear and bulk moduli. Representative minerals within Mars’ subsurface and their respective $\mu_m$ and $\kappa_m$ are in Table 1. We treat basalt and clay as single mineral constituents. We estimate porosity $\phi$ changes with depth using

$$\phi = \phi_0 e^{-\frac{z}{k}},$$

where $z$, $k$, and $\phi_0$ are depth in km, a porosity reduction constant scaled for Mars’ gravitational field, and $\phi$ at the surface, respectively. Clifford (1987) estimated $k = 2.82$ km based on scaling lunar observations; we consider values that range from 1 to 10 km. We assume that $\phi_0$ is between 0.3 and 0.5, consistent with studies that constrained $\phi_0$ from rover measurements and analog Earth studies (Golombek et al., 2018; Lewis et al., 2019; Smrekar et al., 2019; Lognonné et al., 2020). Effective stress $P$ is

$$P = \rho gh - P_f,$$

where $g$, $h$, and $P_f$ are gravitational acceleration on Mars (3.71 m/s$^2$), depth, and fluid pressures, respectively. Coordination number $c_n$ is from Murphy (1982)

$$c_n = 20 - 34\phi + 14\phi^2.$$

We use the input parameters described above to calculate $\mu_e$ from the rock physics model equations, then $V_s$ from equation 1.

<table>
<thead>
<tr>
<th>Mineral</th>
<th>$\mu_m$ (GPa)</th>
<th>$\kappa_m$ (GPa)</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calcite</td>
<td>28.2</td>
<td>71.6</td>
<td>2710</td>
<td>Mavko et al. (2020)</td>
</tr>
<tr>
<td>Basalt</td>
<td>40.0</td>
<td>80.0</td>
<td>2900</td>
<td>Christensen (1972); Heap (2019)</td>
</tr>
<tr>
<td>Clay</td>
<td>6.0</td>
<td>12.0</td>
<td>2650</td>
<td>Vanorio et al. (2003)</td>
</tr>
<tr>
<td>Halite</td>
<td>15.3</td>
<td>25.2</td>
<td>2160</td>
<td>Zong et al. (2017)</td>
</tr>
<tr>
<td>Ice</td>
<td>3.8</td>
<td>8.7</td>
<td>1220</td>
<td>Toksöz et al. (1976)</td>
</tr>
<tr>
<td>Plagioclase</td>
<td>25.6</td>
<td>75.6</td>
<td>2630</td>
<td>Woebel et al. (1963)</td>
</tr>
</tbody>
</table>

To compare measured and modeled $V_s$, we create a rock physics template that relates $V_s$ to $\phi$ (0-0.5), grain-contact friction (100% rough or smooth grain contacts), and pore ice percentage (0-100%) for ice deposited at grain contacts or surrounding grains in contact. Then, we identify the combinations of $\phi$, grain contact friction, and or pore ice percentage that are consistent with the measured $V_s$. We also compare measured and modeled $V_s$ directly; these models assume a porosity reduction profile (equation 4) and
that pores host either 99% ice and 1% gas, 100% gas, 100% liquid water, 2% calcite cement and 98% gas, or 2% calcite cement and 98% liquid water. We model a cryosphere as 99% ice and 1% gas because the cementation model breaks down for the 100% ice limit, where 0% porosity introduces indeterminacy into the equations. We use a 10,000 realization Monte Carlo simulation to incorporate input parameter uncertainties into the results from the models used for direct comparisons with measured \( V_s \). During the Monte Carlo simulation, we randomly select a new \( \phi \)-depth profile for each realization. Selected \( \phi \) values influence coordination number, bulk density, and effective stress.

### 2.2 Modeling \( V_s \) for fractured rocks

We estimate \( \mu_e \) for fractured rocks using the self-consistent model of Berryman (1980). The model’s equations can be found in the Supporting Information Method S1, Berryman (1980), and Mavko et al. (2020). The model’s input parameters are \( \mu_m \), \( \kappa_m \), \( \phi \), and pore shape (defined by the aspect ratio, \( \alpha \) – i.e., the pore’s short axis divided by the long axis).

We calculate \( \mu_e \) assuming that the fractures within a basalt contains either 100% gas, 100% water, 98% gas and 2% calcite cement, 98% water and 2% calcite cement, and 100-100% ice. We then use \( \mu_e \) and \( \rho \) to calculate \( V_s \) from equation 1. We use these results to create rock physics templates relating \( V_s \) to \( \phi \) (0.1-0.5), \( \alpha \) (0.01-1), and pore-filling media that best explain measured \( V_s \).

### 2.3 Sensitivity Analyses

We conduct sensitivity analyses to assess how model parameter uncertainties could influence interpretations as well as to identify which rock properties are resolvable with our models. Here, we first assume that \( \phi_0 \) is 0.4, \( \phi \) exponentially decays with depth, gas fills the pores, and there is no cement within the pores. Then, we vary a single input parameter to assess how its uncertainty influences modeled \( V_s \). For granular media models, we vary mineralogy (100% basalt, plagioclase feldspar, or clay), coordination number (8, 12, 16, or 20), porosity decay constant (1, 2.82, or 10 km), cement type (98% gas and 2% calcite, halite, or ice), and cement location (at grain contacts or entirely surrounds grains). For fractured media models, we vary host rock composition (100% basalt, plagioclase feldspar, or clay).

### 3 Results

#### 3.1 Sediments

The ability to resolve changes in subsurface properties of sediment layers is most affected by uncertainties in cement type and location, followed by \( \mu_m \) and \( \kappa_m \), \( \phi \), and \( \alpha \) (Figure 2). Assuming 100% basalt or plagioclase feldspar grains as representative of the compositional diversity of igneous rocks (and all else equal) results in a \( V_s \) difference of \( \sim 0.20 \) km/s (Figure 2c), which is within the measured \( V_s \) uncertainties (Figure 1). Thus, it is challenging to use the granular media rock physics models alone to distinguish between plausible igneous compositions. Clay layers may be seismically distinguishable from igneous rock layers since the differences in their \( V_s \) predictions are 0.95 km/s and 0.72 km/s, respectively. Uncertainties in the porosity decay constant \( k \) (i.e., 1, 2.82, and 10) produce a \( V_s \) range of \( \sim 0.28 \) km/s (Figure 2b); this result implies that the assumed decay constant does not significantly influence the interpretations of measured \( V_s \). The range for the modeled \( V_s \) difference for coordination numbers of 8, 12, 16, and 20 is 0.65 km/s, which is within the uncertainty of measured \( V_s \) (Figure 2a). Assuming 2% calcite, ice, and halite mineral cement produce \( V_s \) ranges of \( \sim 1.18 \) km/s and \( \sim 0.64 \) km/s for the cement at grain contacts versus on the grain surface, respectively (Figure 2d). Assuming calcite and ice cement at grain surfaces predicts comparable velocities at all depths (i.e.,
Figure 2. Effects of (A) coordination number, (B) porosity decay constant, (C) mineral moduli, and (D) cement type and location on $V_s$. We first assume that the subsurface comprises 100% basalt, porosity reduce exponentially with depth from $\phi_0 = 0.4$, $c_n$ is primarily controlled by $\phi$ as constrained by the Murphy (1982) empirical relationship, and no mineral cements exist between grains and pores. Then, we only change the parameters being assessed in each graph to assess their influence on $V_s$.

Figure 3 shows the granular media rock physics template relating $V_s$, porosity, pore-filling media, and grain contact friction for modeled sediment layers. Measured $V_s$ are consistent with modeled $V_s$ for sediments comprised of 100% rough-grain contacts and sediments that host a few percent ice in their pores (Figure 3). Models for sediment with 100% rough-grain contacts are consistent with measured $V_s$ if the sediments’ porosities are between 0.14 and 0.35. Models with 100% smooth grain contacts underpredict measured $V_s$ for all porosities between 0 and 0.5. Models for sediments that host 10-18% ice that surrounds grains in contact are consistent with the measured $V_s$ if porosities are between 0.2-0.5. If pores are filled with 2% ice deposited at grain contacts, the porosities need to be 0.4-0.5 to explain the measured $V_s$. The measured $V_s$ are consistent with modeled $V_s$ for sediments with ice deposited at grain contacts if the pore-ice percentage is less than 2% and porosities are higher than 0.37.

Assuming a porosity-depth reduction relationship defined by equation 4, where $k = 2.82$ km, provides additional insights into the volume and type of pore-filling materials that could explain measured $V_s$ within the upper crust. Measured $V_s$ are most consistent with modeled $V_s$ for a sediment with basalt grains and whose pores are filled with gas or 2% calcite cement (Figure 4b and 4d). In general, Hertz-Mindlin rough-grained models predict $V_s$ with lower misfits than the smooth-grained models. The smooth-grained model underpredicts $V_s$ by 0.53 km/s for a gas-filled sediment layer in the upper crust (Figure 4b). The liquid water saturated smooth-grained model underpredicts $V_s$ by 0.61 km/s in the upper crust (Figure 4c). The rough-grained models for a gas or liquid water-filled layer predicts higher $V_s$ than the smooth-grained models in the upper crust by 0.4 km/s. Assuming that calcite cement fills 2% of the pores and liquid water or gas fills the remaining 98% predict $V_s$ within $\pm 0.42$ km/s of measured $V_s$, regardless of whether the cement is deposited at grain contacts or surrounds grains (Figure 4d and 4e). Assuming ice-saturated pores overpredicts measured $V_s$ by 1.6 km/s (Figure 4a).
Figure 3. Rock physics template relating $V_s$, porosity, pore ice percentage, and sediments composed of 100% rough or smooth grain contacts.

Figure 4. Comparisons of modeled and predicted $V_s$ assuming basalt grains with pores filled with: A) 99% ice and 1% gas, B) 100% gas, C) 100% water, D) 2% calcite cement and 98% gas, and E) 2% calcite cement and 98% water.
3.2 Fractured Rocks

$V_s$ of fractured rocks are most sensitive to $\alpha$, $\phi$, and elastic moduli of the host rock (Figures 5-7). As expected, $V_s$ increases as porosity decreases and $\alpha$ increases. The difference in $V_s$ between basalt and other host rocks (e.g., plagioclase feldspar and clay) increases with decreasing porosity and increasing aspect ratio. A plagioclase host rock produces a difference of 0-0.5 km/s in $V_s$ compared to a basalt host rock, for all pore-filling media (Figure 6). Thus, we can only distinguish between basalt and plagioclase rocks with $V_s$ differences $>0.3$ km/s. A clay versus basalt host rock lowers $V_s$ by $\sim$0.4-2 km/s for gas, $\sim$0.15-2 km/s for liquid water, $\sim$0.2-2 km/s for 2% calcite cement and 98% gas, and $\sim$0.3-2 km/s for 2% calcite cement and 98% liquid water (Figure 7). Figure 7 shows the combinations of aspect ratio and porosities for when a clay and basalt host rock is resolvable (i.e., contour lines with $V_s$ of at least 1.2 km/s).

A basalt host rock whose fractures are filled with varying percentages of ice, gas, water, and or calcite cement could explain the measured $V_s$ in the upper and deeper crust. The upper crust could be filled with 100% gas, 100% liquid water, or 2% calcite cement with 98% gas or liquid water (Figure 5). Of these, a 100% gas-filled basalt (Figure 5a) produces the smallest number of $\phi$-$\alpha$ combinations ($\phi = 0.1 - 0.47$ and $\alpha = 0.03 - 1$) that could explain the measured $V_s$. A 2% calcite cemented basalt (Figure 5c-d) produces the largest combinations of $\phi$-$\alpha$ that could explain the measured $V_s$. Modeled $V_s$ for basalts with ice that fills 20% to 60% of the pores are consistent with measured $V_s$ if $\phi$ is between 0.2 and 0.5. The measured $V_s$ in the deeper crust are consistent with modeled $V_s$ for a basalt filled with 100% gas or liquid water, 2% calcite cement with 98% gas or liquid water, or 10%-100% ice (Figures 5 and S1). A 100% gas-filled basalt (Figure 5a) produces the smallest number of $\phi$-$\alpha$ combinations ($\phi = 0.1 - 0.4$ and $\alpha = 0.04 - 1.0$) that could explain the measured $V_s$, while 60% ice-filled basalt host rock produces the largest number of $\phi$-$\alpha$ combinations ($\phi = 0.1 - 0.5$ and $\alpha = 0.01 - 1.0$) (Figure 5i).

4 Discussion

Our interpretations are guided by limitations associated with rock physics model assumptions, uncertainties in model parameters and measured $V_s$, available satellite and rover images, gravity-derived bulk density data, and heat flow models. The rock physics models provide end-member $V_s$ estimates for the hypothesized stratigraphy (i.e., either sediments or fractured rocks filled with varying percentages of gas, liquid water, or ice and other mineral cements). Martian subsurface stratigraphy may include a mixture of fractured igneous rocks (e.g., basalts or 100% plagioclase feldspar) emplaced as volcanic lava flows or intrusions, brecciated sedimentary rocks, sands, and clays (Tanaka et al., 2014; Golombek et al., 2018; Pan et al., 2020; Warner et al., 2022). Thus, the measured $V_s$ could be averages from several smaller rock and sediment layers that are not resolvable by the seismic velocity models (Knapmeyer-Endrun et al., 2021). Considering these limitations, our primary interpretations are that the upper crust comprises fractured basalt and cemented sediment layers whereas the deeper crust could comprises gas or water-filled fractured basalt with open, partially cemented fractures or more felsic igneous rock (represented here by 100% plagioclase feldspar) layers with 0-23% porosity.

4.1 Fractured rocks and cemented sediments within the upper crust

Our comparisons of measured and modeled $V_s$ suggest that gas-filled fractured rock and cemented sediment layers may coexist within the upper crust. We interpret that the upper crust is gas-filled because temperatures in the upper crust would freeze water (Clifford et al., 2010). Sediments filled with 2% cement and 98% gas and basalts filled with gas are possible within the upper crust since their modeled $V_s$ are consistent with measured $V_s$ when we parameterize the models with the gravity-derived porosity range (0.10-0.23) (Figure 4) (Goossens et al., 2017). The coexistence of the gas-filled basalt and weakly-
Figure 5. Rock physics template showing the $V_s$ relationship between $\alpha$ (0.01-1.00), $\phi$ (0.1-0.5), varying pore-filling media, and varying pore-filling ice percentage. The black shading shows $V_s$ for the upper crust; the white shading shows $V_s$ for the deeper crust. The pore spaces are filled with either (A) gas, (B) water, (C) 2% calcite cement and 98% gas, (D) 2% calcite cement and 98% water, or (E) 20%, (F) 30%, (G) 40%, (H) 50%, or (I) 60% ice. Y-axis is logarithmic.
Figure 6. Rock physics sensitivity template showing the $V_s$ difference between a basalt and plagioclase feldspar host rock. $V_s$ changes with $\alpha$ (0.01-1.00), $\phi$ (0.1-1), and pore-filling media. The pores are filled with either (A) gas, (B) water, (C) 2% calcite cement and 98% gas, (D) 2% calcite cement and 98% water, or (E) 20%, (F) 30%, (G) 40%, (H) 50%, or (I) 60% ice. Y-axis is logarithmic.
Figure 7. Rock physics sensitivity template showing the $V_s$ difference between a basalt and clay host rock. $V_s$ changes with $\alpha$ (0.01-1.00), $\phi$ (0.1-0.5), and pore-filling material. The pores are filled with either (A) gas, (B) water, (C) 2% calcite cement and 98% gas, (D) 2% calcite cement and 98% water, or (E) 20%, (F) 30%, (G) 40%, (H) 50%, or (I) 60% ice. Y-axis is logarithmic.
cemented sediment layers would be resolved as one seismic velocity layer in seismic velocity models since differences in the layers’ $V_s$ would not produce a large impedance contrasts. Additional support for the potential coexistence of igneous rock and sediment layers in the upper crust comes from (1) Martian meteorites and images of surface-exposed stratigraphic columns that evidence basalts, sandstones, and sediments in the upper 1 km of the crust (Carr & Head, 2002; Edwards et al., 2011; McSween, 2015; Golombek et al., 2018; Hobiger et al., 2021; Knapmeyer-Endrun et al., 2021) and (2) Insight-derived high-resolution seismic velocities that are consistent with gas-filled basalt and sediment layers down to 0.3 km below the surface of the landing site (Hobiger et al., 2021; Wright et al., 2022). Other layers with different lithologies and pore-filling media, and hence different $V_s$, may exist within the upper crust. If so, these layers are likely too thin to be detected by the longer period marqsquake waves used by Knapmeyer-Endrun et al. (2021) to constrain the $V_s$-depth structure.

### 4.2 No ice-saturated cryosphere in the upper crust

There is likely no cryosphere within the upper crust, beneath InSight. The lack of a cryosphere is indicated by observations that when ice-filled, the granular media models overpredict measured $V_s$ by 0.5-2 km/s (Figure 4a) and that, for fractured basalts, there are no modeled combinations of porosity and pore shape that would explain the measured $V_s$ if we restrict porosity to 0.10-0.23, as estimated by Goossens et al. (2017) (Figure 5c-i). One possibility not captured by our models is that there exists mushy ice (i.e., mix of soft, snow-like, ice and brine) within the pores of rocks and sediments deposited at depths greater than a few hundred meters, depending on regional heat flow. Detecting a mushy ice may require improved constraints on attenuation beneath InSight and developing models connecting seismic velocities, attenuation, and mushy ice concentrations (Dou et al., 2017). This can be done using lab and permafrost experiments. Seismic attenuation may also provide constraints on fracture orientations (Li et al., 2022).

### 4.3 Mineralogy, pore collapse, and pore-filling media in the deeper crust

Our model comparisons, the reliability of fractured media models for predicting $V_s$ of very low $\phi$ (<3%) basalts, and differences in the effective pressures and heat flow between Earth and Mars lead us to interpret that, if basaltic, the deeper crust has 10-23% porosity. The measured $V_s$ in Mars’ deeper crust are 0.4-1.7 km/s lower than $V_s$ for modeled unfractured basalts. Fractured media rock physics models successfully predict $V_s$ of very low $\phi$ (<3%) basalts with as little as 0-2% misfit (Tsuji & Iturrino, 2008). Our model predicts $V_s$ of ~3.7 km/s for unfractured ($\phi=0$) Martian basalts, assuming the mineral elastic moduli and densities listed in Table 1. If the deeper crust is basaltic and pores are filled with gas, liquid water, or 2% cement and 98% gas or liquid water, Mars’ comparatively lower $V_s$ may be explained by basalt layers with $\alpha=0.15-0.8$, $\phi=0.10-0.23$ (Figures 1, 5), and bulk density $\rho=2,318-2,713$ kg/m$^3$ (equation 2). These $\rho$ and hence $\phi$ ranges are consistent with Goossens et al. (2017) gravity-derived bulk density and porosity ranges for the deeper crust. The porosity of Earth ocean basalts reduce by ~90% from 0-6 km depth (Chen et al., 2020). At 6 km below the surface of Mars, assuming $\phi = 0.10$ (the lower $\phi$ limit proposed by Goossens et al. (2017)) and gas or liquid water fills the host rock, Earth’s effective pressure is ~3 times greater than Mars’ (equation 5). Earth’s average heat flux is ~4.8 times greater than Mars’ average (Davies & Davies, 2010; Parro et al., 2017). Manning and Ingebritsen (1999) estimates that viscous creep-induced pore collapse occurs at an average depth of ~12 km on Earth. Thus, we infer that Mars’ lower effective stress, heat flow, and gravitational acceleration would cause elastic pore closure to occur at depths deeper than 12 km. Our findings and interpretations imply that a basaltic host rock would require 10-23% porosity and effective pressure-induced pore collapse may occur at the same depth as the second seismic discontinuity, located at the base of the deeper crust. This interpreted depth of pore collapse does not preclude the possibility...
that thermally-activated pore collapse occurred in the deeper crust in the past (Gyalay et al., 2020), and currently open pores may have been created by subsequent surface impacts or other stresses.

Our model comparisons show that the $V_s$ in the deeper crust is also consistent with a plagioclase feldspar host rock with 0-23% porosity. Wieczorek et al. (2022) proposed that Mars’ $V_s$ may be lower than unfractured basalt’s because the Martian crust comprises more felsic, feldspar-dominated igneous rocks whose density and shear moduli, and hence $V_s$, are lower than those of basalt. We represent the felsic end-member igneous rock as a 100% plagioclase feldspar host rock. Our modeled $V_s$ of an unfractured plagioclase host rock is $\sim$3.1 km/s, which falls in the upper 75% quartile of the InSight-derived $V_s$ range for the deeper crust (Figure 1) and supports the idea that the measured $V_s$ on Mars could be explained by zero porosity (unfractured) plagioclase feldspar. We note that the two groups of InSight-derived measured $V_s$ on Mars, based on P-S receiver function inversion from four versus two marsquakes, overlap at $\sim$2.6-2.75 km/s (Figure 1). If the deeper crust comprises 100% plagioclase and it is fractured and filled with gas, liquid water, or 2% cement and 98% gas or liquid water, the measured $V_s$ in the overlapping range may also be explained by layers of 100% plagioclase feldspar whose $\phi$=0.07-0.97, $\rho$=2.277-2.601 kg/m$^3$ (equation 2). These $\phi$ and $\rho$ ranges are consistent with Goossens et al. (2017) gravity-derived $\phi$ and $\rho$ ranges for the deeper crust. If the host rock is 100% plagioclase and unfractured, viscous creep-induced pore closure may occur at the same depth as the shallower seismic discontinuity, located at the top of the deeper crust. If up to 23% porosity exists, the onset of viscous creep-induced pore closure may occur at the same depth of the deeper seismic discontinuity, located at the base of the deeper crust; the porosity may also be due to post-pore-closure impact fracturing or other stresses. Together, our analyses suggest that if the host rock is 100% plagioclase, the deeper crust is unfractured with pore closure occurring at the shallow seismic discontinuity or hosts up to 23% porosity, which is filled with gas, liquid water, or 2% cement and 98% gas or liquid water. In the latter scenario, pore closure occurs at the deeper seismic discontinuity.

If porosity exists, pores in the deeper crust could be filled with gas, liquid water, or 2% cement and 98% gas or liquid water. We can not distinguish between gas and water-filled pores in the deeper crust since the $V_s$ difference between a gas- and water-filled host rock at this depth is less than 0.1 km/s (Figure 5a-d, S2) and the modeled geothermal gradient on Mars suggest that liquid water could be stable beneath 8 km (Clifford et al., 2010). Though the measured $V_s$ are consistent with a fractured host rock whose pores are ice filled, we infer that ice does not fill the pores because temperatures in the deeper crust are too high to freeze water (Clifford et al., 2010). Pores are likely filled with at least 2% cement because models for a 2% calcite cemented crust with 98% gas or liquid water are consistent with measured $V_s$ within the gravity-derived porosity range (0.10-0.23) (Goossens et al., 2017). Apart from calcite, other non-ice mineral cements could exist within the pores since the differences between the elastic mineral moduli of calcite and other expected cements within Mars’ crust (Table 1) result in $V_s$ differences no greater than $\sim$0.1 km/s if cement fills 2% of the pores. Cements usually precipitate from liquid water solutions. If the source of liquid water in the deeper crust is from the surface or upper crust, this liquid water needed to percolate to the deeper crust before temperatures in the upper crust became cold enough to freeze liquid water. Modest amounts of deeper crustal liquid water could also be supplied by intrusive magma below or within the deeper crust (Black et al., 2022).

5 Conclusions

This study uses rock physics models and shear wave velocities $V_s$ to constrain the volume and distribution of subsurface liquid water, mineral cements, and lithology beneath InSight on Mars. The upper crust (0-8 km) most likely comprises gas-filled frac-
tured basalts and minimally cemented (up to 2% in pores) sediment layers. Measured $V_s$ in the upper crust are too low for an ice-saturated cryosphere layer. The deeper crust (8-20 km) comprises consolidated basalts or plagioclase feldspar whose fractures have not closed entirely and may be filled with gas, water, or 2% non-ice mineral cements and 98% gas or liquid water. The range of measured $V_s$ in the deeper crust are also consistent with an unfractured plagioclase feldspar. The presence and quantity of liquid water in the pores would be better resolved by integrating our results with constraints from compressional wave velocities and seismic wave attenuation.

The results of this study have implications for the thermal and hydrogeological history of the Martian subsurface, beneath InSight. Pores within the deeper crust could remain open because the processes promoting porosity creation (e.g., chemical reactions such as dissolution or impact cratering) are more dominant than thermally-activated viscous creep-induced pore collapse. Pores could also be currently open because they were created by impacts after the rocks experienced pore collapse induced by viscous creep. Open pores could host liquid water that, if sourced from the surface or the upper crust, percolated to the deeper crust before temperatures became colder, freezing the water on its way down. Alternatively, liquid water could be introduced to the deeper crust via magmatic processes.

6 Data Availability Statement

No new data were used in this study. The InSight-derived seismic velocities that we used in this study are available in Knapmeyer-Endrun et al. (2021).

Acknowledgments

R. Kilburn, J. Dasent, and V. Wright acknowledge support from National Science Foundation grant 2136301. M. Manga acknowledges support from the CIFAR Earth 4D program and NASA grant 80NSSC19K0545. The authors thank NASA and the InSight team for their dedication, hard work, and vision, especially during this time when COVID-19 is real. The authors also thank the Mars, No Structure, Just Vibes retreat for providing a welcoming environment and support during the writing of this manuscript.

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Supporting Information for “Lithology, pore-filling media, and pore closure depth beneath InSight on Mars inferred from shear wave velocities”

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1. Method S1
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3. Tables S1 to S2

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Introduction

This Supporting Information contains the equations for the rock physics models that we use, two supporting figures, and two supporting tables. The rock physics models that we use are the Hertz-Mindlin granular media model (Mindlin, 2021), the cementation model (Dvorkin & Nur, 1996), and the Berryman self-consistent fractured media model (Berryman, 1980). We refer interested readers to the papers cited above and Mavko, Mukerji, and Dvorkin (2020) for more detailed derivations and descriptions of each model. The figures in this Supplementary Information present results from the Berryman self-consistent fractured media model. Tables S1 and S2 in this Supplementary Information outline and describe the notation for the equations, symbols, and functions presented in this work.
Method S1

We use granular and fractured media rock physics models to calculate dry-frame elastic moduli. We use the Hertz-Mindlin rock physics model (Mindlin, 2021) for cementless sediments, the cementation model (Dvorkin & Nur, 1996) for cemented sediments (whether the cements are at grain contacts or surrounding grains in contact), and the Berryman self-consistent fractured media model (Berryman, 1980) for fractured rocks. The notations for all the equations are in Table S2.

The equations for the Hertz-Mindlin rock physics model (Mindlin, 2021) are,

\[ K_{HM} = \left[ \frac{C^2(1 - \phi)^2 \mu_m^2}{18 \pi^2 (1 - v_m)^2} P \right]^{1/3}, \]  
\[ \mu_{HM} = \frac{2 + 3f - v_m (1 + 3f)}{5(2 - v_m)} \left[ \frac{3C^2(1 - \phi)^2 \mu_m^2}{2 \pi^2 (1 - v_m)^2} P \right]^{1/3}. \]  

The equations for the cementation model (Dvorkin & Nur, 1996) are,

\[ K_{CEM} = \frac{1}{6} C (1 - \phi_c) M_c \hat{S}_n, \]  
\[ \mu_{CEM} = \frac{3}{5} K_{CEM} + \frac{3}{20} C (1 - \phi_c) \mu_c \hat{S}_\tau, \]  
\[ M_c = \rho_c V_{pc}^2, \]  
\[ \mu_c = \rho_c V_{sc}^2. \]
\[ \hat{S}_n = A_n \alpha_c^2 + B_n, \alpha_c + C_n \]  \hspace{1cm} \text{(7)}

\[ A_n = -0.024153 \Lambda_n^{-1.3646}, \]  \hspace{1cm} \text{(8)}

\[ B_n = 0.20405 \Lambda_n^{-0.89008}, \]  \hspace{1cm} \text{(9)}

\[ C_n = 0.00024649 \Lambda_n^{-1.9864}, \]  \hspace{1cm} \text{(10)}

\[ \hat{S}_\tau = A_\tau \alpha_c^2 + B_\tau, \alpha_c + C_\tau, \]  \hspace{1cm} \text{(11)}

\[ A_\tau = -10^{-2} \left( 2.26 v_g^2 + 2.07 v_g + 2.3 \right) \Lambda_\tau^{0.079 v_g^2 + 0.1754 v_g - 1.342}, \]  \hspace{1cm} \text{(12)}

\[ B_\tau = (0.0573 v_g^2 + 0.0937 v_g + 0.202) \Lambda_\tau^{0.0274 v_g^2 + 0.0529 v_g - 0.8765}, \]  \hspace{1cm} \text{(13)}

\[ C_\tau = 10^{-4} \left( 9.654 v_g^2 + 4.945 v_g + 3.1 \right) \Lambda_\tau^{0.01867 v_g^2 + 0.4011 v_g - 1.8186}, \]  \hspace{1cm} \text{(14)}

\[ \Lambda_n = \frac{2 \mu_c (1 - v_g) (1 - v_c)}{\pi \mu} \frac{1}{(1 - 2v_c)}, \]  \hspace{1cm} \text{(15)}

\[ \Lambda_\tau = \frac{\mu_c}{\pi \mu}, \]  \hspace{1cm} \text{(16)}

\[ \alpha_c = \frac{a}{R}. \]  \hspace{1cm} \text{(17)}
The equations for the Berryman self-consistent fractured media model (Berryman, 1980) are,

\[ \sum_{i=1}^{N} x_i (K_i - K_{SC}^*) P^{*i} = 0, \]

\[ \sum_{i=1}^{N} x_i (\mu_i - \mu_{SC}^*) Q^{*i} = 0, \]

\[ P = \frac{1}{3} T_{iijj}, \]

\[ Q = \frac{1}{5} (T_{ijij} - \frac{1}{3} T_{iijj}), \]

where \( T_{ijkl} \) is a strain tensor relating far-field and intra-ellipsoidal strains given by Te Wu (1966). The tensors \( T_{iijj} \) and \( T_{ijij} \) are given by,

\[ T_{iijj} = \frac{3F_1}{F_2}, \]

\[ T_{ijij} - \frac{1}{3} T_{iijj} = \frac{2}{F_3} + \frac{1}{F_4} + \frac{F_4 F_5 + F_6 F_7 - F_8 F_9}{F_2 F_4}, \]

where,

\[ F_1 = 1 + A \left[ \frac{3}{2} (f_{sc} + \theta) - R \left( \frac{3}{2} f_{sc} + \frac{5}{2} \theta - \frac{4}{3} \right) \right], \]

\[ F_2 = 1 + A \left[ 1 + \frac{3}{2} (f_{sc} + \theta) - \frac{1}{2} R (3 f_{sc} + 5 \theta) \right] + B(3 - 4R) + \frac{1}{2} A(A + 3B)(3 - 4R) \left[ f_{sc} + \theta - R \left( f_{sc} - \theta + 2\theta^2 \right) \right], \]
\[ F_3 = 1 + A \left[ 1 - \left( f_{sc} + \frac{3}{2} \theta \right) + R(f_{sc} + \theta) \right], \] \quad (26)

\[ F_4 = 1 + \frac{1}{4} A \left[ f_{sc} + 3\theta - R(f_{sc} - \theta) \right], \] \quad (27)

\[ F_5 = A \left[ -f_{sc} + R \left( f_{sc} + \theta - \frac{4}{3} \right) \right] + B\theta(3 - 4R), \] \quad (28)

\[ F_6 = 1 + A[1 + f_{sc} - R(f_{sc} + \theta)] + B(1 - 0)(3 - 4R), \] \quad (29)

\[ F_7 = 2 + \frac{1}{4}[3f_{sc} + 9\theta - R(3f_{sc} + 5\theta)] + B\theta(3 - 4R), \] \quad (30)

\[ F_8 = A \left[ 1 - 2R + \frac{1}{2} f_{sc}(R - 1) + \frac{1}{2} \theta(5R - 3) \right] + B(1 - \theta)(3 - 4R), \] \quad (31)

\[ F_9 = A [(R - 1)f_{sc} + R\theta] + B\theta(3 - 4R), \] \quad (32)

and the variables \( A, B, \) and \( R \) are,

\[ A = \frac{\mu_i}{\mu_m} - 1, \] \quad (33)

\[ B = \frac{1}{3} \left( \frac{K_i}{K_m} - \frac{\mu_i}{\mu_m} \right), \] \quad (34)

\[ R = \frac{(1 - 2v_m)}{2(1 - v_m)}, \] \quad (35)
\( \theta \) and \( f_{sc} \) are functions given by,

\[
\theta = \begin{cases} 
\frac{\alpha_{sc}}{(\alpha_{sc}^2 - 1)^{3/2}} \left[ \alpha_{sc} (\alpha_{sc}^2 - 1)^{1/2} - \cosh^{-1} \alpha_{sc} \right] \\
\frac{\alpha_{sc}}{(1 - \alpha_{sc}^2)^{3/2}} \left[ \cos^{-1} \alpha_{sc} - \alpha_{sc} (1 - \alpha_{sc}^2)^{1/2} \right],
\end{cases}
\tag{36}
\]

for oblate spheroids (\( \alpha_{sc} < 1 \)),

\[
f_{sc} = \frac{\alpha_{sc}^2}{1 - \alpha_{sc}^2} (3\theta - 2).
\tag{37}
\]
References


Figure S1. Rock physics templates showing relationships between $V_s$, aspect ratio $\alpha$ (0.01 - 1.00), porosity $\phi$ (0.1 - 0.5), and varying percentages of ice in the pores of a basalt host rock, either (A) 10%, (B) 70%, (C) 80%, (D) 90%, or (E) 100% ice. Y-axis is logarithmic.
Figure S2. Rock physics templates showing relationships between $V_s$, aspect ratio $\alpha$ (0.01 - 1.00), porosity $\phi$ (0.1 - 0.5), and pore-filling media in a 100% plagioclase feldspar host rock. The yellow shading shows Knapmeyer-Endrun et al. (2021)'s measured $V_s$ overlap range in the deeper crust (also see Figure ??). The pores are filled with either (A) gas, (B) water, (C) 2% calcite cement and 98% gas, or (D) 2% calcite cement and 98% water. Y-axis is logarithmic.
Table S1.  $P$ and $Q$ coefficient equations for the background material ($m$) and inclusions ($i$).

<table>
<thead>
<tr>
<th>Inclusion shape</th>
<th>$P^m$</th>
<th>$Q^m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spheres</td>
<td>$\frac{K_m + \frac{1}{4} \mu_m}{K_i + \frac{1}{4} \mu_i}$</td>
<td>$\frac{\mu_m + \zeta_m}{\mu_i + \zeta_i}$</td>
</tr>
<tr>
<td>Needles</td>
<td>$\frac{K_m + \mu_m + \frac{1}{2} \mu_i}{K_i + \mu_m + \frac{1}{2} \mu_i}$</td>
<td>$\frac{1}{5} \left( \frac{4 \mu_m}{\mu_m + \mu_i} + 2 \frac{\mu_m + \gamma_m}{\mu_i + \gamma_i} + \frac{K_i + \frac{1}{2} \mu_m}{K_i + \mu_m + \frac{1}{4} \mu_i} \right)$</td>
</tr>
<tr>
<td>Disks</td>
<td>$\frac{K_m + \frac{1}{4} \mu_i}{K_i + \frac{1}{4} \mu_i}$</td>
<td>$\frac{\mu_m + \zeta_i}{\mu_i + \zeta_i}$</td>
</tr>
<tr>
<td>Penny cracks</td>
<td>$\frac{K_m + \frac{1}{4} \mu_i}{K_i + \frac{1}{4} \mu_i + \pi \alpha \beta_m}$</td>
<td>$\frac{1}{5} \left[ 1 + \frac{8 \mu_m}{4 \mu_i + \pi \alpha (\mu_m + 2 \beta_m)} + \frac{2 K_i + \frac{7}{2} (\mu_i + \mu_m)}{K_i + \frac{3}{2} \mu_i + \pi \alpha \beta} \right]$</td>
</tr>
</tbody>
</table>

$\beta = \mu \frac{3 K + \mu}{3 K + 4 \mu}$, $\gamma = \mu \frac{3 K + \mu}{3 K + 4 \mu}$, $\zeta = \mu \frac{9 K + 8 \mu}{6 (K + 2 \mu)}$, $\alpha =$ inclusion aspect ratio
Table S2. Notation for equations, variables, and formulas.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{HM}$</td>
<td>Dry-frame bulk modulus from Hertz-Mindlin</td>
</tr>
<tr>
<td>$\mu_{HM}$</td>
<td>Dry-frame shear modulus from Hertz-Mindlin</td>
</tr>
<tr>
<td>$C$</td>
<td>Coordination number</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Porosity</td>
</tr>
<tr>
<td>$\mu_m$</td>
<td>Mineral shear modulus</td>
</tr>
<tr>
<td>$v_m$</td>
<td>Mineral Poisson’s ratio</td>
</tr>
<tr>
<td>$P$</td>
<td>Hydrostatic confining pressure</td>
</tr>
<tr>
<td>$f$</td>
<td>Volume fraction of rough versus smooth grains</td>
</tr>
<tr>
<td>$K_{CEM}$</td>
<td>Dry-frame bulk modulus from the cementation model</td>
</tr>
<tr>
<td>$\mu_{CEM}$</td>
<td>Dry-frame shear modulus from the cementation model</td>
</tr>
<tr>
<td>$\phi_c$</td>
<td>Critical porosity</td>
</tr>
<tr>
<td>$\mu_c$</td>
<td>Shear modulus of the cement</td>
</tr>
<tr>
<td>$\hat{S}_n$</td>
<td>(proportional to) normal stiffness</td>
</tr>
<tr>
<td>$\hat{S}_\tau$</td>
<td>(proportional to) shear stiffness</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>Cement density</td>
</tr>
<tr>
<td>$V_{Pc}$</td>
<td>Cement P-wave velocity</td>
</tr>
<tr>
<td>$V_{Sc}$</td>
<td>Cement S-wave velocity</td>
</tr>
<tr>
<td>$v_c$</td>
<td>Cement Poisson’s ratio</td>
</tr>
<tr>
<td>$v_g$</td>
<td>Mineral Poisson’s ratio</td>
</tr>
<tr>
<td>$a$</td>
<td>Cement layer radius</td>
</tr>
<tr>
<td>$R$</td>
<td>Grain radius</td>
</tr>
<tr>
<td>$K_i$</td>
<td>Inclusion bulk modulus</td>
</tr>
<tr>
<td>$K_{SC}$</td>
<td>Self-consistent background medium effective bulk modulus</td>
</tr>
<tr>
<td>$\mu_i$</td>
<td>Inclusion shear modulus</td>
</tr>
<tr>
<td>$\mu_{SC}$</td>
<td>Self-consistent background medium effective shear modulus</td>
</tr>
<tr>
<td>$P, Q$</td>
<td>Geometric factors (Table S1)</td>
</tr>
<tr>
<td>$*_{i}$</td>
<td>$i^{th}$ inclusion material</td>
</tr>
<tr>
<td>$x_i$</td>
<td>$i^{th}$ background material volume fraction</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of phases</td>
</tr>
<tr>
<td>$T_{ijkl}$</td>
<td>Strain tensor (Te Wu, 1966)</td>
</tr>
<tr>
<td>$K_m$</td>
<td>Background material bulk modulus</td>
</tr>
<tr>
<td>$\mu_m$</td>
<td>Background material shear modulus</td>
</tr>
<tr>
<td>$v_m$</td>
<td>Background material Poisson’s ratio</td>
</tr>
<tr>
<td>$\alpha_{sc}$</td>
<td>Aspect ratio</td>
</tr>
<tr>
<td>$\theta, f_{sc}$</td>
<td>Functions</td>
</tr>
</tbody>
</table>

(Note: $f = 1$ means 100% rough grains)