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Dynamics of fluid displacement in mixed-wet porous media

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We identify a distinct two-phase flow invasion pattern in a mixed-wet porous medium. Time-resolved high-resolution synchrotron X-ray imaging is used to study the invasion of water through a small rock sample filled with oil, characterized by a wide non-uniform distribution of local contact angles both above and below 90°. The water advances in a connected front, but throats are not invaded in decreasing order of size, as predicted by invasion percolation theory for uniformly hydrophobic systems. Instead, we observe pinning of the three-phase contact between the fluids and the solid, manifested as contact angle hysteresis, which prevents snap-off and interface retraction. In the absence of viscous dissipation, we use an energy balance to find an effective, thermodynamic, contact angle for displacement and show that this angle increases during the displacement. Displacement occurs when the local contact angles overcome the advancing contact angles at a pinned interface: it is wettability which controls the filling sequence. The product of the principal interfacial curvatures, the Gaussian curvature, is negative, implying well-connected phases which is consistent with pinning at the contact line while providing a topological explanation for the high displacement efficiencies in mixed-wet media.

1. Introduction

If the angle that the interface between two fluids forms with a solid surface (the contact angle), is both lower and higher than 90° , meaning that there is a mix of hydrophobic and hydrophilic regions, a material is defined as being mixed-wet [1]. Flow in porous mixed-wet media is omnipresent in nature [2]. For example, lotus and rice leaves [3,4], butterfly wings and gecko feet [5,6], and also human skin [7] are natural porous systems which are not wetted by water and show different grades of mixed wettability (or hydrophobicity). Mixed wettability is studied for several applications: in the fabric industry, the wettability of textiles is altered to form anti-fogging, self cleaning materials [8]; in the medical and cosmetic sectors, wettability controls skin friction and lubrication [9], while in earth science the wettability of the subsurface governs secure storage of CO_2 [10,11], as well as oil recovery [12,13].

To investigate two-phase flow invasion patterns in a mixed-wet porous medium we will use high resolution X-ray microtomography which provides non-destructive, three dimensional (3D) visualisation of the fluids inside different porous materials, from rocks [14,15] to termite nests [16]. This technology has significantly increased the understanding of flow in porous media, including wettability characterisation with direct measurement of *in situ* contact angles [1,17].

Studies of mixed-wet porous rocks using laboratory-based microtomography show that the wettability has an impact on secure trapping of CO_2 [18] and oil recovery, which is favoured in mixed-wet media [19]. While these studies give valuable insights on the end states, they do not capture the displacement or pore-filling sequence, as several minutes (or hours) are required to obtain a single scan. The high photon flux available at synchrotron radiation sources, instead, allows for fast imaging and it can be used to study the dynamics of fluid invasion in porous media [20,21].

The physics of invasion patterns has been studied in systems with uniform contact angles [22–25]. Drainage, where a non-wetting phase displaces the wetting phase, is an invasion percolation process where fluid advances in a connected front from pore to pore through the restrictions, or throats, between pores [24,25]. Filling a wide region of the pore space requires a lower capillary pressure, P_c . The Young-Laplace equation defines the capillary pressure which, for throats with a cylindrical cross-section of radius r , gives:

$$P_c = \frac{2\sigma \cos \theta}{r}, \quad (1.1)$$

where σ is the interfacial tension between the two fluids and θ is their contact angle [2].

Synchrotron and confocal imaging studies have directly observed the dynamics of drainage at the pore scale, showing that this process is characterised by fast invasions of multiple pores (Haines jumps) [26,27], which cause interface recession and distal snap-off in other pores [28]. Snap-off is the filling of throats by the wetting phase, which can disconnect the non-wetting phase [29,30].

If we decrease the contact angle, and the invading phase becomes more wetting, there is a transition from invasion percolation towards frontal advance with displacement controlled by cooperative pore filling in the absence of wetting layer flow – the Cieplak-Robbins transition [31–33].

However, in most natural systems, the wetting phase can also flow in corners and roughness in the pore space (layer flow) [34]. In this case, the displacement of the non-wetting phase by a wetting phase (imbibition) has a more complex dynamics with snap-off [24,35–37]. The displacement becomes percolation-like as the wetting phase fills the narrowest pores and throats throughout the domain [2,34].

A recent study of displacement patterns, in micromodels with uniform contact angle, identified this transition from invasion percolation to connected advance, with layer flow for the most wetting systems. Varying the capillary number and the contact angle of the system, it was shown that the 3D structure has a strong impact on multiphase flow even in quasi-2D patterned microfluidic flow cells and that layer flow affected the dynamics [38].

51 However, the displacement dynamics have not been studied in a 3D mixed-wet porous
 52 material. In particular, we study displacement patterns in a porous rock with a non-uniform
 53 distribution of contact angles both above and below 90° . We dynamically image capillary-
 54 dominated waterflooding, after primary drainage, at high spatial and temporal resolution, using
 55 a synchrotron X-ray source. Our objective is to understand pore-scale displacement physics
 56 in mixed-wet media using observations and analysis of displacement patterns, and several
 57 multiphase flow descriptors including contact angle, interfacial curvatures, capillary pressure and
 58 interfacial area measured on the X-ray images.

59 From the experiments we observe a distinct type of displacement pattern, dissimilar to that
 60 seen either in drainage or imbibition: there is advance of a connected water front, through the
 61 centre of the pores, without evidence of layer flow. The filling sequence is not in order of pore
 62 size, while contact angle hysteresis does not allow for interface retraction and snap-off.

63 Pinning and depinning of the contact line had been previously related to the thermodynamics
 64 of wetting of 2D surfaces [39,40]. We show that, inside the 3D mixed-wet medium studied, the
 65 filling order is controlled by the thermodynamic contact angle [41], which represents the energetic
 66 threshold to be overcome for depinning of the interface for displacement.

67 Local pinning of the contact line also drives the interfaces to arrange as minimal surfaces, with
 68 the principal curvatures of opposite signs in orthogonal directions [42]. The consequent negative
 69 Gaussian curvature indicates well-connected phases, favouring high displacement efficiency [22].

70 Our findings can be extended to other kinds of porous materials. They have numerous
 71 practical applications such as evaporation of liquids inside hydrophobic materials [43], spray
 72 cooling, nanoassembly and DNA stretching, for instance, where a mixed-wet state could be
 73 designed for optimal process performance.

74 2. Materials and methods

75 (a) Materials

76 We used a small cylindrical sample of Ketton limestone, 16.1 *mm* long and with a diameter of
 77 5.8 *mm*. This was drilled from a larger core, where the permeability was measured to be 2.8
 78 $\times 10^{-12}$ m^2 [44]. The Helium porosity of the sample was 28%, and it is chemically composed of
 79 >99% calcite [35]. The experimental fluids were doped to increase their X-ray attenuation values,
 80 to enhance the image contrast [45] by using 15% by weight iododecane in *n*-decane for the oil
 81 phase and 20% by weight potassium iodide (KI) in water, which we call the water phase. Table 1
 82 provides the physical properties of the rock and fluids.

Table 1. Physical properties of the fluids and the rock used in the experiment. Density were measured at 40° and 7.6 *MPa*. Viscosity of water was measured at 50° and 10 *MPa* from [46] and of decane at ambient condition from [47]. The interfacial tension was measured directly using the pendant drop method [48,49]. The total porosity was measured using a Helium porosimeter, while the macro porosity was computed from the images, considering only the pore space which could be resolved with the given resolution (3.5 μm). The pore volume, PV, was computed considering the macro porosity. For the chemical composition refer to [28].

Fluid	Density [kg/m^3]	Viscosity [$\mu\text{Pa} \times \text{s}$]	Interfacial tension [mN/m]
Water + 20%w KI	1154	547	$\sigma_{ow} = 52.1 \pm 5$
Decane + 15%w C10H21I	715	1088	
Rock	Porosity	Chemical composition	Dimensions
Ketton	Total 28% Macro pores 14.3%	>99% Calcite	$d = 5.8 \text{ mm}$, $L = 16.1 \text{ mm}$ $\text{PV} = 6.03 \times 10^{-8} \text{ m}^3$

83 (b) Establishing wettability

84 The wettability of the rock was altered prior to the experiment. The sample was first cleaned
85 using methanol and dried in an oven for 24 *h*, to remove any impurities. Then, the pore space
86 was fully saturated with formation brine, replicating the chemical composition of brine in a
87 giant producing reservoir in the Middle East while the pressure and temperature were raised
88 to reservoir conditions (80 °C and 10 MPa). Crude oil, extracted from the same reservoir, was
89 injected, with increasing flow rate starting at 0.01 mL/min, up to 0.1 mL/min, for a total of 40
90 pore volumes (PV), from the bottom of the sample. The flow was then reversed and fresh oil
91 was injected from the top of the sample, with the same flow rates and total volume. During the
92 first week, 5 PV of oil were injected each day, at a flow rate of 0.05 mL/min. After 4 weeks, the
93 sample was removed from the core holder and left in a closed crude oil bath at 80 °C for 3 more
94 months. This process alters the wettability of the rock from being water-wet, where, as discussed
95 in the introduction, waterflooding is an imbibition process, to a mixed-wet state with a distinct
96 displacement dynamics which we analyse in this paper.

97 (c) Experiment and imaging

98 The sample was transferred to the I13-2 beamline of the Diamond Light Source synchrotron
99 facility (Harwell campus, Didcot, Oxfordshire) in the crude oil bath, at ambient temperature. It
100 was then mounted in a core holder and the crude oil was replaced with the oil phase (doped
101 decane), to avoid the formation of an emulsion during the experiment [50], by flushing 20 PV
102 of oil at a flow rate of 0.1 mL/min. 2 hours were then waited for the fluids to reach equilibrium.
103 Temperature and pressure were then increased to the experimental conditions (50 °C and 8 MPa)
104 and we started to inject water at low flow rate (0.50 μL/min), corresponding to a capillary number
105 of $Ca = \mu q / \sigma$ of 3.3×10^{-9} where μ is the injected water viscosity, q is the Darcy flux (volume
106 injected per unit area per unit time) and σ is the interfacial tension (Table 1 shows the physical
107 properties of rock and fluids).

108 65 tomograms were acquired during water injection, every 74 *s*, for a total of 4706 *s* (78.4
109 minutes), with an exposure time of 0.06 *s* and using 900 projections. The time required for each
110 scan includes 54 *s* of image acquisition and 20 *s* of file transfer and back-rotation of the sample.
111 Before and after water injection, static high quality images were taken using a higher number
112 of projections (3000) and exposure time (0.08 *s*). Each image contains $1280 \times 1284 \times 1280$ voxels
113 with a side length of 3.5 μm.

114 (d) Image processing and segmentation

115 The tomograms were reconstructed, obtaining 3D grey scale images of the rock and the fluids
116 within it during the injection (Figure 1). Image segmentation - the classification of each image
117 voxels into either water, oil or rock - was performed using a machine learning method, called
118 WEKA [51], which provides high quality results, if correctly trained [52] (Figure 2). We subtracted
119 the images before water invasion (Figure 1A) from those with water in the pores (Figure 1B)
120 to create a differential image. The result, shown in Figure 2C, is an image where the voxels
121 invaded by water can be clearly distinguished. The training dataset was created with manual
122 labelling of the differential image (Figure 2A). This was provided as an input, together with the
123 differential images, to the WEKA segmentation random forest algorithm. The resulting binary
124 images (Figure 2B) were combined with the segmented image of the rock to obtain the final
125 segmented images in which voxels of oil, water and rock were assigned to three discrete values
126 (Figure 2D).

127 To assess the accuracy of this segmentation method, we compared its results against those
128 obtained with the more established watershed segmentation technique [53], applied to the
129 static high quality images obtained at the end of the injection, used as a benchmark. The
130 two segmentation methods compared well, confirming previous work [52]. The watershed

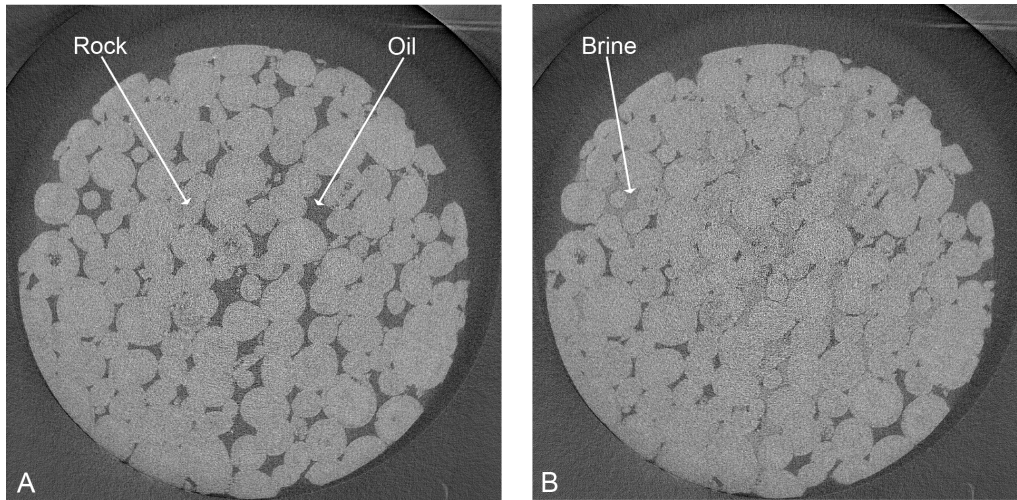


Figure 1. 2D slices of the 3D tomograms as they appear after reconstruction. Panel A shows a slice of the image before water injection, while the image in panel B was taken at the end of water injection.

131 segmentation algorithm could not be applied to the dynamic images, due to the higher level
 132 of noise caused by lower number of projections (Section 2(c)).

133 (e) Throat occupancy and Pearson correlation coefficient

134 We used the maximal ball extraction code of a generalized network model [54] on segmented dry
 135 images of the porous medium. We determined occupancy as the phase residing in the centre of
 136 the throats, local restrictions of the pore space. The detailed procedure to obtain throat occupancy
 137 is described elsewhere [45] and is based on relating any point in the pore space to a maximal
 138 ball, which is the largest sphere that fits in the pore space at that point. The throat radius is the
 139 radius of the maximal ball that lies in the centre of a restriction in the pore space. We also studied
 140 the correlation between throats radius and the time at which they were filled, using the Pearson
 141 correlation coefficient p between two variables x and y :

$$p = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}}, \quad (2.1)$$

142 where \bar{x} and \bar{y} are the estimated mean values and x_i and y_i are single observations of the two
 143 variables. r can range between -1 and +1 representing perfect negative and positive correlation
 144 respectively.

145 (f) Pore scale descriptors, contact angle and capillary pressure

146 The segmented images, obtained as described in Section 2(d), allow the computation of a large
 147 number of pore-scale quantities, which quantify the physics of displacement in mixed-wet media.
 148 Saturations were computed using the pore space which can be resolved with the voxel size of 3.5
 149 μm . We used previously-developed algorithms to find the geometric contact angle [55], interfacial
 150 curvature, specific interfacial area, capillary pressure [42] and the thermodynamic contact angle
 151 [41].

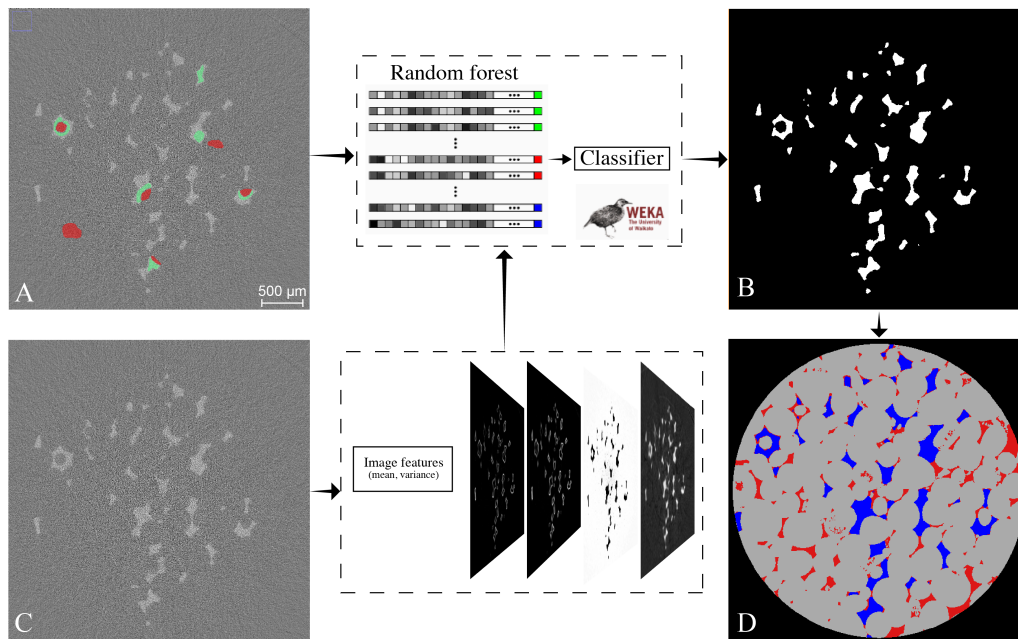


Figure 2. Workflow for image segmentation. The images before water injection (Figure 1A) were subtracted from the images with water (Figure 1B). The resulting differential image is shown in panel C of this image: brighter areas correspond to the places invaded by water. We created a training dataset with manual classification of invaded voxels (green in panel A) versus non invaded by water (red). The machine learning WEKA algorithm computes image features (mean and variance) from the differential image, and these are given as input, together with the training dataset, to the random forest algorithm. The result is a classifier. We applied this classifier to the differential images obtained at each time step and obtained – panel B – binary images with 1 (white) where water invaded the sample and 0 (black) elsewhere. Combining these with the dry segmented image of the rock, we obtained the final three-phase images (D). In D, grey is rock, red is oil and blue is water.

152 3. Results and discussion

153 We start with (a) experimental observation of invasion patterns, and (b) analysis of the order
 154 of throat invasion. Then, (c) the geometric contact angle was computed to define the wettability
 155 in comparison with previous work, followed by (d) saturation, interfacial area, curvature and
 156 capillary pressure measurements. Next, (e) the Gaussian curvature was estimated to assess
 157 connectivity and finally (f) the thermodynamic contact angle was computed using an energy
 158 balance [41].

159 (a) Experimental observation of invasion patterns

160 To observe invasion patterns we acquired images containing $1280 \times 1284 \times 1080$ voxels with a
 161 side length of $3.5 \mu\text{m}$ every 74 s over a period of 78.4 minutes, giving 65 3D images in total,
 162 while water was injected at a flow rate of $0.5 \mu\text{L}/\text{min}$. This corresponds to a capillary number
 163 $Ca = \mu q / \sigma$ of 3.3×10^{-9} (Section 2(c)). The top row of Figure 3 shows that it took 28.3 minutes for
 164 the water front to reach the other side of the imaged domain, which accounted for about 1/3 of the
 165 total rock volume. Until breakthrough, 0.24 pore volumes (PV) were injected. PV are computed
 166 considering the total macro pores of the whole sample which could be resolved with the image
 167 resolution of $3.5 \mu\text{m}$ (porosity and dimensions of the sample are available in Table 1).

168 Firstly, we notice that waterflooding in this mixed-wet rock happened differently to imbibition
 169 in water-wet systems: pore centres were invaded first, and oil was not isolated by snap-off [11,35].

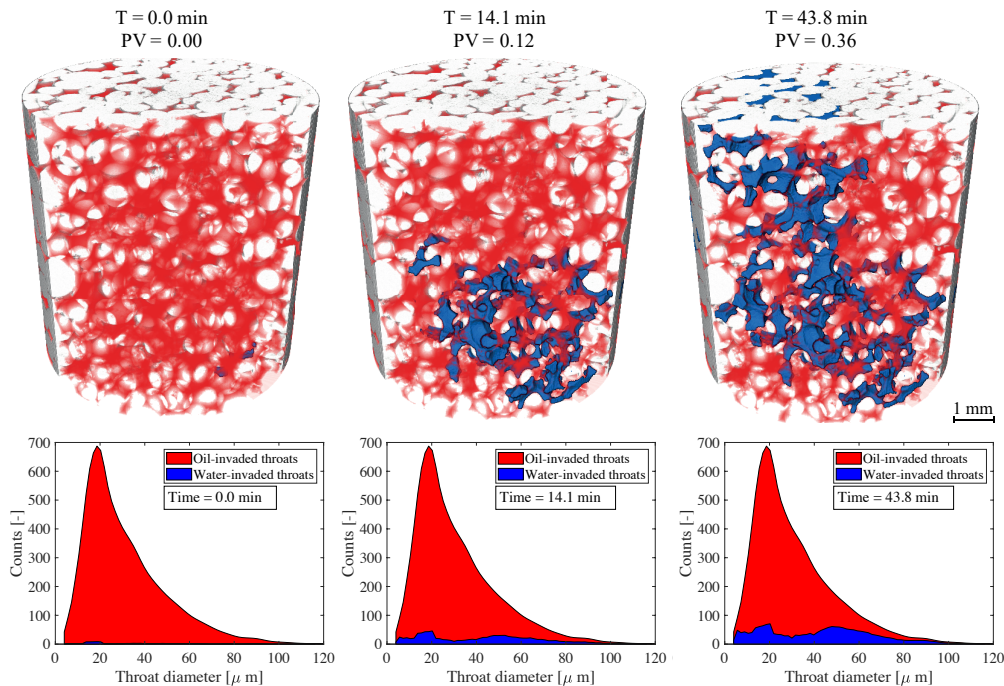


Figure 3. Top row: 3D rendering of rock (grey), oil (semi-transparent red) and water (blue) during waterflooding in the mixed-wet rock, at four time steps. Bottom row: Relation between the dimension of the throats and the phase occupying their centre at four times, during water invasion. Throats of a wide range of radii are invaded over time, showing that throat size is not the main parameter controlling filling. We started to count time since the onset of water invasion in the imaged domain. Pore volumes (PV) were computed considering the total macro pores of the whole sample.

170 Secondly, water displaced oil advancing as a connected phase, and once the pores were
 171 occupied, after breakthrough, water kept flowing in the same connected path, without further
 172 advancing and receding of the interfaces, due to interface pinning. This differs from previous
 173 visualisations of drainage, where distal snap-off and oil-filling events were observed [26,28,50].

174 Thirdly, Figure 3 shows that the order in which pores were invaded differs from what had
 175 been previously observed during either imbibition or drainage, as throats with a wide range of
 176 size were invaded throughout the displacement: there was no filling preference based on throat
 177 radius, as further discussed in Section 3(b).

178 Finally, local pinning of the oil-water interface, see Figure 4, provided evidence of a large
 179 contact angle hysteresis. Unlike in drainage [26], this pinning prevented the recession of the
 180 injected phase after invasion of a wide region of the pore space.

181 After pore invasion, the contact line found an equilibrium position and the contact points
 182 (where oil and water are in contact with the solid, shown in white in Figure 4) did not move. The
 183 reason for this is the adhesion of surface-active components of the crude oil to the solid combined
 184 with roughness which limits the movement of the contact line [1]. However, the pressure of
 185 the water increased as it was injected. This caused a change in curvature and interfacial area
 186 (described next) and an increase in the local contact angle. Once the contact angle overcomes
 187 a certain threshold value, the interface is unpinning and it moves further in the pore space [39,40].
 188 The system was characterized by contact angle hysteresis. In the example of Figure 4 contact angle
 189 increased by about 11° without unpinning and interface movement.

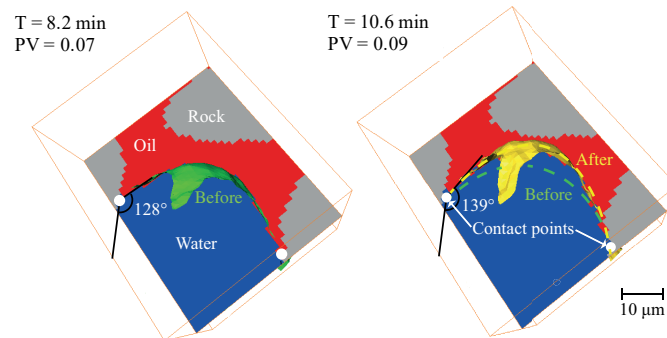


Figure 4. Visualisation of a detail of the pore space where we can observe the pinning of the oil-water interface. During water injection, while the three-phase contact points (white dots) did not move, the interface changed its shape (from green to yellow), with a consequent increase in contact angle. Pore volumes (PV) were computed considering the total macro pores of the whole sample.

190 (b) Order of throat invasion

191 As mentioned before, the classic invasion-percolation theory requires throats to be filled in
 192 decreasing order of size, as larger throats require lower capillary pressure to be invaded (Eq. 1.1).
 193 Pore-network models have been grounded on this basis and are able to predict the displacement
 194 sequence in water-wet media [56]. However, the bottom row of Figure 3 seems to contradict
 195 this hypothesis for this mixed-wet porous medium, as also throats of relatively small size were
 196 invaded at early stages (e.g. $T = 14.1$ minutes, $PV = 0.12$).

197 To investigate further this phenomenon, we considered the throats available to be filled at each
 198 time step t and compared their size with that of the throats which were actually filled by water at
 199 the following time step $t + 1$. Available throats are defined as throats which are connected to the
 200 water front at t . Figure 5 shows that, although there is a tendency for water to invade throats with
 201 a radius slightly higher than the average value (box-plots of Figure 5D), a considerable number
 202 of throats with small radius were invaded, even when larger ones were available for invasion
 203 (Figure 5C).

204 To quantify this observation we used the Pearson correlation coefficient p , Eq. 2.1, and tested
 205 the correlation between the radius of the invaded throats and the time at which they were filled.
 206 Only throats available to be filled were considered at each time step. The Pearson correlation
 207 coefficient p (Eq. 2.1) was -0.03 , which means that there is no correlation between throat radius
 208 and the order in which they are invaded, while invasion-percolation theory in a purely oil-wet
 209 medium would require a strong negative correlation. Figure 6A also shows that the data are
 210 scattered and no visual correlation can be noticed, confirming the quantitative result.

211 A number of pore-scale descriptors provide a complete physical and topological description
 212 of multiphase flow in porous media [11,57]. In the next sections we use these to deepen
 213 our understanding of the observed invasion patterns, and ultimately to understand what is
 214 controlling flow in mixed-wet porous media.

215 (c) Geometric contact angle

216 The *in situ* geometric contact angle, θ_g , computed inside the pore space using 3D images at
 217 high resolution, can be used to define the wettability [2,17,55]. θ_g is usually defined at the end
 218 of waterflooding, imposing the constraint of constant curvature and computing the geometric
 219 angle between the oil-water interface and the solid [17,55]. We computed contact angles, using the
 220 method developed in [55] on the static images taken at the end of water injection (Section 2(c)).
 221 The resulting distribution is shown in Figure 6B: the mean contact angle is 109° and the standard

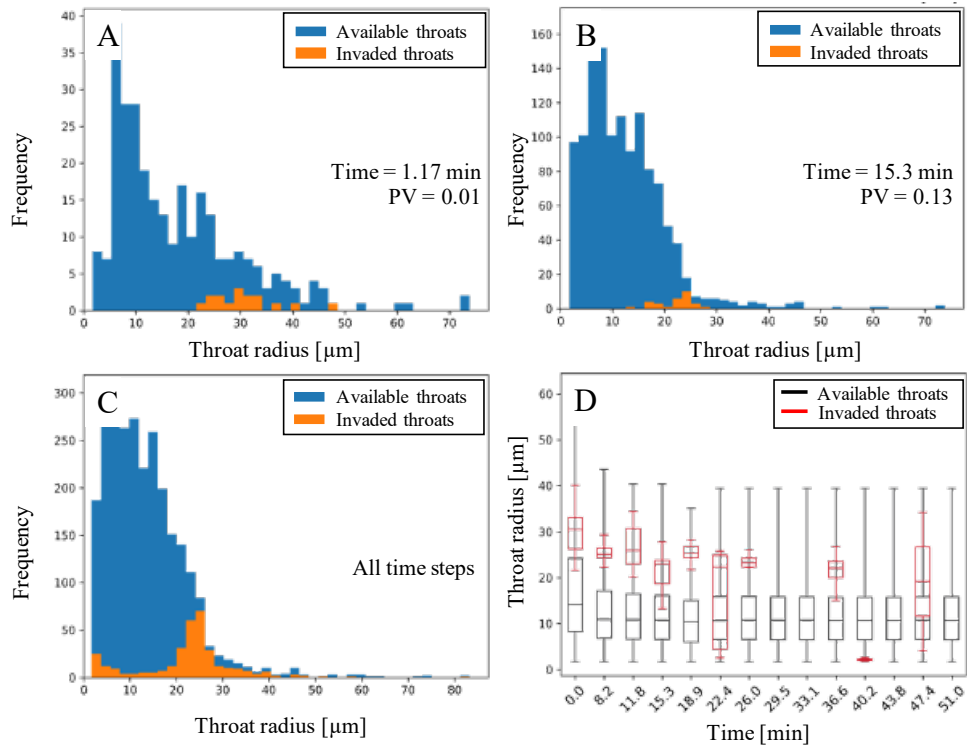


Figure 5. A-C: At each time step, we identified the throats available for invasion (connected to the water front) and compared these with the ones which were actually invaded. D shows box-plot of available and invaded throats at each time step.

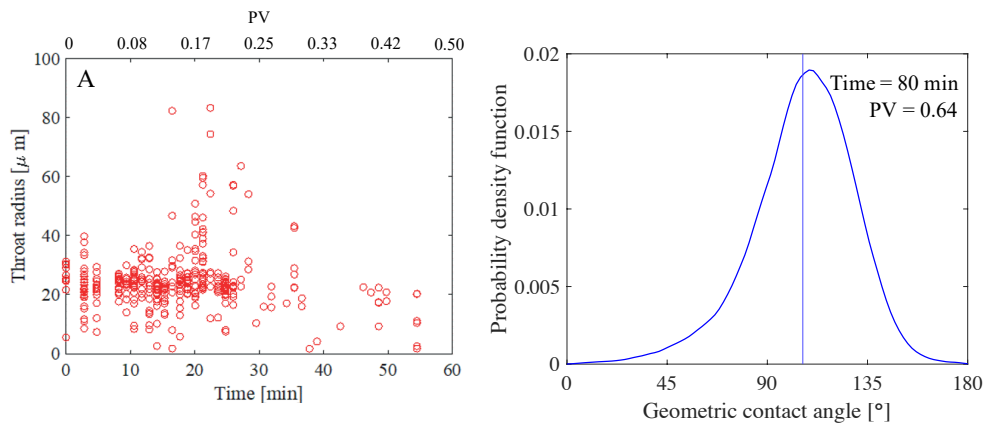


Figure 6. (A) Scatterplot where the radius of invaded throats is the dependent variable and the time at which they were filled is the dependent variable. (B) Geometric contact angle between water and oil on the high quality image taken the end of waterflooding. The vertical line shows the average contact angle (109°)

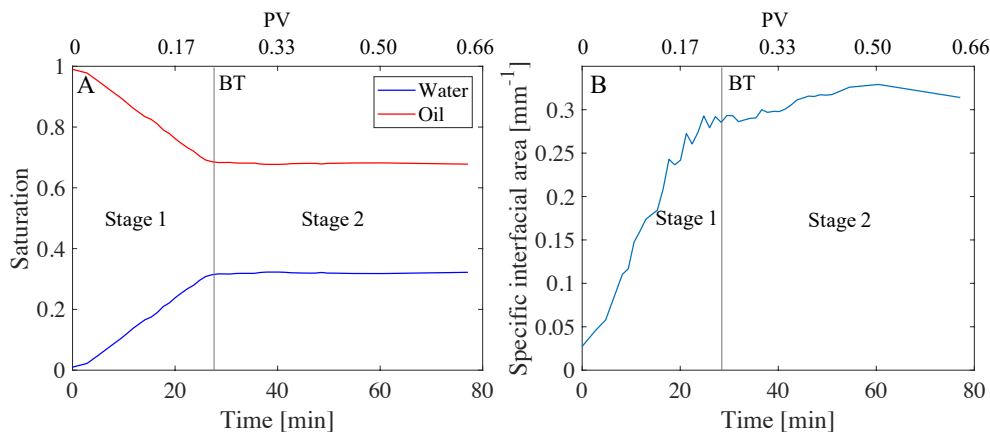


Figure 7. (A) Change in time of water and oil saturation, (B) specific interfacial area between oil and water. First and second stage are defined before and after breakthrough (BT) respectively.

222 deviation 23.2° . This indicates that the sample is indeed mixed-wet such that the solid does not
 223 display a strong preference for either water or oil, consistent with previous measurements on
 224 reservoir rocks [1].

225 (d) Saturation, interfacial area, curvature and capillary pressure

226 The saturation of water and oil was computed from the segmented images at each time step.
 227 Figure 7A shows that the water saturation linearly increased until breakthrough (28.3 minutes, or
 228 0.24 PV, after the onset of water invasion in the imaged domain) and then it did not change further.
 229 The final oil saturation is related to the very low flow rate in the experiment, which was chosen
 230 to be able to follow water invasion pore by pore. The behaviour of saturation over time defines
 231 two stages of the injection: the first stage, from the beginning to breakthrough, with noticeable
 232 changes in volume fractions, and the second stage, where the injected water flows through the
 233 sample with little additional displacement, with pinned three-phase contact lines, as shown in
 234 Figure 4. We also computed the saturation profile along the vertical direction, which did not show
 235 any significant variation. The pressure difference between the fluids due to the density difference
 236 is at most 69 Pa, which is much less than the capillary pressure, and so gravitational effects have
 237 a negligible influence on the saturation.

238 We extracted the oil-water interface which was smoothed to avoid voxelization artefacts [42].
 239 A rigorous assessment of the accuracy of the smoothing and its impact on estimated areas and
 240 curvatures is provided elsewhere [58] we can determine curvature to within 10% if the radius of
 241 curvature is 3 voxels or larger (more than $10 \mu\text{m}$) or a curvature less than 100 mm^{-1} , which, as
 242 we see later, is much larger than the average values we measure. The resulting surfaces were used
 243 to compute the specific interfacial area between oil and water (interfacial area per unit volume),
 244 as a function of time. The principal curvatures κ_1 and κ_2 were also found: the mean curvature
 245 is defined as $\kappa = (\kappa_1 + \kappa_2)/2$. Figure 7B and Figure 8A show that, while during the second stage
 246 volumes did not change, there was some relaxation of the fluid interfaces leading to an increase
 247 in curvature and interfacial area. The negative values of mean curvature show that on average it
 248 is water which bulges into oil, indicating slightly oil-wet or hydrophobic conditions on average,
 249 consistent with an average contact angle, Figure 6, above 90° .

250 The capillary pressure was found from the Young-Laplace equation, $P_c = 2\sigma\kappa$, using the
 251 measured interfacial tension, $\sigma = 52.1 \text{ mN/m}$ and plotted as a function of saturation in Figure 8B.
 252 The values are slightly lower than the one obtained in a sandstone sample whose contact angles
 253 were closer to 90° [42]. The shape of the capillary pressure-saturation relationship is in line with

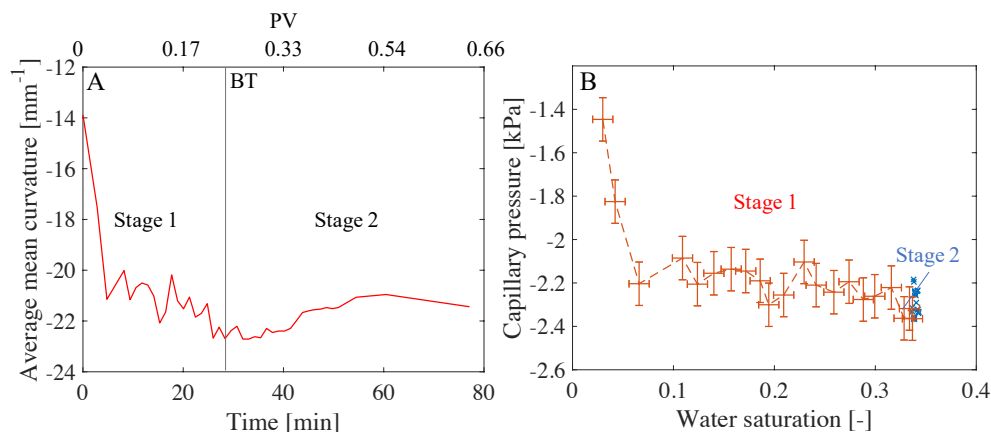


Figure 8. (A) Average mean curvature of the oil-water interface, (B) capillary pressure, before (dashed red line, 1st stage) and after (blue crosses, 2nd stage) breakthrough (BT).

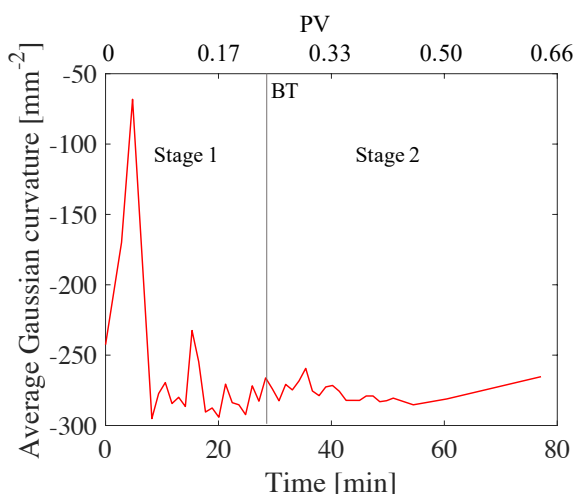


Figure 9. Gaussian curvature of the oil/water interface. Negative values imply high connectivity and favourable flow [57]. First and second stage are defined before and after breakthrough (BT) respectively.

254 the expected behaviour for a slightly oil-wet medium, starting from values approaching zero for
 255 low water saturation and plateauing afterwards, with a gradual decrease with saturation. After
 256 breakthrough (stage 2 of water invasion, blue crosses), the capillary pressure increases slightly,
 257 due to local relaxation of the interfaces.

258 (e) Gaussian curvature

259 We computed the Gaussian curvature $G = \kappa_1 \times \kappa_2$, shown in Figure 9 as the average over the
 260 entire oil-water interface. Remarkably, we observe that the Gaussian curvature is negative: this
 261 means that in most cases the curvatures have opposite signs in orthogonal directions [42].

262 This observation of negative Gaussian curvature has some interesting consequences for flow.
 263 A negative value indicates a well-connected object [57] implying that oil and water can both
 264 flow, providing an explanation for the favourable recoveries seen in many mixed-wet systems

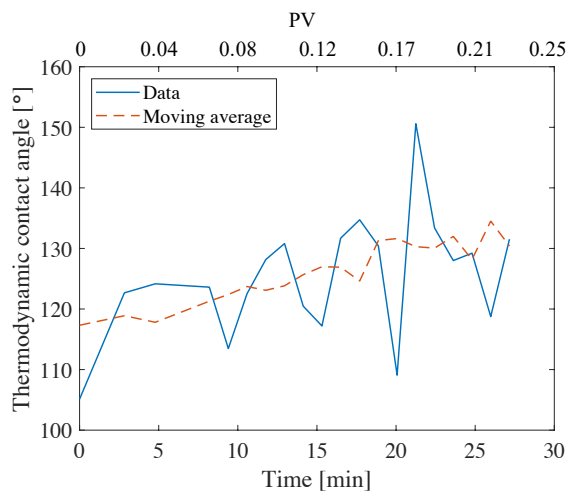


Figure 10. Thermodynamic contact angle, computed using Eq. (3.1) between two subsequent time steps. The 3-points moving average (dashed line) shows the increasing trend as the displacement proceeds.

265 [1,42]. Minimal surfaces are a special case with a mean curvature of zero: these form to
 266 minimize energy at pinned contacts in a range of circumstances from soap films, to foams and
 267 cell structures [42,59]. In our case we indeed have pinned contacts, which drives the interfaces to
 268 be approximately minimal surfaces; however, displacement at a contact angle of greater than 90°
 269 forces the curvature to be slightly negative to allow the water to displace oil in weakly oil-wet
 270 regions of the pore space.

271 (f) Thermodynamic contact angle controls interface movement

272 The geometric contact angle θ_g shown in Figure 6 characterizes the wettability of the system at
 273 rest after waterflooding. Since the fluid-fluid interfaces can hinge at fixed contact points, Figure 4,
 274 θ_g does not necessarily indicate the contact angles associated with displacement, or those values
 275 that should be used in a pore-scale numerical model [60].

276 In contrast, the thermodynamic contact angle θ_t is computed from an energy balance,
 277 considering the changes in saturation and interfacial areas and assuming that there is little viscous
 278 dissipation caused by interface recession and rearrangement, which is valid for this experiment
 279 [41]:

$$\Delta a_{ws} \cos \theta_t = 2\kappa\phi\Delta S_w + \Delta a_{wo}, \quad (3.1)$$

280 where ϕ is the porosity (considering macro pores only), and ΔS_w , Δa_{wo} and Δa_{ws} are the
 281 differences, between two subsequent time steps, in water saturation and in specific interfacial
 282 areas between water and oil, and water and solid.

283 In our experimental dataset θ_t can be computed only during the first stage of water invasion
 284 – before breakthrough – when both saturations and interfacial areas experience significant
 285 differences.

286 Figure 10 shows that θ_t has a value of approximately 110° at the beginning of waterflooding.
 287 While water invades the pore space, an increasing trend is noticed, with a value close to 130°
 288 at breakthrough. With respect to θ_g , the values of θ_t are in the upper region of the probability
 289 distribution, higher than the mean geometric contact angle at the end of waterflooding. Previous
 290 work has shown that the agreement between simulation and experiment is improved by using
 291 contact angles in oil-wet pores that are higher than the geometric value. [61].

292 During a displacement in a three-dimensional mixed-wet porous medium with a wide range of
293 contact angle, it is the advancing contact angle which determines the threshold capillary pressure
294 at which water can displace oil. Contact angle hysteresis causes this angle to be higher than θ_g ,
295 which is measured at the end of waterflooding, when the fluids are at rest [39,40]. The reason
296 for the large contact angle hysteresis is interface pinning, as described previously and shown in
297 Figure 4.

298 This contact angle hysteresis is why we did not observe interface recession and consequent
299 distal and Roof snap-off events [28,50]. On the other hand, interface pinning explains why, while
300 the invasion progresses, θ_t has to rise (Figure 10). An increase in the local energy is required to
301 overcome the interfacial forces caused by interface pinning, with a consequent increase of the
302 thermodynamic contact angle as water invades more oil-wet pores.

303 This shows that, in mixed-wet systems, the dominant parameter which controls water invasion
304 is the contact angle θ , and that the contact angle to be considered is θ_t , as this encapsulates the
305 energy required for the movement of the interface.

306 4. Conclusions

307 We have studied two-phase flow invasion patterns in a mixed-wet porous medium, using
308 dynamic high-resolution X-ray synchrotron imaging. Water invades the pores through their
309 centres, as a connected phase, without evidence of wetting layer flow. We identify a key signature
310 of invasion patterns in three-dimensional mixed-wet media – a wide range of non-uniformly
311 distributed local contact angles. We observe that the movement of water in the initially oil-filled
312 medium is limited by interface pinning, responsible for contact angle hysteresis. This prevents
313 interface recession and snap-off during the displacement. Water invasion does not happen in
314 decreasing order of throat size, meaning that other parameters must control the filling sequence.
315 The thermodynamic contact angle, which encapsulates an energy balance for pore invasion,
316 increases until breakthrough, showing that it constrains pore filling in mixed-wet media. This
317 new finding will be crucial for increasing the predictive abilities of pore-network models which
318 simulate the flow in such mixed-wet porous media.

319 The presence of pinned interfaces drives the oil-water interfaces to become nearly minimal
320 surfaces with a low mean curvature and a negative Gaussian curvature, meaning that the oil
321 bulges into the water in one direction, while water bulges into oil in the other direction. This
322 leads to well-connected phases and is the topological origin of high displacement efficiency in
323 mixed-wet media [1,42].

324 We suggest that we could engineer a mixed-wet state in either natural or artificial materials
325 to facilitate the simultaneous flow of two immiscible phases over a wide range of saturation. A
326 control on the pinning of the fluid/fluid interface would allow the design of optimal conditions
327 in porous materials, from oil recovery or CO₂ storage in rocks, to favourable drying of fabrics, the
328 manufacturing of nano devices and DNA stretching [43].

329 Future work can include the study of different porous media, to investigate the role of
330 geometry and rock heterogeneity. Also, models such as Cassie-Wenzel wetting transition can be
331 used to study the behaviour of contact angles including microstructural features such as surface
332 roughness.

333 **Data Accessibility.** All the 3D images of the sample and the fluids within it during invasion have been
334 uploaded to the Digital Rocks Portal (www.digitalrockportal.org/projects/253) [62]. The scripts
335 for image processing have been made open-access via GitHub (github.com/alessioscanziani).

336 **Authors' Contributions.** All authors were involved in the conceptualization of the research. BB and MJB
337 supervised the research. AS, QL, and AA performed the investigations. All authors were involved in the
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