Cover Sheet: False positives are common in single-station template matching

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False positives are common in single-station template matching

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Abstract Template matching has become a cornerstone technique of observational seismology. By taking known events, and scanning them against a continuous record, new events smaller than the signal-to-noise ratio can be found, substantially improving the magnitude of completeness of earthquake catalogues. Template matching is normally used in an array setting, however as we move into the era of planetary seismology, we are likely to apply template matching for very small arrays or even single stations. Given the high impact of planetary seismology studies on our understanding of the structure and dynamics of non-Earth bodies, it is important to assess the reliability of template matching in the small-n setting. Towards this goal, we estimate a lower bound on the rate of false positives for single-station template matching by examining the behaviour of correlations of totally uncorrelated white noise. We find that, for typical processing regimes and match thresholds, false positives are likely quite common. We must therefore be exceptionally careful when considering the output of template matching in the small-n setting.

Non-technical summary Many signals of interest to seismologists are so small that they cannot be easily seen on seismograms. In order to identify these signals, seismologists have developed the technique of template matching, which takes a large signal and runs it over a seismogram. If the template signal matches the seismogram under a certain mathematical definition, then we consider it to be a match, and we add that part of the seismogram to the catalogue of signals. Normally, seismologists cross-check this process using multiple seismograms recorded at different instruments, but this is not necessarily possible on other planets where it is too expensive to deploy many seismometers. Without this cross-checking, it is possible that many of the "matches" are in fact false positives. We performed a statistical experiment to show that these false positives are in fact likely to be quite common, which means that we must be careful when handling template matching with single seismometers.

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1 Introduction

One of the most important goals in observational seismology is to observe the smallest interesting signals possible. As codified in the Gutenberg-Richter law, the number of seismic events decreases exponentially with magnitude. This implies that the overwhelming majority of events create seismic signals smaller than can be observed above the noise that contaminates seismic observations. Access to these small events gives us great insight into tectonic processes across timescales, including the geometry of buried faults, fault heterogeneity, earthquake statistics etc.

Correlation based methods have proven to be one of the most successful ways of extracting small signals from the noise. This class of methods relies on the fact that interesting seismic signals typically have different structure to both instrumental noise and ambient ground motions produced by environmental processes. Furthermore, within the elastic regime ground motions are linear, so events with different magnitudes will still look similar (albeit with different amplitudes) if they occur at approximately the same location and are filtered appropriately. The cross-correlation class of methods scans the seismic record with templates — snippets of known high-amplitude signals that will match lower amplitude signals buried in the noise. Correlation based techniques using previously observed or calculated templates are therefore also known in the literature as template matching or matched filter analysis. These methods have been prominent in geophysics for many decades, especially in exploration settings, as comprehensive early reviews will attest (Anstey, 1964).

In observational and monitoring settings, the collation of suitable template catalogues had to wait until the proliferation of broadband digital seismograms, but the technique is now ubiquitous across distance ranges and period bands (e.g., Shearer, 1994; Gibbons and Ringdal, 2006; Bobrov et al., 2014). Template matching is extremely computationally intensive, although the calculations are simple. The advent of general-purpose graphical processing units (GPGPUs) has thus benefited template matching analyses immensely, and allowed large continuous waveform databases to be scanned efficiently with many templates, resulting in a huge increase in the number of catalogued events (e.g., Ross et al., 2019).

Template matching studies are potentially especially useful in planetary seismology contexts, which suffer from the constraints of temporary single-station deployments where extracting all possible events from the limited data available is particularly advantageous. In the Martian context, which has been the prime recent focus of planetary seismology, the InSight single-station Mars seismometer demonstrated that a larger-than-terrestrial fraction of the seismicity comes about from events which are very similar to each other. These include events of geological (thermal/tectonic) origin (Dahmen et al., 2021; Sun and Tkalčić, 2022) which are identified through matching, and those of impact origin which display very similar infrasonic chirps (Garcia et al., 2022); similar techniques have recently been re-applied to Apollo data to isolate diurnal variations in crustal properties (Tanimoto et al., 2008) and identify new deep moonquakes (Sun et al., 2019). Given the paucity of data in planetary settings, all successful detections of seismic sources are incredibly useful, and are likely to be influential in our understanding of the planetary target.

An interesting additional application of template matching in a planetary seismology context would be in the search for signals which are expected and which would have predictable waveforms, but are likely to be at or near the noise floor. Such signals are exceedingly rare, but can include cases such as expected impact events (Fernando et al., 2022). Although not currently used by any planetary seismology missions, the potential for automated triggering (e.g., to switch into high-sampling mode) upon detection of seismic precursor phases exists. Similarly, the current procedure of downlinking low-resolution data from spacecraft to Earth, uplinking requests for specific data segments back to the spacecraft, and downlinking these back to Earth may be made substantially more efficient through on-board event detection and selection. In both cases, these improvements would require robust template matching via cross-correlation for single stations, and a minimal rate of false positives. In return, savings may be made in the power and communications budgets. Whilst current limitations of power and on-board processing capacity mean that these techniques have not been used to date, they are likely to become more advantageous as more sophisticated geophysical networks are deployed off-world.

In light of these opportunities for advancing both the instrumental methodology, and interpretation, of planetary seismology, it is of vital importance to thoroughly understand the failure modes of template matching so that we have confidence in proposed detections. In this short manuscript, we investigate a basic issue in template matching — the rate of false positives. It is immediately apparent that any finite length template correlated against an infinitely long target signal will eventually result in a match that is arbitrarily good — the question is, under realistic data processing conditions, does this happen sufficiently quickly as to pose an issue for the interpretation of template matches?

2 Template Matching Definitions

The normalized cross-correlation between two signals of equal length $\mathbf{X} = [x_1, x_2, \ldots, x_n]^T$ and $\mathbf{Y} = [y_1, y_2, \ldots, y_n]^T$ is defined to be

$$CC(\mathbf{X}, \mathbf{Y}) = \frac{\langle \mathbf{X} - \hat{\mathbf{X}}, \mathbf{Y} - \hat{\mathbf{Y}} \rangle}{\sqrt{\langle \mathbf{X} - \hat{\mathbf{X}}, \mathbf{X} - \hat{\mathbf{X}} \rangle \langle \mathbf{Y} - \hat{\mathbf{Y}}, \mathbf{Y} - \hat{\mathbf{Y}} \rangle}}, \tag{1}$$

where

$$\langle \mathbf{X}, \mathbf{Y} \rangle = \sum_{i=1}^{n} x_i y_i. \tag{2}$$

This definition produces a value in [-1,1], where 1 is perfectly correlated and -1 is perfectly anticorrelated, independent of the relative amplitude of the signals or any static offsets. The normalized three-component cross-correlation between two three-component signals $\mathbf{X} = (\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3)$ and $\mathbf{Y} = (\mathbf{Y}_1, \mathbf{Y}_2, \mathbf{Y}_3)$ is then defined to be the average

$$CC_3(\mathbf{X}, \mathbf{Y}) = \frac{CC(\mathbf{X}_1, \mathbf{Y}_1) + CC(\mathbf{X}_2, \mathbf{Y}_2) + CC(\mathbf{X}_3, \mathbf{Y}_3)}{3}. \tag{3}$$

To calculate the cross-correlation time series when $\mathbf{X}$ and $\mathbf{Y}$ are not the same length, we scan the cross-correlation function along the longer signal. Specifically, assume $\mathbf{X}$ is the shorter signal, and that it has M samples, while $\mathbf{Y}$ has N samples. Denoting $\mathbf{Y}^i = [y_i, y_{i+1}, \ldots, y_{i+M-1}]^T$, then $CC(\mathbf{X}, \mathbf{Y}) = \left[CC(\mathbf{X}, \mathbf{Y}^1), CC(\mathbf{X}, \mathbf{Y}^2), \ldots, CC(\mathbf{X}, \mathbf{Y}^{N-M+1})\right]^T$, and similarly for CC_3 for 3 component signals. The Median Absolute Deviation (MAD) of a signal $\mathbf{X}$ is defined to be

$$MAD(\mathbf{X}) = median(|\mathbf{X} - median(\mathbf{X})|). \tag{4}$$

Template-matches are typically defined by a threshold that is some multiple of the MAD of the cross-correlation signal, that is, $\mathbf{X}$ is a match to a segment of $\mathbf{Y}$ at starting index i if

$$CC_{(3)}(\mathbf{X}, \mathbf{Y}^i) \geq cMAD(CC_{(3)}(\mathbf{X}, \mathbf{Y})), \tag{5}$$

for some constant c, where $c = 7$ is a typical choice for 3-component seismograms.

Simulation Results and Discussion

We investigated the base rate of expected false-positives for three-component, single-station template matching. We considered pairs of signals $\mathbf{X}$ and $\mathbf{Y}$ that are completely white-noise, that is, the underlying signals before processing are totally uncorrelated. The rate of production of false positives for white noise signals will therefore give a lower bound on the true rate of false positives for general signals. Due to the timescale invariance of white noise, it would be possible to perform this analysis in a non-dimensional units, however we have chosen to present results in physical units to aid intuition. We considered a typical setup for teleseismic planetary applications, with signals recorded at 20 Hz, bandpass filtered with lower corner frequency 0.1 Hz and upper corner frequencies of $f_{max} =$ 0.4, 0.8, and 1.6 Hz, using a 4 pole zero-phase Butterworth filter. The shorter signal $\mathbf{X}$ has a variable window length of $w_{len} =$ 5, 10, or 20 s, while the longer signal $\mathbf{Y}$ is 100 (Earth) days long. When initially generating signals, we added 40 s of padding to either end (4 times the lower bandpass period) to avoid filter edge effects, before cutting to the required lengths. For each of the 9 combinations of upper corner frequency and window length, we generated 100 pairs of three-component white noise signals $\mathbf{X}$ and $\mathbf{Y}$. We then calculated the MADs and running maximums of the cross correlation signals $CC_3(\mathbf{X}, \mathbf{Y})$. By calculating the results for 100 random pairs, we can also calculate the standard deviation of the resulting estimates.

Figure 1 shows the running maximum cross-correlations and MADs for the 9 combinations of filter and window length. Figure 2 shows the cross-correlations normalized by MAD. Combinations with narrow filter bands and short window lengths, which are seen in the top left corner of the figures, unsurprisingly result in large maximum cross-correlations relatively quickly. However, they also result in relatively high MAD (i.e., there are relatively many periods with high cross-correlation, due to the quasi-sinusoidal nature of the signals over a short time window). As a result, the MAD normalized cross-correlations saturate quickly for these combinations. Conversely, combinations with longer windows and wider passbands, found in the bottom left of the figures, have overall lower maximum cross-correlations, but also lower MADs and so the MAD normalized cross-correlations continue to grow even after 100 days. In particular, the worst-case (f_{max} = 1.6 Hz, $w_{len} =$ 20 s), the maximum MAD normalized cross-correlation exceeds 7 after one day, and 8 after 100 days. As seen in Figure 1, the estimates of the MAD of the cross correlations is very stable by the end of the 100-day correlation period for all cases. This allows us to estimate the maximum possible multiplier of MAD achievable for the different filter / window configurations, which is shown in Table 1.

This experiment considers random pairs of three-component signal $\mathbf{X}$ and $\mathbf{Y}$. A more typical experiment is to hold the longer signal $\mathbf{Y}$ fixed (we only record one seismogram), and to scan multiple templates across it. For the white noise case, because $\mathbf{X}$ and $\mathbf{Y}$ are uncorrelated, the effect of multiple templates is simple to calculate. If the average time between cross-correlations exceeding the MAD threshold of c is T_c for a single template (i.e., matches occur at a rate of $1/T_c$), then for N templates the average time between matches is T_c/N (i.e., a rate of N/T_c). For example, taking the lower-right case of Figure 2, scanning 100 white noise templates would result in a false positive match with MAD normalized cross-correlation exceeding $c = 8$ approximately once a day.

Modern workflows for template matching in observational seismology normally further consider the averaged cross correlation across an array, up to and including arrays with extremely large numbers of instruments such as Distributed Acoustic Sensors (DAS) (e.g., Gibbons and Ringdal, 2006; Li and Zhan, 2018). Array deployments implic-

f_{max} (Hz) \ w_{len} (s)	5	10	20
0.4	5	7	9
0.8	7	10	14
0.4	10	14	20

Table 1 Estimated maximum multiple of MAD to the nearest unit for each configuration of filter corner frequency f_{max} and window length w_{len}.

itly create a "barcode" of relative arrival time patterns for each potential source location that must be generally be satisfied for a signal to count as a match. As such, array deployments are much more resilient to false positives in general. This is not to say that false positives are not an issue; in particular, for arrays with narrow apertures relative to the content of waveform frequency, coherent noise sources can correlate well. Likewise, templates containing common noise phenomenon (such as passing cars, or electronic 'glitch' noise as with InSight on Mars, (Kim et al., 2021)) may match waveform segments that do not contain any interesting seismic signals but do contain a similar noise signal. These effects should be considered as additive to the basic analysis of random noise false-positives investigated here, and are almost certainly more important for larger arrays. The key takeaway of this paper is to emphasize that for single stations, that are the current state-of-the-art for planetary applications (as well as some circumstances on Earth), *the baseline rate of false-positive detection is significant under realistic processing choices.*

3 Conclusions

In this work, we investigated the rate of false-positive detection of template matching for snippets of white noise scanned across white noise records. We used realistic processing for 3-component traces for pre-processing, and found that the rate of false-positive detection is significant. Because white noise is on average totally uncorrelated by definition, these results act as a lower bound on the rate of false positives. Real seismic signals will contain features that may induce "spurious" correlations (in the sense that they are not related to seismic activity), and the relationship between the spectra of real seismic noise and pre-processing filter choices will also have implications for the rate of false positives in excess of the baseline considered here.

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Data and code availability

We have included the Pluto notebook used to generate the results in the submission. This notebook, as well as example datasets, will be uploaded to Zenodo after acceptance so that the assigned DOI corresponds to the final version used for the publication.

Competing interests

The authors declare no competing interests.

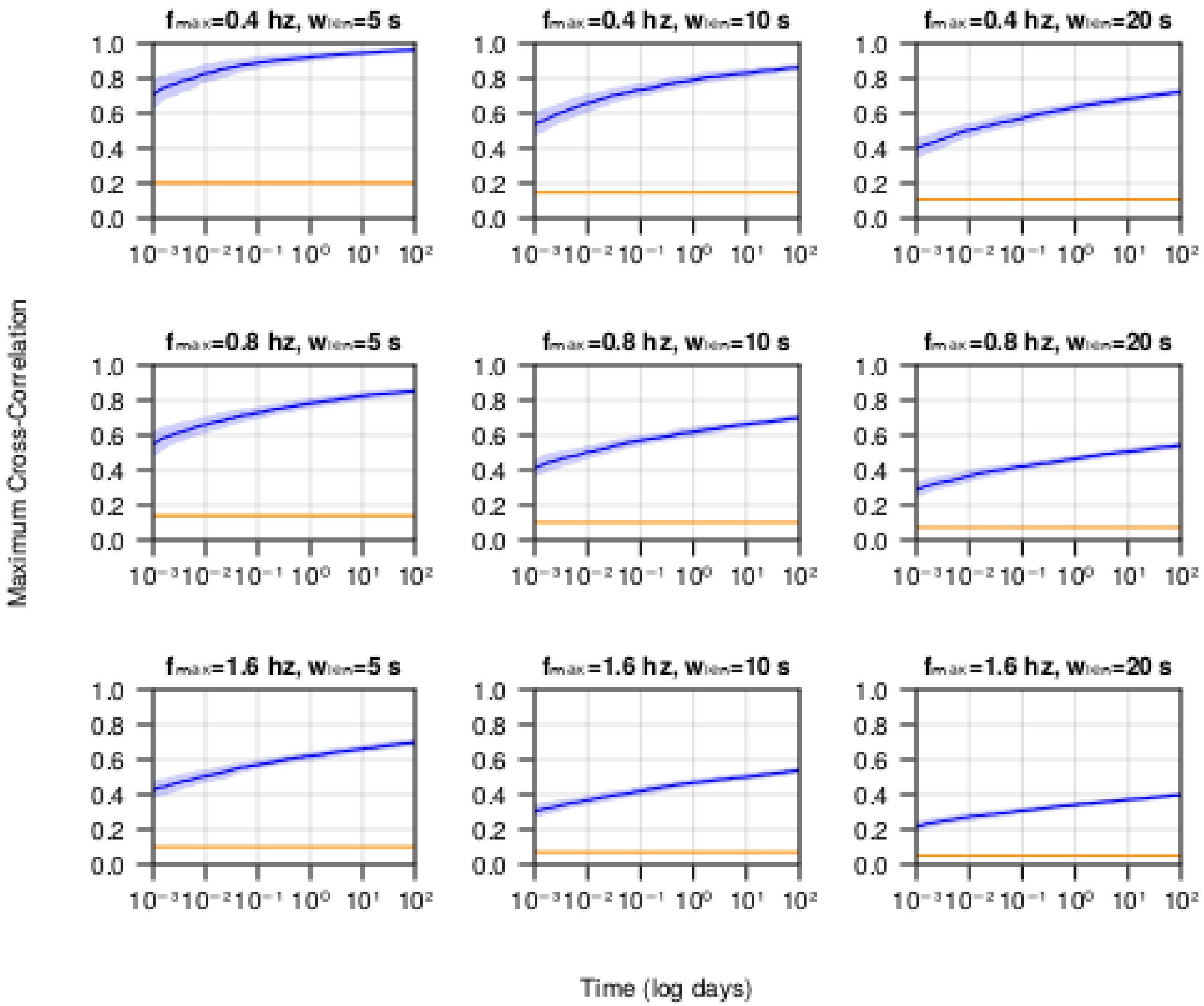

Figure 1 Maximum [-1,1] normalized cross-correlations between three-component random noise segments. Blue lines show the maximum cross-correlation up to some time, with the $\pm 1\sigma$ shown in light blue. Orange lines show the Median Absolute Deviation (MAD) over 100 days, with the $\pm 1\sigma$ shown in light orange (not visible due to narrow uncertainty over this interval).

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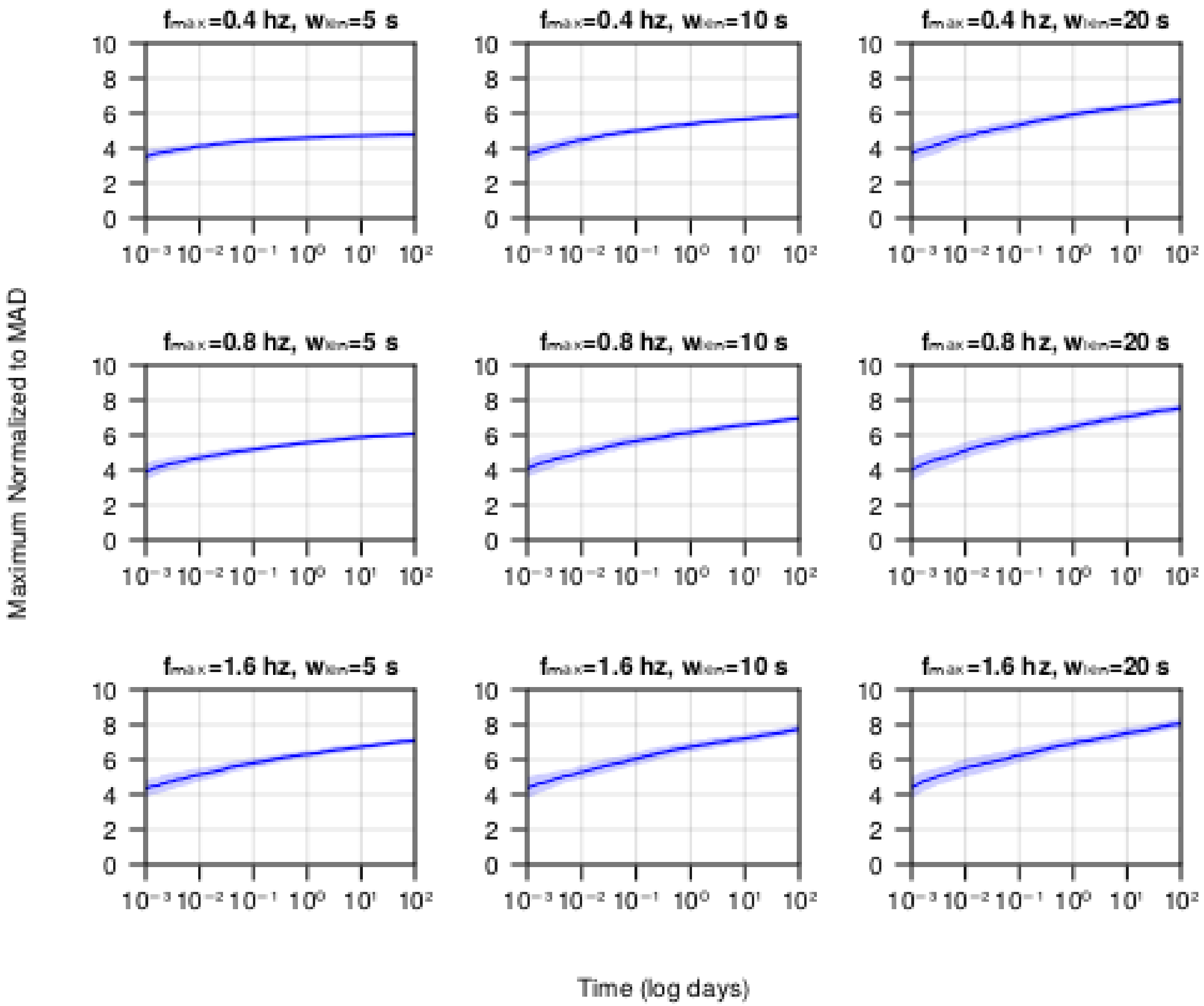

Figure 2 Maximum cross-correlation between three-component random noise segments, normalized by the Median Absolute Deviation (MAD) over 100 days. Blue lines show the maximum MAD normalized cross-correlation up to some time, with the $\pm 1\sigma$ shown in light blue.

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