FIND: A Synthetic weather generator to control drought Frequency, Intensity, and Duration

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Abstract

Water systems worldwide are experiencing climate change-induced shifts in drought properties like frequency, intensity, and duration, affecting water security and reliability. To develop and test effective drought preparedness plans, researchers often use synthetic weather generators to create hydrological scenarios that explore drought variability beyond historical records. Existing weather generators typically allow users to adjust streamflow statistics like percentiles or temporal correlation but do not directly control drought properties of frequency, intensity, and duration. To fill this gap, we propose FIND (Frequency, INtensity, and Duration) synthetic weather generator. FIND incorporates a standardized drought index to directly and independently control drought frequency, intensity, and duration in generated streamflow time series while preserving observed hydrological variability. Use cases for FIND include i) water systems analysis applications that seek to train and test drought strategies under historical and plausible future drought conditions, and ii) bottom-up vulnerability studies relating system vulnerability outcomes to specific changes in drought properties of frequency, intensity, and duration. We demonstrate FIND's versatility through three experiments: replicating historically observed drought properties, generating streamflow scenarios for multiple sites preserving correlation between their drought conditions, and generating a set of scenarios with direct and independent changes in drought properties. FIND source code is openly available for applications beyond the scope of this paper.

Keywords: synthetic weather generator, drought properties, bottom-up

vulnerability analysis

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1. Introduction

Changes in drought frequency, intensity, and duration are expected to challenge water systems worldwide. However, shifts in these drought properties are expected to occur at different rates and magnitudes (Naumann et al., 2018) and produce different impacts. First, future changes in drought frequency, intensity, and duration may be fueled by different climate mechanisms. For example, changes in atmospheric circulation patterns and cyclic climate phenomena such as El Niño Southern Oscillation (ENSO) can lead to longer and more intense droughts (Singh et al., 2022). Land use change and deforestation can contribute to faster and more intense droughts (de Jager et al., 2022), while local evapotranspiration increase may drive more frequent and intense droughts (Aadhar and Mishra, 2020). Second, changes in certain drought properties may yield disproportionate effects on a water system. For instance, some systems may be more susceptible to rising drought intensity compared to longer drought duration (Zaniolo et al., 2023).

In water system literature, climate adaptation studies often rely on sampling future climate scenarios to test system resilience to climate change. Two approaches are commonly recognized. The top-down approach simulates the system under an ensemble of future climate scenarios derived from global circulation models and ran under different greenhouse gas emission scenarios. These ensembles are a lower bound on the uncertainty in climate impacts (Stainforth et al., 2007), and underestimating uncertainty can make planning decisions vulnerable to failure (Bryant and Lempert, 2010; Brown et al., 2012). In addition, these ensembles focus on capturing long-term climate change trends, but they are known to underestimate the impact of short-term extremes like droughts (Johnson et al., 2011; Rocheta et al., 2014; Tallaksen and Stahl, 2014). Bottom-up, vulnerability-based approaches offer an alternative for water system adaptation in the near term (Borgomeo et al., 2015b). Instead of aiming for precise predictions of future climate, these approaches sample relevant hydroclimatic variables within predefined plausible ranges to assess the system's response to changes (Herman et al., 2015). Bottom-up approaches can identify changes in specific variables or combinations of variables that drive water system vulnerability, including changes in drought properties.

Bottom-up methods rely on synthetic weather generators for the generation of a large sampling of plausible hydroclimatic scenarios. These generators aim to preserve certain characteristics of the local climate, such as annual variability, while modifying specific variables of interest for the bottom-up analysis. Some studies focus on changes to relevant hydroclimatic statistics, for example by applying a change factor to simulate shifts in the mean or lower percentiles of precipitation, temperature, or streamflow (Hall and Borgomeo, 2013; Yang et al., 2016; Ray et al., 2018; Giuliani et al., 2022). Other methods alter the temporal dependence structure of hydroclimatic time series, for instance by modifying the seasonality or the persistence of wet and dry conditions. Various techniques are used for this purpose, including Markov chain models (Breinl et al., 2015; Ullrich et al., 2021), spectral analysis and wavelet transforms (Steinschneider and Brown, 2013; Quinn et al., 2018; Fletcher et al., 2023), and copula methods (Borgomeo et al., 2015b; Nazemi et al., 2020). Lastly, Borgomeo et al. (2015a) proposes a versatile tool that lets the user choose the objective function of the streamflow generator to optimize the streamflow properties of interest.

Current bottom-up approaches have limitations when modeling changes in drought properties. The manipulation of specific hydroclimatic statistics can impact the drought properties of the generated scenarios, but only indirectly. For instance, shifting the mean of a streamflow scenario can lead to more intense and longer droughts, and altering the streamflow temporal structure can result in longer or more frequent droughts compared to historical observations. However, the relationship between the change in a hydrological statistic and the change in drought property is not linear and not quantified. Moreover, changes in a hydrological statistic may typically affect more than one drought property. Therefore, the precise quantification, independent manipulation, and systematic evaluation of the effects of changes in drought properties on a system remain challenging. As a result, it becomes difficult to parse out the impacts of comparable changes in drought frequency, intensity, and duration on system vulnerability directly and independently.

Drought indices offer a way to quantify the magnitude and change of drought properties. These indices are functions of hydroclimatological variables (e.g., precipitation, temperature, streamflow) and they can provide a standardized measure of drought based on statistical analysis and comparisons with historical data. One notable streamflow generator that incorporates a drought index is the approach presented in Herman et al. (2016), that uses the Standardized Streamflow Index (SSI, Vicente-Serrano and López-

Moreno (2005)) to quantify drought frequency and severity. However, this generator does not allow for independent manipulation of these two attributes.

We propose FIND (Frequency, INtensity, and Duration), a synthetic drought generator designed to generate streamflow or precipitation scenarios with specific drought properties, which can be quantified and controlled directly and independently as measured via a drought index. FIND utilizes an iterative optimization technique in which portions of a synthetic streamflow time series are sampled and replaced at every iteration according to 5 optimization objectives. These objectives involve reaching the target drought frequency, intensity, and duration, while also preserving the historical monthly streamflow autocorrelation and hydrological distribution during non-drought periods. FIND utilizes standardized drought indices as a standardized measure for quantifying drought properties, namely SSI when generating streamflow scenarios, and the Standardized Precipitation index (SPI, (McKee et al., 1993)) for precipitation scenarios.

FIND can support a variety of applications in water resources systems analysis. In general, weather generators have long been used to sample hydrological variability beyond historical records to create larger datasets for training and testing water management strategies. Unlike existing generators, FIND allows targeted sampling of drought frequency, intensity and duration, making it ideal for evaluating drought planning and management strategies specifically. Extending drought sampling is particularly important as the historical record may only contain a limited number of drought events, which could lead to overfitting drought strategies to a few drought occurrences. Additionally, FIND enables the simulation of non-stationarity in drought properties, including changes in frequency, intensity, and duration. This allows users to train and test a system under more severe conditions than historically observed. Lastly, by systematically assessing a system's response to independent changes in drought properties, FIND can support bottom-up vulnerability analysis whose goal is to draw a clear understanding of the relationship between changes in a specific drought property and its associated impact.

In this work, we demonstrate several FIND applications in hydrological time series generation, including sampling streamflow time series with drought statistics comparable to the historical record, generating streamflow scenarios for multiple correlated sites while preserving their cross-site correlation, and independently perturbing specific drought properties for bottom-up vulnerability analysis. We showcase these functionalities through experiments conducted on a streamflow location on the Pit River in northern California. The code developed for these experiments is openly accessible online and its applicability is intended to extend beyond what presented in this paper.

The remaining sections of the paper are structured as follows. The Methods section details the calculation of the adopted drought index and drought characteristics, provides an overview of the FIND algorithm and its objectives. The Case Study section introduces the streamflow sites used in the analysis, and outlines the experimental design. The Results section presents the findings of the experiments, and the Conclusion section, discusses the usability of the tool and highlights potential applications of FIND.

2. Methods

This chapter is structured in 5 sections. First, we define quantitative measures of droughts and their properties by introducing the calculation of a drought index in section 2.1. Second, we present the FIND algorithm in section 2.2. Third, we formulate its objective functions in section 2.3. Fourth, section 2.4 presents an application of FIND for correlated multisite generation. Finally, we present the experimental design for this paper's numerical analysis.

2.1. Quantification of drought characteristics and SSI calculation

Standardized drought indices offer a quantitative and consistent way to assess drought properties of frequency, intensity, and duration, allowing for comparisons across different regions and time periods. One widely used drought index is the Standardized Precipitation Index (SPI), which measures the deviation of precipitation from its long-term average over a specific time period (McKee et al., 1993). Similar standardized indices have been developed for various hydrometeorological variables, including the Standardized Streamflow Index (SSI) also known as Standardized Runoff Index (SRI) (Vicente-Serrano and López-Moreno, 2005). The experiments contained in this paper use streamflow data, so we will refer to SSI in the text that follows, noting that the proposed concepts hold true for SPI as well.

The SSI is calculated as follows. First, long-term monthly streamflow data for a particular location are aggregated over a desired time length, typically ranging from a few months to a year. A probability distribution function (PDF), such as the Gamma distribution, is selected to model the data. The parameters of the Gamma distribution are estimated using statistical methods like the maximum likelihood estimation. Next, the observed streamflow values are standardized by converting them to standard normal distribution values based on the estimated Gamma parameters. This transformation allows us to compare the observed streamflow to the long-term average in terms of standard deviations. The SSI is then calculated for each month by subtracting the long-term average cumulative distribution function value from the observed standardized value. The resulting SSI values represent the standard deviation of the aggregated streamflow from the long-term average, and they can be positive (indicating wetter conditions) or negative (indicating drier conditions).

Using the SSI time series, it is possible to identify drought events within the specified time period as a prolonged period of negative SSI whose intensity and duration are higher than a given critical threshold. While some standard values of these critical thresholds have been proposed, e.g., the Joint European Commission's definition of meteorological drought (Spinoni et al., 2015), they are widely understood to be application specific. In the case of the FIND algorithm, they can be set by users.

More formally, given the SSI time series, we identify a total of N_{DE} drought events where the i_{th} drought event is denoted as DE_i. For DE_i is classified as drought event if its intensity is higher than the minimum intensity threshold $In(DE_i) > In_{min}$, and its duration is higher than the minimum duration threshold $D(DE_i) > D_{min}$.

Specifically, drought intensity $In(DE_i)$ is measured as the average value of the SSI time series during the duration of the drought, and drought duration $D(DE_i)$ refers to the number of months during which a drought persists. A drought event ends when followed by a wet spell (positive SSI) of a duration of $nmonths\_end\_drought$ months.

Lastly, the drought frequency $F(DE)$ in a time series is calculated as the number of drought occurrences over the time series, divided by its length in years Ny.

2.2. FIND drought generator algorithm

FIND is an iterative synthetic streamflow generator where a streamflow time series is altered over thousands of iterations with Simulated Annealing (SA, Kirkpatrick et al. (1983)) until it reaches the desired drought properties while maintaining observed hydrological variability. SA has long been used to solve combinatorial optimization problems in the water resources literature (Dougherty and Marryott, 1991; Cunha and Sousa, 1999; Thyer et al., 1999), particularly to reconstruct time series that satisfy specified properties (Bárdossy, 1998). Each iteration generates a new, *swapped* streamflow time series by replacing a portion of the original, *parent* time series. The two series are compared across the optimization objectives of drought frequency, intensity, duration, monthly autocorrelation, and hydrological distribution during non-drought periods. One of the two time series is selected to become the next iteration's parent time series according to their objective values. The algorithm proceeds iteratively until a termination criterion is met.

Below, we provide more details on each step of the FIND algorithm, following the schematic in Figure 1.

a. **Parameter and time series initialization:** the user selects the target frequency, intensity, and duration of droughts, either as an absolute value, or as a fraction of historically observed drought characteristics. In addition, a number of preset optimization parameters can be adjusted. These include objective weights, a tolerance parameter that determines convergence, the initial temperature T parameter for SA and its decrease rate, the initial number of consecutive months *n_months* to replace in the parent time series and its decrease rate. The initial parent time series is generated by randomly extracting monthly values from historically calibrated monthly streamflow distributions. The length of the generated time series is controlled by the parameter *nyear_to_generate* set to 100 years.

b. **Swapped time series generation:** A swapped time series is generated by replacing a portion of length *n_months* from the parent time series. We achieve this in 4 steps (Figure 1b.). First, we aggregate historical streamflow with a rolling window of *n_months*, obtain the *n_months*-cumulative historical distribution, and extract a random value from it. This will be the new cumulative streamflow value for the *n_months* segment for the swapped time series. Second, we disaggregate the cumulative value to monthly values using the k-nearest neighbor (k-NN) method (Fix, 1985). This method searches historical *n_months*-long periods with a cumulative streamflow value that is closest to the extracted value and applies the same disaggregation factors to the extracted value. Third, we extract a random timestamp in the parent time series following which the portion of length *n_months* is replaced, generating a swapped time series as a fourth

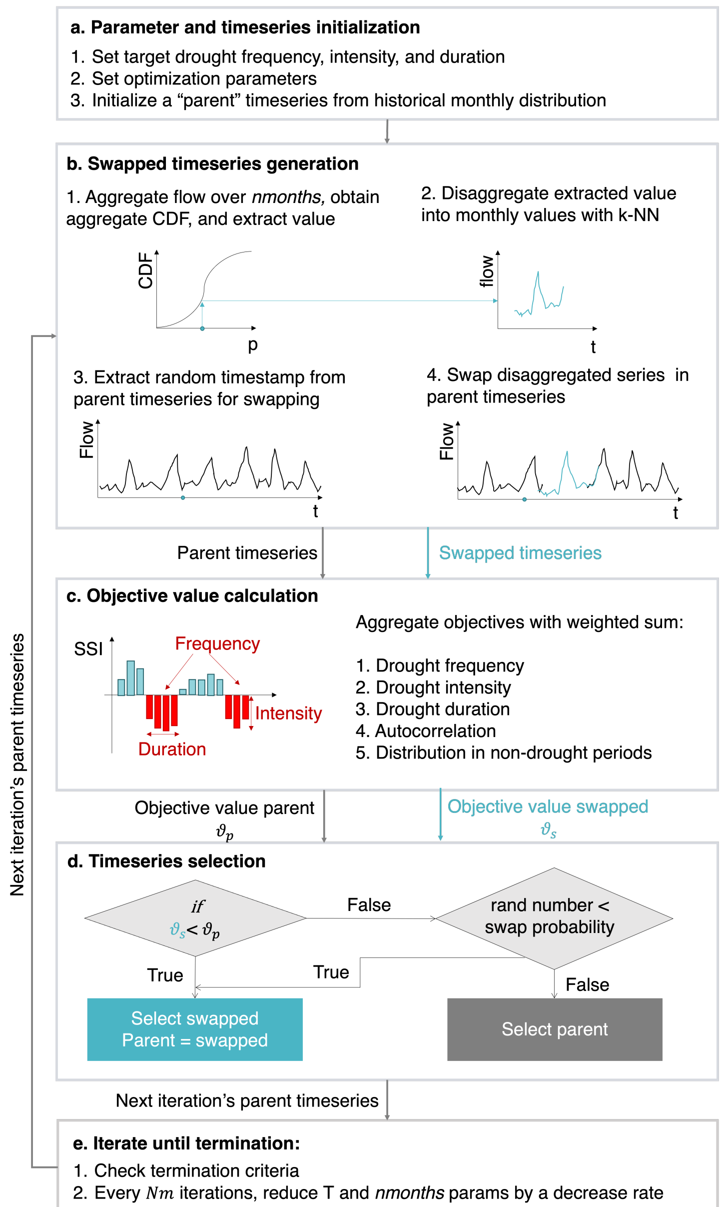

Figure 1: Schematic representation of the 5 steps in the FIND algorithm.

step.

c. **Objective value calculation:** the aggregate objective value $\mathcal{J}$ is calculated for both the parent and swapped time series as a weighted sum of 5 single objective values $\mathbf{J_i}$. The single objectives include the time series' deviation from the target frequency, intensity, duration, monthly autocorrelation over a 12-month period, and the 25th, 50th, and 75th percentiles during non-drought periods. The mathematical formulation of each objective is presented in section 2.3. The aggregated objective represents a measure of distance between the desired streamflow characteristics and those of the current time series, and lower values are preferred.

d. **time series selection:** A time series is selected between the parent and swapped to become the new parent time series for the next iteration. According to SA selection principles, if the swapped time series has a lower (better) objective value, the swapped time series becomes the new parent. If the parent time series has lower objective, the algorithm can occasionally select non-improving swaps with a probability *pmov* determined by the ratio between the parent and swapped aggregate objective values $\mathcal{J}_p$ and $\mathcal{J}_s$ respectively, and the temperature parameter T:

$$pmov = \left| \exp\left(\frac{\mathcal{J}_p - \mathcal{J}_s}{\mathcal{J}_p} * T \right) \right| \tag{1}$$

SA has been demonstrated to be more resistant than regular greedy selections (i.e., strictly minimizing the objective value) in escaping local minima (Dougherty and Marryott, 1991; Borgomeo et al., 2015a).

e. **Iterate until termination:** the time series selected during the previous step becomes the new parent time series. The algorithm proceeds by iterating through steps b.-e. until one of the two terminating criteria is met, namely the parent time series aggregated objective is lower than a tolerance parameter $\mathcal{J}_p < tol$, or the maximum Number of Function Evaluations NFE is reached. NFE depends on 2 user-defined parameters, as typical in SA applications: NFE $= m * Nm$, where m is the number of temperature drops, and Nm, is the number of iterations for each temperature. In FIND, both of temperature and *n_months* are lowered every Nm iterations by a fraction determined by the decrease rate DR, where $0 < DR < 1$. The rationale of the parameter change is that as the optimization proceeds, the search can move from a larger *exploration* of the optimization space to a more targeted *exploitation*, or refinement, of the current solution.

The FIND algorithm draws inspiration from a synthetic streamflow generator introduced by Borgomeo et al. (2015a), which also uses SA to iteratively swap values in an initialized synthetic streamflow time series. However, there are significant differences between the two methods. The previous generator swaps the position of two elements in the synthetic time series during each iteration, restricting the reorganization to values that are already present in the synthetic series, and allowing to swap only two values at a time. FIND, instead, can replace portions of varying length in the time series, allowing us to more efficiently explore the optimization space. In addition, the new swapped values are extracted from a calibrated distribution rather than from a different portion of the same series. This is critical when trying to generate a synthetic time series with, for instance, longer or more intense droughts, as it can happen that no recombination of the initialized time series values can achieve the desired drought properties. Furthermore, FIND introduces the calculation of drought indices in the optimization and employs different objective functions that focus on controlling drought properties rather than streamflow time series statistics.

2.3. Objective calculation

In this section, we formulate the objective functions calculated at step 4 of the FIND algorithm. The 5 single objectives considered in this algorithm are the deviation from target frequency, intensity, duration, observed monthly autocorrelation, and observed non-drought periods quartiles. Only one objective, the autocorrelation, is calculated directly on the streamflow time series while the other 4 are calculated on the relative SSI index time series. In FIND, the SSI of a synthetic streamflow time series is always calculated with reference to historical long-term averages rather than synthetic averages. This allows to maintain comparability across different synthetic time series as well as relevance for the site of interest.

- Drought frequency deviation: defined as the deviation between the target drought frequency F_T and the drought frequency obtained in the synthetic time series. Because all the time series generated in the code have the same length of Ny = 100 years, for simplicity we define the frequency objective directly on the number of drought events, rather than their frequency over the 100-year period.

$$J_F = |N_{DE} – F_T| \tag{2}$$

- Drought intensity deviation: defined as the average difference between the intensity of each drought event $In(DE_i)$ and the target intensity In_T, plus the difference between the average drought intensity and the target. The last element is added to penalize biased deviation, for instance in the case that the intensity of all generated drought events is lower than the target.

$$J_{In} = \sum_{i=1}^{N_{DE}} |In(DE_i) - In_T| + \left| \frac{\sum_{i=1}^{N_{DE}} In(DE_i)}{N_{DE}} - In_T \right| \tag{3}$$

- Drought duration deviation: analogously to the intensity objective, it is defined as the average difference between the duration of each drought event $D(DE_i)$ and the target duration D_T, plus the difference between the average drought duration and the target.

$$J_D = \sum_{i=1}^{N_{DE}} |D(DE_i) - D_T| + \left| \frac{\sum_{i=1}^{N_{DE}} D(DE_i)}{N_{DE}} - D_T \right| \tag{4}$$

- Autocorrelation deviation: this objective penalizes the deviation between the 12-month intermonthly autocorrelation between the historical time series and the synthetic one. The duration of 12 months is chosen with the aim of preserving in-year autocorrelation as well as year-to-year autocorrelation (Herman et al., 2016).

  For a generic time series y, the autocorrelation value for a lag time k is the correlation between values that are k time periods apart: $Corr(y_t, y_{t-k})$. As follows, the monthly autocorrelogram is the array of autocorrelation values from lag time 1 to 12 as in $AC = [Corr(y_t, y_{t-k})]$ for $k = 1, 2, ...12$.

  We call the 12-month autocorrelogram calculated on the historically observed time series as target autocorrelogram AC_T, and the synthetic time series' as AC_{synt}. Finally, the objective value J_{AC} is calculated as the sum of deviations between the two autocorrelogram series at each lag time.

$$J_{AC} = \sum_{k=1}^{12} |AC_{synt} - AC_T| \tag{5}$$

- non-drought quartiles deviation: defined as the summed deviation between the 25th, 50th, and 75th percentiles calculated for the historical SSI and the synthetic time series in non-drought periods. This objective aims to preserve historical hydrological distribution during non-drought periods even when the drought properties are modified. Non-drought events nde are here defined as the entire time series $t = 1 : H$ except the time segments occupied by drought events. The objective is thus formulated as:

$$J_{NDE} = |q25_{nde} - q25_T| + \\ |q50_{nde} - q50_T| + \\ |q75_{nde} - q75_T| \tag{6}$$

  where $q25_{nde}$ is the synthetic time series 25th percentile during non-drought periods, and $q25_T$ is the historical target percentile value, with analogous notations for the 50th and 75th percentiles.

Finally, the aggregated objective $\mathcal{J}$ for a time series is the weighted sum of the 5 single objectives with a convex user-defined set of weights ω_i.

$$\mathcal{J} = \omega_1 * J_F + \omega_2 * J_{In} + \omega_3 * J_D + \omega_3 * J_{AC} + \omega_5 * J_{NDE} \tag{7}$$

Although it is not strictly necessary, in FIND's software the duration objective is divided by a factor of 100 in order to align its order of magnitude to that of the other objectives. We find that this choice simplifies the identification of a suitable set of weights for the problem.

2.4. Multisite generation

In this section, we present the method used by FIND to generate synthetic scenarios for multiple sites while preserving the correlation between the sites' hydrological conditions, represented by the SSI values. We prioritize the correlation between SSI indices, rather than streamflow, to more accurately propagate drought conditions across correlated sites.

The algorithm employed for this analysis differs from the one described in Section 2.2 only for the objective function used. In this case, the objective is to minimize the deviation between the cumulative squared dispersion of the SSI values for the two sites $s1$ and $s2$ and the target dispersion Dis_T. The target dispersion is defined as the historically observed SSI squared dispersion for the two sites, ensuring that the synthetic scenarios closely align with the historical data:

$$Dis_T = \sum_{t=1}^{H} (SSI_{s1_H} - SSI_{s2_H})^2 \tag{8}$$

First, FIND generates a synthetic streamflow scenario and the relative SSI time series for site 1 SSI_{s1} using the algorithm presented in section 2.2. Then, FIND generates the correlated streamflow scenario for site 2 by iterative recombining a randomly sampled streamflow time series for the site, until matching the dispersion between SSI_{s1} and SSI_{s2} with the target Dis_T . The objective function is formulated as:

$$\mathcal{J}_{Dis} = \left| \sum_{t=1}^{H} (SSI_{s1} - SSI_{s2})^2 - Dis_T \right| \tag{9}$$

2.5. Use cases and Experiments

FIND is a versatile tool that supports hydrological time series generation for multiple purposes, including sampling hydrological variability beyond the historical record, generating correlated multi-site scenarios, and perturbing specific hydrological characteristics for bottom-up vulnerability analysis. In this paper, we demonstrate FIND's suitability for each of these objectives in three experiments.

In the first experiment, we utilize FIND to sample historical drought variability beyond the observed record. Our goal is to generate synthetic streamflow time series that exhibit comparable drought properties to the historical data while maintaining the site's historical temporal properties (monthly autocorrelation) and hydrological distribution during non-drought periods. In follow-up studies, these scenarios may be used to augment the sample of historical droughts with synthetic droughts within a similar range of frequency, intensity, and duration. A larger drought sample may be used to more robustly assess the system's response to droughts and the efficacy of drought mitigation policies.

The second experiment demonstrates FIND's ability to generate synthetic scenarios for two sites with correlated hydrology. Water resources planning often involves modeling a spatial extent, such as a watershed, that contains multiple sites of interest. These sites may include, for instance, multiple correlated inflow points to one or more reservoirs, or upstream and downstream flows. FIND allows to generate streamflow scenarios for multiple sites while preserving the cross-site correlation of hydrological characteristics.

Lastly, synthetic streamflow generators are used in bottom-up vulnerability analysis studies where relevant hydrological properties are perturbed to simulate plausible climate change effects and assess system vulnerability to these changes. FIND is the first tool capable of directly and independently controlling drought frequency, intensity, and duration in generated streamflow time series thus enabling future bottom up vulnerability studies to related changes in drought conditions to vulnerability outcomes. In experiment 3, we demonstrate this capability by running the algorithm 25 times intersecting 5 increments of the drought duration and intensity properties, and generating a wide range of drought conditions for the site of interest.

Each experiment requires manual tuning of FIND's optimization parameters, including objective weights for experiments 1 and 3, and termination criteria. Tuning objective weights is often required in single-objective optimization algorithms like SA to convert multiple objectives into a single aggregate objective function. By tuning these parameters, the modeler tries to achieve the desired tradeoff between multiple objectives by adjusting their scale and importance. The tuning of termination criteria balances computational time and desired performance.

The appropriate parameterization depends on the characteristics of the historical record for the case study and the specific goal of hydrological time series generation, as illustrated in the examples above. In the next section, we provide details on the case study adopted for the experiments in this paper and the parameterizations applied to each experiment.

3. Case study

This study examines two sites along the Pit River in northeastern California. The Pit River is a major river that drains from northeastern California into the state's Central Valley crossing the Cascade Range. It is the longest tributary of the Sacramento River and contributes up to eighty percent of the combined water volume into the Shasta Lake reservoir.

The selected sites are located approximately 100 miles apart in the towns of Big Bend and Candy, where long-running USGS monitoring stations have collected Pit river flow data for decades. The analysis of this paper focus on the Big Bend site, utilizing the unimpaired monthly flow record from May 1944 to June 2022. The Candy site, located northeast of Big Bend along the Pit River, is only used in experiment 2 to demonstrate the algorithm's capability to generate streamflow scenarios for multiple sites while preserving cross-site correlation. It is important to note that these two sites were chosen for demonstrative purposes, and the software is designed to accommodate any monthly streamflow or precipitation time series uploaded by the user, as long as the recorded time series is long enough to capture hydrological trends and variability. Generally, an historical record of at least 30 years is necessary, and 50 years is recommended (McKee et al., 1993).

To demonstrate FIND's versatility and usability with precipitation data, the SI reports the same experiments shown in the main paper, but for monthly precipitation data rather than streamflow data. The main precipitation site is located in Santa Rosa, CA, and the secondary site for the multisite experiment is the neighboring Fort Ross.

3.1. Experimental parameters

Table 1 summarizes FIND parameters and their adopted values for each experiment.

The first set of parameters is the drought threshold. Their selection depends on the goal of the application at hand. For instance, some applications may focus on capturing only the most severe droughts to inform emergency drought planning strategies, while other applications might want to capture many different dry spells to devise a routine drought management strategy.

Second, the selection of the optimization parameters depends on the complexity and features of the optimization problem. In general, low values of n_months and T allow small targeted improvements of the time series rather than large-scale exploration of the optimization space. Setting low values for these parameters may therefore expose to local minima traps. Conversely, high values of n_months and T allow large-scale exploration of the space but hinder the fine-tuning, or "exploitation", of solutions, potentially slowing down convergence significantly. The parameter DR controls the rate at which the values of n_months and T decrease during the optimization managing the transition between the initial exploration phase and the final exploitation phase. Parameters Nm, m, and tol control the termination criteria and are selected to balance computational time and final objective value.

Lastly, optimization weights are selected to allow the optimization process to appropriately prioritize specific objectives based on the context and requirements of the application. For instance, in Experiment 3, where drought intensity and duration are perturbed with respect to historical observations, achieving good performance requires assigning higher weights to intensity and duration objectives compared to Experiment 1, which only replicates historical droughts.

	Parameter	Explanation	Exp 1	Exp 2	Exp 3
drought thresholds	D_{min}	minimum drought duration in months	24	24	24
	In_{min}	minimum drought intensity	-0.5	-0.5	-0.5
optimization parameters	n_months	initial number of months replaced at each iteration	48	48	60
	T	initial SA temperature	0.001	0.001	0.001
	DR	decrease rate for parameters $nmonth$ and T	0.8	0.8	0.8
	Nm	iterations for each T and $nmonth$ state	600	600	1000
	m	number of T and $nmonth$ changes	15	15	15
	tol	optimization stops when objective is lower than tol	0.02	0.02	0.005
optimization weights	ω_1	intensity weight	0.1		0.1
	ω_2	duration weight	0.4		0.6
	ω_3	frequency weight	0.1		0.1
	ω_4	autocorrelation weight	0.2		0.1
	ω_5	non-drought distribution weight	0.2		0.1
	W	dispersion weight for multisite experiment		1	

Table 1: FIND parameters and adopted parameterization for each experiment.

4. Results

This chapter presents the results of the three experiments discussed in Section 2.5. In the first experiment, we use FIND to sample drought variability beyond the historical record.

For this experiment, we generate 10 synthetic time series for the Big Bend streamflow site (Figure 2). Panel a. displays the historical streamflow time series in green, and the SSI computed for the site with 12-month rolling time window, highlighting the three identified historical droughts in red. The average observed drought intensity is equal to -0.88, the drought duration is 55.6 months or 4.6 years, and the frequency is 3 drought events in 78 years, which equals one drought every 26 years.

We plotted the streamflow and SSI time series for one of the generated synthetic scenarios (Figure b). An interactive version of this figure is available on GitHub (see code availability section), allowing users to browse through the 10 series. Panels a. and b. have been rescaled to have comparable spacing on the horizontal axis, as the historical time series spans 78 years while the generated time series span 100 years. Lastly, panel c. illustrates the median and quartiles for each month of the year of the historical and

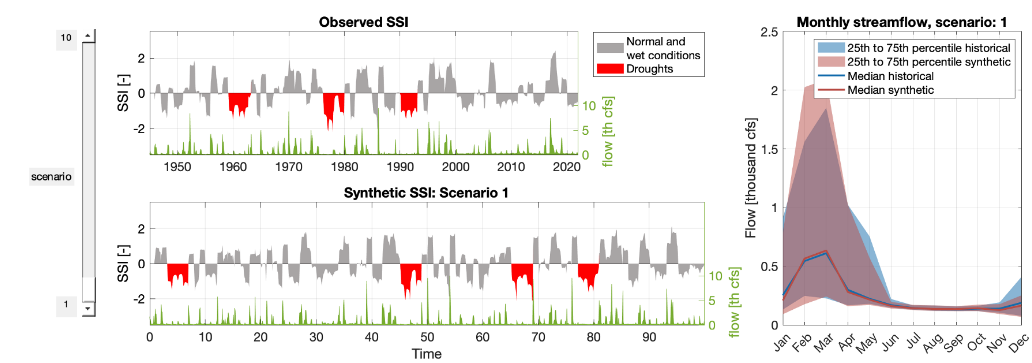

Figure 2: Synthetic drought scenario generated for the Big Bend streamflow site along the Pit River. Panel a: observed streamflow time series, and SSI drought index calculated on observed streamflow. Three droughts are identified according to the parameters defined in section 3.1. Panel b: one of the generated synthetic streamflow scenario and its relative SSI time series. Panel c. monthly median and quartiles of historical and synthetic streamflow series. An interactive version of this figure that allows to move the slider on the left is available on GitHub.

synthetic streamflow series, demonstrating a good overlap between historic and simulated data, and confirming the preservation of expected hydrological properties.

Figure 3 validates the observations made in Figure 2 by presenting the performance of the 10 generated scenarios across five optimization objectives: drought intensity, duration, frequency, monthly autocorrelation, and quartiles during non-drought periods. Panels a. and b. showcase the intensity and duration of drought events in the scenarios (red) compared to the historical scenario (blue). FIND successfully generates drought events with intensities and durations that are comparable in magnitude and range to historical droughts. Panel c. shows the desired drought frequency through a barplot. Historically, we observed 3 droughts in 78 years, which we approximate to a frequency of 4 droughts in 100 years, the length of the generated scenarios. The autocorrelogram in panel d. displays the 24-month streamflow autocorrelation of the synthetic scenarios in red alongside the historical streamflow (blue), serving as the target. We note a slight underestimation of the synthetic autocorrelation at a lag time of 1 month but overall, the synthetic series performs well in capturing the historical time structure. Lastly, panel e. presents the 25th, 50th, and 75th percentiles of the SSI index during non-drought periods. In this case as well, the algorithm demonstrates its

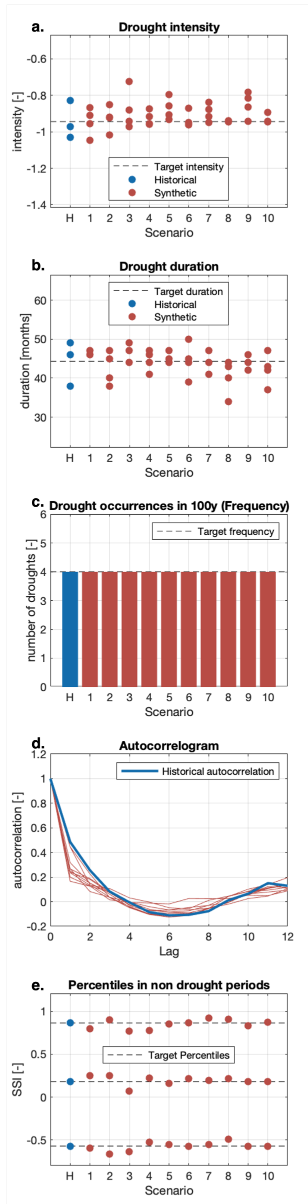

Figure 3: Visualization of the performance across the 5 objectives of the 10 generated scenarios (red) alongside historical observation (blue). Panel a: drought intensity, b: duration, c: frequency, d: autocorrelation, and e: percentiles during non-drought periods.

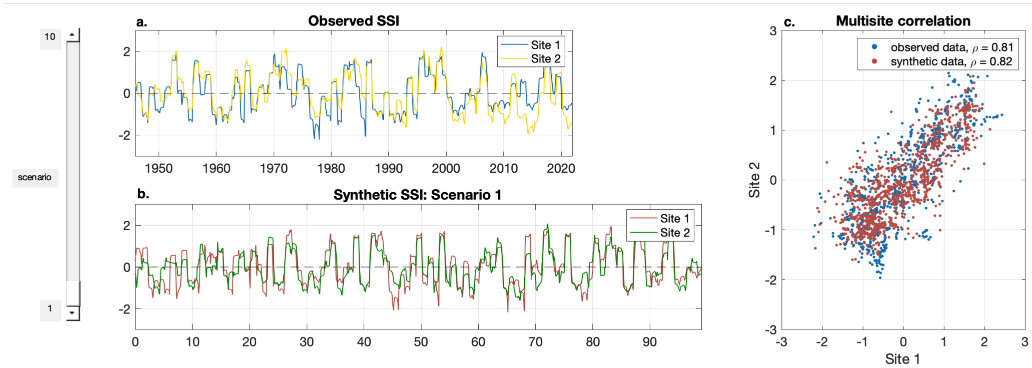

Figure 4: Synthetic drought scenarios generated for two correlated sites along the Pit River, namely Big Bend (site 1) and Canby (site 2). Panel a: SSI drought index calculated on observed streamflow for the two sites. Panel b: SSI time series for a pair of synthetic scenarios for the 2 sites. Visually, they display a similar correlation as the observations. Panel c: scatterplot of the two time series to display their correlation. The cloud of points for the observed and synthetic scenarios overlap, indicating a similar cross-site correlation. An interactive version of this figure that allows to move the slider on the left is available on GitHub.

ability to adequately reproduce historical statistics.

Figure 4 presents the results of experiment 2, which focuses on correlated multisite streamflow generation. Panel a. shows the historical SSI for the two sites, namely Big Bend and Canby. The correlation between the sites is visualized in the scatterplot of panel c, where the cloud of blue dots is distributed along the main diagonal, indicating the a positive correlation. Panel b. shows one SSI scenario generated in the previous exercise in red, alongside the relative correlated synthetic SSI for site 2. The red dots in panel c demonstrate that the synthetic data presents a similar dispersion as the historically observed data. The interactive version of this figure, available on GitHub, enables users to browse through the 10 generated scenarios, providing a more comprehensive exploration of the results.

In the third experiment, illustrated in Figure 5, we demonstrate FIND's capability to generate synthetic streamflow scenarios with user-specified drought properties, distinct from the historical record. This use mode is particularly valuable for future bottom-up vulnerability analysis that aim to quantify the vulnerability outcomes of specific climate changes in drought properties. It's important to note that in experiment 1, our focus was on generating droughts that captured the magnitude and range of historical events. However, in this experiment, our goal is to generate droughts that closely align with a specific target of interest, emphasizing a narrow range over historical range. This targeted approach is more suitable to a vulnerability analysis whose goal is to draw a clear understanding of the relationship between a specific drought property and its associated impact.

The proposed experiment focuses on changes in drought intensity (vertical axis in Figure 5a.) and duration (horizontal axis). We consider change factors ranging from 0.75 (representing a 25% decrease from the historical average) to 1.75 (representing a 75% increase), with increments of 25%. The top axis and right axes represent change factors of duration and intensity, respectively, while the left and bottom axes display the absolute values of duration and intensity. Within each square, a small white dot represents the target for each generated drought, with the square delimiting a 12.5% (half of an increment) deviation around the target multiplier.

For this figure, we generated three drought scenarios for each combination of duration and intensity. Each scenario contains three drought occurrences, represented by dots in the duration-intensity space. The color of each dot corresponds to the combination of duration and intensity for which the scenario was generated: green to yellow shades indicate increasing intensity, while dark to light shades indicate increasing duration. Dots located within squares of the same color indicate that the corresponding drought scenario aligns with the desired characteristics of duration and intensity within a narrow range. This holds true for 97% of the generated droughts (218 out of 225 generated), indicating a high level of success. The few exceptions, located near the boundaries of the target squares, can still be considered within an acceptable range.

Panels b. and c. show examples of generated SSI and streamflow time series for different drought properties. We show Short intense droughts in panel b., namely the intensity-duration combination in the bottom-left of the matrix, which corresponds to high intensity (+75% with respect to historical), and low duration (-25% with respect to historical). In panel c., we shows examples of Long mild droughts, with long duration (+75% with respect to historical) and low intensity (-25% with respect to historical).

5. Conclusions and usability

This study presents FIND (Frequency, INtensity, and Duration) drought generator, the first synthetic streamflow and precipitation generator that

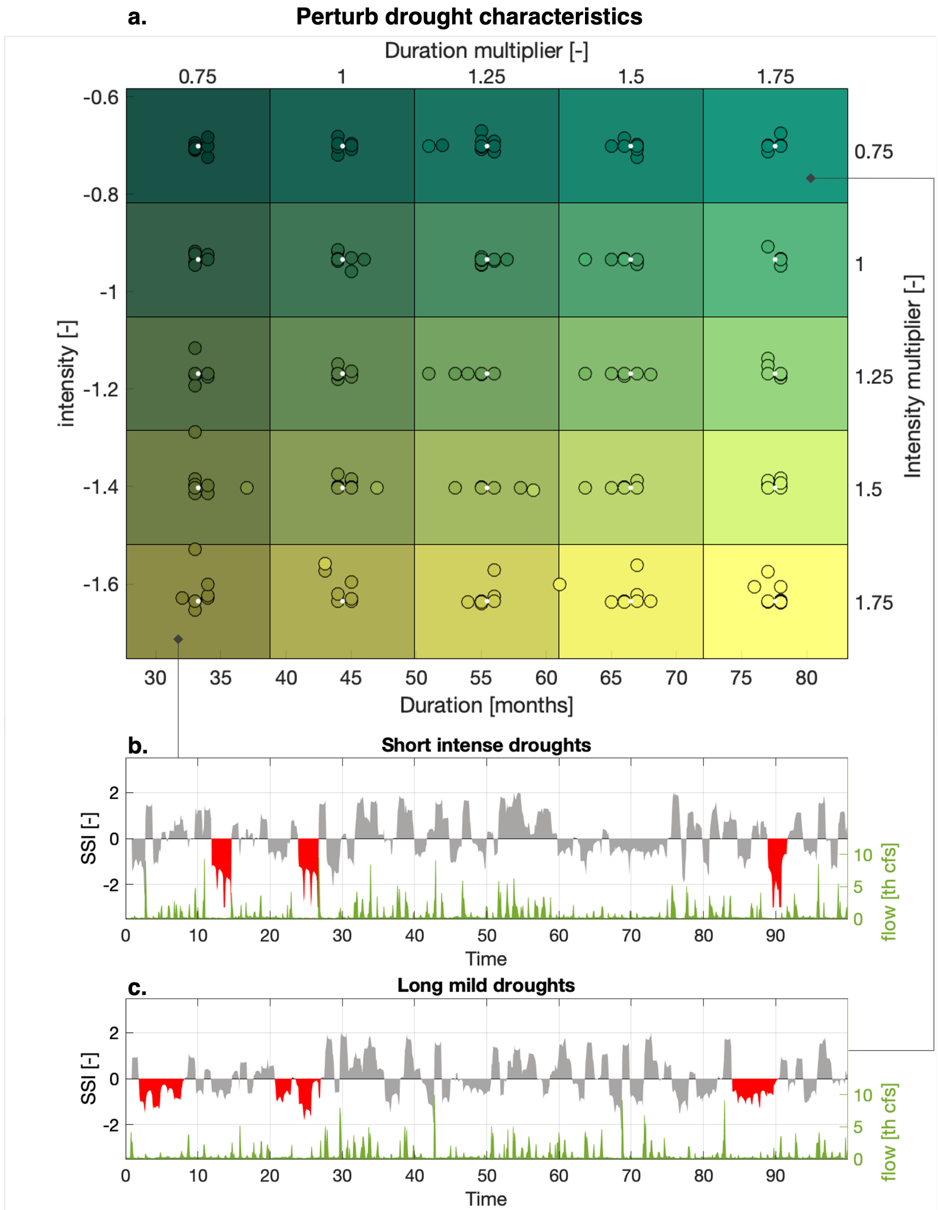

Figure 5: Panel a. Three drought scenarios including 3 drought each are generated for 25 different increments of target intensity and duration and plotted in the space of drought intensity (vertical axes) and duration (horizontal axes). The white dots indicate the targets, and are located in a square delineating a narrow range around the target amounting to half a target increment. Each colored dot represents a single drought in a drought scenario. Dots that match their background color are located in a narrow range of the target duration and intensity they were designed for, indicating success. Panel b. shows one of the generated scenarios for the combination of drought properties in the bottom-left of the matrix, namely the Short intense droughts, with high intensity (+75% with respect to historical), and low duration (-25% with respect to historical). Panel c. shows one of the generated scenarios for the combination of drought properties in the top-right of the matrix, namely Long mild droughts, with long duration (+75% with respect to historical) and low intensity (-25% with respect to historical).

allows users to control the drought properties of a synthetic streamflow scenario directly and independently. This advances current synthetic generation capabilities as existing methods only allow perturbation of statistics of hydroclimatological time series, which are indirectly linked to drought properties, rather than directly controlling drought properties. It is therefore the first synthetic streamflow generator that enables bottom-up vulnerability studies to explicitly relate changes in drought properties to system vulnerability outcomes.

FIND is designed to support water resources systems analysis applications that seek to train and test drought planning and management strategies under historical and plausible future drought conditions by augmenting the available sample of drought events. Varying drought frequency, intensity, and duration directly and independently allows modelers to identify the drought characteristics that pose the greatest threat to water systems. For example, previous work in Santa Barbara, CA found that drought intensity, not duration or frequency, led to the greater water supply deficits (Zaniolo et al., 2023). This insight, which cannot be achieved with a traditional streamflow generator, can be used by planners to design drought management approaches that target the types of droughts that pose the greatest threats.

We demonstrate FIND's applicability to a series of tasks, including sampling hydrological variability beyond the historical record, generating correlated multi-site scenarios, and perturbing specific drought characteristics for bottom-up vulnerability analysis a streamflow site in northeastern California. Beyond the applications demonstrated in this paper, FIND is intended to be a tool freely shared with the community for a variety of hydrological generation problems. In addition to streamflow scenarios, FIND can also be applied to the generation of monthly precipitation scenarios as shown in the SI, with no modification to the code.

Anyone interested in using FIND can freely download the open-source code from a GitHub repository and follow the instructions in the README file which provides guidance on loading the streamflow or precipitation record of interest, and tuning some application-specific parameters. In our experience, a few attempts are needed to tune the set of objective weights that balance the 5 optimization objectives adequately for a new case study, but future work can extend FIND to include techniques to automatically tune weights and other optimization parameters.

Appendix A. Code Availability

The open-source code developed for these experiments is made available on an online repository at `https://github.com/m-zaniolo/FIND-drought-generator`. FIND runs on recent Matlab installations (2021 and beyond).

Appendix B. Acknowledgment

This work is supported by the National Alliance for Water Innovation (NAWI), funded by the U.S. Department of Energy, Energy Efficiency and Renewable Energy Office, Advanced Manufacturing Office under Funding Opportunity Announcement DE-FOA-0001905. The views expressed herein do not necessarily represent the views of the U.S. Department of Energy or the United States Government. We thank Matteo Giuliani for assistance with beta testing of the published code.

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