Fuzzy classification of countries according to their sustainable economic potential: application of the Solow-Swan and Green Solow models.

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Abstract

The pollution and the progressive depletion of resources are global problems that affect all humanity, but their approach is affected by the different sociopolitical and economic conditions present in the different regions. Thus, the objective of this work is to implement and analyze a methodological structure based on the Solow/Green-Solow models that facilitates the identification and organization of countries according to their economic capacity and the polluting emissions associated with their development, with the purpose of promoting the application of more appropriate and effective solutions, considering the diversity of challenges faced by different economies. To achieve this objective, a preliminary guide is proposed to compare and group viable strategies in different territories, evaluating them according to the parameters established by the Solow-Swan and Green Solow models. For the recognition of the parameters, the Levenberg-Marquardt algorithm has been used, the result of which is classified using the fuzzy artificial intelligence method known as c-means, which provides a category for the different countries. The key findings of this article highlight how the particularities of countries are influenced not only by their geographic location, but also by their level of income and the calculated elasticity related to product exports, together with their ecological potential. It also challenges the widely held belief that enrichment is linked to increased carbon emissions.

Author summary

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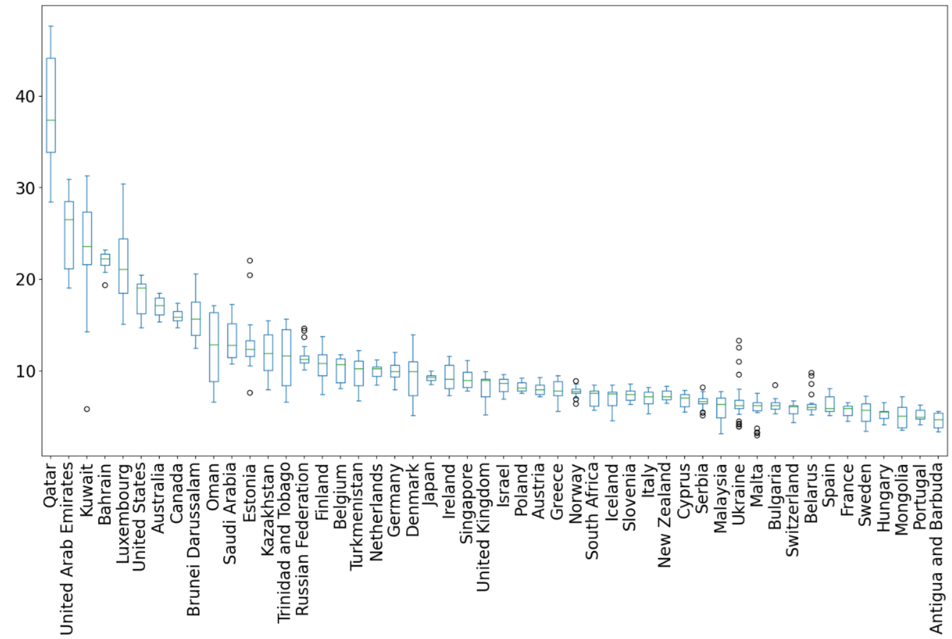
1 Introduction

The development of globalization has had a positive impact on areas such as the economy, culture, technology, and ecology. Since the mid-20th century, the

Fig 1. Distribution of CO2 emissions in the fifty most polluting countries, measured in metric tons per capita, from 1990 to 2020 [5].



interdependence between factors, actors and nations has been a prominent feature of this phenomenon [1].

In this context, globalization has fostered cultural synergies between towns and cities, with recent advances driven by the use and development of Information and Communication Technologies (ICT). However, the impact of globalization has been more pronounced in developed economies than in emerging ones, and has been susceptible to economic crises and protectionist policies. [2].

Consequently, the economic characteristics of each country have undergone a unique and diverse evolution in each territory. As a result, the development of each nation focuses on different aspects related to the common objective of increasing its Gross Domestic Product (GDP), which represents the total value of goods and services produced in a specific period. [3].

Despite the notable differences between territories, common factors have been identified that seem to contribute to the progressive growth of profits in each country. According to authors such as [4], the increase in CO2 emissions is a distinctive feature in GDP growth, due to waste from industrial activities and land use for energy, agricultural and urban conversion. As a result, carbon dioxide (CO2) emissions have been a concern for the past few decades due to their impact on climate change and global warming.

- United States: This country is among the biggest polluters due to its extensive industry and high energy consumption. The extraction of oil and natural gas, together with mining, represent important economic activities in its territory [4, 5].
- Russia: The Russian economy is highly dependent on natural resources, including oil, natural gas, and minerals. The exploitation of these resources has caused environmental problems, such as water and soil contamination and deforestation. [4, 5].
- Australia: is a major exporter of coal, iron ore and natural gas. The exploitation of these resources has had significant impacts on the environment, including

ecosystem degradation and increased greenhouse gas emissions [4,5].

Aware of this situation, many countries have addressed this challenge by implementing government policies tailored to their specific economic situation and context. These customized strategies have demonstrated important advances in the commitment to reduce their environmental impact, such as the following:

- Costa Rica: It is known for its focus on environmental conservation. It has implemented solid policies for the protection of its forests, proposing an extensive system of protected areas. It has also actively promoted ecotourism and renewable energy. As a result, Costa Rica has managed to reverse deforestation and is considered one of the most sustainable countries in the world [6,7].

- Norway: Recognized for its management of natural resources, especially in the energy sector. It has used its oil revenues to finance the Government Pension Fund, while also investing in renewable energy. Norway is a major producer of hydropower and is working to reduce its dependence on fossil fuels through electric mobility policies [7].

- Germany: has demonstrated a strong commitment to the energy transition and the reduction of carbon emissions. It has implemented effective policies to encourage the use of renewable energy, such as the Renewable Energy Law, which has encouraged the generation of solar and wind energy. In addition, the country has set ambitious goals to reduce emissions and is currently on track to achieve them. [7,8].

Despite the efforts made by all organizations in the world to limit consumption in their economic activities, the advance of global warming continues to limit resources and affect the quality of life of millions of people [9–11].

While the solutions so far have focused on technological improvements to reduce pollutant emissions per person, it is also important to recognize that part of this change in basic assumptions involves control policies in industries. However, it is crucial to consider that some environmental solutions may not be viable due to the sociopolitical and economic context of the territory, or the impossibility of substituting certain economic activities.

To effectively address this problem, it is necessary to progressively apply successful alternative ecological practices from countries with similar sociopolitical contexts, which allow economies to reduce polluting emissions without directly affecting the development of their economic activity.

Several authors [12–14] have attempted to produce a general solution, where communication and comparison of ideas can establish guidelines to help improve sustainable economic development. However, many of these structures fail in their implementation because the characteristics of success in other territories are not met in the places where the strategy is intended to be applied.

It is difficult to know for sure if a strategy is valid for its application in other territories. For this reason, it is essential to have a guide or information repository that contains comparable data from all countries. This will allow more informed decisions to be made and will facilitate the exchange of successful experiences between different regions, favoring the adoption of effective practices in the search for solutions to common problems.

However, due to the characteristics of environmental phenomena and the nature of economic approaches, centralizing all the information is difficult due to failures in the measurement instruments and lack of uniformity in the measurements by the different institutions. Each country collects information according to its specific needs, which makes it difficult to compare and analyze the situation globally.

Mathematical models based on economic theories offer a feasible solution to address this decentralization of information. Since the equations designed to model phenomena are adjusted using easily recognizable parameters and generate approximations of reality from the available evidence [15, 16].

Unlike other approximation methods such as stochastics or artificial intelligence, mathematical models are based on the direct correlation of phenomena that are coupled, generating correlations in their behaviors that, in addition to determining their behavior trends through their parameters, also allow predicting these same behaviors and evaluating these future characteristics [17–19].

A highly accepted and proven economic model for the coupling of real data based on economic growth is the Solow-Swan model, also known as the exogenous growth model, which aims to identify the economic growth of an economic system assuming a technological evolution that does not is necessarily constant [20, 21].

$$\frac{dk(t)}{dt} = p \times k(t)^a - q \times k(t). \tag{1}$$

Being k the quotient between capital (K) and labor (L), the parameter (p) represents the product between the savings ratio (s) and the productive factor (A). The parameter (q) is the sum of the capital depreciation ratio (d) and the exponential growth ratio (n). Finally, parameter (a) represents the elasticity of production.

Scientists such as [22] found this model universally applicable to the ecological field of study and, inspired by the Kuznets curve phenomenon, extended the Solow concept to an environmental Solow model (the green Solow model), based on the initial assumptions of the environmental Kuznets curve.

The Green-Solow model establishes a relationship between the ratio (k) and the immediate change in polluting emissions, which generates an index that relates the increase in goods generated per person with the global polluting emissions of the population. These data are readily available, and their relationship structure is based on the following equation:

$$\frac{\dot{E}}{E} = g_E + a\frac{\dot{k}}{k} \tag{2}$$

Being E the variable that represents the pollution emitted and the point of the variables E and k represents the first derivative with respect to time. On the other hand (g_E) represents the growth rate of polluting emissions based on balanced economic growth.

The Green-Solow model has shown great performance in modeling territories and relating the effects of enrichment on the territorial environment [23–25]. Due to this, the parameters associated with this model (g_E, a) show a very promising value to establish monitoring metrics based on the clean enrichment capacity of the territories.

As mentioned above, an initial step to support corporate environmental dispute resolution is the implementation of benchmarking metrics that allow countries to determine which environmental practices have been most effective without negatively affecting their bottom-line growth.

Thus, the objective of this work is to implement and analyze a methodological structure based on the Solow/Green-Solow models that facilitates the identification and organization of countries according to their economic capacity and the polluting emissions associated with their development.

The paper will be divided into three main sections: methodology, where the data implemented and the approach of the study will be explained in detail; results, where the most relevant characteristics of the data obtained will be presented; and finally, conclusions and final reflections.

Materials and methods

Global technological evolution has driven significant advances for the benefit of humanity and revolutionized the production of goods. However, it is undeniable that some of this progress has been achieved at the expense of environmental wealth.

Today, the development of globalization and industrialization has intensified the ecological deficiencies linked to production, which has aggravated certain practices in all countries of the world.

Thus, this methodological development presents the data used to measure the economic development of the territories studied and their impact on the carbon footprint. In addition, the adjustment model implemented, which seeks to establish a relationship between both behaviors. Finally, the empirical development process carried out to categorize each country according to its economic growth and environmental impact will be made explicit.

1.1 Description of data

For the development of this work, four sets of data were used, taken from different sources:

The first data set was taken from [26] which is a database with information on relative levels of income, production, inputs, and productivity, covering 183 countries between 1950 and 2019. From this database we extracted emp (Number of persons employed in millions) and rgdpo (Output-side real GDP at chained PPPs) between 1990 and 2019.

The second data set was extracted from [5]. It is an association committed to finding solutions to a wide range of social, economic, and environmental problems. It also carefully monitors these issues and shares the data collected free of charge. From these repositories, we have obtained Co2 emissions data measured in metric tons per capita, which are fully available for more than 90

The following data were taken as support and validation of the results obtained from the methodological development of the study.

The third data set was drawn from [27]. The Organization for Economic Cooperation and Development (OECD) is an international organization dedicated to improving land policies. It works as a knowledge center, where it analyzes data and promotes the economic development and public policies of its member countries. From this repository we extracted the saving rate, which is equal to the difference between disposable income and final consumption spending, reflecting the portion of income used to acquire financial and non-financial assets. This savings rate is expressed as a percentage of Gross Domestic Product (GDP).

The latest dataset was obtained from "Our World in Data," [28] a project of the Global Change Data Lab, a UK-based non-profit organization that seeks to make knowledge more accessible and understandable on major global issues. The productivity factor was extracted from this repository.

To process the first two data sets, a filtering was performed to include only countries with complete information, resulting in a total of 166 countries with available data from 2010 to 2019. From these data, a pivot table with countries was created as indices and years as columns to facilitate the calculations of the variables necessary for the study.

For the most recent data sets, data integrity was checked again, resulting in a total of 37 countries with valid data for the period from 2010 to 2019. These values will support the validity of this study.

Regarding the implementation of these data for the application of the models, in the Solow-Swan model (see eq 1), the main equation involves the variable k (relationship between the gross domestic product and the total number of people in the territory) in relation to time. On the other hand, in the Green Solow model (see eq 2), in addition to the calculation of k, the consideration of a function E (polluting emissions) is also added as a function of time, for which it is crucial to make explicit the calculation of these.

Real GDP on the production side in chained PPPs was used to calculate k, since it represents real economic quantities without considering the effects of inflation. In addition, to obtain the total number of people by territory, the number of employed people was used, since these individuals have a significant impact on the final GDP of each region.

To quantify polluting emissions E, CO2 emissions by territory were taken as a reference, due to their direct impact both on global warming [29] and the excessive production of industries [30].

Once the metrics related to the variables of interest were obtained, the derivative or immediate change over time was calculated. Since the measurements were made annually and discretely, the derivative of the phenomena was calculated using the difference between one year and the previous year. In this way, the derivative of the variables E and K was calculated.

To conclude the data processing, the product between the savings ratio and the productive factor is calculated, thus obtaining a p, as observed in the Solow-Swan model (see eq 1, 2). This parameter p will serve as validation of the results obtained in the study.

1.2 Methodological approach

1.2.1 Explanation of the models and their parameters

The methodological approach of this study focused on analyzing two models: Solow-Swan and Green Solow. The objective is to estimate the parameters of each equation for each country and, based on the results obtained, use an unsupervised ranking method to classify countries into similar groups. Finally, the amount of GDP generated per unit of CO2 will be calculated in metric units per capita per country to assess which countries are using their economic capacities more efficiently in relation to their polluting emissions. This analysis will provide supporting information to identify strategies applicable to other countries belonging to the same category, considering the equivalent context in which they are found.

Both the Solow-Swan and Green Solow models are considered long-term approximations. Given the nature of data collection and the lack of an explicit relative length in the theory, it is decided to evaluate these models in time subdivisions, using groups of three consecutive time periods. This process is iterated from these periods, resulting in a set of 6 approximations per model, each with its respective set of parameters.

In short, for each model, the corresponding parameters (p, a, q) in the case of Solow-Swan, and (a, g_E) in the case of Green Solow will be adjusted. For each country, a set of 6 data will be obtained for each parameter and the average of these values will be calculated. The result will be a measure of central tendency that will represent the general behavior of the country in relation to the parameters evaluated. At the end of the process, 5 representatives of the average countries will be obtained, one for each parameter.

Since the coefficient a (output elasticity) is assumed to be the same measure in both models, which are defined with the same assumptions, this value would be expected to be approximately similar in both cases. However, their calculation is based on evidence of k in one model and evidence of polluting emissions E in the other.

Therefore, the elasticity of the product becomes crucial, as this process determines how closely the two elements are aligned and how they might affect each other. Their comparison provides us with important information about the relationship between economic growth and polluting emissions, allowing us to better understand their impact and interaction in the models evaluated.

1.2.2 Parameter approximation

The method used to calculate these parameters was based on the Levenberg-Marquardt algorithm [31, 32], widely recognized for its effectiveness in parameter analysis and estimation, as demonstrated in previous research [33–35]. The implementation of this algorithm was done in Python, using the scipy library [36] allowing the calculation process to be performed iteratively.

To validate the calculations, comparisons will be made between the values obtained using the Solow-Swan model and the values from the databases obtained in [27, 28]. These comparisons will allow the accuracy of the model to be assessed in several countries included in the available data. Due to the difficulty involved in monitoring, evaluating, and measuring the rest of the parameters for all territories, only the p variable will be used for validation, since there is insufficient information on the other coefficients.

1.2.3 non-supervized fuzzy categorization

Once all the parameters are obtained, a categorization is carried out using an unsupervised model known as c-means [37, 38]. This model is widely recognized for generating accurate categorizations in comparative studies due to its fuzzy characteristics in categorization, which extends the concept of unsupervised artificial intelligence [39–41]. The c-means classification method was implemented in Python, specifically making use of the fuzzy-c-means library [42].

To train the model, it is provided with 4-element vectors, consisting of 3 model parameters (p, q, g_E) and the relative error between the two elasticities $\frac{|a_k - a_E|}{a_E}$. Where a_k is the elasticity of the Solow-Swan model and a_E is the elasticity of the Green Solow model.

The advantage of using a_E as a reference point is that it is derived directly from the economic data of each country, without the intervention of exogenous intermediate variables or complex assumptions about ideal economic behavior. This will provide us with a more accurate and realistic view of the relationship between production and economic growth in each country, improving the quality of the categorization performed by the c-means model and allowing us to draw informed conclusions about patterns of economic and environmental behavior in different countries.

Finally, using the categories generated by the c-means method, the amount of GDP generated per unit of CO2 will be evaluated in metric units per capita to determine which country has been most effective in optimizing its process of economic enrichment by limiting the levels associated contaminants emissions. In this way, we can obtain a clear and comparative vision of the economic and environmental sustainability of each country, and to what extent these characteristics allow us to understand their performance in a more sustainable economic development.

Based on this methodology, studies were carried out and the results are presented in the following section.

Results

Table 1. Table comparing actual and model-approximated values for the 34 countries with actual data available

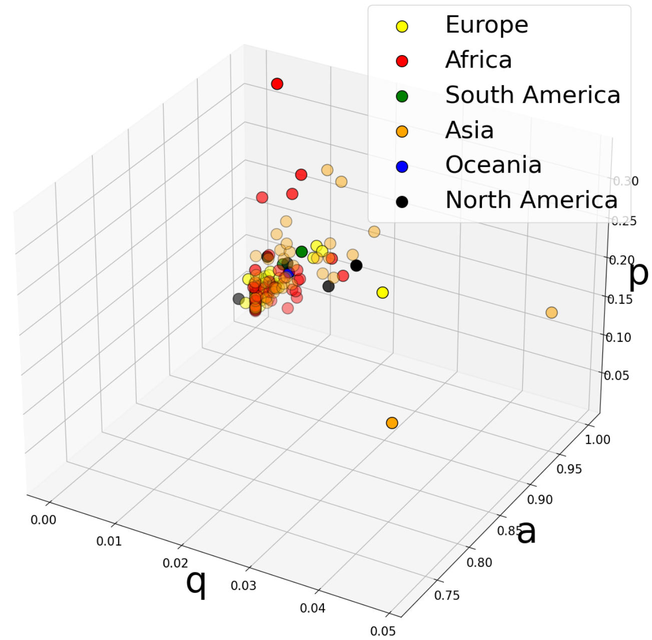
Countries	Real p	Aprox p	Error
Australia	0,0474	0,0222	0,03
Austria	0,0843	0,0402	0,04
Belgium	0,0845	0,0299	0,05
Brazil	0,1771	0,0931	0,08
Canada	0,0468	0,0216	0,03
Chile	0,2469	0,0658	0,18
China	0,3968	0,0775	0,32
Costa Rica	0,0855	0,0202	0,07
Denmark	0,0778	0,0307	0,05
Estonia	0,1008	0,0580	0,04
Finland	0,0596	0,0361	0,02
France	0,0567	0,1167	0,06
Germany	0,0768	0,0215	0,06
Greece	-0,0715	0,0280	0,10
Hungary	0,0345	0,0488	0,01
Ireland	0,0769	0,0639	0,01
Israel	0,0956	0,0234	0,07
Italy	0,0449	0,0253	0,02
Japan	0,0639	0,0226	0,04
Latvia	-0,0348	0,0476	0,08
Lithuania	0,0282	0,0647	0,04
Luxembourg	0,1425	0,0401	0,10
Netherlands	0,0965	0,0269	0,07
New Zealand	0,0418	0,0236	0,02
Norway	0,1753	0,0602	0,12
Poland	0,0442	0,0401	0,00
Portugal	0,0147	0,0354	0,02
Slovenia	0,0498	0,0460	0,00
South Africa	0,0165	0,0324	0,02
Spain	0,0761	0,0270	0,05
Sweden	0,0971	0,0408	0,06
Switzerland	0,1066	0,0476	0,06
United Kingdom	0,0087	0,0327	0,02
United States	0,0291	0,0540	0,02

As an initial step in the results, we will evaluate the p of the Solow-Swan parameter, which has been calculated based on real evidence. We will also consider approximate mean values for some countries for which data were available. This analysis will allow us to study the characteristics of the approximation of the parameters and determine how close the model is in terms of productivity factors and the savings rate.

The results are summarized in table 1, which shows that the model has a consistent, though not 100% accurate, approximation of the grand total. Specifically, more pronounced errors are observed in countries such as China, Chile, Norway, Luxembourg, Greece, Brazil, Latvia, Israel, the Netherlands, and Costa Rica. These countries show outliers that exceed the mean and median error, which is approximately 0.06.

Significant errors in the model indicate that the estimates fell below the expected

Fig 2. Results of the Solow-Swan model parameter calculation



behavior. Although there does not seem to be a precise pattern to this behavior, it can be attributed to the fact that countries come from different parts of the world with different sociopolitical backgrounds. It is possible that abrupt changes due to specific social issues or endogenous characteristics in each territory are the main cause of the variation in the parameters.

Relative to the other approximate countries that are close according to the model, a similar pattern of underestimation is also observed. This information is relevant to understand the particularities of Solow and the method of calculating the p coefficients. Despite this, the trends appear to be satisfactory enough to consider the model as a good trend checker and an adequate estimator of the overall coefficients of each country.

As for the other parameters for which there are no real data, it would be difficult to say that they will remain as close to the real values without concrete tests. However, the p approximation shows favorable results by maintaining a proportionality with the real trend, suggesting that the other factors could maintain a similar characterization. Although it cannot be guaranteed with certainty, the good performance of the model in estimation of "p" increases the confidence in its ability to approximate the other parameters with a reasonable approximation.

The figure 2 shows the results of the calculation of the parameters of the Solow-Swan model. Elements such as the product of the savings rate by the productivity factor (p) show notable variability in this study. Despite these fluctuations, some continental features show significant similarities. For example, African countries present the highest generational factors, despite not having a high spatial density between them compared to other territories.

These findings suggest that the characteristics of Africa appear to be favorable for generating savings and productivity according to the Solow-Swan model. In addition, the Asian countries show a well-defined characterization in terms of the value of the parameter (p) and seem to have a more consistent behavior, occupying a prominent position in the graph in relation to these values.

In contrast, validating with the real values available in the data, it is observed that the economy with the highest average value of (p) is China. This result shows significant consistency with respect to the results provided by the study, indicating a strong tendency for the data to fit the model, despite the slight year-to-year bias and constant changes in context.

This finding reinforces the validity and robustness of the Solow-Swan model to predict economic parameters, especially in the case of China. Although there are fluctuations and variations in the data over time, the model has demonstrated a consistent ability to capture broad trends in the relationship between savings, productivity, and economic growth in this country.

Regarding the growth parameter (q), once again the results favor Asian and European countries. Three countries representing these continents stand out as being the most skewed to the right compared to the others. This indicates that these economically leading countries lead the growth parameters, although they are also the least stable in terms of predictions. Due to their high-growth swings, it is more difficult to make an accurate estimate of their long-term behavior, indicating their remoteness from other economies and the distinct absence of a clear pattern within the continents they represent.

On the other hand, in the behaviors that lead the most grouped data we have the countries in North America and some regions of Asia, which show high growth but are grouped together with the rest of the countries. This makes them characteristic profiles that inspire confidence in their economic progress. These results suggest that some countries show greater stability and consistency in their growth rates, which may be a positive indicator of their ability to maintain sustainable economic development over time.

Concluding the analysis of parameters for the results of the Solow-Swan model, the value of parameter (a), also known as elasticity, stands out. This parameter is the most particular of the three as it shows a significant similarity between the different components. The results indicate that the characteristics of the values of (a) provided by the Solow-Swan model tend to be close to inelasticity, that is, approximations to one or an ideal equilibrium where supply and demand have been trivialized to an ideal point.

Naturally, most of these values are less than one, suggesting that the demand for the product does not change significantly after a price increase. This could be explained because the Gross Domestic Product was calculated based on chained PPPs, whose values limit the variability of the process and generate a desired stability effect.

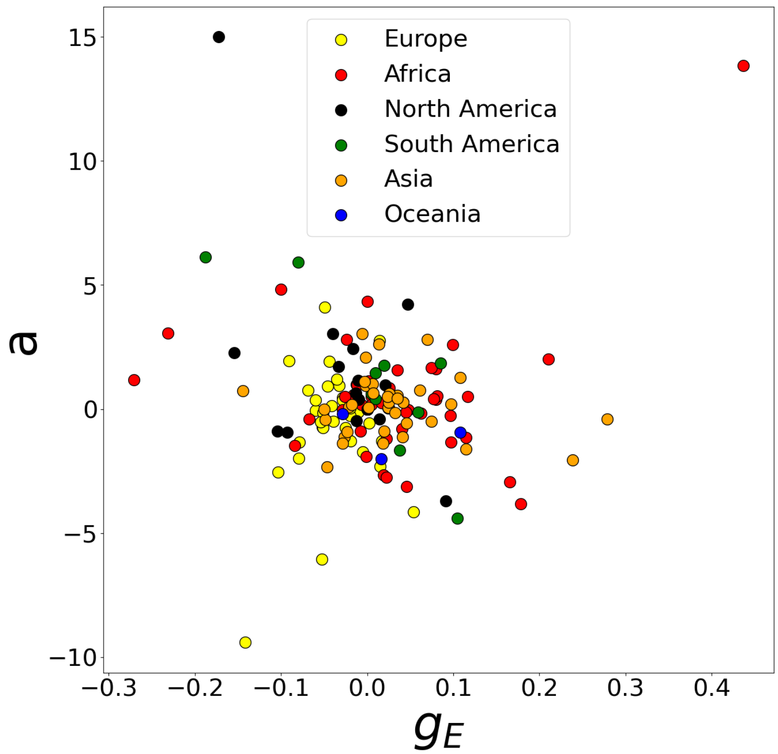
On the contrary, the next results provided by the Green Solow model may provide a different perspective on the elasticity of production, considering polluting emissions (E) as an important variable in the analysis. This will allow a more complete and detailed comparison of the relationship between production and emissions, providing valuable information to better understand environmental impact and economic sustainability in the global context.

For the Green Solow model, different results are expected for the elasticity variable, since this model is outside of a controlled environment and is affected by the apparent randomness of carbon emissions. This implies that there may be considerable growth in parameter differences between the Solow-Swan model and the Green Solow model.

This variability in the results may also be due to the nature of carbon emissions, which may be influenced by various factors, such as environmental policies, industrialization and the technology used in each country.

Figure 3 presents the results of the parameter estimation of the Green Solow model. As can be seen, the elasticity values (a) in this model can vary significantly with respect to those of the Solow-Swan model. These differences in the values of (a) suggest that the Green Solow model considers more directly and explicitly the impact of polluting

Fig 3. Results of the Green Solow model parameter calculation



emissions on economic growth.

In the individual analysis, the growth terms based on elasticity differ considerably from the Solow-Swan model. The values range from -10 to 15, being considerably higher compared to the more uniform values of the Solow-Swan model. Despite this variability, it is observed that the behavior of the parameter seems to maintain a trend, with the countries represent Africa still showing a characteristic elasticity, but also adding a leading country in elasticity: the United States, representing North America.

The presence of higher elasticity values suggests that carbon emissions can have a significant impact on the economy, affecting productivity and economic development to a greater extent than in the Solow-Swan model.

By analyzing the variable g_E, which represents the growth rate of carbon emissions under balanced economic growth, we can see that the Green Solow model shows different representatives on different continents, and several leading countries in each territory. It should be noted that Africa stands out in this aspect, having as representatives countries such as Togo, Lesotho and Nigeria, which present the lowest pollution rates in relation to their ideal economic activity.

In addition, there is a large group of countries that, despite engaging in a variety of clearly differentiated economic activities, experience economic development outcomes that are consistently related to environmental responsibility, according to the Solow-Swan model. Although CO2 emissions vary between these territories and can sometimes be considerably higher than in others, this does not alter the general relationship between economic development and environmental responsibility.

Despite this, it is important to note that countries that focus on these aspects are not necessarily experiencing significant growth in their Gross Domestic Product, and may also face economic challenges, as shown in the figure 3, for so these parameters, while informative, are not sufficient on their own to generate adequate estimates of the characteristics of their territory.

Therefore, we will conclude the results section by presenting the categorizations obtained by applying the c-means algorithm, and then we will organize the results according to the metrics explained in section 1.

Table 2. Table containing the countries in category 0

Country	Continent	Pib per Co2	Category
Suriname	South America	1.363,94	0
Luxembourg	Europe	1.431,34	0
Malta	Europe	1.754,09	0
Maldives	Asia	2.337,36	0
Montenegro	Europe	2.504,83	0
Mauritius	Africa	8.954,92	0
Latvia	Europe	9.751,17	0
Lebanon	Asia	14.628,30	0
Jordan	Asia	16.889,17	0
Kazakhstan	Asia	21.167,71	0
Belarus	Europe	22.261,24	0
Panama	North America	25.858,69	0
Togo	Africa	34.573,01	0
Honduras	North America	35.327,13	0
Norway	Europe	38.877,07	0
Zimbabwe	Africa	41.007,97	0
Paraguay	South America	59.473,73	0
Ukraine	Europe	62.345,22	0
Romania	Europe	66.070,49	0
Netherlands	Europe	72.384,61	0
Poland	Europe	86.814,82	0
Canada	North America	87.619,28	0
Angola	Africa	126.392,05	0
Spain	Europe	210.141,53	0
United Kingdom	Europe	271.331,63	0
Nepal	Asia	284.966,37	0
Nigeria	Africa	679.554,45	0
Indonesia	Asia	1.010.164,03	0

Table 3. Table containing the countries in category 1

Country	Continent	Pib per Co2	Category
Trinidad and Tobago	North America	2.527,91	1
Armenia	Asia	12.128,54	1
Georgia	Asia	16.163,36	1
Tajikistan	Asia	33.455,22	1
Central African Republic	Africa	81.602,16	1
Sri Lanka	Asia	235.669,17	1
Ghana	Africa	270.160,78	1
France	Europe	403.385,21	1
Pakistan	Asia	864.977,05	1

The classifications provided by the algorithm generated 5 categories that express the main characteristics of each country. Each of the 5 categories will be explained in detail

Table 4. Table containing the countries in category 2

Country	Continent	Pib per Co2	Category
Brunei Darussalam	Asia	1.629,50	2
Guyana	South America	2.403,10	2
Cabo Verde	Africa	2.871,20	2
Mongolia	Asia	3.477,24	2
Qatar	Asia	3.879,36	2
Equatorial Guinea	Africa	4.950,73	2
North Macedonia	Europe	4.983,61	2
Bhutan	Asia	5.220,84	2
Gabon	Africa	5.281,31	2
Kuwait	Asia	7.044,84	2
Oman	Asia	7.254,20	2
Serbia	Europe	13.255,73	2
Lithuania	Europe	13.765,72	2
Guinea-Bissau	Africa	14.698,84	2
Bulgaria	Europe	16.979,44	2
Azerbaijan	Asia	21.675,05	2
Ireland	Europe	22.725,30	2
El Salvador	North America	26.356,80	2
Singapore	Asia	27.498,47	2
Syrian Arab Republic	Asia	29.211,47	2
Iraq	Asia	68.408,08	2
Saudi Arabia	Asia	71.336,26	2
Sierra Leone	Africa	88.086,14	2
Morocco	Africa	135.499,51	2
Argentina	South America	169.108,41	2
Burkina Faso	Africa	183.739,32	2
Mozambique	Africa	186.069,56	2
Mali	Africa	187.987,01	2
Malawi	Africa	193.468,63	2
Russian Federation	Europe	231.373,49	2
Thailand	Asia	238.768,75	2
Sudan	Africa	284.452,77	2
Myanmar	Asia	510.172,94	2
Uganda	Africa	570.018,78	2
Brazil	South America	1.171.056,82	2
Ethiopia	Africa	1.179.125,03	2
India	Asia	3.685.459,03	2

below:

Category 0: Economies in this category strive to progressively improve their carbon emissions, even though their economic growth or enrichment has not changed significantly in recent years. Regarding the depreciation of their currency, the countries in this category show a positive behavior, although this may vary slightly according to the different contexts.

Category 1: The countries in this category exhibit a mostly average behavior but stand out especially in terms of economic growth and depreciation of their currencies, presenting positive characteristics in these aspects. However, it is important to note that they also display a highly unstable market elasticity, the instability of which has

Table 5. Table containing the countries in category 3

Country	Continent	Pib per Co2	Category
Belize	North America	1.132,64	3
Cyprus	Asia	3.009,71	3
Jamaica	North America	5.518,83	3
Lesotho	Africa	5.618,83	3
Bosnia and Herzegovina	Europe	6.428,85	3
Slovenia	Europe	7.261,30	3
Albania	Europe	17.327,91	3
Croatia	Europe	18.198,44	3
Finland	Europe	18.487,87	3
Denmark	Europe	23.889,34	3
Israel	Asia	25.762,34	3
Uruguay	South America	27.502,55	3
Nicaragua	North America	32.900,71	3
Greece	Europe	35.611,02	3
Hungary	Europe	38.170,49	3
Belgium	Europe	39.088,50	3
Austria	Europe	42.438,92	3
Uzbekistan	Asia	43.484,33	3
Portugal	Europe	50.034,62	3
Dominican Republic	North America	50.164,81	3
Australia	Oceania	53.274,28	3
Sweden	Europe	70.854,38	3
Chile	South America	73.231,94	3
Switzerland	Europe	74.525,62	3
Guinea	Africa	113.786,68	3
Burundi	Africa	177.909,22	3
Niger	Africa	226.812,12	3
Italy	Europe	298.821,40	3
United States	North America	833.311,41	3

become increasingly apparent over time. Unlike other profiles, it is not so common to identify notable patterns of carbon emission reductions in these countries.

Category 2: This profile includes countries with notable entry rates, which suggests that they have implemented effective production policies. However, some of these countries also have high inflation rates, which could affect their economic stability. Despite these variations, the countries with this profile show an average stability in the sustainability index, maintaining a close relationship between their economic activity and the pollution generated by it.

Category 3: This classification groups countries with a progressive profile in reducing their carbon footprint, although their economic characteristics do not allow them to achieve fully effective savings in relation to their economic activity. These countries tend to experience monotonous economic growth that is hard to shake off. In addition, they have the greatest differentiation in the instability of their products, which makes them territories with high market elasticity.

Category 4: This category includes the most polluting countries that are in the first places of the emission indices according to their economic activity. In addition, they are characterized by low or no economic growth over time and minimal savings rates in relation to their economic activities. The instability in the evolution of the elasticity of its products is also notorious, which causes constant disturbances and values that are

Table 6. Table containing the countries in category 4

Country	Continent	Pib per Co2	Category
Grenada	North America	496,85	4
Sao Tome and Principe	Africa	1.008,61	4
Iceland	Europe	1.826,02	4
Estonia	Europe	2.095,23	4
Djibouti	Africa	5.901,41	4
Fiji	Oceania	6.042,58	4
Turkmenistan	Asia	6.985,69	4
Botswana	Africa	10.073,74	4
Namibia	Africa	11.485,25	4
Liberia	Africa	15.648,46	4
New Zealand	Oceania	18.184,91	4
Mauritania	Africa	22.765,88	4
Bolivia	South America	33.532,76	4
Costa Rica	North America	37.894,38	4
Tunisia	Africa	44.239,34	4
Benin	Africa	53.439,44	4
Ecuador	South America	56.969,22	4
Malaysia	Asia	76.481,09	4
South Africa	Africa	76.550,59	4
Haiti	North America	79.504,17	4
Cambodia	Asia	96.043,77	4
Guatemala	North America	103.370,63	4
Zambia	Africa	111.781,59	4
Cameroon	Africa	147.770,13	4
Rwanda	Africa	169.679,94	4
Peru	South America	171.263,24	4
Madagascar	Africa	272.581,64	4
Colombia	South America	283.681,24	4
Germany	Europe	332.577,61	4
Kenya	Africa	355.414,74	4
Tanzania	Africa	514.865,59	4
Japan	Asia	520.164,94	4
Philippines	Asia	529.907,97	4
Bangladesh	Asia	1.212.088,61	4
China	Asia	2.195.513,85	4

difficult to predict. The results of the categories can be identified in the tables 2,3,4,5,6, where it is explicitly shown to which category they belong and the order assigned in ascending order, considering an ordering based on the GDP generated per unit of CO2 in metric units per capita.

Conclusion

The study carried out has shown that the resulting categories are really promising when it comes to modeling ecological evolution based on the economic characteristics of each country. For example, Italy belongs to category 3 which shows a significant environmental impact by limiting carbon dioxide emissions in relation to its economic activity. This approach contrasts with the perspective of some authors [43, 44] who consider Italy as a prominent leader in the development of innovative green technologies, as well as in the implementation of sustainability indices.

Another example of the different categories identified in this study are Croatia, Cyprus, Sweden and Belgium, since they have been classified in category 3, which refers to countries with high sustainable economic development. This ranking is supported by [44], which lists these countries as having the highest average value of green economic growth.

Although different authors have suggested that there is a direct relationship between polluting emissions and the enrichment of the countries of the world, it has been suggested that there is a direct relationship between emissions and the enrichment of the countries of the world [45], this statement is not necessarily true.

In fact, there are characteristics and patterns of behavior between economies that can limit this relationship. A prominent example is Germany [7, 8, 46], which belongs to category 4. Despite its economic growth and high pollution rates, this country is in the process of improving by keeping its polluting emissions under control and reducing them. This questions the commonly accepted notion that economic enrichment inevitably translates into higher levels of pollution. Germany demonstrates that it is possible to move towards more sustainable practices despite significant economic growth.

However, the evidence indicates that there is a significant number of countries whose elasticity is directly affected by their sustainability (category 3 and 4 countries). This implies that economic activities that reduce polluting emissions and seek greater environmental awareness seem to influence their ability to respond and adapt to economic and commercial changes.

On the other hand, studies such as [46] have shown that countries such as Estonia, Japan, and Germany, which are representative of category 4, have a high elasticity in terms of trade. These results coincide with the classification made, since category 4 is defined for countries with unstable elasticity.

Similarly, research such as [47] supports the notion of remarkable economic growth of India and some South regions of South Asia, classified in category 2 due to their high economic productivity. In this category of countries, the popular belief seems to prevail that there is a direct relationship between polluting emissions and gross domestic product.

The implies that the applied method not only adapts to the economies and their environmental characterization, but also covers much more specific aspects, such as elasticity. This suggests that the classification and categories obtained are capable of capturing and holistically evaluating the complex relationship between economic development and environmental sustainability of each country, thus allowing a more complete and detailed vision of the situation of each nation.

It is important to point out that, despite the simplicity of the models used, the results produced by the parameter-based classification are strongly linked to the specific characteristics of the territories, as demonstrated in the previous validations. Although the Solow model is general and lacks customization by country, it is still useful not only for general unification studies of countries with lack of information, but also for establishing guidelines for monitoring and evaluating economies.

These models offer a solid preliminary foundation for understanding the relationships between economic growth and sustainability in different contexts, providing guidelines for establishing recommendations for informed economic and environmental decision-making. The flexibility and applicability of these approaches allow more freedom to work with than a robust model whose assumptions focus on specific territories.

This means that countries could use these easily replicable methodologies to compare their economic peers and generate strategies and trade-offs that, in their social contexts, facilitate the replication of ideas without the latent risk of failure due to wrong initial assumptions of the strategies. Ultimately, these approaches open the door to a more comprehensive and adaptable analysis to improve economic and environmental sustainability in different nations.

Finally, as future work that could complement the structure of this methodology, it is important to highlight that the fundamental basis of this study lies in its simplicity, which allows it to cover most countries. However, to maintain this stability without losing sight of the initial assumptions of the model, which allow it to adapt to different contexts, the fractional differential equations methodology emerges as an option that could expand the results of this research, especially if they are not non-homogeneous orders are applied to individualize the capacity of each variable in the models, which have demonstrated great adaptive capacities in natural phenomena [48].

From a theoretical approach, the Solow model and the Green Solow model could function as highly applicable fractional expansions [49, 50] that generalize the individual growth ratio of each country and determine, through an additional set of parameters, the immediate change over time in the evolution of pollution and capital per country.

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