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2 **Verifying pore network models of imbibition in rocks using**
3 **time-resolved synchrotron imaging**

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11 **Key Points:**

- 12 • A new method to compare pore network simulations of two-phase flow to time-resolved
13 micro-CT data
- 14 • Evolution of fluid distributions were validated with respect to pore size, connectivity
15 and flow paths
- 16 • Quasi-static pore network models put the fluids in approximately the right pores to
17 predict flow properties

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18 Abstract

19 At the pore scale, slow invasion of a wetting fluid in porous materials is often modelled with
20 quasi-static approximations which only consider capillary forces in the form of simple pore
21 filling rules. The appropriateness of this approximation, often applied in pore network mod-
22 els, is contested in literature, reflecting the difficulty of predicting imbibition relative per-
23 meability with these models. However, validation by sole comparison to continuum-scale
24 experiments is prone to induce model overfitting. It has therefore remained unclear whether
25 difficulties generalizing the model performance are due to network extraction, the pore fill-
26 ing rules, or whether a quasi-static description is useful at all. Here, we address this by ex-
27 amining whether such a model can predict the pore-scale fluid distributions underlying the
28 behaviour at the continuum scale. To this end, we compare the fluid arrangement evolution
29 measured in fast synchrotron micro-CT experiments on two rock types to quasi-static simu-
30 lations which implement capillary-dominated pore filling and snap-off, including a sophisti-
31 cated model for cooperative pore filling. The results indicate that pore network models with
32 appropriate pore filling rules can in principle obtain a good first-order prediction of the up-
33 scaled flow properties of strongly-wetted rocks at low capillary numbers.

34 1 Introduction

35 Dunking a biscuit into tea, which then soaks up the liquid, is an everyday example of
36 a wetting fluid (tea) displacing a non-wetting fluid (air) in a porous medium (biscuit). This
37 process, termed imbibition, occurs in nature in the topmost layer of soil during rainfall, dur-
38 ing the flow of immiscible pollutants in ground water resources, the geological storage of
39 carbon dioxide or hydrogen, and the management of petroleum reservoirs [*Bickle, 2009; At-*
40 *teia et al., 2013; Blunt, 2017*]. To understand the link between pore structure, wettability and
41 upscaled multi-phase flow parameters, a large amount of effort has been put into simulating
42 the pore-scale fluid distribution and calculating the corresponding macroscopic properties
43 [*Meakin and Tartakovsky, 2009; Bultreys et al., 2016a*].

44 The key challenge in this field is to describe the intricate arrangement of the two flu-
45 ids in the pore space during the fluid invasion [*Herring et al., 2013; Armstrong et al., 2016;*
46 *Zhao et al., 2016*]. Models struggle to capture the effect of the many fluid-fluid interface
47 movements on the fluid flow over sufficiently large length and time scales [*Meakin and Tar-*
48 *takovsky, 2009*]. While Navier-Stokes solvers are becoming mature enough to perform useful
49 numerical experiments on domains tens of pores across over relatively short time scales [*Fer-*

50 *rari and Lunati, 2013; Shams et al., 2018; Alpak et al., 2018; McClure et al., 2018*], it re-
51 mains desirable to define a more conceptual model which describes the emergence of struc-
52 tures at the scale of fluid clusters for upscaling [*Hilfer et al., 2015*].

53 Pore network models (PNMs) conceptualize the pore space as a network of larger void
54 volumes, pores, bounded by restrictions, called throats (Figure 2). In Earth science cases, the
55 global flow rates are typically so low that the capillary force is often assumed to be domi-
56 nant at the pore scale [*Blunt, 2017*]. The most computationally efficient type of pore network
57 models therefore adopts the quasi-static assumption: only capillary forces are taken into ac-
58 count and the fluids are assumed in capillary quasi-equilibrium at each moment [*Wilkinson*
59 *and Willemsen, 1983; Blunt et al., 1992*]. The fluid invasion process is abstracted into filling
60 rules which aim to produce similar emerging fluid distribution patterns as when the capillary,
61 viscous and inertial forces drive the process in reality [*Blunt et al., 2013*]. The pore space is
62 invaded as a sequence of invasion events of three types: snap-off, piston-like displacement
63 and cooperative pore filling [*Lenormand et al., 1983*]. Predicting the evolution of the pore-
64 scale fluid distribution therefore reduces to calculating the entry capillary pressure for each
65 of these events in all available pores, and gradually filling the pore space with the invading
66 fluid in order of this pressure. Dynamic pore network models, on the other hand, explicitly
67 capture the visco-capillary force balance of immiscible transport [*Lenormand et al., 1983;*
68 *Joekar-Niasar and Hassanizadeh, 2012*], but come at the price of increased complexity and
69 computational demands.

70 While a number of early studies have showed that quasi-static PNMs can match exper-
71 imental relative permeability curves in specific cases [*Øren et al., 1998; Valvatne and Blunt,*
72 *2004; Blunt et al., 2013*], substantial efforts over several decades have not led to satisfying
73 predictive capabilities in a general sense [*Bondino et al., 2012*]. It has been unclear whether
74 the shortcomings originate from uncertainties in the extracted network, the pore filling rules,
75 or whether it is adequate to use a quasi-static description at all. Newly available fast time-
76 resolved X-ray micro-CT experiments [*Wildenschild and Sheppard, 2013; Youssef et al.,*
77 *2013; Berg et al., 2013; Andrew et al., 2015; Bultreys et al., 2016b; Schlüter et al., 2017;*
78 *Singh et al., 2017*] have challenged the quasi-static assumption, as viscous and inertial effects
79 such as ganglion dynamics and intermittency have been observed [*Rücker et al., 2015; Arm-*
80 *strong et al., 2016; Gao et al., 2017; Reynolds et al., 2017*]. However, it is currently not well
81 understood how strongly the quasi-static approximation deviates from the experimental real-
82 ity in rock samples, and if this produces fundamentally flawed predictions at the intermediate

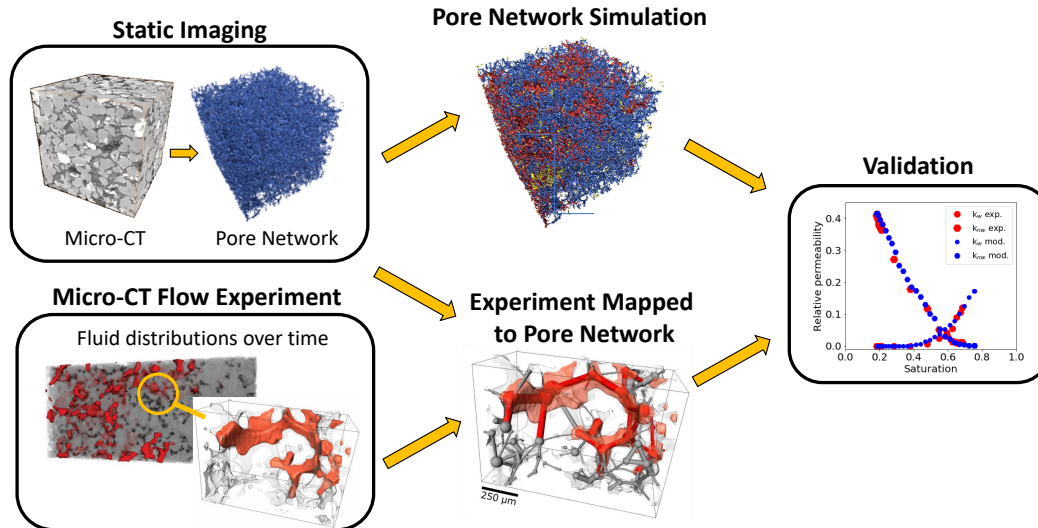
83 scale of the fluid clusters. The validation question is complicated by the fact that model sim-
84 plifications lead to a large number of internal microscopic degrees of freedom, but the output
85 consists of relatively few macroscopic parameters that can be used for validation. Therefore,
86 there is a risk of overfitting the model when tuning it to experimentally measured continuum
87 scale flow properties, e.g. relative permeability and capillary pressure-saturation functions
88 [Bondino *et al.*, 2012; Aghaei and Piri, 2015; Masalmeh *et al.*, 2015; Jerauld *et al.*, 2017].
89 Instead, criteria which contain enough information to reconstruct the relevant aspects of the
90 system’s internal state are required [Liu *et al.*, 2013].

91 In a previous study, we showed how PNM predictions can be mapped to fluid distribu-
92 tions measured with micro-CT to facilitate direct pore-by-pore comparisons [Bultreys *et al.*,
93 2018]. Here, we extend this to time-resolved micro-CT datasets to validate the filling se-
94 quence rather than a static snap-shot, resulting in a more complete picture. The experimental
95 data in this study are two publicly available fast synchrotron micro-CT datasets of water-
96 flooding in strongly water-wet Ketton limestone [Singh *et al.*, 2017] and Gildehauser sand-
97 stone [Rücker *et al.*, 2015]. This study investigates whether a quasi-static assumption can
98 result in realistic filling sequences for strong imbibition at low capillary numbers. Contrary
99 to previous studies, e.g. Berg *et al.* [2016], the models used here implement snap-off and
100 cooperative pore filling processes on experimentally based geometries [Valvatne and Blunt,
101 2004; Ruspini *et al.*, 2017]. We used a realistic cooperative pore filling algorithm [Ruspini
102 *et al.*, 2017], and minimized the influence of the specific PNM extraction and of the bound-
103 ary conditions on the analysis. The general workflow is depicted in figure 1. We find that
104 the quasi-static model generally puts the fluids in the right pores to approximate mm-scale
105 averaged properties, at least in strongly water-wet samples at low capillary numbers.

111 2 Materials and methods

112 2.1 Experiments

113 Details of the Ketton and Gildehauser unsteady-state (constant injection flow rate)
114 imbibition experiments are described in Singh *et al.* [2017] and Rücker *et al.* [2015]. Both
115 datasets are publicly available [Singh *et al.*, 2018; Berg *et al.*, 2018]. Ketton and Gildehauser
116 (the latter a local variety of Bentheimer) respectively have a porosity of 23 % and 20 %; and
117 a permeability of $2.8 \times 10^{-12} \text{ m}^2$ and $1.48 \times 10^{-12} \text{ m}^2$ [Andrew *et al.*, 2014]. In the experi-
118 ments, KI-brine was used as the wetting phase and decane as the non-wetting phase. As the



106 **Figure 1.** The workflow for validation of pore scale multi-phase flow models proposed in this paper. After
 107 extracting a pore network model from a micro-CT scan of the sample, we generated a series of fluid distribu-
 108 tions in the pore network model by either numerical simulation or experimental micro-CT imaging. We then
 109 investigate whether the simulation succeeds at filling pores in approximately the right order compared to the
 110 experiment, particularly with regards to predicting upscaled flow properties.

119 samples were clean quarry stones, they were water-wet. Cylindrical samples with a diame-
 120 ter of approximately 4 mm and a height of 10 mm were inserted into flow cells. First, a high
 121 quality dry micro-CT scan was obtained. Next, the samples were fully saturated with brine
 122 and drained by injecting decane. Then, imbibition was initiated by injecting brine at very
 123 low capillary numbers (1.26×10^{-9} in the Ketton case, 1.8×10^{-8} in the Gildehauser case).
 124 During imbibition, each sample was partly imaged along its height by fast micro-CT (Ketton:
 125 400 scans at 38 seconds/scan, voxel size $3.28 \mu\text{m}$, image height 3.28 mm; Gildehauser: 40
 126 scans at 45 seconds/scan, voxel size $4.4 \mu\text{m}$, image height 2.49 mm). Imaging was performed
 127 at I13-2 (Diamond Light Source) and at TOMCAT (Swiss Light Source). As detailed in the
 128 aforementioned references, the images were treated with a non-local means filter and seg-
 129 mented with the watershed algorithm in Avizo 9 (ThermoFisher/FEI). The final Gildehauser
 130 image contained an oil cluster connected to the top and the bottom of the image, i.e. the oil
 131 in the sample may still have been mobile.

2.2 Quasi-static modelling

The pore network models used here were extracted from the segmented dry scans using the maximal ball method [Raeini *et al.*, 2017]. This method employs the distance map of the pore space image to find pores and throats as constrictions and dilations. The radii of these elements were found by looking for the maximum inscribed spheres at these locations. Shape factors were used to describe the angularity of pores and throats as described in *Bultreys et al.* [2018]. The pores and throats had circular, triangular and square cross-sections, based on their shape factor [Valvatne and Blunt, 2004].

After network extraction, imbibition was simulated using a quasi-static invasion percolation approach, based on *Øren et al.* [1998], *Valvatne and Blunt* [2004] and *Ruspini et al.* [2017]. Following these works, entry pressures were calculated for each pore and throat which could be accessed by the invading fluid and from which the defending fluid had an escape path (i.e. was not trapped). The pressure calculations are summarized in the Supporting Information. The wetting phase was injected from the top and bottom face, and the non-wetting phase was allowed to escape through both, to account for backflow in the porous medium outside the field of view. We used the fluid occupancy at the start of imbibition in the experimental data to assign the initial condition of the model. Advancing contact angles were randomly assigned to each pore or throat in the range between 35 and 45 degrees for Ketton [Scanziani *et al.*, 2017] and 35 and 55 degrees for Gildehauser [Khishvand *et al.*, 2016], based on in situ contact angle measurements. We list here the PNM assumptions and parameters which can influence the simulated filling sequence:

- The segmentation of the micro-CT scan. While this can have an important effect on the network extraction and simulations, the large pores and the good image quality in the dry scans was deemed to lead to high quality segmentations. We used the segmented images provided in the public domain data sets, without tuning the segmentation to improve the PNM results.
- The detection of pore and throat centres as local minima and maxima of the distance map. The main user-defined parameter is the minimum inscribed radius for a point to be called a pore centre, set to 1.5 voxel lengths. This does not affect the results for the well-resolved, rather simple pore structures in this work. In any event, the same network extraction was used to analyse both the experiment and the model.

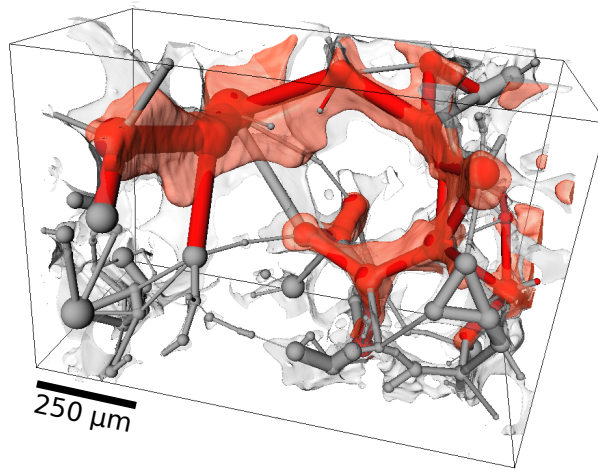
- 163 • Pore and throat shape factor determination, and the simplification of pore shapes to
164 (predominantly) triangles with the same shape factor and inscribed radius [*Øren et al.*,
165 1998]. This induces an approximation, but it is fully defined by the detection of pore
166 centres and throat surfaces.
- 167 • Advancing contact angle assignment. These were randomly assigned in each pore or
168 throat in the range between 35 degrees and 45 degrees for Ketton [*Singh et al.*, 2016;
169 *Scanziani et al.*, 2017] and 35 degrees and 55 degrees for Gildehauser [*Khishvand*
170 *et al.*, 2016], based on in situ contact angle measurements. The sensitivity within
171 likely values (several ranges spanning 25 degrees to 65 degrees) was found not to in-
172 fluence the conclusions in this paper, as shown in the Supporting Information.
- 173 • The capillary pressure reached during drainage, prior to imbibition. This is inferred
174 from the radius of the smallest pore or throat that is oil-filled in the micro-CT image
175 after drainage.
- 176 • One parameter in the cooperative pore filling algorithm, which determines when the
177 interface becomes unstable while entering the throats. This parameter has a very lim-
178 ited influence on the simulations results [*Ruspini et al.*, 2017].

179 **2.3 Data analysis**

180 To allow consistent comparison of the experiments to PNM simulations, we used the
181 PNM as an image analysis tool on the imbibition datasets. As detailed in *Bultreys et al.*
182 [2018], the inscribed spheres at each pore and throat centre in the dry scan were determined
183 during network extraction. These were used to define the fluid occupying the pore or throat
184 centre. For each time step in the imbibition experiment, a pore or throat was called wetting-
185 filled if more than half of the voxels of the associated inscribed sphere are segmented as wet-
186 ting fluid. The simulations were run on the same PNM used to analyse the experiments, en-
187 abling a like-for-like comparison (Figure 2). The measures shown in this paper were calcu-
188 lated on the PNM structure, with fluid distributions mapped from the experiments, or from
189 simulations on the PNM.

195 **2.3.1 Filling size**

196 Pores and throats in the experiment were considered to be invaded at time step t if they
197 were filled with wetting fluid in image t and with non-wetting fluid in image $t - 1$. If they



190 **Figure 2.** Voids in Ketton limestone, with the non-wetting fluid observed in one of several hundred snap-
 191 shots of an imbibition experiment indicated in transparent red (the figure shows a small subvolume of a
 192 micro-CT scan). The balls and sticks represents the corresponding pore network model, colored according
 193 to the fluids occupying each pore or throat in a simplified model of fluid invasion. We use this one-to-one
 194 mapping to compare the evolving fluid filling pattern to the experiment.

198 changed filling state more than 3 times, they were not listed here, as these were a small num-
 199 ber of mainly narrow throats affected by image noise. The filling time was then translated to
 200 the filling sequence rank number (the rank of the pore or throat filling event when ordered
 201 according to the filling time). This was necessary because the simulations do not have a time
 202 scale, but only predict in which order pores and throats fill. In the Gildehauser case, the rate
 203 of filling was relatively large compared to the time resolution of the imaging. Therefore, fill-
 204 ing sizes at certain time steps in this experiment are shown using box plots with outliers.

205 **2.3.2 Connectivity**

206 To characterize the connectivity, we calculate the Euler characteristic of the shape
 207 formed by the throats (conceptualized here as cylinders) and pores (conceptualized as spheres)
 208 that are filled with the non-wetting phase from either the experiment or the simulation. We
 209 first remove isolated non-wetting phase-filled throats to reduce the dependency on imaging
 210 noise. Then, we modify this shape by adding a sphere to the end of any cylinder which does
 211 not already end in a sphere. This does not change the Euler characteristic as it is invariant
 212 under continuous deformation of the shape. The Euler characteristic χ can be calculated us-

213 ing the Betti numbers β_i :

$$\chi = \beta_0 - \beta_1 + \beta_2 \quad (1)$$

214 Here, β_0 equals the number of connected components k and β_2 is the number of holes
215 in the shape, the latter by construction equaling zero. β_1 is the number of loops in the shape
216 [Wildenschild and Sheppard, 2013], equalling $N_e - N_v + k$, with N_e the number of edges and
217 N_v , the number of vertices of the corresponding graph [Berge, 2001]. Therefore, the calcula-
218 tion of the Euler characteristic reduces to:

$$\chi = N_v - N_e \quad (2)$$

219 Note that the calculation does not depend on the exact pore and throat shapes. Detect-
220 ing the connected components in the non-wetting phase graph allows to calculate the amount
221 and sizes of clusters, after which the amount of loops can be calculated as $k - \chi$.

222 **2.3.3 Flow paths**

223 The relevance of discrepancies in the filling sequence to relative permeabilities is a
224 question of key concern [Berg *et al.*, 2016]. However, the calculation of volumes and fluid
225 flow conductances for each pore and throat induces non-uniqueness associated with user-
226 defined parameters [Sorbie and Skauge, 2011]. Since we are only interested here in the fluid
227 displacements and not in improving conductivity or saturation calculations, we circumvent
228 this problem by calculating relative permeabilities for both the experimental and the simu-
229 lated fluid distributions on the same PNM, using the same (possibly imprecise) conductivi-
230 ties and volumes. This yields a flow-weighted connectivity metric which is a proxy to (but
231 not necessarily equal to) physical relative permeability measurements.

232 For the experiments, relative permeabilities were calculated by mapping the experi-
233 mentally observed fluid distribution on the same PNM as used in the imbibition simulations.
234 The Hagen-Poiseuille conductivity model was used to calculate the flow rates through the
235 wetting and the non-wetting phase separately, using the OpenPNM pore network modelling
236 package [Gostick *et al.*, 2016]. We assumed that throats filled with one phase have zero con-
237 ductivity for the other phase. Flow rates for the simulations were calculated on the same
238 PNM with the same conductivities, and discrepancies compared to the experiment therefore
239 came solely from differences in the fluid distribution. A good match means the flow distri-
240 bution in the model is approximately correct to predict the upscaled properties. However, to

241 match experimentally measured relative permeabilities (based on pressure measurements),
242 it may still be necessary to improve the volume and conductivity assignment of pores and
243 throats. This less fundamental issue is outside the scope of the study.

244 **Results and discussion**

245 Previous studies have shown that the pore-scale fluid distribution in experiments is not
246 fully reproducible [Ferrari *et al.*, 2015; Ling *et al.*, 2017; Bultreys *et al.*, 2018]. The averaged
247 properties are, however, much more reproducible, as different realisations of the fluid distri-
248 bution can lead to consistent averaged results if they are statistically similar. Therefore, sim-
249 ply comparing the filling sequence on a pore-by-pore basis does not offer a relevant answer
250 [Bultreys *et al.*, 2018]. Recognizing that a good prediction of the upscaled multi-phase flow
251 properties needs to capture the geometry and the connectivity of the fluid phases throughout
252 the filling sequence, we use the following criteria to validate the model:

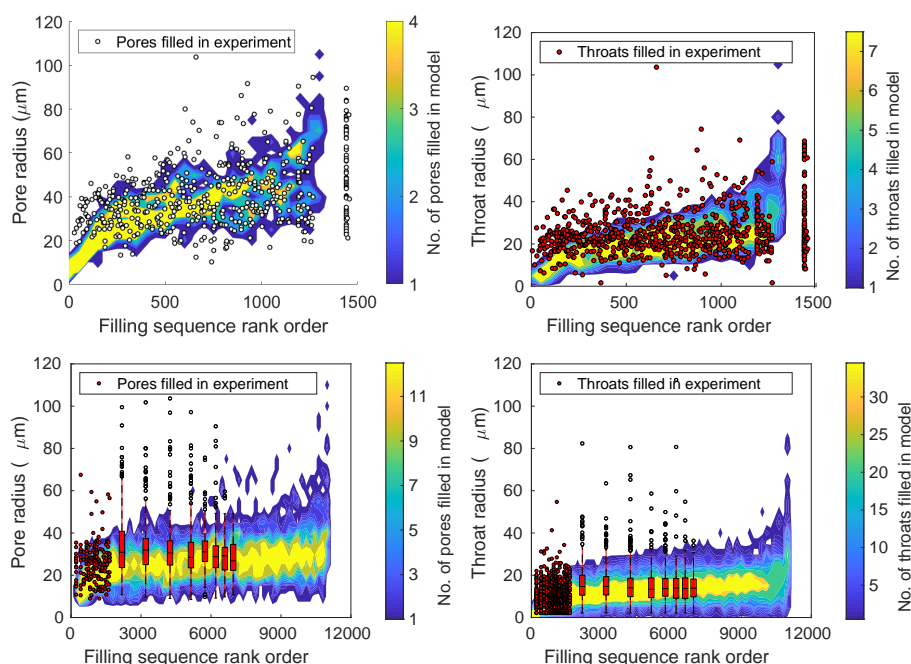
- 253 • In terms of size (radius), which pores and throats are filled throughout the filling se-
254 quence?
- 255 • How does the connectivity of the non-wetting phase change during the filling se-
256 quence?
- 257 • How do the flow paths of the wetting and the non-wetting phase change throughout
258 the filling sequence?

259 As our goal was to evaluate physical concepts rather than model implementation, the
260 influence of PNM non-uniqueness on the validation was reduced as much as possible. Pa-
261 rameters that affect the filling sequence were set based on physical principles and were sensitivity-
262 checked.

263 **Filling size**

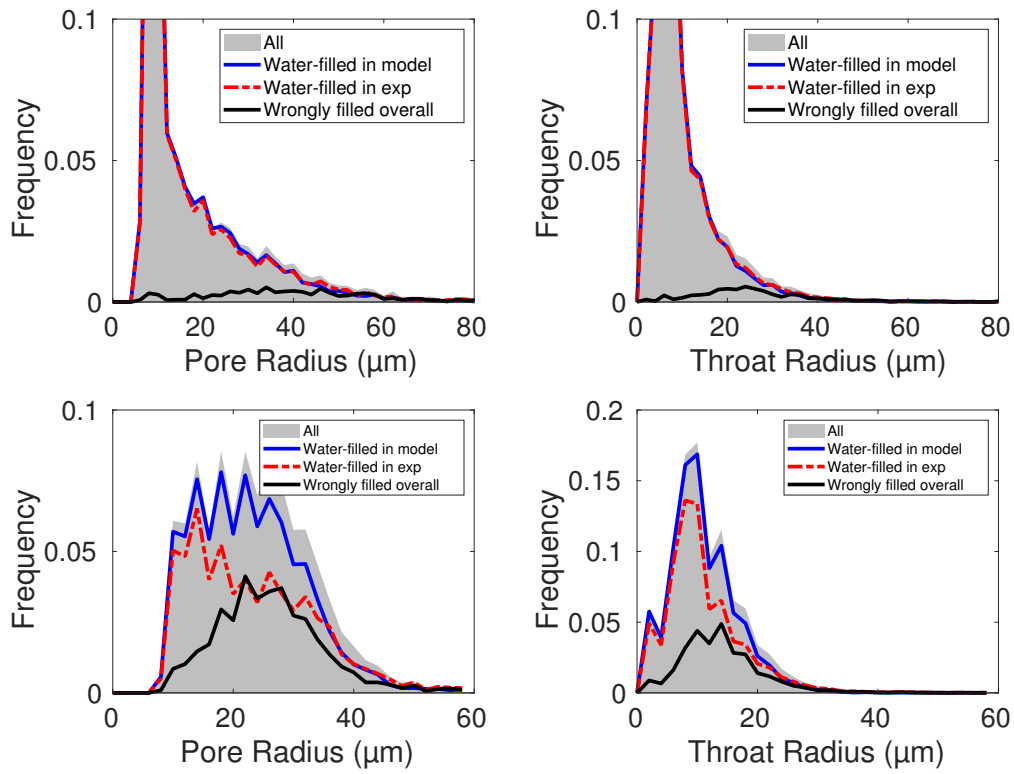
264 The PNM used in this study explicitly defines pores as local dilations and throats as
265 local constrictions [Raeini *et al.*, 2017]. In terms of the associated capillary energy, pores
266 should locally be less favourable places for the invading wetting phase to pervade, while
267 throats should be more favourable. Therefore, we tracked the invasion of the wetting phase
268 in these locations in the experiment and the model, using the same network extraction on
269 both. We define filling in the experiment as events where the image shows a change in oc-

270 cupancy, such that the voxels associated with the maximal sphere centred on a pore or throat
 271 first become occupied principally by the wetting phase in the segmented image.



272 **Figure 3.** Pore and throat filling sizes in the the model and the experiment for Ketton (top row) and Gilde-
 273 hauser (bottom row) matched well, although there is a tendency for a small number of larger elements to be
 274 filled in the experiment and not by the model. Dots and box plots (for time steps where many network ele-
 275 ments were invaded at the same time) present the radii of pores and throats filled with the invading wetting
 276 phase during the experiment in rank order. The contour plots show the pore or throat radii filled in the model.

277 Figure 3 shows the radii of pores and throats that were invaded during the experiment
 278 as a function of their rank order in the filling sequence. In general, pores and throats of simi-
 279 lar sizes were filled throughout the filling sequence in the experiment and the model. In both
 280 cases, filling did not strictly happen in order of increasing radius due to the occurrence of the
 281 three types of filling (snap-off, piston-like advance and cooperative pore filling) and the vari-
 282 ation in pore and throat shapes. A small number of pores and throats with large radii were
 283 filled in the experiment, but not in the simulations. This could be related to geometry simpli-
 284 fications in the network model, to local variations in wettability [AlRatrou et al., 2018], or
 285 to viscous and/or inertial effects which are not captured in the quasi-static PNM. A decrease
 286 of the local capillary pressure caused by the latter two effects may have led to the invasion of
 287 pores with larger characteristic size than predicted by the quasi-static model.



288 **Figure 4.** Pore and throat sizes that are filled with water at the end of imbibition, for Ketton (top row) and
 289 Gildehauser (bottom row). The black line indicates the frequency of pore and throat sizes that were filled with
 290 fluid in the model different than in the experiment.

291 Figure 4 shows the size distribution of the water-filled pores and throats in the experi-
292 ments and the model after imbibition, and indicates pores and throats filled with a fluid in the
293 model different from the experiment, similar to *Bultreys et al.* [2018] and *Øren et al.* [2019].
294 In the Ketton experiment both model and experiment reached residual saturation. There is
295 a very close agreement, with a filling discrepancy of 9 % in the pores and 6 % in the throats
296 (the percentage of pores or throats that were filled with a different fluid in the model as in the
297 experiment).

298 The Gildehauser experiment did not reach the residual saturation in the experimen-
299 tal field of view, and therefore we compared the simulated fluid distribution with the same
300 saturation as the final state in the experiment. The model overpredicted the number of water-
301 filled pores and throats, resulting in a filling discrepancy of 30 % in the throats and 38 % in
302 the pores. This is explained by the fact that the simulation was stopped at the same saturation
303 as the experiment, while a number of large pores were filled earlier in the experiment than
304 in the model (Figure 3). A previous study showed filling discrepancies of 18 % and 21 % for
305 PNM simulations compared to steady-state Bentheimer experiments at the end of imbibition
306 [*Bultreys et al.*, 2018], and a standard deviation of up to 25 % on the pore occupancy for the
307 intermediately-sized pores in repeated experiments. A different study on Paaratte sandstone
308 found filling discrepancies of 15 % and 13 % [*Øren et al.*, 2019].

309 The validation of pore and throat filling sizes presented here improves on the approach
310 in *Bultreys et al.* [2018] and *Øren et al.* [2019] because it took the dynamics of the filling
311 process into account. Overall, a more favourable agreement was found here than in *Bultreys*
312 *et al.* [2018] due to the use of a different cooperative pore filling algorithm [*Ruspini et al.*,
313 2017] and initial and boundary conditions that more closely resembled those of the experi-
314 ment. The cooperative pore filling algorithm developed by *Ruspini et al.* [2017] is a signifi-
315 cant improvement over the previously available algorithms because it accounts for the local
316 geometry of the surrounding throats to assign the capillary invasion pressures.

317 In repeated drainage-imbibition experiments on Bentheimer, the standard deviation of
318 the filling state was found to be up to 25 % for the most variable pores, while the large-scale
319 behaviour did not vary significantly [*Bultreys et al.*, 2018]. The filling discrepancy between
320 the PNM simulation and the experiment is thus of similar magnitude to the difference be-
321 tween two repeat experiments that are macroscopically nearly indistinguishable.

Connectivity

The upscaled flow characteristics are strongly influenced by the phase connectivity [Hilfer, 2006], which can be characterized by the Euler characteristic [Armstrong *et al.*, 2016; Schlüter *et al.*, 2016]. We compared the evolution of the Euler characteristic during imbibition in the model and the experiment (Figure 5). A consistent comparison to the simulations was achieved by mapping the experimental distributions on the PNM and calculating the Euler characteristic there. The experimental trend is consistent with direct image-based calculations [Alpak *et al.*, 2018].

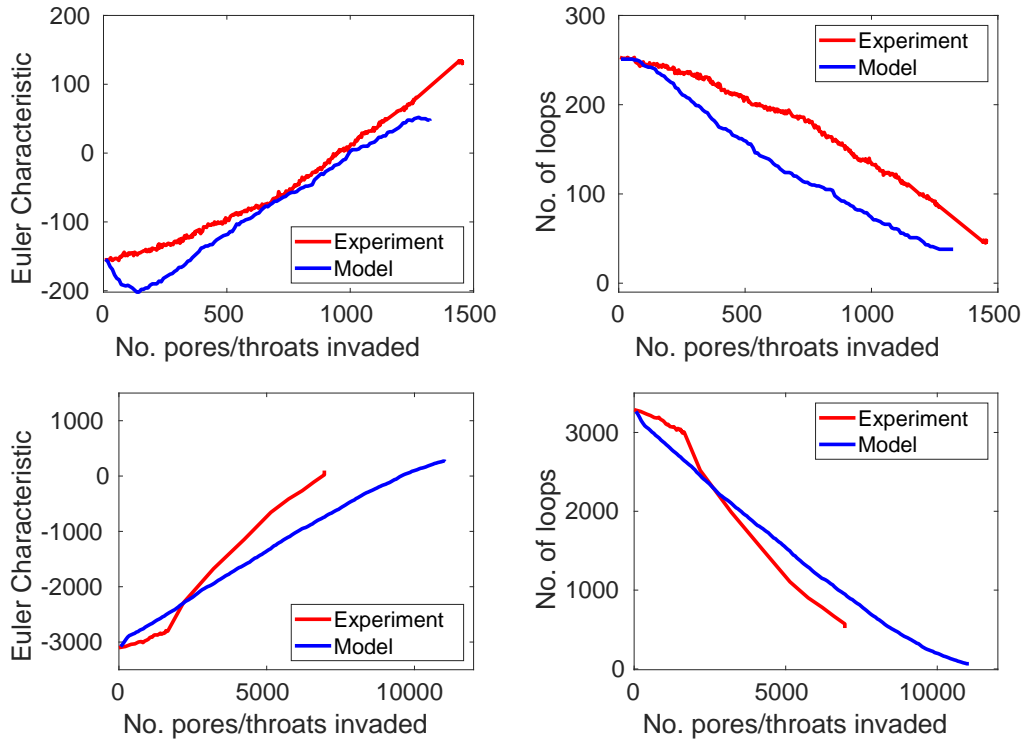


Figure 5. Comparison of the evolution of the non-wetting phase Euler characteristic in the experiment and in the model (left) and the amount of loops in the non-wetting phase (right), for Ketton (top) and Gildehauser (bottom). A large negative Euler characteristic indicates that the non-wetting phase is well connected with many loops in the structure.

Figure 5 shows that magnitude and trends of the Euler characteristic are correctly represented by the model. There are, however, some discrepancies. In the Ketton simulation the model shows a local minimum at the start of imbibition. This is caused by an image processing artifact: small pores that were wrongly identified as non-wetting-filled after drainage.

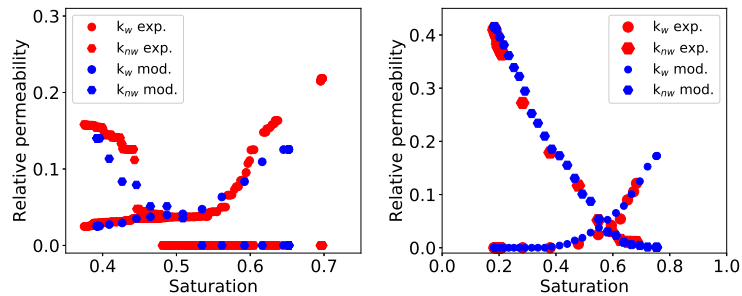
338 The effect of this on the Euler characteristic was partly compensated by redundant loops in
339 the modelled non-wetting phase distribution being removed too quickly. The latter may have
340 been caused by a mismatch in the amount of cooperative pore filling compared to snap-off
341 between the simulation and the experiment, due to geometric simplifications in the model.
342 Imbibition in Ketton is more sensitive to cooperative pore filling than most other rock sam-
343 ples, due to Ketton's pore structure resembling that of a bead pack [Ruspini *et al.*, 2017]. In
344 this particular case, the model may therefore overestimate the amount of capillary trapping.

345 In the Gildehauser case, the Euler characteristic increased more quickly in the experi-
346 ment than in the model. In the experiment, there was a change in the slope of the Euler char-
347 acteristic and the number of loops after approximately 1600 pores and throats were filled.
348 This was likely related to the occurrence of two different filling regimes observed by *Rücker*
349 *et al.* [2015]: first pathway flow and film swelling, and then a regime which includes clus-
350 ter dynamics. This was not reproduced in the model. Furthermore, the faster increase in the
351 experimental Euler characteristic is partly due to misidentification of the fluid occupancy in
352 smaller pores and throats caused by imaging noise in the Gildehauser experiment. This re-
353 sulted in an excessive amount of small non-wetting phase clusters being detected. However,
354 the average sizes of multi-throat clusters (excluding single-throat clusters to reduce imaging
355 noise) were in fair agreement between the model and the experiment, as were the sizes of the
356 largest non-wetting phase clusters which dominate the flow (see Supporting Information). As
357 will be shown in the next section, the discrepancies discussed here had a limited influence on
358 the flow paths through each fluid.

359 **Flow paths**

360 The filled pore/throat sizes and the connectivity together control the flow paths which
361 are accessible to each fluid, and consequently the relative permeability. We compared the
362 flow paths in the model and the experiment by calculating relative permeabilities on the im-
363 aged and the modelled fluid distributions. This relative permeability can be seen as a flow-
364 weighted connectivity of the fluid distributions. Since both the experimental and the mod-
365 elled results used the same conductivity and volume assignments in pores and throats, mis-
366 matches in the presented relative permeabilities can only come from discrepancies in the
367 modelled fluid distributions.

368 The resulting relative permeabilities indicated a close match between the model and
 369 the experiment (Figure 6). This implies that the quasi-static model provided a good estima-
 370 tion of the fluid distribution with respect to its impact on the upscaled flow properties. There
 371 were some discrepancies near the end of imbibition in Ketton. This was likely related to a
 372 small mismatch of cooperative pore filling versus snap-off, and to the boundary conditions,
 373 which were unknown in the experiment due to the limited field of view of the scan. While
 374 ganglion dynamics have been seen to play an important role near the residual non-wetting
 375 phase saturation in the Gildehauser experiment [Rücker *et al.*, 2015], there is no direct evi-
 376 dence of this in the Ketton dataset. We do not draw conclusions about the behaviour near the
 377 end point of imbibition in the Gildehauser case here, as the experiment did not continue until
 378 residual non-wetting saturation.



379 **Figure 6.** Imbibition relative permeability calculations based on the modelled and the experimental fluid
 380 distributions for Ketton (left) and Gildehauser (right) allow to consistently compare the flow paths in the
 381 model and the experiment and indicate a good match.

382 Conclusions

383 A key question concerning imbibition in permeable materials is how to formulate con-
 384 ceptual models that decrease the degrees of freedom compared to an explicit description,
 385 while still capturing the salient behaviour of the process. Quasi-static PNMs abstract the pro-
 386 cess by simplifying the local pore shapes and by assuming that capillary forces are dominant,
 387 allowing imbibition to be described as a sequence of simple pore and throat filling processes.
 388 However, one of the main problems has been validation: the complex relation between the
 389 fluid occupancy of the system and the experimentally measured properties (e.g. relative per-
 390 meability) typically used for validation makes it difficult to understand the origin and gener-

391 ality of any observed (mis-)match [*Sorbie and Skauge, 2011; Bondino et al., 2012; Bultreys*
392 *et al., 2018*].

393 The analysis we have developed here allowed the comparison of pore-scale experimen-
394 tal data to a quasi-static PNM with minimal influence from anything but the fluid distribu-
395 tions. This indicated that the quasi-static assumption provided a good approximation of the
396 order in which pores with different sizes were invaded. The model did misinterpret the filling
397 behaviour of some larger pores, and overestimated the amount of loop-breaking in the non-
398 wetting phase. However, the pores and throats causing these discrepancies showed to be of
399 minor importance for the flow paths, which closely matched the experiments.

400 The conclusion is that an abstracted, conceptually simple model based on quasi-static
401 physics *can* give a good first order approximation of continuum scale properties such as rel-
402 ative permeability, at least in strongly-wetted rocks with relatively homogeneous pore struc-
403 tures and at low capillary numbers. Whether or not it *does*, depends on model implementa-
404 tion (e.g. volume and conductivity assignment) and accurate input concerning pore geometry
405 and advancing contact angles. These less fundamental considerations are outside the scope
406 of this study.

407 In summary, PNM as a concept appear a valuable starting point for building concep-
408 tual models. However, it must be stressed that more validation needs to point out whether
409 this holds in circumstances where more influence of ganglion dynamics has been observed
410 or can be expected, e.g. near the end of imbibition [*Rücker et al., 2015*], in intermediate /
411 mixed-wet samples [*Zou et al., 2018*] and in samples with very wide pore size distributions.

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