Too many streams and not enough time or money? Analytical depletion functions for streamflow depletion estimates

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Article Impact Statement: Analytical depletion functions are accurate tools to estimate impacts of pumping on streams over different hydrologic landscapes.
Abstract

Groundwater pumping can cause streamflow depletion by reducing groundwater discharge to streams and/or inducing surface water infiltration. Analytical and numerical models are two standard methods used to predict streamflow depletion. Numerical models require extensive data and efforts to develop robust estimates, while analytical models are easy to implement with low data and experience requirements but are limited by numerous simplifying assumptions. We have pioneered a novel approach that balances the shortcomings of analytical and numerical models: analytical depletion functions (ADFs), which include empirical functions expanding the applicability of analytical models for real-world settings. In this paper, we outline the workflow of ADFs and synthesize results showing that the accuracy of ADFs compared against a variety of numerical models from simplified, archetypal models to sophisticated, calibrated models in both steady-state and transient conditions over diverse hydrogeological landscapes, stream networks, and spatial scales. Like analytical models, ADFs are rapidly and easily implemented and have low data requirements but have significant advantages of better agreement with numerical models and better representation of complex stream geometries. Relative to numerical models, ADFs have limited ability to explore non-pumping related impacts and incorporate subsurface heterogeneity. In conclusion, ADFs can be used as a stand-alone tool or part of decision-support tools as preliminary screening of potential groundwater pumping impacts when issuing new and existing water licenses while ensuring streamflow meets environmental flow needs.
The importance of streamflow depletion

Groundwater pumping has caused dramatic groundwater depletion globally over the past decades, and anticipated growing demands from agricultural irrigation and other human activities may further stress groundwater sustainability (Döll et al. 2014; Gleeson et al. 2020; de Graaf et al. 2019; Rodell et al. 2018; Wada et al. 2014; Wada et al. 2012). In addition to declines in aquifer storage, groundwater pumping affects surface water by reducing groundwater discharge to surface water and/or inducing surface water infiltration, which together are defined as “streamflow depletion” (Barlow and Leake 2012; Gleeson and Richter 2018; Konikow and Leake 2014; Theis 1941). At the onset of pumping, most of the pumped water is from groundwater storage (Figure 1a). As pumping continues, an increasing proportion of water comes from capture rather than storage (Leake 2011; Leake et al. 2010) (Figure 1b). Assuming pumping has limited effects on groundwater recharge or non-stream discharge (e.g., groundwater evapotranspiration), pumped water inevitably depletes streamflow in aquifers that are hydraulically connected to surface water features, but the timing and magnitude of streamflow depletion depend on local hydrogeological conditions and well location and pumping rate (Bredehoeft and Durbin, 2009; Walton, 2011). In extreme cases, streamflow depletion can lead surface water to disconnect from groundwater systems and even dry up streams (Barlow and Leake 2012; Bierkens and Wada 2019). Streamflow depletion threatens environmental flow needs (defined as volume and timing of streamflow required for the proper functioning of the aquatic ecosystem) and thus affects the health of the aquatic ecosystem at watershed (Essaid and Caldwell 2017; Zeng and Cai 2014; Zipper et al. 2019a), regional (Forstner and Gleeson 2019; Maxwell and Condon 2016; Reba et al. 2017), and global scales (de Graaf et al. 2019). Therefore, understanding and quantifying streamflow
depletion is critically important for conjunctive water management, particularly for regions with intensive groundwater consumption.

We have developed analytical depletion functions (ADFs) to advance streamflow depletion assessment. The ADFs have been tested in study sites with different hydrogeologic landscapes, stream networks, and spatial scales and proved to be accurate tools for estimating streamflow depletion in real-world settings. To date, there is no synthetic review to summarize the performance and limitation of this new tool. In addition, guidelines for using numerical model to assess streamflow depletion, especially in relation to simpler tools, is lacking. Therefore, the main objective of this paper is to describe and advance ADFs for streamflow depletion prediction as an effective and efficient new tool in the water sustainability toolbox for researchers, consultants and industry. For ease of use, ADFs have been implemented in the open-source ‘streamDepletr’ package for R software (Zipper 2019), but can be calculated in a relatively straightforward manner in any programming language. In this paper, we review the methods that have been used for streamflow depletion assessment, highlight the workflow of a single ADF which has performed best across multiple domains as an example, and review the performance of ADFs in tested domains with diverse hydrogeological settings to enhance the confidence of applying ADF.
Figure 1 a) Streamflow depletion and groundwater depletion caused by a groundwater pumping well. b) Status of streamflow depletion since the start of pumping: decreased groundwater discharge or induced infiltration of water from the river. c) Source of pumping groundwater from streamflow depletion and groundwater depletion through time since the start of pumping. Figures from Gleeson and Richter (2018).

Methods to quantify streamflow depletion

In practice, it is not feasible to quantify streamflow depletion based solely on field measurements because (1) sufficient monitoring networks are often lacking, and natural flow variability creates
challenges for isolating the changes in flow from pumping (e.g., Flores et al. 2020); (2) where there are sufficient measurements available, streamflow depletion must be a significant fraction of streamflow so that changes in streamflow caused by pumping can be measured (Barlow and Leake 2012; Flores et al. 2020); and (3) it may take days to years for a well stress to be manifested in a stream depending on local hydrogeological conditions, such as surface water and groundwater interactions, and the distance from the well to nearby streams (Alley et al. 1999; Barlow and Leake 2012; Bredehoeft and Durbin 2009; Konikow and Leake 2014; Walton 2011). Due to these challenges, field-based measurement of streamflow depletion is only feasible for localized sites at the scale of an individual reach (e.g., Sophocleous et al. 1988; Hunt et al. 2001; Kollet and Zlotnik 2003). Often the knowledge gained from a small scale cannot directly be applied at large watershed/regional ones due to the spatial heterogeneity in hydrological conditions and groundwater use.

Therefore, streamflow depletion is evaluated with a variety of non-field-based methods from local experts’ guesses to advanced numerical models (Figure 2). Local experts’ guesses – for example, often assigning all streamflow depletion to the nearest stream segment with no time lags between the pumping and impacted streams – can provide rough estimates of streamflow depletion but do not account for the spatial and/or temporal scales that influence pumping impacts on streamflow and are therefore assumed to be a low accuracy approach in Figure 2. Numerical groundwater models and analytical models are the two most common approaches to quantify streamflow depletion (Barlow and Leake 2012). Numerical models are widely used for site-specific assessment due to their more sophisticated representation of surface and subsurface flow processes compared to analytical models (Ahlfeld et al. 2016; Feinstein et al. 2016). As a result, streamflow depletion
estimates from numerical models are often considered to be the “gold standard”, but they are often limited by data availability, and require significant human and financial resources to develop, calibrate, and validate them (Rathfelder 2016). In general, numerical models are calibrated for study sites based on historical field observations which are treated as a baseline scenario. To assess streamflow depletion, pumping scenarios can be simulated by either turning off existing pumping wells, changing their pumping schedules/volumes, and/or adding new pumping wells. Streamflow depletion due to groundwater pumping can then be calculated as the difference in streamflow and stream-aquifer exchange between the pumping and baseline scenarios.

In contrast to numerical models, analytical models are relatively easy to implement with low data and experience requirements and are thus well-suited to provide initial estimates of groundwater pumping effects on streamflow (Flores et al. 2020; Reeves et al. 2009), though they rely on numerous simplifying assumptions. For instance, the commonly-used Glover model assumes an infinite horizontal homogenous and isotopic aquifer bounded by a single linear stream with full penetration of the aquifer (Glover and Balmer 1954). Therefore, it is commonly accepted that the accuracy of analytical models is generally lower than numerical models. Thus far, analytical models for a myriad of subsurface conditions and stream geometry types have been derived [see the details in review paper of Huang et al. (2018)], including confined (Theis 1941; Glover and Balmer 1954; Hunt 1999; Chen and Yin 2004; Sun and Zhan 2007; Singh 2009), unconfined (Huang et al. 2011; 2012), and leaky aquifers (Hunt 2003, 2008; Butler et al. 2007; Zhan and Park 2003). Based on early analytical models, Jenkins (1968a, 1968b) introduced the concept of a streamflow depletion factor, which is calculated as \((d^2S)/(T)\), where \(d\), \(S\), and \(T\) are distance, storativity, and transmissivity between pumping wells and streams, respectively. Jenkins (1968a,
1968b) showed how the principle of superposition could be applied to analytical models to estimate streamflow depletion under cyclic pumping schedules. Mathematically, Jenkins’ stream depletion factor is equal to the amount of time it would take for depletion to equal 28% of the pumping rate at a nearby stream using the Glover and Balmer (1954) analytical model (Barlow and Leake, 2012). As a result, the stream depletion factor can be mapped within an area of interest (such as an aquifer or watershed) to show how much the time it would take for streamflow depletion to have an appreciable impact on a stream based on different pumping locations.

A widespread limitation of analytical models is that they consider pumping impacts on a single stream, with a few exceptions (Sun and Zhan, 2007; Huang et al. 2014, 2018), and streams in these analytical models are assumed to be linear. As a result, when analytical models are implemented in practice, all streamflow depletion is assigned to a single stream segment (typically the segments nearest to the well). As such, the spatial impacts of pumping on streams cannot be evaluated which can lead to an overestimate of streamflow depletion in some stream segments that are close to pumping wells and an underestimate of streamflow depletion for stream segments that are further away from the pumping wells. This may be more pronounced when multiple pumping wells exist in the study domain. To our best knowledge, theory for use of analytical models in complex, sinuous stream networks is lacking (Huang et al. 2014, 2018). Therefore, there is an apparent gap between analytical and numerical models and a need to advance application of analytical models for the real-world settings with a complex stream network of multiple and non-linear stream segments to improve streamflow depletion estimation.
Figure 2 Diagram showing methods and accuracy in assessing streamflow depletion in conjunctive water management. Accuracy in this figure refers to the method that can estimate the streamflow depletion best in real-world settings with multiple streams. Inset figures are from Zipper et al. (2019b) and Harbaugh (2005).
Analytical depletion functions

To date, limited work has been done to advance analytical models’ application in real-world settings with multiple and sinuous stream segments. Before we developed analytical depletion functions (ADFs), as far as we know, the Michigan Water Withdrawal Assessment Tool (Reeves et al. 2009) is the only study that integrates the concept of depletion apportionment to distribute the depletion by a pumping well to multiple stream segments - a significant advancement for the practical application of analytical models. However, this method was tested in only a single watershed. Inspired by this concept, Zipper et al. (2018) tested five depletion apportionment equations across wide ranges of stream networks in British Columbia and found that a new depletion apportionment approach which explicitly considers stream geometry mostly closely matched results from a numerical model. This promising test demonstrated that inclusion of stream geometry and depletion apportionment can improve the accuracy of analytical models in the real-world settings.

Subsequently, Zipper et al. (2019b) introduced the concept of ADFs, which combine (1) stream proximity criteria used to determine which stream segments are most likely to be affected by a pumping well; (2) depletion apportionment equation which is a geometric method to distribute depletion among the affected stream segments; and (3) analytical models which are to calculate the amount of depletion for all impacted stream segments based on the previous two components. In that study, Zipper et al. (2019b) compared 50 different combinations of stream proximity criteria, depletion apportionment equations, and analytical models, and found that the choice of depletion apportionment equations had the greatest impact on the match between numerical models and ADFs. Streamflow depletion predicted by ADFs is significantly different from analytical models.
alone (Zipper et al. 2019b, 2021). Here, we use the best performing ADF from prior studies to show the workflow of applying and using an ADF. But it is important to note that a myriad of ADFs are possible with different combinations of the stream proximity criteria, depletion apportionment equation and analytical models.

A practitioner’s workflow of analytical depletion functions

Data Acquisition

Geospatial data including stream network and well locations are the two primary input parameters, which are often available from government agencies and remotely sensed products (e.g., MERIT Hydro, Yamazaki et al. 2019; HydroSHEDS, Lehner and Grill 2013). The availability of hydrostratigraphic data (i.e., hydraulic conductivity, water table depth, and storativity) are highly variable across global regions for various spatial scales. For small scales, ideal estimates of hydrostratigraphic properties are derived from field data, such as pumping tests at specific study sites. For large regions, global data compilation of permeability and porosity [e.g., GLobal HYdrogeology MaPS, GLHYMPS (Gleeson et al. 2014; Huscroft et al. 2018)], for instance, can provide useful inputs for the ADFs. The choice and resolution of data should be consistent with research and management objectives, as the quality and scale of this data will strongly influence streamflow depletion predictions.
Figure 3. Diagram showing workflow to implementation of analytical depletion functions, which include (a) stream proximity criteria, (b) depletion apportionment equations, and (c) analytical model. Inset figures are from Zipper et al. (2019b).

Data processing for analytical depletion functions

The different combinations of ADFs can derive different streamflow depletion estimates. The preliminary assessments showed that the choice of depletion apportionment equation has the largest impact on streamflow depletion estimates with ADFs, followed by stream proximity criteria, and analytical model, which highlights the importance of ADFs over standalone analytical models (Zipper et al. 2019b). Zipper et al. (2019b) compared 50 ADFs concluding that a combination of “adjacent + expanding” stream proximity criteria, “web squared” depletion apportionment equation, and Hunt analytical model (Figure 3) performed best in the Navarro River Watershed, California, USA. Subsequent work in two different hydrologic landscapes in British Columbia, Canada (Li et al. 2020) and the Republican River basin, USA (Zipper et al. 2021) further supported that this ADF performed best across different stream network and hydrogeological conditions.
However, ADF intercomparisons have only been carried out in these four settings to date, and it should be noted that other ADFs may perform better in other regions or settings. We, therefore, suggest that more case studies are needed to advance the application of ADFs, and future studies can test the best-performed ADFs derived first.

The “Adjacent+Expanding” stream proximity criteria (first described in Zipper et al. 2019b) selects any stream segment that is in a catchment adjacent to the well or is within the maximum radial distance where depletion would be at least 1% of the pumping rate at a given time step. From these stream segments, depletion apportionment equations then calculate the fraction of total depletion allocated to each stream segment. The Web Squared depletion apportionment equation (first described in Zipper et al. 2018) splits each stream segment into a finite number of points (e.g., space between each point is 5 meters) and apportion depletion based on the square of the inverse distance of each stream segment to the well as shown in Eqn. (1).

\[
 f_i = \frac{\sum_{p=1}^{P_i} \frac{1}{d_{i,p}^2}}{\sum_{j=1}^{n} \sum_{p=1}^{P_j} \frac{1}{d_{j,p}^2}}
 \]

where \( f_i \) is the depletion fraction of total streamflow depletion from a well apportioned to a stream segment, \( P \) is the total number of points which a stream segment is divided into the web squared equation, \( d \) is the distance from a well to a stream segment, and \( n \) is the total number of stream segments meeting the stream proximity criteria.

In the current “streamDepletr” package (Zipper 2019), the Glover (Glover and Balmer 1954) and Hunt analytical models (Hunt 1999) are included due to the simplicity of implementation, as well as the Jenkins (1968a) superposition approach for irregular pumping schedules. These two models
have similar assumptions, which include 1) a homogenous and isotropic aquifer that extends an
infinite distance away from the stream with no vertical groundwater flow; 2) The aquifer is
confined, or is assumed to quasi-confined with transmissivity and saturated thickness constant over
the pumping period; 3) The pumping well fully penetrates the saturated aquifer and the water is
instantaneously released from aquifer storage with no lag times. These two models have different
assumptions of stream and aquifer interaction, which allows ADFs that can be applied in settings
with variable hydrogeologic characteristics and data availability. Specifically, the Glover model
assumes that streams fully penetrate the aquifer, i.e., no resistance to flow through the streambed,
while the Hunt model assumes the streams partially penetrate the aquifer with a streambed
clogging layer of finite thickness (b_r) and hydraulic conductivity (K_r) impede water exchange
between the aquifer and streams. The volumetric streamflow depletion rate, Q_a of a stream segment
can be calculated by Eqns. (2) and (3) for the Glover model and Hunt model, respectively.

\[ Q_a = Q_w \cdot \text{erfc} \left( \sqrt{\frac{S d^2}{4Tt}} \right) \]  \hspace{2cm} (2)

\[ Q_a = Q_w \cdot \left( \text{erfc} \left( \sqrt{\frac{S d^2}{4Tt}} \right) - \exp \left( \frac{\lambda^2 t}{4ST} \right) \text{erfc} \left( \frac{\lambda^2 t + \lambda d}{4ST} \right) \right) \]  \hspace{2cm} (3)

Where, \( Q_w \) is the pumping rate (L^3/T); \( t \) is the time since the start of pumping (Time); \( d \) is the
well-stream distance (L). \( S \) is the aquifer storage coefficient (e.g., specific yield in an unconfined
aquifer, unitless); \( T \) is aquifer transmissivity (L^2/Time). Theoretically, \( T \) and \( S \) are the effective
values averaged between pumping wells and affected stream segments. Zipper et al. (2018, 2019b,
2021) used the weighted average values of \( T \) and \( S \) of the shortest distance between well and
stream segments while Li et al. (2020) adopted \( T \) and \( S \) at the well locations. As a result, two ways
of calculations provided consistently accurate estimates of streamflow depletion. In the Appendix,
we outline the method in detail to calculate the effective $T$ and effective $S$ for homogenous and heterogenous conditions. $\lambda$ is the streambed conductance. The streambed conductance is defined as $\lambda = w_r \frac{K_r}{b_r}$, where $w_r$ is the width of the stream segments (L). It should be noted that streambed conductance in the Hunt model has a unit of Length$^2$/Time and can be interpreted as the conductance per unit length of a stream (Li et al. 2020).

**Potential applications, output analysis, and data visualization**

The output of streamflow depletion can be used in various ways. 

1) ADFs can be used as a pre-screening tool to prioritize areas with potential streamflow depletion concern for more detailed study using field study and/or numerical models in order to better target developing a local numerical model. 

2) Based on the spatial impacts of streamflow depletion, watershed maps, for example, to identify which areas would lead to the greatest streamflow depletion in stream segments of interest to facilitate a targeted management approach. This is analogous to the “capture maps”, which is similar to the one in Leake et al. (2010), but allows for the specific investigation of impacts on multiple stream segments which is not possible using analytical models. 

3) The simple implementation of ADFs can serve as a basis for developing the decision support tool due to their low computational needs (Huggins et al. 2018). 

4) ADFs offer water managers a useful tool to consider spatially-distributed streamflow depletion impacts when considering new applications for groundwater rights. The importance of streamflow depletion has been well-recognized in water research and management communities. For instance, the *Sustainable Groundwater Management Act* (2014) in California, USA, identifies the depletion of interconnected surface waters as one of six core undesirable results. Similarly, the *Water Sustainability Act* (2016) in British Columbia, Canada mandates that groundwater pumping must
not reduce streamflow below the environmental flow needs where streams and aquifers are hydraulically connected. ν) ADFs also provide a user-friendly approach to explore various scenarios, such as different pumping schedules (e.g., pumping rate and well depth), well-stream distance, and sensitivity tests of local hydrogeological parameters to understand streamflow depletion and potential mitigation options.

**Performance of analytical depletion functions in tested domains and future research**

The performance of ADF has been tested across a wide variety of hydrologic landscapes, streamflow networks, and spatial scales in four watersheds (Table 1). The accuracy of ADFs has been tested by comparing against numerical models from archetypal (Zipper et al. 2018) to calibrated ones (Li et al. 2020; Zipper et al. 2021), from steady-state (Zipper et al. 2018) to transient conditions (Zipper et al. 2019b; Li et al. 2020), and have been adopted for theoretical tests (Zipper et al. 2018) to decision-support tools (Huggins et al. 2018; Li et al. 2020). Rather than provide a detailed description of each of four models we compare, Table 1 synthesizes the location, hydrologic landscapes (Winter 2001), domain size, elevation range, hydraulic conductivity ranges, specific yield ranges and number of hydrostratigraphic units. The models cover hydrological landscapes, including *plateau and highlands, mountain valley, and riverine valley*. The tested domains’ size ranges from small (165 km²) to large watershed (77868 km²) with hydrostratigraphic materials from simple (1 unit) to complex (6 units) and hydraulic conductivity spans from magnitudes of $1 \times 10^{-12}$ to $1 \times 10^{-3}$ m/s.

We consider three metrics that can comprehensively assess ADF performance: 1) *correct identification of most affected streams*, which is shown as the percentage of wells for which ADFs
and MODFLOW agree on the most affected stream segment; 2) mean absolute error (MAE) for most affected stream segments normalized by pumping rate, which is the MAE between the ADF and numerical model for the most affected streams; 3) MAE for all affected streams normalized by pumping rate, which quantifies the overall accuracy of ADF for all affected stream segments.

As shown in Table 1, ADFs can capture all most-affected streams in a small watershed (BX Creek) with simple hydrostratigraphic units and stream networks, while its capability reduces to more than 50% in large watersheds with complex hydrostratigraphic units and stream network (Republican River). In addition, the MAE of the most affected streams is less than 15% of the pumping rate, indicating that ADF can accurately identify and estimate the streamflow depletion for the most affected streams. Furthermore, the MAE for all affected streams is less than 5% of the pumping rate, which is much smaller than those of most-affected streams as streams experiencing small depletion and hence deriving small errors are included in the assessment. Based on the above summary, the performance of ADFs proves that ADF can be an accurate tool for the streamflow depletion assessment over diverse hydrogeological landscapes and scales tested to date. We also suggest that future studies without numerical models, the best-performed ADFs with a combination of “adjacent + expanding” stream proximity criteria, “web squared” depletion apportionment equation, and Hunt analytical model can be applied for streamflow assessment, but the potential uncertainties should be acknowledged.

Despite the overall acceptable performance of the ADFs in a limited number of studies, we also found that their performance varies with different hydrogeological factors.
1) **Pumping rate and schedule.** ADFs had small errors with a continuous pumping schedule in Navarro River watershed, and also had a good performance for the intermittent pumping, and tended to perform better during the pumping season than the non-pumping season in three domains (Zipper et al. 2019b; Li et al. 2020). Moreover, test in the Republican River domain found that the performance of ADFs is insensitive to the pumping rate (Zipper et al., 2021). Altogether, ADFs can be applied for both continuous and intermittent pumping schedule, but performance is best for the pumping season when streamflow depletion is often larger.

2) **Hydrostratigraphy.** Transmissivity and storativity are the two primary hydrostratigraphic input parameters for both analytical models and ADFs. Studies showed analytical models and ADFs are more sensitive to transmissivity than storativity (Zipper et al. 2018; Li et al. 2020). Furthermore, transmissivity and storativity play a more dominant role than other landscape factors such as distance to surface water, indicating that these model input parameters should be carefully selected to ensure the accuracy of streamflow depletion, and regional estimates should be supplemented with local estimates (such as pumping tests) where possible. However, variation in ADFs in response to transmissivity is not consistent. For example, ADF performed better in higher conductivity (transmissivity) areas in the BX Creek and Republican River domains, while the inverse trend was shown in the Peace region (Table 1), which is likely due to the differences in hydrological conditions.

3) **Well-stream geometry.** Distance between affected streams and wells is a key variable controlling the timing of impacts which influences the accuracy of ADFs. In BX Creek and Peace region domains, better performance is found for wells within ~2 kilometers of a stream. Further, the best-performed areas are detected within 3 km in the Navarro River watershed. However, ADFs performance near streams is also variable as these regions often have higher streamflow depletion
rates and therefore higher uncertainties; in other words, the potential impact of uncertainty increases closer to streams because of the higher streamflow depletion. Therefore, the uncertainties within a few kilometers between stream and wells should be considered in the decision-making process.

4) **Streambed conductance** is an input required for ADFs using the Hunt analytical model. In the Navarro River watershed, considering streambed conductance can improve the accuracy of ADFs, while streambed conductance was not a significant factor leading to the differences between Glover and Hunt model in both BX Creek and Peace region domains. Therefore, the choice of ADFs with different analytical models should consider local stream and hydrogeological conditions, while acknowledging that streambed conductance is an exceedingly difficult parameter to measure in the field (Christensen 2000).

5) **Other landscape parameters** can also affect the performance of the ADFs, which adds another layer of uncertainty in applying ADFs in the real-world settings. In BX Creek, Navarro River, and Republican River domains, ADF performed the best in small topographic relief where a shallow water table exists. Conversely, we detected inconsistent responses in the Peace region. Besides, Zipper et al. (2018) showed that ADF performance decreased with increases in drainage density and recharge rates, and Zipper et al. (2021) found that performance degraded near phreatophytic vegetation. Therefore, our synthesis highlights that response of streamflow depletion to hydrogeological characteristics could be region-specific and additional testing in other hydrogeological environments is needed.

Here, we suggest the focus of future testing can be prioritized for the following issues.
1) Evaluate ADFs against actual field measurements of streamflow depletion, potentially from past experiments that have been conducted (e.g., Flores et al. 2020) in addition to further comparisons against numerical models.

2) Evaluate ADFs in hydrologic landscapes that have not yet been examined specifically, such as coastal and hummocky terrain (Winter 2001). As shown in Table 1, the previous assessments have focused on certain hydrologic landscapes (mountain valley, plateau and highlands and riverine valley).

3) Examine the appropriateness of the assumption of many analytical models that groundwater recharge and phreatophytic evapotranspiration do not change over the pumping period. Groundwater evapotranspiration and recharge have not been considered in the preliminary assessment of the performance of ADFs. Groundwater pumping, however, can alter groundwater hydrological processes in the regions with substantial phreatophytic evapotranspiration. Zipper et al. (2021) found lower agreement between ADFs and a calibrated MODFLOW model when pumping was occurring near cells with phreatophytic evapotranspiration.

4) Examine the cumulative impacts of multiple pumping wells on streamflow depletion rather than one-well-at-a-time, like most of our current assessments of ADF performance.

5) In this technical note, we consistently compared streamflow depletion estimated by ADFs to MODFLOW models. Streamflow depletion can be modelled in a number of different ways in MODFLOW as well as other numerical models such as HydroGeoSphere (Brunner and Simmons 2012), ParFlow (Maxwell and Condon 2016), and GFLOW (Haitjema 1995). Future studies are encouraged to compare streamflow depletion estimated by ADFs and other numerical models.
Table 1 Summary of the performance of analytical depletion function over the tested domains.

<table>
<thead>
<tr>
<th>Domains</th>
<th>Location</th>
<th>Hydrologic landscapes (Winter 2001)</th>
<th>Domain size (km²)</th>
<th>Elevation range (meter)</th>
<th>Hydraulic conductivity ranges (m/s)</th>
<th>Specific Yield Ranges</th>
<th>Number of Hydrostratigraphic Units</th>
<th>ADFs performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>BX Creek Watershed, Canada (Li et al., 2020)</td>
<td>British Columbia, Canada</td>
<td>plateau and highlands</td>
<td>165</td>
<td>350~1850</td>
<td>$1 \times 10^{-8} - 1 \times 10^{-5}$</td>
<td>0.02~0.15</td>
<td>4</td>
<td>100</td>
</tr>
<tr>
<td>Navarro River watershed, USA (Zipper et al., 2019b)</td>
<td>California, USA</td>
<td>mountain valley</td>
<td>816</td>
<td>0~211</td>
<td>$1 \times 10^{-5}$</td>
<td>0.1</td>
<td>1</td>
<td>70</td>
</tr>
<tr>
<td>Peace Region, Canada (Li et al., 2020)</td>
<td>British Columbia, Canada</td>
<td>riverine valleys</td>
<td>1952</td>
<td>440~1580</td>
<td>$7.7 \times 10^{-12} - 3 \times 10^{-3}$</td>
<td>0.02~0.15</td>
<td>6</td>
<td>83</td>
</tr>
<tr>
<td>Republican River watershed, USA (Zipper et al., 2021)</td>
<td>Colorado, Kansas, Nebraska, USA</td>
<td>plateau and highlands</td>
<td>77,868</td>
<td>450~1800</td>
<td>$3.5 \times 10^{-5} - 1.1 \times 10^{-3}$</td>
<td>0.17~0.23</td>
<td>3</td>
<td>&gt;54</td>
</tr>
</tbody>
</table>
Conclusions

Quantification of groundwater pumping on streamflow depletion is critical for water sustainability and management. In this paper, we introduced an emerging approach, analytical depletion functions, which include capabilities that expand the utility of analytical models for real-world settings to evaluate the spatial and temporal groundwater pumping effects on streamflow depletion. The performance of analytical depletion functions has been tested by comparing them against the numerical models in different hydrological landscapes and stream networks. We conclude that analytical depletion functions can provide comparable accurate estimates of streamflow depletion to numerical models while requiring less data and experience to implement. This does not imply that analytical depletion functions can replace numerical models, but we see the potential benefits of using analytical depletion function as a preliminary screening tool to identify where the numerical model is needed to address the environmental issues resulting from streamflow depletion. Therefore, we highly recommend that water management can put analytical depletion functions in their toolbox to assess the potential environmental impacts of streamflow depletion on water resources sustainability, environmental flow needs, and aquatic functioning.

Authors’ Contribution

T.G. and L.Q. and S.C.Z. proposed idea for this note and conceptualize the figures. L.Q. wrote the first draft of the manuscript and drew the figures. S.C.Z. and T.G. developed the methods and figure for Appendix. All authors contributed to the writing.

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Appendix

*Calculating effective transmissivity and storativity in heterogeneous conditions*

Transmissivity ($T$) and storativity ($S$) are the two primary hydrostratigraphic inputs for the ADFs. Zipper et al. (2020) stressed that accurate inputs are the key to ensure robustness of streamflow depletion estimation. In homogeneous conditions, the $T$ and $S$ from any point, such as the location of the well, can be used as inputs for the ADFs. In the presence of subsurface heterogeneity, multiple hydrostratigraphic units, and/or variable aquifer thickness, the estimation of appropriate $T$ and $S$ values is more complicated (Figure 4). Hence, the effective transmissivity ($T_E$) and effective storativity ($S_E$) are typically used to represent the integrated hydrostratigraphic conditions between the well and affected streams.

In practice, there are often a mixture of datasets with different spatial resolutions within a domain. A typical case may be a gridded global dataset available in raster data format, and some regions also have localized data (e.g., aquifer boundary) with finer discretization than the global data. To facilitate the calculation of appropriate input parameters for ADFs, we explain here an approach to calculate $T_E$ and $S_E$ in heterogenous conditions, including mixed-resolution input data. In our
description, we assume that all input data are in raster format. For regions with finer localized polygon data, they should be rasterized at an appropriate resolution (i.e., resolution should not be finer than the data source used to derive the polygon) and intersected into the coarser dataset (Figure 4b).

To derive the $T_E$ and $S_E$, we first calculate cell effective transmissivity ($T_C$, Eqn. A1) and cell effective storativity ($S_C$, Eqn. A2) for each raster cell intersected by a straight line between the well and the stream (inclusive) using the assumption of flow parallel to layering following Kollet and Zlotnik (2003).

\[
T_C = (m_1 + m_2 + \ldots + m_n) \left( \frac{K_1 m_1 + K_2 m_2 + \ldots + K_n m_n}{m_1 + m_2 + \ldots + m_n} \right) \quad (A1)
\]

\[
S_C = \frac{S_1 m_1 + S_2 m_2 + \ldots + S_n m_n}{m_1 + m_2 + \ldots + m_n} \quad (A2)
\]

where, $m$ and $K$ are the thickness and hydraulic conductivity, respectively, of layer $n$ at that grid cell. We then can calculate $S_E$ as the arithmetic mean of all $S_C$ between well and stream, and $T_E$ as the average of $T_C$ (Eqn. A3) using the assumption of lateral flow perpendicular to the cells (Domenico and Schwartz 1998)

\[
T_E = \frac{\sum_{C}^{}c_T d_C}{\sum_{C}^{}c_T d_C} \quad (A3)
\]

where $C_T$ is the total number of cells between the well and the stream and $d_C$ is the width of each cell $C$. 
This approach allows calculations to take advantage of any available data source, regardless of resolution, and can also be used to calculate the $T_E$ and $S_E$ when comparing the ADFs to numerical models.

**Figure 4.** Examples of cross-section between a well and stream showing how to calculate the effective transmissivity and effective storativity. (a) shows data of consistent resolution, for example, from a single global dataset. (b) shows how local higher-resolution data (hydrostratigraphic unit 3) can be integrated in the coarser data resolution (e.g., global dataset).
is the thickness of a specific layer. $n$ is the total number of layers between well and streams. $t_i$ is the total number of layers data with finer resolution.

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