Using deformation to recover displacement with Distributed Acoustic Sensing

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SUMMARY

Over a period of less than a decade, Distributed Acoustic Sensing (DAS) has become a well-established technology in seismology. For historical and practical reasons, DAS manufacturers usually provide instruments that natively record strain (rate) as the principal measurement. While at first glance strain recordings seem related to ground motion waveforms (displacement, velocity, acceleration), not all the seismological tools developed over the past century (e.g., magnitude estimation, seismic beamforming, etc.) can be readily applied to strain data. Notably, the directional sensitivity of DAS differs from conventional particle motion sensors, and DAS experiences an increased sensitivity to slow waves, often highly scattered by the subsurface structure and challenging to analyse. To address these issues, several strategies have been already proposed to convert strain rate measurements to particle motion. In this study we focus on strategies based on a quantity we refer to as "deformation". Deformation is defined as the change in length of the cable and is closely related to displacement, yet both quantities differ from one another: deformation is a relative displacement measurement along a curvilinear path. We show that if the geometry of the DAS deployment is made of sufficiently long rectilinear sections, deformation can be used to recover the displacement without the need of additional instruments. We validate this theoretical result using full-waveform simulations and by comparing, on a real dataset, the seismic velocity recovered from DAS with that recorded by collocated seismometers. The limitations of this approach are discussed, and two applications are shown: enhancing direct P-wave arrivals and simplifying the magnitude estimation of seismic events. While using deformation is in some respects more challenging, converted displacement provides better sensitivity to high velocity phases, and permits the direct application of conventional seismological tools that are less effective when applied to the strain (rate) data.

1. Introduction

In recent days, Distributed Acoustic Sensing (DAS) has rapidly gained traction in the seismological community. The technology is used in a wide range of applications across diverse disciplines. The widespread adoption has been greatly accelerated thanks to the availability of ready-to-use telecom dark fibres, and its sensing capabilities in harsh/underwater environments where regular seismometers would have been difficult to deploy and real-time acquisition would be challenging. The DAS technique typically consist in measuring variations of the phase of back-scattered light at each location along an optical fibre to estimate the longitudinal dynamic strain (Hartog, 2017). This provides the equivalent of a uniform, dense array of sensors while deploying only one instrument at one end of the cable. For historical and practical reasons, DAS interrogators natively record strain (or strain rate, its temporal derivative) while traditional seismologic instruments record translational ground motions (displacement, velocity or acceleration). Because strain is a spatial derivative of the displacement, it differs from particle motion in several respects. First, strain measurements have a characteristic directional sensitivity (i.e., a dependence on the direction of propagation and on the particle motion of the recorded wave) with narrower directional sensitivity for P-waves and a clover-like response pattern for S-waves (Martin et al., 2021). Second, strain measurements are more sensitive to slow waves, amplifying highly scattered waves that are difficult to analyse (Trabattoni et al., 2022). And last, finite-gauge length DAS measurements exhibit spectral notches in their instrument response at integer multiples of the apparent phase velocity over gauge length (J. Yang et al., 2022). Most of the tools developed in seismology are adapted to translational ground motion measurements, which may need modification to be used with strain. For instance, magnitude estimation was historically calibrated on the Wood-Anderson seismometer and all magnitude scales require ground displacement inputs. It has also been shown that beamforming can fail on strain data due to the predominance of scattered waves that reduce the wavefield coherence (van den Ende & Ampuero, 2021).

For those reasons, several studies have proposed different ways to convert strain to displacement (or other combinations of time derivatives of those two quantities). A first approach is to use the dominant apparent velocity of the observed phases (Daley et al., 2016). The reference velocity is generally estimated to correspond to the moveout of the most energetic phases. This approach has been successfully used for magnitude estimation in both offline and real-time early-warning contexts (Lior et al., 2021, 2023). The main limitation is, however, that only dominant phases are correctly converted. In particular, the relative amplitude between phases with different apparent velocities remains unmodified. This is problematic when the phases of interest have lower amplitudes than the dominant phases. Exact conversion should rebalance the respective amplitude of phases with different apparent velocities. This is particularly useful to enhance body waves that have a much higher apparent velocity relative to shallow scattered waves (van den Ende & Ampuero, 2021).

In this study we focus on an alternative way to convert strain to ground motion based on numerical integration along the space dimension. FK rescaling (Daley et al., 2016; Wang et al., 2018; Lindsey et al., 2020; Y. Yang et al., 2022) was a commonly adopted first step in this direction. This method allows one to convert strain to velocity by simultaneously integrating in space and differentiating in time in the FK domain. Here, the amplitudes are weighted by the quantity $-\omega/k$, where ω is the pulsation and k the apparent wavenumber along the cable. Because k can be close to zero all studies that used this approach proposed to dampen small wavenumbers to avoid instabilities and the emergence of spurious low-wavenumber signals. These methods are limited to finite 2D time-space windows, and simultaneously apply a time and a space transformation while sometimes only a space integration would be required (e.g., from strain to displacement or from strain-rate to velocity).

Temporal and spatial differentiation/integration can be done separately instead of combining time and space transformation by FK rescaling (note that the FK transformation can be done by applying the Fourier transform twice, in any order, over the time and the space dimensions). We will leave aside the temporal transformation needed to convert strain rate into strain because this problem is the same as converting between displacement, velocity and acceleration, and can be addressed with common methods in the time or spectral domains. The problem of the spatial integration is that the (time-varying) initial value (or the integration constant) is unknown. Van den Ende & Ampuero (2021) showed that by using co-located seismometers as reference, the local particle velocity can be estimated along rectilinear segments through spatial integration of the recorded strain rate. This method is hence limited to setups where co-located sensors are available. On the other hand, Yang et al. (2022) used a new generation of DAS that can natively record integrated strain rate, hereafter called "deformation rate". They observed that integrated measurements show spurious low-wavenumber signals that can empirically be removed by spatial or FK filtering methods.

Fichtner et al. (2022) laid down a mathematical foundation that explains the origin of the spurious signals observed in strain-integrated recordings along curvilinear paths. The study focusses on transmission measurements where the pulse emitted at one end of the cable is recorded by a unique sensor at the other end of the cable. This type of measurement is not distributed (i.e., sensitive to the strain at each point of the cable) but integrated (i.e., sensitive to the integrated strain along the cable). The authors show that strain-integrated measurements are mostly insensitive to the displacement wavefield except at very specific locations: at the two extremities of the cable (the relative displacement between the two endpoints is measured) and at each curve or kink of the cable. While this is beneficial for transmission measurements (Marra et al., 2018), because it allows to retrieve information along the cable (and not only at the endpoints), the consequences for DAS are not developed.

In this study, the theory of deformation measurements along non-rectilinear cables will be presented. Then concepts will be validated by full-waveform simulations and by comparing DAS recordings with co-located seismometers deployed at the Stromboli volcano (Italy). Finally, two applications will be presented: body wave enhancement in a telecom submarine cable laying offshore on the Chilean margin; and direct magnitude estimation for events recorded by a dedicated cable deployed along the Irpinia fault system (Southern Italy).

2. THEORY

The displacement of a body can be decomposed into a rigid-body displacement component and a deformation component (Fossen, 2016). Rigid-body displacement is the translation and rotation of the body as a whole. Deformation is the change in shape/size of the body. DAS is sensitive to the latter but only longitudinally along the cable geometry. In the following, we refer to this curvilinear deformation along the cable as "deformation", as proposed by Yang et al. (2022). This quantity is then related to usual DAS quantities (e.g., strain) and to seismological ground motion quantities (e.g., displacement).

2.1. DAS natively measures deformation

Let us consider a model fibre-optic cable. Each point of the cable is identified by its curvilinear coordinate s which is the distance at rest $L_0(s)$ from the DAS interrogator $L_0(s) = s$. If ground displacement occurs (e.g., due to the passage of a seismic wave) the distance $L(s,t)$ varies. We define the deformation $\delta(s,t)$ as the variation of that distance compared to the undeformed resting distance:

$$\delta(s,t) \equiv L(s,t) - L_0(s) \tag{1}$$

For small ground motion, deformation is equal to the curvilinear integration of $\varepsilon(s,t)$ the strain component along the cable (Fichtner et al., 2022). This latter quantity is equal to the projection along $\boldsymbol{e}(s)$, the cable directional vector, of the curvilinear derivative of the displacement $\boldsymbol{u}(s,t)$:

$$\delta(s,t) = \int_0^s \varepsilon(s',t)ds' = \int_0^s \boldsymbol{e}(s') \cdot \frac{\partial \boldsymbol{u}}{\partial s'}(s',t)ds' \tag{2}$$

Note that, if not otherwise mentioned, the regular typed symbols are the along-the-cable component of their bold counterparts, and, that when omitted in a sentence, we refer to those projected quantities.

The operating principle of DAS consists in measuring the phase of the backscattered field generated by a laser pulse to measure scatterers' displacements. At rest, the phase of the light backscattered from position

s presents a random structure $\phi_0(s)$ due to the small and spatially randomised variations of the optical index of the fibre glass. For a small deformation of the fibre, the optical distances of the heterogeneities relative to the DAS interrogator change, shifting the phase $\phi(s,t)$ as follows (Hartog, 2017):

$$\phi(s,t) = \frac{4\pi n\xi}{\lambda}\delta(s,t) + \phi_0(s) \tag{3}$$

where n is the effective optic index of the fibre, ξ is the photo-elastic coefficient that relates the deformation of the fibre to changes of its optical index, and λ is the central wavelength in the vacuum of the coherent laser pulse. Finally, the coefficient $4\pi = 2 \times 2\pi$ considers the fact that light travels back and forth in the cable and is affected twice by the cable deformation. The unknown $\phi_0(s)$ prevents the recovery of the static deformation. To get rid of this constant, high pass filtering can be used with a cut-off frequency related to the frequency content of the signal of interest.

More commonly, the temporal derivative of the phase is used to remove the unknown phase reference. It gives the deformation rate $\dot{\delta}(t,s)$:

$$\dot{\phi}(s,t) = \frac{4\pi n\xi}{\lambda}\dot{\delta}(s,t) \tag{4}$$

Further applying a spatial derivative gives the strain rate $\dot{\varepsilon}(s,t)$ which has the advantage of being a local measurement (see later):

$$\frac{\partial\dot{\phi}}{\partial s}(s,t) = \frac{4\pi n\xi}{\lambda}\dot{\varepsilon}(s,t) \tag{5}$$

Most modern DAS interrogators use coherent detection techniques and measure phase as a primitive (Hartog, 2017). They compute derivatives numerically in contrast to direct detection techniques, where optical interferometric methods are used to make phase comparisons. Spatial differentiation implies a transfer function that is proportional to the wavenumber. This degrades the low wavenumber sensitivity of DAS. Numerical differentiation is also known to produce noisier outputs. Smooth derivative schemes are generally used, (e.g., by averaging the derivative over a spatial window). For most DAS interrogators, the user can tune the spatial smoothing through the choice of a parameter called gauge length. This latter quantity damps the high wavenumber signals and can introduce notches to the instrumental response (J. Yang et al., 2022). The same issues are encountered in the time domain. We will show that deformation, which is proportional to phase, can be used to directly recover displacement.

2.2. Illustration of deformation recordings

For didactive purposes, we ran a simple numerical simulation (Fig. 1). A synthetic cable deployment is exposed to an incoming impulsive plane P-wave made of a 7.5 Hz Ricker wavelet. The medium is homogenous with P-wave velocity of 1 km/s. The geometry of the cable includes perfect and perturbed linear sections, kinks, and a curved section (Fig. 1a). It also includes a free-hanging section that is oscillating (e.g., driven by ocean currents; Mata Flores, Mercerat, et al., 2022; Mata Flores, Sladen, et al., 2022). Strain (Fig. 1g) appears similar to displacement (Fig. 1d) but it is affected by its comparatively high-pass wavenumber response. Cable segments where waves arrive with an almost normal incidence (e.g., segment BC), and consequently a longer apparent wavelength, record lower amplitudes. Deformation was computed using eq. (7) and curvatures where approximated by an infinitesimal segment-wise rectilinear geometry (see later). Deformation (Fig. 1b) appears as the superposition of the true displacement field (Fig. 1d) and horizontal spurious features (Fig. 1c) that prevent the use of deformation as is for seismic waves analysis. Those features are referred as "non-local" effects.

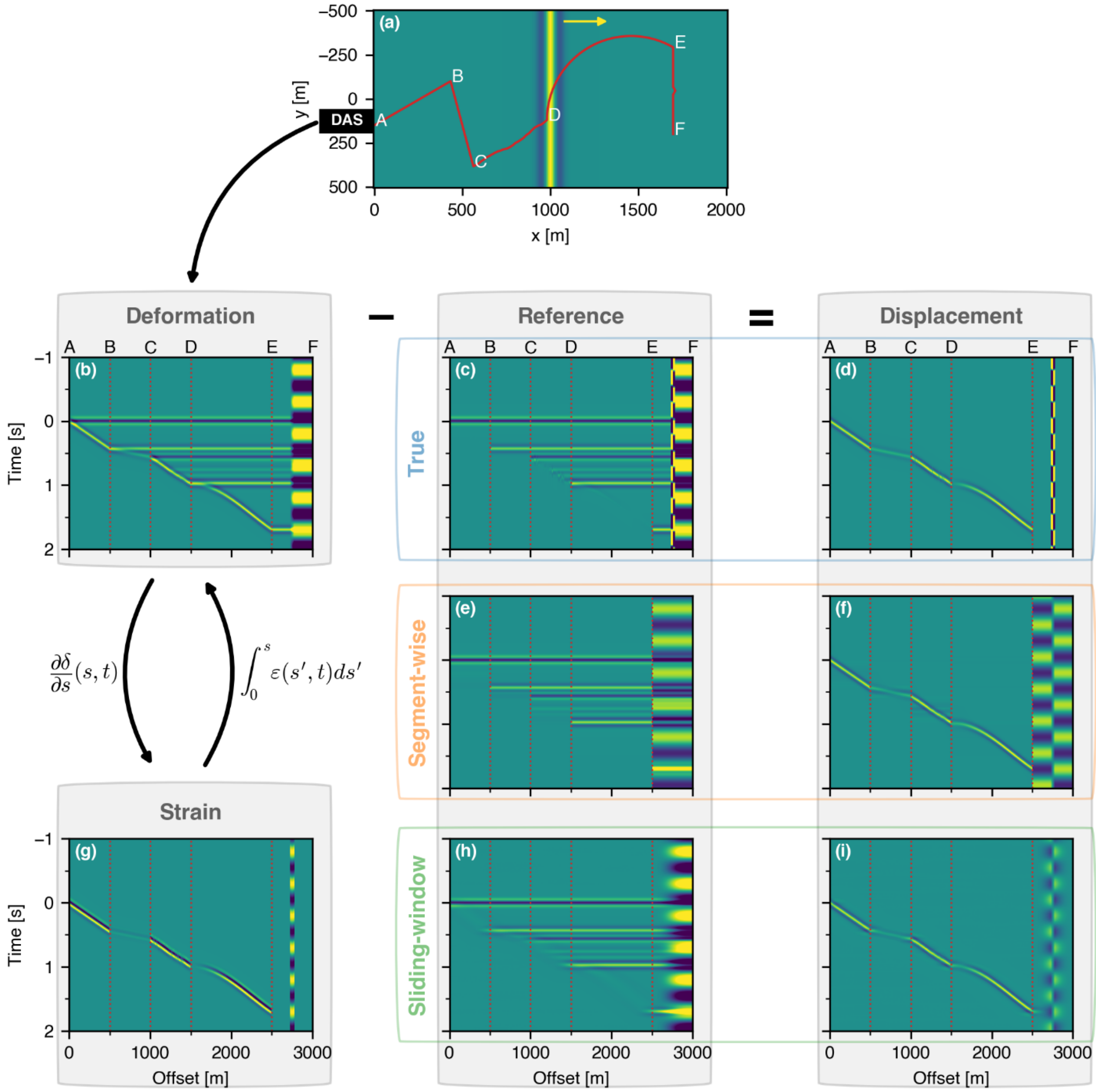

Figure 1. Illustrative simulation. (a) An impulsive plane P-wave propagating along the x-axis produces a horizontal displacement wavefield (blue-green-yellow background raster). A fibre-optic cable (red line) records the deformation along a given geometry. DAS interrogators that use coherent detection techniques measure phase which is closely related to (b) deformation. Deformation is the sum of (d) the true displacement and (c) a reference term (also referred to as "non-local effects") that must be estimated and removed. Non-local effects can be observed as horizontal features. They are space invariant signals starting at a given offset. The origin is either the beginning of the cable (marker A), a kink (markers B, C, D and E), a perturbation of the rectilinear segment (section CD), the curvature of the cable (section DE), or a non-linear effect, here produced by a 50-meter hanging section at the middle of the segment EF. The simulation was tailored so that non-local effects can be visually distinguished but in general they accumulate an overlap, resulting in data that is difficult to interpret. Segment-wise method: The displacement can be closely recovered (f), and the locality of the measurement greatly improved by removing (e) the averaged deformation on each segment (red vertical dotted lines). Few horizontal artifacts can be seen (e.g., on segment CD). Sliding-window method: An alternative solution is to remove (h) a sliding average (here 500 m is used with a Hann tapering) to get (i). Artifacts (or border effects) can be observed at some kinks. Spatial differentiation of the deformation gives (g) strain which is a local measurement.

2.3. Non-locality of the deformation

A non-local measurement implies that a measurement made at a given location depends on the value of the field of interest at other locations. Deformation is non-local for two reasons: the lack of a proper reference (referred to as "reference error") and changes in cable direction (referred to as "geometric effects").

Let us consider the simple case of a rectilinear geometry (Fig. 2a and segment AB in Fig. 1). Integrating eq. (2) with constant direction $\boldsymbol{e}(s)$ shows that, in this configuration, the deformation is the difference of the displacement $u(s,t)$ between the two integration limits:

$$\delta(s,t) = u(s,t) - u(0,t) \tag{6}$$

Deformation gives the displacement relative to a reference which is the displacement of the beginning of the cable. If $u(0,t)$ is zero, the deformation is equal to the displacement (Fig. 2c). Otherwise, the motion of the reference point produces a spatially-constant (but time-varying) additive term on all measurement points (Fig. 2b). This reference error appears as a horizontal space-invariant signal that affects the whole cable (from marker A to F in Fig. 1b).

Along a rectilinear cable, the displacement of one point of the fibre generates an equal amount of positive strain on one side and negative strain on the other side (Fig 2c). When integrating through, the two contributions cancel out making subsequent measurement points insensitive to the displacement of that point. This is no longer the case when the cable is not rectilinear: the displacement of a point of the cable located at a kink produces an unbalanced amount of strain (Fig. 2d).

For a segment-wise rectilinear geometry, the overall deformation is the sum of the deformation of each rectilinear segment up to the point of interest s. Splitting the integral in eq. (2) per segment and applying eq. (6) per segment, this can be expressed as:

$$\delta(s,t) = u(s,t) - \sum_{k=1}^{n} [u(s_k^+, t) - u(s_k^-, t)] - u(0,t) \tag{7}$$

where s_k are the locations of the kinks and n is the number of kinks preceding the location s. At each kink a new additive constant term appears and impacts all subsequent measurement points. Those terms are equal to the difference between the displacement along the direction before (subscript $-$) and after (subscript $+$) the kinks. It changes the reference against which the displacement is measured. Those geometrical effects appear as horizontal space invariant features that start at each kink location (at markers B, C, D and E, and along the non-rectilinear CD segment in Fig. 1b).

For the general case of a curved cable, it can be shown that integrating eq. (2) by parts (Fichtner et al., 2022) gives:

$$\delta(s,t) = u(s,t) - \int_0^s \frac{\partial \boldsymbol{e}}{\partial s}(s)\boldsymbol{u}(s,t)ds - u(0,t) \tag{8}$$

The term $\partial\boldsymbol{e}/\partial s(s)$ represents the sharpness in change of direction. This implies that the radius of curvature plays a central role in the amplitude of this effect (the geometric effect related to the smoothly curved section DE is for example not very pronounced in Fig. 1c).

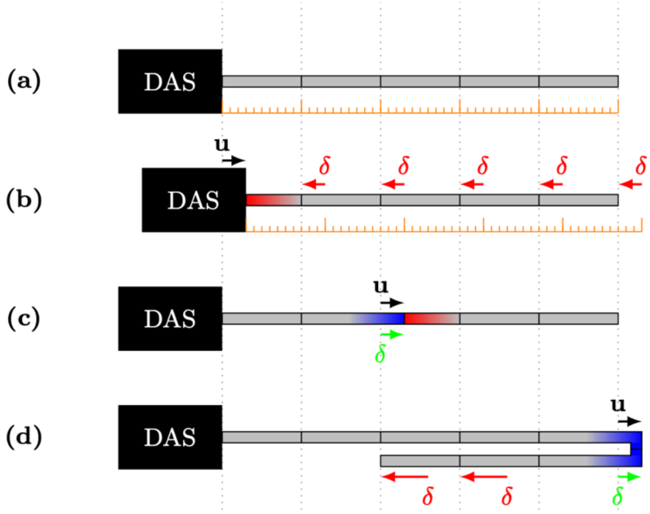

Figure 2. Non-locality of the deformation. (a) A rectilinear fibre-optic cable (grey line) that is at rest. Reference markers (black ticks) are placed at regular distances of the DAS (orange ruler). Let observe three snapshots of the passage of a zero-average localised impulsive wavelet propagating through the cable from left to right. (b) The ground displacement starts to reach the beginning of the cable (blue is elongation, red contraction). The displacement of the instrument produces a contraction of the beginning of the cable that reduces the distance to all subsequent points of the cable. A constant deformation $\delta(s,t) = -u(0,t)$ is added to the entire cable. (c) The deformation is then confined inside the linear section. Because the contraction on one side is equal to the elongation at the other side, the integrated strain cancels out and subsequent point remain unaffected. Since in that case the reference point (the DAS interrogator) also does not move, the measurement is local: $\delta(s,t) = u(s,t)$. (d) Eventually, the impulse reaches the end of the rectilinear section. If the cable continues with a kink (here with a 180° angle), the elongation is no longer compensated by an equal contraction (that here lies outside of the cable). In the case presented here, it is the opposite: the backward part of the kink is elongated with the same amount. The arc length to reach points located after the kink is augmented by twice the displacement of the kink $\delta(s,t) = 2u(s,t)$. This is a typical example of geometric effect showing that integration cannot be applied as is across a kink.

Non-local effects disappear once spatial differentiation is applied (eq. 5) making strain (rate) a local measurement (Fig. 1g) since every measurement is compared to its neighbours. Next, we show that it is possible to reduce the non-locality of deformation and approximate the displacement by applying suitable processing.

2.4. Recovering displacement from deformation

We have seen that deformation is a measure of displacement relative to a reference that is unknown and related to the displacement of the start and the kinks (or curved portions) of the cable. This leads to spurious signals that appear as additive terms which are space-invariant over each rectilinear segment (Fig. 1b). By contrast, seismic signals exhibit a finite wavelength of variation (Fig. 1d) that – if averaged over a sufficient length scale – has zero spatial mean. To approximate the non-local effects (Fig. 1c), one solution is then to compute the spatial mean over each rectilinear segment (Fig. 1e). This estimate of the reference can then be removed from the deformation to estimate the displacement (Fig. 1f). We will refer to this first approach as the "segment-wise" method. Applying this process to eq. (7) show that the obtained estimate of the displacement $\hat{u}(s,t)$ is the displacement $u(s,t)$ minus its spatial average $\overline{u}_n(t)$ over the n-th segment related to the location s:

$$\hat{u}(s,t) = \delta(s,t) - \overline{\delta}_n(t) = u(s,t) - \overline{u}_n(t) \tag{9}$$

We set as reference the spatial average, essentially replacing an arbitrary reference with one that approximates to zero if the wavelength of the signals of interest is much shorter that the length of the rectilinear section of the cable (see later). The mean can be computed using a weighting (or tapering) window $w_n(s)$ that integrates to one and has zero values outside the n-th segment (see later):

$$\bar{\delta}_n(t) = \int w_n(s')\delta(s',t)ds' \tag{10}$$

Unfortunately, deploying linear sections of cable is often impractical and, in many cases, the user has only limited knowledge on the cable geometry (e.g., for submarine cables). In those cases, a more flexible approach is to choose a characteristic length over which we expect the cable to be approximately linear. Then for each cable location, the reference is estimated by removing a local sliding average (Fig. 1g-h):

$$\hat{u}(s,t) = \delta(s,t) - (w * \delta)(s,t) = u(s,t) - (w * u)(s,t) \tag{11}$$

were $*$ denotes the convolution along space and w is a chosen weighting window. This second approach will be referred to as the "sliding-window" method. To get the values at the ends of the cable, padding is necessary with this approach. Several padding strategies can be used: filling with zeros, with the edge value, or by reflecting values (using mirrored values on the edge). The choice of the padding mode only affects the beginning and the end of the cable and do not appear to have significant consequences. The reflecting mode seemed to generally work better and was used in this study (see later, supplementary material, Fig. S1).

The two proposed methods reduce the non-locality of the deformation measurements. The segment-wise scheme limits the non-locality to the size of each segment: the value of the displacement at one location of the cable only affects the values measured within its segment. The sliding-window scheme limits the non-locality to the width of the chosen sliding window. This can be important for example in case of highly corrupted channels (e.g., the hanging cable in Fig. 1f, i) that will only impact a finite width of the cable. On the other hand, the use of Infinite Impulse Response (IIR) filters to perform average removal or (high-pass filtering) theoretically spreads those errors indefinitely.

2.5. DAS sensitivity to displacement

The displacement that is recovered in the manner described above only approximates the true displacement. In particular, the mean removal affects the low wavenumber response of the recovered displacement. We will focus on the sliding-window method, but similar results can be showed for the segment-wise method. Using the convolution theorem in eq. (11), the sensitivity can be expressed as:

$$\hat{U}(k,t) = \big(1 - W(k)\big)U(k,t) \tag{12}$$

where uppercase letters are the spatially Fourier transformed versions of their lowercase counterparts, and k is the apparent wavenumber along the cable.

Depending on the choice of $w(s)$ the response varies, but some common features are always retrieved. Owing to the relative measurement principle of DAS, rigid-body displacements cannot be recovered. Consequently, displacements of wavelengths that exceed the length of rectilinear sections (approaching the limit of rigid-body translation), cannot be retrieved. On the other hand, wavelengths that are much shorter than the typical rectilinear length tend to average out along the cable length and relative motion of the different parts of the cable can be correctly measured. This mandates the use of long linear segments for studies focused on measuring long apparent wavelengths.

The choice of the optimal weighting function w depends on the nature of the signal and the objective of the analysis. Using a rectangular window implies a sinc weighting function in the wavenumber domain which ensures minimal low wavenumber sensitivity but generates prominent side lobes (Fig. 3a). The use of a smoother weighting function like the Hann window deteriorates the smallest recoverable wavenumbers but significantly reduces the presence of side-lobes, ensuring a much flatter response (Fig. 3a).

Because the apparent wavenumber depends on the angle of incidence at which the incoming wave strikes the cable, eq. (12) implies a directional sensitivity (Fig. 3 b-e). Waves orthogonal to the cable have infinite wavelength and cannot be recovered, regardless of their frequency content. This is already the case for P-waves where a single component displacement measurement results in zero sensitivity in the orthogonal direction. For S-waves this produces the presence of a notch whose width depends on the relative size of the wavelength of interest with the length of the rectilinear section, and on the chosen tapering (Fig. 3c, e).

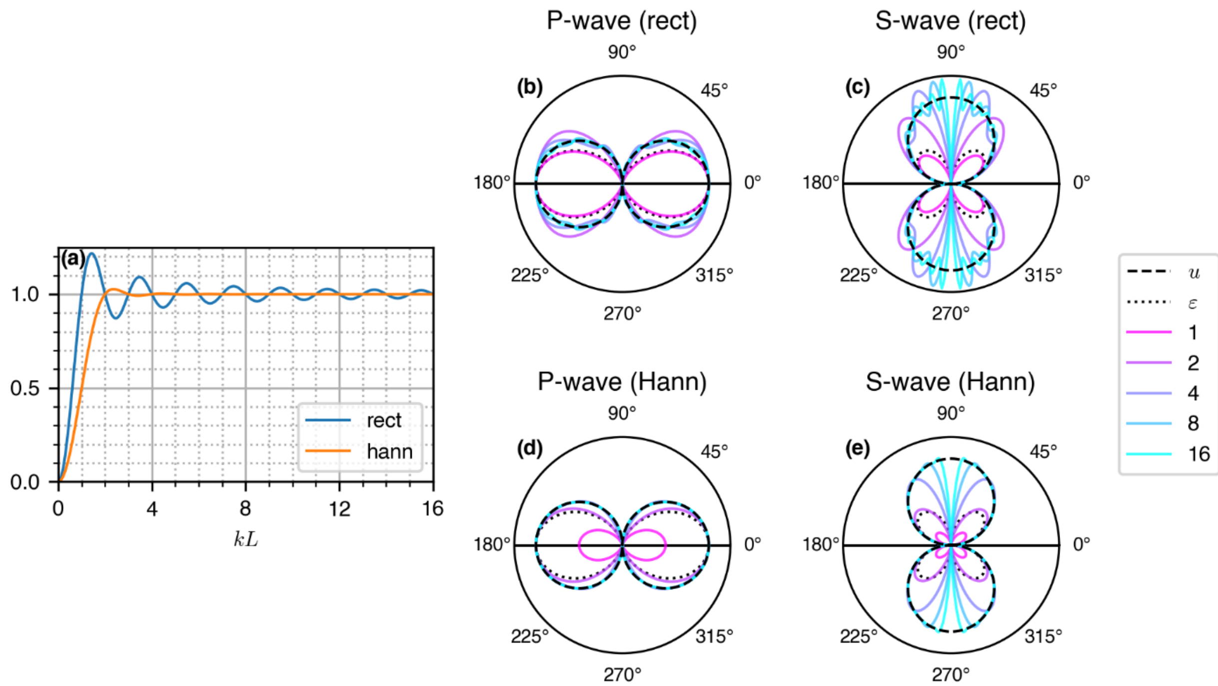

Figure 3. Sensitivity characteristics of deformation-recovered displacement. (a) The sensitivity is a function of the product between the apparent wavenumber and the length of the rectilinear section (kL); and the shape of the window (here rectangular and Hann windows are shown). Rigid-body displacement cannot be recovered (zero wavenumbers) by DAS. The shape of the window affects the recovered displacement, and its selection requires a compromise between a flat response (no side-lobes) and a strict low cut-off wavelength. Because the apparent wavenumber is a function of the angle of incidence, a directional sensitivity pattern is implied. (b), (c), (d) and (e): Directivity patterns for plane P- and S-waves for the rectangular and the Hann windows for different kL values (from 1, magenta lines, to 16, cyan lines – see legend on the right). The horizontal black line indicates the cable direction. For P-waves, the sensitivity converges toward the response of the desired displacement (black dashed line) as kL increases. For S-waves, waves with orthogonal incidence to the cable have infinite apparent wavelength making their recovery physically impossible. Consequently, the sensitivity of S-waves exhibits a notch at orthogonal directions of the cable that get thinner as kL increases. Note that the rectangular window converges twice faster and has a twice smaller notch than the Hann window but generates significantly more ripples.

3. Validation

In this section, we validate the proposed displacement (or velocity) recovery method using full waveform simulations, and through comparison with data acquired on the Stromboli volcano by both DAS and co-located seismometers.

3.1. Full-waveform simulation

To evaluate the capacity of DAS to recover velocity instead of the natively provided strain rate, a full-waveform simulation was used to model complex and realistic wavefields. We modelled waves propagating through a shallow sedimentary basin that represents a common situation for DAS deployments both on-land and off-shore. Sedimentary basins produce complex wavefields because ballistic waves are distorted, amplified, trapped, and cause interference among each other.

We used the geometry proposed in Trabattoni et al. (2022) that corresponds to the case of a dedicated deployment located in the Irpinia Near-Fault Observatory (INFO, Southern Italy). It consists of a two-layer basin of 25 m depth resting on the bedrock (Fig. 4e). To test the influence of shallow heterogeneities beneath the cable, the original model was modified by laterally extending the basin and by including an abrupt lateral change in phase velocity in the upper layer.

We simulated the wave propagation in the basin using SPECFEM2D (Tromp et al., 2008). The source consisted of two plane waves (P and S) with identical angle of incidence (40°) and source time function (5 Hz Ricker) but delayed by 2 s and with different polarities. A 350 m long virtual cable was located at the centre of the basin. The horizontal component of the velocity was evaluated at the surface every 0.5 m with a sampling rate of 200 Hz (Fig. 4a) and used to estimate the deformation (see Appendix B), the DAS measured strain rate (Fig. 4b), and the DAS recovered velocity (Fig. 4c and d). The latter was obtained using both the segment-wise (eq. 9) and the sliding-window (eq. 11) approaches with the same Hann window whose width was set to the entire length of the cable. Reflection-mode padding was performed to obtain estimates at the cable's extremities (see other padding modes in the supplementary material, Fig. S1).

The velocity wavefield is dominated by high apparent velocity direct and multiple P and S waves (Fig. 4a). The strain rate wavefield enhances low velocity surface and refracted S waves propagating in the basin and increases the amplitude contrast between both sides of the basin (Fig. 4b). Both recovered velocities visually match well with the true velocity. Only the phases with the highest apparent velocities are not perfectly retrieved. The first arrival, which has a particularly high apparent velocity and low energy, is partially recovered especially with the segment-wise approach.

To quantify the error, several metrics were applied at each channel (Fig 4d): the Mean Square Error (MSE), Percentage MSE (PMSE: the MSE divided by the mean square of the reference) and the correlation coefficient (CC). The MSE – which is minimized by mean removal – is constant for the segment-wise approach because the same reference error occurs at each location (i.e., a single reference value is estimated for the entire cable). For the sliding-window approach its value varies with slightly worse results on the high amplitude/low velocity first half of the basin but with much smaller errors in the low amplitude/high velocity section. This result highlights that the segment-wise estimation is driven by the high amplitude areas and that the sliding-window approach better adapts to subsurface wave speed variations. Because of this amplitude contrast, relative metrics (PMSE and CC) better capture the recovery performance. Both the PMSE and CC highlight the difference between the two approaches but also reveal increased errors at the extremities of the cable and at the location of the subsurface discontinuity. The median CC and PMSE are respectively 0.90 and 20% for the segment-wise scheme and, 0.95 and 11% for the sliding-window scheme. Note that the relative effectiveness of one scheme over the other is scenario-dependent.

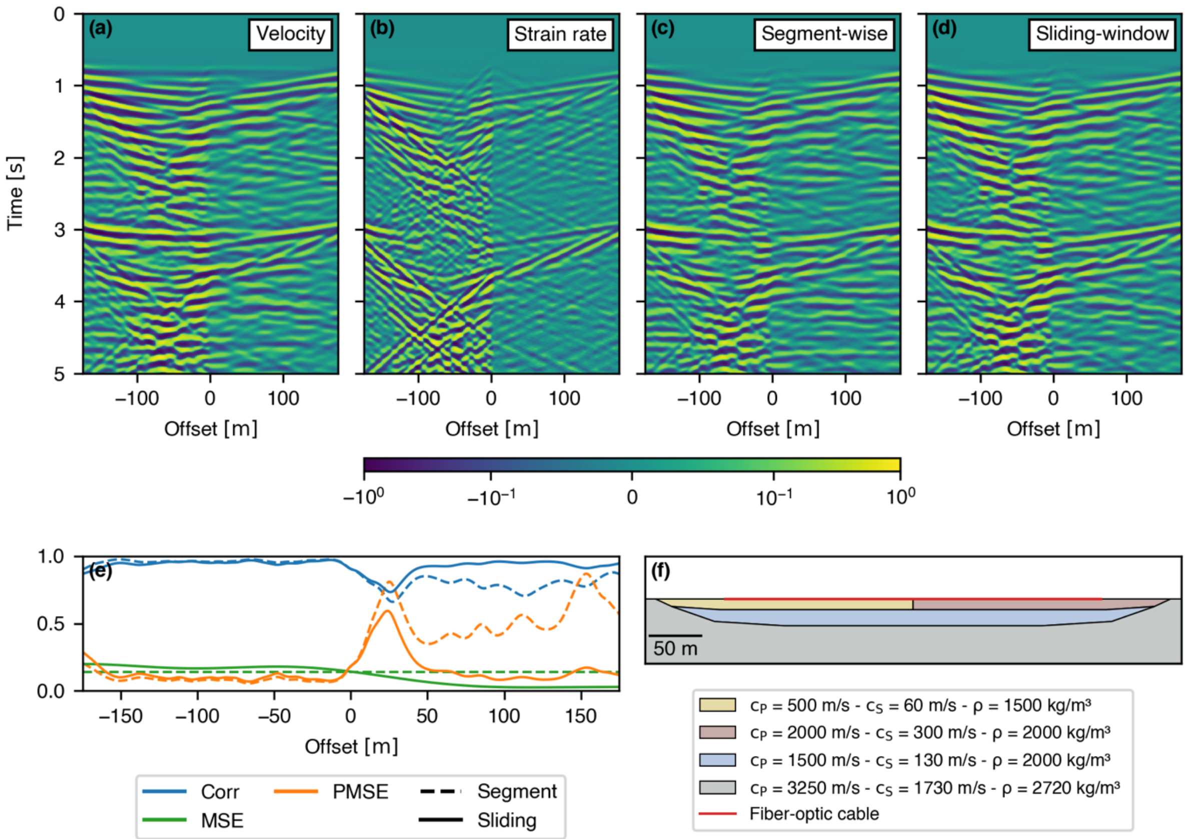

Figure 4. Full-waveform simulation for a shallow sedimentary basin configuration. (a) Simulated particle velocity along the cable. (b) Strain rate along the cable. (c) and (d) Recovered particle velocity along the cable with the segment-wise and the sliding-window methods. While strain rate is almost insensitive to long wavelengths, the phases with high apparent speed are remarkably well reconstructed after the conversion process. Yet, near horizontal arrivals are hardly retrieved since they are associated with high apparent wavelengths, comparable or longer than the fibre extension. (e) Error metrics along the fibre. The error is larger in the higher speed area because waves have inherently longer wavelengths and lower amplitudes (f) The geometry of the basin, composed of two soft layers superimposed on the bedrock. The speed of the most superficial layer is increased over one half of the basin, to simulate an abrupt wave speed change beneath the cable.

3.2. Comparison with co-located seismometers

To further validate and better illustrate our theoretical developments, we applied our methodology to an experiment with a dedicated fibre-optic cable deployed on the Stromboli volcano. In this experiment, DAS-recovered velocities could be compared with traces recorded by co-located seismometers. A 3 km fibre-optic cable was deployed on the northeast flank of Stromboli to monitor the volcano activity (Biagioli et al., submitted). The data presented here was acquired in September 2021 by a Febus A1-R DAS interrogator parametrised with a gauge length of 4.8 m, a differentiation time of 10 ms, a repetition rate of 10 kHz decimated to a sampling rate of 200 Hz and a channel spacing of 2.4 m. This study focuses on a 600 m portion of the cable composed of relatively rectilinear segments where six co-located three-component nodes (SmartSolo IGU-16HR with 5 Hz corner frequency and sampling at 250 Hz) were deployed at the middle of the segments (Fig 5a). The recordings of an explosive event of mild intensity that occurred at 20:47:03 UTC on 26 September 2021 were studied.

To compare DAS with nodal seismometer data, the recordings were decimated to a common sampling rate (50 Hz) and filtered between 2.5 and 15 Hz (limited by the noise floor of the DAS instrument). The instrumental response of the seismometers was removed and the 3D particle velocity was projected along the direction of the cable. The strain rate recorded by DAS (Fig. 5b) was multiplied by a calibration factor of 2.5 (see Biagioli et al., submitted for more details). Strain rate was integrated spatially to get the deformation rate (Fig. 5c). The latter is severely affected by spurious signals which accumulate at each change in cable direction. Velocity was then recovered by applying both the segment-wise (Fig. 5d) and the sliding-window (Fig. 5e) approaches. For the segment-wise approach, the segment limits were manually identified (noted A-E). Because it reduced the recovery performance metrics (see later), one gradual change in the orientation of the cable (between B and C) was not considered as a kink, and the BC segment was treated as a single straight section. This suggests that a compromise between the segment length and its rectilinearity must be found. For the sliding-window scheme, the choice of the optimal Hann window length (250 m) was estimated by computing, for varying window lengths, the PMSE and the correlation coefficient (CC) between the velocity recovered from DAS and that recorded by the seismometers for each seismometer (Fig. 6). The window length must ideally be longer than the apparent wavelength of the recorded waves and smaller than the characteristic rectilinear length of the cable geometry. Results show that indeed the optimal window length is constrained to the longest rectilinear segment length that can be centred on the channel of interest. Finally, the DAS channels closest to each seismometer (indicated as #1-6) were extracted for a direct comparison with the co-located seismometers (Fig 5e and g).

Using either recovery approach, the recovered DAS velocity waveforms exhibit an acceptable agreement, both in phase and in amplitude, with the seismometer recordings. The worst results are found at station #6. This receiver was located on a segment featuring a strong bend in the cable made to avoid an outcrop of more competent rocks. We therefore attribute the poor performance at this station to strong geometrical effects. Likewise, station #5 was located along the shortest segment (DC) and compares poorly to the recovered DAS velocities. This is likely owing to the short length of the segment and the presence of a connection box that introduced a small T-shape like geometric perturbance. For the other stations, the correlation coefficients (CC) typically range between 0.7 and 0.8, reaching 0.9 at the station #1, with the segment-wise scheme. Both the sliding-window and segment-wise approaches provides similar results in terms of PMSE and CC. Looking at the DAS data in space-time plots, the segment-wise approach visibly exhibits horizontal artifacts (e.g., before the event, close to the borders of the BC segment in Fig. 5d) and wavefield discontinuities at the segment limits (e.g., for the strongest arrival at marker B in Fig. 5d), that arise from the fact that the average is removed simultaneously on all channels and that each segment is treated independently. The performance of the two approaches depends on several factors, such as the geometry of the deployment, the lithology, the coupling and the selection of one approach depends on the wavefield of interest and the aim of the analysis.

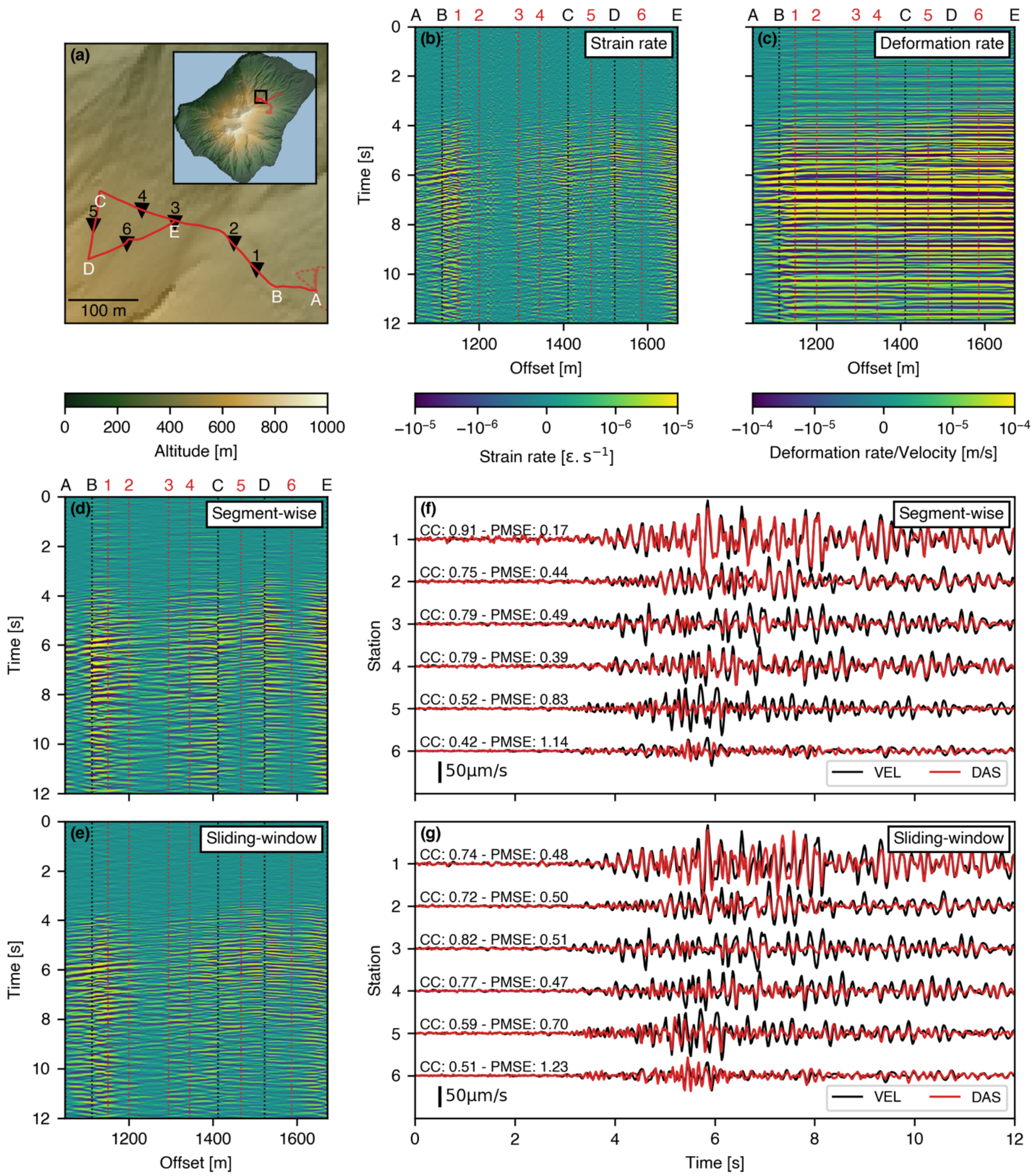

Figure 5. Comparison between velocities recovered from DAS and co-located-seismometers. (a) Deployment of the fibre-optic cable across the north-east flank of Stromboli volcano, with a detail of the fibre section considered in this study (solid red line). Six nodes were deployed (#1-6) at the midpoints of the straight segments (black inverted triangles). Letters from A (i.e., the beginning of the cable section) to E (its end) mark the sharp changes in orientation. (b) Strain rate recorded along the considered cable segment. Dashed black lines individuate the changes in cable direction marked in (a). Red dotted lines represent the offsets of the DAS channel closest to each seismometer. (c) Deformation rate without any processing. (d) Velocity recovered with the segment-wise approach using the A-E markers as segment limits. (e) Velocity recovered with the sliding-window approach with a 250 m Hann window. (f-g) Comparison between the velocity waveforms from the seismometers (in black) and recovered from DAS

(in red), described by means of correlation coefficient (CC) and the PMSE, for the segment-wise and sliding-window approaches respectively.

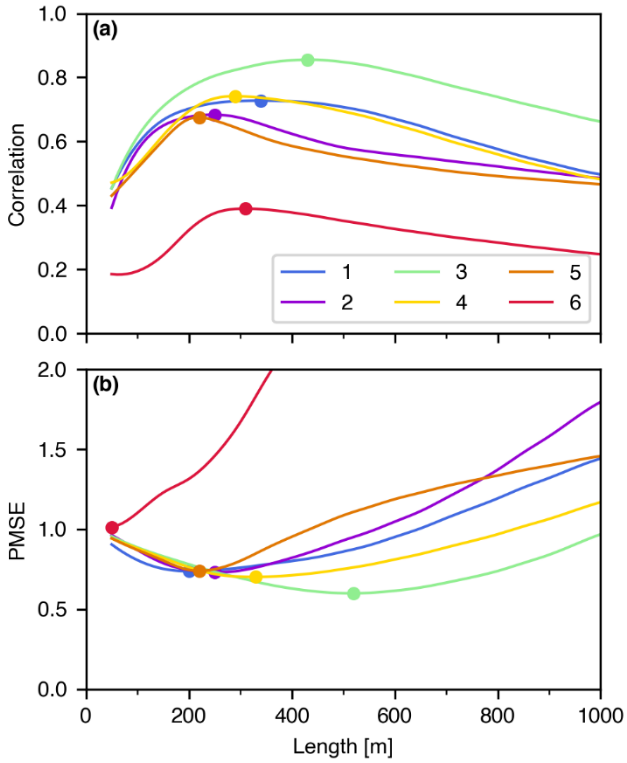

Figure 6. Search of the optimal window length L^*. DAS-recovered velocity computed with different window lengths using the sliding window approach was compared to the ground velocity recorded at the six co-located seismometers (#1-6). Two comparison metrics were used: (a) PMSE; and (b) Correlation Coefficients (CC). Circles mark the optimal values for each channel. We found a correlation between the segment lengths and the value of L^*. Channel #5 is located on the smaller segment (DE) and have the smaller L^*. Its L^* is twice its segment length because the effective length of the Hann window is half its size. Channel #3 is in the middle of the longest section (BC) and present the longer L^* of 450 m. Channels #1 and #4 – being closer to their segment end – have smaller L^*. Channel #2 is almost on a smooth turn and also have intermediate L^*.The distance to the closest edge seems to be the main factor dictating L^*.

4. Applications

Two applications of the use the deformation will be presented. The first one shows that deformation allows to enhance direct body P-waves that are otherwise difficult to observe in strain rate data. The second one shows how magnitude estimation is facilitated using deformation because it permits the use of traditional seismologic methods.

4.1. Direct P-waves enhancement

In and around sedimentary basins, distributed strain rate measurements are known to be sensitive to site-effects and mainly record highly scattered/refracted phases (Trabattoni et al., 2022). We will show here that deformation measurements improve the sensitivity to waves with higher apparent speeds and allow one to observe phases that are otherwise indistinguishable.

Sedimentary basins can be found both on land and off-shore. We will focus on a small regional earthquake recorded during a deployment that took place offshore Chile (Fig. 7a). A DAS interrogator (OptoDAS – Alcatel Submarine Networks) was connected in Concón during the month of November 2021 to the submarine fibre optic telecom cable (operated by GTD group) that links Concón to La Serena. A gauge length of 8.16 m was used with a repetition rate of 625 Hz decimated to a sampling rate of 125 Hz and a spatial sampling of 4.08 m.

Both P and S phases arriving to the bottom of the basin are either converted or transmitted. Because of the very slow velocity of S-waves in sediments and hence the high P to S velocity ratio, S-waves get much more amplified and distorted than P-waves (Fig. 7b). This results in the dominance of slow refracted/scattered waves. Because of the high sensitivity to slow waves of strain rate measurements, this is further exacerbated (Fig 7c). When converting data to displacement (using the sliding-window scheme with a 1 km Hann window) the increased sensitivity to faster waves allows the observation of faint waves that travel as direct P-waves in the sediments (Fig. 7d). Also, the overall signal-to-noise ratio improves (see supplementary material, Fig. S2). The analysis of the direct, coherent P arrival potentially enables the use of array processing techniques such as beamforming (van den Ende & Ampuero, 2021).

Because the P transmitted P arrival (Pp) is less distorted, it provides a much better estimate of the P-wave arrival time. Studying the arrival time difference between Pp and the P converted S (Ps) phases should provide information on the sedimentary structure. Finally, the use of the Pp phase should enable more accurate location procedures.

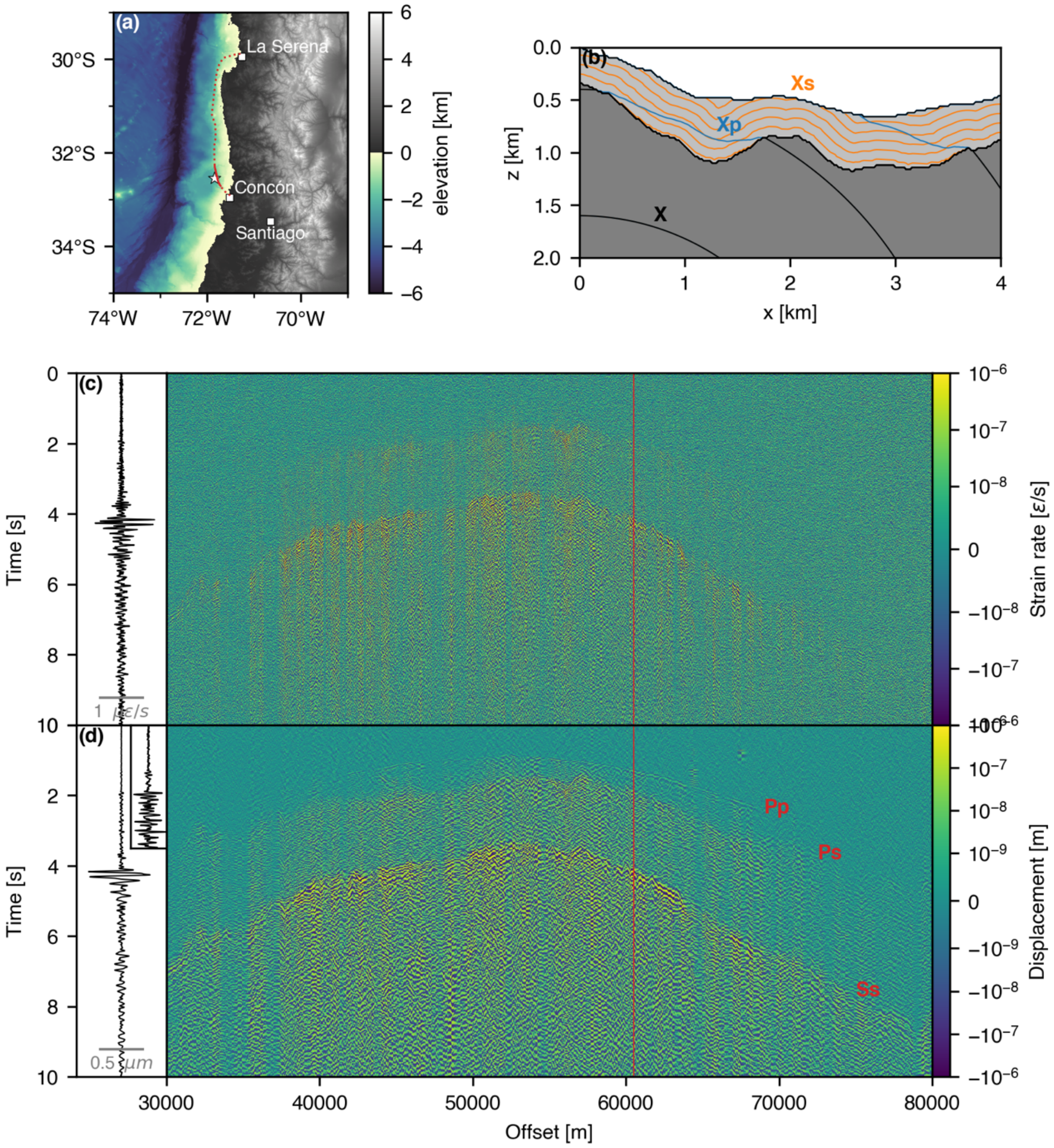

Figure 7. Application of the use of the deformation to recover displacement. (a) Geographic context of the deployment. A submarine telecom cable (dashed red line) was instrumented from Concón. Here only the data coming from the section between 30 and 80 km away from the instrument was used (red solid line) to record a small regional event that occurred very close to the cable (white star: probable location; magnitude was not estimated). (b) Illustration of the strong deformation and amplification of any incoming phase X into a converted/transmitted S-wave (noted Xs) while the P-wave (noted Xp) get less distorted. (c) Strain rate. We mainly see the increased high frequency and wavenumber instrumental noise along with slow, strongly scattered waves. On the left inset, the trace of the channel located at offset 61 km (red solid line) is displayed. We mainly see the Ss arrival. (d) Displacement recovered using a 1 km sliding Hann window. The noise is reduced, and slow apparent velocity waves are enhanced. This allows for the direct P-wave phase (noted Pp) to emerge from the noise. On the left inset, the trace of the channel located at 61 km (red solid line) is displayed. We see three arrivals. A zoom on the first 3.5 s is showed to highlight the Pp arrival. Both recordings (strain rate and displacement) have been filtered above 5 Hz to remove the micro seismic noise contribution.

4.2. Magnitude estimation

Most methods that estimate the magnitude of an earthquake rely on the amplitude of particle motion recordings. We propose to use DAS recovered displacement to directly estimate the local magnitude (M_L) of small earthquakes recorded during the experiment presented in Trabattoni et al. (2022). A 1.1 km long, L-shaped fibre (Fig. 8c) was installed in the active tectonic area of the Southern Apennines (Italy), in the region affected by the 1980 Irpinia M6.9 earthquake. It was integrated within the Irpinia Near Fault Observatory (INFO) which is composed of 31 permanent seismological stations. The experiment continuously recorded earthquakes and ambient noise for almost five months from September 2021 to January 2022.

Seismic events were extracted from the DAS records using origin times from the INFO catalogue. Strain rate recordings were filtered between 1-30 Hz, and then converted to deformation rate through spatial integration (eq. 2). The sliding-window scheme was used with a window length of 250 m to recover the velocity. Assuming a flat frequency response for the fibre, velocity traces were transformed into Wood-Anderson displacements by time integration and by applying the specific Wood-Anderson instrumental response. We independently estimated the local magnitude for all the channels along the fibre using the scale tailored for the Irpinia area $M_L = \log_{10} A + 1.79 \log_{10} R - 0.58$ (Bobbio et al., 2009). Here A is the maximum peak amplitude of the Wood-Anderson displacement, and R is the hypocentre distance (in km).

We selected DAS usable channels according to their Signal to Noise Ratio (SNR), evaluated as the ratio between the maximum amplitude A and the Root Mean Square (RMS) of the 20 s period preceding the origin time of the event. Only channels meeting a SNR criterion of 5 were considered. Events where less than 30 channels did meet that criterion were discarded, reducing the number of usable earthquakes from about one hundred to 29 events. The final magnitude was estimated as the median value of the M_L distribution at all the usable channels $\widetilde{M_L} = \mathrm{median}\big(M_L(s)\big)$. To quantify the uncertainties, the SMAD (Standard Median Absolute Deviation) over the different available channels were computed as $\mathrm{SMAD}(s) = 1.4826 \times \mathrm{median}\big(\big|M_L - \widetilde{M_L}\big|\big)$. For comparison, the same process was applied to the traces recorded by the INFO network.

Magnitude estimates provided by the DAS match those computed from the INFO network (Fig. 8a) showing that using DAS recovered velocity enables simple and accurate M_L estimation. This workflow avoids the need of inverting any effective velocity, provides correct magnitude estimation and is computationally efficient.

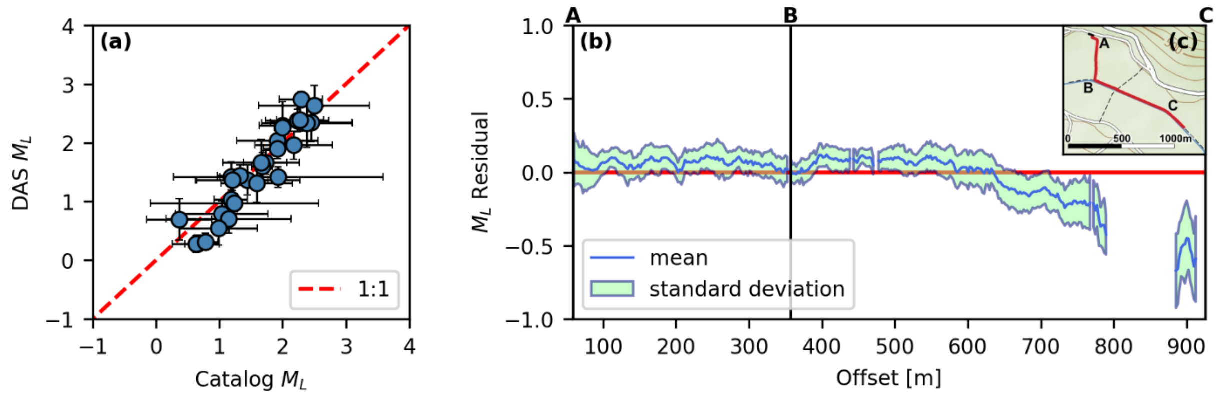

Figure 8. M_L estimation from DAS-recovered velocity. (a) Comparison between DAS and INFO estimates, with associated uncertainties (SMAD). The results are compatible with a 1:1 curve. DAS estimations have lower uncertainty probably because DAS samples a small spatial extent with more correlated hence potentially biased measurements. (b) Mean residual at each channel along the fibre. The curve is represented in between the standard deviation of the residuals, and only channels where M_L has been estimated for a minimum of 10 events have been used for this analysis. The SNR decreases at the last channels of the fibre, likely due to the influence of the basin structure. The vertical lines represent the separation between the main segments of the Irpinia DAS array, showed in (c). DAS estimations are quite stable along the majority of the cable despite a change of the nature of the subsurface between section AB (cultivation field) and BC (dirt road).

Looking at the variability of the estimated magnitude along the cable for different events the estimated magnitudes are quite stable for a major part of the cable (Fig. 8b): The estimates of the last segment decrease progressively, maybe due to local site effects and attenuation, for which the calibrated magnitude scale may be inappropriate. This trend could be removed to improve the magnitude estimation.

5. CONCLUSION

While the standard output provided by most DAS interrogators is provided in strain (rate), here we propose to use the deformation (rate) which is the spatial integral of the strain (rate) along the cable. The deformation is closely related to the displacement but presents crucial differences. Deformation is a measure of the change in length of the cable and provides a displacement measurement relative to a reference. If the reference is non-zero, the inferred deformation no longer equals the true particle motion at a given location on a DAS cable. As a result, spatially-constant offsets appear in the estimated displacement data, and accumulate along the cable. To recover the true displacement from the deformation, without direct access to a reference (as e.g., provided by co-located seismometers), two methods are proposed. When the cable geometry is known, estimating and removing the spatial mean of the signal for each rectilinear segment effectively eliminates the reference; we refer to this method as the segment-wise method; If the cable geometry is poorly known or cannot be readily approximated by linear sections, a sliding average (sliding-window method) continuously estimates and removes the reference from the deformation data. The performance of each method is comparable but situationally-dependent. Nevertheless, owing to its flexibility and the reduced presence of artifacts we recommend the sliding-widow scheme as standard conversion procedure.

Displacements recovered from deformation provide an instrumental sensitivity that, compared to strain, presents several benefits: (i) it is compatible with standard seismological tools, (ii) it is more sensitive to long apparent wavelength signals and has a broader directivity pattern, (iii) it is proportional to the phase of the backscattered light which is the primitive measurement provided by most modern DAS interrogators that uses coherent detection techniques hence. On the other hand, deformation-based methods are limited by the rectilinearity of the deployed fibre optic cable. The maximum recoverable wavelength by a rectilinear section is directly linked to its length. This should encourage the use of geometries with long rectilinear segments.

The benefits of deformation-recovered displacement are highlighted for two use cases. It allows direct non-scattered P-waves with high apparent velocity to emerge for a telecom submarine deployment in a very active subduction context, permitting improved analyses in sedimentary basins context. Furthermore, it allows to use traditional magnitude estimation methods based on DAS data to estimate the magnitude of small local events using existing attenuation relationships calibrated with a local seismic network; we demonstrated this in an on-land active normal fault context. Because of the low computational cost of the methods, deformation recovered displacement could be used for earthquake early warning. With the proposed data conversion schemes, we bring DAS data closer to traditional seismological data, permitting the re-use of conventional seismological tools.

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Supplementary Material

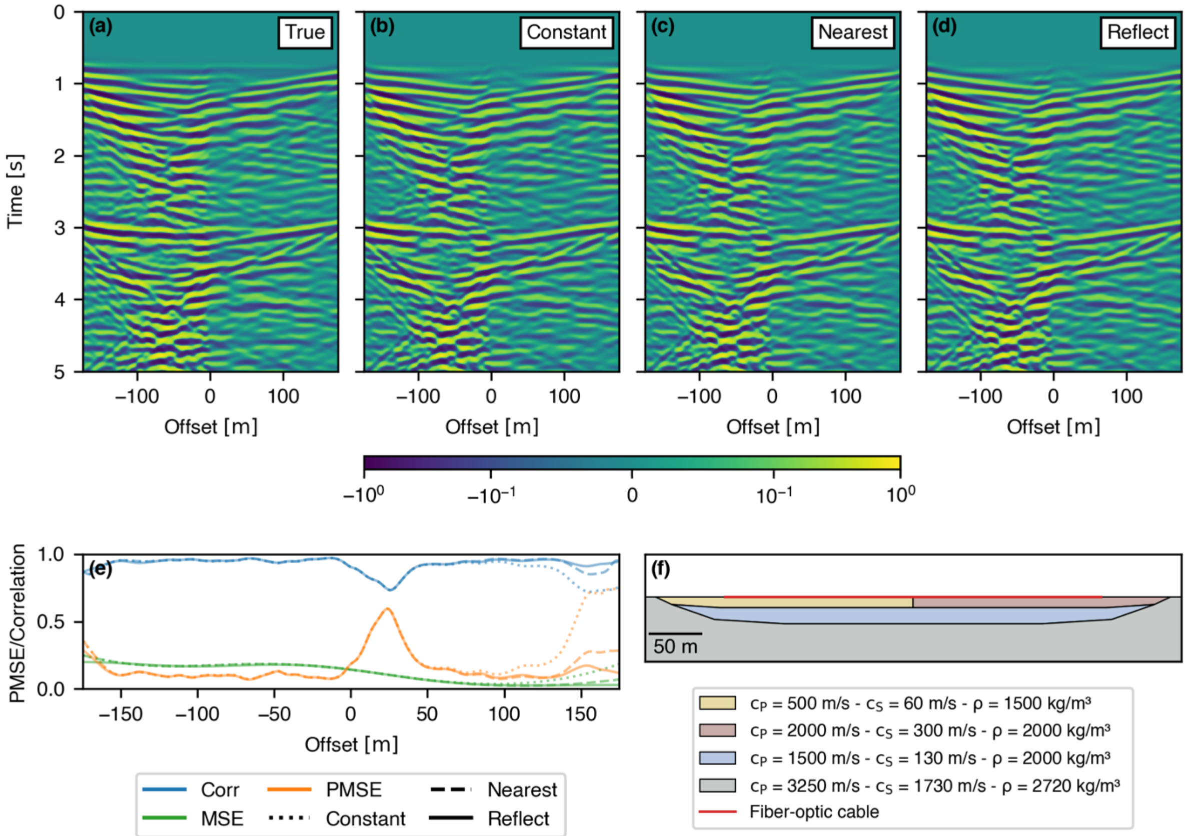

Figure S1. Effects of different padding modes used in the sliding-window scheme. The true velocity (a) is compared with recovered velocity using: (b) constant zero padding, (c) constant closest value padding, and, (d) reflection (or mirror) symmetric padding. Results are mostly identical except at the cable ends. (e) Error metrics for each padding mode. The reflect mode has overall better performances for this case.

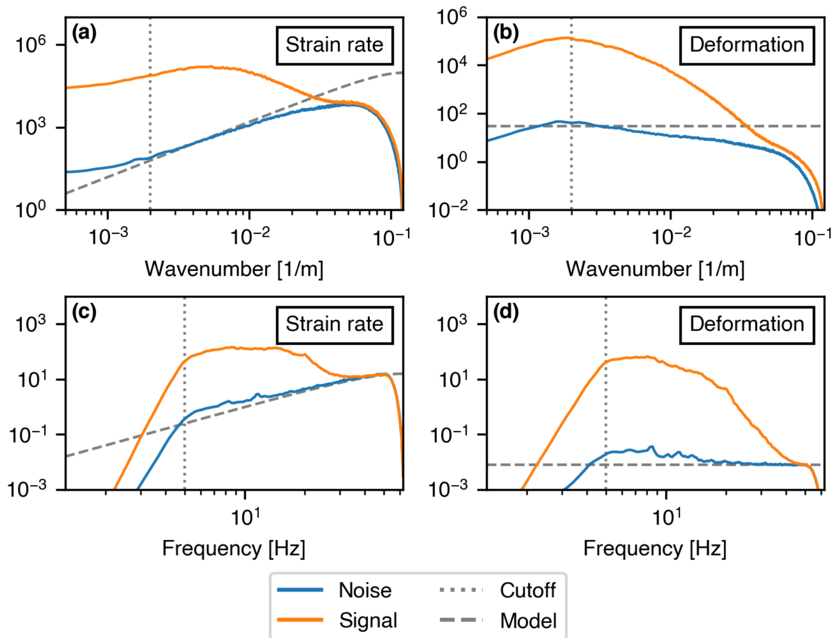

Figure S2. Noise levels of the OptoDAS interrogator. (a) Average raw strain rate levels in the wavenumber domain for 20 s time windows before (noise in blue) and during (signal in orange) the event presented in Fig 7. (b) Same but for the recovered displacement with a cut-off (vertical dotted grey line) wavelength of

500 m associated with the sliding 1 km Hann window used. (c) Raw strain rate levels but in the frequency domain with a clear cut-off frequency of 5 Hz implied by the filtered meant to remove the micro-seismic noise. (d) Same for the deformation-recovered displacement. The strain-rate noise level both increases with the wavenumber and the frequency but less than expected (grey dashed line). If noise was an additive white term in deformation and if strain-rate was computed by simple spatial and temporal differentiation we would expect linearly. It is probable that other derivation schemes are used, that the noise characteristic in deformation is not white and that more advances proprietary processing technics are used.