

Acoustic full waveform inversion for 2-D ambient noise source imaging

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SUMMARY

We present a method for estimating seismic ambient noise sources, by acoustic full waveform inversion of interstation cross-correlations. The method is valid at local scales for laterally heterogeneous media, and ambient noise sources confined to the Earth's surface. Synthetic tests performed using an actual field array geometry, are used to illustrate three unique aspects of our work. First: the method is able to recover noise sources of arbitrary spatial distribution, both within and outside the receiver array, with high fidelity. This holds true for complex velocity models and does not require a good initial guess for inversion, thereby bridging a clear gap in the existing research literature. Second: we analyse the extent of biases in source inversion, that arise due to inaccurate velocity models. Our findings indicate that source inversion using simplified (e.g. homogeneous) velocity models may work reliably when lateral variations in velocity structure are limited to 5 or 10% in magnitude, but is vitiated by strong variations of 20% or higher. Finally, our technique is implemented without the adjoint method, which is usually inextricably linked to full waveform inversion. Inversions are performed using source

kernels computed for each receiver pair, and this approach is computationally tractable for real-world problems with small aperture seismic arrays.

Key words: Seismic noise; seismic interferometry; waveform inversion; inverse theory

1 INTRODUCTION

2 The study of the ambient seismic field, or seismic ambient noise as it is popularly known, is now
3 firmly entrenched in mainstream seismological research. Ambient seismic sources can shed light
4 on such natural phenomena as ocean wave coupling with the seafloor (e.g. Juretzek & Hadziioan-
5 nou 2016), sediment transport in rivers (Tsai et al. 2012), glacier hydrology and dynamics (e.g.
6 Aso et al. 2017; Labedz et al. 2022), tropical cyclones (e.g. Retailleau & Gualtieri 2019) and
7 underground hydrothermal activity (e.g. Cros et al. 2011). On the other hand, seismologists are
8 widely interested in using ambient noise as a tool for studying Earth structure, typically by apply-
9 ing interferometric techniques to extract meaningful signals from noise recordings (e.g. Shapiro &
10 Campillo 2004). Even in this case, source information is essential because the spatial distribution
11 of noise sources effectively determines whether inter-station Green's functions can be accurately
12 recovered from noise cross-correlations (Roux et al. 2005; Snieder 2004). There is ample evidence
13 for biases in structure estimation, arising from realistic distributions of ambient noise sources on
14 Earth (e.g. Kimman & Trampert 2010; Yao & van der Hilst 2009). If one chooses instead to lever-
15 age the power of full waveform inversion (FWI), the assumption of Green's function retrieval is
16 dropped and both sources and structure must be simultaneously estimated (Sager et al. 2018b;
17 Zhou et al. 2022). Thus, regardless of one's particular interest in seismic ambient noise, the ability
18 to determine the strength and locations of the noise sources is of vital importance.

19 Traditional, computationally cheap methods for locating ambient sources include beamform-
20 ing (e.g. Gal et al. 2015; Gerstoft & Tanimoto 2007), matched-field processing (e.g. Cros et al.
21 2011) and backprojection (e.g. Liu et al. 2016) as well as cross-correlation based imaging (e.g.
22 Ermert et al. 2016; Tian & Ritzwoller 2015). The last few years have witnessed the emergence of
23 FWI methods, which seek to match observed noise cross-correlations with theoretically modelled
24 ones, incorporating their finite frequency sensitivity to spatially distributed sources (Fichtner et al.

25 2017; Tromp et al. 2010). This paper focusses on FWI to recover noise source distribution with
26 the assumption of a known structure model (Datta et al. 2019; Ermert et al. 2017, 2021; Igel et al.
27 2021; Xu et al. 2019, 2020). These ‘source inversion’ techniques are useful in their own right, and
28 essential components in the toolkit for the larger problem of full waveform noise cross-correlation
29 tomography (e.g. Sager et al. 2020).

30 FWI, by definition, entails wave-equation based forward modelling. However, different ap-
31 proximations to the seismic wave equation (e.g. acoustic vs. elastic) or different assumptions about
32 the medium of propagation, lead to a family of methods achieving varying degrees of modelling
33 rigour. From analytical modelling in homogeneous or laterally homogeneous media (Datta et al.
34 2019; Xu et al. 2019, 2020) to numerical simulations in spherically symmetric (Igel et al. 2021) or
35 full 3-D Earth models (Ermert et al. 2017, 2021) at global scales, a variety of methods have been
36 proposed. In this study, we present an approach using acoustic modelling in 2-D media that incor-
37 porates laterally heterogeneous structure information. Xu et al. (2020) used classic surface wave
38 analysis on ambient noise data (Bensen et al. 2007) to estimate a uniform phase velocity required
39 for forward modelling. With our approach, one can go a step further by using the 2-D phase or
40 group velocity maps obtained from ambient noise surface wave tomography (Shapiro 2019).

41 Independent of our choice of modelling scheme, another highlight of this study, achieved by
42 building on the work of Datta et al. (2019), is the ability to localize sources outside the sensor
43 array. Previous non-global studies have had limited success in this regard. Xu et al. (2019), for
44 example, reported that both traveltimes and waveform inversion are only able to estimate rough
45 source directions, when sources are outside the array. Our method recovers external source shapes
46 and sizes fairly accurately, given a moderate inter-sensor path density. This holds good even when
47 the inversion is initialised with a uniform source model.

48 The two aforementioned features of our method lend themselves readily to a scrutiny of the
49 source-structure trade-off in seismic interferometry (Fichtner 2014), which is not very well doc-
50 umented. Xu et al. (2019) reported the failure of source inversion in case of inaccurate structure
51 models, but only for homogeneous halfspace models devoid of lateral variations. Other studies
52 have argued that misfit functions defined using measurements of waveform energy ought not to

be sensitive to lateral structural variations (Igel et al. 2021; Sager et al. 2018a). We work with a waveform energy misfit, only slightly different from that used in said studies, and find that structure (velocity) information does impact source inversion.

In the following, we first present the methodology for forward and inverse modelling (Section 2), followed by the synthetic tests (Section 3) which form the basis of our conclusions (Section 4).

2 CROSS-CORRELATION MODELLING AND INVERSION

Datta et al. (2019) introduced a method for estimating noise source directionality in a homogeneous medium. The limitation of estimating only directions was imposed by the choice of model parameterization, and the homogeneous medium assumption allowed for cross-correlations to be modelled analytically. In this study, we extend their method on both fronts. A similar but less restrictive model parameterization allows actual source locations and shapes to be estimated, and integration with a numerical solver eliminates the need to assume a homogeneous medium. However the choice of measurement, calculation of kernels and inversion strategy, remain essentially unchanged.

2.1 Forward modelling

The foundation of the method is a forward model for computing the cross-correlation $C(\mathbf{x}_\alpha, \mathbf{x}_\beta)$ between any pair of receivers α and β :

$$C(\mathbf{x}_\alpha, \mathbf{x}_\beta; \omega) = P(\omega) \int d^2\mathbf{x} G^*(\mathbf{x}_\alpha, \mathbf{x}; \omega) G(\mathbf{x}_\beta, \mathbf{x}; \omega) \sigma(\mathbf{x}), \quad (1)$$

where ω is angular frequency, P is the source power spectrum, G is the Green function for the medium, σ is the source strength, and the position coordinate \mathbf{x} is limited to a horizontal plane (approximating an area on the Earth's surface). The asterisk denotes complex conjugation. This follows from the formulation of Tromp et al. (2010), subject to the following assumptions (e.g. Malkoti et al. 2021; Xu et al. 2019):

(i) all ambient seismic sources are confined to the Earth's surface, so that the integral in (1) is a surface integral

75 (ii) all sources have the same spectral shape $P(\omega)$, so that the power spectral density (S) of
 76 noise sources can be separated into its positional and frequency contributions, i.e. $P(\omega)\sigma(\mathbf{x}) =$
 77 $S(\mathbf{x}, \omega)$.

78 The frequency-domain expression (1) can be evaluated semi-analytically when closed form solu-
 79 tions for G are available — such as in a homogeneous (Datta et al. 2019) or 1-D (Malkoti et al.
 80 2021) medium.

In anticipation of numerically solving the wave equation for heterogeneous media, we recast the expression for crosscorrelation in equation (1) in terms of the wavefields u :

$$C(\mathbf{x}_\alpha, \mathbf{x}_\beta; \omega) = \int d^2\mathbf{x} u^*(\mathbf{x}, \mathbf{x}_\alpha; \omega)u(\mathbf{x}, \mathbf{x}_\beta; \omega)\sigma(\mathbf{x}), \quad (2)$$

with $u(\mathbf{x}, \mathbf{x}_\alpha; \omega) = P(\omega)^{1/2}G(\mathbf{x}, \mathbf{x}_\alpha; \omega)$. Note that source-receiver reciprocity has also been invoked for computational efficiency, turning every receiver location into a source (e.g. Hanasoge 2014; Xu et al. 2019). The monochromatic wavefields in equation (2) can be computed in the frequency domain (e.g. Kumar et al. 2022). However, we use a time-domain finite difference solver implemented using the “Devito” package (Louboutin et al. 2019; Luporini et al. 2020). The uniform finite-difference stencil is second-order in time and fourth-order in space. Equation (2) in the time domain is

$$C(\mathbf{x}_\alpha, \mathbf{x}_\beta; t) = \int d^2\mathbf{x} \int_{-\infty}^{\infty} d\tau u(\mathbf{x}, \mathbf{x}_\alpha; \tau)u(\mathbf{x}, \mathbf{x}_\beta; t + \tau) \sigma(\mathbf{x}). \quad (3)$$

81 We obtain $u(\mathbf{x}, \mathbf{x}_\alpha; \tau)$ as the solution to the acoustic wave equation with a source wavelet equal to
 82 the inverse Fourier transform of $P(\omega)^{1/2}$. Provided that the computed wavefields $u(\mathbf{x}, \mathbf{x}_\alpha)$ can be
 83 stored in memory, N numerical simulations are required to obtain all possible cross-correlations
 84 for an N -receiver array. In the inverse problem of estimating $\sigma(\mathbf{x})$, these N simulations, once
 85 performed at the start, are not required to be repeated every iteration, because the velocity model
 86 is held fixed.

87 2.2 Model parameterization

As in Datta et al. (2019), the model space is parameterized using a set of 2-D Gaussian basis functions, $B_j(\mathbf{x})$. In this study, we introduce a non-negative parameterization to ensure that the

iterative optimization scheme (Section 2.3) does not lead to unphysical, negative values for $\sigma(\mathbf{x})$:

$$\sigma(\mathbf{x}) = \sum_j m_j^2 B_j(\mathbf{x}), \quad (4)$$

88 m_j being the basis function coefficients that are directly inverted for. We note that alternate means
89 of enforcing positivity are available, such as in Xu et al. (2019). A second feature of our parameter-
90 ization is that the basis functions are present uniformly throughout the domain, rather than merely
91 in a ring surrounding the receiver array (Datta et al. 2019). The size and spacing of the Gaussian
92 basis are user-controlled parameters, held fixed during inversion. The specifications used to obtain
93 the results presented in this paper, are listed in Table 3.1.

94 2.3 Inversion Strategy

We follow the inversion strategy detailed in Datta et al. (2019, Appendix A), subject to minor changes necessitated by the model parameterization, equation (4). To summarize, we use a misfit function defined by Hanasoge (2013):

$$\chi = \frac{1}{2} \sum_i \left(\ln \frac{E_i^{obs}}{E_i^{syn}} \right)^2, \quad (5)$$

where the index i runs over receiver pair combinations, and E , the measurement, is the waveform energy in a time window of interest $w(t)$:

$$E_i = \sqrt{\int w(t) C_i^2(t) dt} \quad (6)$$

95 For inversion of noise-free synthetic data, as done in this study, we use a window spanning the
96 entire positive or negative correlation branch.

Local optimization techniques require the gradient of χ , or a misfit kernel K satisfying $\delta\chi = -\int K(\mathbf{x})\delta\sigma(\mathbf{x})d^2\mathbf{x}$. This misfit kernel is the sum of individual source kernels for each receiver pair (K_i), weighted by the corresponding misfit (Hanasoge 2013):

$$K = \sum_i \ln \left(\frac{E_i^{obs}}{E_i^{syn}} \right) K_i(\mathbf{x}) \quad (7)$$

Using (4), we get the following expression for the gradient (\mathbf{g}) of χ :

$$g_j = \frac{\partial\chi}{\partial m_j} = - \int 2K(\mathbf{x})m_j B_j(\mathbf{x}) d^2\mathbf{x} \quad (8)$$

97 K , and therefore \mathbf{g} , can be obtained by either evaluating (7), which requires all the individual
 98 source kernels K_i , or by adjoint techniques, which build K from adjoint wavefields and ‘event
 99 kernels’ (Tromp et al. 2010), without access to the individual K_i . We use the former, non-adjoint
 100 approach. Using an analogy from earthquake traveltime tomography, this is akin to the difference
 101 between calculating individual banana doughnut kernels (e.g. Dahlen et al. 2000) or not (Tromp
 102 et al. 2005).

Our approach remains computationally feasible because numerical simulations are not re-
 quired to obtain the K_i in every iteration, as explained in Section 2.1. Furthermore, we paral-
 lelise the implementation over the individual receivers. The advantage of this inversion strategy is
 that in addition to the gradient, it gives us access to the Jacobian through the individual kernels.
 Combining (7) and (8), the elements of our Jacobian matrix (\mathbf{J}) are:

$$J_{ij} = \int 2K_i(\mathbf{x})m_jB_j(\mathbf{x}) d^2\mathbf{x} \quad (9)$$

103 The Jacobian can in turn be used to build an approximation to the Hessian operator, and optimiza-
 104 tion methods such as the Gauss-Newton method, can be employed. For details of how this is done,
 105 and complete derivations of equations (8) and (9), the reader is referred to Appendix A of Datta
 106 et al. (2019).

107 3 SYNTHETIC TESTS

108 We conduct a series of synthetic tests using the field array geometry of Datta et al. (2019), which
 109 corresponded to an exploration seismic deployment by Shell. This choice is motivated by the
 110 relatively large number of receivers (289) in the array, which allows us to explore the effects of
 111 different array sizes and path densities, by randomly picking different subsets of receivers. We
 112 present results in this paper for two sub-array geometries, one with 20 and the other with 50
 113 receivers (Figure 1, parts b and c, respectively). Synthetic tests with more than 50 receivers did not
 114 produce significantly different results.

115 Our modelling domain for all tests is 50 km \times 50 km, and other simulation parameters are
 116 listed in Table 3.1. These specific choices result in source distributions represented by a total of

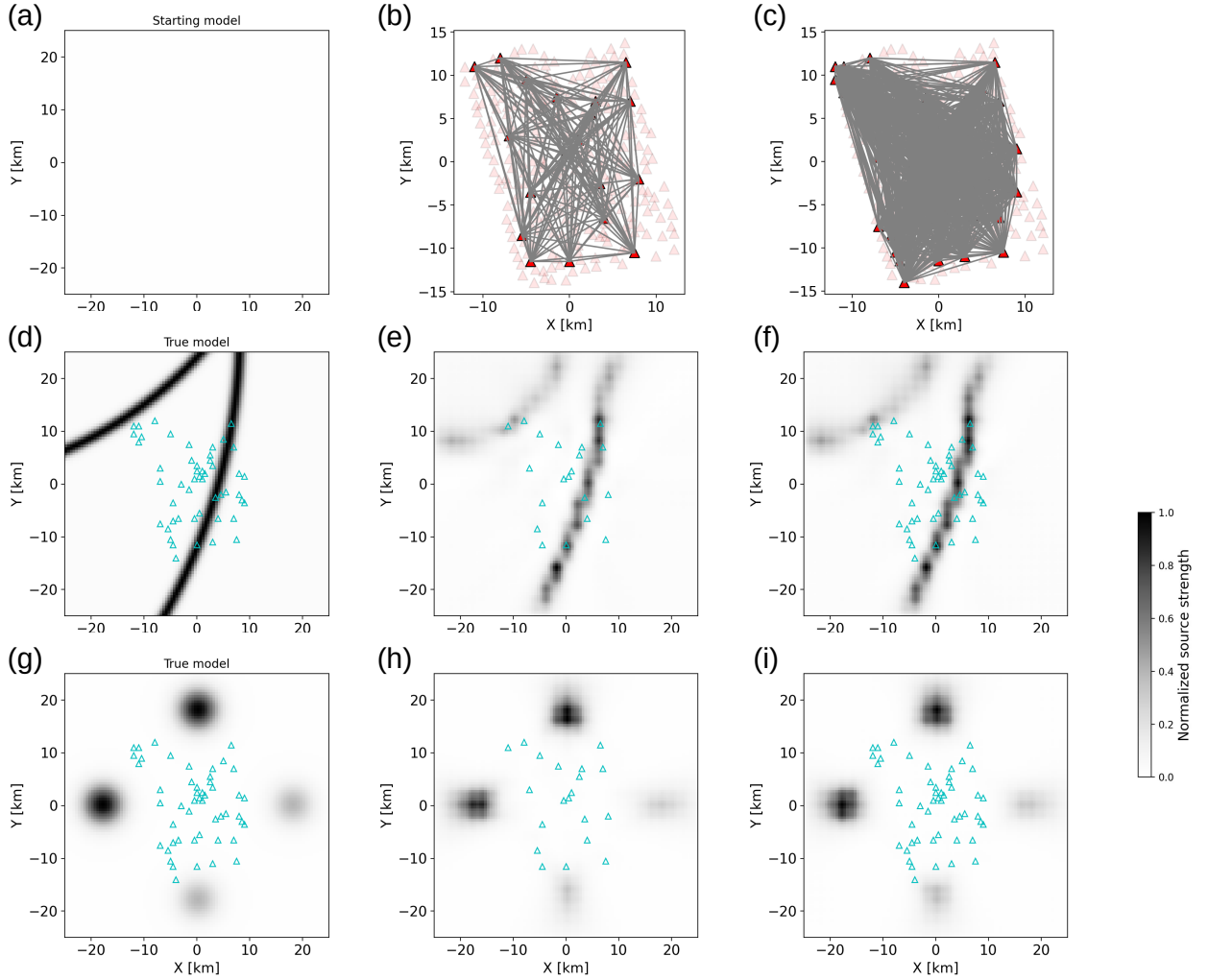


Figure 1. Source inversion results with homogeneous velocity models. (a) Starting model for all inversions. (b)-(c) Tomographic setup used for inversion: map with 20 and 50 receivers selected respectively (red triangles), along with inter-receiver ray paths (grey lines). (d) TSM-1, shown with the 50 receivers selected in (c). (e)-(f) Inversion results for TSM-1, using 20 and 50 receivers respectively. (g) TSM-2, shown with the 50 receivers selected in (c). (h)-(i) Inversion results for TSM-2, using 20 and 50 receivers respectively. All source distributions are shown normalized by their maximum value (to focus on relative source strength), with the exception of the starting model (a), which is normalized with respect to the test models to illustrate its low amplitude.

117 625 basis functions. We present results for two test source models, TSM-1 (Fig. 1d) and TSM-2
 118 (Fig. 1g). In actual field scenarios, TSM-1 may represent, for example, roads situated close to or
 119 passing through, the receiver array. TSM-2 is more of a toy model, containing sources of varying
 120 strengths located entirely outside the array. The test models are built independently, without using
 121 the parameterization (4), which is reserved for the inverse problem.

122 For both test models, we generate noise-free synthetic data to serve as “observed data”. A
 123 uniform source distribution (Figure 1a) is used as a prior as well as the initial model for inversion.
 124 This initial model has very low amplitude compared to the test models, but it cannot be zero
 125 because of the logarithmic misfit function (5) used in the inversion.

126 Figure 1 shows that the inversion recovers true source locations and shapes fairly well, for both
 127 test models. As expected, results with 50 receivers are better than those with 20, but the improve-
 128 ment is modest, especially for sources lying outside the receiver array (TSM-2). The pixellated
 129 appearance of the inversion results as compared to the test models, is due to the parameterization
 130 (4) and the size of basis functions used (Table 3.1). Finally, we note that either analytical or nu-
 131 merical modelling may be used in these inversions, because a homogeneous medium is assumed.
 132 For consistency with the rest of the paper, we have presented results obtained with numerical mod-
 133 elling. The results with analytical Green functions were similar to those in Figure 1 and have not
 134 been shown.

135 3.1 Impact of heterogeneous velocity structure

136 To investigate the effects of heterogeneous structure, we use two types of velocity models to gen-
 137 erate the test data: a single low velocity anomaly (Figure 2) and a checkerboard (Figure 3). In both
 138 cases we consider perturbation amplitudes of 5, 10 and 20% relative to the reference velocity of
 139 2 km/s (used in Fig. 1). Inversions are then performed both with and without accurate velocity
 140 models. In this section, we present tests performed with the 50-receiver array geometry only.

141 When the velocity information provided in inversion is accurate, we obtain results that are
 142 all nearly identical to the homogeneous velocity case (Fig. 2). The inverse modelling can toler-
 143 ate errors in velocity information up to 10%, but artefacts appear for the case of 20% velocity
 144 heterogeneity. Also, sources outside the array are not retrieved accurately in this case. With the
 145 checkerboard velocity model, inversion artefacts are less pronounced (Figure 3), but the sources
 146 outside the array are very faintly recovered for a magnitude of 20% heterogeneity. Similar tests
 147 with the second source model, TSM-2, are presented in Appendix A. In contrast to TSM-1, TSM-2
 148 is a simpler model and the sources exist only outside the array. The stronger sources to the North

Parameter	Value
Uniform grid spacing	0.5 km
Basis function width	5 km
Basis function spacing	2 km
Reference wavespeed	2 km/s
Time series length	50 s
Central frequency	0.2 Hz

Table 1. Simulation parameters used.

149 and West of the array have been recovered for both the Gaussian anomaly (Figure A1) and the
 150 checkerboard model (Figure A2). The weaker sources (to the East and South) are not recovered
 151 well in both the instances, particularly for the case of 20% velocity heterogeneity.

152 It should be emphasised that the $\sigma(\mathbf{x})$ starting model is the same (see Fig. 1a) for all tests in this
 153 section and in Appendix A,. On the other hand, the optimal value of the damping parameter used
 154 in inversion (Datta et al. 2019, Appendix A), is determined separately in each case by independent
 155 L-curve analysis (Hansen & O’Leary 1993).

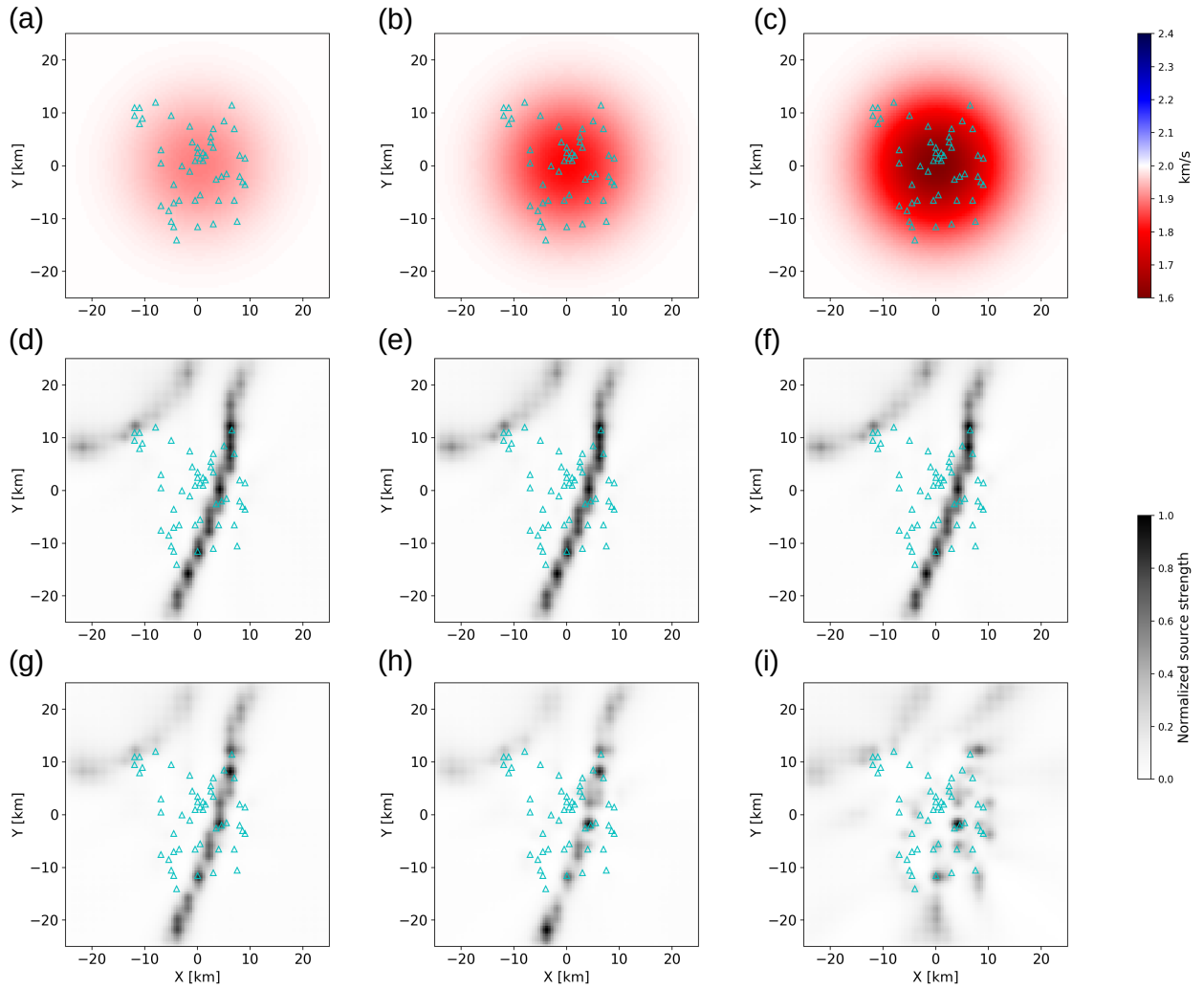


Figure 2. Source inversion results with 2-D velocity models. Top row: Velocity models with a perturbation amplitude of (a) 5% (b) 10% (c) 20%. Middle row: Inversion results for TSM 1, obtained using an accurate velocity model, i.e. velocity models (a)-(c) are used to generate synthetic test data, as well as in source inversion leading to the corresponding results (d)-(f). Bottom row: Results of inversion performed without an accurate velocity model, i.e. velocity models (a)-(c) are used to generate synthetic test data, but source inversion is performed using a homogeneous velocity model. In all cases, the starting (source) model for inversion is the same as in Figure 1a.

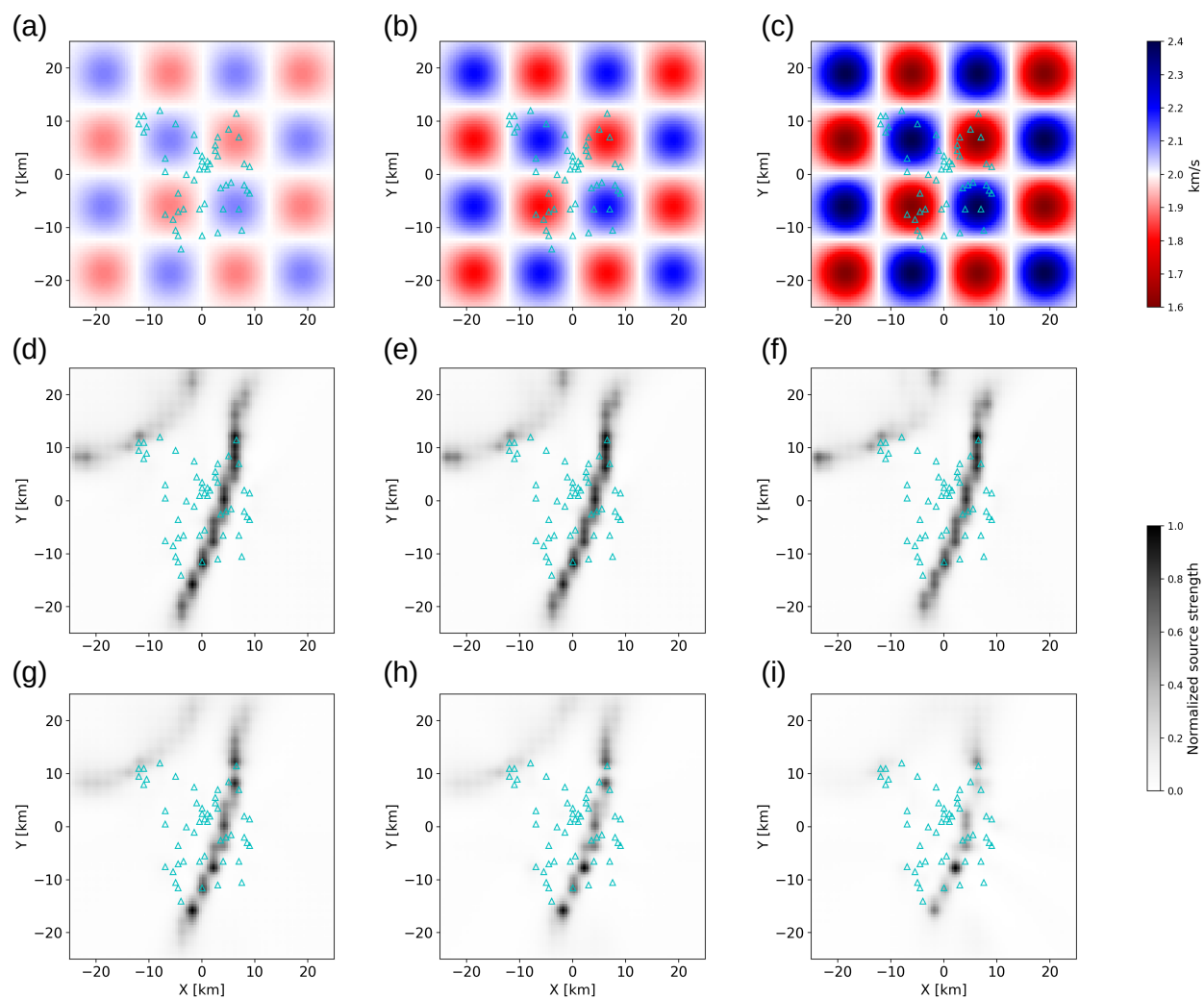


Figure 3. Similar to Figure 2, but using checkerboard-style velocity models.

156 4 CONCLUSIONS

157 Our synthetic tests have shown that:

158 (i) the inversion technique presented in this paper is able to recover source distributions any-
159 where in the modelling domain, under ideal conditions of error-free modelling.

160 (ii) accurate modelling of velocity structure, within the acoustic approximation, has limited
161 impact on ambient noise source inversion. Detrimental effects of assuming a simplistic velocity
162 model are seen only when the assumed model is a very poor approximation to the true structure.
163 For media with weak lateral heterogeneity, even the simplest approximation of a homogeneous
164 medium (with a reference wavespeed accurate to within 10%) is acceptable. It becomes unaccept-
165 able when true wavespeed variations exceed 20% of the assumed reference value.

166 The second point above is relevant for realistic scenarios where the velocity structure is not known
167 *a priori*. With real data, one can expect unmodelled lateral heterogeneity to have a greater impact
168 on source inversion, because real data would likely contain noise as well as ‘modelling error’ (both
169 of which were absent in the synthetic data of Section 3).

170 5 DISCUSSION

171 We have introduced an acoustic full waveform inversion technique for ambient noise source dis-
172 tribution. Although more sophisticated techniques, which account for Earth’s three-dimensional,
173 elastic (and anelastic) structure already exist (e.g. Ermert et al. 2017, 2021), our method occu-
174 pies a niche within the field. Compared to 3-D elastic FWI, it incorporates a significant amount
175 of modelling rigour, at a fraction of the computational cost — all simulations presented in this
176 paper were performed on a desktop computer. Further, it affords an opportunity to leverage results
177 from the immensely popular field of ray-based ambient noise tomography. Surface wave group
178 velocity maps produced by ray-based methods (e.g. Shekar et al. 2022) can be used to characterize
179 Earth structure in our source inversion scheme. More generally, such maps can be used as starting
180 structure models in a future joint source-structure acoustic inversion scheme, which may serve
181 to refine traditional ambient noise tomographic models. The obvious limitation of our technique

182 is the acoustic regime, which limits us to membrane wave simulations and precludes the use of
183 any multicomponent information from ambient noise cross-correlations (e.g. Xu & Mikesell 2017;
184 Malkoti et al. 2021).

185 In this paper, we have used the developed technique to investigate the impact of velocity het-
186 erogeneity on ambient noise source inversion. We also used it to demonstrate recovery of noise
187 sources located outside the receiver array, an important capability of small-aperture arrays de-
188 ployed for local-scale investigations. We surmise that the reason for our success is our Hessian-
189 based optimization scheme, as opposed to just gradient-based optimization. Indeed, the inverse of
190 the approximate Hessian can act as a focusing operator that compensates for uneven illumination
191 and finite bandwidth of the wavefields (Pratt et al. 1998). However, we note that the Gauss-Newton
192 approach is only feasible for small aperture arrays used in local- to (small) regional- scale studies.

193 Application of our technique to real data is a subject of ongoing research. In the future, its
194 technical scope can be widened in several ways. For example, the assumption of uniform spectral
195 character of all ambient sources can be relaxed to invert for space- as well as frequency- dependent
196 ambient source distribution (e.g. Ermert et al. 2017, 2021). While this considerably expands the
197 parameter space, techniques like sparsity promotion (e.g. Shekar & Sethi 2019) can be employed to
198 constrain the inversion. Another line of development would be to extend to 3-D elastic modelling,
199 and we note that this is possible with the “Devito” package itself.

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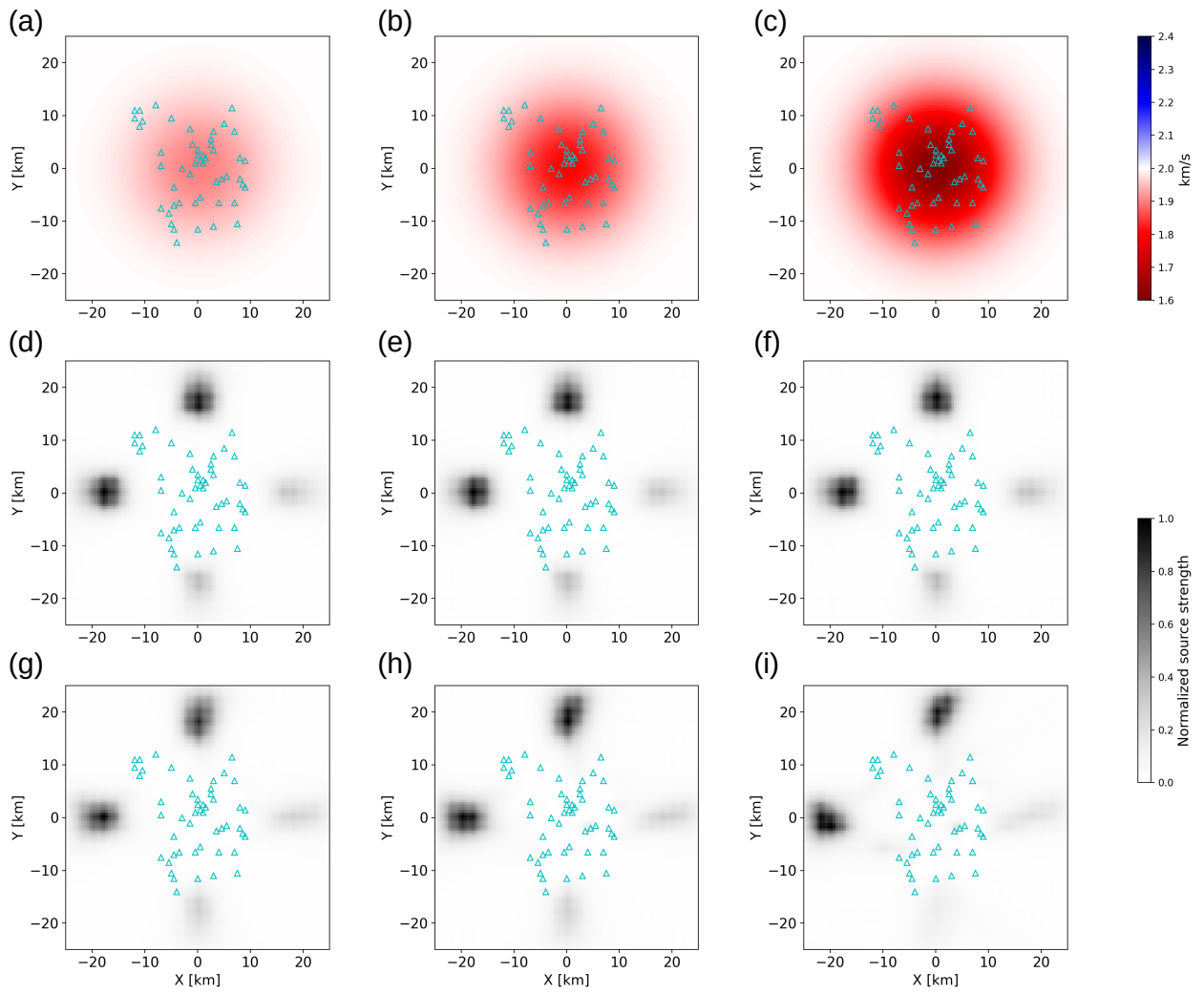


Figure A1. Similar to Figure 2 in the main text but for TSM-2.

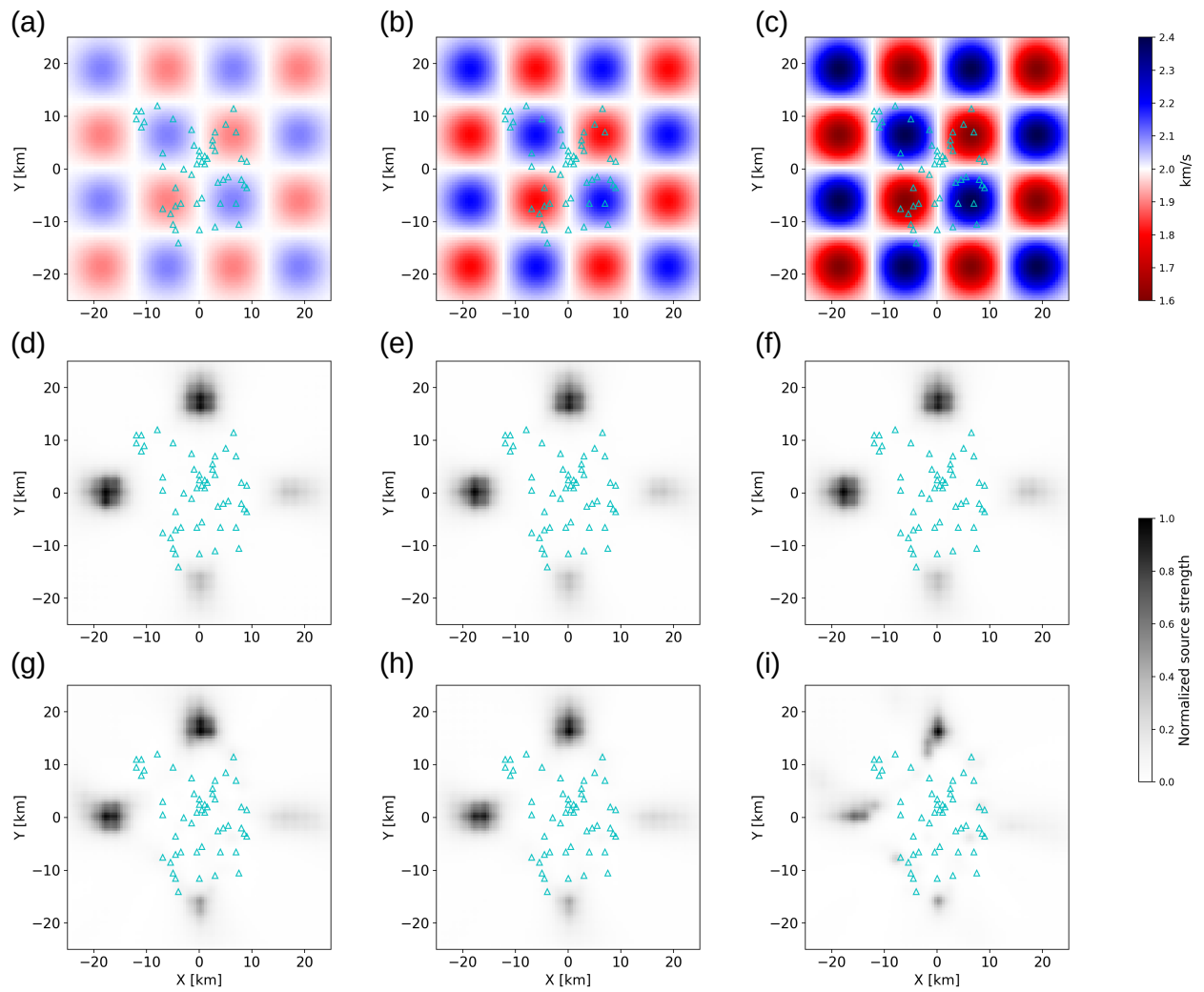


Figure A2. Similar to Figure 3 in the main text but for TSM-2.

205 **DATA AVAILABILITY**

206 No new data were generated or analysed in support of this research. The source inversion code
 207 used in this work will be made available on github: [https://github.com/ arjundatta23/cc_kern_inv](https://github.com/arjundatta23/cc_kern_inv).

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