

Coversheet for “Investigating the Relationship Between Sea Surface Temperature
and the Mechanical Efficiency of Tropical Cyclones”

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1 **Investigating the Relationship Between Sea Surface Temperature and the**
2 **Mechanical Efficiency of Tropical Cyclones**

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7 ABSTRACT: Previous studies have investigated how sea surface temperature (SST) affects the
8 potential intensity of tropical cyclones (TCs). However, this is an upper bound only on the
9 maximum near-surface azimuthal winds, and does not fully account for the effects of atmospheric
10 moisture. Potential intensity might not vary in the same way as the total kinetic energy (W_{KE})
11 of a TC would with changing SST. W_{KE} is related, via the conceptualization of the TC as a heat
12 engine, to TC mechanical efficiency. We investigate how TC mechanical efficiency varies with
13 SST in a series of moist, axisymmetric, radiative-convective numerical experiments with constant
14 SSTs ranging from 295K to 307.5K. We find a -2.1 \%K^{-1} decrease in the mechanical efficiency
15 with SST. While the increase in the net heat energy gained by the TC heat engine acts to increase
16 W_{KE} , the mechanical efficiency still decreases with SST due to the effects of moisture on W_{KE}
17 and on the total heat input to the TC. Moist convection in an unsaturated atmosphere is associated
18 with substantial irreversible entropy production, which detracts from the energy that the TC can
19 use to power its winds. The increasing moisture content in a warmer atmosphere predicted by
20 Clausius-Clapeyron scaling leads this irreversibility to increase in an unsaturated atmosphere,
21 presenting a larger penalty on W_{KE} and decreasing the mechanical efficiency. Our results highlight
22 the importance of giving full consideration to the effects of moisture on the TC heat engine in
23 studies of how climate affects TCs.

24 SIGNIFICANCE STATEMENT: The purpose of this study is to investigate how the “mechanical
25 efficiency” of tropical cyclones varies with sea surface temperature. This matters because the
26 conceptualization of the tropical cyclone as a heat engine implies that the kinetic energy of its
27 wind field is generated at this efficiency. The mechanical efficiency may be affected by changes in
28 climate. Our results demonstrate that the mechanical efficiency decreases at -2.1 \%K^{-1} with sea
29 surface temperature, and in particular highlight the important role of moisture in this result.

30 **1. Introduction**

31 Tropical cyclones (TCs) are often destructive and dangerous phenomena, frequently causing loss
32 of life and extensive property damage (Zhang et al. 2009; Klotzbach et al. 2018). A common
33 conceptual model of the TC in a steady state is that of the TC as a heat engine (Emanuel 1986;
34 Bister and Emanuel 1998). In this framework, energy is injected into the TC via latent and sensible
35 heat fluxes from the warm, moist ocean surface, and lost to the environment via radiative cooling,
36 primarily in the upper troposphere at a comparatively cool effective temperature. As with the
37 Carnot heat engine, this heat transport allows the TC heat engine to generate work that balances
38 the action of dissipation in steady state (Emanuel 1986; Bister and Emanuel 1998). Yet, unlike a
39 classical heat engine, the total heat input to the TC heat engine is the same as the total heat output
40 in steady state, which implies that the TC heat engine cannot generate any useful work. This is not
41 incompatible with the TC heat engine generating nonzero work, including the kinetic energy of its
42 wind field, because this constraint only applies to useful work that is used and dissipated externally
43 to the TC, as in the classical heat engine. However, in a steady-state TC all generated work is used
44 and dissipated inside the system (Bister et al. 2011; Bister and Emanuel 1998). This modified heat
45 engine framework has proved illuminating in studying the thermodynamics and energetics of TCs
46 (Pauluis 2016; Pauluis and Zhang 2017; Fang et al. 2019; Wang and Lin 2021). A major departure
47 from the Carnot engine conceptual model is the moisture-associated irreversibility inherent in a
48 real TC, which reduces the kinetic energy work that the TC heat engine can produce compared to
49 its upper bound. Nearly all of this irreversibility is associated with phase changes of water in an
50 unsaturated moist atmosphere and frictional dissipation associated with precipitation (Pauluis et al.
51 2000; Pauluis and Held 2002a,b; Pauluis 2011; Pauluis and Zhang 2017; Wang and Lin 2021).

52 The mechanical efficiency of a TC (Pauluis and Zhang 2017; Fang et al. 2019) is a property of
53 the TC heat engine, defined as the fraction of the total heat input that is available to drive the full
54 3D wind field, and may vary with climate change. Changes in the mechanical efficiency are set by
55 the combined variability of several energetic terms that reflect the impact of the distributions of
56 temperature and moist entropy in a TC on its energetics, the effects of irreversibility associated with
57 moisture on the kinetic energy produced by the TC heat engine, and the total heat input to the TC
58 heat engine. Although some studies have examined aspects of the variability of the TC mechanical
59 efficiency and related factors (e.g. Pauluis 2016; Pauluis and Zhang 2017; Fang et al. 2019; Wang
60 and Lin 2021), much about this remains poorly understood. In this paper, we investigate how
61 and why sea surface temperatures (SSTs), which are expected to generally increase with climate
62 change, affect the mechanical efficiency of mature TCs.

63 Hakim (2011) created the first phase plot of a TC in the classical temperature-entropy space of
64 heat engine diagrams, using the mean trajectory of a series of air parcels released at the point of
65 maximum wind of a simulated axisymmetric TC. More recently, Pauluis (2016) developed the Mean
66 Air Flow as Lagrangian Dynamics Approximation (MAFALDA), using the modified isentropic
67 averaging procedure developed by Mrowiec et al. (2016), which allows the comprehensive study
68 of TC mechanical efficiency using a moist, irreversible heat engine framework.

69 The question of how energetic terms related to the mechanical efficiency are affected by SSTs
70 has not been answered comprehensively for TCs. Such a study has been performed for the global
71 atmosphere (LaLiberté et al. 2015; Lembo et al. 2019; Knietzsch et al. 2015; Held and Soden
72 2006; Pan et al. 2017; Lucarini et al. 2010a,b) and for local atmospheric convection (Romps 2008;
73 Singh and O’Gorman 2016; Pauluis 2016). Khairoutdinov and Emanuel (2013) study how the
74 kinetic energy per unit surface area of an individual TC’s wind field varies across a series of
75 “TC-world” experiments with different SSTs. Other aspects of the thermodynamic behavior of
76 TCs, such as the variability of their total kinetic energy and mechanical efficiency as the TCs
77 intensify, have been studied using MAFALDA (Fang et al. 2019; Mrowiec et al. 2016; Pauluis and
78 Zhang 2017; O’Neill and Chavas 2020; Li et al. 2023). This approach involves constructing cyclic
79 thermodynamic trajectories that are assumed to represent a set of mean air parcel trajectories in the
80 TC (Pauluis 2016). These trajectories range from thermodynamically very efficient trajectories that
81 pass through the TC’s eyewall (similar to those in Hakim (2011)) to very inefficient trajectories,

82 which represent convection that occurs beyond the eyewall in rainbands. Previous studies mainly
83 focus on one or two thermodynamic trajectories in a TC to perform these calculations, typically
84 the most thermodynamically efficient trajectory. Relatively little attention has been paid to the
85 behavior of all trajectories. Wang and Lin (2021) quantified a TC mechanical efficiency that
86 effectively accounts for all trajectories via an entropy budget, and derived a relationship between
87 the mechanical efficiency, the degree of irreversibility, and the compactness of the TC, where the
88 latter is the focus of their study. The authors briefly discussed how changes in SST might affect
89 the upper limit of the mechanical efficiency, defined according to classical TC heat engine theory
90 (Emanuel 1986; Bister and Emanuel 1998). A mechanical efficiency effectively accounting for
91 all trajectories was also defined by both Singh and O’Neill (2022) and Pan et al. (2017), although
92 these studies focused on localized convection and the global atmospheric circulation, respectively.

93 In this paper, we apply MAFALDA to a series of simulations with a range of constant SSTs to
94 determine how SST affects the mechanical efficiency of TCs. The paper is organized as follows:
95 in section 2 we describe our model setup, definitions of terms, and analysis procedure. We present
96 our results in section 3, including both analyses of the whole TC circulation and separate analyses
97 of the overturning circulation passing through the TC eyewall and of the TC rainband circulations.
98 Finally, in section 4 we discuss our results in the context of previous studies and present some
99 conclusions.

100 **2. Experimental Design and Method**

101 *a. Experimental Design*

102 *(i) Axisymmetric Simulations* For this study we use the Bryan Cloud Model 1 (CM1) (Bryan
103 and Fritsch 2002; Bryan and Rotunno 2009), version 20.2. CM1 is a non-hydrostatic model and
104 integrates the moist, compressible governing equations. Experiments are run on an f-plane at
105 20°N in a 1500 km × 25 km axisymmetric domain with a horizontal grid spacing of 1km. The
106 domain has a vertically stretched grid spacing (Wilhelmson and Chen 1982), ranging from 50m
107 at a height of 0km to 500m above 5.5km. Rayleigh damping layers are imposed in the top 3km
108 and outer 100km of the domain. CM1 is run in the mesoscale modeling setup with a Bryan and
109 Rotunno (2009) planetary boundary layer parametrization. The microphysics parametrization is the
110 Morrison double-moment moisture scheme with graupel as the large-ice category. The radiation

111 scheme is the RRTMG scheme. Following Bretherton et al. (2005), we remove the diurnal and
112 seasonal cycles from RRMTG by fixing the solar constant at 650.83 W m^{-2} and the solar zenith
113 angle at 50.5° . The model includes dissipative heating and conserves energy and mass. It accounts
114 for the heat capacity of hydrometeors in the equation set for moist microphysics (Bryan and Fritsch
115 2002), and includes vertical transport of energy associated with hydrometeor sedimentation, all of
116 which improve the energy conservation of the model.

117 We conduct a series of 200-day simulations of TCs in radiative-convective equilibrium (RCE)
118 with the lower boundary of the model in each set to a different, constant SST, ranging from 295K to
119 307.5K in steps of 0.5K. The model is initialized with a modified Rankine vortex¹ with a maximum
120 wind speed of 15 m s^{-1} at a radius of 75km, a radial decay parameter of -0.35 between radii of 75km
121 and 200km, and the radius of zero winds at 500km. The initial environmental state sounding for
122 each simulation was constructed using the horizontal- and time-mean vertical profiles of potential
123 temperature and water vapor mixing ratio from the last 20 days of a corresponding 100-day RCE
124 simulation, together with an imposed zero-wind profile for the initial state.

125 *(ii) RCE simulations* Each 100-day RCE simulation had an SST that matched that of the cor-
126 responding axisymmetric simulation. Settings for the RCE simulations generally matched those
127 of the axisymmetric simulations, except that the RCE simulations were conducted in a 120km x
128 120km doubly-periodic domain with a model top at 26km, 1km higher than for the axisymmetric
129 simulations. This was due to the CM1 requirement that input environmental soundings, such
130 as those constructed from these RCE simulations, must extend higher than the chosen model top.
131 Since the domain was doubly-periodic, no lateral Rayleigh damping layer was imposed. The upper-
132 level Rayleigh damping layer was 1km thicker than for the axisymmetric simulations, to ensure
133 that it began at the same level. All RCE simulations were initialized with the Dunion (2011) moist
134 tropical sounding. No initial perturbation was added to the domain. The Bryan and Rotunno (2009)
135 planetary boundary layer parametrization, which was used for the axisymmetric simulations, could
136 not be used for these simulations since it was developed for axisymmetry. Instead, the planetary
137 boundary layer parametrization in these simulations was the Yonsei University parametrization
138 (Hong et al. 2006).

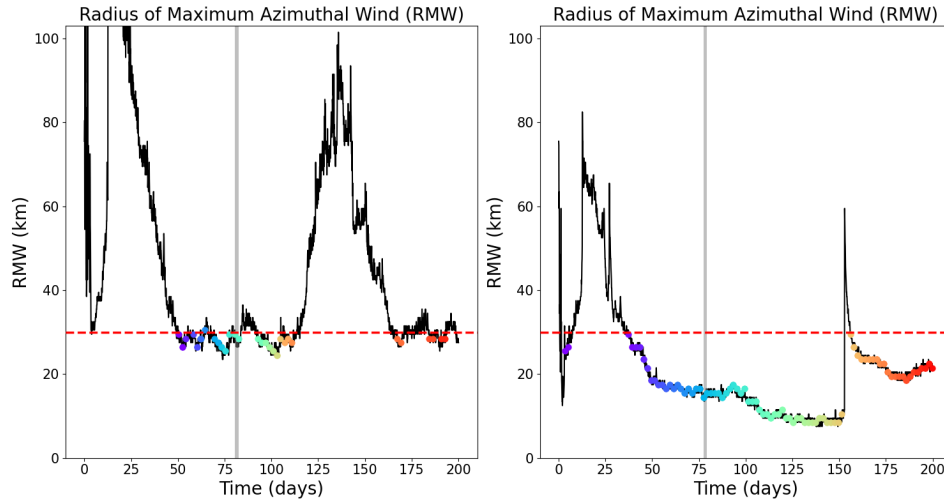
¹A Rankine vortex has $Vr^{-1}=\text{constant}$ inside the radius of maximum winds (RMW) and $Vr=\text{constant}$ outside the RMW. In a modified Rankine vortex, the latter relationship is modified to $Vr^X=\text{constant}$ (Holland 1980). In our simulations, X is set to 0.35 (i.e., the radial decay parameter in CM1 was -0.35).

139 (iii) *500 parcels simulation* In addition to the main axisymmetric simulations presented here,
140 we additionally run a simulation that is identical to the axisymmetric simulation with an SST of
141 299.5K and includes 500 Lagrangian air parcels. This simulation is run for 100 days in total and is
142 used to validate our analysis methodology, see section 2b. In this work, the SST of 299.5K is used
143 as a reference SST for quantifying changes in variables in $\% K^{-1}$, as described in section 3a, and
144 so was also selected as the SST for our parcel simulation.

145 *b. MAFALDA Procedure*

146 (i) *Selection of “single-eyewall” and “realistic” time periods* Because axisymmetric simulated
147 TCs can exhibit unphysical behavior (O’Neill and Chavas 2020), from each individual 200-day
148 simulation we selected for our main analyses all non-overlapping 2-day-long periods where the
149 radius of maximum azimuthal winds (RMW) did not exceed a threshold that we subjectively set to
150 30km. These are henceforth referred to as the “realistic” periods. The evolution of the RMW in
151 two of the constant-SST experiments is shown in Fig. 1, as well as both the realistic periods that
152 we selected from these simulations and the 30km threshold. This threshold was set based on our
153 observations of how the RMW evolved in the different simulations, which often included periods
154 where the RMW suddenly grew. During such periods, the TC was either undergoing an unrealistic,
155 axisymmetric version of an eyewall replacement cycle (ERC), or was spinning up. The 30km
156 threshold was chosen to exclude these periods while still yielding a nonzero number of realistic
157 time periods for all experiments. We excluded the ERC and spinup periods in order to study
158 approximately steady-state TCs, and because the unrealistic ERCs may influence the energetics of
159 the TCs.

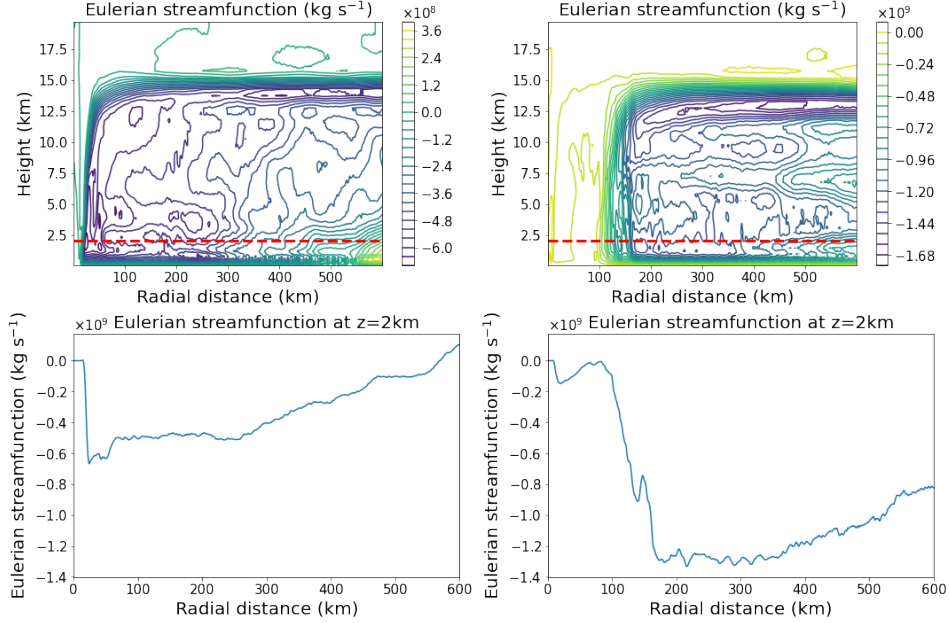
160 The analysis subdomain sometimes contained strong secondary eyewalls during the realistic
161 periods. From the set of 2-day realistic periods for each simulation, we selected for additional
162 analyses (see Figs. S1-S3 and Tables S1-S3 in the online Supplemental Material) one of these as
163 the “single-eyewall” period, where there were no strong secondary eyewalls in the inner subdomain
164 in which we performed our analysis, denoted by the gray shading in Fig. 1. This subdomain will
165 be discussed later. To identify the single-eyewall periods, for each realistic period, we calculated
166 the radius at which the minimum value of the time-mean streamfunction at a height of 2km
167 occurred within the analysis subdomain. The single-eyewall period was the period during which



179 FIG. 1. Selected 2-day “realistic” (color-coded, pairs of rainbow points) and “single-eyewall” time periods
 180 (gray shading) for the simulations with an SST of 295K (left) and 305K (right). The $RMW=30\text{km}$ threshold is
 181 denoted by the dotted red line. In both plots, the radius of maximum azimuthal wind at a height of 10m (RMW)
 182 is shown in black.

183
 184 this minimum occurred at the smallest radius, on the condition that this radius was less than 100km
 185 (Fig. 2). The TC’s time-mean eyewall at this height, which is associated with net upward vertical
 186 mass flux, should not extend further than the radius where the minimum in the time-mean Eulerian
 187 streamfunction occurs. As such, this radius presents an upper bound on the radial extent of the outer
 188 edge of the eyewall at this height. We implemented this restriction at a height above the boundary
 189 layer, so that this constraint would act at a height where the primary eyewall was present. Given
 190 this, we arbitrarily chose a height of 2km. This methodology provided an additional restriction that
 191 ensured that the strongest eyewall in the TC was the primary eyewall. As expected, the selected
 192 single-eyewall periods contained no strong secondary eyewalls in the analysis subdomain, whereas
 193 very strong secondary eyewalls existed in some realistic periods, which can be seen by inspection
 194 of the time-mean Eulerian streamfunctions (Fig. 2).

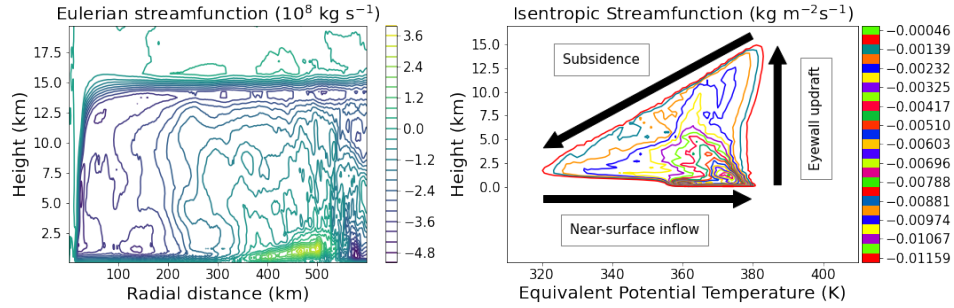
194 (ii) *Reconstruction of thermodynamic cycles and mechanical efficiency derivation* We analyzed
 195 the output data from our simulations during both the realistic and single-eyewall sets of 2-day
 196 time periods using the Mean Air Flow As Lagrangian Dynamics Approximation (MAFALDA).
 197 MAFALDA was initially developed by Pauluis (2016) for generalized convection, and was further
 198 developed for TCs by Mrowiec et al. (2016). MAFALDA involves four steps (Pauluis 2016):



183 FIG. 2. Top: Time-mean Eulerian streamfunctions, calculated based on the density of dry air and the vertical
 184 and radial wind fields, for the single-eyewall period from the 305K simulation (left) and for a different, realistic
 185 time period from the same simulation. A height of 2km is indicated by the dashed red line on both plots. In
 186 this example, the TC contains a strong secondary eyewall during the realistic period that dominates over the
 187 primary eyewall, whereas it contains no strong secondary eyewalls during the single-eyewall period. Bottom:
 188 Corresponding values of the time-mean Eulerian streamfunctions at a height of 2km as a function of radius.
 189 Comparing the upper and lower plots demonstrates that, as expected, the radius where the minimum of the time-
 190 mean Eulerian streamfunction occurs acts as an upper bound on the radial extent of the strongest eyewall’s outer
 191 edge at this height. For the single-eyewall periods, the minimum value in the time-mean Eulerian streamfunction
 192 at a height of 2km must occur at a radius of less than 100km. Given this condition, we pick the time period
 193 where this minimum occurs at the smallest radius.

- 208 1. Compute the TC’s isentropic streamfunction, $\psi(z, \theta_e)$ ² (Fig. 3): the time-average of the net
 209 upward mass flux binned by the equivalent potential temperature value at each altitude (Pauluis
 210 and Mrowiec 2013; Mrowiec et al. 2016). For this, we compute the isentropic integral for an
 211 axisymmetric setup, where the differential area is equal to $r dr d\vartheta$. Here, r and ϑ represent the
 212 radial and azimuthal coordinates, respectively. We also integrate radially outward from the
 213 TC center to an outer radius R_{max} (the “subdomain”), which is equal to 600km for this study.

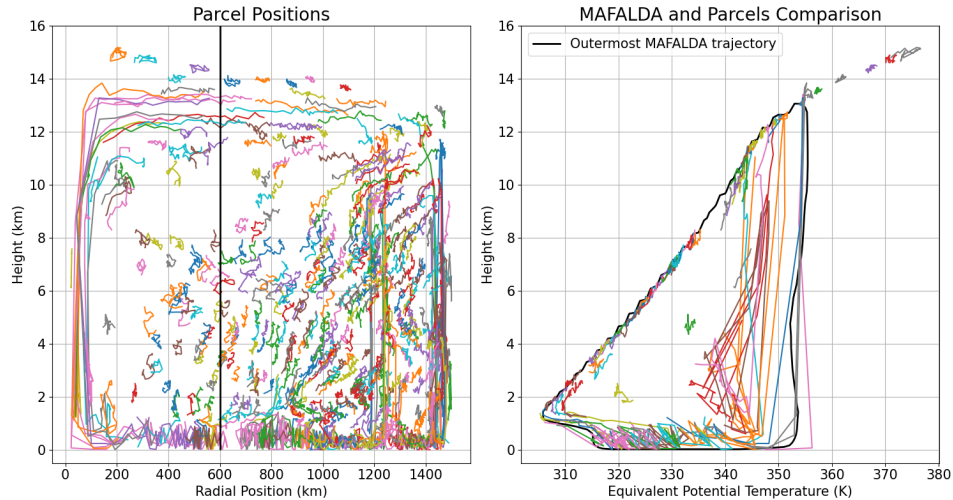
²We refer to ψ as the “isentropic” streamfunction following the standard terminology in the literature. However, it does not represent constant-entropy conditions.



199 FIG. 3. Comparison of the time-mean Eulerian streamfunction (left), calculated based on the density of dry
 200 air and the vertical and radial wind fields; and the isentropic streamfunction, ψ (right) for the single-eyewall
 201 time period of the 305.5K simulation. Black arrows in the right-hand plot show the direction of the overturning
 202 circulation that passes through the eyewall in z - θ_e space. For the right-hand plot, each trajectory is color-coded
 203 according to the corresponding level of the isentropic streamfunction. Each of the levels of the isentropic
 204 streamfunction is associated with at least one “MAFALDA trajectory”, which for many of the levels is quite
 205 large, as can be seen in the right-hand plot. However, the same plot shows that, due to the complexity of the TC
 206 circulation, some levels of ψ that contain one large MAFALDA trajectory sometimes also contain one or more,
 207 typically small, MAFALDA trajectories, as described in Section 2.

214 Since the TCs in the experiments with lower SSTs were smaller than at higher SSTs (see Fig.
 215 S4 in the online supplemental material), this constant radius was subjectively chosen so that
 216 it was not so large that it included too much of the environmental, or non-TC, circulation for
 217 the smallest storms and not so small that it missed important dynamics in the largest storms
 218 (see Text S1 in the online supplemental material).

- 219 2. Find the isentropic-mean values of thermodynamic variables, $\tilde{f}(z, \theta_e)$, defined as the mass
 220 weighted conditional average of the variable f .
- 221 3. Construct a set of N cyclic “trajectories” in z - θ_e coordinates from ψ , henceforth referred to as
 222 the “MAFALDA trajectories”. These are the closed loops in z - θ_e space corresponding to the
 223 individual levels of ψ when plotted as a contour plot with N levels. There may be more than
 224 one MAFALDA trajectory for a given level of the streamfunction (Fig. 3). While MAFALDA
 225 trajectories account for the mean flow in z - θ_e coordinates, they do not necessarily capture
 226 the trajectory of any individual air parcel. However, we implicitly assume this in using
 227 MAFALDA (Pauluis and Mrowiec 2013; Pauluis 2016). Figure 4 presents a comparison



239 FIG. 4. Left: Trajectories of all 500 parcels during a 2-day period from an 100-day simulation with the same
 240 setup as the main simulation with an SST of 299.5K. Each parcel trajectory is color-coded. The black vertical
 241 line indicates the radial extent of the analysis subdomain. Right: Trajectories of parcels within the analysis
 242 subdomain only, during the chosen 2-day period, cast into height-equivalent potential temperature space. Each
 243 parcel trajectory is color-coded. The black line indicates the outermost MAFALDA trajectory derived from the
 244 isentropic streamfunction that was calculated during this period.

228 between the trajectories of Lagrangian air parcels in $z-\theta_e$ space and the largest MAFALDA
 229 trajectory derived from a 2-day period in the simulation with 500 parcels. This 2-day period
 230 met the requirements applied to select the realistic periods from the main simulations. For
 231 this comparison, we used only the trajectories of air parcels that were within the analysis
 232 subdomain ($r \leq R_{max}$) during this time. By visual inspection, the MAFALDA trajectories
 233 derived from the isentropic streamfunction provide a good approximation to the trajectories
 234 of real air parcels in the TC.

- 235 4. Interpolate the values of the thermodynamic variables, f , along the MAFALDA trajectories
 236 using the isentropic-mean variables, $\tilde{f}(z, \theta_e)$, as a starting point. This results in a “timeseries”
 237 of these variables following along each MAFALDA trajectory. These “timeseries” can be
 238 used to cast ψ into other thermodynamic spaces, for example temperature-entropy space.

245 MAFALDA allows us to quantify the mechanical efficiency of simulated TCs, as a function of
 246 energy reservoirs within the system. These include:

- 247 1. W_{Max} , equivalent to the useful work that a Carnot engine would produce if it were operating
248 in equivalent temperature and entropy conditions (Pauluis 2016). It can also be thought of as
249 the net heat energy gained by the TC including all heat sources and sinks (Fang et al. 2019),
250 which can then be used either as kinetic energy of the TC’s wind field or may be consumed
251 by moist processes in the TC.
- 252 2. W_{KE} , the total kinetic energy generated along the MAFALDA trajectories. The net heat energy
253 gained by the TC W_{Max} acts as a maximum bound on W_{KE} . The generated W_{KE} is dissipated
254 by viscosity along the same trajectory that generates it. In a steady state, the generation of
255 W_{KE} and its dissipation by viscosity are equal.
- 256 3. W_{Moist} , a two-part energetic penalty on W_{KE} due to the presence of moisture in a TC (Pauluis
257 2016; Pauluis and Zhang 2017) that acts to reduce W_{KE} compared to its maximum bound, the
258 net heat energy gained by the TC W_{Max} . W_{Moist} is the sum of W_P and ΔG , and can be thought
259 of as relating approximately to the irreversible entropy production from moist processes in the
260 TC.
- 261 4. W_P , the energy that is used to lift water mass from where it enters the TC system—typically
262 the surface—to the level where it leaves the system as precipitation. This term can be
263 conceptualized as relating to irreversible entropy production from precipitation-associated
264 dissipation in the TC (Pauluis et al. 2000; Pauluis and Held 2002a,b).
- 265 5. ΔG , the Gibbs penalty, representing the energetic cost of maintaining the TC hydrological
266 cycle and associated phase transitions in subsaturated or supersaturated conditions (Pauluis
267 2011; Pauluis and Zhang 2017). The main contributor to ΔG is the entropy production by
268 irreversible evaporation. Although the remainder of ΔG does not relate directly to moist
269 irreversible entropy production, this term can still be thought of as approximately relating
270 to irreversible entropy production due to the phase transitions of water in subsaturated or
271 supersaturated conditions (Pauluis and Held 2002a,b).

272 All quantities are in J kg^{-1} , measured as per unit mass of dry air circulating in each “cycle”, or
273 level, of the isentropic streamfunction.

274 In order to construct a mechanical efficiency—and related energetic quantities—that reflect the
275 energetics of the TC’s whole circulation in the analysis subdomain, we must begin by calculating

276 these variables for each MAFALDA trajectory separately. For a given MAFALDA trajectory,
 277 j , we obtain “timeseries” of thermodynamic quantities (e.g. the temperature \tilde{T}) following each
 278 trajectory. Given the set of timeseries derived for a specific trajectory, j , we can calculate the
 279 energetic quantities as in Pauluis (2016):

$$\begin{aligned}
 W_{Max,j} &= \oint \tilde{T} d\tilde{s} \\
 \Delta G_j &= - \oint \sum_{w=v,l,i} \tilde{g}_w d\tilde{r}_w \\
 W_{P,j} &= \oint \Gamma \tilde{r}_t d\tilde{z} \\
 W_{KE,j} &= - \oint \tilde{\alpha}_d d\tilde{p} - W_{P,j}
 \end{aligned} \tag{1}$$

280 T is the absolute temperature, s is the moist entropy per unit mass of dry air, α_d is the specific
 281 volume per unit mass of dry air, p is the pressure, r_v , r_l , and r_i are the mixing ratios of water in its
 282 vapor, liquid, and solid phases, respectively, and g_v , g_l , and g_i are the specific Gibbs free energies
 283 of water in its vapor, liquid, and solid phases, respectively. Γ is the acceleration due to gravity
 284 and $r_t = r_v + r_l + r_i$ is the total mixing ratio of water. All thermodynamic variables are defined in
 285 Appendix A.

286 The first and second laws of thermodynamics can be combined to derive the Gibbs relationship
 287 (Pauluis 2011, 2016):

$$T ds = dh - \alpha_d dp - \sum_{w=v,l,i} g_w dr_w, \tag{2}$$

288 where h is the total enthalpy per unit mass of dry air, defined as in Pauluis and Zhang (2017).

289 Based on this relationship, the energetic quantities can be related to each other as follows:

$$W_{Max,j} = W_{KE,j} + W_{Moist,j}. \tag{3}$$

290 where

$$W_{Moist,j} = W_{P,j} + \Delta G_j \tag{4}$$

291 The kinetic energy generated along a trajectory, $W_{KE,j}$, is simultaneously dissipated by viscous
 292 forces. This dissipation can be represented by a quantity $D_{v,j} = \oint d\tilde{d}_v$, where d_v is the dissipation

293 per unit mass of dry air (J/kg). Assuming the dissipation occurs mostly near the surface, the
 294 associated dissipation rate can be approximated by the bulk formula in Bister and Emanuel (1998)'s
 295 equation (6). Multiplying this by $\frac{1}{q_d}$, where q_d is the ratio of dry to total air mass, gives the
 296 dissipation rate per unit mass of dry air, which can then be used to calculate d_v . In steady state,
 297 $W_{KE,j} = D_{v,j}$.

298 Thus far, we have only defined the energetic quantities for a specific MAFALDA trajectory. We
 299 can now define energetic terms representing the whole circulation of the TC in the subdomain.
 300 We define the energetic quantities, X , for the TC circulation within our subdomain using the mean
 301 values of these quantities over all N levels of ψ :

$$X = \frac{\sum_{l=0}^N \sum_{j=0}^{M_l} X_{j,l}}{N} \quad (5)$$

302 Here, l represents each of the N levels of the isentropic streamfunction. As mentioned previously,
 303 due to the complexity of the TC circulation, several MAFALDA trajectories, j , may belong to the
 304 same level of the isentropic streamfunction (Fig. 3). Simply taking the mean over all MAFALDA
 305 trajectories would then weight levels of ψ containing more than one trajectory more heavily than
 306 others. Because, by construction, each level of ψ represents the same fraction of the mass transport
 307 (Pauluis 2016), we calculate the sum of each energetic quantity over all trajectories at a given level
 308 of ψ before taking the mean of each energetic variable over all levels.

309 Following (3) and (4), we can relate these mean energetic quantities to each other as (Pauluis
 310 2016; Fang et al. 2019):

$$W_{Max} = W_{KE} + W_{Moist}. \quad (6)$$

311 where

$$W_{Moist} = W_P + \Delta G \quad (7)$$

312 MAFALDA additionally allows us to calculate the mean heat input and output to the TC circu-
 313 lation, Q_{in} and Q_{out} , defined as:

$$Q_{in} = \frac{\sum_{l=0}^N \sum_{j=0}^{M_l} Q_{in,j,l}}{N}$$

$$Q_{out} = \frac{\sum_{l=0}^N \sum_{j=0}^{M_l} Q_{out,j,l}}{N}$$

314 where:

$$d\tilde{q} = \tilde{T}d\tilde{s} + \sum_{w=v,l,i} \tilde{g}_w d\tilde{r}_w$$

$$Q_{in,j} = \oint d\tilde{q} \Big|_{d\tilde{q}>0}$$

$$Q_{out,j} = \oint d\tilde{q} \Big|_{d\tilde{q}<0}$$

315 Here, $d\tilde{q}$ represents the external heating of an air parcel. Q_{in} and Q_{out} account for all sources
 316 and sinks, respectively, of heat in the TC, including heating or cooling from phase transitions of
 317 water vapor and from surface heat flux. Finally, we define the mechanical efficiency, η_{mech} , of the
 318 TC following Pauluis and Zhang (2017):

$$\eta_{mech} = \frac{W_{KE}}{Q_{in}}, \quad (8)$$

319 Pauluis and Zhang (2017) demonstrated that the Carnot efficiency is an upper bound on η_{mech} .
 320 The Carnot efficiency is defined as:

$$\eta_C = \frac{T_{in} - T_{out}}{T_{in}}, \quad (9)$$

321 T_{in} and T_{out} are, respectively, the mean temperatures at which heat enters and leaves the TC
 322 system. They are defined based on Q_{in} and Q_{out} , following Pauluis and Zhang (2017):

$$T_{in} = \frac{Q_{in}}{\bar{T}_{in}},$$

323 where:

$$\frac{Q_{in}}{T_{in}}|_j = \oint \frac{d\tilde{q}}{\tilde{T}} \Big|_{\frac{d\tilde{q}}{\tilde{T}} > 0},$$
$$\frac{Q_{in}}{T_{in}} = \frac{\sum_{l=0}^N \sum_{j=0}^{M_l} \frac{Q_{in}}{T_{in}}|_{j,l}}{N},$$

324 and T_{out} is defined similarly.

325 Henceforth, unless stated otherwise, all MAFALDA-derived energetic variables are calculated
326 in J kg^{-1} . In some instances, we additionally calculate MAFALDA-derived energetic variables, X ,
327 in W m^{-2} , defined as (Pauluis 2016):

$$X(\text{Wm}^{-2}) = \sum_{l=0}^N \sum_{j=0}^{M_l} \Delta\psi_l X_{j,l}, \quad (10)$$

328 where $\Delta\psi_l$ is the mass circulation ($\text{kg m}^{-2} \text{s}^{-1}$) at each level of ψ , defined as follows (Pauluis
329 2016):

$$\Delta\psi_l = -N^{-1}\psi_{min}, \quad (11)$$

330 where ψ_{min} is the minimum value of ψ . $\Delta\psi_l$ has the same value for all levels by construction.

331 When using MAFALDA, isolating the thermodynamic cycles associated with the TC from cycles
332 associated with convection far from the center requires a choice of outer radius, which defines the
333 subdomain in which we reconstruct these cycles. For this study, the outer radius was 600km (see
334 Text S1, Table S4, and Fig. S4 in the online supplemental material). We checked the sensitivity
335 of the results derived from the realistic and single-eyewall periods to the outer radius, considering
336 values between 400km and 1200km (see Text S1 and Tables S5-S14 in the online supplemental
337 material). In general, the signs and statistical significance of trends in the variables with increasing
338 SST are not sensitive to the choice of outer radius. In the following two sections, we will note
339 instances where the sign or statistical significance of a trend is not robust to the choice of outer
340 radius.

341 3. Results

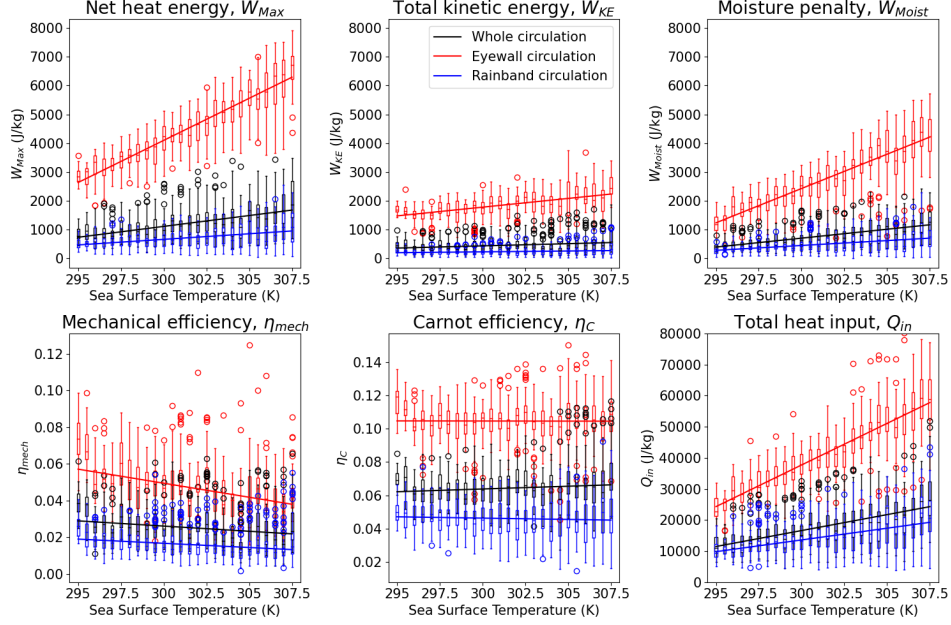
342 a. Variability of the MAFALDA-derived variables for the whole TC circulation

343 We first examine how the mechanical efficiency of the whole TC circulation within the subdomain
344 varies with SST, as well as the variability of related energetic quantities. These results are presented
345 as black boxplots and trend lines in figures 5 and 6. For each boxplot, the unfilled circles represent
346 outlier points, which are separated from the median value by more than 1.5 interquartile ranges.
347 Only the results from the realistic periods will be discussed in the remainder of this text—results
348 from the single-eyewall periods are presented in the online supplemental material. Henceforth,
349 all trends are quantified in units of $\% \text{ K}^{-1}$, calculated for the realistic periods as percentages of
350 the mean value of each variable at a reference SST of 299.5K. This SST is closest to the most
351 recently-published global SST threshold for TC genesis (26.4°C) (Defforge and Merlis 2017).

352 W_{KE} increases with SST at $3.4 \% \text{ K}^{-1}$ (Fig. 5, Table 1). Accompanying this, the net heat energy
353 gained by the TC, W_{Max} , increases at a higher rate of $7.0 \% \text{ K}^{-1}$. We also observe a strong increase
354 in moist processes with SST, associated with the increase in atmospheric moisture implied by
355 Clausius-Clapeyron scaling at constant relative humidity of about $7 \% \text{ K}^{-1}$ (Trenberth et al. 2003).
356 The related increased irreversibility is reflected by an increase in the associated energetic penalty,
357 W_{Moist} , of $9.6 \% \text{ K}^{-1}$. Because of this strong increase, W_{Moist} represents a larger portion of W_{Max}
358 at higher SST. This reduces the magnitude of the $3.4 \% \text{ K}^{-1}$ increase in W_{KE} compared to the 7.0
359 $\% \text{ K}^{-1}$ increase in its maximum bound, the net heat energy gained by the TC, W_{Max} .

360 The $9.6 \% \text{ K}^{-1}$ W_{Moist} increase exceeds the Clausius-Clapeyron scaling of $7 \% \text{ K}^{-1}$ that applies
361 to the atmospheric water content. This is because this energetic penalty is not only dependent
362 on changes in atmospheric moisture content: it is also dependent on changes in the depth of the
363 circulation, and changes in the Gibbs free energy of water in all its phases—as can be seen by the
364 definitions of W_P and ΔG . Changes in relative humidity may also play a role in the magnitude of
365 the increase.

366 Additionally, both components of W_{Moist} , W_P and ΔG , increase with SST, at $12 \% \text{ K}^{-1}$ and 8.2%
367 K^{-1} , respectively (Fig. 6, Table 1). Since W_P is related to the precipitation in the TC, the increase
368 in W_P may reflect an increase in TC precipitation with SST.



352 FIG. 5. Top: W_{Max} , W_{KE} , and W_{Moist} versus SST. Bottom: η_{mech} , η_C , and Q_{in} versus SST. Results are
 353 derived from the set of realistic periods. Several realistic periods were chosen for each simulation, as described in
 354 Section 2b. The boxplots in this figure show the spread of the results calculated for all realistic periods from each
 355 simulation. Black boxplots and trend lines are for the whole circulation of the TCs, red boxplots and trend lines
 356 are for the eyewall circulation only, and blue boxplots and trend lines are for the rainband circulation only. For
 357 each boxplot, the unfilled circles represent outlier points. Outlier points are identified as those that are separated
 358 from the median value by more than 1.5 interquartile ranges.

383 In spite of the increase in W_{KE} with SST, the mechanical efficiency decreases at $-2.1\% \text{ K}^{-1}$ (Fig.
 384 5, Table 1). This is due to the increase in the heat input Q_{in} , which grows at $6.3\% \text{ K}^{-1}$. Although
 385 the increase in W_{KE} acts to increase the mechanical efficiency, the increase in Q_{in} acts to decrease
 386 it, as can be seen from equation (8). Here, the effect of the increase in the heat input dominates,
 387 resulting in a decreased mechanical efficiency.

388 In contrast, the Carnot efficiency, η_C , increases with SST at a rate of $0.53\% \text{ K}^{-1}$ ³ (Fig. 5, Table
 389 1). The increase in η_C is due to the increase in $T_{in}-T_{out}$ of $0.86\% \text{ K}^{-1}$ ⁴, which dominates over the
 390 $0.32\% \text{ K}^{-1}$ increase in the input temperature, T_{in} (Table 2, Fig. 7). Because η_{mech} decreases while

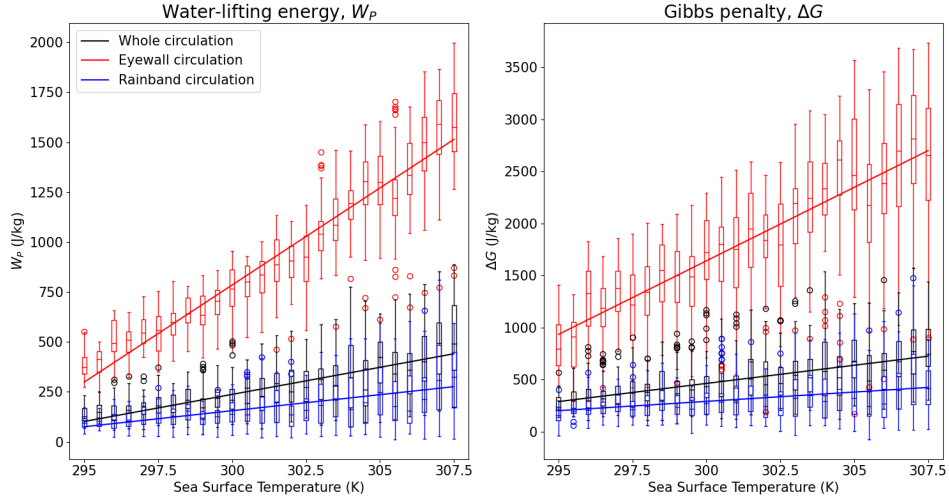
³The Carnot efficiency trend changes sign at large outer radius values, and its statistical significance is somewhat sensitive to outer radius (Table S5)

⁴The $T_{in}-T_{out}$ trend changes sign at large outer radius values, and its statistical significance is somewhat sensitive to outer radius (Table S5)

359 TABLE 1. Trends for the MAFALDA-derived energetic and efficiency variables during all realistic periods.
 360 Values are shown for the whole circulation of the TCs, as well as for only the eyewall and rainband circulations.
 361 All linear trend values are statistically significantly different from zero at the 99 % confidence level except for
 362 the eyewall η_C trend, which is not statistically significant, and the rainband η_C , which is statistically significantly
 363 different from zero at only the 90 % confidence level. To determine this, we used a two-sided t-test, using the
 364 HC1 estimator (MacKinnon and White 1985) to correct for heteroskedasticity (Text S3 in the online supplemental
 365 material).

Variable	Circulation type	Trend (% K ⁻¹)
W_{Max}	Whole	7.0
	Eyewall	7.8
	Rainband	6.8
W_{KE}	Whole	3.4
	Eyewall	3.5
	Rainband	2.8
W_{Moist}	Whole	9.6
	Eyewall	11
	Rainband	9.3
W_P	Whole	12
	Eyewall	14
	Rainband	12
ΔG	Whole	8.2
	Eyewall	9.7
	Rainband	7.5
Q_{in}	Whole	6.3
	Eyewall	7.4
	Rainband	5.9
η_{mech}	Whole	-2.1
	Eyewall	-3.2
	Rainband	-2.8
η_C	Whole	0.53
	Eyewall	-0.013
	Rainband	-0.37
$\frac{\eta_{mech}}{\eta_C}$	Whole	-2.7
	Eyewall	-3.0
	Rainband	-2.2

391 η_C increases with SST, the ratio $\frac{\eta_{mech}}{\eta_C}$ —the “relative efficiency”—decreases at -2.7 % K⁻¹ (Fig. 7,
 392 Table 1).



404 FIG. 6. W_P and ΔG versus SST. Results are derived from the set of realistic periods. Black boxplots and trend
 405 lines are for the whole circulation of the TCs, red boxplots and trend lines are for the eyewall circulation only,
 406 and blue boxplots and trend lines are for the rainband circulation only. For each boxplot, the unfilled circles
 407 represent outlier points. Outlier points are identified as those that are separated from the median value by more
 408 than 1.5 interquartile ranges.

393 *b. Effect of SST on the eyewall and rainband circulations*

394 In addition to our analysis of the mean behavior of energetic and efficiency quantities for the
 395 TC's whole circulation, we differentiate between the more efficient eyewall circulation and the
 396 circulation in the non-eyewall rainbands⁵ (Text S2). Because the TC eyewall is uniquely saturated
 397 and characterized by the deepest convection within the storm, this aspect of the TC circulation may
 398 respond quite differently to changes in SST compared to the rainband circulation. The eyewall
 399 circulation results are shown as red boxplots and trend lines in Figures 5-7, whereas the rainband
 400 circulation results are color-coded in blue. Changes in energetic and efficiency variables for the
 401 eyewall and rainband circulations are consistently of the same sign as changes for the whole
 402 circulation (Figs. 5-7, Table 1) and are highly significant, with the exception of η_C for both the
 403 eyewall and rainband circulations.

⁵While the equality in equation (3) holds well—to within 5%—for most of the time periods we examine, it occasionally holds to within only 10%. For the single-eyewall periods, this is the case for several MAFALDA trajectories belonging to levels of ψ corresponding to the rainband circulation for the 295K simulation, and for trajectories at one level belonging to the rainband circulation for the 296K simulation. For the realistic periods, (3) holds to within only 10% for the eyewall circulation in 5 of the 1835 total realistic periods examined across all simulations, and in one realistic period for both the whole and rainband circulations. Additionally, in one of the realistic periods, the equality does not hold for the eyewall circulation.

409 The $-3.2\% \text{ K}^{-1}$ decrease in the mechanical efficiency of the eyewall circulation is of a similar
410 size as the $-2.8\% \text{ K}^{-1}$ decrease in the mechanical efficiency of the rainband circulation (Fig. 5,
411 Table 1), although the absolute magnitude of the decrease for the eyewall circulation is larger, as
412 can be seen by inspection of Fig. 5. Accompanying these decreases, the Carnot efficiency for the
413 eyewall circulation does not statistically significantly change⁶, whereas the Carnot efficiency of the
414 rainband circulation decreases with a weak level of statistical significance at $-0.37\% \text{ K}^{-1}$ ⁷. Because
415 of this, the decrease in the relative efficiency is more pronounced for the eyewall circulation, at
416 $-3.0\% \text{ K}^{-1}$, than the $-2.2\% \text{ K}^{-1}$ decrease for the rainband circulation (Table 1). As a result, there
417 is a smaller difference between the relative efficiencies of the eyewall and rainband circulations at
418 higher SST (Fig. 7).

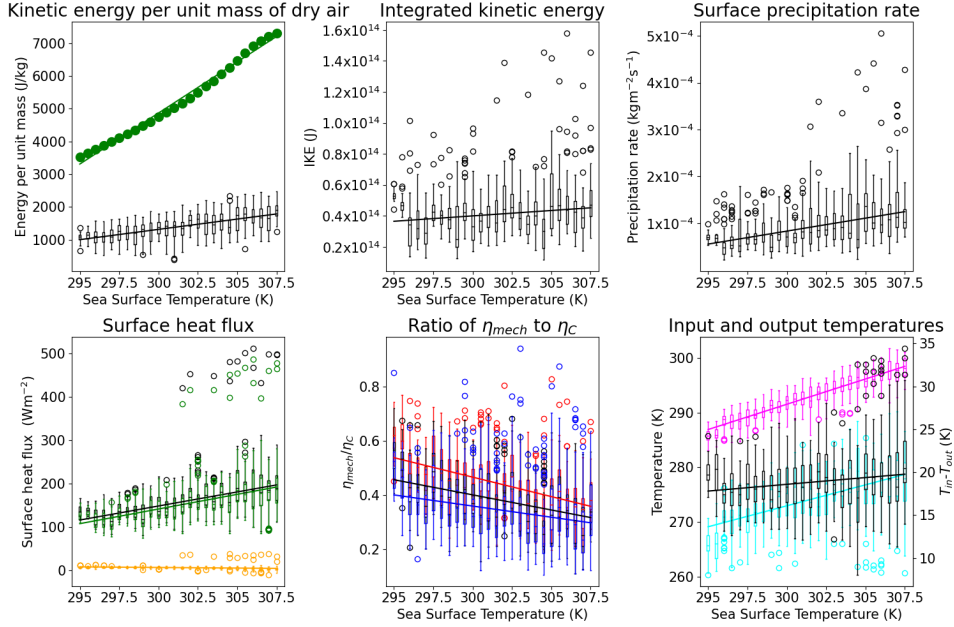
419 *c. Variability of quantities related to the MAFALDA-derived energetic terms*

420 Finally, we examine the variability of more familiar terms in our experiments that relate to the
421 MAFALDA-derived energetic quantities (Fig. 7, Table 2). We examine the variability of the kinetic
422 energy (KE) per unit mass of dry air based on the maximum azimuthal wind speed at 10m, v_{max} ,
423 in each experiment, defined in Appendix B. Dry air was used as the reference air mass to provide a
424 meaningful comparison to the MAFALDA-derived energetic quantities, which are also defined per
425 unit dry air mass. We used a similar definition for the KE per unit mass of dry air based instead on
426 the potential intensity, v_{PI} , which is an upper bound on v_{max} (Emanuel 1986; Bister and Emanuel
427 1998) (Appendix B). We additionally examine the variability with SST of the integrated kinetic
428 energy (IKE) (Powell and Reinhold 2007), surface precipitation rate, and surface total, latent, and
429 sensible heat fluxes (Appendix B). All quantities are computed only within the subdomain that we
430 used to calculate the MAFALDA-derived energetic and efficiency variables.

449 The KE associated with v_{max} increases at a rate of $5.1\% \text{ K}^{-1}$ with SST (Fig. 7, Table 2).
450 Accompanying the increase in the v_{max} -associated KE, the KE associated with the potential
451 intensity, v_{PI} , also increases at a rate of $6.8\% \text{ K}^{-1}$. The $6.8\% \text{ K}^{-1}$ increase in the v_{PI} -associated
452 KE is very close to the increase in W_{Max} of $7.0\% \text{ K}^{-1}$ that we observe for the whole circulation
453 (Table 1). Additionally, the increase in the v_{max} -associated KE of $5.1\% \text{ K}^{-1}$ is reduced compared
454 to the $6.8\% \text{ K}^{-1}$ increase in the v_{PI} -associated KE, similar to the reduced magnitude of the

⁶The sign and statistical significance of the eyewall trend are sensitive to outer radius (Table S6)

⁷The statistical significance and sign of the rainband trend are somewhat sensitive to outer radius (Table S7)



431 FIG. 7. Top: Maximum tangential wind speed-derived (black) and potential intensity-derived (green) kinetic
 432 energy per unit mass of dry air, integrated kinetic energy (IKE), and surface precipitation rate versus SST. Bottom
 433 left: Surface sensible (orange), latent (green), and total (black) heat flux versus SST. Bottom center: η_{mech}/η_C for
 434 the whole circulation (black), eyewall circulation (red), and rainband circulation (blue) versus SST. Bottom right:
 435 temperatures associated with the heat input (magenta) and output (cyan), as well as the difference between these
 436 temperatures (black), for the whole circulation of the TC heat engine, versus SST. Results for all variables except
 437 the potential intensity-derived kinetic energy are calculated for the realistic periods. The potential intensity-
 438 derived kinetic energy is calculated with output from our RCE simulations and so is not specific to any given
 439 time period from each simulation. For each boxplot, the unfilled circles represent outlier points. Outlier points
 440 are identified as those that are separated from the median value by more than 1.5 interquartile ranges.

455 increase in W_{KE} compared to the increase in W_{Max} that we observe for all circulation types (Table
 456 1). Finally, the IKE, which accounts for the full 10-meter wind field of the TC in the subdomain,
 457 also increases at $1.6\% \text{ K}^{-1}$. These results are all in qualitative agreement with the increases in
 458 W_{KE} that we observed for all circulation types (Fig. 5, Table 1).

459 The surface precipitation rate increases with SST at $6.5\% \text{ K}^{-1}$ (Fig. 7, Table 2). This is
 460 consistent with the $12\% \text{ K}^{-1}$ increase in W_P (Fig. 6, Table 1).

461 The total surface heat flux (THF), which represents a portion of the total heating of the TC, Q_{in} ,
 462 increases at $4.3\% \text{ K}^{-1}$ with SST (Fig. 7, Table 2). In order to make a meaningful comparison

441 TABLE 2. Trends for quantities related to the MAFALDA-derived variables during the realistic periods, as
 442 well as for MAFALDA-derived temperature variables calculated for the whole circulation during the realistic
 443 periods. The potential intensity-derived kinetic energy is calculated from the output of RCE simulations and so
 444 is not calculated during the realistic periods. All trend values are constructed from a linear fit to the data and
 445 are calculated as percentages of the mean value of each variable at an SST of 299.5K. All linear trend values are
 446 statistically significantly different from zero at the 99 % confidence level. To determine this, we used a two-sided
 447 t-test, using the HC1 estimator (MacKinnon and White 1985) to correct for heteroskedasticity (Text S3 in the
 448 online supplemental material).

Variable	Trend (% K ⁻¹)
Kinetic energy derived from v_{max}	5.1
Kinetic energy derived from v_{PI}	6.8
Integrated Kinetic Energy	1.6
Surface precipitation rate	6.5
Total surface heat flux	4.3
Surface latent heat flux	4.7
Surface sensible heat flux	-4.0
Input temperature, T_{in}	0.32
Output temperature, T_{out}	0.28
$T_{in}-T_{out}$	0.86

463 between the changes in the surface heat fluxes and the response of Q_{in} to SST, we additionally
 464 calculated the trend in Q_{in} when this variable was instead computed in $W m^{-2}$ rather than in $J kg^{-1}$.
 465 Q_{in} ($W m^{-2}$) increases at 6.4 % K⁻¹ (Table 3), in agreement with the increase in the THF. While
 466 the THF increases with SST, its latent and sensible components show opposite-signed changes with
 467 SST. The surface latent heat flux (LHF) increases at 4.7 % K⁻¹, but the surface sensible heat flux
 468 (SHF) decreases at -4.0 % K⁻¹. This difference may be due to the moistening of the atmosphere
 469 implied by Clausius-Clapeyron scaling as SST increases. Singh and O’Neill (2022) found, in
 470 radiative-convective equilibrium (RCE) experiments of localized convection, that the SHF was
 471 larger in dry RCE than in moist RCE. Because the SHF represents a small portion of the THF (Fig.
 472 7), the change in the THF is almost entirely due to the increase in the LHF. This increase in the
 473 LHF with SST is qualitatively consistent with Clausius-Clapeyron scaling, at constant near-surface
 474 relative humidity, of the mixing ratios in the LHF bulk formula shown in Appendix B.

475 TABLE 3. Trends for the MAFALDA-derived energetic variables, calculated in W m^{-2} during all realistic
476 periods. Values are shown for the whole circulation of the TCs only. All linear trend values are statistically
477 significantly different from zero at the 99 % confidence level. To determine this, we used a two-sided t-test,
478 using the HC1 estimator (MacKinnon and White 1985) to correct for heteroskedasticity (Text S3 in the online
479 supplemental material).

Variable	Circulation type	Trend (% K^{-1})
W_{KE}	Whole	3.9
W_P	Whole	12
ΔG	Whole	8.3
Q_{in}	Whole	6.4

480 4. Discussion

481 We have investigated how the mechanical efficiency, η_{mech} , of mature tropical cyclones is affected
482 by sea surface temperature (SST). We found that η_{mech} decreases with SST, at a rate of -2.1%
483 K^{-1} (Table 1). This decrease was driven by a 6.3% K^{-1} increase in the total heating of the TC,
484 Q_{in} , that dominates over the 3.4% K^{-1} increase in the kinetic energy of the wind field, W_{KE} . The
485 increase in W_{KE} is smaller than the 7.0% K^{-1} increase in its maximum bound, the net heat energy
486 gained by the TC W_{Max} . The majority of the increase in the net heat energy is instead reflected
487 by the strong increase in the moist processes in the TC. This increase occurs in part because of
488 an increase in the atmospheric moisture content that is expected from Clausius-Clapeyron scaling.
489 The related larger irreversibility at higher SST is shown by the increase in the associated moisture
490 penalty, W_{Moist} , of 9.6% K^{-1} . W_{Moist} represents a larger portion of W_{Max} at higher SST, which
491 reduces the magnitude of the increase in W_{KE} compared to the increase in W_{Max} . This is true
492 regardless of the subset of the TC circulation that we consider (Table 1, Figure 5).

493 In contrast to our results, Pan et al. (2017) find, using primarily the NCEP-DOE R2 (Kalnay
494 et al. 1996; Kistler et al. 2001; Kanamitsu et al. 2002) and ERA-Interim (Uppala et al. 2005;
495 Berrisford et al. 2011; Dee et al. 2011) reanalysis datasets, that the mechanical efficiency of the
496 global atmosphere increased over the period 1979-2013. However, reanalysis products are not well
497 suited to evaluating climate trends (Thorne and Vose 2010; Chemke and Polvani 2019). Together
498 with the fact that the global atmospheric heat engine and the TC heat engine may behave quite

499 differently in response to warming, this may help explain the discrepancy between our results and
500 theirs.

501 An upper bound of the mechanical efficiency is the Carnot efficiency, η_C . In our simulations,
502 η_C increases with SST at a rate of $0.53\% \text{ K}^{-1}$ (Table 1), inconsistent with previous studies of the
503 global atmosphere (Lembo et al. 2019; Lucarini et al. 2010a,b; Knietzsch et al. 2015). However,
504 the responses to warming of the TC heat engine and global atmospheric heat engine may be quite
505 different. The increase in the Carnot efficiency combined with the decrease in the mechanical
506 efficiency results in a decrease of $-2.7\% \text{ K}^{-1}$ in the relative efficiency, $\frac{\eta_{mech}}{\eta_C}$. Because this decrease
507 is more pronounced for the eyewall circulation, at $-3.0\% \text{ K}^{-1}$, than the $-2.2\% \text{ K}^{-1}$ decrease for the
508 rainband circulation, there is a smaller difference between the relative efficiencies of the eyewall
509 and rainband circulations at higher SST (Fig. 7). The relative efficiency of the whole circulation
510 ranges from about 0.2 to 0.6 for the majority of the periods examined. Ozawa and Shimokawa
511 (2015) used observations of 663 North Pacific TCs to estimate the relative efficiency of a TC to
512 be about 0.6 on average. Although our relative efficiency values generally fall below 0.6, our
513 TC relative efficiency is consistently greater than the estimated relative efficiency of the global
514 atmosphere of 0.1 (Golitsyn 1970), which is in agreement with Ozawa and Shimokawa (2015).
515 Our results may differ with those of Ozawa and Shimokawa (2015) for several reasons: first, the
516 authors assumed a uniform surface wind speed in their estimation of the relative efficiency, which
517 is not the case in reality; second, we examine highly idealized, axisymmetric TCs whereas Ozawa
518 and Shimokawa (2015) used data from real TCs. It is possible that the asymmetric aspects of
519 the TC circulation, or other factors such as interactions between TCs and their environment, are
520 important in setting the relative efficiency of real TCs.

521 W_{KE} increases with SST at a rate of $3.4\% \text{ K}^{-1}$ (Table 1, Fig. 5). This increase is consistent across
522 all circulation types⁸. An increase in the kinetic energy is also supported by a $5.1\% \text{ K}^{-1}$ increase
523 in the v_{max} -associated KE, and by a $1.6\% \text{ K}^{-1}$ increase in the IKE (Table 2). These increases are
524 consistent with the increases in comparable or related quantities with SST found in many previous
525 studies of the global atmosphere (Lembo et al. 2019; Pan et al. 2017), of localized convection
526 (Romps 2008; Singh and O’Gorman 2016; Pauluis 2016), and of TCs (Wang and Toumi 2021;
527 Kreussler et al. 2021; Khairoutdinov and Emanuel 2013; Knutson and Tuleya 2004), although some
528 studies of the global atmosphere instead find a decrease in similar variables (LaLiberté et al. 2015;

⁸The rainband trend changes sign at large outer radius values, and its statistical significance is sensitive to outer radius (Table S7)

529 Lucarini et al. 2010a,b; Knietzsch et al. 2015). In particular, the increase in W_{KE} is comparable to
530 a key result of Khairoutdinov and Emanuel (2013), who study how the kinetic energy per unit area
531 per storm varies across a series of constant-SST “TC-world” experiments. Across a suite of these
532 simulations, the SST varied from 294-309K. They found that the kinetic energy (KE) per unit area
533 at the surface per storm increased dramatically with SST, at a rate of about $17\% \text{ K}^{-1}$.

534 The magnitude of the increase in W_{KE} is consistently lower than the increase in W_{Max} for all
535 circulation types (Table 1). This is due to a strong increase in the energetic penalty associated with
536 moist irreversible entropy production in a TC, W_{Moist} . The increase in W_{Moist} is in agreement with
537 the increases of comparable quantities in previous studies of the global atmosphere and of localized
538 convection (Singh and O’Gorman 2016; Lembo et al. 2019; Romps 2008; Knietzsch et al. 2015;
539 Pauluis 2016).

540 Both components of the moisture penalty—the precipitation-associated portion, W_P , and the
541 portion associated with the TC hydrological cycle, ΔG —increase with SST, at $12\% \text{ K}^{-1}$ and 8.2
542 $\% \text{ K}^{-1}$, respectively (Fig. 6, Table 1). This is true for all circulation types, and is qualitatively in
543 agreement with increases in similar quantities found in studies of both localized convection and the
544 global atmosphere (LaLiberté et al. 2015; Singh and O’Gorman 2016; Romps 2008; Pauluis 2016).
545 The increase in W_P is associated with an increase in the surface precipitation rate of $6.5\% \text{ K}^{-1}$
546 (Table 2), consistent with the results of previous studies for both TCs and the global atmosphere
547 (Knutson and Tuleya 2004; Jeevanjee and Romps 2018; Khairoutdinov and Emanuel 2013). W_P
548 increases more strongly than the surface precipitation rate with SST, which may be due to some
549 portion of the precipitation re-evaporating before it reaches the surface, so that some of the increase
550 in the total precipitation would be reflected in W_P but not in the surface precipitation rate. W_P also
551 depends on the depth of the TC circulation, which increases with SST (See Fig. S4 in the online
552 supplemental material).

553 Finally, the $-2.1\% \text{ K}^{-1}$ decrease in the mechanical efficiency occurs in spite of the increase in
554 W_{KE} due to an accompanying $6.3\% \text{ K}^{-1}$ increase in the heat input, Q_{in} (Table 1). Recall that Q_{in}
555 does not only include the surface heat fluxes- it also accounts for radiative heating, heating due to
556 the phase transitions of water vapor, and dissipative heating associated with both precipitation and
557 kinetic energy dissipation. In what follows, we will examine further the increase in the heat input
558 with SST, where all variables are calculated in W m^{-2} for consistency. Motivated by the strong

559 role of moisture in modulating the kinetic energy increase, we examine separately the variability
560 with SST of different “moist” and “dry” heat sources that contribute to the 6.4 % K⁻¹ increase
561 in Q_{in} (W m⁻²) (Tables 2 and 3). The total surface heat flux should represent the majority of
562 Q_{in} . We find that the increase in total surface heat flux of 4.3 % K⁻¹ is almost entirely due to a
563 4.7 % K⁻¹ increase in the surface latent heat flux, whereas the surface sensible heat flux exhibits
564 a decrease of -4.0 % K⁻¹ with SST. There is a noticeable difference in magnitude between the
565 increase in the total surface heat flux and the increase in the total heating. We speculate that, based
566 on the increases in W_{KE} (W m⁻²) and W_P (W m⁻²) of 3.9 % K⁻¹ and 12 % K⁻¹, respectively, both
567 the dissipative heating associated with dry frictional dissipation and the precipitation-associated
568 dissipative heating increase with SST. However, these increases in dissipative heating alone are
569 unlikely to explain the discrepancy between the increases in Q_{in} (W m⁻²) and the total surface heat
570 flux. Since Q_{in} also includes radiative heating, we postulate that an increase in radiative heating
571 must occur with increasing SST, though calculating this with MAFALDA will be left for future
572 work. In summary, the increase in Q_{in} with SST is dominated by the increases in the moist heat
573 sources—surface latent heat flux and precipitation-associated dissipation—with additional positive
574 contributions due to the dry heating sources from kinetic energy dissipation and a likely increase
575 in radiative heating, whereas the decrease in the dry surface sensible heat flux acts to decrease Q_{in} .

576 Our experimental setup and analyses include several idealizations. A highly idealized aspect of
577 our simulations is the assumption that TCs are axisymmetric, which is far from true in reality. Our
578 results may omit important changes in the asymmetric circulation of TCs with SST. Our simulations
579 also represent the ocean as a constant-SST surface, and so important feedbacks between the TC
580 and the ocean are not captured in our simulations. Future studies should verify that our results are
581 robust in less idealized models, including non-axisymmetric models. Importantly, our methodology
582 assumes that the circulation of the TC in the subdomain where we perform our analysis is not open.
583 However, this is far from the case in the real world, and even in our simulations. Future work
584 should test whether our results are robust when analytical techniques that do not assume a closed
585 circulation are used. Finally, the effect of SST on the global distribution and frequency of TCs was
586 not considered in this study, since we only examined the impacts of SST on single TCs.

587 The conceptualization of the tropical cyclone as a heat engine has long been used to study factors
588 affecting the intensity of TCs (e.g. Emanuel 1986; Bister and Emanuel 1998). The efficiency of the

589 TC heat engine is a key factor in determining how the intensity of TCs, as well as their full wind
590 fields, vary in different climates. However, the frameworks used by many previous studies fail to
591 fully account for the effects of moisture on the efficiency of TCs, and often focus on the changes in
592 TC intensity or in the surface wind field, ignoring changes in other portions of the wind field (e.g.
593 Emanuel 1986; Bister and Emanuel 1998; Wang and Toumi 2021; Kreussler et al. 2021). Despite
594 the idealizations inherent to our model and analysis procedure, we conclude that the mechanical
595 efficiency of mature TCs decreases with increasing SST. The large increase in moist processes and
596 related irreversibility in the TC with SST results in the associated moisture penalty consuming a
597 larger portion of the net heat energy gained by the TC, which also increases with SST, in warmer
598 simulations. In this way, the increase in the moisture penalty modulates the magnitude of the
599 increase in the kinetic energy produced by the TC heat engine. We also find that moist heating
600 sources are the primary contributors to the large increase in the total heat input to the TC, which
601 ultimately drives the decrease in the mechanical efficiency in spite of the increase in the kinetic
602 energy. Our results highlight the importance of considering in full the effects of moisture on the
603 TC heat engine in studies of the TC's relationship to climate.

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611 *Data availability statement.* Modified source code and model inputs required to rerun our simu-
 612 lations can be found at <https://doi.org/10.5281/zenodo.7983011>. The CM1 homepage is
 613 <https://www2.mmm.ucar.edu/people/bryan/cm1/>, from where all other required inputs and
 614 source code can be downloaded. All analysis code is available at [https://doi.org/10.5281/](https://doi.org/10.5281/zenodo.7302058)
 615 [zenodo.7302058](https://doi.org/10.5281/zenodo.7302058).

616 APPENDIX A

617 Definitions of thermodynamic quantities

618 Following Bolton (1980), the equivalent potential temperature, θ_e , is defined as follows:

$$\theta_e = T \left(\frac{p_{ref}}{p} \right)^{(0.2854(1.0-0.28r_v))} e^{\left(\frac{3376.0}{T_{LCL}} - 2.54 \right) r_v (1.0+0.81r_v)},$$

619 where T is the temperature (K), p_{ref} is a reference pressure, set to 1000hPa, p is the total pressure
 620 (hPa), r_v is the water vapor mixing ratio (kg kg^{-1}), and T_{LCL} is the temperature (K) at the lifting
 621 condensation level:

$$T_{LCL} = 55.0 + \frac{2840.0}{3.5 \ln T - \ln(1.0 \times 10^{-20} + e_p) - 4.805}.$$

622 The water vapor pressure in hPa, e_p , is defined as:

$$e_p = \frac{pr_v}{\frac{R_d}{R_v} + r_v}.$$

623 R_d ($\text{J kg}^{-1}\text{K}^{-1}$) is the specific gas constant for dry air and R_v ($\text{J kg}^{-1}\text{K}^{-1}$) is the specific gas constant
 624 for water vapor.

625 Henceforth, unless noted otherwise, thermodynamic terms are defined following Pauluis (2016).

626 Moist entropy, s ($\text{J kg}^{-1}\text{K}^{-1}$), per unit mass of dry air, is defined as:

$$\begin{aligned}
 s &= s_d + r_v s_v + r_l s_l + r_i s_i \\
 s_d &= c_{p,d} \ln \frac{T}{T_{ref}} - R_d \ln \frac{p_d}{p_{ref}} \\
 s_v &= c_{p,l} \ln \frac{T}{T_{ref}} + \frac{L_v}{T} - R_v \ln \mathcal{H} \\
 s_l &= c_{p,l} \ln \frac{T}{T_{ref}} \\
 s_i &= c_{p,i} \ln \frac{T}{T_{ref}} - \frac{L_{f,0}}{T_{ref}}
 \end{aligned}$$

627 $c_{p,d}$, $c_{p,l}$, and $c_{p,i}$ ($\text{J kg}^{-1}\text{K}^{-1}$) are the specific heat capacities of dry air, liquid water, and ice,
 628 respectively. $L_{f,0}$ (J kg^{-1}) is the latent heat of freezing at 273.15K. T_{ref} (K) is the reference value
 629 of temperature, set here to 273.15K. $p_d = p - e_p$ is the partial pressure of dry air, while r_l and r_i
 630 (kg kg^{-1}) are the mixing ratios of water in its liquid and solid phase, respectively. L_v is the latent
 631 heat of vaporization as a function of temperature, defined as:

$$L_v = L_{v,0} + (c_{p,v} - c_{p,l})(T - T_{ref})$$

632 $L_{v,0}$ (J kg^{-1}) is the latent heat of vaporization at 273.15K and $c_{p,v}$ is the specific heat capacity of
 633 water vapor.

634 \mathcal{H} is the relative humidity with respect to liquid water, defined as:

$$\mathcal{H} = \frac{e_p}{e_s},$$

635 where:

$$e_s = \min(e_{s,i}, e_{s,l}),$$

636 where $e_{s,l}$ is the saturation vapor pressure with respect to liquid water and $e_{s,i}$ is the saturation
 637 vapor pressure with respect to solid water, where both are defined as in Goff and Gratch (1946).

638 Following convention, the entropy as defined here assumes that the specific entropies of dry air
 639 and of liquid water are zero in the reference state. A brief discussion of how the MAFALDA-derived
 640 variables depend on the choice of reference entropy can be found in Appendix C.

641 The specific volume per unit mass of dry air, α_d ($\text{m}^3 \text{kg}^{-1}$), is defined:

$$\alpha_d = \frac{R_d T + R_v r_v T}{p}.$$

642 The specific Gibbs free energy of water vapor, liquid water, and solid water (J kg^{-1}) are defined,
 643 respectively, as:

$$\begin{aligned} g_v &= c_{p,l}(T - T_{ref} - T \ln \frac{T}{T_{ref}}) + R_v T \ln \mathcal{H} \\ g_l &= c_{p,l}(T - T_{ref} - T \ln \frac{T}{T_{ref}}) \\ g_i &= c_{p,i}(T - T_{ref} - T \ln \frac{T}{T_{ref}}) - L_{f,0}(1 - \frac{T}{T_{ref}}) \end{aligned}$$

644

APPENDIX B

645

Definitions of quantities related to MAFALDA-derived variables

646 We define the kinetic energy per unit mass of dry air based on the maximum tangential wind
 647 speed at a height of 10m, v_{max} (m s^{-1}). The reference mass was that of dry air to ease comparison
 648 with the MAFALDA- derived energetic quantities, which are defined per unit mass of dry air. The
 649 v_{max} -derived kinetic energy is defined as:

$$KE_{v_{max}} = \frac{1}{2} \frac{\rho_m}{\rho_d} v_{max}^2$$

650 ρ_m and ρ_d are computed at the point of maximum 10m azimuthal wind speed v_{max} . ρ_m and ρ_d
 651 are the densities of humid and dry air:

$$\rho_m = \frac{p_d}{R_d T} + \frac{p_v}{R_v T},$$

$$\rho_d = \frac{p_d}{R_d T}.$$

652 Here, p_v (Pa) is the partial pressure of water vapor. p_v is defined as equal to e_p . ρ_d is equivalent
 653 to ρ but is denoted ρ_d in this Appendix for clarity.

654 The potential intensity, v_{pi} (m s^{-1}), is an upper bound on v_{max} . For each SST simulation, it
 655 is computed based on the time- and horizontal-mean vertical profiles of pressure, temperature,
 656 and water vapor mixing ratio from the last 20 days of the corresponding RCE simulation. For the
 657 calculation, we use an improved version of the Bister and Emanuel (2002) algorithm (Gilford 2020,
 658 2021) assuming that $\frac{c_k}{c_d}=0.9$ and that any ascent was pseudoadiabatic. A kinetic energy per unit
 659 mass of dry air, $KE_{v_{pi}}$ is then defined based on v_{pi} . The calculation of $KE_{v_{pi}}$ is largely analogous
 660 to the definition of $KE_{v_{max}}$, except that we use interpolated values of ρ_d and ρ_m at 10m that are
 661 calculated based on domain-average values of ρ_m and ρ_d at the surface and at 25m in the RCE
 662 simulations. In contrast, ρ_m and ρ_d in the calculation of $KE_{v_{max}}$ are interpolated specifically to the
 663 point of the maximum 10m azimuthal wind in the axisymmetric simulations.

664 The surface precipitation rate ($\text{kg m}^{-2} \text{s}^{-1}$) is included in the output of CM1. Using this, we
 665 calculate the horizontal-mean, time-average surface precipitation rate in the subdomain for each
 666 2-day period. The integrated kinetic energy (IKE) is defined as in Powell and Reinhold (2007),
 667 with the exception that the horizontal wind speed at 10m, rather than the total wind speed at 10m,
 668 is used because the former is an output of CM1 while the latter is not, and because the vertical
 669 wind speed should be much smaller than the horizontal wind speed at this level:

$$IKE = \int_V \frac{1}{2} \rho_m U^2 dV.$$

670 dV is the volume element centered on a height of 10m and has a thickness of 1 meter in the
 671 vertical direction. ρ_m is the density of humid air and U is the horizontal wind speed, both at a
 672 height of 10m.

673 The surface sensible (SHF), latent (LHF), and total heat fluxes (THF) (W m^{-2}) are defined using
 674 the bulk formulae from CM1:

$$SHF = \rho_m c_p c_k U (\theta_s - \theta_{25})$$

$$LHF = \rho_m L_v c_k U (r^*(T_s) - r_{25})$$

$$THF = SHF + LHF$$

675 Here, ρ_m is the density of humid air at the surface, U is the horizontal wind speed at 10m, and
 676 c_k is the enthalpy exchange coefficient. $r^*(T_s)$ (kg kg^{-1}) is the saturation water vapor mixing
 677 ratio at the surface (Bolton 1980), θ_s is the potential temperature (K) at the surface, and θ_{25} and
 678 r_{25} are the potential temperature and water vapor mixing ratio at the lowest model level: 25m.
 679 In the formula for the SHF, the absolute temperature difference is approximated by the potential
 680 temperature difference.

681 For this study, we examine the variability of the horizontal- and time-averages of all quantities
 682 except the IKE and $KE_{v_{max}}$ during the time period under consideration. For the IKE and $KE_{v_{max}}$,
 683 only the time-average value is computed.

684 APPENDIX C

685 Sensitivity of MAFALDA-derived variables to the reference entropy choice

686 When defining the entropy, we must choose its value in the reference state. This reference entropy
 687 can be defined fully by choosing the values of the specific entropies of dry air and liquid water in the
 688 reference state, $s_{d,0}$ and $s_{l,0}$. In our work, we use a definition of the moist entropy that assumes $s_{d,0}$
 689 and $s_{l,0}$ are zero. However, other definitions of entropy have been used in the literature, including
 690 the absolute entropy of Marquet (2017). Any study using MAFALDA with a different definition
 691 of the entropy must account for the ways this impacts some of the MAFALDA-derived variables.

692 The MAFALDA-derived terms that would change under a different entropy definition are W_{Max} ,
 693 ΔG , and W_{Moist} . W_{Max} is calculated as $\oint \tilde{T} d\tilde{s}$ for each MAFALDA trajectory. Because of this, it
 694 will be directly affected by a different definition of the reference entropy. By definition, the specific
 695 Gibbs free energy of water in each of its phases, w , is $g_w = h_w - T s_w$. From this, we can see that
 696 the specific Gibbs free energies of water in all its phases will be impacted by both the choice of

697 the reference entropy and the choice of the reference enthalpy. Since ΔG depends on the specific
 698 Gibbs free energies of water, the value of ΔG will change if a different entropy definition is chosen.
 699 Since W_{Moist} is dependent on ΔG , this variable will also be impacted.

700 In what follows, X_{gen} will denote the value of a MAFALDA-derived variable when some general
 701 definition of the entropy is used. If a different definition of the entropy were to be used then,
 702 assuming $s_{l,0}$ and $s_{d,0}$ are constant, the MAFALDA terms discussed above would be modified as
 703 follows:

$$\begin{aligned}
 W_{Max,gen} &= W_{Max} - s_{l,0} \oint \tilde{r}_t d\tilde{T} \\
 \Delta G_{gen} &= \Delta G - s_{l,0} \sum_{w=v,l,i} \oint \tilde{T} d\tilde{r}_w \\
 W_{Moist,gen} &= W_{Moist} - s_{l,0} \sum_{w=v,l,i} \oint \tilde{T} d\tilde{r}_w
 \end{aligned}$$

704 Although the differential heating defined in section 2b is a function of entropy, it can be rewritten,
 705 using the Gibbs relationship, in such a way that it does not depend on entropy (Pauluis and Zhang
 706 2017):

$$dq = T ds + \sum_{w=v,l,i} g_w dr_w = dh - \alpha_d dp$$

707 As such, the total heating, Q_{in} , will not be affected by changes in the entropy definition. As a
 708 result of this, the efficiencies considered in this work are also independent of the choice of reference
 709 entropy. However, both Q_{in} and the efficiencies do depend on the choice of reference enthalpy.

710 In MAFALDA, the mechanical work performed by the atmospheric flow, $W_P + W_{KE}$, is related
 711 both to the entropy transport via W_{Max} , and to the Gibbs penalty. The sensitivity of W_{Max} to the
 712 reference entropy reflects the fact that the choice of the reference state affects the entropy transport.
 713 However, it should be stressed here that the entropy transport captured by W_{Max} is not the total
 714 entropy transport, but represents only that associated with air motions. A second component of
 715 the atmospheric entropy transport is associated with the entropy transport by falling hydrometeors.
 716 When the atmospheric circulation transports water from the surface ocean to the colder atmosphere,
 717 falling rainfall carries the same amount of water back from the atmosphere to the ocean surface.
 718 The entropy transport associated with falling hydrometeors depends on the reference entropy as
 719 well - but this sensitivity cancels out with that of the entropy transport that contributes to W_{Max} .

720 There are two possible approaches in dealing with the entropy transport by falling hydrometeors.
721 One option is to convert the MAFALDA cycles into closed cycles, by carrying condensed water
722 on the descending part of the cycle, following the suggestion of Pauluis (2011). This has the
723 advantage of making the cycles fully independent of the reference state, albeit at the expense of
724 making the computation slightly more complicated. A second option, adopted here following the
725 original formulation of MAFALDA, is to use liquid water as the reference state with the practical
726 goal of minimizing the entropy transport by falling precipitation. Doing so ensures that the entropy
727 transport by the air motions captured in W_{Max} is a good approximation of the total entropy transport.
728 For other choices for the reference states - such as the absolute entropy of Marquet (2017) - falling
729 precipitation carries a substantial amount of entropy that should be properly accounted for in the
730 computation of thermodynamic cycles.

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