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Improved representation of laminar and turbulent sheet flow in subglacial drainage models

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	transition model and its improved performance relative to the standard turbulent model, we recommend its use for transient simulations of subglacial drainage.

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Improved representation of laminar and turbulent sheet flow in subglacial drainage models

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ABSTRACT.

Subglacial hydrology models struggle to reproduce seasonal drainage patterns that are consistent with observed subglacial water pressures and surface velocities. We modify the standard sheet-flow parameterization within a coupled sheet–channel subglacial drainage model to smoothly transition between laminar and turbulent flow based on the locally computed Reynolds number in a physically consistent way (the “transition” model). We compare the transition model to standard laminar and turbulent models to assess the role of the sheet-flow parameterization in reconciling observed and modelled water pressures under idealized and realistic forcing. Relative to the turbulent model, the laminar and transition models improve seasonal simulations by increasing winter water pressure and producing a more prominent late-summer water pressure minimum. In contrast to the laminar model, the transition model remains consistent with its own internal assumptions across all flow regimes. Based on the internal consistency of the transition model and its improved performance relative to the standard turbulent model, we recommend its use for transient simulations of subglacial drainage.

27 1 INTRODUCTION

28 The subglacial drainage system beneath the flanks of the Greenland Ice Sheet is subject to seasonal vari-
29 ations in surface melt input, resulting in strong seasonal cycles in subglacial water pressure and ice flow
30 (e.g., Joughin and others, 2008; Moon and others, 2014; Davison and others, 2020; Vijay and others, 2021).
31 The seasonal velocity patterns, and how they vary with increasing volumes of surface melt, are key to
32 understanding ice-discharge-related sea-level contributions from Greenland (e.g., King and others, 2020).
33 However, it remains difficult to model seasonal water pressure and corresponding ice-flow velocities (e.g.,
34 Koziol and Arnold, 2018; Cook and others, 2022; Ehrenfeucht and others, 2023) that are consistent with
35 observations of water pressure and ice velocity (e.g., Andrews and others, 2014; Moon and others, 2014;
36 Nienow and others, 2017), limiting the ability of existing models to explain ice-flow patterns and their
37 seasonal variations.

38 Modern subglacial hydrology models represent water flow through a variety of flow elements, most
39 commonly including efficient drainage through R-channels (Röthlisberger, 1972) and inefficient drainage
40 through linked cavities (Kamb, 1987). Models take different forms (e.g., Flowers, 2015), including those with
41 coupled distributed-channelized flow and spatially extensive channel networks (e.g., Werder and others,
42 2013; Hewitt, 2013; Hoffman and others, 2018), as well as those comprised of a single set of flow elements
43 that transition between inefficient and efficient drainage (Schoof, 2010; Sommers and others, 2018; Felden
44 and others, 2023). Some models represent physical processes in more detail, for example by including a
45 weakly connected drainage system (e.g., Hoffman and others, 2016), while others trade process detail for
46 computational efficiency (e.g., de Fleurian and others, 2014; Bueler and van Pelt, 2015).

47 Models that explicitly represent distributed and channelized flow elements (e.g., Werder and others,
48 2013; Hewitt, 2013; Hoffman and others, 2018) capture much of the presently understood physics of real
49 subglacial drainage and have had success when applied to steady-state ice-sheet hydrology (e.g., Hager and
50 others, 2022), with modelled drainage pathways resembling those inferred from radar data (e.g., Dow and
51 others, 2020). However, these models have difficulty producing realistic water-pressure variations when
52 applied to ice-sheet-scale domains and forced with seasonally varying surface melt inputs. Specifically,
53 models tend to (1) underpredict winter water pressures (de Fleurian and others, 2018; Poinar and others,
54 2019; Ehrenfeucht and others, 2023) compared to winter water pressure inferred from seasonal velocity
55 patterns (e.g., Moon and others, 2014; Vijay and others, 2021) and observed in borehole water pressures

(e.g., Andrews and others, 2014; Wright and others, 2016) (c.f., Downs and others, 2018), (2) fail to capture the late-summer pressure minimum (e.g., Koziol and Arnold, 2018; Cook and others, 2020) that is inferred from typical Greenland outlet glacier velocity records (e.g., Davison and others, 2020), and (3) require a priori assumptions about distributed flow being fully laminar or turbulent (e.g., Werder and others, 2013; Hewitt, 2013). It is unclear whether the assumptions in (3) hold across the typical spatiotemporal domain of these models. Resolving the discrepancies enumerated above is important for capturing the complete relationship between surface melt, subglacial drainage, and ice flow in numerical models.

Most subglacial drainage models require specification of the relationship between water flux or discharge and the hydraulic potential gradient driving flow at the scale of drainage elements. Here we investigate the role of this relationship within distributed drainage components in controlling seasonal pressure variations as modelled with the Glacier Drainage System (GlaDS) model (Werder and others, 2013), a representative example of an explicitly channel-resolving model. We compare seasonal water-pressure variations modelled for different flux models to assess the influence on the shortcomings identified above. On the basis of our results, we make recommendations for the parameterization of distributed water flux in this popular class of channel-resolving drainage models.

2 METHODS

2.1 Subglacial hydrology model

Subglacial drainage is modelled with the Glacier Drainage System (GlaDS) model (Werder and others, 2013) as implemented in MATLAB (commit 040032e). GlaDS conceptualizes subglacial water flow occurring through a distributed drainage system composed of linked cavities and through an efficient drainage system composed of R-channels (Schoof and others, 2012; Hewitt and others, 2012; Werder and others, 2013). GlaDS is a representative example of the broader class of multi-component models that share common physical processes (e.g., Hewitt, 2013; Hoffman and others, 2018), and primarily differs in the discrete nature of subglacial channels from models that represent individual elements as transitioning between distributed and channelized flow (e.g., Schoof, 2010; Sommers and others, 2018; Felden and others, 2023).

GlaDS requires specification of a number of parameters that control the formation of subglacial cavities, water flow within distributed and channelized drainage elements, basal sliding, englacial water storage, and the strength of sheet-channel coupling. Constraining drainage model parameters with direct measurements is difficult and has only rarely been done for a few model parameters (e.g., Werder and others, 2009; Pohle

85 and others, 2022). Inferring parameter values via drainage model inversions (e.g., Irrazaval and others,
86 2021; Brinkerhoff and others, 2021) is a promising direction given a variety of observational data sources
87 (e.g., Nanni and others, 2021; Derkacheva and others, 2021; Rada Giacaman and Schoof, 2023), however,
88 the limited availability of observational data continues to make parameter inference challenging. In this
89 study, model parameter values (Table 1; Section S1.1) are chosen to obtain summer water pressures near
90 overburden with channels extending ~ 30 km inland. These values are similar to existing model applications
91 to Greenland-scale catchments with seasonal melt forcing (e.g., Gagliardini and Werder, 2018; Downs and
92 others, 2018; Cook and others, 2022). The size of the controlling bed obstacle (including both the bump
93 height h_b and the bump length l_b), the width of sheet flow contributing to channel discharge (l_c), and the
94 channel conductivity (k_c) in particular are larger here than typically used for alpine glaciers (e.g., Werder
95 and others, 2013) or steady state Antarctic applications (e.g., Dow and others, 2022; Hager and others,
96 2022), potentially reflecting the physically larger scale compared to alpine glaciers and the increased size
97 of drainage elements compared to Antarctic applications. Of these key parameters (h_b , l_b , and k_c), the
98 greatest sensitivity is to the bed bump height since it controls the rate of cavity opening and sets the
99 typical sheet thickness.

100 We intentionally disallow cavities from opening by ice creep when water pressure exceeds ice overburden
101 by setting the ice creep constant $\tilde{A}_s = 0$ when $p_w > p_i$. We expect that unrepresented physical mechanisms
102 would take over when p_w exceeds p_i (e.g., Tsai and Rice, 2010; Schoof and others, 2012; Dow and others,
103 2015). Based on model experiments, allowing cavities to creep open as a rough approximation of these
104 mechanisms leads to undesirable behaviour: cavities grow arbitrarily large within overpressurized regions,
105 preventing channels from developing and leading to persistent and extensive pressure above overburden.
106 Disabling creep opening is therefore a suitable modelling choice for the configuration presented here (Section
107 S1.2).

108 While GlaDS is a representative example of a channel-resolving subglacial drainage model, there are
109 physical processes that are missing in its formulation, especially the representation of hydraulically uncon-
110 nected or weakly connected bed patches (e.g., Murray and Clarke, 1995; Andrews and others, 2014; Hoffman
111 and others, 2016). Since GlaDS represents only hydraulically connected drainage, winter water pressures
112 may be expected to be lower than observations of winter water pressure within disconnected patches. For
113 example, Rada Giacaman and Schoof (2023) report mean winter water pressure $\sim 90\%$ of overburden within
114 hydraulically connected boreholes and $> 100\%$ of overburden for hydraulically unconnected boreholes for

Table 1. Constants (top group) and model parameters (bottom group) for GlaDS simulations.

Symbol	Description	Value	Units
ρ_w	Density of water	1000	kg m^{-3}
ρ_i	Density of ice	910	kg m^{-3}
g	Gravitational acceleration	9.81	m s^{-2}
c_w	Specific heat capacity of water	4.22×10^3	J kg^{-1}
c_t	Pressure melting coefficient	-7.50×10^{-8}	K Pa^{-1}
ν	Kinematic viscosity of water at 0°C	1.793×10^{-6}	m s^{-2}
k_s	Effective laminar sheet conductivity	0.05	Pa s^{-1}
α_s	Sheet-flow exponent	$[\frac{5}{4}, \frac{3}{2}, 3]$	
β_s	Sheet-flow exponent	$[\frac{3}{2}, 2]$	
k_c	Channel conductivity	0.5	$\text{m}^{3/2} \text{s}^{-1}$
α_c	Channel-flow exponent	5/4	
β_c	Channel-flow exponent	3/2	
h_b	Bed bump height	0.5	m
l_b	Bed bump length	10	m
l_c	Width of sheet-flow contributing to channel	10	m
e_v	Englacial porosity	1×10^{-4}	
ω	Laminar-turbulent transition parameter	1/2000	
u_b	Basal velocity	30	m a^{-1}
\tilde{A}^a	Rheological parameter for creep closure	1.78×10^{-25}	$\text{s}^{-1} \text{Pa}^{-3}$
\tilde{A}_s	Rheological parameter for creep when $N < 0$	0	$\text{s}^{-1} \text{Pa}^{-3}$
n	Ice-flow exponent	3	
\dot{m}_s	Basal melt rate	0.05	m w.e. a^{-1}

^a \tilde{A} differs from the canonical rheology parameter A by a factor of $\frac{2}{27}$. The listed value for \tilde{A} corresponds to the recommended value $A = 2.4 \times 10^{-24} \text{ s}^{-1} \text{Pa}^{-3}$ for temperate ice (Cuffey and Paterson, 2010)

115 a small alpine glacier.

116 **2.2 Sheet-flow model**

117 The distributed water flux parameterization used by GlaDS and similar models (e.g., Hewitt, 2013; Hoffman
 118 and others, 2018) assumes that water flow is exclusively laminar or turbulent. We aim to test the validity
 119 of this assumption and develop a parameterization that can transition between laminar and turbulent
 120 flow depending on local drainage characteristics. Previously, progress has been made in addressing the
 121 shortcomings listed above through adjustments to the distributed drainage parameterization, including
 122 representing flow within the distributed drainage system as laminar (Hewitt, 2013; Banwell and others,
 123 2016; Gagliardini and Werder, 2018; Cook and others, 2022), by explicitly parameterizing sheet conductivity
 124 as a function of surface melt rate (e.g., Downs and others, 2018) or by including a Reynolds-number-
 125 dependent transmissivity (Sommers and others, 2018, 2023). These models share the common feature that
 126 resistance to water flow in the distributed drainage system is sensitive to the volume of water supplied
 127 from surface and basal melt to the subglacial system. This sensitivity is obtained in different ways, but
 128 with similar impacts on the modelled winter water pressure. Therefore, adjusting the distributed water
 129 flux parameterization to include both laminar and turbulent flow (e.g., Sommers and others, 2018) is a
 130 promising direction to improve modelled seasonal water pressure variations.

131 *2.2.1 Standard sheet-flow model*

132 We consider two primary forms for the distributed water flux parameterization with GlaDS. The standard
 133 discharge-per-unit-width (\mathbf{q}) parameterization for subglacial drainage models intends to represent the aver-
 134 age flux through many sub-grid-scale linked cavities (e.g., Werder and others, 2013; Hewitt, 2013; Hoffman
 135 and others, 2018) and can be written

$$\mathbf{q} = -k_s h^{\alpha_s} |\nabla \phi|^{\beta_s - 2} \nabla \phi, \quad (1)$$

136 for conductivity k_s , water thickness h , hydraulic potential ϕ , and exponents α_s and β_s .

137 Choosing values for α_s and β_s requires an assumption about the relationship between water flux, cavity
 138 height, and the hydraulic potential gradient. Values of $\alpha_s = 3$ and $\beta_s = 2$ correspond to purely laminar
 139 flow (e.g., Creyts and Schoof, 2009; Hewitt, 2013; Cook and others, 2022), while $\alpha_s = 5/4$ and $\beta_s = 3/2$ are
 140 typically explained as representing fully turbulent flow according to the Darcy–Weisbach relationship (e.g.,

141 Schoof and others, 2012; Werder and others, 2013; Hoffman and others, 2018). It is worth noting that the
 142 parameterization for channel discharge is written in an analogous way, with the same interpretation of the
 143 exponents α_c and β_c .

144 The validity of the laminar or turbulent assumption can be assessed by inspecting the Reynolds number,
 145 Re . In the context of standard fluid dynamics, the Reynolds number predicts whether a specified flow is
 146 laminar or turbulent. For a general flow with representative velocity V , length scale D , and for a fluid
 147 with kinematic viscosity ν , the Reynolds number is the ratio of the inertial and viscous forces, $Re = \frac{VD}{\nu}$.
 148 In the context of the discharge-per-unit-width parameterization (Eq. 1), the length scale D is set to the
 149 water sheet thickness h , so the Reynolds number becomes $Re = \frac{q}{\nu}$, where $q = |\mathbf{q}| = VD$.

150 The transition between laminar and turbulent flow is best understood for the simple case of flow
 151 through circular pipes. In this case, the empirical relationship between Re and the Darcy friction factor
 152 f_D is summarized by the Moody diagram (Moody, 1944), which demonstrates the clear differences in the
 153 behaviour of laminar and turbulent flows (Fig. 1). Laminar flow results in an inverse relationship between
 154 Re and f_D that is independent of roughness (straight line in Fig. 1). Fully turbulent flow is represented
 155 by the friction factor being independent of Re as $Re \rightarrow \infty$. The transition from laminar to fully turbulent
 156 flow can be approximated by the Colebrook–White equation (Colebrook and White, 1937).

157 The fully turbulent behaviour from the Moody diagram can be carried over to the context of distributed
 158 subglacial water flow through a macroporous sheet by writing the Darcy–Weisbach equation (e.g., Moody,
 159 1944) for flow between parallel plates and in terms of the flux q instead of the flow velocity (Section S1.3.1,
 160 Eq. S.4). By doing this, fully turbulent flow would require a flow exponent $\alpha_s = 3/2$, as in the SHAKTI
 161 model (Sommers and others, 2018) and in contrast to the assumed value of $5/4$ for GlaDS and similar
 162 models; however, given the conceptual differences between flow through rough pipes, on which the Moody
 163 diagram is based, and the subglacial linked cavity system, we test the sensitivity of modelled water pressure
 164 to turbulent flow exponent values $\alpha_s = 3/2$ and $\alpha_s = 5/4$. We denote the model using Eq. (1) with $\alpha_s = 5/4$
 165 “turbulent $5/4$ ”, with $\alpha_s = 3/2$ “turbulent $3/2$ ”, and with $\alpha_s = 3$ and $\beta_s = 2$ as “laminar” (Table 2). All
 166 models use $\beta_s = 3/2$ to represent turbulent flow.

167 2.2.2 Sheet-flow model with laminar–turbulent transitions

168 Equation (1) assumes that water flow everywhere and at all times is either purely laminar or purely
 169 turbulent. To remove this limitation and develop a model appropriate for the entire Re range, we replace

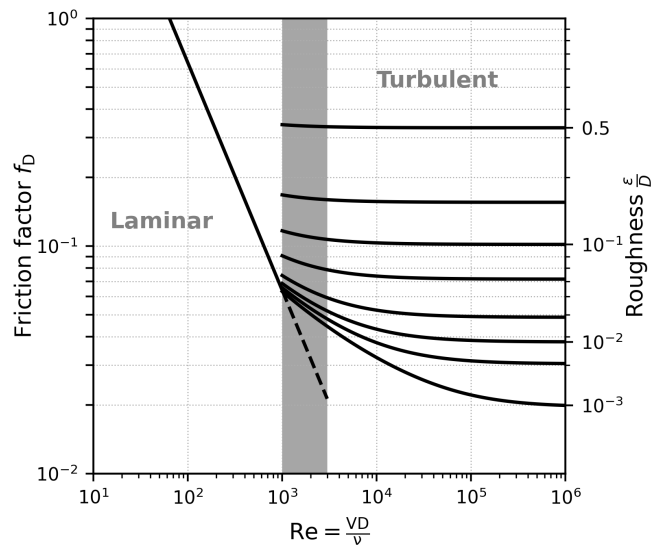


Fig. 1. Moody diagram, representing the friction factor $f_D = \frac{h_1}{(\frac{L}{D}) \frac{V^2}{2g}}$ (for head loss h_1 over a pipe of length L , diameter D , and with flow velocity V), as a function of the Reynolds number $Re = \frac{VD}{\nu}$ for different relative roughness scales (ε). The transition region (shaded grey, $1000 \leq Re \leq 3000$) separates regions of laminar flow and turbulent flow. The laminar friction factor is $f_D = \frac{64}{Re}$ (Moody, 1944), and the friction factor in the transition and turbulent regimes is computed using the Colebrook-White equation (Colebrook and White, 1937).

Eq. (1) with a model that represents both laminar and turbulent flow, with the partitioning governed by the local Reynolds number:

$$-k_s h^3 \nabla \phi = \mathbf{q} + \omega Re \left(\frac{h}{h_b} \right)^{3-2\alpha_s} \mathbf{q}, \quad (2)$$

for bed bump height h_b . Substituting $Re = \frac{q}{\nu}$ yields a quadratic equation that can be solved exactly for q (Eq. S6–S8, Table 2). The transition parameter ω governs partitioning between laminar and turbulent flow, with the transition occurring at approximately $Re = 1/\omega$. The exponent α_s controls the behaviour of the model in the fully turbulent limit ($\omega Re \gg 1$).

We call Eq. (2), which transitions between laminar and turbulent flow based on the local Reynolds number, the “transition” model. In the laminar regime ($\omega Re \ll 1$), the first term on the right hand side dominates and Eq. (2) reduces to the laminar model (Eq. 1 with $\alpha_s = 3$ and $\beta_s = 2$). In the turbulent regime ($\omega Re \gg 1$), the second term on the right hand side dominates and Eq. (2) reduces to the turbulent model (Eq. 1 with α_s specified by the turbulent assumption and $\beta_s = 3/2$) with an effective turbulent conductivity given by $k_t^2 = k_s \frac{\nu}{\omega} h_b^{3-2\alpha_s}$. In the intermediate regime ($\omega Re \sim 1$), Eq. (2) smoothly blends laminar and turbulent flow. Table 2 summarizes the five flux parameterizations obtained by applying Eqs. (1) and (2) with turbulent flow exponents $\alpha_s = 5/4$ and $\alpha_s = 3/2$.

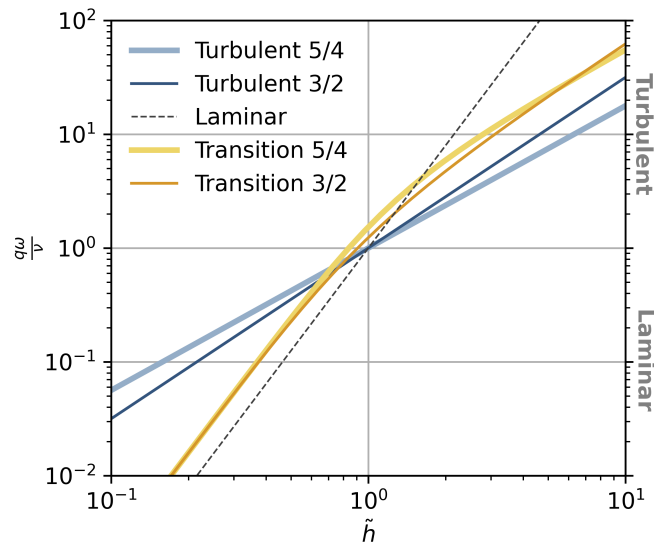


Fig. 2. Scaled sheet thickness $\tilde{h} = \frac{h}{h_{\text{crit}}}$ and scaled sheet discharge $\frac{q_w}{\nu}$ for the five flux parameterizations in Table 2 and with a fixed hydraulic potential gradient. The sheet thickness is scaled by h_{crit} , the sheet thickness that produces a Reynolds number equal to the transition threshold ($\omega\text{Re} = 1$) for turbulent and laminar models.

184 Figure 2 compares the flux dependence on sheet thickness for the transition (Eq. 2), laminar and
 185 turbulent models (Eq. 1) for a fixed hydraulic potential gradient. The nondimensional sheet thickness,
 186 $\tilde{h} = \frac{h}{h_{\text{crit}}}$, is scaled using the critical sheet thickness, defined as the sheet thickness that produces the
 187 critical Reynolds number ($\omega\text{Re} = 1$). That is, h_{crit} is defined to satisfy

$$1 = \frac{\omega}{\nu} k_s h_{\text{crit}}^3 \overline{\nabla\phi}, \quad (3)$$

188 where $\overline{\nabla\phi}$ is the mean hydraulic potential gradient assuming water pressure is equal to overburden for a
 189 given ice geometry. Equation (3) is derived from the laminar model, but with the sheet conductivity chosen
 190 for the turbulent model (Section 2.2.3), the critical sheet thickness is identical for laminar and turbulent
 191 models. Sheet flux is represented by $\omega\text{Re} = \frac{q_w}{\nu}$, such that values < 1 correspond to laminar flow and values
 192 > 1 represent turbulent flow.

193 Transitioning between laminar and turbulent flow in this way means that Eq. (2) is consistent with the
 194 Moody diagram (Fig. 1). The flux is more sensitive to changes in cavity height h and potential gradient $\nabla\phi$
 195 in the laminar regime than in the turbulent regime. By changing the sensitivity to h and $\nabla\phi$ as a function
 196 of Re , the transition model (Eq. 2) should allow for restricted flow during winter compared to a turbulent
 197 model. If the Reynolds number reaches or exceeds the transition point (set by $1/\omega$), the flux becomes less
 198 sensitive to h and $\nabla\phi$, such that the minimum flow resistance (measured by the friction factor f_D) is set

Table 2. Summary of sheet-flow parameterizations with parameter values substituted in the general forms (Eq. 1 and 2).

Model	Equation	Equation number	Parameters
Turbulent 5/4	$\mathbf{q} = -k_s h^{5/4} \nabla\phi ^{-1/2} \nabla\phi$	(1)	$\alpha_s = 5/4, \beta_s = 3/2$
Turbulent 3/2	$\mathbf{q} = -k_s h^{3/2} \nabla\phi ^{-1/2} \nabla\phi$	(1)	$\alpha_s = 3/2, \beta_s = 3/2$
Laminar	$\mathbf{q} = -k_s h^3 \nabla\phi$	(1)	$\alpha_s = 3, \beta_s = 2$
Transition 5/4	$\mathbf{q} = -\frac{\nu}{2\omega} \left(\frac{h_b}{h}\right)^{1/2} \left(-1 + \sqrt{1 + 4\frac{\omega}{\nu} \left(\frac{h}{h_b}\right)^{1/2} k_s h^3 \nabla\phi }\right) \frac{\nabla\phi}{ \nabla\phi }$	(2)	$\alpha_s = 5/4$
Transition 3/2	$\mathbf{q} = -\frac{\nu}{2\omega} \left(-1 + \sqrt{1 + 4\frac{\omega}{\nu} k_s h^3 \nabla\phi }\right) \frac{\nabla\phi}{ \nabla\phi }$	(2)	$\alpha_s = 3/2$

199 by the fully turbulent limit, in contrast to the laminar model where there is no lower bound on the friction
 200 factor (e.g., the ‘‘Turbulent’’ region of the Moody diagram; Fig. 1).

201 The transition parameterization (Eq. 2) is similar in form to the Forchheimer equation used for non-
 202 Darcy flow through porous media, where the potential gradient is balanced by the sum of a linear term
 203 (with respect to flux, or equivalently velocity) representing laminar flow, and a quadratic term repre-
 204 senting turbulent flow (e.g., Ward, 1964; Bear, 1972; Venkataraman and Rao, 1998). In the glaciological
 205 context, Stone and Clarke (1993) applied the Forchheimer equation to represent drainage within till be-
 206 neath Trapridge Glacier. The result of Eq. (2) has a similar effect as the Flowers and Clarke (2002) model,
 207 where sheet conductivity is a non-linearly increasing function of water thickness, such that the flux param-
 208 eterization accommodates a large range in flux magnitudes and approximates both laminar and turbulent
 209 flows. Equation (2) is most closely related to the flux parameterization used by the SHAKTI (Sommers
 210 and others, 2018) and SUHMO (Felden and others, 2023) models and the rock fracture-flow models these
 211 parameterizations are based on (e.g., Zimmerman and others, 2004; Chaudhuri and others, 2013). However,
 212 compared to SHAKTI and SUHMO, we apply this parameterization to represent flow exclusively within
 213 the distributed drainage system, whereas Sommers and others (2018) and Felden and others (2023) apply
 214 a similar parameterization to represent flow within the drainage system as a whole. We have further in-
 215 troduced a free conductivity parameter k_s to the transition model (Eq. 2) in order to recover the standard
 216 GlaDS model in laminar and turbulent limits. We retain the standard turbulent flux parameterization
 217 for subglacial channels (Werder and others, 2013, Eq. 12) based on modelled Reynolds number within the
 218 turbulent regime ($\text{Re} \gtrsim 2000$) for channels discharge above a minimum discharge $Q = 10^{-2} \text{ m}^3 \text{ s}^{-1}$.

2.2.3 Turbulent model sheet conductivity

The turbulent models in Table 2 prescribe the conductivity k_s in units that depend on the value of α_s , and differ from the units of k_s in the laminar and transition models. The conductivity for the turbulent models must therefore be scaled appropriately to obtain a fair comparison between models (Section S1.3.3). For the same reason, implementation of the transition model must be done with caution for models that choose to non-dimensionalize the governing equations (e.g., Werder and others, 2013). The conductivity for the turbulent models, k_t , is computed by setting the turbulent and laminar flux models equal with $h = h_{\text{crit}}$ (Eq. 3) and with the mean hydraulic potential gradient (allowing for $\alpha_s = 3/2$ or $5/4$ for the turbulent model),

$$k_t h_{\text{crit}}^{\alpha_s} |\nabla\phi|^{1/2} = k_s h_{\text{crit}}^3 |\nabla\phi|. \quad (4)$$

This scaling choice sets the laminar and turbulent models to intersect at $h = h_{\text{crit}}$ and $\omega\text{Re} = 1$ in Fig. 2. The turbulent models could, instead, be set to match the trajectory of the transition model in the fully turbulent limit. Matching the turbulent trajectories, however, would result in the turbulent models significantly overestimating sheet flux relative to the transition and laminar models for the entire range shown in Fig. 2, rendering the models incomparable. A similar scaling could be done to set the transition model to intersect the laminar and turbulent models at $h = h_{\text{crit}}$ and $\omega\text{Re} = 1$; however, we have chosen to match the laminar and transition models in the laminar regime (the slight offset in Fig. 2 for $\tilde{h} < 1$ represents the small contribution of the second term in Eq. 2 and is a consequence of the log-scale).

2.3 Synthetic experiment design

We apply GlaDS with the flux parameterizations in Table 2 to a synthetic ice-sheet margin domain with both synthetic and realistic temperature forcings. The synthetic domain and temperature forcing isolates differences between the models by reducing external controls on the drainage configuration, while the realistic temperature forcing allows us to assess differences in seasonal pressure patterns given plausible variations in surface melt rate that impact the development of efficient drainage in summer.

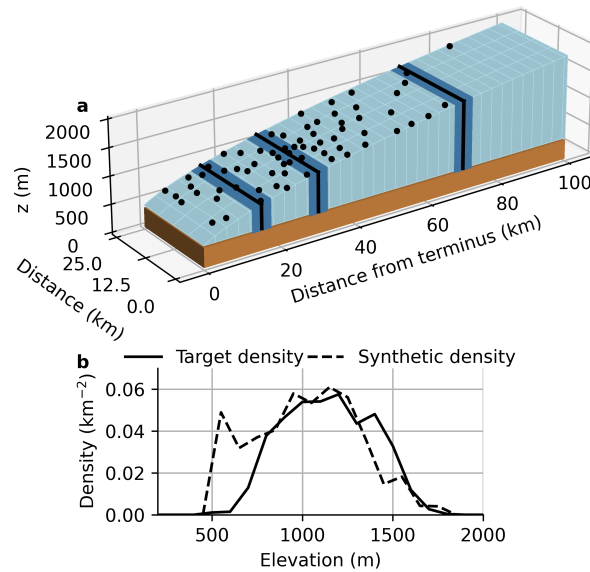


Fig. 3. Overview of synthetic model domain and moulin distribution. (a) Surface and bed elevation with moulins indicated by black circles. The bands at 15, 30, and 70 km indicate where model variables are aggregated in other figures. (b) Target moulin density (derived from Yang and Smith 2016) and density of randomly generated synthetic moulin design as a function of surface elevation.

242 2.3.1 Domain and geometry

243 The model is applied to a 100 km × 25 km domain with ice thickness similar to the SHMIP experiment
 244 (de Fleurian and others, 2018) (Fig. 3a). The domain is adapted to coarsely represent the K-transect in
 245 western Greenland to ensure the surface melt forcing (Section 2.3.2) and geometry are consistent. The
 246 bed is flat with an elevation of 350 m, which approximates the ice-margin elevation near the K-transect
 247 (Smeets and others, 2018). The minimum ice-surface elevation is 390 m at the terminus (approximately
 248 equal to the elevation of the lowest K-transect station; van de Wal and others, 2005). The surface elevation
 249 is computed as

$$z_s = 6 \left(\sqrt{x + 5000} - \sqrt{5000} \right) + 390 \quad (5)$$

250 for x measured in metres from the terminus. The maximum surface elevation is 1909 m, which is near
 251 or above the modern-day ELA of >1700 m a.s.l. (Smeets and others, 2018).

252 2.3.2 Melt forcing

253 The subglacial model is forced with steady basal melt (0.05 m w.e. a⁻¹, Table 1) and seasonally varying
 254 surface melt. The basal melt rate, representing the total melt by external heat sources (i.e., geothermal

flux and basal drag), is chosen to be in line with measured (e.g., 3 to 8 cm w.e. a⁻¹, Harper and others, 2021) and inferred basal melt rates (Karlsson and others, 2021) in west Greenland. Since our focus is on seasonal evolution of subglacial drainage, we neglect diurnal variations in surface melt rate. We have found that seasonal water pressure patterns and the relative performance of the flux models (Table 2) are not sensitive to diurnal variations (Fig. S7). Spatially distributed surface melt rates are computed from a prescribed sea-level temperature $T_0(t)$ using a temperature-index model,

$$\dot{m}(z, t; \Gamma) = \max(0, f_m(T_0(t) - \Gamma z)), \quad (6)$$

for melt factor f_m , temperature lapse rate Γ , and elevation above sea level z . The melt factor $f_m = 0.01$ m w.e. a⁻¹ °C⁻¹ is taken from the SHMIP experiment (de Fleurian and others, 2018), and the temperature lapse rate $\Gamma = 0.005$ °C m⁻¹ is chosen to be consistent with summer lapse rates observed in west Greenland (Fausto and others, 2009).

GlaDS is forced with two sea-level temperature timeseries:

1. “Synthetic” forcing using a sea-level temperature parameterization adapted from the SHMIP experiment case D3 (de Fleurian and others, 2018):

$$T_0(t) = -a \cos\left(\frac{2\pi t}{T_{\text{year}}}\right) + b, \quad (7)$$

where constants a and b control the intensity and duration of surface melt, and T_{year} is the number of seconds in a year.

2. “KAN” forcing using daily mean air temperatures recorded at the PROMICE KAN_L weather station (How and others, 2022). We use temperatures from 2014, a representative year in terms of total volume and duration of surface melt over the 2009–2022 period (Section S1.4, Fig. S1)

Prior to applying the above forcings, we forced the model with surface melt identical to that of the SHMIP experiment case D3. Modelled subglacial drainage for the turbulent 5/4 model (as used in the SHMIP experiment) recreates the key features of the published SHMIP outputs (Fig. S1) (de Fleurian and others, 2018).

The constants a and b for the synthetic forcing scenario presented here are computed to retain the same duration of positive sea-level temperatures as the SHMIP experiment and to result in the same total

279 melt volume as the KAN scenario so that only the temporal variations in surface melt rate, and not the
280 total melt volume, vary by scenario. We also tested the sensitivity to total melt volume by increasing the
281 temperatures in the KAN timeseries to produce the same total melt volume as the original SHMIP case
282 D3 (Fig. S9), but present the results for the observed melt volume since these results are expected to be
283 more realistic.

284 *2.3.3 Moulins*

285 Surface meltwater drains into the subglacial system through discrete moulin locations. Supraglacial catch-
286 ments are generated by randomly placing catchment centroids throughout the domain according to a
287 space-filling maximin design (i.e., a design that maximizes the minimum distance between moulins) and
288 with an elevation-dependent density derived from supraglacial mapping (Yang and Smith, 2016) (Fig. S2).
289 The moulin density is greatest at 1138 m a.s.l., and we assign a total of 68 supraglacial catchment centroids,
290 computed from the product of the observation-derived density and the hypsometry of our domain.

291 Supraglacial catchments are generated by drawing a Voronoi diagram from the catchment centroids
292 (i.e., assigning each node in the mesh to the catchment of the nearest centroid), and moulins are placed
293 as the node with the lowest surface elevation within each catchment subject to the constraints: (1) the
294 minimum distance between neighbouring moulins is 2.5 km, and (2) moulins can not be placed on boundary
295 nodes or within 5 km of the terminus. Fig. S2 illustrates the moulin and catchment generation scheme in
296 more detail (Section S1.5)

297 Surface meltwater is accumulated within catchments and instantly routed into moulins. This scheme
298 neglects the impact of supraglacial hydrology, which characteristically delays the diurnal peak and reduces
299 the diurnal amplitude of surface inputs to moulins compared to the diurnal cycle of surface melt rate (e.g.,
300 Muthyala and others, 2022). This simplification is appropriate in our synthetic model setup considering
301 the idealized nature of our experiments and since we are not attempting to resolve diurnal cycles in water
302 pressures in response to diurnal variations in moulin inputs.

303 *2.3.4 Boundary and initial conditions*

304 The subglacial model is posed on an unstructured triangular mesh. We apply GlaDS on a mesh with 4156
305 nodes and a mean edge length of 883 m. This mesh resolution was chosen from mesh refinement tests as
306 a suitable tradeoff between precision and computation time (Fig. S3). Boundary conditions consist of a

307 zero-pressure boundary condition at the terminus ($x = 0$ km) and a zero-flux condition elsewhere.

308 GlaDS simulations involve a steady-state spin-up used as initial conditions for periodic runs. The
309 spin-up is accomplished in three phases to ensure numerical stability: (1) 25 years with no surface inputs,
310 starting with a uniform water depth equal to half the bed bump height and no subglacial channels (a
311 sufficient duration for the model to evolve to an intermediate winter-like state that is independent of
312 the uniform initial condition); (2) 25 years with a linear ramp-up of surface melt intensity; and (3) 50
313 years with constant melt rates to reach a final steady state (evaluated based on the rate-of-change of
314 average water pressure). Each of these steps is longer than strictly necessary. For example, a steady state
315 drainage configuration is typically reached well before the end of step (3), but with implicit and adaptive
316 timestepping the extra spin-up time is associated with negligible increases in runtime.

317 Periodic simulations are run for two years, and only results from the second year are analyzed. It would
318 also be possible to begin seasonal simulations directly from the uniform initial condition. This, however,
319 would require the transient simulations to be run for several melt seasons to reach a periodic state as
320 remnant channels with areas up to $S \approx 12 \text{ m}^2$ persist through the winter near the terminus and require
321 multiple melt seasons to reach their equilibrium size. Given this nonuniform winter condition, it is faster
322 to approach the periodic state from a channelized system, i.e., from the steady simulation.

323 3 RESULTS

324 3.1 Synthetic scenario

325 To illustrate the differences between modelled water pressure for the five flux parameterizations (Table 2),
326 we first present modelled subglacial water pressure (normalized by ice overburden) and channel discharge
327 for the synthetic forcing scenario (Fig. 4). The primary differences in modelled subglacial drainage are found
328 during winter and above the maximum extent of surface melt (i.e., above ~ 70 km). The most significant
329 differences are a result of the flux parameterization family (i.e., turbulent, laminar, and transition), with
330 only minor differences related to α_s (i.e., between turbulent $5/4$ and turbulent $3/2$, and transition $5/4$ and
331 transition $3/2$).

332 These model outputs confirm the well-known winter water pressure problem for the standard turbulent
333 $5/4$ model, which tends to produce unrealistically low winter and high summer water pressures (e.g.,
334 de Fleurian and others, 2018; Poinar and others, 2019; Ehrenfeucht and others, 2023). For this scenario,
335 the turbulent models predict winter water pressures of 43% of overburden at 30 km with $\alpha_s = 5/4$ and

Table 3. Water pressure normalized by overburden (i.e., floatation fraction) for synthetic and KAN temperature-forcing scenarios. Winter floatation fraction is computed as the average value within $x = 30 \pm 2.5$ km (Fig. 3) during the two months preceding the initial onset of surface melt. Summer floatation fraction is computed as the 95th-percentile width-averaged water pressure produced during the melt season within $x = 30 \pm 2.5$ km. The bracketed number beside summer floatation fractions for the KAN scenario indicates the number of days water pressure exceeded overburden. Water pressure does not exceed overburden in the Synthetic scenario.

Scenario		Floatation fraction (number of days above overburden)				
		Turbulent 5/4	Turbulent 3/2	Laminar	Transition 5/4	Transition 3/2
Synthetic	Winter	0.430	0.511	0.670	0.671	0.672
	Summer	0.829	0.826	0.811	0.828	0.847
KAN	Winter	0.436	0.513	0.670	0.671	0.672
	Summer	1.04 (9)	1.02 (10)	0.997 (6)	1.01 (9)	1.01 (11)

51% with $\alpha_s = 3/2$ (Table 3). These modelled winter water pressures are low compared to borehole observations close to overburden (e.g., winter water pressure higher than 95% of overburden 7 km from the ice margin (van de Wal and others, 2015); ~ 80 – 100% of overburden 27 to 33 km from the ice margin (Wright and others, 2016)), even after accounting for the difference in pressure between connected and disconnected bed patches (e.g., Rada Giacaman and Schoof, 2023). The winter water pressure is improved for the laminar, transition 5/4 and transition 3/2 models (each produce water pressure 67% of overburden at 30 km). Summer water pressure is between 81% to 85% of overburden for all models (Table 3). The relative performance of the five models in Fig. 4 is the same as that obtained with higher surface melt rates used in the SHMIP experiment D3 (Fig. S9). The increased melt volume in the SHMIP forcing scenario compared to the our synthetic forcing experiment results in summer water pressure above overburden for all except the laminar model.

The differences in water pressure between the flux parameterizations can be understood by considering the spatial and seasonal pattern in modelled Reynolds number, transmissivity, water depth, hydraulic potential, and conductivity (Fig. 5). The turbulence index (ωRe) highlights regions and times where the turbulent and laminar assumptions are inconsistent (Fig. 5a, b). The turbulent model assumes $\omega\text{Re} \gg 1$ everywhere and for all times, so that the turbulent model is applied inappropriately above $x = 20$ km and outside of the peak summer season. On the other hand, the laminar model is inappropriate near $x = 20$ km, near the terminus, and during elevated summer water pressures.

The transmissivity, $T = \rho_w g \frac{q}{|\nabla\phi|}$, measures the discharge-per-unit-width associated with a specified potential gradient (Fig. 5c, d). It has similar spatial and seasonal patterns as the turbulence index ωRe .

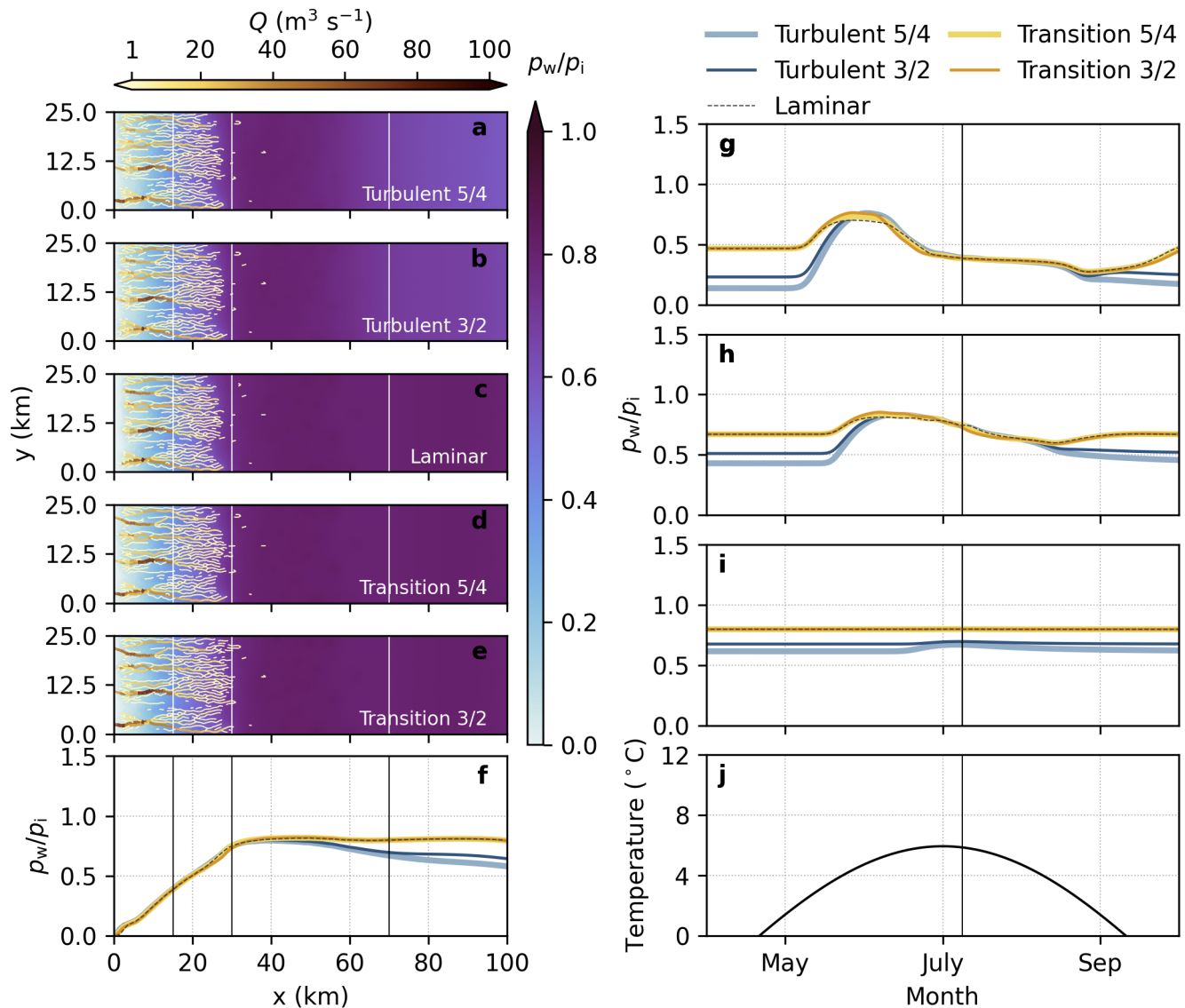


Fig. 4. Synthetic forcing scenario. Floatation fraction $\frac{p_w}{p_i}$ and channel discharge on 9 July (a–e) for turbulent 5/4 (a), turbulent 3/2 (b), laminar (c), transition 5/4 (d) and transition 3/2 (e) models, and width-averaged floatation fraction on 9 July (f). Width-averaged pressure in bands at $x = 15 \pm 2.5$ km (g), $x = 30 \pm 2.5$ km (h), and $x = 70 \pm 2.5$ km (i) and imposed air temperature at 390 m a.s.l. used to force the temperature-index model (j). The centre of bands used for (g–i) are indicated by vertical lines in (a–f), and the time of (a–f) is shown by vertical lines in (g–i).

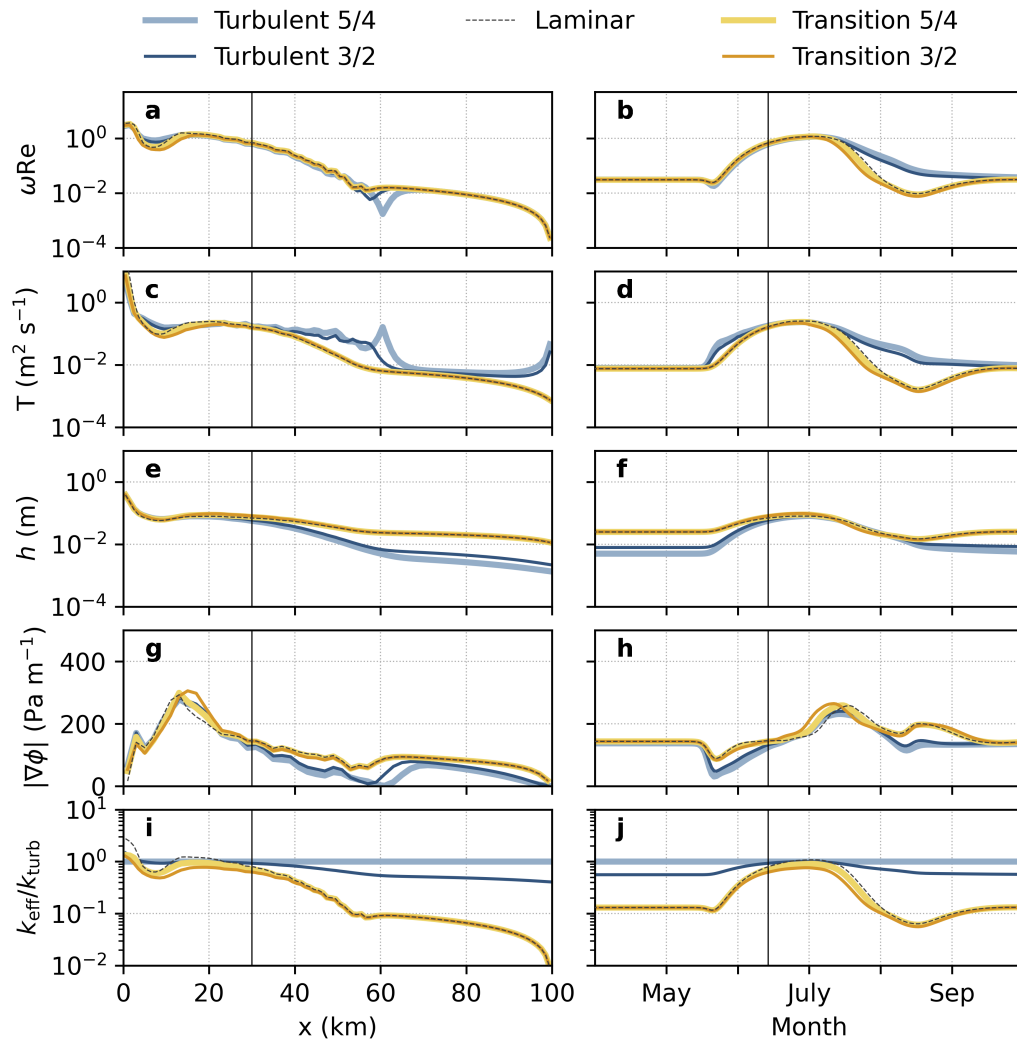


Fig. 5. Turbulence index ωRe (log scale; a, b), transmissivity T (log scale; c, d), water depth h (log scale; e, f), potential gradient $|\nabla\phi|$ (linear scale; g, h), and effective turbulent conductivity (log scale; i, j) on 14 June (left column), and averaged for the band $x = 30 \pm 2.5$ km (right column).

356 The four order-of-magnitude spatial variation produced by the laminar and transition models is the same
 357 as that produced by the SHAKTI model in winter for Helheim Glacier (Sommers and others, 2023).

358 The spatial and seasonal patterns in turbulence index ωRe can be decomposed into individual contri-
 359 butions from the water depth h (Fig. 5e,f) and potential gradient $|\nabla\phi|$ (Fig. 5g,h). Of the two components,
 360 the water depth h more strongly controls the turbulent index than the potential gradient. This is in line
 361 with the mathematically stronger dependence on h than the potential gradient, especially for the laminar
 362 and transition models.

363 The differences in seasonal water pressure variations between the turbulent, laminar, and transi-
 364 tion models are largely explained by variations in the effective turbulent conductivity, defined as $k_{\text{eff}} =$

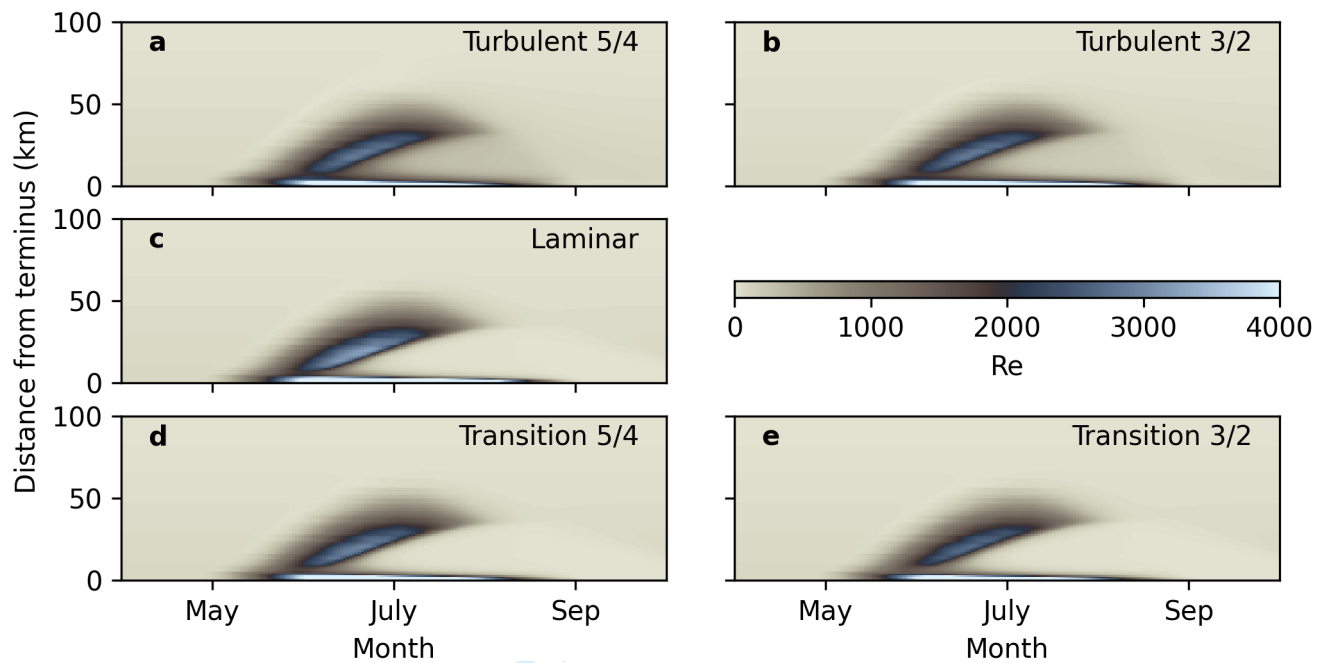


Fig. 6. Reynolds number and channel discharge for synthetic scenario for turbulent 5/4 (a), turbulent 3/2 (b), laminar (c), transition 5/4 (d), and transition 3/2 (e) models.

365 $q/h^{5/4}|\nabla\phi|^{1/2}$ (Fig. 5i,j). By this definition, $k_{\text{eff}} = k_s$ for the turbulent 5/4 model, meaning that variations
 366 in the effective turbulent conductivity for other models allow them to be directly compared to the more
 367 commonly used turbulent 5/4 model. For the remaining models, k_{eff} is a function of the water thickness
 368 and potential gradient, with h again being the main driver based on its higher exponent. The k_{eff} for the
 369 laminar and transition models varies over two orders of magnitude in space (Fig. 5i) and more than one
 370 order of magnitude in time (Fig. 5j). The reduced effective conductivity for the laminar and transition
 371 models in winter explains the higher winter water pressure compared to the turbulent models, while the
 372 large seasonal changes in effective conductivity explain the reduced seasonal amplitude in water pressure
 373 compared to the turbulent models.

374 The modelled Reynolds number further highlights the conceptual inconsistencies with purely turbulent
 375 or laminar models (Fig. 6). Distributed water flow is mostly laminar for all models, with turbulent flow
 376 limited to a narrow band that migrates upglacier during the first half of the melt season. The laminar
 377 model is inconsistent around the distributed–channelized drainage transition, introducing uncertainty into
 378 the onset of channelization predicted with this model.

379 The results in Fig. 4 and 5 align with what is expected based on the Moody diagram (Fig. 7). Here
 380 the spread in the curves for the turbulent 5/4 (lighter blue) and transition 5/4 (lighter yellow) models is a

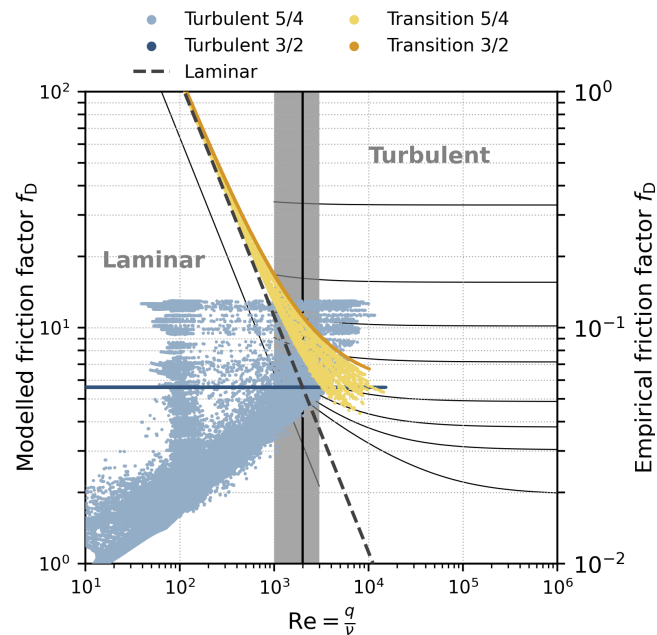


Fig. 7. Moody diagram computed from model outputs in the synthetic scenario for the five flux parameterizations (Table 2). The turbulent 3/2 model appears as a horizontal line since its friction factor is independent of Re and $\nabla\phi$. The results from the laminar model are displayed as a thick dashed line to distinguish the modelled results from the theoretical curves. The transition Reynolds number is shown as the solid black line at $Re = 2000$. For reference, the classical pipe-flow Moody diagram from Fig. 1 is shown in the background (thin black lines, right axis). Note that the scaling between the two axes is arbitrary.

381 result of the Re - f_D relationship depending on the hydraulic potential gradient, which varies in space and
 382 time. As shown by the effective turbulent conductivity (Fig. 5i,j), the turbulent models have significantly
 383 less flow resistance compared to the laminar and transition models except when the Reynolds number
 384 approaches the laminar-turbulent transition point. The opposite slope of the turbulent 5/4 cluster of
 385 points further suggests a structural problem where flow resistance decreases with decreasing water supply
 386 (e.g., flow resistance decreases without bound during winter), regardless of the chosen model parameters.
 387 This behaviour is not supported by the other models or the empirical friction factor curves. Of all the
 388 models, the transition 3/2 model (darker yellow) is closest to the empirical friction factor curves.

389 3.2 KAN scenario

390 The evolution of summer water pressure is sensitive to the temporal pattern of surface melt (Fig. 8).
 391 Despite identical total melt volumes between the synthetic and KAN temperature forcing scenarios, peak
 392 summer water pressures are higher with KAN temperature forcing (Table 3, S1).

393 The turbulent 5/4 and turbulent 3/2 models once again predict low winter water pressure (44% and 51%

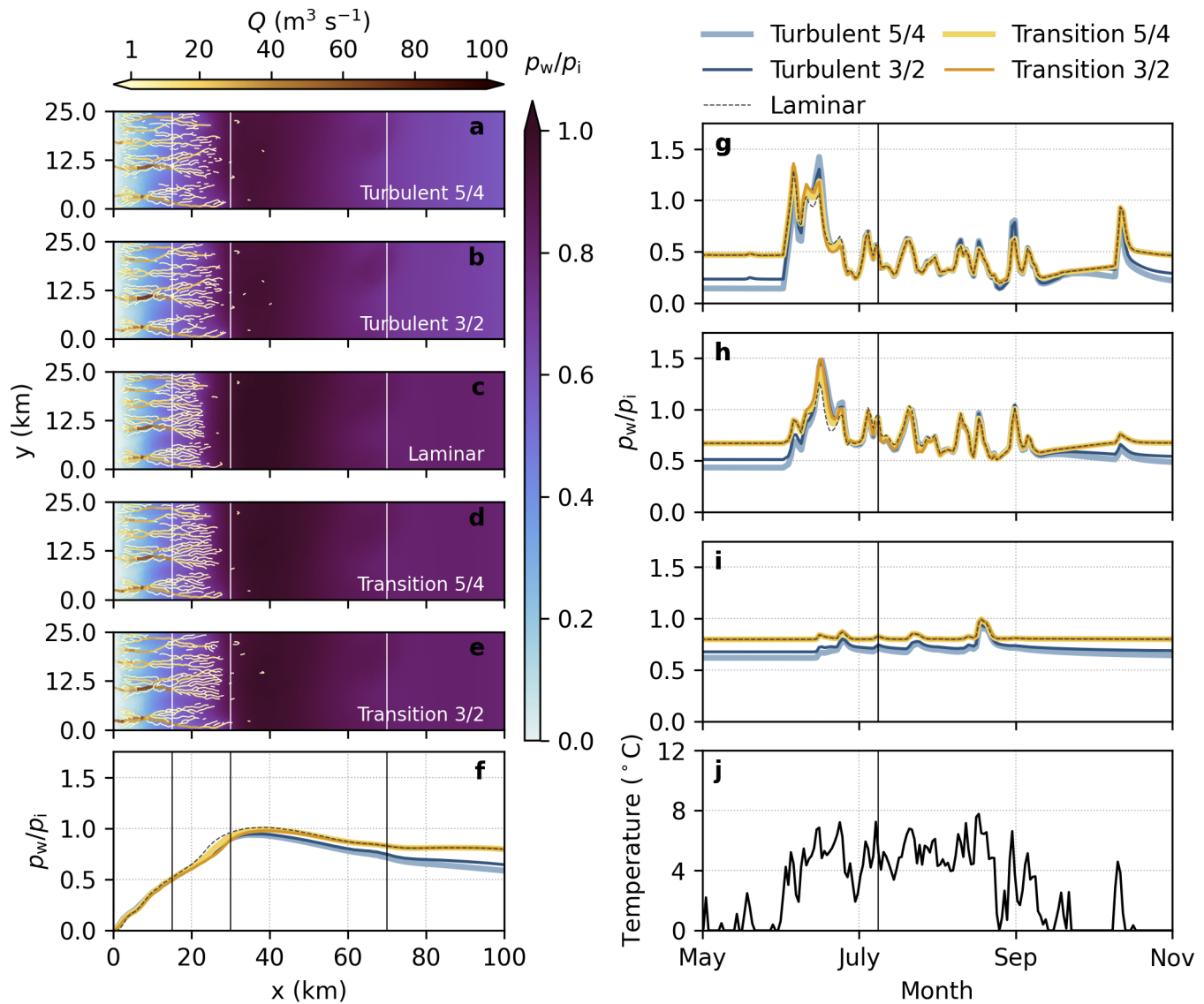


Fig. 8. KAN forcing scenario. Floatation fraction $\frac{p_w}{p_i}$ and channel discharge on 9 July (a-e) for turbulent 5/4 (a), turbulent 3/2 (b), laminar (c), transition 5/4 (d) and transition 3/2 (e) models, and width-averaged floatation fraction on 9 July (f). Width-averaged pressure in bands at $x = 15 \pm 2.5$ km (g), $x = 30 \pm 2.5$ km (h), and $x = 70 \pm 2.5$ km (i) and imposed air temperature at 390 m a.s.l. used to drive the temperature-index model (black curve, right axis g-i). The centre of bands used for (g-i) are indicated by vertical lines in (a-f), and the time of (a-f) is shown by vertical lines in (g-i).

394 of overburden at $x = 30$ km) compared to the laminar and transition models (67%) (Table 3). Compared
395 to the synthetic forcing case, there is a slightly more prominent late-summer water-pressure minimum
396 observed with the laminar and transition models. At 15 km and 30 km from the terminus, the late-
397 summer water pressure minima are below the winter baseline value, and from September to November the
398 water pressure is generally increasing. This trend is not observed as strongly for the turbulent models
399 given their lower winter baseline water pressure. Peak summer water pressures are broadly similar for all
400 models, exceeding overburden by $\sim 50\%$ in the spring at 30 km. The laminar model has the lowest peak
401 pressure (126% of overburden), perhaps as a result of the breakdown of the laminar assumption (Fig. S13).
402 Following the spring pressure maximum, peak water pressure during melt events remains near overburden,
403 with the 95th-percentile summer water pressure at 30 km between 100% and 104% of overburden (Table 3).

404 The controls on differences in water pressure between the flux parameterizations are the same as for
405 the synthetic scenario. The opposing sensitivity of the friction factor (i.e., flow resistance) to the Reynolds
406 number (i.e., flow intensity) for the turbulent models compared to the laminar and transition models
407 (Fig. 7) results in significantly lower winter water pressure and a larger variation between winter and
408 summer water pressure for the turbulent models.

409 To ensure the qualitative differences observed between the synthetic and KAN forcings are not a function
410 of seasonal melt volume, we re-ran the KAN simulations with the original SHMIP D3 (larger) seasonal
411 melt volume. To do this, we increased the temperatures in the KAN timeseries by 2.43°C and adjusted the
412 lapse rate to $\Gamma = -0.0075^\circ\text{C m}^{-1}$ to produce the desired seasonal melt volume. The qualitative differences
413 related to the flux parameterizations are robust with respect to this change in total melt volume, however
414 the modelled water pressure is unrealistically high during melt events, reaching $\sim 200\%$ of overburden for
415 all models (Fig. S10).

416 3.3 Parameter sensitivity

417 The results for the synthetic (Fig. 4) and KAN (Fig. 8) scenarios represent a single set of parameter values.
418 To see how the performance of these models varies across parameter space, especially as we push the
419 laminar and turbulent models further outside their regimes of applicability, we test parameter settings that
420 represent high and low channel-extent end-members. To achieve this, we tune the sheet (k_s) and channel
421 (k_c) conductivities and l_c , which controls channel initialization (Table S3)

422 3.3.1 High channelization end-member

423 To increase the propensity for channelized drainage, the sheet conductivity k_s is decreased from 0.05 to
424 0.02 Pa s^{-1} , channel conductivity k_c is increased from 0.5 to $1.0 \text{ m}^{3/2} \text{ s}^{-1}$, and l_c is increased from 10 to 25 m.
425 Compared to the reference case (Fig. 8), this sensitivity test results in minimum summer water pressures
426 below the steady winter value for all models. The turbulent model once again produces the lowest winter
427 water pressure, while all models exhibit similar overall behaviour in summer (Fig. S12).

428 3.3.2 Low channelization end-member

429 The low channelization end-member uses a sheet conductivity $k_s = 0.1 \text{ Pa s}^{-1}$, channel conductivity $k_c =$
430 $0.2 \text{ m}^{3/2} \text{ s}^{-1}$, and $l_c = 10 \text{ m}$. These parameters result in channels limited to the lowest 20 km for the
431 turbulent and transition models, with only scattered channels below 15 km produced by the laminar model.
432 All models produce higher peak summer water pressure than the reference case (Fig. 8), with the laminar
433 model producing notably lower pressures closer to overburden (Fig. S14).

434 Across the sensitivity tests (Fig. S12, Fig. S14) and the KAN scenario (Fig. 8), the turbulent model
435 has the behaviour closest to that of the laminar and transition models in the high channelization end-
436 member. However, this scenario produces nearly entirely laminar flow (Fig. S13), rendering the turbulent
437 model physically inconsistent. Similarly, the laminar model outperforms the transition models in the low
438 channelization end-member, where the Reynolds number indicates nearly fully turbulent flow (Fig. S15).

439 4 DISCUSSION

440 4.1 Distributed water flux parameterizations

441 We have presented modelled subglacial drainage configurations for five flux parameterizations (Table 2).
442 With both synthetic and KAN surface melt forcing, the laminar and transition models tend to show
443 more desirable behaviour than the turbulent models. The laminar and transition models result in higher
444 winter water pressures, while still exhibiting late-summer water pressure minima below the winter baseline.
445 These desirable features are more clear in the KAN scenario (Fig. 8), since the smooth melt forcing in the
446 synthetic scenario results in muted seasonal pressure variations (Fig. 4). Given the consistently lower
447 performance of the turbulent model across parameter settings intended to represent extremes of low and
448 high channelization (Fig. S12, S14), these findings do not appear to be a consequence of the particular

449 parameter values used throughout.

450 The laminar and transition models have generally similar performance in terms of summer and winter
451 water pressure. However, the laminar model produces lower peak water pressures and fewer instances
452 of water pressure above overburden than the transition model. These desirable features produced by
453 the laminar model occur when water flow is demonstrably turbulent, and the laminar model therefore
454 physically inconsistent (Fig. 7). This contradiction points to missing model physics that would act to
455 increase drainage capacity, as accomplished by the laminar model, in the spring to reduce water pressures
456 to closer to overburden.

457 Unrealistic modelled winter water pressure has previously been addressed using the turbulent 5/4
458 model by prescribing the sheet conductivity k_s as a linear function of surface melt rates to allow for
459 reduced conductivity during winter and increased conductivity during summer (Downs and others, 2018).
460 The result of this conductivity parameterization is a similar seasonal pattern of turbulent conductivity as
461 reproduced by the laminar and transition models (Fig. 5j). The major difference between the laminar and
462 transition models and the Downs and others (2018) parameterization is the magnitude of variation. Downs
463 and others (2018) prescribe the conductivity to vary on the order of $\mathcal{O}(10^4)$ in time but remain constant in
464 space, whereas we have a variation of order $\mathcal{O}(10^1)$ in time, and order $\mathcal{O}(10^2)$ in space. These variations in
465 our model results have not been prescribed, but emerge naturally as a result of the flux parameterizations,
466 as has also been demonstrated for the SHAKTI transition parameterization (Sommers and others, 2023).

467 Seasonal pressure variations have been shown to depend on the evolving connectivity of distributed
468 drainage elements, where low winter water pressure in connected bed regions may be compensated for
469 by high pressure within disconnected bed regions (e.g., Andrews and others, 2014; Hoffman and others,
470 2016; Rada Giacaman and Schoof, 2023). By comparing a coupled hydrology–dynamics model to sliding
471 speed, subglacial discharge, and ice thickness data from Argentière Glacier, Gilbert and others (2022)
472 found a turbulent flow exponent $\alpha_s \geq 5$ provided the best fit to observed velocities. The high value for
473 the turbulent flow exponent was interpreted as possibly representing a switch in bed connectivity as a
474 function of the water thickness h (e.g., Flowers, 2000; Helanow and others, 2021). In other words, Gilbert
475 and others (2022) suggest that some of the net effects of changing bed connectivity can be included by
476 increasing the sheet-flow exponent α_s . In this context, some of the poor performance of the turbulent model
477 can be attributed to its failure to represent decreased hydraulic connectivity (i.e., taking f_D^{-1} as a proxy
478 for connectivity) in winter. Based on these considerations, the possibility that $\alpha_s > 3$ for sub-turbulent

479 flows, in particular for the transition model, should be investigated if further data suggest that $\alpha_s > 3$ can
480 reproduce key features related to changes in bed connectivity.

481 These advantages in the laminar and transition models over the turbulent model come with minimal
482 costs in terms of the difficulty running the model and in the computational burden. Running the laminar
483 model only requires a trivial change in parameters (α_s , β_s , and appropriately scaling the conductivity k_s in
484 Eq. (1)). Running the transition model requires a simple modification of the model source code to replace
485 Eq. (1) with Eq. (2). The laminar and transition models differ from the turbulent models in terms of
486 computation time by up to $\sim 20\%$, with the models with $\alpha_s = 3/2$ taking the longest to run (Table S2).

487 4.2 Turbulent flow exponent

488 The turbulent flow exponent (α_s in Eqs. (1) and (2), Table 1) has a secondary impact on modelled water
489 pressure and drainage configuration relative to the primary control of the form of the flux parameterization.
490 However, winter water pressure for the turbulent model is sensitive to the value of α_s , with the turbulent
491 $3/2$ model predicting higher (slightly more realistic) winter water pressure (e.g., Fig. 8h). Sensitivity is
492 very low for the transition model, since the turbulent exponent α_s only applies in fully turbulent $\omega\text{Re} \gg 1$
493 limit, which is rarely reached in our model configuration (Fig. 7).

494 Given that the fully turbulent limit is not reached in our model outputs (Fig. 7), the choice of α_s for
495 the turbulent and transition models can not be assigned strictly from Darcy–Weisbach pipe flow theory.
496 However, the upwards slope of the envelope of modelled friction factors for the turbulent $5/4$ model in
497 Fig. 7 is inconsistent with the other flux models and with empirical friction factor curves, suggesting that
498 $\alpha_s = 3/2$ is a more reasonable choice than $\alpha_s = 5/4$.

499 Our model outputs and theoretical considerations suggest that $\alpha_s = 3/2$ yields marginally more realistic
500 outputs than $\alpha_s = 5/4$ (i.e., $\sim 10\%$ higher winter water pressure for comparable parameter values). For
501 modelling studies that take the turbulent flow assumption, we recommend α_s be treated as an uncertain
502 parameter and tuned where possible (e.g., Gilbert and others, 2022) rather than prescribed as $\alpha_s = 5/4$
503 based on precedent. Given the minimal sensitivity for the transition model, and since the turbulent
504 exponent α_s is only applied in the transition model in the true turbulent limit ($\omega\text{Re} \gg 1$), it should be
505 appropriate to use the transition $3/2$ model, instead of transition $5/4$, by default.

506 4.3 Choosing an appropriate flux parameterization

507 Considering the discussion of both the form (Section 4.1) and turbulent exponent (Section 4.2) of the
508 distributed flux parameterization, we recommend the following:

- 509 1. Use the transition 3/2 model by default based on its theoretical (i.e., unlimited Re range of applicability;
510 Fig 7) and practical (i.e., desirable features in modelled water pressure compared to the turbulent model;
511 Fig. 8) attributes.
- 512 2. If only aggregate model outputs (e.g., spatio-temporally averaged basal effective pressure) are important,
513 the laminar model may be appropriate as an approximation of the transition model. In this case, it
514 should be verified that the modelled Reynolds number does not reach the turbulent regime, since the
515 model is physically inconsistent and overestimates sheet capacity with $\omega\text{Re} > 1$. However, if modelled
516 water pressures far exceed overburden, it may be practically beneficial to use the laminar model precisely
517 because of this overestimation of the sheet capacity compared to the transition model, while accepting
518 the physical inconsistencies as a symptom of remaining model discrepancy.
- 519 3. Avoid the turbulent model for seasonally varying subglacial drainage simulations, unless theoretical (i.e.,
520 modelled Reynolds number) and/or practical (i.e., demonstrated sensitivity of quantities of interest to
521 the flux model) reasons are discovered that make its performance superior to the transition model. In
522 this case, the turbulent 3/2 model is recommended over the turbulent 5/4 model, but sensitivity of any
523 quantities of interest to the value of α_s should be assessed.

524 4.4 Study limitations

525 4.4.1 Model geometry and domain

526 There are a number of limitations related to the idealized model setup utilized here. We have presented
527 results for a flat bed, which is not broadly representative of topography beneath Greenlandic outlet glaciers
528 (e.g., Morlighem and others, 2017). To address this, we additionally tested the sensitivity of model outputs
529 to different realizations of bed topography, including a bed with a ~ 6 km-wide and 350 m-deep trough both
530 along the centre of the domain and following a sinusoidal path, and U-shaped bed topography (Fig. S17).
531 These tests show no difference in the relative performance of each model since topography has a similar
532 influence on water pressure for all flux parameterizations (Fig. S18, S19, S20). These tests suggest that the

533 performance of the parameterizations are not sensitive to the choice of synthetic bed topography. We have
534 also assessed the sensitivity of our results to boundary conditions by imposing a full floatation boundary
535 condition. Based on this test, the boundary condition imposed at the terminus has a limited impact on
536 water pressure and channel discharge beyond the lowest ~ 5 km (Fig. S21). Future work should investigate
537 differences in modelled water pressure and the performance of the flux parameterizations when applied to
538 real topography, including for marine-terminating glaciers. Based on these synthetic tests, bed and surface
539 topography or the outlet boundary condition are not expected to change the relative performance of each
540 model.

541 *4.4.2 Meltwater forcing*

542 The synthetic and KAN surface melt forcings and uniform basal melt rate used here are simplified relative
543 to realistic melt rates. The KAN surface meltwater forcing scenario, derived from daily mean temperatures
544 recorded at the KAN_L AWS (How and others, 2022) and a simple temperature-index model, elicits a more
545 realistic water pressure response than the unrealistic synthetic scenario. For the KAN forcing, we have
546 used a single sample of temperature forcing measured at the KAN_L PROMICE station. This timeseries
547 was chosen to be representative in terms of the total melt volume and melt season duration, however
548 different temperature timeseries will result in different modelled water pressures. Given the consistency
549 of the differences between the flux parameterizations between the synthetic (Fig. 4) and KAN scenarios
550 (Fig. 8), it is unlikely the performance differences of the flux parameterizations are a function of the choice
551 of temperature timeseries.

552 Since we are focused on subglacial water pressure on seasonal timescales, we have chosen to omit diurnal
553 variations in forcing the subglacial drainage model. We have also ignored supraglacial (e.g., Poinar and
554 Andrews, 2021; Hill and Dow, 2021) and englacial (e.g., Andrews and others, 2022) hydrologic processes
555 that impact the diurnal evolution of water pressure (e.g., Andrews and others, 2018). Neglecting diurnal
556 oscillations has previously been shown to have only a limited impact on the seasonal development of the
557 subglacial drainage system (e.g., Werder and others, 2013), and experiments with prescribed diurnal forcing
558 show a minimal impact (Fig. S7).

559 We have prescribed a uniform basal melt rate of $0.05 \text{ m w.e. a}^{-1}$ in our simulations. Since GlaDS does
560 not allow for melt by potential energy dissipation within the distributed drainage system (Werder and
561 others, 2013), this rate represents melt by external heat sources including geothermal flux and basal drag

(e.g., Harper and others, 2021; Karlsson and others, 2021). Based on tests with a lower basal melt rate, representing a low friction regime, the winter water pressure decreases for all models (Fig. S11), such that our specific conclusions do not depend on the chosen basal melt rate. The increased winter water pressure we observe with higher basal melt rates corroborates previous findings that accurate accounting of the basal heat balance (e.g., Harper and others, 2021; Karlsson and others, 2021) is important for reproducing high winter water pressure (Sommers and others, 2023).

4.4.3 Reynolds number and transition parameter

The partitioning between laminar and turbulent flow (Fig. 5) has been based on the Reynolds number computed using the distributed flux q , which represents the average flux through many subglacial cavities within each model element. It is therefore not exactly clear how representative this bulk-averaged Re metric is of flow through physical subglacial drainage elements comprising the “distributed water sheet” as represented in models. The problem of determining a representative Reynolds number is shared by models of non-Darcy porous flow (e.g., Ward, 1964; Bear, 1972; Venkataraman and Rao, 1998). In this context, the problem can be partially addressed by direct numerical simulation of flow through a particular medium (e.g., Wood and others, 2020). Given the uncertainty in the exact form of subglacial drainage elements, this is not a question that can be answered within the framework of current subglacial hydrology models, but it is important to consider when assigning the transition parameter ω , since the Reynolds number cannot be interpreted as precisely as for simple flows. We have assumed that the transition from laminar to turbulent flow occurs at $Re \approx 2000$, but it remains to be shown what transition threshold yields the best agreement with velocity or subglacial water pressure data in more realistic model settings.

5 CONCLUSIONS AND RECOMMENDATIONS

Subglacial drainage models are key to understanding the relationship between surface and basal melt, basal motion, and ultimately grounded-ice contributions to sea level (e.g., King and others, 2020). However, these models have important shortcomings when applied to ice-sheet-scale domains with seasonally varying melt forcing. Subglacial models (1) underpredict winter water pressures, (2) fail to capture the late-summer pressure minimum, and (3) require a priori assumptions about distributed flow being fully laminar or turbulent, among other limitations. We have demonstrated that these three problems can be measurably addressed by modifying the parameterization controlling water flux in the distributed (linked-cavity)

590 drainage system while maintaining purely turbulent flow within subglacial channels.

591 We have tested five flux parameterizations (Table 2), including the standard turbulent model (e.g.,
592 Schoof and others, 2012; Werder and others, 2013), the fully laminar model (e.g., Hewitt, 2013; Gagliardini
593 and Werder, 2018; Cook and others, 2022), and a parameterization that transitions between laminar and
594 turbulent flow based on the local Reynolds number, for two values of the turbulent flow exponent ($\alpha_s =$
595 $5/4, 3/2$) where appropriate. The flux parameterizations are tested within the GlaDS model (Werde and
596 others, 2013) using synthetic and realistic seasonally varying air temperature forcing on a synthetic ice-sheet
597 margin domain.

598 Laminar and transition models outperform turbulent models on all identified criteria. Winter water
599 pressure is increased by 18–33% of overburden across the domain by using the laminar and transition
600 models for comparable parameter values. When forced with realistic seasonally varying air temperatures,
601 the laminar and transition models produce late-summer water pressures slightly below the winter baseline.
602 Fundamentally, the turbulent and laminar models are inconsistent with their underlying assumptions over
603 the full range of Reynolds numbers that must be represented by the sheet-flow parameterization (e.g.,
604 Fig 7).

605 We suggest using the transition ($\alpha_s = 3/2$) model where possible based on its desirable features and
606 physical consistency in representing flows over a realistic range of Reynolds numbers. The laminar model
607 produces similar results for seasonal-scale simulations, but suffers from conceptual inconsistencies. However,
608 it may be beneficial to use the laminar model despite its conceptual limitations when modelled spring water
609 pressure are unexpectedly high. The turbulent model should be used with caution and an appreciation of
610 its structural limitations.

611 The practical and conceptual improvements made by the transition model are encouraging for mod-
612 elling transient subglacial water-pressure variations. However, a gap remains between models and obser-
613 vations, especially for winter water pressure and in explaining why the physically inconsistent laminar
614 model sometimes performs best in the scenarios we have presented. Promising areas to direct efforts to
615 improve modelled winter water pressure include the representation of spatiotemporal heterogeneity in basal
616 hydraulic connectivity (e.g., Andrews and others, 2014; Hoffman and others, 2016; Helanow and others,
617 2021) and transmissivity (e.g., Gilbert and others, 2022) in subglacial models, and establishing routine
618 two-way hydrology–dynamics coupling (e.g., Cook and others, 2022). Further aspects of the physics cap-
619 tured by subglacial drainage models that are open for development, and should lead to improved realism

620 in model outputs, include coupling basal hydrology with englacial (e.g., Koenig and others, 2014; Andrews
621 and others, 2022) and supraglacial processes (e.g., Das and others, 2008; Hill and Dow, 2021); the tran-
622 sition to open-channel flow when water pressure becomes negative (e.g., Röthlisberger, 1972; Hewitt and
623 others, 2012; Sommers and others, 2018); representing spatially variable basal materials (e.g., Muto and
624 others, 2019; Maier and others, 2021) and the corresponding impact on the appropriate cavity opening
625 parameterization and the form of distributed water flow; and the physics of over-pressurization, including
626 the mechanical response of the ice overhead (e.g., Tsai and Rice, 2010) and the englacial and supraglacial
627 hydrologic implications (e.g., St Germain and Moorman, 2019; Andrews and others, 2022).

628 **CODE AND DATA AVAILABILITY**

629 Code to run GlaDS and analysis scripts are available online at [https://doi.org/10.5281/zenodo.](https://doi.org/10.5281/zenodo.10019656)
630 10019656. GlaDS-Matlab code is available by request to Mauro Werder. PROMICE AWS data is available
631 online at <https://doi.org/10.22008/FK2/IW73UU> (How and others, 2022).

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638 **AUTHOR CONTRIBUTIONS**

639 TH, GF, DB, and MH conceived of the idea of modifying the subglacial sheet-flow parameterization and
640 designed the experiments. TH implemented the transition parameterization within GlaDS and ran the
641 model, including analyzing and visualizing model outputs. TH, GF, and MH interpreted the model results
642 with input from MW. TH prepared the manuscript with contributions from GF, MH, and MW.

643 **REFERENCES**

644 Andrews LC, Catania GA, Hoffman MJ, Gulley JD, Lüthi MP, Ryser C, Hawley RL and Neumann TA (2014) Direct
645 observations of evolving subglacial drainage beneath the Greenland Ice Sheet. *Nature*, **514**(7520), 80–83 (doi:

- 646 10.1038/nature13796)
- 647 Andrews LC, Hoffman MJ, Neumann TA, Catania GA, Lüthi MP, Hawley RL, Schild KM, Ryser C and Morriss BF
648 (2018) Seasonal evolution of the subglacial hydrologic system modified by supraglacial lake drainage in western
649 Greenland. *Journal of Geophysical Research: Earth Surface*, **123**(6), 1479–1496 (doi: 10.1029/2017JF004585)
- 650 Andrews LC, Poinar K and Trunz C (2022) Controls on Greenland moulin geometry and evolution from the Moulin
651 Shape model. *The Cryosphere*, **16**(6), 2421–2448 (doi: 10.5194/tc-16-2421-2022)
- 652 Banwell A, Hewitt I, Willis I and Arnold N (2016) Moulin density controls drainage development beneath the Green-
653 land ice sheet. *Journal of Geophysical Research: Earth Surface*, **121**(12), 2248–2269 (doi: 10.1002/2015JF003801)
- 654 Bear J (1972) *Dynamics of fluids in porous media*. American Elsevier Publishing Company, ISBN 9780486131801
- 655 Brinkerhoff D, Aschwanden A and Fahnestock M (2021) Constraining subglacial processes from surface velocity
656 observations using surrogate-based Bayesian inference. *Journal of Glaciology*, **67**(263), 385–403 (doi: 10.1017/jog.
657 2020.112)
- 658 Bueler E and van Pelt W (2015) Mass-conserving subglacial hydrology in the Parallel Ice Sheet Model version 0.6.
659 *Geoscientific Model Development*, **8**(6), 1613–1635 (doi: 10.5194/gmd-8-1613-2015)
- 660 Chaudhuri A, Rajaram H and Viswanathan H (2013) Early-stage hypogene karstification in a mountain hydrologic
661 system: A coupled thermohydrochemical model incorporating buoyant convection. *Water Resources Research*,
662 **49**(9), 5880–5899 (doi: <https://doi.org/10.1002/wrcr.20427>)
- 663 Colebrook CF and White CM (1937) Experiments with fluid friction in roughened pipes. *Proceedings of the Royal*
664 *Society of London. Series A-Mathematical and Physical Sciences*, **161**(906), 367–381 (doi: 10.1098/rspa.1937.0150)
- 665 Cook SJ, Christoffersen P, Todd J, Slater D and Chauché N (2020) Coupled modelling of subglacial hydrology
666 and calving-front melting at Store Glacier, West Greenland. *The Cryosphere*, **14**(3), 905–924 (doi: 10.5194/
667 tc-14-905-2020)
- 668 Cook SJ, Christoffersen P and Todd J (2022) A fully-coupled 3D model of a large Greenlandic outlet glacier with
669 evolving subglacial hydrology, frontal plume melting and calving. *Journal of Glaciology*, **68**(269), 486–502 (doi:
670 10.1017/jog.2021.109)
- 671 Creyts TT and Schoof CG (2009) Drainage through subglacial water sheets. *Journal of Geophysical Research: Earth*
672 *Surface*, **114**(F4) (doi: 10.1029/2008JF001215)
- 673 Cuffey KM and Paterson WSB (2010) *The Physics of Glaciers*. Elsevier, Butterworth-Heinemann, Oxford, 4 edition

- 674 Das SB, Joughin I, Behn MD, Howat IM, King MA, Lizarralde D and Bhatia MP (2008) Fracture propagation
675 to the base of the Greenland Ice Sheet during supraglacial lake drainage. *Science*, **320**(5877), 778–781 (doi:
676 10.1126/science.1153360)
- 677 Davison BJ, Sole AJ, Cowton TR, Lea JM, Slater DA, Fahrner D and Nienow PW (2020) Subglacial drainage
678 evolution modulates seasonal ice flow variability of three tidewater glaciers in southwest Greenland. *Journal of*
679 *Geophysical Research: Earth Surface*, **125**(9), e2019JF005492 (doi: 10.1029/2019JF005492)
- 680 de Fleurian B, Gagliardini O, Zwinger T, Durand G, Le Meur E, Mair D and Råback P (2014) A double continuum
681 hydrological model for glacier applications. *The Cryosphere*, **8**(1), 137–153 (doi: 10.5194/tc-8-137-2014)
- 682 de Fleurian B, Werder MA, Beyer S, Brinkerhoff DJ, Delaney I, Dow CF, Downs J, Gagliardini O, Hoffman MJ,
683 Hooke RL, Seguinot J and Sommers AN (2018) SHMIP the subglacial hydrology model intercomparison project.
684 *Journal of Glaciology*, **64**(248), 897–916 (doi: 10.1017/jog.2018.78)
- 685 Derkacheva A, Gillet-Chaulet F, Mouginot J, Jager E, Maier N and Cook S (2021) Seasonal evolution of basal
686 environment conditions of Russell sector, West Greenland, inverted from satellite observation of surface flow. *The*
687 *Cryosphere*, **15**(12), 5675–5704 (doi: 10.5194/tc-15-5675-2021)
- 688 Dow CF, Kulessa B, Rutt IC, Tsai VC, Pimentel S, Doyle SH, van As D, Lindbäck K, Pettersson R, Jones GA and
689 Hubbard A (2015) Modeling of subglacial hydrological development following rapid supraglacial lake drainage.
690 *Journal of Geophysical Research: Earth Surface*, **120**(6), 1127–1147 (doi: 10.1002/2014JF003333)
- 691 Dow CF, McCormack FS, Young DA, Greenbaum JS, Roberts JL and Blankenship DD (2020) Totten Glacier
692 subglacial hydrology determined from geophysics and modeling. *Earth and Planetary Science Letters*, **531**, 115961
693 (doi: 10.1016/j.epsl.2019.115961)
- 694 Dow CF, Ross N, Jeofry H, Siu K and Siegert MJ (2022) Antarctic basal environment shaped by high-pressure flow
695 through a subglacial river system. *Nature Geoscience*, 1–7 (doi: 10.1038/s41561-022-01059-1)
- 696 Downs JZ, Johnson JV, Harper JT, Meierbachtol T and Werder MA (2018) Dynamic hydraulic conductivity reconciles
697 mismatch between modeled and observed winter subglacial water pressure. *Journal of Geophysical Research: Earth*
698 *Surface*, **123**(4), 818–836 (doi: 10.1002/2017JF004522)
- 699 Ehrenfeucht S, Morlighem M, Rignot E, Dow CF and Mouginot J (2023) Seasonal acceleration of Petermann glacier,
700 Greenland, from changes in subglacial hydrology. *Geophysical Research Letters*, **50**(1), e2022GL098009 (doi: 10.
701 1029/2022GL098009)
- 702 Fausto RS, Ahlstrøm AP, Van As D, Bøggild CE and Johnsen SJ (2009) A new present-day temperature parameter-
703 ization for Greenland. *Journal of Glaciology*, **55**(189), 95–105 (doi: 10.3189/002214309788608985)

- 704 Felden AM, Martin DF and Ng EG (2023) SUHMO: an adaptive mesh refinement SUBglacial Hydrology MOdel v1.
705 0. *Geoscientific Model Development*, **16**(1), 407–425 (doi: 10.5194/gmd-16-407-2023)
- 706 Flowers GE (2000) *A multicomponent coupled model of glacier hydrology*. Ph.D. thesis, University of British Columbia
707 (doi: 10.14288/1.0053158)
- 708 Flowers GE (2015) Modelling water flow under glaciers and ice sheets. *Proceedings of the Royal Society A: Mathe-*
709 *matical, Physical and Engineering Sciences*, **471**(2176), 20140907 (doi: 10.1098/rspa.2014.0907)
- 710 Flowers GE and Clarke GKC (2002) A multicomponent coupled model of glacier hydrology 1. Theory and synthetic
711 examples. *Journal of Geophysical Research: Solid Earth*, **107**(B11), ECV–9 (doi: 10.1029/2001JB001122)
- 712 Gagliardini O and Werder MA (2018) Influence of increasing surface melt over decadal timescales on land-terminating
713 Greenland-type outlet glaciers. *Journal of Glaciology*, **64**(247), 700–710 (doi: 10.1017/jog.2018.59)
- 714 Gilbert A, Gimbert F, Thøgersen K, Schuler TV and Käab A (2022) A consistent framework for coupling basal
715 friction with subglacial hydrology on hard-bedded glaciers. *Geophysical Research Letters*, **49**(13), e2021GL097507
716 (doi: 10.1029/2021GL097507)
- 717 Hager AO, Hoffman MJ, Price SF and Schroeder DM (2022) Persistent, extensive channelized drainage modeled
718 beneath Thwaites Glacier, West Antarctica. *The Cryosphere*, **16**(9), 3575–3599 (doi: 10.5194/tc-16-3575-2022)
- 719 Harper J, Meierbachtol T, Humphrey N, Saito J and Stansberry A (2021) Generation and fate of basal meltwater
720 during winter, western Greenland Ice Sheet. *The Cryosphere*, **15**(12), 5409–5421 (doi: 10.5194/tc-15-5409-2021)
- 721 Helanow C, Iverson NR, Woodard JB and Zoet LK (2021) A slip law for hard-bedded glaciers derived from observed
722 bed topography. *Science Advances*, **7**(20), eabe7798 (doi: 10.1126/sciadv.abe7798)
- 723 Hewitt IJ (2013) Seasonal changes in ice sheet motion due to melt water lubrication. *Earth and Planetary Science*
724 *Letters*, **371**, 16–25 (doi: 10.1016/j.epsl.2013.04.022)
- 725 Hewitt IJ, Schoof C and Werder MA (2012) Flotation and free surface flow in a model for subglacial drainage. Part
726 2. Channel flow. *Journal of Fluid Mechanics*, **702**, 157–187 (doi: 10.1017/jfm.2012.166)
- 727 Hill T and Dow CF (2021) Modeling the dynamics of supraglacial rivers and distributed meltwater flow with the Sub-
728 aerial Drainage System (SaDS) model. *Journal of Geophysical Research: Earth Surface*, **126**(12), e2021JF006309
729 (doi: 10.1029/2021JF006309)
- 730 Hoffman MJ, Andrews LC, Price SF, Catania GA, Neumann TA, Lüthi MP, Gulley J, Ryser C, Hawley RL and
731 Morriss B (2016) Greenland subglacial drainage evolution regulated by weakly connected regions of the bed. *Nature*
732 *communications*, **7**(1), 13903 (doi: 10.1038/ncomms13903)

- 733 Hoffman MJ, Perego M, Price SF, Lipscomb WH, Zhang T, Jacobsen D, Tezaur I, Salinger AG, Tuminaro R and
734 Bertagna L (2018) MPAS-Albany Land Ice (MALI): a variable-resolution ice sheet model for Earth system modeling
735 using Voronoi grids. *Geoscientific Model Development*, **11**(9), 3747–3780 (doi: 10.5194/gmd-11-3747-2018)
- 736 How P, Abermann J, Ahlstrøm A, Andersen S, Box JE, Citterio M, Colgan W, RS F, Karlsson N, Jakobsen J, Langley
737 K, Larsen S, Mankoff K, Pedersen A, Rutishauser A, Shield C, Solgaard A, van As D, Vandecrux B and Wright
738 P (2022) PROMICE and GC-Net automated weather station data in Greenland (doi: 10.22008/FK2/IW73UU)
- 739 Irrarrazaval I, Werder MA, Huss M, Herman F and Mariethoz G (2021) Determining the evolution of an alpine glacier
740 drainage system by solving inverse problems. *Journal of Glaciology*, **67**(263), 421–434 (doi: 10.1017/jog.2020.116)
- 741 Joughin I, Das SB, King MA, Smith BE, Howat IM and Moon T (2008) Seasonal speedup along the western flank
742 of the Greenland Ice Sheet. *Science*, **320**(5877), 781–783 (doi: 10.1126/science.1153288)
- 743 Kamb B (1987) Glacier surge mechanism based on linked cavity configuration of the basal water conduit system.
744 *Journal of Geophysical Research: Solid Earth*, **92**(B9), 9083–9100 (doi: 10.1029/JB092iB09p09083)
- 745 Karlsson NB, Solgaard AM, Mankoff KD, Gillet-Chaulet F, MacGregor JA, Box JE, Citterio M, Colgan WT, Larsen
746 SH, Kjeldsen KK, Korsgaard N, Benn DI, Hewitt IJ and Fausto RS (2021) A first constraint on basal melt-water
747 production of the greenland ice sheet. *Nature Communications*, **12**(1), 3461 (doi: 10.1038/s41467-021-23739-z)
- 748 King MD, Howat IM, Candela SG, Noh MJ, Jeong S, Noël BPY, van den Broeke MR, Wouters B and Negrete A
749 (2020) Dynamic ice loss from the Greenland Ice Sheet driven by sustained glacier retreat. *Communications Earth
750 & Environment*, **1**(1), 1 (doi: 10.1038/s43247-020-0001-2)
- 751 Koenig LS, Miège C, Forster RR and Brucker L (2014) Initial in situ measurements of perennial meltwater storage
752 in the Greenland firn aquifer. *Geophysical Research Letters*, **41**(1), 81–85 (doi: 10.1002/2013GL058083)
- 753 Koziol CP and Arnold N (2018) Modelling seasonal meltwater forcing of the velocity of land-terminating margins of
754 the Greenland Ice Sheet. *The Cryosphere*, **12**(3), 971–991 (doi: 10.5194/tc-12-971-2018)
- 755 Maier N, Gimbert F, Gillet-Chaulet F and Gilbert A (2021) Basal traction mainly dictated by hard-bed physics over
756 grounded regions of Greenland. *The Cryosphere*, **15**(3), 1435–1451 (doi: 10.5194/tc-15-1435-2021)
- 757 Moody LF (1944) Friction factors for pipe flow. *Trans. Asme*, **66**, 671–684 (doi: 10.1115/1.4018140)
- 758 Moon T, Joughin I, Smith B, Van Den Broeke MR, Van De Berg WJ, Noël B and Usher M (2014) Distinct patterns of
759 seasonal Greenland glacier velocity. *Geophysical Research Letters*, **41**(20), 7209–7216 (doi: 10.1002/2014GL061836)
- 760 Morlighem M, Williams CN, Rignot E, An L, Arndt JE, Bamber JL, Catania G, Chauché N, Dowdeswell JA, Dorschel
761 B, Fenty I, Hogan K, Howat I, Hubbard A, Jakobsson M, Jordan TM, Kjeldsen KK, Millan R, Mayer L, Mouginot

- 762 J, Noël BPY, O’Cofaigh C, Palmer S, Rysgaard S, Seroussi H, Siegert MJ, Slabon P, Straneo F, van den Broeke
763 MR, Weinrebe W, Wood M and Zinglensen KB (2017) BedMachine v3: Complete bed topography and ocean
764 bathymetry mapping of Greenland from multibeam echo sounding combined with mass conservation. *Geophysical
765 Research Letters*, **44**(21), 11–051 (doi: 10.1002/2017GL074954)
- 766 Murray T and Clarke GKC (1995) Black-box modeling of the subglacial water system. *Journal of Geophysical
767 Research: Solid Earth*, **100**(B6), 10231–10245 (doi: 10.1029/95JB00671)
- 768 Muthyala R, Rennermalm ÅK, Leidman SZ, Cooper MG, Cooley SW, Smith LC and Van As D (2022) Supraglacial
769 streamflow and meteorological drivers from southwest Greenland. *The Cryosphere*, **16**(6), 2245–2263 (doi: 10.
770 5194/tc-16-2245-2022)
- 771 Muto A, Alley RB, Parizek BR and Anandakrishnan S (2019) Bed-type variability and till (dis)continuity beneath
772 Thwaites Glacier, West Antarctica. *Annals of Glaciology*, **60**(80), 82–90 (doi: 10.1017/aog.2019.32)
- 773 Nanni U, Gimbert F, Roux P and Lecointre A (2021) Observing the subglacial hydrology network and its dynamics
774 with a dense seismic array. *Proceedings of the National Academy of Sciences*, **118**(28), e2023757118 (doi: 10.1073/
775 pnas.2023757118)
- 776 Nienow PW, Sole AJ, Slater DA and Cowton TR (2017) Recent advances in our understanding of the role of meltwater
777 in the Greenland Ice Sheet system. *Current Climate Change Reports*, **3**, 330–344 (doi: 10.1007/s40641-017-0083-9)
- 778 Pohle A, Werder MA, Gräff D and Farinotti D (2022) Characterising englacial R-channels using artificial moulins.
779 *Journal of Glaciology*, **68**(271), 879–890 (doi: 10.1017/jog.2022.4)
- 780 Poinar K and Andrews LC (2021) Challenges in predicting Greenland supraglacial lake drainages at the regional
781 scale. *The Cryosphere*, **15**(3), 1455–1483 (doi: 10.5194/tc-15-1455-2021)
- 782 Poinar K, Dow CF and Andrews LC (2019) Long-term support of an active subglacial hydrologic system in Southeast
783 Greenland by firn aquifers. *Geophysical Research Letters*, **46**(9), 4772–4781 (doi: 10.1029/2019GL082786)
- 784 Rada Giacaman CA and Schoof C (2023) Channelized, distributed, and disconnected: spatial structure and temporal
785 evolution of the subglacial drainage under a valley glacier in the Yukon. *The Cryosphere*, **17**(2), 761–787 (doi:
786 10.5194/tc-17-761-2023)
- 787 Röthlisberger H (1972) Water pressure in intra-and subglacial channels. *Journal of Glaciology*, **11**(62), 177–203 (doi:
788 10.3189/S0022143000022188)
- 789 Schoof C (2010) Ice-sheet acceleration driven by melt supply variability. *Nature*, **468**(7325), 803–806 (doi: 10.1038/
790 nature09618)

- 791 Schoof C, Hewitt IJ and Werder MA (2012) Flotation and free surface flow in a model for subglacial drainage. Part
792 1. Distributed drainage. *Journal of Fluid Mechanics*, **702**, 126–156 (doi: 10.1017/jfm.2012.165)
- 793 Smeets PC, Kuipers Munneke P, Van As D, van den Broeke MR, Boot W, Oerlemans H, Snellen H, Reijmer CH and
794 van de Wal RS (2018) The K-transect in west Greenland: Automatic weather station data (1993–2016). *Arctic,
795 Antarctic, and Alpine Research*, **50**(1), S100002 (doi: 10.1080/15230430.2017.1420954)
- 796 Sommers A, Rajaram H and Morlighem M (2018) SHAKTI: subglacial hydrology and kinetic, transient interactions
797 v1.0. *Geoscientific Model Development*, **11**(7), 2955–2974 (doi: 10.5194/gmd-11-2955-2018)
- 798 Sommers A, Meyer C, Morlighem M, Rajaram H, Poinar K, Chu W and Mejia J (2023) Subglacial hydrology modeling
799 predicts high winter water pressure and spatially variable transmissivity at Helheim Glacier, Greenland. *Journal
800 of Glaciology*, 1–13 (doi: 10.1017/jog.2023.39)
- 801 St Germain SL and Moorman BJ (2019) Long-term observations of supraglacial streams on an Arctic glacier. *Journal
802 of Glaciology*, **65**(254), 900–911 (doi: 10.1017/jog.2019.60)
- 803 Stone DB and Clarke GKC (1993) Estimation of subglacial hydraulic properties from induced changes in basal
804 water pressure: a theoretical framework for borehole-response tests. *Journal of Glaciology*, **39**(132), 327–340 (doi:
805 10.3189/S0022143000015999)
- 806 Tsai VC and Rice JR (2010) A model for turbulent hydraulic fracture and application to crack propagation at glacier
807 beds. *Journal of Geophysical Research: Earth Surface*, **115**(F3) (doi: 10.1029/2009JF001474)
- 808 van de Wal RSW, Greuell W, van den Broeke MR, Reijmer CH and Oerlemans J (2005) Surface mass-balance
809 observations and automatic weather station data along a transect near Kangerlussuaq, West Greenland. *Annals
810 of Glaciology*, **42**, 311–316 (doi: 10.3189/172756405781812529)
- 811 van de Wal RSW, Smeets CJPP, Boot W, Stoffelen M, van Kampen R, Doyle SH, Wilhelms F, van den Broeke
812 MR, Reijmer CH, Oerlemans J and Hubbard A (2015) Self-regulation of ice flow varies across the ablation area in
813 south-west Greenland. *The Cryosphere*, **9**(2), 603–611 (doi: 10.5194/tc-9-603-2015)
- 814 Venkataraman P and Rao PRM (1998) Darcian, transitional, and turbulent flow through porous media. *Journal of
815 hydraulic engineering*, **124**(8), 840–846 (doi: (ASCE)0733-9429(1998)124:8(840))
- 816 Vijay S, King MD, Howat IM, Solgaard AM, Khan SA and Noël B (2021) Greenland ice-sheet wide glacier clas-
817 sification based on two distinct seasonal ice velocity behaviors. *Journal of Glaciology*, **67**(266), 1241–1248 (doi:
818 10.1017/jog.2021.89)
- 819 Ward JC (1964) Turbulent flow in porous media. *Journal of the hydraulics division*, **90**(5), 1–12 (doi: 10.1061/
820 JYCEAJ.0001096)

- 821 Werder MA, Loye A and Funk M (2009) Dye tracing a jökulhlaup: I. Subglacial water transit speed and water-storage
822 mechanism. *Journal of Glaciology*, **55**(193), 889–898 (doi: 10.3189/002214309790152447)
- 823 Werder MA, Hewitt IJ, Schoof CG and Flowers GE (2013) Modeling channelized and distributed subglacial drainage
824 in two dimensions. *Journal of Geophysical Research: Earth Surface*, **118**(4), 2140–2158 (doi: 10.1002/jgrf.20146)
- 825 Wood BD, He X and Apte SV (2020) Modeling turbulent flows in porous media. *Annual Review of Fluid Mechanics*,
826 **52**, 171–203 (doi: 10.1146/annurev-fluid-010719-060317)
- 827 Wright PJ, Harper JT, Humphrey NF and Meierbachtol TW (2016) Measured basal water pressure variability of
828 the western Greenland Ice Sheet: Implications for hydraulic potential. *Journal of Geophysical Research: Earth
829 Surface*, **121**(6), 1134–1147 (doi: 10.1002/2016JF003819)
- 830 Yang K and Smith LC (2016) Internally drained catchments dominate supraglacial hydrology of the southwest Green-
831 land Ice Sheet. *Journal of Geophysical Research: Earth Surface*, **121**(10), 1891–1910 (doi: 10.1002/2016JF003927)
- 832 Zimmerman RW, Al-Yaarubi A, Pain CC and Grattoni CA (2004) Non-linear regimes of fluid flow in rock fractures.
833 *International Journal of Rock Mechanics and Mining Sciences*, **41**, 163–169