

Learning generative models for geostatistical facies simulation based on a single training image

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Abstract

Characterization of subsurface reservoirs often requires geological facies models to identify areas with favorable rock properties. With the development of computing powers, deep learning approaches, such as the generative adversarial networks (GANs), became widely used for simulating complex geological models. However, training of the GANs typically requires a large quantity of training data for updating neural parameters. This process is generally done using traditional geostatistical methods based on multiple-point statistics or process-based models to build the training data. In this study, we propose to train the GANs using one single training image, a conceptual model from which the statistics of the geological patterns can be extracted. The training image is first down-sampled to different scales, and the generator and the discriminator are trained alternately for each scale. The training process is implemented from the coarsest to the finest scale to learn the spatial statistics from the training image progressively. We apply the proposed GANs to simulate the 2D Lena river delta and 3D Descalvado aquifer analog model, in which complex geological patterns and structures from the training image are successfully learned and reproduced by GANs. The gradual deformation method is further applied to iteratively calibrate the random realizations by the generator to

23 observed data, in an optimization workflow. The optimization scheme is implemented many
24 times to obtain multiple independent models that all match the observed data.

25 Keywords: Training image; GANs; Geostatistical simulations; Multiple scales; Model
26 calibration

27 **1 Introduction**

28 Geological facies modelling consists in generating spatial models in two- or three-dimensions
29 of a categorical variables representing the spatial distribution of geological facies or rock types
30 with realistic geological patterns. These models are commonly used in groundwater, carbon
31 storage, and hydrocarbons exploration studies to identify geo-bodies with rock and fluid
32 properties of interest. Many geostatistical approaches for facies modelling have been proposed
33 such as two-point statistics, multiple-point statistics, object-based and process-based methods
34 (Strebelle, 2002; Hu and Chugunova, 2008; Mariethoz et al., 2010; Mariethoz and Caers, 2014;
35 Pyrcz and Deutsch, 2014). In recent years, deep learning methods have been applied for
36 geostatistical modeling of categorical and continuous variables such as facies or rock and fluid
37 properties. Various generative models have been trained to reproduce the complex geometrical
38 patterns representing realistic realizations of the spatial distribution of geological facies. One
39 of the most popular methods is the generative adversarial networks (GANs) (Goodfellow et al.,
40 2014), which can generate realistic models and have been widely used in geoscience
41 applications (Dupont et al., 2018; Chan and Elsheikh, 2019; Azevedo et al., 2020). Laloy et al.
42 (2018) use GANs to generate two- and three-dimensional geological facies models in an
43 unconditional way. Sun (2018) presents a state-parameter identification GAN for learning
44 bidirectional relations between parameter space and corresponding model space. Chen et al.
45 (2022) model subsurface sedimentary facies using an advanced self-attention GAN to capture

46 and reproduce large scale features of the training image. Feng et al. (2022) investigate GANs
47 within the Bayesian framework for geological facies modelling.

48 The application of GANs in facies modelling has also been extended to conditional simulation,
49 in which random realizations are constrained to physical measurements (i.e. direct
50 observations). The conditional simulation by GANs can be treated as a semantic image
51 inpainting problem, in which the task to generate constrained models is formulated as an
52 optimization problem (Yeh et al., 2017). The optimization process can be divided into two
53 steps (Zhang et al., 2021), to train unconditional GANs for producing plausible facies models
54 when inputting latent vectors (step 1) and to update the location of the input vectors in the latent
55 space based on the computed error between generated model and conditioning data (step 2).
56 Mosser et al. (2018) apply the content loss for constraining the conditioning data and generate
57 conditional pore and reservoir-scale models. Zhang et al. (2019) use GANs to learn the
58 sedimentary architecture and simulate geologically realistic three-dimensional reservoir facies
59 models. Zhong et al. (2019) formulate conditional GANs to learn the dynamic functional
60 mappings in multiphase models and predict the CO₂ plume migration in heterogeneous storage
61 reservoirs. In contrast to the two-step approach (Yeh et al., 2017), Zhang et al. (2021) propose
62 to perform the conditioning of facies models in the training process of GANs, adopting the idea
63 of image-to-image translation (Isola et al., 2018). The progressive growing of GANs has been
64 applied by Song et al. (2021) for conditioning geophysics-interpreted probability maps, and
65 multiple geological facies models are generated, which are consistent with input conditions.

66 However, in the aforementioned studies, a large quantity of training data is typically required
67 to update neural parameters of GANs in the training process, and some traditional geostatistical
68 methods are commonly applied to generate such training data (e.g. Song et al., 2021; Zhang et
69 al., 2021; Feng et al., 2022; Sun et al., 2022). Therefore, the training of GANs models heavily

70 relies on the geostatistical methods, even though the trained GANs can reproduce complex
71 geological patterns more computationally efficiently. In order to address this problem, we
72 propose to train the GANs model based on a single training image, without using any
73 geostatistical method for building the training data. The proposed GANs perform unconditional
74 simulations based on a multi-scale architecture (Shocher et al., 2018; Shaham et al., 2019).
75 Moreover, we apply the gradual deformation method (GDM) with an optimization workflow
76 to iteratively calibrate the random realizations generated by GANs to the observed data (Hu,
77 2000; Caers, 2007). The novelty of the proposed method is that the training of GANs only uses
78 one single training image according to the simulation approach used in multiple-point statistics
79 (MPS); in addition, the proposed method does not require the stationary assumption of the
80 training image. The proposed GANs model is applied to simulate complex 2D river delta and
81 3D geochemical facies models, and the random realizations are then conditioned/calibrated to
82 honor observed data using GDM.

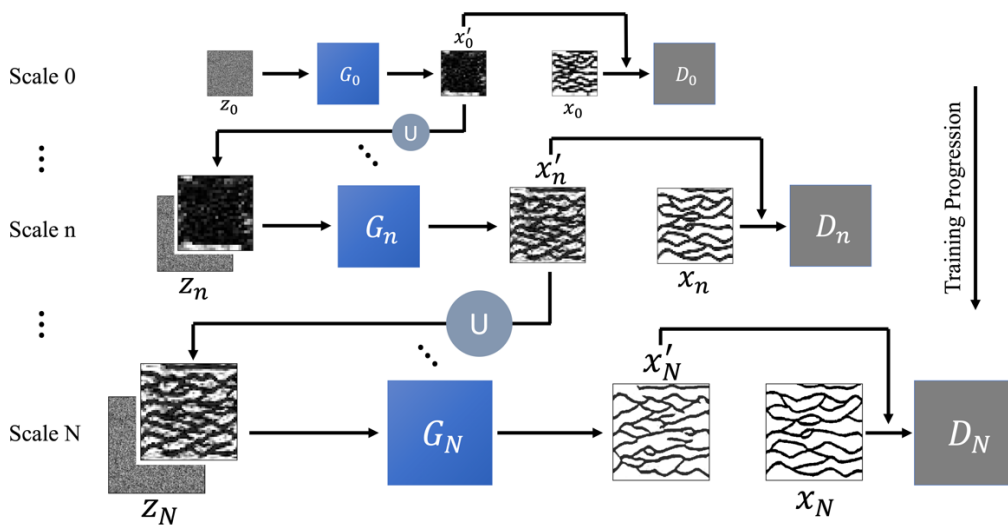
83 **2 Methodology**

84 **2.1 GANs based on a single training image**

85 GANs are networks consisting of two components: the discriminator (D) and the generator (G).
86 During the training process, the generator tries to generate images as realistic as possible, while
87 the discriminator aims to distinguish the real images from the fake ones (Goodfellow et al.,
88 2014). The training process of GANs is iteratively implemented. Once trained, the generator
89 of GANs is given with latent vectors that are drawn from a pre-defined distribution, and
90 plausible and diverse models are generated to reproduce the probability distribution of the
91 training data (Zhang et al., 2021).

92 In this work, in the context of geological facies modeling, instead of using multiple training
93 images as the training data, we only use one single training image for updating neural

94 parameters of GANs. This is achieved by introducing a pyramid of fully convolutional GANs,
 95 where each is responsible for capturing the spatial statistical distribution of down-sampled
 96 patches of the training image at different scales (Shaham et al., 2019). Figure 1 shows the high-
 97 level architecture of the proposed GANs, consisting of a pyramid of generators and
 98 discriminators to be trained at different scales (Shaham et al., 2019). The single training image
 99 x is first down-sampled to different scales by a factor of r , making the image pyramid of
 100 $\{x_0, \dots, x_n, \dots, x_N\}$. The final image x_N has the same size of the image x . Correspondingly, the
 101 proposed GANs model has a pyramid of generators $\{G_0, \dots, G_n, \dots, G_N\}$ and discriminators
 102 $\{D_0, \dots, D_n, \dots, D_N\}$, which are trained with the down-sampled image patches individually (Fig.
 103 1).



104
 105 Fig. 1 Schematic view of the multi-scale architecture of GANs where the training of the
 106 generator and discriminator is performed from the coarse to the fine scale (Scale 0 to Scale
 107 N). U stands for the up-sampling operation.

108 All the generators and discriminators are trained sequentially, starting from Scale 0 (Fig. 1),
 109 and once G_n and D_n are trained, they are kept fixed, and the training process is moved to the
 110 next scale level. The generation process at the n^{th} level involves all the generators $\{G_0, \dots, G_n\}$,

111 as well as the random noise maps $\{z_0, \dots, z_n\}$ up to the current level (Shaham et al., 2019). At
 112 the n^{th} scale level, an adversarial training process is performed separately: the generator G_n
 113 tries to generate fake image x'_n to fool the discriminator D_n and the discriminator D_n attempts
 114 to distinguish the real image x_n from the fake one x'_n . Different from other GANs where the
 115 1D latent vectors are considered as inputs, in the proposed implementation, the generator G_n
 116 here takes an image sample generated by the up-sampled trained generator at the previous scale
 117 x'_{n-1} and the 2D random noise map z_n as inputs.

118 At the coarsest scale (Scale 0 in Fig. 1), there is no image sample, and only the random noise
 119 map z_0 is given to the generator G_0 for generating an image sample x'_0 . Each generator G_n is
 120 trained to learn the internal structure of the training image at different scales, and the finer
 121 details from the training image are learned sequentially, as the finer scale details cannot be
 122 generated by the previous generators $\{G_0, \dots, G_{n-1}\}$. Indeed, this multi-scale strategy is similar
 123 to the multiple grid simulation approach used in MPS methods for capturing large scale
 124 structures of geological models of interest (Tran, 1994). The formulation for generating an
 125 image sample at the n^{th} level is given by:

$$x'_n = \begin{cases} G_0(z_0) & n = 0 \\ G_n(z_n, (x'_{n-1}) \uparrow) & 0 < n \leq N \end{cases} \quad (1)$$

126 in which \uparrow represents the up-sampling operation (U in Fig. 1).

127 The loss function at the n^{th} scale level for G_n and D_n is formulated as (Shaham et al., 2019):

$$\min_{G_n} \max_{D_n} \mathcal{L}(G_n, D_n) = \mathcal{L}_{adv}(G_n, D_n) + \alpha \mathcal{L}_{rec}(G_n), \quad (2)$$

128 where \mathcal{L}_{adv} is the adversarial loss used in common GANs training for penalizing the
 129 distribution distance between down-sampled image x_n and generated image sample x'_n ; α is a
 130 weighting factor to balance the two loss functions; and \mathcal{L}_{rec} is the reconstruction loss to ensure

131 that x_n can be reproduced given a specific set of random noise maps. A random noise map z^*
 132 is drawn once at the coarsest level (Scale 0, Fig. 1) and is kept fixed in the training process
 133 afterwards. The reconstructed image at the n^{th} scale is denoted as x_n^{rec} , and the reconstruction
 134 loss function is given by:

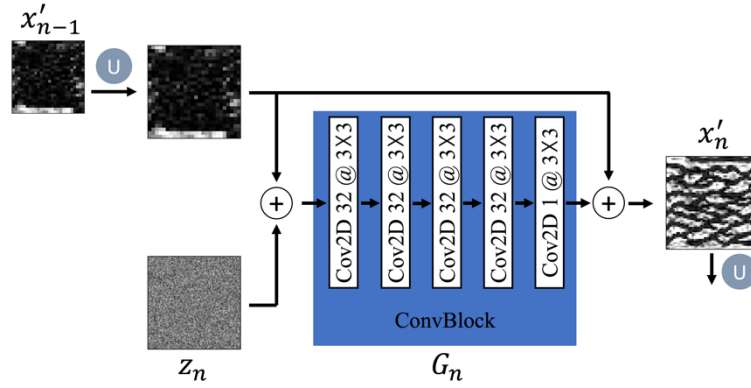
$$\mathcal{L}_{rec} = \|x_n^{rec} - x_n\|^2, \quad 0 \leq n \leq N \quad (3)$$

135 where x_n^{rec} can be generated as:

$$x_n^{rec} = \begin{cases} G_0(z^*) & n = 0 \\ G_n(0, (x_{n-1}^{rec}) \uparrow) & 0 < n \leq N \end{cases} \quad (4)$$

136 The deep convolutional neural networks are applied to design the generators and discriminators
 137 at different scales using PyTorch, and they all have the same network architecture. Figure 2
 138 shows the neural network architecture for the generator and discriminator. At the n^{th} scale
 139 level, the image sample generated by the generator from the previous scale x'_{n-1} is first up-
 140 sampled to the current resolution and added to the random noise map z_n . This result is then
 141 considered as the input to the convolutional block with 5 layers. The output from the last
 142 convolutional layer is added with the up-sampled x'_{n-1} to obtain x'_n . The first four
 143 convolutional layers in the ConvBlock (Fig. 2a) have 32 kernels whereas only 1 kernel is used
 144 in the last convolutional layer. The kernel size for the two-dimensional filters is 3×3 , with a
 145 stride step of 1×1 . Each convolutional layer is followed by a batch normalization (Ioffe and
 146 Szegedy, 2015), and the Leaky rectified linear unit (ReLU) is used as the activation function,
 147 except in the last layer where the hyperbolic tangent (tanh) function is applied (Fig. 2a). The
 148 discriminator D_n at the n^{th} scale has a similar network architecture with the generator G_n (Fig.
 149 2b), for distinguishing between the generated fake image x'_n and real image x_n down-sampled
 150 to the current resolution. Moreover, a Markovian discriminator (Li and Wand, 2016) is applied
 151 to classify the input as real or fake, by calculating the mean of the patch output from the last

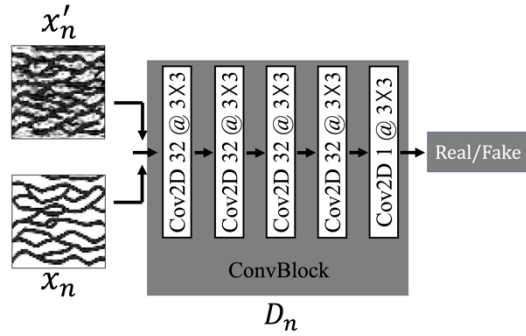
152 convolutional layer of D_n . The number of kernels in the ConvBlock for the generator and
 153 discriminator (Fig. 2) is increased by a factor of 2 for every 4 scales, in order to capture more
 154 details at the finer scale.



155

156

(a)



157

158

(b)

159 Fig. 2 Network architecture for the generator (a) and discriminator (b) at the n^{th} scale level.

160 The number of training epoch is 2,000 to train each generator and discriminator per scale, with
 161 a learning rate of 0.0005 and the Adam optimizer (Kingma and Ba, 2014). The hyperparameter
 162 α (Eq. 2) is set as 10 to weight the loss contribution between \mathcal{L}_{adv} and \mathcal{L}_{rec} . The total scale
 163 number N for the multi-scale training as well as for down-sampling the training image (Fig. 1)
 164 is determined by the scale factor r and original size of the training image:

$$N = \text{ceil} \left[\log_r \left(\frac{25}{\min(\text{size of image})} \right) \right] + 1, \quad (5)$$

165 where the function of ceil rounds a real number to the nearest integer, the min function returns
 166 the lowest value of the size of the image, and r is the scale factor assumed to be 0.75.

167 **2.2 Data calibration/conditioning using gradual deformation method**

168 The presented GANs can only generate random realizations, without any data
 169 conditioning/calibration. Then, the gradual deformation method (GDM) is applied to
 170 condition/calibrate the GANs random realizations to observed data \mathbf{d} as a result of an
 171 optimization problem where the realizations are stochastically perturbed until they match the
 172 data \mathbf{d} (Hu, 2000; Hu et al., 2001; 2004). The principle of GDM is to generate perturbations of
 173 initial model realizations in such a way that the realizations match the observed data at the data
 174 locations (Caers, 2007). Given two independent Gaussian functions \mathbf{Y}_1 and \mathbf{Y}_2 , a Gaussian
 175 random function \mathbf{Y} is built as a linear combination of \mathbf{Y}_1 and \mathbf{Y}_2 as

$$\mathbf{Y}(\theta) = \mathbf{Y}_1 \cos \theta + \mathbf{Y}_2 \sin \theta, \quad (6)$$

176 with θ being the perturbation parameter (Hu, 2000). When $\theta = 0$, then $\mathbf{Y} = \mathbf{Y}_1$; and when θ is
 177 gradually increased to $\pi/2$, $\mathbf{Y} = \mathbf{Y}_2$. Given two independent realizations \mathbf{y}_1 and \mathbf{y}_2 of \mathbf{Y}_1 and
 178 \mathbf{Y}_2 , then $\mathbf{y}(\theta)$ represents a set of realizations:

$$\mathbf{y}(\theta) = \mathbf{y}_1 \cos \theta + \mathbf{y}_2 \sin \theta, \quad (7)$$

179 Consider \mathbf{y} a realization of \mathbf{Y} and \mathbf{g} the forward operator that relates \mathbf{y} to \mathbf{d} , then an objective
 180 function \mathcal{L} can be formulated in the optimization process for finding model parameters \mathbf{y} to
 181 match observed data \mathbf{d} :

$$\mathcal{L} = \|\mathbf{d} - \mathbf{g}(\mathbf{y})\|, \quad (8)$$

182 and minimizing the difference between the observed data \mathbf{d} and the simulated data $\mathbf{g}(\mathbf{y})$. When
 183 \mathbf{y} is parameterized with the perturbation parameter θ , then \mathcal{L} in Eq. (8) becomes a function of
 184 θ :

$$\mathcal{L}(\theta) = \|\mathbf{d} - \mathbf{g}(\mathbf{y}(\theta))\|, \quad (9)$$

185 and the multi-dimensional optimization problem in Eq. 8 becomes a one-dimensional
 186 optimization problem (Eq. 9), to find the optimal value of the perturbation parameter θ^{opt} such
 187 that $\mathbf{g}(\mathbf{y}(\theta^{opt}))$ best matches \mathbf{d} (Caers, 2007).

188 In practice, a single optimization of θ could not result in a model that matches \mathbf{d} satisfactorily,
 189 and a repeated workflow is required by generating new realizations and repeating the
 190 optimization in an iterative procedure (Caers, 2007):

- 191 1. Generate two independent Gaussian realizations \mathbf{y}_1 and \mathbf{y}_2
- 192 2. Iterate until the observed data \mathbf{d} are matched
 - 193 • Search θ in a range between 0 and $\pi/2$ for optimization

$$\theta^{opt} = \min_{\theta} \{\mathcal{L}(\theta) = \|\mathbf{d} - \mathbf{g}(\mathbf{y}_1 \cos \theta + \mathbf{y}_2 \sin \theta)\|\}, \quad (10)$$

- 194 • Set (i) $\mathbf{y}_1 = \mathbf{y}(\theta^{opt})$

195 (ii) Generate a new random realization \mathbf{y}_{new}

196 (iii) $\mathbf{y}_2 = \mathbf{y}_{new}$

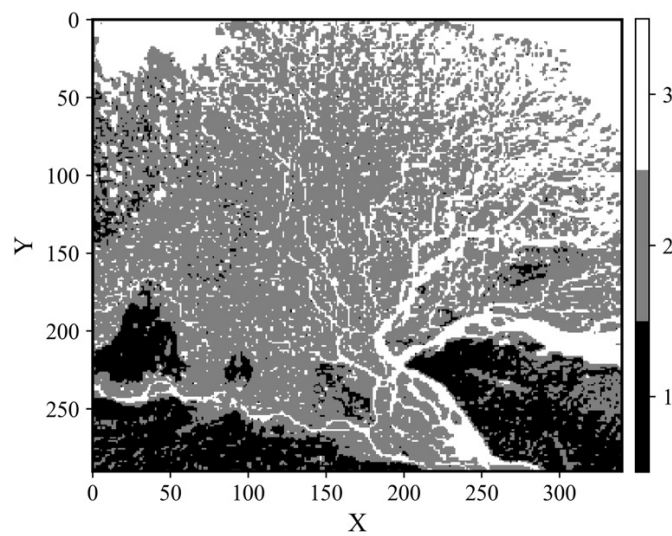
197 For obtaining N_c conditional/calibrated realizations, the workflow is implemented N_c times.

198

199 **3 Application**

200 **3.1 Two-dimensional Lena river delta**

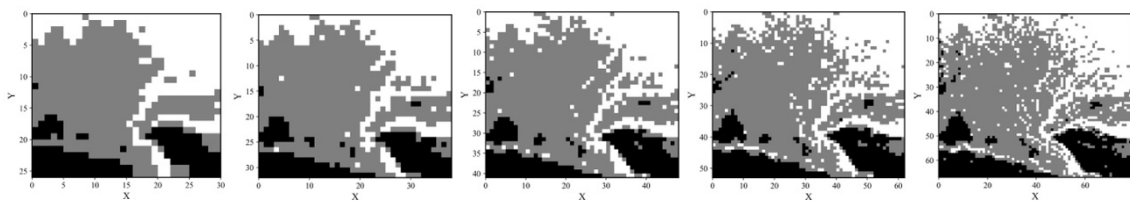
201 The first example for the facies simulation by the proposed GANs is a complex fluvial model
202 from the Lena river delta (Hu et al., 2014). The training image represents the spatial distribution
203 of facies derived from a satellite image (Fig. 3) with a cutoff operation on the digital values
204 (<http://earthobservatory.nasa.gov/IOTD/view.php?id=2704>).



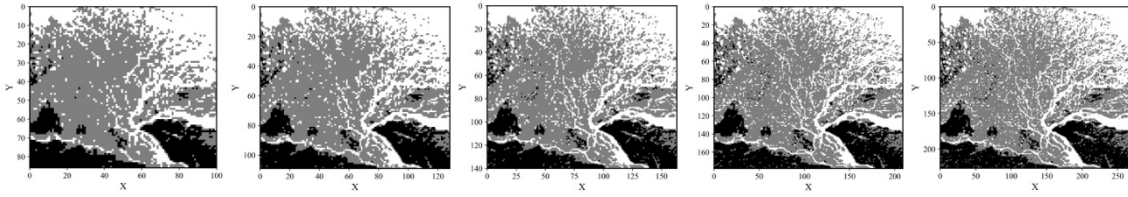
205

206 Fig. 3 Training image of the 2D Lena river delta.

207 The original training image with a pixel size of 340×290 is then down-sampled to different
208 scales (Eq. 5). Fig. 4 shows the image pyramid of $\{x_0, \dots, x_5, \dots, x_9\}$ from the coarsest to the
209 finest scale. In total, there are 11 scales, and the training image at the original resolution (Fig.
210 3) is considered as x_{10} .



211

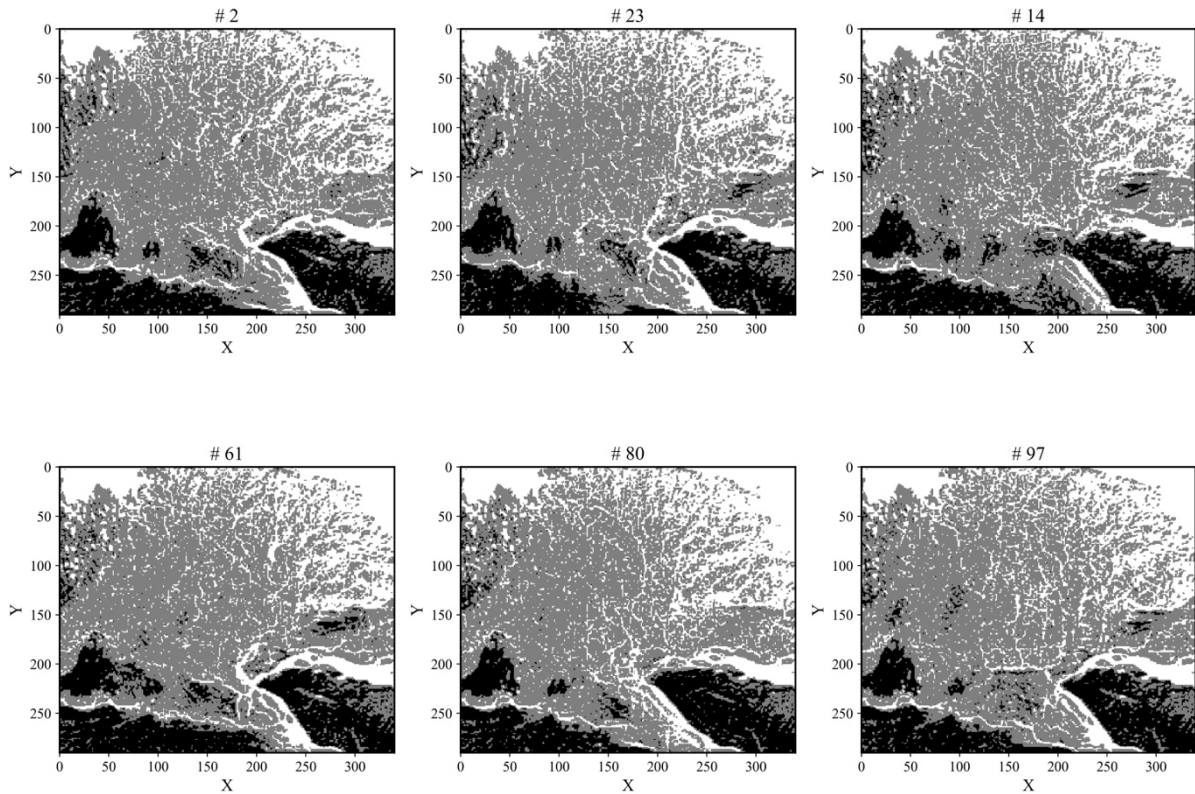


212

213 Fig. 4 The pyramid of training image that has been down-sampled to different scales

214 $\{x_0, \dots, x_5, \dots, x_9\}$.

215 These down-samples images (Fig. 4) are used as training data for updating the GANs at each
 216 scale (Fig. 1), and the total training time is about 2 hours using the GPU of NVIDIA Quadro
 217 RTX 6000. The trained generator is then used to randomly generate 100 facies models, which
 218 only takes approximately 10 s. Figure 5 shows six randomly selected realizations of the 2D
 219 Lena river delta, and all of them can successfully capture the geometry of the river delta with
 220 diverse variations.

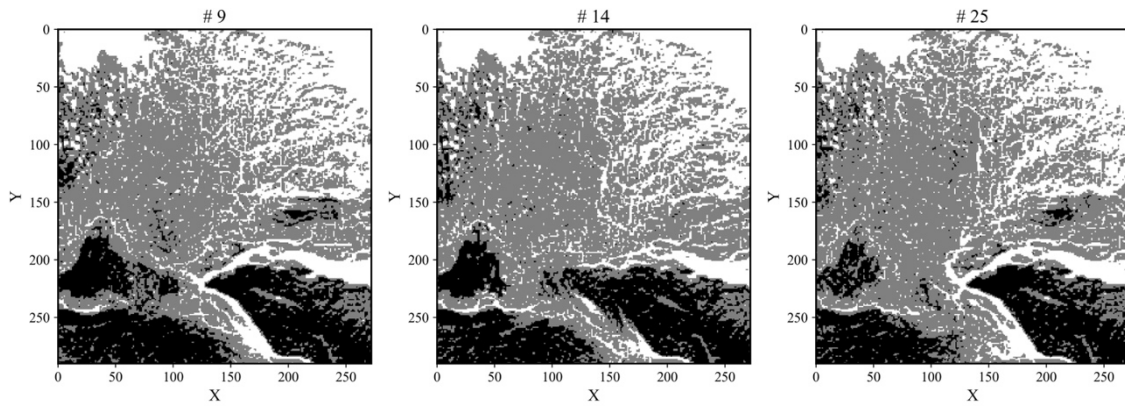


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222

223 Fig. 5 Six randomly selected realizations of the Lena river delta by the trained GANs.

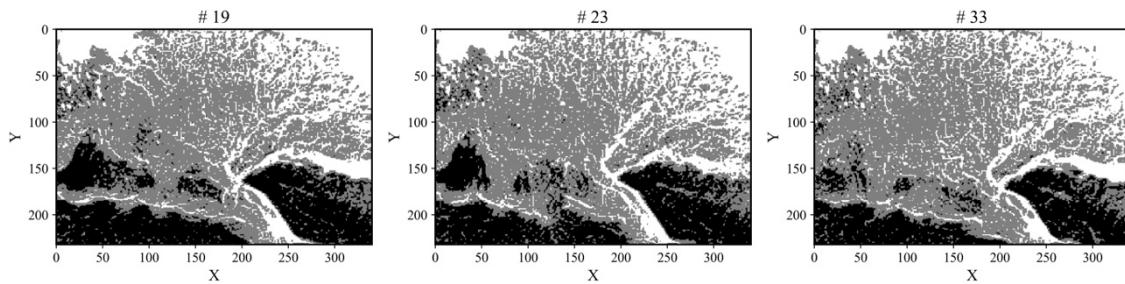
224 Furthermore, by changing the input size of the random noise map, the trained GANs are able
225 to generate realizations of any arbitrary image size, as the full convolutional layers are used in
226 the generator (Shaham et al., 2019). Figure 6 shows the random realizations for different image
227 sizes, and all of them show similarities with the training image (Fig. 3).



228

229

(a)



230

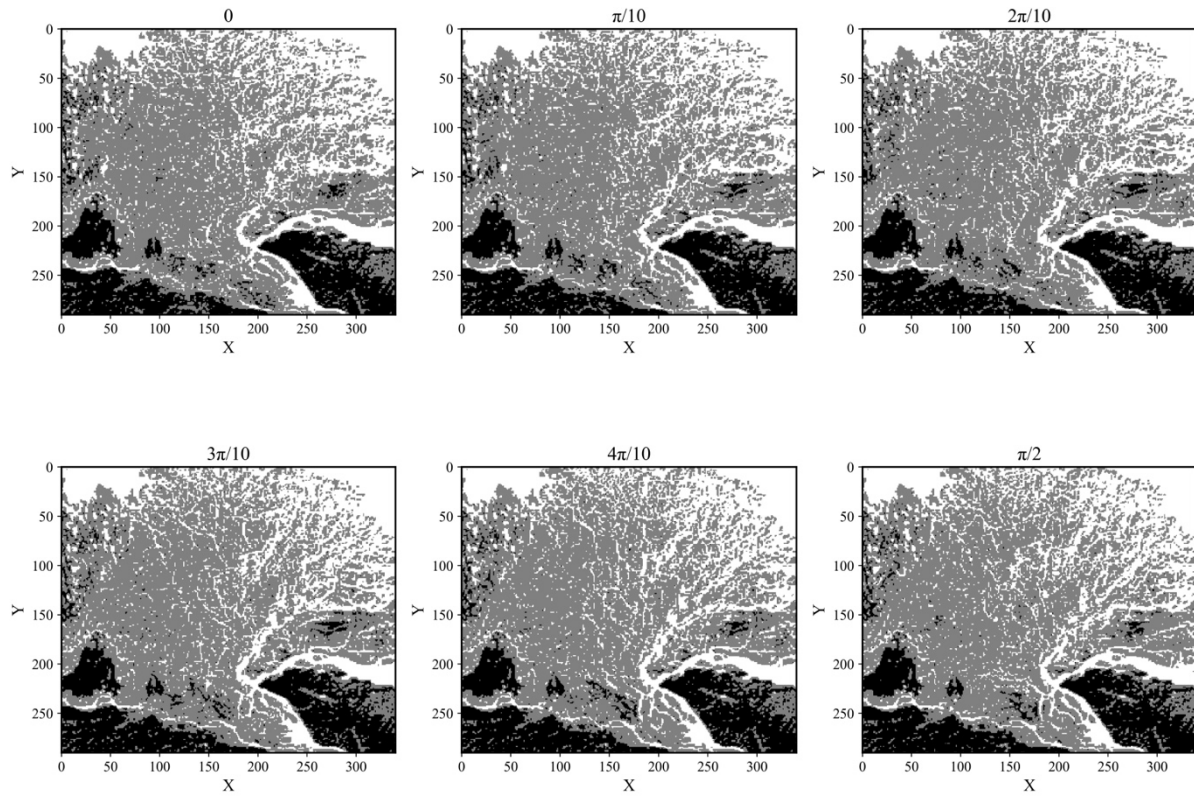
231

(b)

232 Fig. 6 Random realizations of image size of 272×290 (a) and 340×232 (b).

233 The randomly generated realizations do not honor any observed data, and the proposed GDM
234 is then used to condition/calibrate these models to observational data. Within the multi-scale
235 architecture of GANs, the random noise maps are drawn independently from a pre-defined
236 Gaussian distribution and are kept at each scale. The generation of the facies model by the
237 trained GANs is considered as the forward operator \mathbf{g} in the optimization problem (Eq. 10),

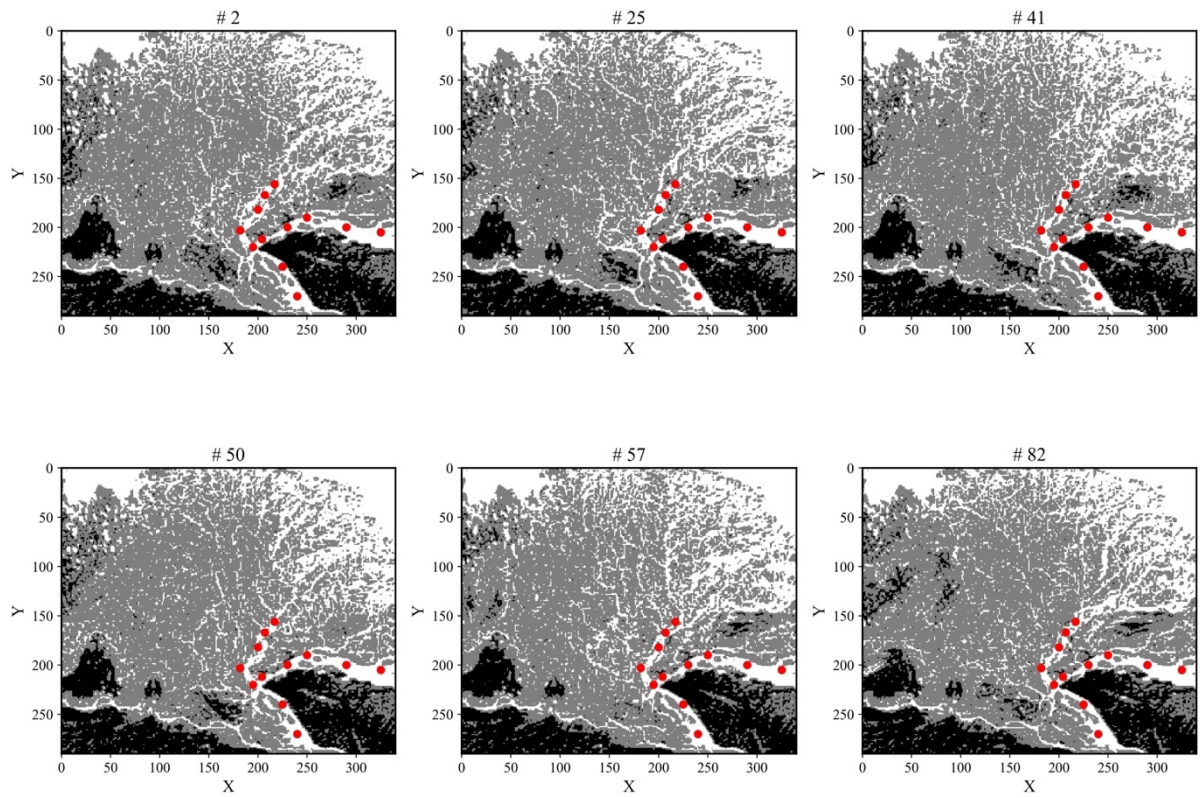
238 based on a linear perturbation of the random noise maps at each scale (Fig. 2a). A gradual
239 deformation of the facies models between two random realizations is shown in Fig. 7, in which
240 the facies models are gradually perturbed from $\theta = 0$ to $\theta = \pi/2$.



242

243 Fig. 7 Gradual deformation between two random realizations of facies models ($\theta = 0$ and
244 $\theta = \pi/2$).

245 The conditional/calibrated facies models by the GDM are shown in Fig.8, where the red circles
246 represent the locations where the channel facies (white color) are observed (Fig. 8). Compared
247 to the unconditional realizations (Fig. 5), the conditional/calibrated realizations better capture
248 the channel structures based on the observed data.

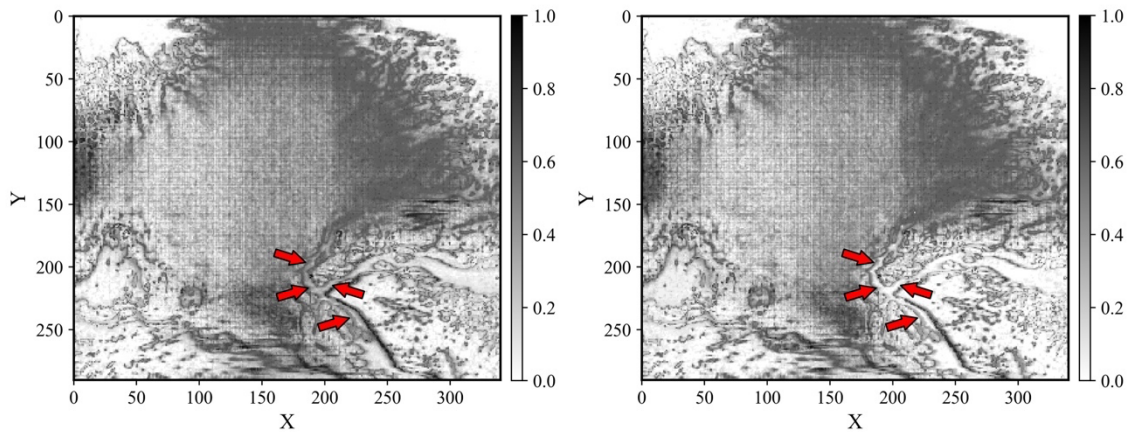


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250

251 Fig. 8 Six conditional/calibrated facies models by the proposed GDM to observed channel
 252 facies (red circles).

253 Figure 9 shows the entropy values based on 100 unconditional realizations and conditional
 254 realizations, respectively. When constrained to the channel facies hard data, the channel
 255 geometry shows smaller entropy values (Fig. 9b, red arrows), which means that the channel
 256 facies are simulated consistently with the observed data, compared to the unconditional
 257 realizations (Fig. 9a, red arrows). The discontinuous artifacts in Fig. 9 are due to the 2D
 258 convolutional filters used and the limited local variability of the facies distribution in the
 259 training image at some areas.



260

261

(a)

(b)

262

Fig. 9 Entropy values based on 100 unconditional realizations (a) and 100 conditional

263

realizations (b). Darker color represents larger uncertainty for the facies simulation.

264

To compare their spatial variability, the training image, unconditional/random realizations and

265

conditional/calibrated realizations are mapped into a low-dimensional space using multi-

266

dimensional scaling (MDS) with Euclidean distance (Cox and Cox, 2008).

267

Unconditional/random realizations and conditional/calibrated realizations overlap with the

268

training image quite well in the 2D space, meaning that there is a good similarity with the

269

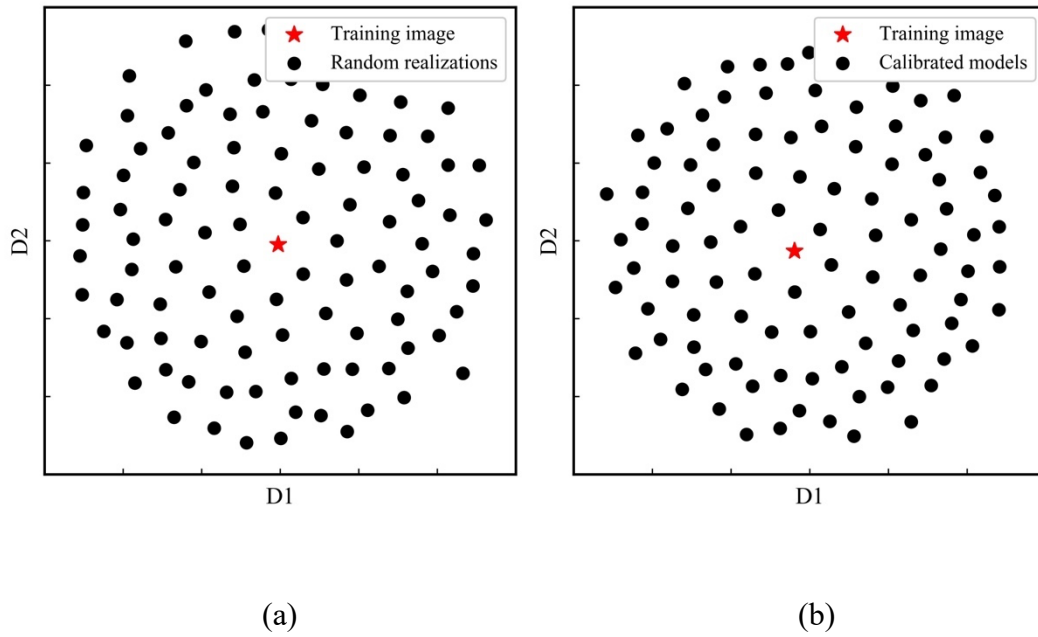
training image (Fig. 10). Furthermore, unconditional/random realizations and

270

conditional/calibrated realizations are scattered well in the 2D space, indicating a large

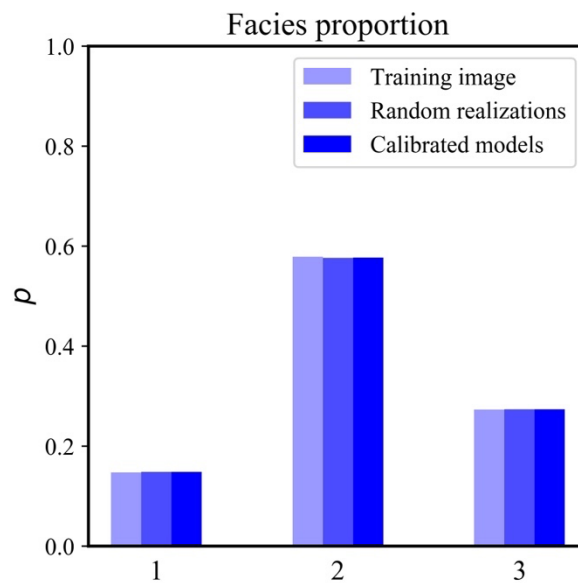
271

diversity among them.



274 Fig. 10 MDS plot of the training image with unconditional/random realizations (a) and
 275 conditional/calibrated models (b).

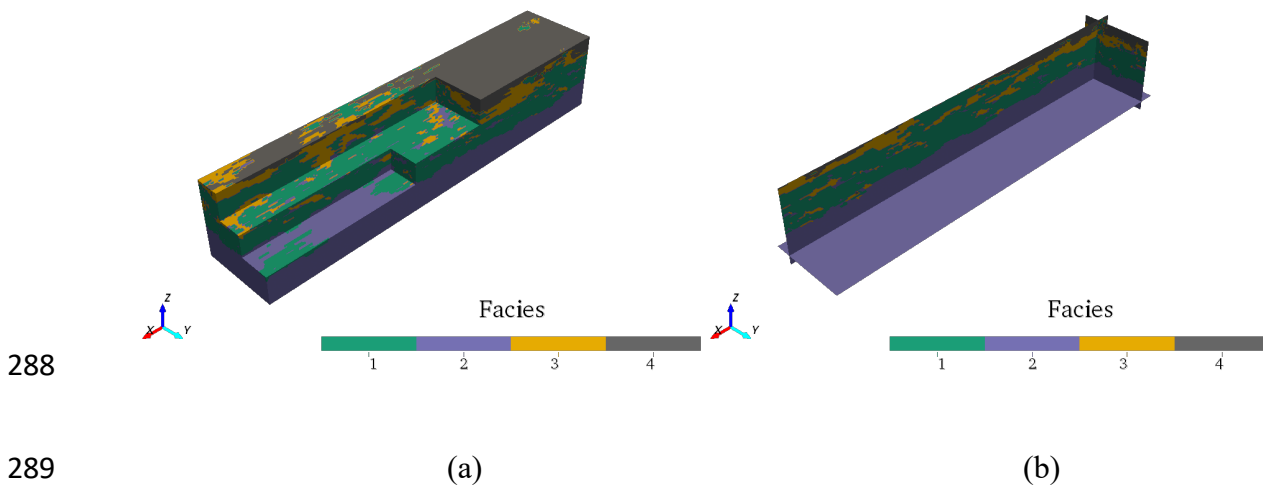
276 The facies proportions are shown in Fig. 11, in which the unconditional/random realizations
 277 and conditional/calibrated realizations can successfully reproduce the histogram of the three
 278 facies in the training image.



280 Fig. 11 Facies proportions of training image, random realizations and calibrated models.

281 3.2 Three-dimensional Descalvado aquifer analog

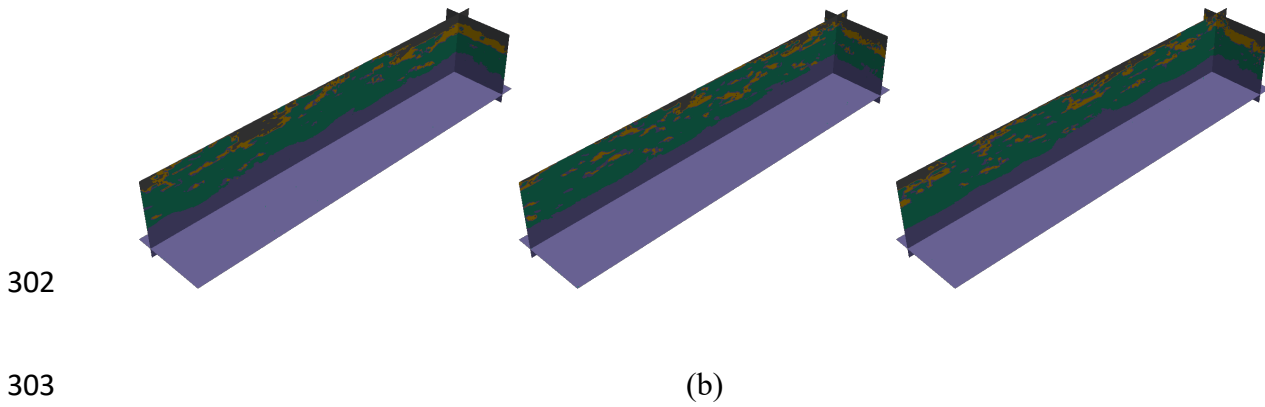
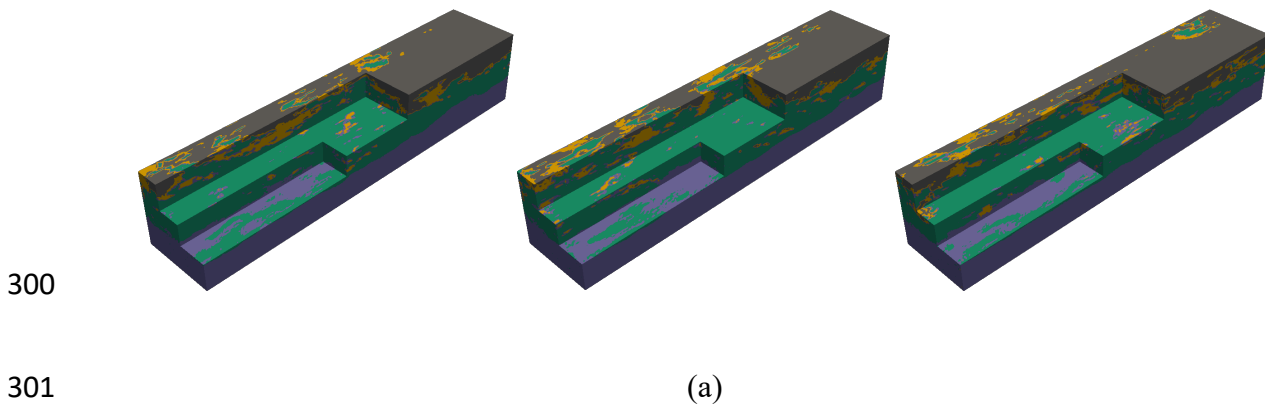
282 The second example for the proposed method is the 3D Descalvado aquifer analog data for
283 characterizing sedimentary structures to analyze groundwater formations in Brazil (Bayer et
284 al., 2015). The original nine facies have been merged into four facies, based on the similarity
285 of grain size, sorting and texture. Fig. 12 shows the training image of the analog dataset in 3D
286 space and slices for inspecting internal structures. The pixel size of the 3D facies model is
287 $250 \times 50 \times 50$ in the X -, Y - and Z -directions.



288
289
290 Fig. 12 Training image of the Descalvado analog dataset in 3D space (a) and slices (b). 1:
291 Trough-cross-bedded sand and gravel; 2: Planar-cross-bedded aeolian sand; 3: Horizontally-
292 laminated to planar cross-stratified sand; 4: Trough-cross-bedded sand and clay intraclasts.

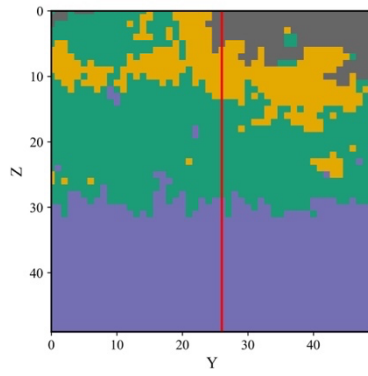
293 The training image in Fig. 12 is then down-sampled to different scales (Eq. 5) for training the
294 proposed GANs (Fig. 1), in which the convolutional kernels are extended to 3D accordingly.
295 The hyperparameters such as training epoch, learning rate, and number of hidden layers are the
296 same with the ones used in the 2D case study. It takes about 12 hours for training the multi-
297 scale GANs, and approximately 60 s for generating 100 unconditional realizations. Figure 13

298 displays three randomly selected realizations by the trained GANs, showing a similar structure
299 with the training image (Fig. 12)



304 Fig. 13 Three randomly selected realizations in 3D space (a) and their corresponding slices
305 (b).

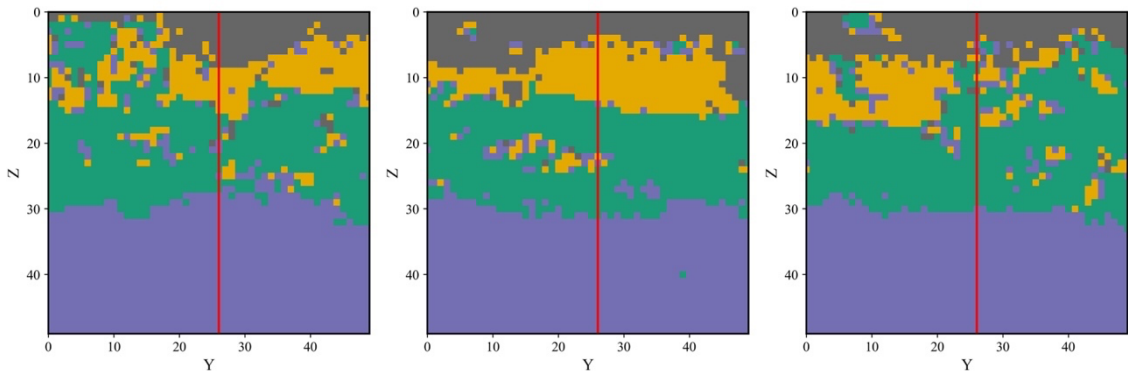
306 The proposed GDM is then applied to condition/calibrate the random realizations to observed
307 data. The data consists of a pseudo well with known facies distribution at the location $X = 126$
308 and $Y = 26$. Figure 14 shows three conditional/calibrated realizations, together with the
309 training image and three selected unconditional/random realizations of the cross section at $X =$
310 126. After data conditioning/calibration, the facies at the well location (Fig. 14, red line) are
311 reproduced in the conditional/calibrated realizations (Fig. 14c), compared to the
312 unconditional/random realizations (Fig. 14b).



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314

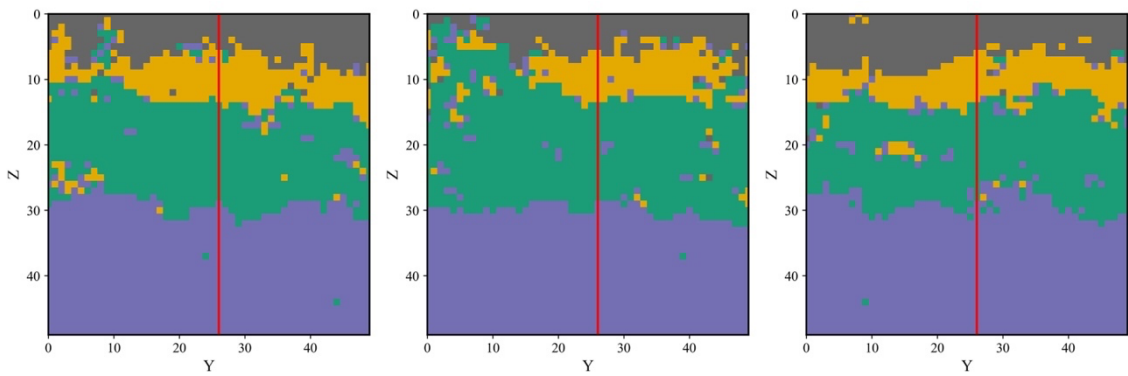
(a)



315

316

(b)



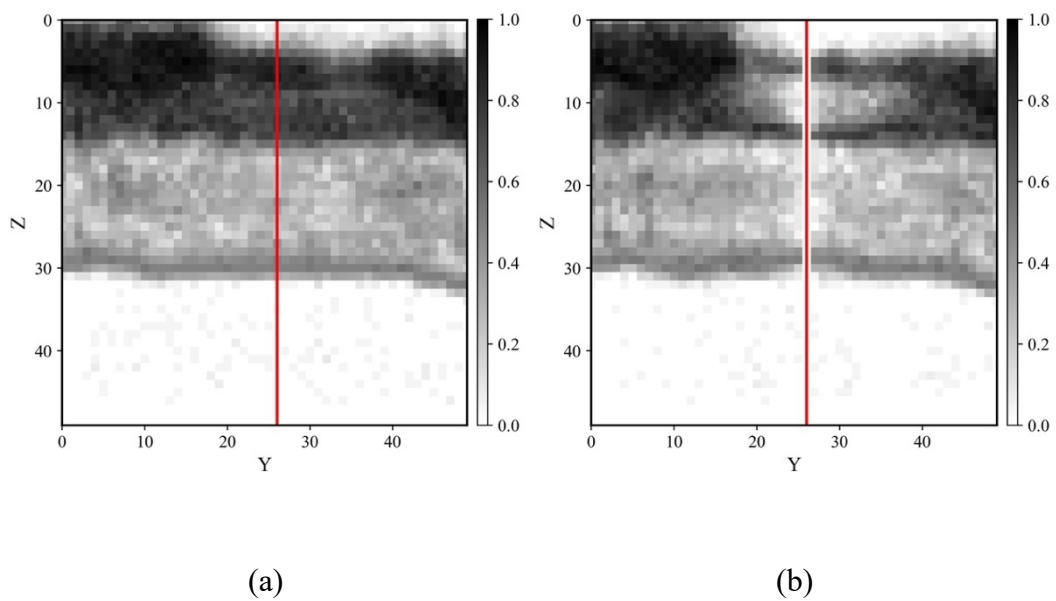
317

318

(c)

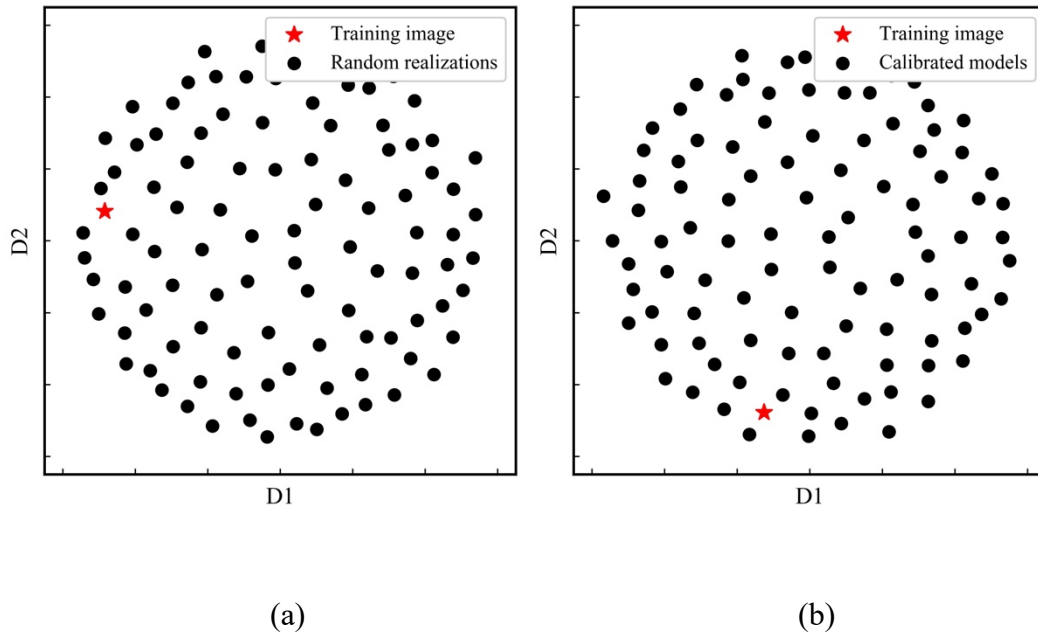
319 Fig. 14 Training image (a), three unconditional/random realizations (b) and three
320 conditional/calibrated realizations (c) of the cross section at $X = 126$. The red line represents
321 the well log location where facies are assumed to be known.

322 The entropy map of the cross section for 100 unconditional realizations and conditional
323 realizations is shown in Fig. 15. As constrained by the observed data, the entropy value at the
324 well location is 0, meaning no uncertainty.



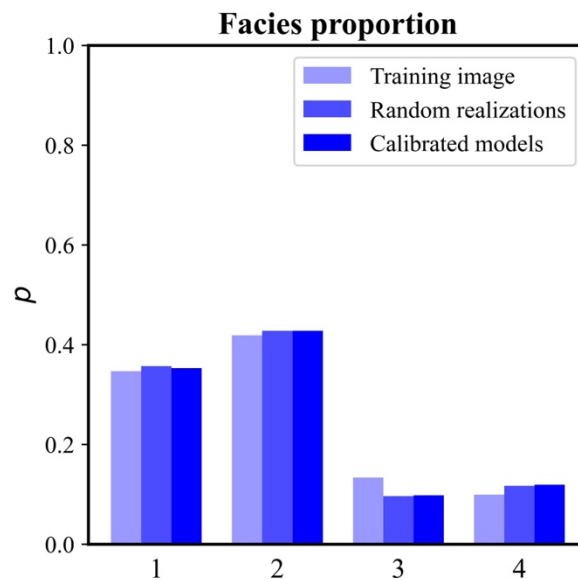
327 Fig. 15 Entropy map of the cross section at $X = 126$ for 100 random realizations (a) and 100
328 calibrated realizations using GDM (b).

329 The MDS plot of the unconditional/random realizations and conditional/calibrated realizations
330 is shown in Fig. 16. The training image is overlapped with the unconditional realizations and
331 calibrated realizations in the low-dimensional space, indicating a good similarity between them.



334 Fig. 16 MDS plot of the unconditional/random realizations (a) and conditional/calibrated
 335 realizations (b), together with the training image.

336 The histogram of the facies proportion is displayed in Fig. 17. The unconditional/random
 337 realizations and conditional/calibrated realizations show similar facies proportions with the
 338 ones of the training image.



340 Fig. 17 Facies proportions of training image, random realizations and calibrated models.

341 **4 Discussion**

342 This work presents a novel approach to train generative models using a single training image,
343 instead of a large quantity of training images required in other studies. Thus, the traditional
344 geostatistical methods are not needed for building the training data. The proposed GANs are
345 applied to simulate the geological facies models in 2D and 3D applications. A cutoff operation
346 with different threshold values is applied to the output of the last layer of the generator to obtain
347 categorical variables. In principle, the one-hot encoding scheme could be used, leading to
348 increased computational cost. Compared to traditional geostatistical methods such as the single
349 normal equation simulation requiring stationary training images (Strebelle, 2002), the proposed
350 GANs can simulate more complex geological models, such as those based on geological
351 processes without any stationary assumption. Moreover, in this study, a categorical random
352 variable representing geological facies is applied; however, the proposed GANs can be
353 extended to simulate continuous variables such as rock and fluid properties in subsurface
354 models by changing the activation function in the last layer of the generator accordingly.

355 The gradual deformation method is applied to calibrate the random realizations generated by
356 trained GANs to observed data, as a result of an optimization process. Compared to the
357 unconditional realizations, the conditional/calibrated realizations by GDM can better preserve
358 the geological structures by honoring direct measurements as shown in the presented
359 applications (Figs. 8, 9, 14 and 15). However, conditioning/calibrating the models to observed
360 data might require additional computational time. Alternative to the GDM, conditioning to the
361 observed data can be directly performed in the simulation process by the trained generator, in
362 which an extra loss function should be pre-defined to account for the content loss (Yeh et al.,
363 2017). Moreover, the conditional probabilities or indirect measurements (i.e. soft data) can be

364 integrated in the simulation by adopting the nu/tau model (Polyakova and Journel, 2007;
365 Krishnan, 2008), and the softmax activation function should be used in the last layer of the
366 generator for probabilities calculation.

367 **5 Conclusion**

368 We presented a new implementation of a deep learning model based on GANs to simulate
369 geological facies models in the subsurface. Compared to other applications of the GANs on the
370 geostatistical simulation, the proposed GANs do not require a large quantity of training data
371 for updating neural parameters, and only one single training image is sufficient. Thus, the
372 training of the proposed GANs does not rely on geostatistical methods to generate training data.
373 The training image is down-sampled to different scales, and the training of GANs is performed
374 in a multi-scale architecture, as the spatial statistical information from the training image is
375 progressively captured. The random realizations by the trained generators are then
376 conditioned/calibrated to observed data by gradually deforming the random noise maps using
377 the gradual deformation method. The proposed methodology is applied for unconditional and
378 conditional simulations to two case studies, in which a 2D river delta and a 3D aquifer analog
379 model are successfully modelled, as shown by the MDS plot and facies proportions. The
380 proposed methods can be extended to model continuous variables such as porosity and
381 permeability. Future research will focus on performing hard and/or soft data conditioning in
382 the simulation process directly with the trained generator, instead of using the gradual
383 deformation method.

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387 **Code availability**

388 The source code and trained model are available at <https://github.com/RhFeng/SGANs>

389 Name of Code: SGANs.

390 Developer: Runhai Feng.

391 Contact: runhai.feng@gmail.com.

392 Year first available: 2023.

393 Software: PyTorch 1.4

394 Program language: Python 3.6

395 **Authorship statement**

396 Runhai Feng developed the methodology and the code, and worked on the application; Dario

397 Grana provided scientific advising and helped with the scientific writing.

398 **Declaration of competing interest**

399 The authors declare that they have no known competing financial interests or personal

400 relationships that could have appeared to influence the work reported in this paper.

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403 **References**

404 Azevedo, L., Paneiro, G., Santos, A., Soares, A., 2020. Generative adversarial network as a
405 stochastic subsurface model reconstruction. *Comput. Geosci.* 24 (4): 1673–1692

406 Bayer, P., Comunian, A., Höyng, D., Mariethoz, G., 2015. High resolution multi-facies
407 realizations of sedimentary reservoir and aquifer analogs. *Scientific Data* 2: 150033.
408 <https://doi.org/10.1038/sdata.2015.33>

409 Caers, J., 2007. Comparing the gradual deformation with the probability perturbation method
410 for solving inverse problems. *Mathematical Geology* 39: 27–52

411 Chan, S., Elsheikh, A.H., 2019. Parametric generation of conditional geological realizations
412 using generative neural networks. *Comput. Geosci.* 23(5): 925–952

413 Chen, M., Wu, S., Bedle, H., Xie, P., Zhang, J., Wang, Y., 2022. Modeling of subsurface
414 sedimentary facies using Self-Attention Generative Adversarial Networks (SAGANs). *Journal*
415 *of Petroleum Science and Engineering.* 214: 110470

416 Cox, M., Cox, T., 2008. Multidimensional scaling. In: *Handbook of data visualization.* Springer
417 *handbooks Comp. statistics.* Springer, Berlin. https://doi.org/10.1007/978-3-540-33037-0_14

418 Dupont, E., Zhang, T., Tilke, P., Liang, L., Bailey, W., 2018. Generating realistic geology
419 conditioned on physical measurements with generative adversarial networks.
420 *arXiv:1802.03065*

421 Feng, R., Grana, D., Mukerji, T., Mosegaard, K., 2022. Application of Bayesian generative
422 adversarial networks to geological facies modeling. *Mathematical Geosciences* 54: 831–855

423 Goodfellow, I., Pouget-Abadie, J., Mirza, M., Xu, B., Warde-Farley, D., Ozair, S., Courville,
424 A., Bengio, Y., 2014. Generative adversarial networks. *arXiv:1406.2661*

425 Hu, L.Y., 2000. Gradual deformation and iterative calibration of Gaussian-related stochastic
426 models. *Mathematical Geology* 32(1): 87–108

427 Hu, L.Y., Blanc, G., Noetinger, B., 2001. Gradual deformation and iterative calibration of
428 sequential simulations. *Math. Geol.* 33: 475–489

429 Hu, L.Y., Le Ravalec-Dupin, M., 2004. An improved gradual deformation method for
430 reconciling random and gradient searches in stochastic optimizations. *Math. Geol.* 36(6): 703–
431 720

432 Hu, L.Y., Chugunova, T., 2008. Multiple-point geostatistics for modeling subsurface
433 heterogeneity: A comprehensive review. *Water Resources Research* 44: W11413

434 Hu, L.Y., Liu, Y., Scheepens, C., Shultz, A.W., Thompson, R.D., 2014. Multiple-point
435 simulation with an existing reservoir model as training image. *Math. Geosci.* 46: 227–240

436 Ioffe, S., Szegedy, C., 2015. Batch normalization: accelerating deep network training by
437 reducing internal covariate shift. *arXiv: 1502.03167*

438 Isola, P., Zhu, J.Y., Zhou, T., Efros, A.A., 2018. Image-to-image translation with conditional
439 adversarial networks. *arXiv: 1611.07004*

440 Kingma, D.P., Ba, J., 2014. Adam: A method for stochastic optimization. *arXiv: 1412.6980*

441 Krishnan, S., 2008. The Tau model for data redundancy and information combination in earth
442 sciences: Theory and Application. *Mathematical Geosciences* 40: 705–727

443 Laloy, E., Héroult, R., Jacques, D., Linde, N., 2018. Training-image based geostatistical
444 inversion using a spatial generative adversarial neural network. *Water Resour. Res.* 54(1): 381–
445 406

446 Li, C., Wand, M., 2016. Precomputed real-time texture synthesis with markovian generative
447 adversarial networks. European Conference on Computer Vision, 702–716

448 Mariethoz, G., Renard, P., Straubhaar, J., 2010. The direct sampling method to perform
449 multiple-point geostatistics simulations. Water Resources Research 46: W11536

450 Mariethoz, G. and Caers, J., 2014. Multiple-point geostatistics: stochastic modeling with
451 training images. John Wiley & Sons.

452 Mosser, L.J., Dubrule, O., Blunt, M.J., 2018. Conditioning of three-dimensional generative
453 adversarial networks for pore and reservoir-scale models. arXiv:1802.05622

454 Polyakova, E., Journel, A., 2007. The Nu expression for probabilistic data integration. Math.
455 Geol. 39(8): 715–733

456 Pyrcz, M.J., Deutsch, C.V., 2014. Geostatistical reservoir modeling. Oxford university press

457 Shaham, T.R., Dekel, T., Michaeli, T., 2019. SinGAN: learning a generative model from a
458 single natural image. arXiv:1905.01164v2

459 Shocher, A., Bagon, S., Isola, P., Irani, M., 2018. Ingan: Capturing and remapping the “DNA”
460 of a natural image. arXiv:1812.00231

461 Song, S., Mukerji, T., Hou, J., 2021. Bridging the gap between geophysics and geology with
462 generative adversarial networks. IEEE Trans Geosci Remote Sensing.
463 <https://doi.org/10.1109/TGRS.2021.3066975>

464 Strebelle, S., 2002. Conditional simulation of complex geological structures using multiple-
465 point statistics. Mathematical Geology 34(1): 1-21

466 Sun, A.Y., 2018. Discovering state-parameter mappings in subsurface models using generative
467 adversarial networks. *Geophysical Research Letters* 45: 11137–11146

468 Sun, C., Demyanov, V., Arnold, D., 2022. Geological realism in fluvial facies modelling with
469 GAN under variable depositional conditions. *Computational Geosciences*,
470 <https://doi.org/10.1007/s10596-023-10190-w>

471 Tran, T.T., 1994. Improving variogram reproduction on dense simulation grids. *Computers &*
472 *Geosciences* 20(7-8): 1161–1168

473 Yeh, R.A., Chen, C., Lim, T.Y., Schwing, A.G., Hasegawa-Johnson, M., Do, M.N., 2017.
474 Semantic Image Inpainting with Deep Generative Models. *Proceedings of the IEEE conference*
475 *on computer vision and pattern recognition*, 5485–5493

476 Zhang, C., Song, X., Azevedo, L., 2021. U-net generative adversarial network for subsurface
477 facies modelling. *Computational Geosciences* 25: 553–573

478 Zhang, T.F., Tilke, P., Dupont, E., Zhu, L.C., Liang, L., Bailey, W., 2019. Generating
479 geologically realistic 3D reservoir facies models using deep learning of sedimentary
480 architecture with generative adversarial networks. *Petroleum Science* 16: 541–549

481 Zhong, Z., Sun, A.Y., Jeong, H., 2019. Predicting CO₂ plume migration in heterogeneous
482 formations using conditional deep convolutional generative adversarial network. *Water Resour.*
483 *Res.* 55(7): 5830–5851