Cosmogenic nuclide dating conundrum for retreat of the Laurentide Ice Sheet and the critical roles of geomagnetic and heliomagnetic modulation of cosmic ray flux

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Cosmogenic nuclide dating conundrum for retreat of the Laurentide Ice Sheet and the critical roles of geomagnetic and heliomagnetic modulation of cosmic ray flux

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Abstract

What we regard as anomalously old ¹⁰Be exposure dates reported from the terminal moraine of the Laurentide Ice Sheet (LIS) in northeastern North America, such as recently published for Allamuchy NJ, ostensibly point to the start of deglaciation at 25 thousand calendar years before present (cal. ka). These dates are well within the conventional age span of the Last Glacial Maximum (LGM) and are in stark contrast with published ¹⁴C accelerator mass spectrometry (AMS) dates for earliest terrestrial plant macrofossils found in LIS deglacial clay deposits that range back to only ~16 cal. ka, which more plausibly coincide with the known timing of the glacio-eustatic rise and meltwater discharge to the North Atlantic and Gulf of Mexico that mark the demise of the LGM in the marine record. To explore possible explanations for this inconsistency, we first employed a statistical model of the geomagnetic field that includes secular variation with nondipole terms and can be applied globally. The model results in a decrease in the magnetic shielding factor by about 10% at mid-latitudes compared to oft-used geomagnetic scaling schemes. However, the time-integrated axial dipole moment estimated separately suggests little overall change in average shielding since about 20 cal. ka. This seems to leave cosmic ray flux modulated by a time-varying heliomagnetic field linked to sunspot activity as an underestimated factor in widely used ¹⁰Be exposure age calculators. If generally
biased by about 23% higher compared to modern levels as reported for the past 9.4 cal. ka, the elevated high cosmic ray flux would make \(^{10}\)Be reference production rates proportionately higher, to about 5.5 at/g/y at sea level-high latitude, and reduce exposure ages to about 3/4 of those that have been previously calculated for LGM and younger rocks (to less than 20 cal. ka in the case of Allamuchy). Varying but generally higher solar modulation will require reevaluation of cosmogenic exposure dates in general, as in the case of Allamuchy, that would allow improved synchronization of marine and terrestrial records of glaciation. Other test cases can result in improved GIA deglaciation models and alternative estimates of effects of shielding in ice-flow models.

1. Introduction

There are widely divergent published results from cosmogenic surface exposure dating and radiocarbon chronologies for the retreat of the Laurentide ice sheet (LIS) from the Last Glacial Maximum (LGM) in northeastern North America (Fig. 1). Recession of the southeastern lobe of the LIS was placed at around 25 cal. ka (calendar kilo-annum or thousands of years ago) according to \(^{10}\)Be measurements on glacial boulders associated with terminal moraines in New England and ostensibly supported by \(^{14}\)C bulk sediment dates ranging from ~22 to 27 cal. ka on deglacial sediments in the region (Balco et al., 2009; Balco and Schaefer, 2006; Balco et al., 2002). However, Peteet et al. (2012) soon afterwards reported calibrated \(^{14}\)C accelerator mass spectrometry (AMS) dates on terrestrial plant macrofossils in earliest deglacial sediments in the region that range back to only ~16 cal. ka. The younger timing was seen as much more compatible with the well-dated sea level record, which implied that melting of the LIS, the largest variable continental ice volume for the LGM (circa. 16–29 cal. ka) and equivalent to 70 to 80 m of the ~120 m sea level drop that characterizes it (Clark and Mix, 2002; Tarasov et al., 2012), did not sensibly proceed until around 20 cal. ka and not in earnest until about 16 cal. ka (Fig. 2).

Nonetheless, a recent \(^{10}\)Be exposure age study (Corbett et al., 2017) that builds on unpublished but widely cited thesis work (Larsen, 1996) on glacial boulders and pavements associated with the terminal moraine in Allamuchy Forest and vicinity in northern New Jersey (NJ) (Fig. 1) also reported an exposure age of 25.2±2.1 cal. ka. The \(^{10}\)Be exposure ages of ~25 cal. ka taken at face value would imply that the southeastern lobe of the LIS started to retreat
during maximum ice volume well within the LGM as denoted by low global sea-level (Fig. 2). As pointed out by Peteet et al. (2012), this would also imply an extraordinarily long delay of 9,000 years (from 25 cal. ka until 16 cal. ka) before introduction of vegetation on the deglaciated landscape. Such an extended delay in temperate latitudes does not seem plausible (Jones and Henry, 2003; Matthews, 1992) because trees today grow in close proximity to (and even on debris-covered) glaciers in southern Alaska including on permafrost (Fickert et al., 2007) with rapid primary succession by plants following deglaciation in decades (Chapin et al., 1994; Cooper, 1923). The oldest \(^{14}C\) AMS-dated terrestrial plant macrofossils found thus far occur in clays of less than 5% organic content that argue for deposition with glacial meltwater during earliest ice retreat.

The broader conflict between old \(^{14}C\) bulk sediment dates and younger \(^{14}C\) AMS dates on terrestrial plant macrofossils in the same clays of deglaciation bog/limnic sequences associated with the LIS terminal moraine is systemic and regional in scope (Fig. 3). For example, reliance is still placed (Corbett et al., 2017) on previously rejected \(^{14}C\) bulk sediment dates of 27.2±1.4 cal. ka at Budd Lake, NJ (unpublished thesis of Harmon, 1968) and 25.8±1.6 cal. ka in a contorted section of the Harbor Hill moraine in Port Washington on Long Island, New York (Sirkin and Stuckenrath, 1980). These dates are from the same terminal moraine associated with published \(^{14}C\) AMS dates on first appearance of tundra plants from basal silts/clays of 14.4±0.4 cal. ka at Tannersville just to the west of Budd Lake and 14.6±0.3 cal. ka at High Rock just to the east (Peteet et al., 2012) (Fig. 4). More pertinently, Corbett et al. (2017) cite evidence from controversial \(^{14}C\) bulk sediment dates of 22.2 and 22.5 cal. ka from nearby Francis Lake (unpublished thesis of Cotter, 1983; but see opposing view by Karrow et al., 1986) in support of the \(^{10}Be\) exposure age of 25.2±2.1 cal. ka that was obtained from glacial pavement and boulders at nearby Allamuchy Forest and environs. Yet at Allamuchy Pond the same litho- and biostratigraphy is recorded as at Francis Lake only 6 km to the west where Dryas and willow leaves screened from basal clays in the basal herb zone transition are \(^{14}C\) AMS dated at 14.4 ±0.8 cal. ka (Peteet et al., 2012; Peteet et al., 1993).

The requisite usage of terrestrial macrofossils for \(^{14}C\) AMS dating in basal clays/silts for timing of deglaciation versus \(^{14}C\) dates on bulk sediment, which is apt to be contaminated by older carbon in the landscape, is widely acknowledged (e.g., Birks, 1993; Curry et al., 2010; Gaglioti et al., 2014; Grimm et al., 2009; Hajdas et al., 1993; Peteet et al., 1993; Peteet and
Mann, 1994; Peteet et al., 1990; Thompson et al., 2017; Zimmerman and Wahl, 2020).

Moreover, usage of terrestrial rather than aquatic plant macrofossils, which tend to give variably older \(^{14}\)C dates (MacDonald et al., 1987), is essential for radiocarbon dating accuracy in these environments (Birks, 2002; Marty and Myrbo, 2014).

Recently updated compilations (Dalton et al., 2020; Wickert et al., 2023) that include \(^{14}\)C AMS terrestrial plant macrofossil dates and reject the earlier bulk dating used by Dyke et al. (2003) indicate that the southern Laurentide margin was at LGM extent from 26.0 to 18.7 cal. ka (Fig. 1). This timing is in tempo with the global sea level record (Fig. 2) and with independent evidence that meltwater to the North Atlantic was minimal prior to 18.5 cal. ka (Keigwin et al., 1991) and started by 16.1 cal. ka in the Gulf of Mexico (Flower et al., 2004). The 25 cal. ka \(^{10}\)Be exposure age for the terminal moraine in NJ (Corbett et al., 2017) thus appears anomalously old even in this broader context and despite the continued usage of old \(^{14}\)C bulk dates for support (Stanford et al., 2020).

2. \(^{10}\)Be production and surface exposure dating

Cosmogenic exposure dating is a long-established technique of more than 30 years and is based on measurements of the concentration of a cosmogenic nuclide, in this case, \(^{10}\)Be produced in quartz in natural rock surfaces (Gosse and Phillips, 2001; Lal, 1991). The basic assumption is that the flux of highly energetic charged particles (~90% protons, ~10% helium nuclei) constituting galactic cosmic rays is constant over long time scales (Aab and others, 2017) although modulated in Earth’s space environment by a varying solar magnetic field (Steinhilber et al., 2012). Production of \(^{10}\)Be takes place overwhelmingly by high-energy spallation from secondary particles produced in the atmosphere and occurs within centimeters of the rock surface with only ~2% by interactions with deeper penetrating muons (Balco, 2017). Production rates depend strongly on the site altitude, an approximation of atmospheric pressure or weight but in the case of spallation the cosmogenic production is also modulated by the local geometry and magnitude of the time-varying geomagnetic field to about 60° in magnetic latitudes (74° in magnetic inclination), poleward of which the geomagnetic dependency becomes negligible.

A variety of scaling schemes have been used to normalize \(^{10}\)Be production at a given site to sea level and high latitude (SLHL). Five scaling schemes (St, De, Du, Li, Lm) are in the various versions of the first online exposure age calculator (v2, v2.2, v2.3; Balco et al., 2008)
and two additional schemes (LSDn, LSD) for a total of seven in the CRONUS-Earth effort (Marrero et al., 2016; Phillips et al., 2016a). The reported differences in applications amongst the various scaling schemes tend to be small and are often averaged although there remain “substantial unresolved difficulties in modeling cosmogenic nuclide production and the calibration of production rates” (Borchers et al., 2016).

The number of scaling schemes has been mercifully reduced to only three in the recent version (v3) of the widely used ‘online exposure age calculator formerly known as the CRONUS-Earth online exposure age calculator’ (http://hess.ess.washington.edu/; https://cosmognosis.wordpress.com/2016/08/01/let-a-hundred-flowers-bloom/). Scheme St continues from the initial version of the online exposure age calculator and is based on the latitude-altitude scaling factors of (Lal, 1991) recast in terms of atmospheric pressure by (Stone, 2000) and for the geomagnetic latitude of the present-day field. Scheme Lm is basically scheme St with a time-varying geomagnetic field intensity model, which according to what documentation is available online for v3 (https://sites.google.com/a/bgc.org/v3docs/), is now a spherical harmonic analysis (SHA) of the geomagnetic field for the past 14 cal. ka (SHA.DIF.14k; Pavón-Carrasco et al., 2014) and a geocentric axial dipole (GAD) field with a prescribed time-varying dipole moment model for earlier periods. A third scheme, LSDn, is a nuclide-dependent variant (Phillips et al., 2016a) of what is described as a physics-based analytical framework for in situ cosmogenic nuclide production (Lifton et al., 2014). LSDn apparently uses the same geomagnetic field model as scheme Lm with look-up tables of precalculated scaling factors for the forward integrations. We could not readily access these tables and instead focus on simple numerical experiments with the longstanding St-Lm schemes based on empirical data and the Desilets-Dunai-Lifton (DeDuLi) schemes that LSDn seems similar to and utilize particle ray trajectory tracing to calculate effective vertical cutoff rigidities as a geomagnetic cutoff parameter, a common parameterization of cosmic ray intensity measurements.

2.1 St and Rc-based scaling schemes

We calculate spallation rate factors for scheme St from scaling equation coefficients in Stone (2000) normalized to the value at 60° latitude and a standard sea level pressure. The DeDuLi schemes are based on analytical estimates of the effective vertical cutoff rigidity, Rc
(here using Equation 2 of Lifton et al. (2014) for consistency). These and other relevant functions are included in Section S1 as routines for heuristic purposes in the R programming language (R_Core_Team, 2018).

The altitude scaling factor for spallation reactions (Eq. 7 in (Desilets et al., 2006)) as given in terms of \( R_c \) and atmospheric depth or weight \( (x) \) relative to sea level (1033 g/cm\(^2\), equal to standard atmospheric pressure of 1023.15 hPa) agrees well with the original empirical altitude scaling factor for the Lal/Stone St scheme from sea level to about 800 hPa (~2000 m altitude) but then the St and \( R_c \)-based scaling factors diverge with decreasing atmospheric pressure (higher altitude) (Fig. 5A). The scaling factor for latitude depends on the geomagnetic field model and has more variants than for altitude. The Lal/Stone St scheme has numerically larger scaling factors at any given latitude, ranging albeit not very regularly from almost 0.6 at the equator to 1.0 at 60° latitude poleward of which cosmogenic production rates become essentially independent of the geomagnetic latitude (Elsasser et al., 1956; Gosse and Phillips, 2001; Lifton et al., 2014) (Fig. 5B). The St scaling factor is cast in terms of geomagnetic latitude (presumably equivalent to geographic latitude in this context although that is not entirely clear) to organize and model the empirical cosmogenic data, a common practice in all scaling schemes, rather than the directly observable local geomagnetic inclination (as in Dunai, 2000; see also informative Comment and Reply of Desilets et al. (2001) and Dunai (2001)).

For a geomagnetic dipole field of comparable modern magnetic moment (~80 ZAm\(^2\)), geomagnetic shielding for spallation reactions expressed in terms of \( R_c \) can then be used to calculate a latitude scaling factor \((f(R_c))\) using Eq. 6 with Dorman function in Desilets et al. (2006). The \( f(R_c) \) factor at sea level takes the canonical sigmoidal form plotted for a stationary GAD field (Fig. 5B) and varies from ~0.54 at the equator to 1.0 at the ‘knee’ at 60° and higher latitudes. Other geomagnetic models have similar sigmoidal curves and are discussed below. The grand scaling factor, \( F \), is then the product of the latitude and altitude scaling factors \((F(R_c,x) = f(R_c) f(x))\), which can be used to estimate the \(^{10}\)Be production rate \((P(R_c,x))\) at a sample site relative to the production rate \((P_0)\) at a SLHL calibration site using Eq. 8 in Desilets et al. (2006): \(P(R_c,x) = F * P_0\).

2.2 \( R_c \) and dipole wobble
The SHA-DIF-14k model (Pavón-Carrasco et al., 2014), which is apparently now used in scheme Lm as well as LSDn in version v3 of the online calculator, is inherently limited by the inhomogeneous distribution of available archeomagnetic and volcanic paleomagnetic data: 97% of the total in this analysis are located in the Northern Hemisphere and 83% of the total are from 3 cal. ka to present. The spherical harmonic model is nonetheless sufficient to calculate virtual geomagnetic poles (VGP) from estimates of the dipole ($g^0_1$, $g^1_1$ and $h^1_1$) coefficients that are provided at 50-year intervals since 14 cal. ka (data listings available in the Earth Ref Digital Archive at [http://earthref.org/ERDA/1897/](http://earthref.org/ERDA/1897/)). The overall mean VGP pole position is located at 89.3°N 337.0°E ($n=279$, angular standard deviation (ASD) = 7.6°, precision parameter (K) = 118, and radius of circle of 95% confidence (A95) = 0.8°), which is not significantly different from the geographic axis despite the very tight grouping of the VGPs. This close correspondence confirms that the GAD provides an appropriate fit to the geomagnetic field averaged over the past 14 cal. ka, and importantly, even within just 2 cal. ka according to Pavón-Carrasco et al. (2014). The average $Rc$ will thus be essentially the same whether calculated with respect to the latitude from the mean VGP pole or with respect to the geographic axis for any site; nonetheless, $Rc$ averaged from constituent VGP distributions will tend to be sensibly different because of nonlinearity in the relationship of $Rc$ and latitude. The VGP dispersion can be regarded as a proxy for the effect of the dipole wobble component of secular variation of the geomagnetic field on $Rc$. The small dispersion of VGP poles from SHA-DIF-14k (K=118), however, hardly captures the full range of secular variation (as discussed below) and thus differs from the singular GAD pattern by only a few percent in mid-latitudes (Fig. 5B).

2.3 $Rc$ and a statistical geomagnetic field model

A more generalized approach to latitude scaling is to use a statistical model of the geomagnetic field that includes the full range of spherical harmonic contributions to the secular variation and that is also conveniently applicable over time scales of arbitrary duration from thousands to even millions of years ago. A candidate model is TK03 (Tauxe and Kent, 2004) where the geomagnetic field is treated as a Giant Gaussian Process (Constable and Parker, 1988) that follows Model G of McElhinny and McFadden (1997), which attributes the observed latitudinal dependence in directional dispersion to independent contributions from spherical harmonic families of odd and even symmetry for dynamo sources (Section S1). Modern studies of dispersion of paleomagnetic directions observed in lava flows from different areas as reliable
instantaneous recorders of the geomagnetic field confirm that the ASD of the calculated VGPs roughly doubles from nominally 12° at the equator (K~46) to around 24° (K~11) by 60° and higher north and south latitudes (Cromwell et al., 2018; Johnson et al., 2008), as modeled by TK03 for a time-averaged GAD field.

Mean $R_c$ can be calculated from individual inclinations converted with dipole formula to virtual geomagnetic latitudes in 5000 realizations of TK03 at every 5° of site latitude (Table S1). The resulting magnetic scaling factors are comparable to those for the other field models at geographic latitudes less than 30°; however, the TK03 scaling factors are appreciably lower at higher geographic latitudes, for example, 0.866 compared to 0.933 at 45° for the singular GAD model (Fig. 5B). Since the geocentric dipole typically represents more than 90% of the strength of the geomagnetic field at Earth’s surface, much of this departure from the singular GAD model can be attributed to greater dipole wobble modeled by VGP poles with a more dispersed Fisherian distribution than SHA, for example, with about a nominal ASD=16° (K=27) (Table S1). Contributions from the much smaller nondipole field components, which TK03 fully represents (to degree and order 8) statistically by design, account for the yet larger departures of $f(R_c)$ values because of averaging over a broader window of virtual geomagnetic latitudes.

Parenthetically, we note that the magnitude of the key axial-dipole ($g_1^0$) term in TK03 has no impact on the VGP scatter produced by the statistical model (Cromwell et al., 2018) although a varying dipole moment is an important element of cosmic ray modulation.

2.4 Varying geomagnetic and heliomagnetic fields

Temporal variation in strength of the geomagnetic field expressed as $M_t/M_0$, the ratio of the average dipole moment from a given time ($M_t$) to its present-day value ($M_0$, ~ 80 ZAm²), is the lead term in calculating $R_c$ at any given site latitude (e.g., Equation 2 of Lifton et al., 2014) (Fig. 6). Continuous empirical models for the axial dipole moment (ADM) such as GGF100k (Panovska et al., 2019), which we have chosen to use here (data listings available in the Earth Ref Digital Archive at https://earthref.org/ERDA/2382/), show that the dominant feature since 100 cal. ka is a distinct low associated with the Laschamp geomagnetic excursion at around 42 cal. ka (Fig. 7A). A cumulative plot of ADM as a proxy for the integrated shielding effect of a fluctuating dipole moment on $^{10}$Be production shows that the time-averaged dipole moment has been within a few percent of a constant present-day fiducial back to around 20 cal. ka, which
happens to encompass the age range of the primary $^{10}$Be calibration sites (see below). The near-
constant time-integrated dipole moment also renders schemes St and Lm as equivalent over this
time frame. The ADM cumulative curve then gradually decreases to about 0.85 of a constant
present-day fiducial from 20 cal. ka to around 50 cal. ka across the Laschamp excursion (Fig.
7B). We note that the time-varying ADM model also provides a broad framework for
understanding well-calibrated production rate variations of cosmogenic $^{14}$C in the atmosphere
such as derived by (Fairbanks et al., 2005) where the long-term pattern of systematic age offsets,
in this case calibrated by precise U-series dating on corals, can be linked to lower overall
geomagnetic shielding of cosmic ray flux from lingering effects of the Laschamp excursion in
conjunction with radiocarbon capture in short-term carbon cycling (Fig. 7C).

Solar modulation ($S$) of the interplanetary magnetic field generated by the Sun can
variably deflect portions of the galactic cosmic ray flux impinging Earth (Gosse and Phillips,
2001; Lifton et al., 2005; Steinhilber et al., 2012). The solar magnetic field is closely associated
with sunspot cycles where a higher solar magnetic field (and greater shielding of Earth’s
neighborhood in the solar system from galactic cosmic rays) occurs when sunspot numbers are
higher, and vice versa. The 11-year sunspot (Schwabe) cycle is modulated by longer-period
variations, such as the Gleissberg and Dalton minima and famously the first-named Maunder
Grand Minimum, when sunspots were largely absent for practically a century. Such long solar
magnetic minima should be times of relatively higher cosmic ray flux impinging Earth and are
expected to be reflected in higher cosmogenic isotope production. This is indeed what has been
reported using a variety of independently dated ice core and tree ring archives of cosmogenic
nuclides ($^{10}$Be and $^{14}$C) for the past 9.4 cal ka (Steinhilber et al., 2012) data available at
The younger part of the record (Fig. 8) allows direct linkages of cosmogenic production rates to
sunspot activity; the inferred relationship between solar magnetic field variations and cosmic ray
intensity is extended to the rest of the available record back to 9.4 cal. ka based on the measured
cosmogenic isotope production rates in the ice core and tree-ring archives.

Compared to the average cosmic ray intensity for 1944-1988 CE corresponding to
relatively lively sunspot activity, most of the earlier part of the record has reduced sunspot
activity that allowed higher cosmic ray flux to Earth. For example, cosmic ray flux for the
Gleissberg, Dalton and Wolf grand solar minima was ~1.5 times higher and the Maunder and
Spörer grand solar minima more than 1.6 times higher than modern levels. A scaling factor, \( S_t/S_0 \), based on this record is incorporated in our R-routines (Section S1) in which relative \(^{10}\text{Be}\) production at a given site varies directly with incremental cosmic ray intensity as estimated for the past 9.4 cal. ka, over which the flux \((S_t/S_0)\) is on average a factor of 1.23 larger. In comparison, a weighted mean solar factor for the past 11.4 cal. ka based on the tree-ring radiocarbon record and used in scaling scheme Li (Balco et al., 2008) is only 1.05 (Lifton et al., 2005) although solar factors ~30% higher were predicted by (Desilets and Zreda, 2001). The same solar modulation framework of (Lifton et al., 2005) as implemented by (Balco et al., 2008) was later adopted in the LSD model by (Lifton et al., 2014), who explicitly chose not to explore alternative frameworks citing Steinhilber et al. (2008). More recent exchanges (e.g., Beer et al., 2018; Cameron and Schüssler, 2019; Usoskin et al., 2011) also indicate that further work is needed to determine how changes in the heliomagnetic field affect cosmic ray deflection.

3. Comparison of scaling schemes with primary \(^{10}\text{Be}\) calibration sites

With these analytical tools in hand, we apply the different scaling schemes to the CRONUS-Earth primary \(^{10}\text{Be}\) calibration sites (Borchers et al., 2016), as lodged in the ICE-D production rate online database (Martin et al., 2017) (Section S2). The \(^{10}\text{Be}\) data were collected by modern sampling, laboratory and measurement protocols (e.g., referenced to 07KNSTD); local shielding and erosion corrections, typically a few percent, are accepted as given. Data relevant to determination of \(^{10}\text{Be}\) production at each calibration locality with various scaling schemes are summarized in Table 1.

For MR (Macaulay Ridge, New Zealand), \(^{10}\text{Be}\) concentrations are reported to average 89900 at/g for 7 boulder samples after taking into account corrections of 1-2% for sample thickness and local shielding, with a tight age constraint from \(^{14}\text{C}\) AMS determinations of 9634±50 cal. years ago on wood fragments immediately beneath the rock slide (Putnam et al., 2010). Our implementation of the St scheme delivers a SLHL \(^{10}\text{Be}\) production rate of 4.00 at/g/y for spallation with a ~2% contribution from muon processes (Balco, 2017), which when discounted gives 3.92 at/g/y that is reassuringly close to the rate of 3.84±0.08 at/g/y determined in more thorough online fashion for the St scheme by Putnam et al. (2010). When the \(^{10}\text{Be}\) concentration is scaled according to \( R_c \) for a constant GAD or the comparable average ADM field and divided by the calibration age, a \( P_{\text{SLHL}} \) of about 4.16 at/g/y is obtained, which when
discounted for ~2% muon contributions (4.1 at/g/y) is within the range of SLHL \(^{10}\)Be production rates (3.74–4.15 at/g/y) quoted by the authors from the five scaling methods in online calculator v2 (Putnam et al., 2010). Scaling schemes that include secular variation of directions give higher total SLHL \(^{10}\)Be production rates, 4.35 at/g/y for SHA and 4.52 at/g/y for TK03. The calibration age of MR is close to the older age limit of 9.4 cal. ka of the relative cosmic ray intensity record determined by Steinhilber et al. (2012), which would indicate that the SLHL \(^{10}\)Be production rates determined by any of the scaling schemes should be increased by a factor of 1.23. This would imply that the estimated bracketing \(P_{SLHL}\) values for the St and TK03 scalings would range from 4.93 to 5.56 at/g/y (4.8 to 5.5 at/g/y for spallation only).

Comparable results are obtained from the primary calibration dataset for PPT (Promontory Point Terrace, Utah), providing \(P_{SLHL}\) bracketing total rates of 4.04 and 4.48 at/g/y for St and TK03 even though the calibration age (18.3 cal. ka) is almost twice as old as the one for MR (Table 1). Although the older calibration age for PPT makes it less clear how to factor in the higher relative cosmic ray intensity determined thus far for only the past 9.4 cal. ka (Steinhilber et al., 2012); a simple extension of the factor of 1.23 would imply that the estimated bracketing \(P_{SLHL}\) values for the St and TK03 scalings would range from 4.97 to 5.51 at/g/y (4.9 to 5.4 at/g/y for spallation only). Results reported for the SCOT (Scotland, United Kingdom) dataset provide similar \(P_{SLHL}\) bracketing total rates of 4.22 and 4.58 at/g/y for St and TK03, respectively; an extrapolation of the factor of 1.23 to the 11.7 cal. ka calibration age would imply that the estimated bracketing \(P_{SLHL}\) values for St and TK03 scalings would range from 5.20 to 5.63 at/g/y (5.1 to 5.5 at/g/y for spallation only).

The MR, PPT and SCOT data sets provide SLHL \(^{10}\)Be production rates within about 5% of each other for any particular scaling scheme. Much more problematic is the primary calibration dataset HU08 based on glacial boulders from the high altitude (4859±9 m) and low latitude (13.9° S) Huancane site in Peru with a calibration age of 12.3 cal. ka. Samples from 10 glacial boulders give average \(P_{SLHL}\) ranging from 3.73 at/g/y for St to only 2.99 at/g/y for TK03, opposite to the low to high sense for these scaling schemes and as little as 60% of the more mutually consistent \(P_{SLHL}\) determined for MR, PPT and/or SCOT. The wide divergence of the Huancane \(P_{SLHL}\) may point to analytical shortcomings at the extreme of altitude ranges (Phillips et al., 2016b) (e.g., Fig. 5A). Contributing factors may be uncompensated effects of boulder surface erosion and weathering evidenced by 5-6 cm-high remnant pedestals (Kelly et al., 2015).
and degraded sample bulk densities of only 2.29 g/cm³ (Phillips et al., 2016a) compared to more
typical bulk sample densities of around 2.7 g/cm³ reported for the other calibration sites.

4. Significance for **10Be** exposure age at Allamuchy

Measured **10Be** concentrations for 13 boulders and glaciated surfaces at Allamuchy average 122000 at/g (Corbett et al., 2017) (**Table 1**). Using SCOT calibrations, for example, **P<sub>SLHL</sub>** values according to the various scaling schemes without the solar-factor would give exposure ages ranging from 22.7 to 24.2 cal. ka (22.3 to 23.8 cal. ka discounted 2% for muon contribution) for St and TK03, respectively, within but at the younger end of the age range of 25.2 ± 2.1 cal. ka reported with one standard deviation by Corbett et al. (2017) using the official CRONUS **10Be** production rates and array of scaling schemes. Similar exposure ages would be obtained for the MR and PPT calibrations, whose **P<sub>SLHL</sub>** are about the same as for SCOT. However, using the anomalously low **P<sub>SLHL</sub>** values determined from the HU08 calibration site would imply implausibly old exposure ages at Allamuchy, for example, 36.4 cal. ka using the **P<sub>SLHL</sub>** rate of 3.0 at/g/y with TK03 even when discounted 2% for muon contribution.

Extending the average solar-factor determined for the past 9.4 years (Steinhilber et al., 2012) effectively decreases the calculated exposure ages for Allamuchy by 3/4 across all calibration schemes to be less than 20 cal. ka. For example, the resulting exposure ages (discounted for 2% muon production) for SCOT would be 18.1 cal. ka for St and 19.3 cal. ka for TK03. These age estimates that factor in the documented solar influence are much closer to the 16 cal. ka **14C** AMS dates on earliest terrestrial plant macrofossils in deglacial sediments on the Laurentide terminal moraine (Peteet et al., 2012).

5. Discussion

A plausible explanation for an exposure age of 25 cal. ka that we regard as anomalously old by some 9,000 years for LIS recession from its terminal moraine in northeastern North America is an undervalued solar modulation factor in estimates of the **10Be** production rate in the widely used online exposure age calculators from the published versions (v2, v2.2, v2.3; Balco et al., 2008) to the current online-only version (v3) of the ‘online exposure age calculator formerly known as the CRONUS-Earth online exposure age calculator’ ([http://hess.ess.washington.edu/](http://hess.ess.washington.edu/); [https://cosmognosis.wordpress.com/2016/08/01/let-a-hundred-flowers-bloom/; last accessed 25May2023](https://cosmognosis.wordpress.com/2016/08/01/let-a-hundred-flowers-bloom/; last accessed 25May2023)). A solar modulation factor for cosmic ray flux determined for the past 9.4 cal. ka
(Steinhilber et al., 2012) increases $^{10}$Be production rates globally by an average of ~23%, which if applied to Allamuchy would reduce the previously calculated exposure age estimates of ~25 cal. ka (Corbett et al., 2017) to less than 20 cal. ka using any of the reliable (i.e., excluding HU08) $^{10}$Be primary calibration data (Table 1). Additional localized factors could further decrease the likely $^{10}$Be exposure ages for Allamuchy. For example, Corbett et al. (2017) pointed out that between-sample $^{10}$Be concentrations for the erratic boulders and glaciated surfaces vary several times more than expected from analytical uncertainties alone, which could reflect the presence of inherited $^{10}$Be in the sample population even though inheritance was ultimately discounted largely because of the widespread occurrence of glacial striations as indication of presumed sufficient abrasion of contaminating material from rock surfaces. Another contributing factor could stem from less shielding due to reduced atmospheric pressure during lowered sea level and/or from katabatic winds at the ice sheet margin in the early LIS recession stage (Staiger et al., 2007).

The inclusion of a solar modulation factor is expected to have broad ramifications to reported $^{10}$Be exposure ages if it is indeed as large on average as the 23% determined for the past 9.4 cal. ka (Steinhilber et al., 2012). We believe such a percentage is already supported by bringing exposure ages at Allamuchy into reasonable alignment with reliable marine (sea-level, meltwater) and continental (earliest deglacial terrestrial plants) dating of Laurentide recession. As shown in Table 1, the St scheme as one of the three remaining favored schemes in v3 of the ‘online exposure age calculator formerly known as the CRONUS-Earth online exposure age calculator’ results in $P(SLHL)$ for the MR, PPT and SCOT calibration sets of 4.0 to 4.2 at/g/y, in the neighborhood of what is currently regarded as the global value for $^{10}$Be exposure dating (Borchers et al., 2016; Phillips et al., 2016a). However, adding the solar modulation factor would increase $P(SLHL+S)$ to around 4.9 to 5.2 at/g/y and thus make $^{10}$Be exposure dates proportionately younger. Scaling schemes that effectively include geomagnetic secular variation, such as the statistical TK03 model, have higher $P(SLHL)$, which with solar modulation (according to Steinhilber et al., 2012) increase to around 5.5 at/g/y for spallation.

Adoption of ~4 at/g/y average $P(SLHL)$ in CRONUS-Earth was partly due to including legacy scheme St, which runs notably low (~3 at/g/y) for the mid-latitude, low to moderate altitude calibration sites (MR, PPT and SCOT) compared to other scaling schemes (Table 1) yet $P(SLHL)$ with scheme St for primary calibration site HU08 (3.73 at/g/y) is beguilingly close to
those of the other calibration sites (4.00, 4.04 and 4.22 at/g/y for MR, PPT and SCOT, respectively). St assumes a static geomagnetic field with no secular variation and yet the magnetic scaling factor shows a more erratic pattern as a function of latitude than the schemes based on effective vertical cutoff rigidity (Fig. 5B). We suggest that an average P(SLHL+S) of 5.5±0.1 at/g/y based on the TK03 scaling scheme for MR, PPT and SCOT calibration sites that includes a solar modulation factor of 1.23 (and is discounted 2% for muon contribution) provides a good working estimate for exposure age determinations as far back as 20 cal. ka, beyond which a lower average geomagnetic dipole moment (that would tend to increase $^{10}$Be production rates) needs to be taken into account. Compared to the currently accepted CRONUS consensus P(SLHL) of ~4.0 at/g/y, this would reduce exposure ages to nominally ~3/4 of the quoted values.

Another indication that the SLHL $^{10}$Be reference production rate is appreciably higher than the currently used level of ~4 at/g/y comes from attempts to include glacial isostatic adjustment (GIA) in exposure dating. For example, Lowell et al. (2021) report $^{10}$Be exposure dates using a SLHL $^{10}$Be production rate of 4.3 at/g/y from Balco et al. (2009) on glacial boulders along a 375 km transect just west of Lake Superior (see Fig. 1 for transect location) perpendicular to the retreating margin of the southwestern Labrador lobe of the LIS as delineated by radiocarbon isochrons (Dalton et al., 2020). According to the $^{10}$Be exposure dates, deglaciation occurred by ~18 cal. ka near the projected southwestern end of the transect at Kylen Lake and by ~10.5 cal. ka near Pillar at the northeastern end at a mean retreat rate of ~50 km/kyr (thick red line in Fig. 9). Over the same transect, isochrons of ice-margin retreat derived from radiocarbon ages of deglacial deposits converted to calendar years in Lowell et al. (2021) are up to 4 ka younger. A related problem emerges with the stated inability to correct the $^{10}$Be ages for changes in elevation due to uplift from GIA. Even updated GIA model ICE-6G (Peltier et al., 2015) produces adjusted $^{10}$Be ages that are deemed to be unacceptably up to ~10% older, for example, increasing the $^{10}$Be age from 17.4 to 19.2 cal. ka for sample AF-109 and 15.3 to 16.4 cal. ka for sample AF-110 near the southern end of the transect (Supplemental Material text 1.3 and Table S4 in Lowell et al., 2021). However, if a SLHL $^{10}$Be production rate of ~5.3 at/g/y that included a solar modulation factor of 1.23 was used, the $^{10}$Be date for sample AF-109 would be only 15.3 cal. ka (and sample AF-110 would be 13.4 cal. ka) after uplift correction with GIA model ICE-6G as implemented in the iceTEA online toolkit (Jones et al., 2019). The average
retreat rate would be faster (~70 km/kyr, dashed line in Fig. 9) and the recession trajectory would be mostly within the radiocarbon isochron error envelope.

The stated anchor of the transect to the southwest was a basal radiocarbon age of 18.1 cal. ka from sediment cores at Kylen Lake. However, Lund and Banerjee (1985) reported a major age reversal of several thousand years in bulk sediment radiocarbon ages at the base of one of the Kylen Lake cores with palynological evidence for the classic landscape ragweed disturbance (Ambrosia) up-core almost 1 cal. ka off, and warned that the radiocarbon dates were likely contaminated and too old. The problem was acknowledged by Lowell et al. (2021) but following an evaluation of radiocarbon dates from prior work as well as radiocarbon dates from a new sediment core from Kylen Lake in an unpublished thesis (Norris, 2019), a modeled basal age of 18.1 cal. ka from Kylen Lake was nevertheless used to anchor the transect in the shifting shoals of bulk sediment radiocarbon dating.

A more recent example of potential implications of recognizing significant solar modulation of cosmic ray flux is in a study of cosmogenic-nuclide concentrations in subglacial bedrock cores between Thwaites and Pope glaciers in Antarctic where Balco et al. (2023) argued that the West Antarctic Ice Sheet (WAIS) at the site was about 35 m thinner several thousand years ago and subsequently thickened to its present thickness. The conclusion basically followed from modeling the concentrations of \(^{10}\text{Be},^{26}\text{Al}\) and \(^{14}\text{C}\) measured in cores of the subglacial bedrock that were higher than expected from shielding by present ice thickness, implicitly assuming a uniform present-day cosmic ray flux over the entire Holocene. However, Steinhilber et al. (2012) showed that the average cosmic ray flux was about 50% higher about 7.5 cal. ka and decreased to modern levels by around 2.5 cal. ka before increasing to another series of peaks during the Spörer and Maunder solar minima in the last millennium before decreasing to present-day levels (Fig. 8B). This suggests an alternative interpretation of the variable cosmogenic-nuclide concentrations whereby the ice thickness for WAIS at that locale may have stayed approximately the same for the past ~6 cal. ka but a varying cosmic ray flux caused commensurate changes in the cosmogenic-nuclide production rate and hence accounted for much of the observed age-dependent pattern of their subglacial bedrock concentrations.

Other implications of a significant solar modulation factor exist but are less immediately obvious considering the wide range of SLHL \(^{10}\text{Be}\) production rates from around 6 to 4 at/g/y that
have been used in more than 30 years of published investigations. Normalization of these results to a consistent SLHL \( ^{10}\text{Be} \) production rate and scaling scheme would be revealing.

6. Conclusions

- The anomalously old \( ^{10}\text{Be} \) exposure dates for LIS recession in northeastern North America of \(~25\) cal. ka, such as at Allamuchy, which are inconsistent with independently documented timing of meltwater production and global sea level rise from the marine record and are not supported by \( ^{14}\text{C} \) AMS dates on terrestrial plant macrofossils in early deglacial sediments, point to a deficiency in the \( ^{10}\text{Be} \) exposure dating methodology.

- The incorporation of a published but apparently unutilized solar modulation factor results in an average increase by about a factor of 1.23 in cosmic ray intensity compared to the modern over the 9.4 cal. ka length of the currently available record. This decreases \( ^{10}\text{Be} \) exposure ages to about \( \frac{3}{4} \) of stated values and in the case of Allamuchy to less than 20 cal. ka, which works toward resolving the glaring age discrepancy with the marine record and reliable radiocarbon dates in the terrestrial realm.

- Secular variation in geomagnetic field directions could be conveniently represented in scaling schemes on time scales of several millennia and longer using a globally valid statistical field model (TK03) that incorporates secular variation with non-dipole components and results in SLHL \( ^{10}\text{Be} \) production rates for mid-latitude sites that are about 10% higher than with conventional models that have little (e.g., SHA.DIF.14k) to no (geocentric axial dipole or GAD) directional dispersion.

- Estimates of the axial dipole moment such as GGF100k have a time-integrated mean at about the present-day field value going back to \(~20\) cal. ka, decreasing to about 0.85 of the modern value only by \(~50\) cal. ka. Geomagnetic field strength thus does not appear to be a critical factor in exposure dating over the latest Pleistocene and Holocene.

- An average \( P(SLHL+S) \) of 5.5 at/g/y based on extending the published solar modulation factor of 1.23 for the past 9.4 cal. ka and using the TK03 magnetic scaling scheme for mid-latitude calibration sites produces a reasonable fit to GIA model ICE-6G of exposure age data from the Labrador Dome of the LIS along a transect just west of Lake Superior.
• Changes in observed $^{10}$Be production in subglacial bedrock due to known variable solar modulation provides an alternative explanation to changes in shielding from variation in thickness of West Antarctic ice-sheet, providing another line of evidence to test implications of large-amplitude solar modulation.

Author contributions

Authorship in alphabetical order. DP initiated the study and assessed radiocarbon dates, LL wrote the R-code and incorporated solar modulation influences, DK incorporated geomagnetic field models and prepared the draft of the manuscript with DP and LL.

Competing interests

The authors declare that they have no conflict of interest.

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References


Harmon, K., 1968. Late Pleistocene Forest Succession in Northern New Jersey. Rutgers University, New Brunswick, NJ.


Fig. 1. Synthesis of southern Laurentide ice sheet (white area) during Last Glacial Maximum from 26.0 - 18.7 cal. ka (figure from Wickert et al., 2023) based on revised dates from Dalton et al. (2020) modified to fit a marine chronology. Lakes are medium gray, land is dark gray, and oceans are light gray; associated Mississippi River drainage-basin extent shown by heavy black line. Box shows location of map in Fig. 3 and includes locale (small red open circle) of terminal moraine at Allamuchy and environs studied by Corbett et al. (2017); thick red line shows the approximate location of transect studied by Lowell et al. (2021) and shown in Fig. 9.
Fig. 2. Contrasting age estimates for deglaciation of southeastern lobe of Laurentide ice sheet (LIS) based on $^{14}$C AMS dating of earliest terrestrial plant macrofossils in deglacial sediments (Peteet et al., 2012) and on $^{10}$Be exposure dating for terminal moraine in Allamuchy area of New Jersey (Corbett et al., 2017), plotted with respect to major climate events (Last Glacial Maximum, (LGM), Heinrich events H1, H2 and H3, the Bolling-Allerod warm period (B-A) and the Younger Dryas cold period (YD), estimated ice-volume equivalent sea-level change (blue curve) since 35 cal. ka (from Lambeck et al., 2014), and summer insolation at 65°N (maroon curve, calculated with Paillard et al. (1996)). See Peteet et al. (2012) for additional climate proxies and discussion.
Fig. 3. $^{14}$C AMS dates on plant macrofossils in earliest deglacial sediments (red triangles; Peteet et al., 2012), $^{14}$C bulk sediment dates (open circles with references in labels), and $^{10}$Be exposure dates from glacial boulders and pavement within the box around Allamuchy Pond that outlines study region in Fig. 2 of Corbett et al. (2017) associated with the retreat of the southeastern lobe of the LIS in northern NJ and southern NY. Figure adapted from Corbett et al. (2017).
Fig. 4. Radiocarbon dates in context of litho- and bio-stratigraphy of cores from ponds and lakes in close proximity to terminal moraine of Laurentide ice sheet at LGM in northern NJ (see Fig. 3 for locations). Bulk sediment $^{14}$C dates from gyttja, silts, or clay (stratigraphic layers) are indicated in black; $^{14}$C AMS dates on identified terrestrial macrofossils are indicated in red (from Peteet et al., 2012).
Fig. 5. Altitude and latitude scaling factors. A) Altitude scaling factor at high latitude (HL) versus air pressure and equivalent altitude for St, the Lal/Stone empirical scaling scheme (Stone, 2000) and the analytical scaling scheme based on effective vertical cutoff rigidity, $R_c$ (Desilets et al., 2006; Lifton et al., 2014). B) Magnetic scaling factor for different schemes including empirical St (Stone, 2000) and those based on effective vertical cutoff rigidity, $R_c$, using Equation 2 of Lifton et al. (2014) to derive a magnetic scaling factor, $f(R_c)$ (Desilets et al., 2006) with modern magnetic moment (~80 Am$^2$) as functions of latitude according to different geomagnetic field models: GAD, geomagnetic axial dipole field with no dispersion; K118 (SHA), mean VGP from SHA.DIF.14k model of Pavón-Carrasco et al. (2014) for 0-14 cal. ka with equivalent Fisher precision parameter K=118; K27, GAD field with Fisher distribution of VGP of K=27; TK03, statistical geomagnetic field model of Tauxe and Kent (2004).
**Fig. 6.** Effective vertical cutoff rigidity ($R_c$) as a function of geomagnetic latitude for a representative range of dipole moments ($M$) relative to the modern value ($M_0$) calculated with Equation 2 of Lifton et al. (2014).
**Fig. 7.** A) Variations in geomagnetic axial dipole moment (ADM) for 0–50 cal. ka from Model GGF100k (blue wiggly line; Panovska et al., 2019). Present-day ADM is shown by horizontal line for reference. B) Cumulative ADM in 50-y intervals for Model GGF100k going back in time from the present (blue curve) and compared to cumulative ADM for a constant present Earth field dipole moment and for +10%, -10% and -15% of present Earth field dipole moment shown for reference (labeled red lines). C) Continuous calibration of high precision $^{14}$C AMS dates to calendar ages based on U-series dates on corals and other calibration data from 0 to 50 cal. ka (from Fairbanks et al., 2005).
**Fig. 8.** Panel A shows time-integrated common production rate of cosmogenic radionuclides for the last 9400 cal. years relative to the present day (1944-1988 CE) from data shown in panel B that is zoomed in to past millennium (panel C) and to last 350 years (panel D). Panels B, C, and D are from Steinhilber et al. (2012) where red circles and green curve are 22-year averages and yearly averages of calculated cosmic ray intensity, and annual sunspot numbers (SSN) plotted at the bottom. Grand solar minima are O: Oort, W: Wolf, S: Spörer, M: Maunder, D: Dalton and G: Gleissberg. Black dashed lines in each panel represent average cosmic ray intensity for 1944-1988 CE. Data from Steinhilber et al. (2012)) is available at https://www.ncei.noaa.gov/pub/data/paleo/climate_forcing/solar_variability/steinhilber2012.txt.
**Fig. 9.** Time-distance plot (adapted as base from Fig. 2 of Lowell et al., 2021) with additions described below) showing chronological constraints on Laurentide Ice Sheet retreat along a transect from Kylen Lake (Minnesota) in SW to Pillar (Ontario) in NE (see Fig. 1 for transect location). Red line with blue error envelope from Lowell et al. (2021) is their interpretation of retreat and hiatus pattern anchored to a $^{14}$C bulk sediment mean basal age of 18.1 cal. ka at Kylen Lake and constrained by $^{10}$Be exposure dates (black circles and error bars) using a SLHL $^{10}$Be production rate of 4.33 at/g/y from Balco et al. (2009). Green shaded area is uncertainty envelope of radiocarbon-based isochrons of LIS retreat from Dalton et al. (2020) converted to calendar years by Lowell et al. (2021). Dark gray open squares are samples from Table S4 of Lowell et al. (2021) with $^{10}$Be ages recalculated here with addition of a solar modulation factor of 1.23, which is equivalent to a SLHL $^{10}$Be production rate of 5.3 at/g/y (implied retreat sketched as solid orange line), and scaled by an average of +8% using glacial isostatic adjustment model ICE-6G (Peltier et al., 2015) as implemented by (Jones et al., 2019) according to (Lowell et al., 2021) (implied retreat sketched as dashed orange line).
Table 1. Primary $^{10}$Be calibration sites and Allamuchy site scaled with various schemes.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>MR</th>
<th>PPT</th>
<th>SCOT</th>
<th>HU08</th>
<th>Allamuchy</th>
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<td>11700</td>
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<td>TBD</td>
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<tr>
<td>P(SLHL+S): Cs/F*1.23, at/g</td>
<td>5.559</td>
<td>5.507</td>
<td>**5.629</td>
<td>3.663</td>
<td>**Age=19705 cal. yr</td>
</tr>
</tbody>
</table>
\(^{10}\)Be exposure dating parameters for the CRONUS-Earth primary calibration sites from Borchers et al. (2016) (Section S2) used to derive \(^{10}\)Be production rates normalized to sea level high altitude (SLHL) according to various scaling schemes: MR (Macaulay Ridge, NZ), PPT (Promontory Point Terrace, Utah), SCOT (Scotland), and HU08 (Huancane, Peru). Selected scaling schemes are St, the original empirical scaling scheme of Lal (1991) modified by Stone (2000); GAD, geocentric axial dipole where geographic and geomagnetic latitudes are equivalent in a static field; SHA, spherical harmonic model of 0–14 cal. ka paleomagnetic measurements (Pavón-Carrasco et al., 2014); TK03, statistical geomagnetic field model of Tauxe and Kent (2004). For GAD, SHA, and TK03, the effective vertical cutoff rigidity (Rc) is calculated using Equation 2 in Lifton et al. (2014), and the magnetic (f(Rc)), altitude (f(x)), and total scaling factor (F) are calculated using equations 6, 7 and 5, respectively, in Desilets et al. (2006). Geomagnetic dipole moment assumed to average to present-day value over exposure times less than ~20 cal. ka (see text). Also listed are published parameters for Allamuchy terminal moraine sites in New Jersey used to estimated \(^{10}\)Be exposure age for retreat of Laurentide ice sheet (Corbett et al., 2017); ages for each scaling scheme based on asterisked production rates according to SCOT primary calibration data for reference; ages corresponding to other primary calibrations can be readily calculated.
Supplementary Material

Section S1. Various routines used for heuristic purposes relevant to $^{10}$Be production rate scaling in the R programming language (R_Core_Team, 2018).

Section S2. CRONUS-Earth primary $^{10}$Be calibration sample data (Borchers et al., 2016) as lodged in evolving versions of the ICE-D production rate online database (Martin et al., 2017). Items highlighted in yellow in attached pages generated by version 1 of the ICE-D infrastructure were used to calculate quantities shown in Table 2. According to the ICE-D document header, “As of April 2022, updates and corrections will only be made in version 2, and version 1 will no longer be updated.” Hence including a copy of version 1 here is deemed useful for stability even though differences appear superficial with version 2 that currently (6/14/2023) resides at: https://version2.ice-d.org/production%20rate%20calibration%20data/cal_data_set/4.

Table S1. Effective vertical cutoff rigidity ($R_c$) and corresponding magnetic scaling factor ($f(R_c)$) as a function of latitude for various geomagnetic field models.

References


# Computes 10Be production rates according to Lal (1991)

lal.eq1 <- function(L, y) {
  L <- abs(L)
  y <- y/1000
  a <- approx(lam.bins, A, xout=L)$y
  b <- approx(lam.bins, B, xout=L)$y
  c <- approx(lam.bins, C, xout=L)$y
  d <- approx(lam.bins, D, xout=L)$y
  q <- a + b*y + c*y^2 + d*y^3
  return(q)
}

# Computes eq.2, eq. 3 and eq.4 from Stone (2000)
# JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 105, NO. B10, PAGES 23,753-23,759, OCTOBER 10, 2000

stone.eq2 <- function(ap, lambda) {
  lambda <- abs(lambda)
  A <- c(31.8518, 34.3699, 40.3153, 42.0983, 56.7733, 69.0720, 71.8733, 71.8733)
  B <- c(250.3193, 258.4759, 308.9894, 512.6857, 649.1343, 832.4566, 863.1927, 863.1927)
  C <- c(-0.083393, -0.089807, -0.106248, -0.120551, -0.160859, -0.199252, -0.207069, -0.207069)
D <- c(7.4260E-05, 7.9457E-05, 9.4508E-05, 1.1752E-04, 1.5463E-04, 1.9391E-04, 2.0127E-04)
E <- c(-2.2397E-08, -2.3697E-08, -2.8234E-08, -3.8809E-08, -5.0330E-08, -6.3653E-08, -6.6043E-08)
M.1013 <- c(0.587, 0.600, 0.678, 0.833, 0.933, 1.000, 1.000)
lam.bins <- c(0, 10, 20, 30, 40, 50, 60, 90)

# interpolate coefficients to the latitude (lambda)
a <- approx(lam.bins, A, xout=lambda)$y
b <- approx(lam.bins, B, xout=lambda)$y
c <- approx(lam.bins, C, xout=lambda)$y
d <- approx(lam.bins, D, xout=lambda)$y
e <- approx(lam.bins, E, xout=lambda)$y
M <- approx(lam.bins, M.1013, lambda)$y

## Altitude (atm. pressure) factor (eq.2)
S.lambda <- a + b*exp(-ap/150) + c*ap + d*ap^2 + e*ap^3

## Factor for isotope production by muon capture (eq.3)
M.lambda <- M * exp((1013.25 - ap)/242)

# fraction of spallogenic production at sea level for 10Be
f.sp <- 0.974

## Combined factor (eq.4)
F.lambda <- f.sp * S.lambda + (1-f.sp) * M.lambda

return(list(S.lambda = S.lambda, M.lambda = M.lambda, F.lambda = F.lambda))

## Computes vertical rigidity using eq.2 from Lifton et al. (2014)
# http://dx.doi.org/10.1016/j.epsl.2013.10.052
#
# lambda: magnetic latitude (deg)
# Mt:     the dipole moment at given time,
# Mo:     the reference dipole moment (2010 DGRF: 7.746e22 A m^2)
#
# lifton.eq2 <- function(lambda, Mt=7.746, Mo = 7.746){
lambda <- abs(lambda)
l <- lambda/180*pi  # in radians
c1 <- 6.89901
c2 <- 103.241
c3 <- 522.061
c4 <- 1152.15
c5 <- 1189.18
c6 <- 448.004
Rc1 <- (c1*cos(l) - c2*cos(l)^2 + c3*cos(l)^3 - c4*cos(l)^4 + c5*cos(l)^5 - c6*cos(l)^6)

# rectify the "bouncing" at high latitudes
zz <- which(lambda > 74.9)
Rc1[zz] <- 0
Rc <- (Mt/Mo) * Rc1

return(Rc)
}

###
# Computes eq.6 and eq.7 from Desilets et al. (2006)
# doi:10.1016/j.epsl.2006.03.051
#
# Computes the altitude scaling factors and
# scaling factor for the given magnetic rigidity.
# Total scaling factor is the product of the 2 factors
#
# ap: atmospheric pressure (hPa)
# Rc: rigidity (GV)
#
desilets.eq7 <- function(ap = 1013.25, Rc = 0){

  # convert hPa to gr/cm^2
  x <- ap * 1.0197

  # Desilets et al 2006 eq.6 (i.e., Dorman function)
  alpha <- 10.275
  kappa <- 0.9615
  f.Rc <- 1 - exp(-alpha * Rc^(-kappa))

  # coefficients from Table 2
  n   <- 1.0177E-02
  a   <- 1.0207E-01
  k   <- -3.9527E-01
  a0  <- 8.5236E-06
  a1  <- -6.3670E-07
  a2  <- -7.0814E-09
  a3  <- -9.9182E-09
  a4  <- -9.9250E-10
  a5  <- -2.4925E-11
  a6  <- 3.8615E-12
  a7  <- -4.8194E-13
  a8  <- -1.5371E-14

  ## polynomial coefficients of eq.5 and eq.7
  # note that c3 and c4 are negligible for any reasonable value of Rc
  c1 <- n/(1 + exp(-a * Rc^(-k)))
  c2 <- (a0 + a1*Rc + a2*Rc^2)/2
  c3 <- (a3 + a4*Rc + a5*Rc^2)/3
  c4 <- (a6 + a7*Rc + a8*Rc^2)/4

  # polynomial values for 1033 g/cm2 and site pressure
  p.1033 <- c1*1033 + c2*1033^2 + c3*1033^3 + c4*1033^4
  p.x  <- c1*x + c2*x^2 + c3*x^3 + c4*x^4

  # Desilets et al 2006 eq.5
  lambda.1033 <- (1033 - x) / (p.1033 - p.x)

  ## Desilets et al. (2006) eq.7 (also eq. 2 of Desilets and Zreda, 2003)
  # note that this equation has typos in Desilets et al. (2006)
  f.x  <- exp((1033 - x)/lambda.1033)
## Computes the averaged $^{10}$Be production factor from Steinhilber et al. 2012
# doi:10.1073/pnas.1118965109
# Need file "steinhilber2012_TSI.txt" in the working folder
# age: age (yr BP)
# Returns the inverse of Steinhilber's production rate averaged between
today and the given age.
# For instance:
# steinhilber.pf(1000) returns a production factor pf = 0.7863477,
# meaning that present day production is 0.7863477 times the averaged
# production of the last 1000 years, hence averaged production in the
# last 1000 yr was almost 30% larger than present day production.
# steinhilber.pf <- function(age){
if(age > 9389) {
  warning("steinhilber.pf: age too old, production factor = 1/1.231\n")
  pr <- 1.231
} else {
  dat <- read.table("steinhilber2012_TSI.txt",
header = TRUE,
skip = 156)
  idx <- which(dat$Year < age)
  pr <- mean(dat$X1.PC[idx])
}
return(sf=1/pr)
}

## Computes total cosmogenic production factor with constant geocentric axial dipole
# (GAD) field, air pressure and age (age is used only to include the variability of
# production rate from Steinhilber et al. 2012).
# Combines lifton.eq2, desilets.eq7, and Steinhilber (2012) data (if age is given).
# By default the "steinhilber factor" is disregarded (i.e. set to 1).
# If age > 9400 the steinhilber factor is set to 1/1.231 (with a warning message)
# lambda: magnetic latitude (deg)
# ap: air pressure (hPa)
# age: age (yrBP) - default NA i.e., no steinhilber correction
# ...: additional arguments passed to lifton.eq2 (e.g. virtual dipole moment Mt)
# cosmogenic.factor.GAD <- function(ap=1013.25, lambda, age = NA, ...){
  Rc <- lifton.eq2(lambda, ...)
df <- desilets.eq7(ap, Rc=Rc)
  return(list(mag.factor=Rc, alt.factor=x, Total.factor=Rc * x))
}
if (is.na(age)){
    sf <- 1
} else {
    sf <- steinhilber.pf(age)
}

return(list(mag.factor = df$mag.factor,
    alt.factor = df$alt.factor,
    solar.factor = sf,
    total.factor=df$mag.factor * df$alt.factor * sf))

###
# Computes total cosmogenic factor with the GAD hypothesis
# and the variable Axial Dipole Moment (ADM) of Panovska et al. (2019)
#
# It needs the following file in the working directory:
# (downloaded from https://earthref.org/ERDA/2384/)
# "ADM_GGF100k.txt"
#
# lambda: magnetic latitude (deg)
# ap:  air pressure (hPa) - default 1013.25
# age: age (yrBP)
# Mo:  present day geomagnetic dipole moment (1e22 A m^2) - default 7.746
#
# cosmogenic.factor.ADM <- function(ap = 1013.25, lambda, age, Mo = 7.746){
    age <- age/1000  # age in kyr
    if(age > 100){stop("Age must be smaller than 100 kyrBP")}
    adm <- read.table("ADM_GGF100k.txt", header = FALSE, skip = 1)
    idx <- which(adm$V1 <= age)
    ## Compute Rc and df for all ages
    Rcs <- lifton.eq2(lambda, Mt = adm$V2[idx], Mo = Mo)
    df <- desilets.eq7(ap, Rc=Rcs)
    ## return the average values (i.e., integrate through time)
    mag.factor <- mean(df$mag.factor)
    alt.factor <- mean(df$alt.factor)
    Rc <- mean(Rcs)
    return(list(mean.Rc=Rc, mag.factor=mag.factor, alt.factor=alt.factor, total.factor=mag.factor * alt.factor))
}

###
# Computes total cosmogenic factor using geomagnetic latitude and field intensity
# from the SHA.DIF.14k geomagnetic model, air pressure and age < 14000 yr BP.
The Steinhilber production factor can be included for ages < 9400 yr BP

Needs the following files in the working directory:

- "coeff_SHA.DIF.14k.dat"
- "steinhilber2012_TSI.txt" (if steinhilber = TRUE)

- ap: air pressure (hPa) - default 1013.25
- lat: geographic latitude (deg)
- long: geographic longitude (deg) - default 0
- age: age (yrBP) - must be < 14000
- steinhilber: if TRUE includes the variability of production rate - default FALSE
- dir.only: if TRUE ignore field intensity - default FALSE

```r
cosmogenic.factor.SHAdif <- function(ap = 1013.25,
                                      lat,
                                      long = 0,
                                      age,
                                      steinhilber = FALSE,
                                      dir.only = FALSE)
{
  if (age > 14000){
    warning("cosmogenic.factor.SHAdif: calculation is inaccurate for age > 14000\n")
  }

  # SHA.DIF.14k uses ages in yrAD
  ageAD <- 1950 - age

  ## This is the complete dataset of Gauss coefficient from file "sha-dif-14k.rar"
  ## (downloaded it from https://earthref.org/ERDA/1897/)
  #
  ## read SHA table of sha_dif_14k
  sha <- read.table("coeff_SHA.DIF.14k.dat", header = T)

  ## select only dipole coefficients
  tmp1 <- subset(sha, subset = order == 1 & rank == 0, select = c(Age, g0=g))
  tmp2 <- subset(sha, subset = order == 1 & rank == 1, select = c(g, h))
  sha.dipole<-data.frame(Age = tmp1$Age, g01 = tmp1$g, g11 = tmp2$g, h11 = tmp2$h)

  ## compute dipole axis from Gauss dipolar coefficients
  dipole <- gauss2dipole(sha.dipole$g01, sha.dipole$g11, sha.dipole$h11)
  dipole$Age <- sha.dipole$Age
  site <- c(long, lat)

  ## select relevant ages (AD)
  idx <- which(ageAD <= dipole$Age)

  ## Magnetic colatitude of site is the angle (great circle distance) between magnetic pole and site
  Mlat <- 90 - angle(cbind(dipole$phi[idx], 90-dipole$theta[idx]), site, all=T)

  idx2 <- which(Mlat >90)
  Mlat[idx2] <- 180-Mlat[idx2]

  if (dir.only){
    # disregards the variability of dipole intensity
    Rc <- lifton.eq2(lambda = Mlat)
  } else {
    # direction and intensity (Mo = 30.1 µT from IGRF2000)
  }
}
```
Rcs <- lifton.eq2(lambda = Mlat, Mt = dipole$Bo[idx], Mo = 30.1)
}
df <- desilets.eq7(ap, Rc=Rcs)

mf <- mean(df$mag.factor) # mean magnetic factor
af <- mean(df$alt.factor) # mean altitude factor
Rc <- mean(Rcs)            # mean Rc

if (steinhilber){
  sf <- steinhilber.pf(age)
} else {
  sf <- 1
}

return(list(mean.Rc=Rc, mag.factor=mf, alt.factor=af, solar.factor=sf, total.factor=mf * af * sf))

##
# Computes total cosmogenic factor using the geomagnetic latitudes
# from TK03 geomagnetic model and air pressure
#
# ** It needs pmag.py installed to compute the TK03 dataset **
# https://pmagpy.github.io/PmagPy-docs/intro.html
#
#
## compose the command string to run tk03.py
# this makes 5000 surrogates and store in temporary file named tk03.tmp
# command string will be: tk03.py -n 5000 -lat xx > tk03.tmp
cmd.str <- paste("tk03.py -n 5000 -lat ", lat)
cmd.str <- paste(cmd.str, " > tk03.tmp")

## loop the multiple command strings (i.e., lat)
# for each cmd.str compute the Rc
r <- length(cmd.str)
mf <- mat.or.vec(nr = r, nc = 1)
af <- mat.or.vec(nr = r, nc = 1)
mRc <- mat.or.vec(nr = r, nc = 1)
for (i in 1:r){
  system(cmd.str[i])
  TK03 <- read.table("tk03.tmp", skip = 7) # skip = 7 remove some warning lines in my computer
  pole <- vgp(TK03[, 1:2], lat[i], 0)
  Mlat <- 90 - angle(cbind(pole[,2], pole[,1]), c(0, lat[i]), all=T) # pls note that latitudes can be negative

  # compute vertical rigidities
  Rc <- lifton.eq2(Mlat)

  # compute scaling factors
  df <- desilets.eq7(ap, Rc=Rc)
mf[1] <- mean(df$mag.factor)  # mean magnetic factor
af[1] <- mean(df$alt.factor)  # mean altitude factor
mRc[1] <- mean(Rc)
}

# make a table with results
results <- data.frame(Command.string = cmd.str,
                       latitude = lat,
                       mean.Rc = mRc,
                       mag.factor = mf,
                       alt.factor = af,
                       total.factor = mf * af)

# remove the temporary file
system("rm tk03.tmp")
return(results)
}

##
# conversion from height or altitude (meters, m), to air pressure (hPa) at temperature = 15 C
h2p <- function(h) {
  ap <- 101325 * (1 - 2.25577e-5 * h)^5.25588
  return(ap/100)
}

##
# Compute the dipole axis direction and Bo given the Gauss coefficient of order 1
#
gauss2dipole <- function(g01, g11, h11){
  Bo <- sqrt(g01^2 + g11^2 + h11^2)
  c1 <- sqrt(g11^2 + h11^2)
  theta <- acos(-g01/Bo) *180/pi
  phi <- acos(-g11/c1) *180/pi
  idx <- which(h11 > 0)
  phi[idx] <- -phi[idx] + 360
  return(list(theta = theta, phi = phi, Bo = Bo))
}

##
# angle between directions dir1 and dir2, in deg
angle <- function(dir1, dir2, all = FALSE){
  P1 <- ai_lmn(dir1)
  P2 <- ai_lmn(dir2)
  A <- acos(P1 %*% t(P2))*180/pi
  if(!all) {A <- diag(A)}
  return(A)
}
```r
##
# from azimuth and anclination to directors cosine
ai_lmn <- function(di){
di <- as.matrix(di)
if(ncol(di) == 1) {di <- t(di)}
di <- di*pi/180  # in rad
l <-cos(di[,2]) * cos(di[,1])
m <-cos(di[,2]) * sin(di[,1])
n <-sin(di[,2])

lmn <- cbind(l, m, n)
colnames(lmn) <- c("l", "m", "n")
return(lmn)
}
##
#  Calculate the (Virtual) Geomagnetic Pole given the site coordinates
#  and the (paleo)magnetic direction.
#  Angles are in deg.
#  dir: column matrix with declination and inclination
#
vgp <- function(dir,  site.lat,  site.long)
{
dec <- dir[,1]
inc <- dir[,2]
r <- pi/180

P <- atan2(2, tan(inc*r)) # magnetic colatitude in radiants

# Buttler eq: 7.2 (radiants)
vgp.lat <- asin(sin(site.lat*r)*cos(P) + cos(site.lat*r)*sin(P)*cos(dec*r))

# Buttler eq: 7.3
a <-round((sin(P)*sin(dec*r))/cos(vgp.lat), 10)
beta <- asin(a)*180/pi

vgp.long <- site.long + 180 - beta
idx <- which(cos(P) >= (sin(site.lat*r)*sin(vgp.lat)))
vgp.long[idx] <-site.long+beta[idx]  # overwrite condition above

vgp.lat <- vgp.lat *180/pi  # VGPlat in deg

return(cbind(lat = vgp.lat, long = vgp.long))
}
```
Table S1. Effective vertical cutoff rigidity ($R_c$) and corresponding magnetic scaling factor ($f(R_c)$) as a function of latitude for various geomagnetic field models.

<table>
<thead>
<tr>
<th>Lat</th>
<th>$R_c$ (GV)</th>
<th>$f(R_c)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GAD</td>
<td>SHA</td>
</tr>
<tr>
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<td>14.665</td>
</tr>
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</tr>
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</tbody>
</table>
GAD is mean effective vertical cutoff rigidity ($R_c$, in GV) as a function of absolute latitude (Lat) at sea level calculated using Equation 2 of Lifton et al. (2014); SHA is mean $R_c$ for VGP distribution calculated from dipole coefficients for 14 cal. ka to present from Pavón-Carrasco et al. (2014); K27 is mean $R_c$ for VGP distribution with Fisher’s precision parameter $K=27$; and TK03 is mean $R_c$ for the statistical geomagnetic field model of Tauxe and Kent (2004). The corresponding magnetic scaling factors ($f(R_c)$) were calculated with mean $R_c$ using Equation 6 with Dorman function in Desilets et al. (2006). Mean and standard deviation (SD) are for GAD, SHA, K27, and TK03 entries for $R_c$ and for $f(R_c)$.