The Spectrum of Fault Slip in Elastoplastic Fault Zones

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Highlights:

- Numerical models for inelastic fault zones show that the spectrum of fault slip depends on the bulk yield strength relative to fault reference frictional strength.
- For a given fault friction, weaker fault zones experience more inelastic deformation and plastic dissipation leading to the emergence of a new stability transition boundary that limits earthquake size regardless of fault length.
- As bulk inelastic dissipation increases, fault slip transitions from fast to slow slip and from throughgoing to localized ruptures, and eventually to complete locking of seismogenic patch.

Abstract

Natural faults are usually surrounded by damage zones that exhibit nonlinear material response. This study investigates the role of fault zone strength in modulating the spectrum of fault slip across different spatio-temporal scales. We carry out long-term simulations of seismic and aseismic slip for an elastoplastic spring slider model with rate-and-state friction as well as a continuum model of a 2D anti-plane rate-and-state fault embedded in an elastoplastic bulk. Results of the elastoplastic spring slider model show the emergence of a new stability boundary, depending on the bulk yield strength relative to fault frictional strength, that limits the rupture size regardless of the fault length. Continuum simulations generate a spectrum of slip analogous to the spring slider model including localized or migrating events of slow and fast slip. A fault may remain locked for yield strength sufficiently low and close to fault reference strength even if it is intrinsically rate weakening and larger than the nucleation length scale predicted by the elastic analysis. These findings shed new light on the nature of fault frictional stability and suggest the critical role of the fault zone rheological properties in modulating the spectrum of fault slip.

1. Introduction

Unstable sliding of geologic faults manifests itself as earthquakes—one of the deadliest and most prevalent yet unpredictable natural hazards. Apart from large earthquakes, faults are also found to host small earthquakes, and slow slip events (Beroza & Ide, 2011; Ito et al., 2013; Burgmann 2018). Usually, earthquakes are associated with frictional sliding on fault surfaces encompassing long-term slow aseismic slip and rapid seismic ruptures (Avouac 2015). However, fault zone complexities, including geometric and material non-linearity (Ben-Zion & Sammis, 2003; Mitchell & Faulkner, 2009; Lewis & Ben-Zion, 2010), influence the fault slip behavior leading to complex pattern of seismicity in space and time (Chen et al., 2020; Ross et al., 2020). Physics-based simulations for sequence of earthquakes and aseismic slip (Ben-Zion & Rice, 1993; Lapusta et al., 2000; Chen & Lapusta 2009; Kaneko et al., 2010; Barbot et al., 2012; Allison & Dunham, 2021; Erickson et al., 2022; Jiang et al., 2022; Abdelmeguid & Elbanna, 2022a, 2022b) are emerging as promising tools for understanding the complex processes associated with different forms of frictional instabilities and resulting slip pattern, as well as in developing seismic hazard models.

Stable and unstable frictional sliding is largely attributed to the frictional properties of the fault whether the steady-state frictional strength increases with slip rate (velocity strengthening) or decreases with slip rate (velocity weakening). A velocity-strengthening patch is generally associated with stable aseismic sliding and may become unstable through enhanced coseismic weakening associated with flash heating or a rapid increase in pore fluid pressure due to shear heating (Noda & Lapusta, 2013). While a fault with velocity weakening friction is locked during the interseismic period, it is potentially unstable and generates different patterns of slip. The style of slip in terms of peak slip rate, spatial extent, and temporal periodicity, depends on the size of the velocity weakening patch relative to the critical length scale associated with nucleation as well as the relative magnitude of frictional parameters associated with static and dynamic stress drop (Barbot 2019; Cattania 2019). Generation of slow slip sequence accompanied by slow earthquakes are shown in Barbot (2019) to depend on the relative magnitude of the frictional parameters and the relative fault size. Specifically, slow slip is found in these studies when the length of the velocity weakening patch is close to the nucleation length.

Additionally, geologic heterogeneities (Skarbek et al., 2012; Bedford et al., 2022) in terms of the different proportions of velocity strengthening and velocity weakening patches, as well as the rate-dependent evolution (Kaproth & Marone, 2013) of frictional parameters are possible mechanism for slow slip generation. Velocity-weakening friction generating stick-slip instabilities may generate slow slip when rate-and-state friction parameters evolve with slip rate and take transition from velocity weaking to velocity strengthening. Numerical simulations using a spring-slider model by Im et al. (2020) show that velocity dependent frictional parameters enable generating slow slip events having similar characteristics to those observed in nature (Dragert et al., 2001; Heki & Kataoka, 2008; Radiguet et al., 2012). However, it is not clear what physical mechanism may lead to this rate dependence of the frictional parameters.

Pore pressure also plays an important role in controlling spectrum of slip. Fluid pressure reduces effective normal stress and increases the nucleation length scale. Accordingly, the size of the seismogenic velocity weakening patch decreases relative to the size of the nucleation patch. This results in slow slip transients (Liu & Rice, 2007). Dilatant strengthening (Segall et al., 2010), resulting in reduced pore pressure, also explains the generation of slow slip events in a seismogenic velocity-weakening region. Slow and fast slip may arise through the relative contribution of dilatancy-induced strengthening and enhanced coseismic weakening due to thermal pressurization of pore fluid. Dilatant hardening itself may be a manifestation of inelastic processes associated with propagating crack tip (French & Zhu, 2017).

These on-fault characteristics controlling the spectrum of slip are investigated mostly with homogeneous elastic bulk. Heterogenous bulk with a low velocity zone near the fault may generate slip complexity (Abdelmeguid 2019; Thakur et al. 2020; Nie & Barbot 2022). Also, a recent study by Collettini et al. (2022) explains the observation of distributed microseismicity through a conceptual model of distributed ductile deformation in the bulk. Incorporation of viscoelastic fault zones (Miyake & Noda, 2019; Goswami & Barbot, 2018) and viscous damping in fault strength (Wu 2021; Nakata et al., 2011) are shown to generate slow slip events. Using numerical simulation with elastoplastic shear zone, Tong & Lavier (2018) shows generation of slow and fast slip events by varying the rate-and-state friction parameters. By reducing the difference between the direct effect parameter and state evolution parameter, they found slip pattern to change from fast slip events to slow slip. In the limit of vanishing difference between the friction parameters, corresponding to the limit of velocity neutral, creeping events are found.

In our previous work on seismic cycle simulations with off-fault plasticity (Mia et al., 2022), we showed that bulk yield strength and post-yield viscous relaxation contribute to the emergence of spatio-temporal clustering of seismicity. Here, we investigate the effect of off-fault bulk strength on fault slip expanding the parameter space for yield strength to also consider values close to the reference frictional strength. This is a critical parameter regime that have not been investigated before and may qualitatively alter the partitioning of slip and energy dissipation between on-fault and off-fault processes. To that end, we first simulate the long-term frictional sliding of an elastoplastic spring slider model with rate-and-state friction. Then, to further corroborate our findings, we investigate sequences of seismic and aseismic slip on a 2D anti-plane rate and state model embedded in a fully continuum elastoplastic model. We evaluate the resulting slip patterns with a special focus on slow slip.

2. Elastoplastic spring slider

We simulate the long-term sliding of a spring-block system under constant load-point velocity applied at the end of the spring. The frictional interface is velocity weakening governed by rate-and-state friction law (Dieterich, 1979; Ruina, 1983) and the state evolution follows the aging law (Ampuero & Rubin, 2008). The friction law is outlined in the Supplementary Information (Text-S1). To investigate the effect of bulk strength on frictional sliding, we vary the yield strength and the stiffness of the spring. Critical stiffness (Rice & Ruina, 1983; Ranjith & Rice, 1999) is related to the frictional properties and normal stress, and is defined by, $k_{cr} = \sigma_n(b - a)/L$. Critical stiffness marks the transition between stable and unstable frictional sliding in a purely elastic setting.

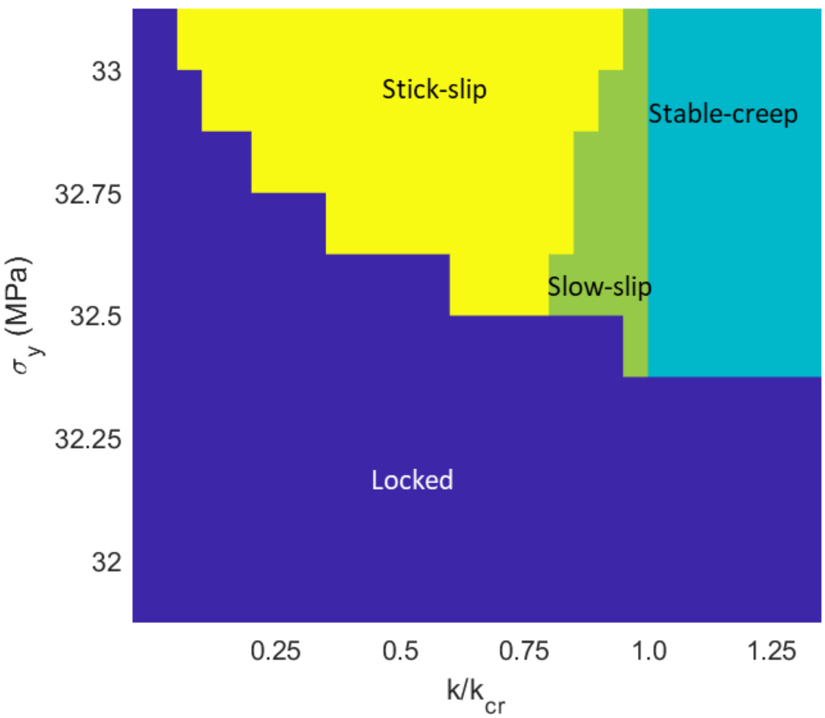

Figure 1. Sliding patterns for an elastoplastic spring block system depending on yield strength and stiffness of the spring. Reference strength of the frictional interface is $\sigma_n f_o = 30\ MPa$. Sliding patterns change from stick-slip to slow-slip as spring stiffness becomes close to a critical stiffness defined for the elastic system from the frictional parameters as $k_{cr} = \sigma_n(b-a)/L$. For stiffness higher than critical value, the sliding is stable. A region of slow-slip exists between stick-slip and stable-creep for a narrow range of stiffness and yield strength values. There is another transition towards lower stiffness, which depends on the magnitude of the yield strength, where the slider remains locked. This new stability boundary does not exist for purely elastic spring block systems.

From stability analysis (Ranjith & Rice, 1999; Rice et al., 2001), it is known that stiffness greater than the critical value correspond to stable sliding which tends to attain steady state in sync with the imposed load point velocity. For elastic spring, stiffness lower than the critical value generate unstable sliding. The elastoplastic spring bock slider shows different sliding patterns ranging from stick-slip to complete locking depending on the yield strength and the elastic stiffness of the spring (Figure-1). We classify the slip patterns based on the amplitude of the block's peak slip rate. Slow slip is identified with slip rate lower than a seismic threshold (taken here to be $0.01\ \mathrm{m/s}$) but higher than the background plate rate (10^{-9} m/s). Stick slip corresponds to slip rate exceeding the seismic threshold. We observe that slow slip emerges for a narrow range of yield strength and stiffness values. The block also creeps stably at the imposed loading rate for stiffness values larger than the critical value predicted by the elastic analysis. However, another new transition boundary emerges with plasticity, in the limit of lower stiffness, that is not observed in purely elastic analysis. This transition from stick-slip to locking (i.e., sliding with slip rates that are orders of magnitudes slower than the imposed plate loading rate) depends on the yield strength. As yield strength decreases, this transition stiffness increases. For sufficiently low yield strength as becomes close to the reference frictional strength ($\sigma_n f_o$), the slip rate asymptotically decreases to $\sim 10^{-15}$ m/s, orders of magnitude lower than the imposed plate rate. We refer to the sliding with this negligible slip rate as locked.

3. Continuum simulation of a 2D anti-plane rate-and-state fault

We model a 2D anti-plane rate-and-state fault embedded in a full-space elastoplastic medium subjected to slow tectonic plate rate. The fault has a central velocity-weakening VW region surrounded by velocity-strengthening VS patches from both sides. Off-fault material response is modeled with J2 plasticity which coincides with Drucker-Prager plasticity for the anti-plane setting with no variation in normal stress. A hybrid scheme combining finite element and spectral boundary integral is employed for spatial discretization (Abdelmeguid et al., 2019; Mia et al., 2022; Abdelmeguid & Elbanna 2022b). We use an adaptive time-stepping algorithm (Lapusta et al. 2000) to efficiently resolve slow and fast slip. The model geometry with the hybrid scheme setup as well as the input parameters for the simulations are outlined in the Supplementary Information (Text-S2, Table-S1, Figure-S1).

We simulate sequences of earthquakes and aseismic slip (SEAS) with different values of yield strength to investigate the effect of bulk strength on long-term fast and slow slip. Resulting patterns, given as space-time contours of the fault slip rate, with different values of yield strength are shown in Figure 2. The temporal evolution of the peak slip rate in different cases is shown in Supplementary Information, Figure-S2. The simulations show sliding patterns analogous to the elastoplastic spring slider model. For lower yield strength the fault remains locked without generating any seismic event. When the yield strength is increased, the fault generates unstable frictional sliding including slow slip, spatially localized seismic events, and partial ruptures distributed over the fault length intermixed with transient episodes of slow slip. For higher values of yield strength, or in the limit of purely elastic bulk, the fault fails in predominantly large, fault-spanning, fast earthquakes. Below we briefly describe these different slip regimes.

Locked fault: When the yield strength (31 MPa) is close to the fault reference frictional strength ($\sigma_n f_o = 30$ MPa), the slip rate of the central VW patch decreases to $\sim 10^{-15}$ m/s (Figure-2a). Relative to the plate loading rate, this slip rate is orders of magnitude smaller which indicates that the seismogenic zone (VW patch) of the fault remains effectively locked or stuck. The VS patch of the fault creeps following the plate loading. Aseismic creep from the VS patch penetrates slightly into the VW patch but the peak slip rate remains close to the imposed plate rate (10^{-9} m/s) in a very limited region adjacent to the VS patch as shown in Figure-2a and Supplementary Information, Figure-S2a.

Slow slip: When the yield strength is increased to 32 MPa, slow slip emerges (Figure-2b, 2g, 2j). Signature of creep penetration exists near the transition between the VS and VW patches. The duration of the slow slip events for yield strength 32 MPa is in the scale of weeks to month.

Spatially, these events are localized near the transition region from VS to VW. Similar spatially localized slow slip events are also observed in between localized partial ruptures for yield strength 32.5 MPa (Figure-2c, 2h, 2k). However, the peak slip rate associated with some of these aseismic transients reaches orders of magnitude higher than the background plate rate. The irregular pattern with yield strength 33.5 MPa also includes slow slip events (Figure-2d, 2i, 2l). The peak slip rates associated with these slow slip events are even higher, but they are still below the seismic threshold. These slow slip events, observed with yield strength 33.5 MPa, are no longer localized in the transition region. Rather they show spatial migration over the full length of the seismogenic (Velocity-Weakening) patch of the fault.

Localized seismic events: A repeating pattern of localized seismic events is observed for yield strength 32.5 MPa as shown in Figure-2c. These seismic events are inter-mixed with slow slip episodes as described above. The seismic events are spatially localized near the boundary between VS and VW. They rupture approximately the same area, but they do not repeat with the exact return period (Supplementary Figure-S2c). Their average stress drop is around 2.3 MPa with standard deviation of 0.5 MPa approximately which is consistent with stress drop measured for earthquake repeaters (Chen & Lapusta, 2009). The slow slip events observed in this case occur mostly before the seismic events.

Stick-slip with partial ruptures: When yield strength is increased to 33.5 MPa, fault sliding includes slow slip and partial seismic ruptures (Figure-2d). In the early stages of the sequence, partial ruptures extend on both sides of fault, but a locked patch remains in the center that gets progressively narrower with time. Later in the cycle, the whole VW region is ruptured by subsequent partial events. Here the partial ruptures are prevalent throughout the VW patch unlike the localized repeating events shown in Fiure-2b. Also, the clustering of events observed with relatively higher yield strength e.g., 36 MPa (Figure-2e) is not found for the case with yield strength 33.5 MPa.

Stick-slip with fault-spanning ruptures: For relatively higher yield strength (e.g., 36 MPa shown in Figure-2e), seismic events, including partial ruptures as well as ruptures spanning the full VW patch, dominate the slip pattern. Evolution of the spatial extent of rupture, discussed above for lower yield strength, showing central locked patches progressively unclamped with time does not exist with relatively higher yield strength. The resulting sequence, including partial and fault-spanning rupture, is aperiodic with clustering of seismic events in space and time. Simulation with elastic bulk (Figure-2f) results in simple periodic cycle with fault-spanning ruptures only. Cycle simulations for a range of high values of yield strength and post-yield viscosity are discussed in our previous study (Mia et al., 2022) where seismicity pattern changes from complex spatio-temporal clustering to simple periodic pattern with increasing yield strength and post-yield viscosity. While here we adopted a quasi-dynamic approximation of inertia effect through a radiation damping term, the results are qualitatively similar even if we consider full inertia effects (Supplementary Information, Figure-S2).

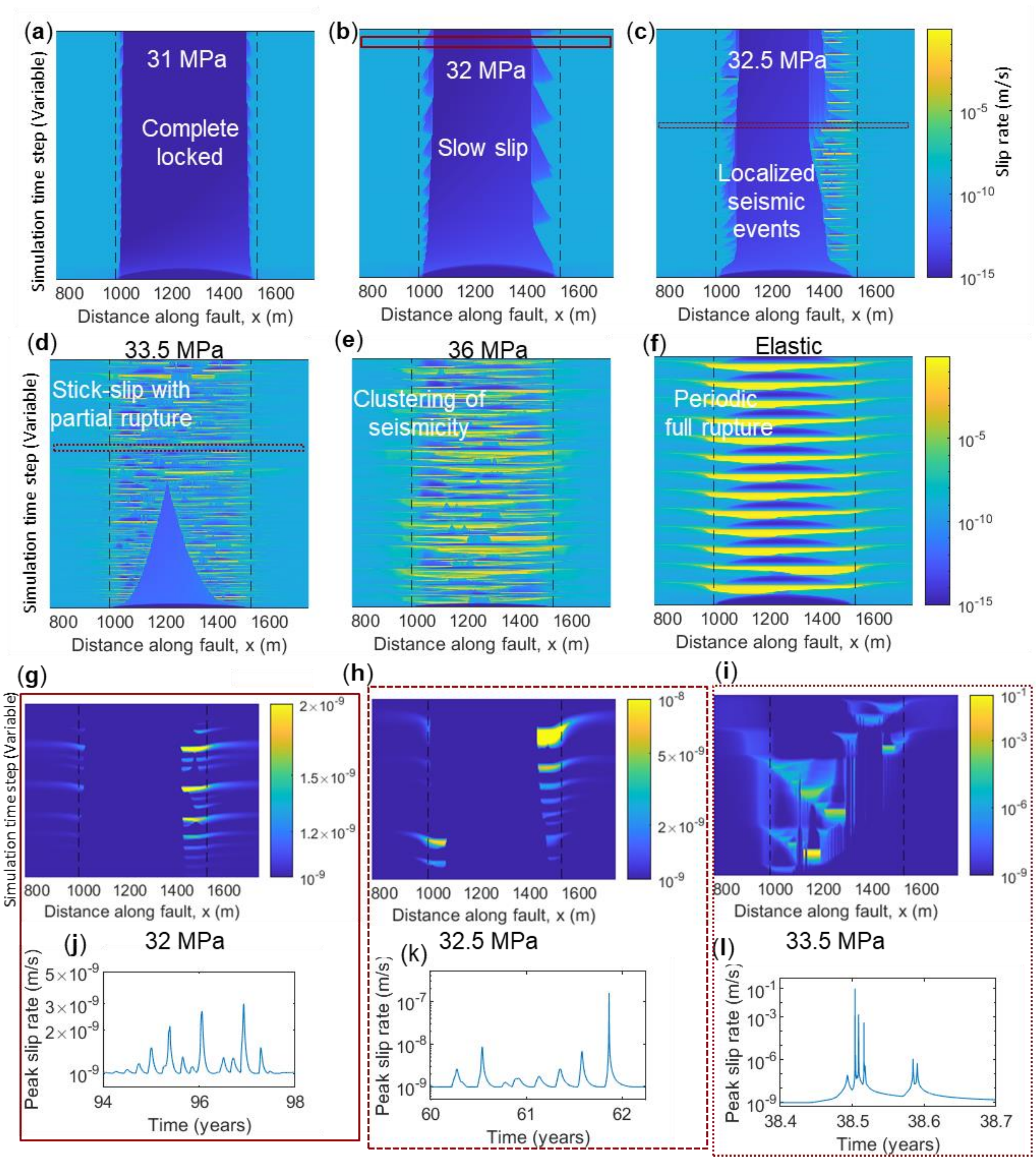

Figure 2. Spectrum of slip for a rate-and-state fault. Spatio-temporal evolution of slip rate illustrating different slip pattern for different yield strength (**a-e**). Elastic case results in periodic seismic cycle as shown in **f**. Region between two vertical dashed lines correspond to velocity weakening friction. Fault reference strength is $\sigma_n f_o = 30$ MPa. Fault remains locked for yield strength, $\sigma_y = 31$ MPa. Locked fault with occasional slow slip is found for $\sigma_y = 32$ MPa. Localized earthquakes near the transition between VS and VW are shown in **c**. Complex pattern including slow slip and seismic events with partial rupture is shown in **d**. Clustering of seismicity is found when yield strength is increased (**e**). Closer examination of example of slow slip events that exist with $\sigma_y = 32, 32.5$ and 33.5 MPa are shown in **g-l**.

Partitioning of deformation

Plasticity results in a slip deficit on the fault surface through partitioning of total deformation into on-fault slip and off-fault plastic deformation. In Figure-3, the spatial distribution of the cumulative slip and off-fault plastic deformation is shown for yield strength 33.5 MPa. Here we calculate an effective measure for the variation of the plastic deformation in the fault zone along the fault length by integrating the equivalent plastic strain, γ, as $\mathrm{u_p}(x,t) = 2\int_0^{L_y} \gamma(x,y,t)dy$, where, L_y is the half width of the computational strip modeled using FEM, and y represents the spatial direction normal to the fault plane. The factor of 2 accounts for the symmetry of the plastic strain distribution about the fault surface. This is characteristics of anti-plane plasticity in homogeneous media where the normal stress does not change with deformation. For the current model geometry, $L_y = 30$ m which is around 1.5 times the process zone size. This width is found sufficient to contain the spatial extent of plasticity in the fault normal direction. Outside this computational strip, the exterior half-spaces are elastic and are modeled using the spectral boundary integral approach. Therefore, they do not experience plastic deformation. Total plastic deformation along the fault, $\mathrm{u_p}(x,t)$ is shown in Figure-3b. Cumulative slip is plotted in Figure-3a. The contour lines for both quantities are plotted every 10 years up to 50 years.

As shown in Figure-3a, cumulative slip in the VS patch is around 1.5 m in 50 years. Slip accumulation is reduced near the transition from VS to VW and is further reduced in the interior of the locked VW region. The central region of the VW patch is locked in early stages. As time passes, seismic events progressively unlock the VW fault through a sequence of partial ruptures as shown in Figure-2d. The non-smooth shape of the cumulative slip lines corresponds to the irregular pattern of slow slip and partial ruptures spreading over the whole VW region. From the plastic deformation plot (Figure-3b), it is evident that plastic deformation in the bulk compensates for the slip deficit in the VW patch. Plastic deformation is maximum in the central part of the VW patch where the fault slip is minimum and gradually vanishes towards the VS region.

Off-fault plastic deformation gets an increasing share of the total deformation budget when the bulk yield strength is lower. To evaluate the partitioning of deformation and associated slip pattern for different values of yield strength, we compute the on-fault deformation by integrating fault slip termed as $\text{Potency}\,(t) = \int_0^{L_f} d(x,\, t)dx$, and off-fault plastic deformation by integrating the equivalent plastic strain over the domain as $\text{PD}(t) = \int_S \gamma(x, y, t)dS$. Where d is the slip, L_f is the seismogenic region of the fault including VW patch and the transition region between VW and VS, and S is the area of the 2D elastoplastic fault zone. For all the cases shown in Figure-3c, there is an initial increase of plastic deformation while the potency is small. This indicates plasticity accumulation when the fault is locked prior to any seismic events. When fault slip occurs, both potency and plastic deformation increases but the increment of plastic deformation is relatively lower. For the cases shown in Figure-3c, the ratio between plastic deformation and potency varies over an order of magnitude ranging from ~0.1 for $\sigma_y = 36$ MPa to ~3.5 for $\sigma_y = 32$ MPa.

For the same amount of potency, plastic deformation is higher with lower yield strength. The pattern of slip with clustered seismic events ($\sigma_y = 36$ MPa , shown in Figure-2,3) is associated with relatively lower plastic deformation to potency ratio (~ 0.1). However, fault slip involving slow slip and partial ruptures spreading over the fault ($\sigma_y = 33.5$ MPa , shown in Figure-2,3) correspond to higher ratios of plastic deformation to potency (~ 0.5). Localized seismic event and slow slip in otherwise locked fault ($\sigma_y = 32.5$ MPa, $\sigma_y = 32$ MPa shown in Figure-2,3) corresponds to relatively higher plastic deformation to potency ratio (> 1). In these limits of low yield strength, more deformation is distributed in the bulk than localized as slip on the fault surface.

Figure-3d shows the ratio of plastic dissipation relative to total dissipation. Total dissipation is the sum of plastic dissipation (PW) and frictional dissipation (FW). Here, frictional dissipation is computed over the region of the fault used to compute the potency. This region includes the VW patch as well as the transition regions between the VW and VS patches. That is, we do not account for frictional dissipation due to slip in the VS region which conforms to the plate loading. Contribution of plastic dissipation is higher with lower yield strength. On one hand, for the case of $\sigma_y = 32$ MPa , generating slow slip events, plastic dissipation is around 80% of the total dissipation. On the other hand, clustered seismic events with $\sigma_y = 36$ MPa is associated with much lower plastic dissipation ($\sim 10\%$ of total dissipation). The decrease in the ratio of plastic to total dissipation is a result of less off-fault plastic deformation and corresponds to higher frictional dissipation through fault slip in large and fast seismic events. Higher off-fault deformation and smaller on-fault slip leads to increased off-fault dissipation. Mia et al., 2022, showed that even if the contribution of plasticity to the energy budget is small, it may still play a significant role in regulating the spatio-temporal clustering of seismic events through limiting the stress concentration ahead of the rupture tip and facilitating rupture arrest. Here, we additionally show that if the energy dissipation is dominated by off-fault plastic dissipation, which happens for lower yield strength, fault-spanning events cannot occur and slip becomes progressively slower. Specifically, we observe that irregular partial ruptures, slow slip, repeaters, and even complete locking of the VW patch emerge in the limit of low yield strength and increased off-fault plastic dissipation irrespective of fault length. This further suggests that plastic dissipation should be low (less than 10%) compared to total dissipation, in order for large fault spanning events to occur.

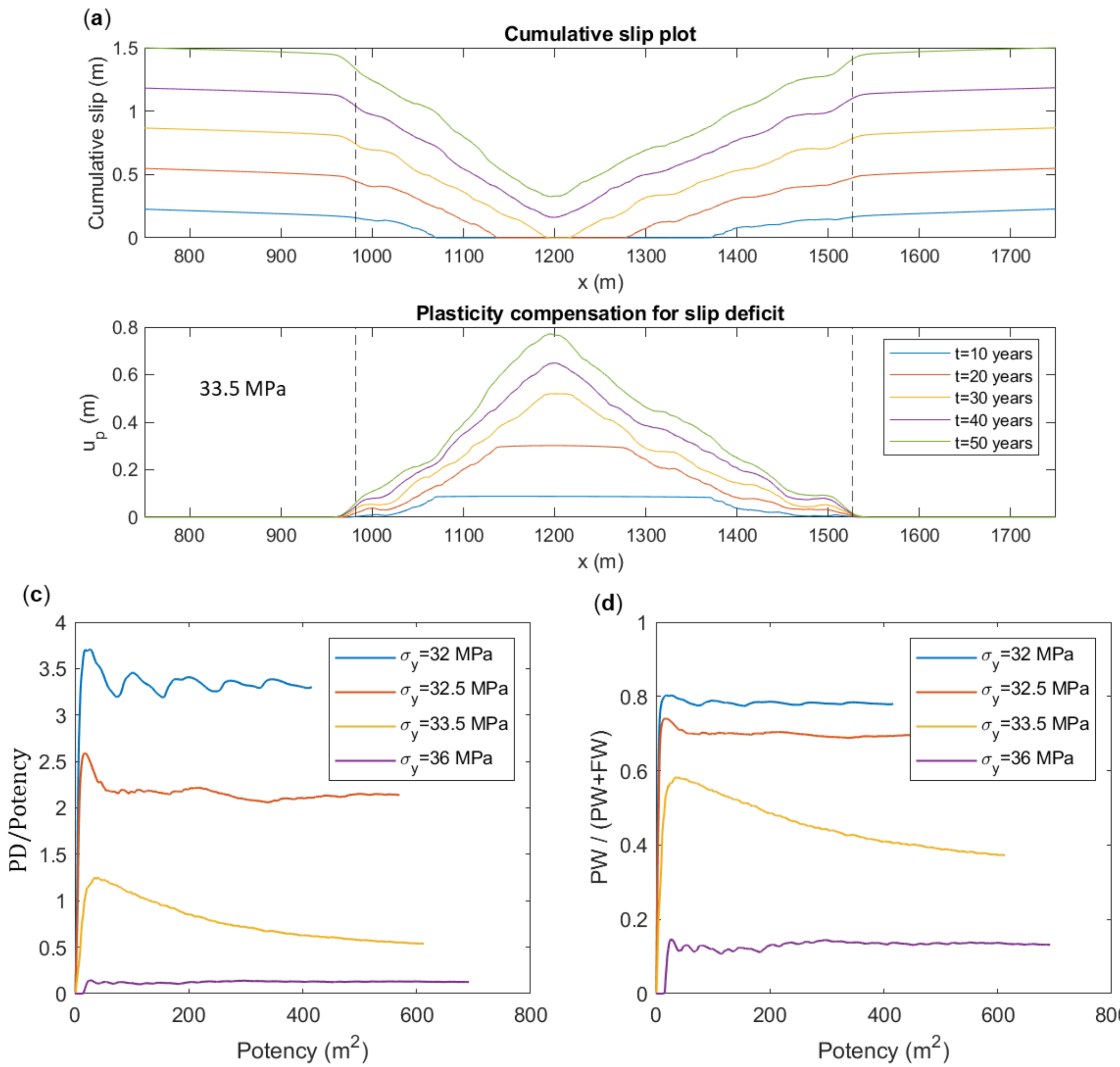

Figure 3. Plasticity compensation for slip deficit and partitioning of deformation as well as energy dissipation between the bulk and the fault. (**a**) Cumulative slip is plotted at an interval of 10 years (for $\sigma_y = 33.5$ MPa). (**b**) Off-fault plastic deformation obtained from integrating the plastic strain in lateral direction is plotted along the fault in the same interval of cumulative slip plot (for $\sigma_y = 33.5$ MPa) . The region within the wo vertical dashed lines corresponds to the fault section with velocity weakening friction. The extent of locked region subsequently shrinks with increased slip. Plasticity compensation is higher in the region where slip accumulation is lower. (c) Relative contribution of on-fault slip and off-fault plastic deformation. Potency is computed by integrating slip, Potency $(t) = \int_0^{L_f} d(x,\ t)dx$ and plastic deformation is computed from integrating the plastic strain over the domain, $\mathrm{PD}(t) = \int_S \gamma(x,y,t)dS$. Lower yield strength corresponds to higher amount of plastic deformation. (**d**) Plot of plastic energy dissipation (PW) relative to total dissipation. FW is the frictional dissipation computed over the velocity weakening patch and the transition region. Higher percentage of plastic dissipation is associated with lower yield strength generating slow slip.

4. Discussion

In this work, we showed, using a simple elastoplastic spring block model, that spring stiffness, even when lower than the critical value for instability predicted by elastic analysis, may not facilitate stick-slip instability if the yield strength is low enough. Since stiffness is inversely proportional to length, this finding suggests that not all perturbations with wavelength greater than the nucleation length are unstable. Rather the maximum wavelength that is unstable may vary depending on yield strength. This implies that there may exist a maximum rupture length, when off-fault plasticity is considered, irrespective of how large the fault is. Similar patterns are observed in continuum 2D anti-plane simulations with different values of yield strength. The rich slip patterns observed in the continuum model include locked fault, slow slip events, localized sequence of seismic event and irregular patterns with partial ruptures spreading over the fault. Neither frictional heterogeneity nor pore pressure perturbation is introduced here. Only the bulk strength is acting as the controlling factor in modulating the slip pattern.

In a characteristic seismic cycle, VS patch accumulates slip during the interseismic period and VW patch catches up with seismic slip accumulation. However, when the bulk strength is relatively low, seismic events are rare or spatially limited with partial ruptures. This results in slip deficit prevailing throughout the seismic cycle. We found that off-fault plastic deformation compensates for the slip deficit in this case. For sufficiently low bulk strength, fault may remain locked without generating seismic events while off-fault bulk accommodates the deformation through plastic deformation. This is also analogous to the spring slider results where plastic deformation in the spring resists and even, in some cases, prevents the sliding of the block. Relative amount of off-fault plastic deformation and on-fault slip is associated with the spectrum of slip from clustered fast slip to a mixture of fast and slow slip as the yield strength decreases. Inelastic dissipation must remain low for fast fault large events to exist.

We note that large amount of shallow off-fault deformation, exceeding 80% of the total surface deformation (i.e., 4 times the on-fault slip) has been reported around some faults recently by Li et al., (2022). They hypothesized that the large amount of off-fault deformation may include some elastic deformation. The determination of the extent of inelastic off-fault deformation would be possible if additional data from seismic refection was available. The plastic strain in our simulation is limited to a narrow region close to the fault (Supplementary Figure-S4). If this holds true for off-fault plasticity accumulation in natural fault zones, particularly at depth, it may be difficult to distinguish between on-fault and off-fault deformation from field observations specially from satellite measurements as there are limitations on high resolution measurements of off-fault deformation (Antoine et al., 2021). This partitioning of deformation is, however, important to quantify as accumulation of off-fault plasticity may have implications for fault weakening. As plastic deformation involves dissipation over narrow but finite region, it can influence the heat diffusion and constrain the rise of temperature associated with slip localization into an extremely thin surface in an otherwise elastic medium (Rice 2006). This may contribute to understanding the scarcity of fault zone melting in some field observations. However, since most of the plastic dissipation is still in the form of heat, we do not think that off-fault plasticity resolves the heat flow anomaly.

Slow slip events recorded in some field observations show a wide range of durations in the scale of days to months, and large earthquakes have been found after the migration of slow slip events and small earthquakes (Ito et al., 2013; Ruiz et. al., 2014; Obara & Kato, 2016). The coexistence of slow slip events and small earthquakes in subduction zone has also been reported by Ito et al.,

2007. In our simulation, fault slip accompanied by a mixture of seismic events with partial ruptures and slow slip events are observed. Some of the slow slip events show spatial migration over the seismogenic patch with duration reaching the scale of months. We observe emergence of spectrum of slip by simulating different cases with small difference in yield strength. In natural fault zone, off-fault bulk property (for example, yield strength) may vary spatially and evolve with time. Since yield strength depends on effective normal stress, physical process like fluid pressure perturbation may alter the yield strength independent of fault strength alteration depending on the hydraulic properties in the fault core relative to the fault zone. Healing of damaged fault zone may also contribute to the evolution of yield strength. Consequently, a fault may host different patterns of slip switching from fast seismic rupture to slow aseismic slip when yield strength transiently decreases.

The localized seismic events observed in our continuum simulations, at the boundary of the VW and VS regions, is analogous to repeating microseismicity model presented in Sammis & Rice (2000) but some differences exist. The similarity is that the location of the seismic events is near the transition between creeping asperity (VS patch) and seismogenic locked asperity (VW patch). The slip deficit in their model is compensated by occasional full-fault spanning events whereas the slip deficit in our simulations is compensated by off-fault plastic deformation. Also, in our simulations, there are slow slip events in between the seismic events.

In this model, we consider the frictional parameters like reference friction coefficient, direct effect, and state evolution coefficient to remain stationary. However, frictional parameters may evolve with coseismic deformation (Kaproth & Marone, 2013; Im et al., 2020) which may get pronounced with off-fault damage or plasticity accumulation. Investigating the interplay between evolving fault friction as well as degradation and subsequent healing of fault zone material warrants future study. Fault zone geometric complexity including nonplanar fault, and interaction of multiple faults in presence of off-fault material non-linearity is planned as future study. Furthermore, evolution of pore pressure associated with volumetric deformation of the fault zone need to be considered for its vast implications on the evolution of strength and deformation in fluid-saturated fault zones.

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Data Availability Statement

The authors accept the data policy. Data generated from the numerical simulations are uploaded on an open access repository (10.5281/zenodo.7718768).

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Supplementary Information for

The Spectrum of Fault Slip in Elastoplastic Fault Zones

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Introduction

The Supplementary Information includes:

- Text S1 outlines the model setup and methods.
- Table S1 provides input parameters for the simulations.
- Figure S1 shows model geometry and hybrid scheme setup.
- Figure S2 shows history of peak slip rate for simulations with different yield strength and elastic case .
- Figure S3 shows results of fully dynamics simulations for yield strength 33.5 MPa, 36 MPa and elastic case. Slip patterns are qualitatively similar to the quasi-dynamics cases presented in the main text.
- Figure S4 shows spatial extent of off-fault plasticity for yield strength 33.5 MPa.

Text S1. Rate-and-state friction

The fault friction is governed by a regularized rate-and-state friction law (Dieterich, 1979; Ruina, 1983; Ben-Zion & Rice, 1997; Lapusta et al., 2000) where the friction coefficient, f, is a function of slip rate, V, and state variable, θ.

$$f(V,\theta) = a\ sinh^{-1}\left[\frac{V}{2V_0} exp\left(\frac{f_0 + bln(V_0\theta/L)}{a}\right)\right] \qquad \text{(Eqn-1).}$$

Here, a and b are non-negative dimensionless frictional parameters related to direct effect and state evolution respectively. $a < b$ indicates velocity weakening friction whereas $a > b$ indicates velocity strengthening friction. f_0 is the reference friction coefficient with a reference slip rate V_0. L is critical slip weakening distance. State variable, θ, refers to the time of contact between the sliding asperities which evolves following a prescribed aging law (Ruina, 1983; Ampuero & Rubin, 2008):

$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{L} \qquad \text{(Eqn-2).}$$

For constant normal stress, σ_n, fault strength is then expressed as $\sigma_n\, f(V,\theta)$. In case of pore pressure perturbation $P(x,\mathrm{t})$, effective normal stress is $\sigma_e(x,t) = \sigma_n - P(x,t)$, and fault strength becomes $\sigma_e(x,t)\ f(V,\theta)$. We model dry case considering no perturbation of pore pressure i.e., $\sigma_e(x,t) = \sigma_n$. Approximating inertia with radiation damping, called quasi-dynamics approach (Rice 1993; Ranjith & Rice, 1999), shear stress in the frictional interface is given by,

$$\tau = \sigma_e\ f(V,\theta) + \eta_r V \qquad \text{(Eqn-3).}$$

Where the radiation damping coefficient is given by $\eta_r = \frac{\mu}{2c_s}$.

Text S2. Model Setup and Methods

The model includes frictional interface (fault) embedded in a 2D whole space (Figure-S1).

Balance of linear momentum leads to the equilibrium equation:

$$\sigma_{ij,j} + b_i = \rho \ddot{u}_i \ \text{ in } \Omega \qquad \text{(Eqn-4).}$$

For 2D anti-plane deformation, displacement at any point (x, y), and time t, is $u_z(x, y, t)$. Corresponding stress components are σ_{xz} and σ_{yz}. Eqn-4 gives wave equation for linear elastic homogeneous material with infinitesimal strain approximation. Slow deformation during interseismic period allows ignoring inertia term and solving a series of static equilibria with tectonic plate deformation and friction boundary condition on the fault surface. Ignoring body force (b_i) and dropping inertia term ($\rho\ddot{u}_i$) Eqn-4 reduces to

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} = 0 \qquad \text{(Eqn-5).}$$

Slip constraint on the frictional interface at $y = 0$ is given by

$$u_z(x, 0^+, t) - u_z(x, 0^-, t) = d(x, t) \qquad \text{(Eqn-6).}$$

Continuity of traction across the fault plane leads to

$$\sigma_{yz}(x, 0^+, t) = \sigma_{yz}(x, 0^-, t) = T^f \qquad \text{(Eqn-7).}$$

For elastoplastic bulk, constitutive relation with additive decomposition of total strain gives

$$\dot{\sigma}_{xz} = 2\mu\,(\dot{\epsilon}_{xz} - \dot{\epsilon}^p_{xz}), \quad \text{and} \quad \dot{\sigma}_{yz} = 2\mu\,(\dot{\epsilon}_{yz} - \dot{\epsilon}^p_{yz}) \qquad \text{(Eqn-8).}$$

where, μ is the shear modulus and components of the symmetric total strain tensor are given by

$$\epsilon_{xz} = \frac{1}{2}\frac{\partial u_z}{\partial x}, \quad \text{and} \quad \epsilon_{yz} = \frac{1}{2}\frac{\partial u_z}{\partial y} \qquad \text{(Eqn-9).}$$

We use J2 plasticity model to capture the inelastic response of the off-fault bulk. For 2D anti-plane problem, it reduces to a yield function:

$$F(\sigma) = \sqrt{(\sigma_{xz}^2 + \sigma_{yz}^2)} - \sigma_y \qquad \text{(Eqn-10).}$$

Where, σ_y is the bulk yield strength. Plastic strain rate is expressed as,

$$\dot{\epsilon}^p_{xz} = \dot{\gamma}\,\frac{\partial F}{\partial \sigma_{xz}}, \quad \text{and} \quad \dot{\epsilon}^p_{yz} = \dot{\gamma}\,\frac{\partial F}{\partial \sigma_{yz}} \qquad \text{(Eqn-11).}$$

Where, $\dot{\gamma}$ is the consistency parameter defining the rate of equivalent plastic strain. We use a radial return mapping algorithm (Simo & Hughes, 2006; Abdelmeguid & Elbanna, 2022b) to update the stresses.

In earthquake cycle simulation, the above equations need to be solved for an effectively unbounded domain. Spectral boundary integral enables truncating the computational domain by replacing the exterior homogeneous linear elastic half spaces with integral relation between the shear stress and displacement history (Breitenfeld & Geubelle, 1998; Geubelle & Breitenfeld, 1997; Geubelle & Rice, 1995; Lapusta et al., 2000; Abdelmeguid et al., 2019; Abdelmeguid & Elbanna 2022a, 2022b). For 2D antiplane problem, the shear stress at the boundary of the homogeneous half space is given by

$$T_3^{s^\pm}(x,t) = T_3^{o\ s^\pm}(x,t) \mp \frac{\mu}{c_s}\dot{u}_3^\pm(x,t) + f_3^\pm(x,t) \qquad \text{(Eqn-12).}$$

Where, the superscripts $+$, and $-$ indicate top and bottom half space respectively. T_3^o represents the initial stress as well as any externally applied stress. $\dot{u}_3$ represent particle velocity and c_s is shear wave speed. f_3 is a functional resulting from space-time convolution of the displacement history at the boundary expressed in Fourier domain. In the velocity representation, the Fourier coefficient of $f_3^\pm(x,t)$ is expressed as

$$F_3^\pm(t;k_n) = \mp\mu|k_n|U_3^\pm(t;k_n) \pm \int_0^t W_{33}(|k_n|c_s t')\dot{U}_3^\pm(t-t';k_n)dt' \qquad \text{(Eqn-13).}$$

The convolution kernel is given by $W_{33}(\xi) = \int_\xi^\infty \frac{J_1(\zeta)}{\zeta}d\zeta$,where $J_1(\zeta)$ is the order one Bessel function of the first kind. Wave number, $k_n = 2\pi n/\lambda$, with replication length λ. U_3 and $\dot{U}_3$ are the Fourier coefficients of displacement and velocity respectively. For the aseismic slow deformation phase, the velocity terms from Eqn-12 and Eqn-13 can be neglected. The first term in Eqn-13 represents the static contribution,

$$\tilde{F}_3^{\pm}(t;k_n) = \mp\mu|k_n|U_3^{\pm}(t;k_n) \quad \text{(Eqn-14).}$$

Also considering the stress increment beyond the initial condition, we can set $T_3^{s^{\pm}} = 0$. Therefore, the boundary stress, $T_3^{s^{\pm}}(x,t)$ can be calculated using inverse Fourier transform of $\tilde{F}_3^{\pm}(t;k_n)$.

We incorporate the fault surface in FEM using domain decomposition technique (Aagaard et al., 2013), where the Lagrange multiplier represents fault traction (T^f). Weak form of the governing equation gives following system of equations (Abdelmeguid et al., 2019; Abdelmeguid & Elbanna, 2022b):

$$\mathbf{K}u - F^p + \mathbf{L}_\mathrm{f}^T T^f + \mathbf{L}_\mathrm{s}^T T^s = F \quad \text{(Eqn-15)}$$

$$\mathbf{L_f}u = \mathbf{L_d}d \quad \text{(Eqn-16).}$$

For elastoplastic bulk, plastic force is obtained by integrating the plastic strain, $F^p = \int_\Omega \mathbf{B^T D}\epsilon^p d\Omega$. Where, **B** is the strain-displacement matrix, **D** is the matrix with material moduli. **K** denotes the stiffness matrix; F is the force vector. $\mathbf{L_f}$, $\mathbf{L_d}$ and $\mathbf{L_s}$ is comes from integrating the shape function.

We use a predictor-corrector approach to solve for the unknown displacements and fault tractions. Fault slip rate is then calculated from the following equation by equating fault total traction with fault strength and including radiation damping approximation for inertia (called quasi-dynamics approach). With radiation damping coefficient, $\eta_r = \frac{\mu}{2c_s}$,

$$T^f + T_o^f = \sigma_e f(V,\theta) + \frac{\mu V}{2c_s} \quad \text{(Eqn-17).}$$

Where, T_o^f is fault initial traction. Slip for next time step is computed by integrating the slip rate.

Solution steps for Quasi-dynamics solver:

1. At any time, t, for given slip, $d(t)$, state variable, $\theta(t)$, applied tectonic deformation, $u_b(t) = V_p t$, displacement $u(t-\Delta t)$, plastic strain, $\epsilon^p(t-\Delta t)$ and hence plastic force, $F^p(t-\Delta t) = \int_\Omega \mathbf{B}^\mathbf{T}\mathbf{D}\epsilon^p(t-\Delta t)d\Omega$ are known.
2. Prediction for SBI boundary traction, T^s, using $u(t-\Delta t)$.
3. Elastic predictor step, $F^p(t) = F^p(t-\Delta t)$.
4. Solve linear system of equations for $u(t)$ and $T^f(t)$:

$$\mathbf{K}u - F^p + \mathbf{L}_\mathrm{f}^T T^f + \mathbf{L}_\mathrm{s}^T T^s = F$$

$$\mathbf{L_f}u = \mathbf{L_d}d$$

5. Corrector for SBI boundary traction using $\frac{1}{2}[u(t) + u(t-\Delta t)]$.
6. Plastic corrector: using radial return algorithm, compute $\epsilon^p(t)$ and $F^p(t)$.
7. Repeat steps $4-6$ until error, $\frac{||u^{n+1}(t)-u^n(t)||}{||u^{n+1}(t)||} < tolerence$
8. Compute slip rate, V, using T^f from: $T^f + T_o^f = \sigma_e\, f(V, \theta) + \frac{\mu V}{2c_s}$
9. Estimate time increment based on slip rate and characteristic slip distance, $\Delta t = min\,[C\,\frac{L}{V}]$ (Lapusta et al., 2000)
10. Update slip and state variable for next time step:

$$d(t+\Delta t) = d(t) + V(t)\Delta t$$

$$\dot{\theta}(t) = 1 - V(t)\,\theta(t)/L$$

$$\theta(t+\Delta t) = \theta(t) + \dot{\theta}(t)\Delta t$$

Fully dynamics solver with coupling of finite element and spectral boundary integral scheme is outlined in our previous studies (Mia et al., 2022; Abdelmeguid & Elbanna, 2022a, 2022b).

Table S1. List of parameters used in the simulations.

Parameter	Symbol	Values
Effective normal stress on fault	σ_n	50 MPa
Critical slip distance	L	500×10^{-6} m
Plate rate	V_p	10^{-9} m/s
Reference Slip rate	V_0	10^{-6} m/s
Initial slip rate	V_{init}	10^{-9} m/s
Reference friction coefficient	f_0	0.6
Shear wave speed	c_s	3464 m/s
Shear modulus	μ	32.038 GPa
Yield strength	σ_y	Variable
Nucleation length	L_{nuc}	100 m
Process zone size	R	21.36 m
Mesh size	dx and dy	0.5 m

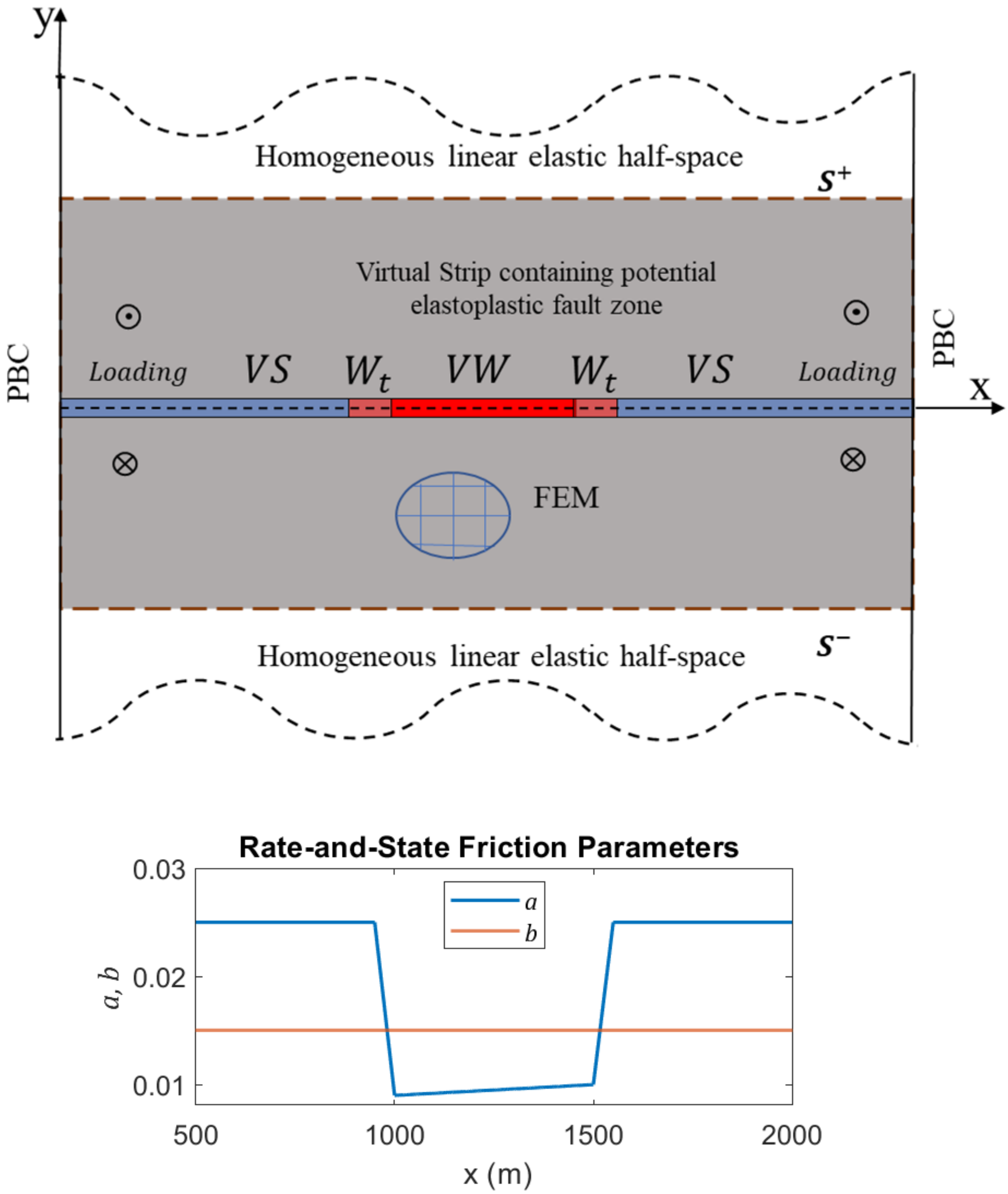

Figure S1. Model geometry and hybrid scheme set-up (Mia et al., 2019). 2D anti-plane rate-and-state fault contains seismogenic velocity weakening (VW) patch and creeping velocity strengthening (VS) patch with a transition region (W_t). Tectonic plate loading is applied through out of plane constant slip rate, $V_p = 10^{-9}\mathrm{m/s}$. Narrow virtual strip containing fault and potential elastoplastic bulk is discretized with FEM. Spectral Boundary Integral (SBI) replaces homogeneous linear elastic half spaces at the virtual boundaries (S^+ and S^-). Periodic Boundary Conditions (PBC) are applied at the lateral boundaries. Bottom figure shows the distribution of rate-and-state frictional parameters. a and b are non-negative dimensionless rate-and-state frictional parameters related to direct effect and state evolution respectively. Velocity weakening (VW) patch is associated with $a < b$ and velocity strengthening (VS) refers to $a > b$.

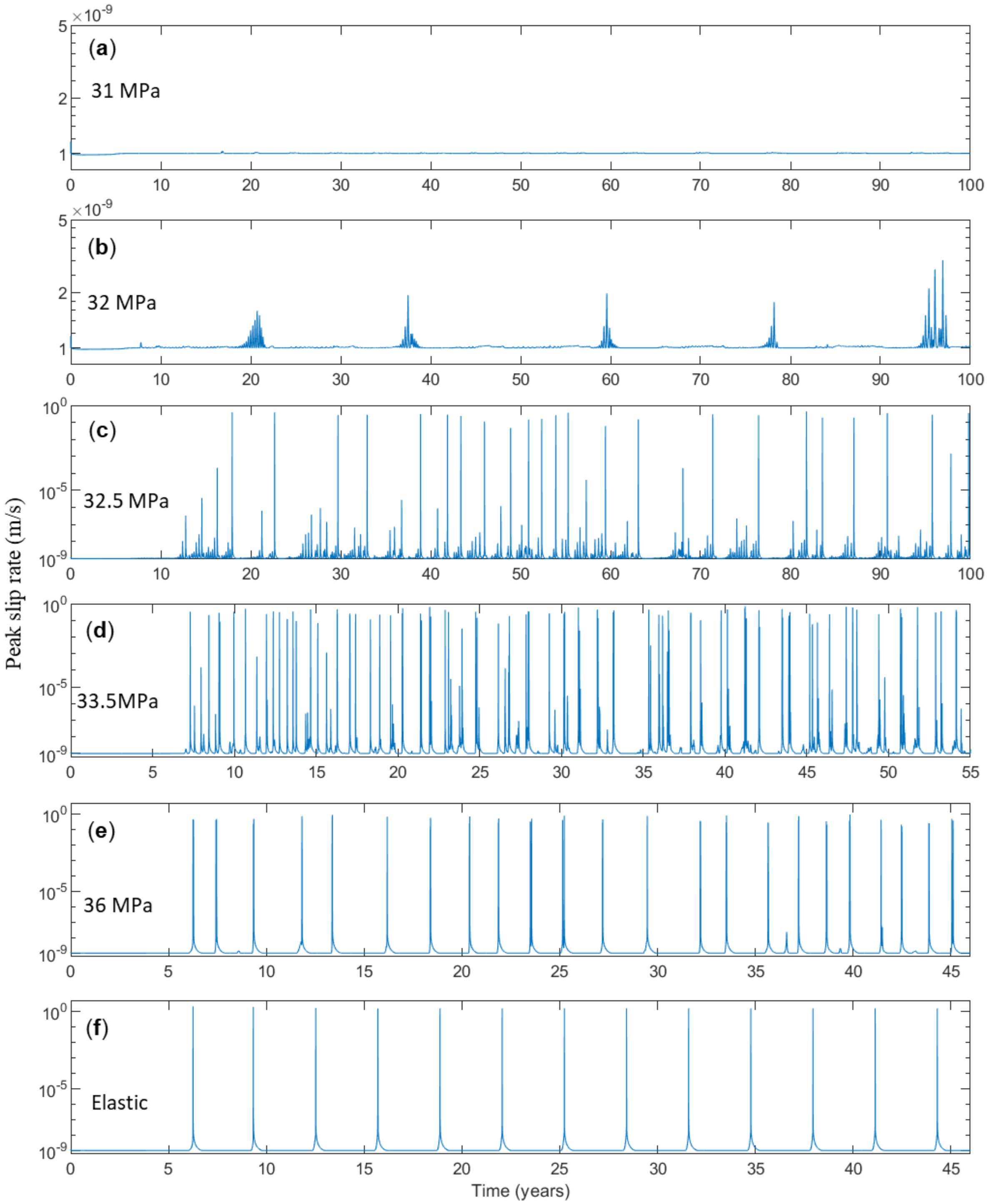

Figure S2. History of peak slip rate for different yield strength (a-e). Elastic reference case is generating periodic pattern is shown in f. Peak slip rate remains close to the plate rate for locked fault with yield strength 31 MPa as shown in a. (b) Slow slip events with 32 MPa. (c) Localized seismic events and preceding slow slip events with 32.5 MPa. (d) Irregular pattern including slow slip and partial rupture with 33.5 MPa. (e) Aperiodic sequence with clustering of seismicity for higher yield strength.

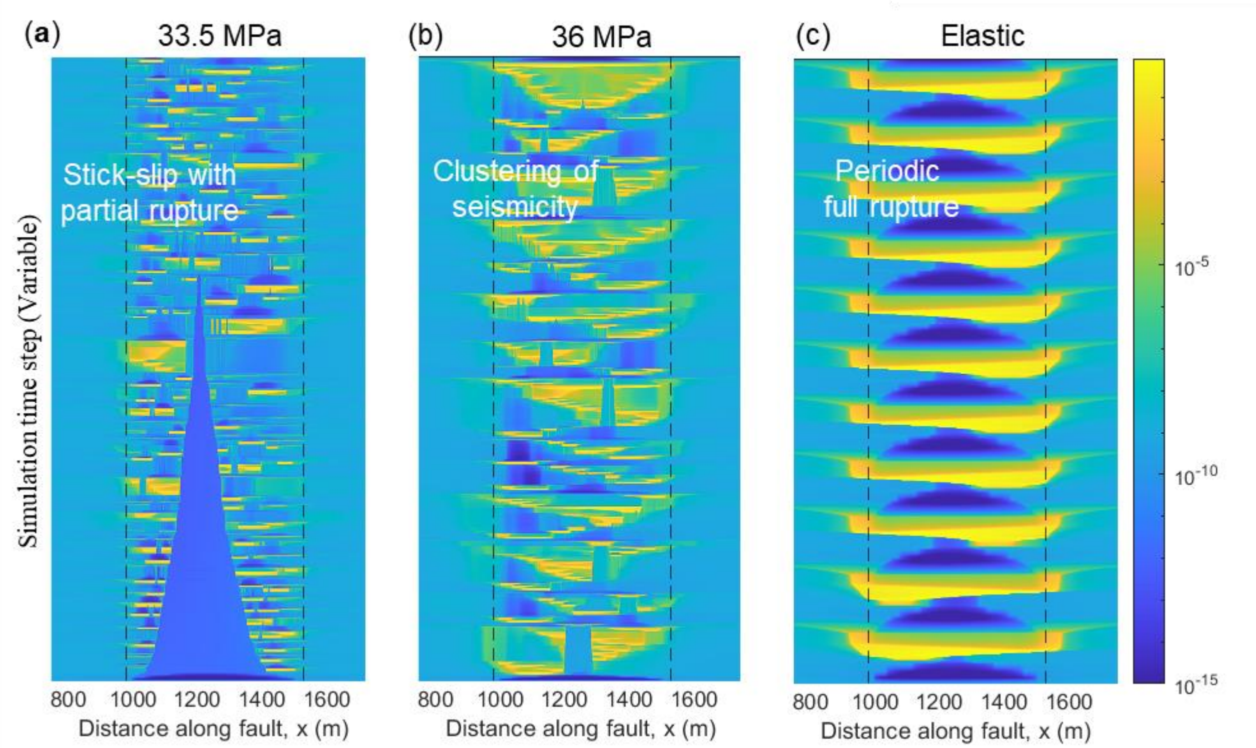

Figure S3. Results of fully dynamics simulation illustrating spatio-temporal evolution of slip rate. Patterns are qualitatively similar to the corresponding quasi-dynamics cases discussed in the main text. (**a**) Yield strength 33.5 MPa results in partial ruptures successively unlock the central VW region and spread over the whole fault. (**b**) Relatively higher yield strength (36 MPa) shows clustering of seismicity. (**c**) Elastic case generates simple periodic fault spanning ruptures.

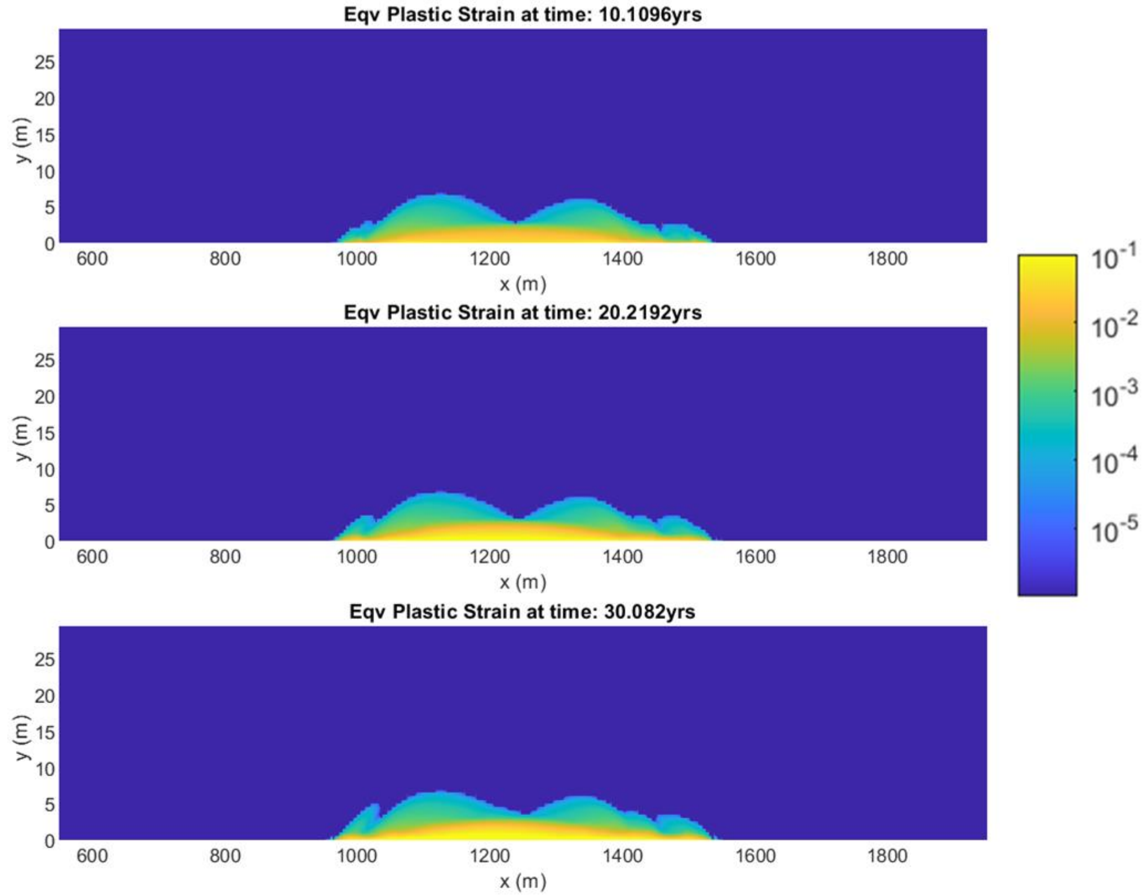

Figure S4. Equivalent plastic strain for yield strength 33.5 MPa. Spatial extent of off-fault plasticity perpendicular to the fault plane is limited to narrow region close to the fault. Plasticity distribution is symmetric across the fault plane as the model is anti-plane with constant normal stress. Horizontally, plasticity spreads along whole VW region of the fault.

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