

Redshift of Earthquakes via Focused Blind Deconvolution of Teleseisms

Pawan Bharadwaj^{1,2}, Chunfang Meng², Aimé Fournier², Laurent Demanet^{1,2},
Mike Fehler²

¹Department of Mathematics, Massachusetts Institute of Technology
²Earth Resources Laboratory, Massachusetts Institute of Technology

Key Points:

- We present a data-driven factorization method of surface teleseisms into source and path-related signals.
- The method outputs the earthquake source spectrum and its variation with azimuth to implicate rupture propagation.
- The method allows us to estimate the rupture modes even in the absence of near-source stations.

Abstract

We present a robust factorization of the teleseismic waveforms resulting from an earthquake source into signals that originate from the source and signals that characterize the path effects. The extracted source signals represent the earthquake spectrum, and its variation with azimuth. Unlike most prior work on source extraction, our method is data-driven, and it does not depend on any path-related assumptions e.g., the empirical Green's function. Instead, our formulation involves focused blind deconvolution (FBD), which associates the source characteristics with the *similarity* among a multitude of recorded signals. We also introduce a new spectral attribute, to be called redshift, which is based on the Fraunhofer approximation. Redshift describes source-spectrum variation, where a decrease in frequency occurs at the receiver in the opposite direction of unilateral rupture propagation. Using the redshift, we identified unilateral ruptures during two recent strike-slip earthquakes. The FBD analysis of an earthquake, which originated in the eastern California shear zone, is consistent with observations from local seismological or geodetic instrumentation.

Plain Language Summary

The hazard assessment of large earthquakes is closely related to the propagation of their associated ruptures. This research responds to numerous fundamental challenges involved in directly measuring source signals that originate from a propagating rupture. It is desirable to directly measure the source pulses at the seismometers and subsequently infer quantities that are related to the rupture propagation. However, the signals measured in place of those pulses are affected by the subsurface properties through which they propagate before reaching these stations. Thus, instead of measuring the earthquake source signal directly, each seismic station records two types of information that are convolved into a single signal: information about the earthquake (source pulse) and information about the unknown subsurface features through which waves passed (path effects). Consequently, an accurate characterization of an

Corresponding author: Pawan Bharadwaj, pawbz@mit.edu

41 earthquake rupture involves reliably analyzing the recorded seismograms to separate
 42 the path effects from the source pulses. Current methods for separating out the two
 43 types of information rely on unreliable assumptions, and may be confounded because
 44 extracting source pulse requires assumptions about the path, but conversely extract-
 45 ing path effects requires assumptions about the source. In this paper, we introduce to
 46 seismology a new analysis method, “focused blind deconvolution”, that can be used to
 47 extract source or path information without relying on traditional assumptions. Instead,
 48 this method compares data from the same source picked up by multiple receivers, and
 49 uses advanced signal processing to identify similarities and differences among the data.
 50 Similarities among the signals can be identified as source effects, while dissimilarities
 51 indicate path effects. Because it does not require the aforementioned assumptions,
 52 this method will provide more accurate and reliable information to seismologists.

53 1 Introduction

54 Identifying the dominant rupture characteristics of earthquakes is important for
 55 evaluating the earthquake hazard (Heaton & Helmberger, 1977; Olson & Apsel, 1982;
 56 Somala et al., 2018; Galovic et al., 2019). In order to do so, we consider source extrac-
 57 tion (Ulrych, 1971; Clayton & Wiggins, 1976) mainly from the first arriving surfaces
 58 waves, termed as R1 (Rayleigh) and G1 (Love) waves, contained in the long-period
 59 records of intermediate-magnitude strike-slip earthquakes. The primary difficulty as-
 60 sociated with this extraction is: instead of measuring the earthquake source signal,
 61 each seismic station records a spatio-temporal convolution between the source s and
 62 the complex subsurface Green’s function g that is unknown. As g depends on the
 63 unknown subsurface characteristics e.g., its structure and intrinsic attenuation, an ac-
 64 curate characterization of the earthquake involves a *blind factorization* of the ground
 65 motion data into two terms that represent the source and path effects separately. To
 66 our knowledge, this paper presents the first demonstration of the required factoriza-
 67 tion, thanks to a recent advance in deconvolution methodology, namely “focused blind
 68 deconvolution” (FBD, introduced by Bharadwaj et al., 2019). Our factorization pro-
 69 vides complementary information on the rupture characteristics compared to existing
 70 methods that rely on isolating (e.g, Tocheport et al., 2007) the P-wave arrivals for
 71 back-propagation (Larmat et al., 2006; L. Meng et al., 2016; Yin & Denolle, 2019)
 72 and/or construction of an empirical Green’s function (EGF, Hartzell, 1978; Lanza
 73 et al., 1999; McGuire, 2004; Vallée et al., 2011; Vallée & Douet, 2016; Kikuchi &
 74 Kanamori, 1982; Lay et al., 2009).

75 The factorization of the seismograms is challenging and generally not solvable,
 76 because of the unknown trade-off between s and g i.e., extracting one requires assump-
 77 tions about the other. However, FBD compares a multitude of records (e.g., Plourde
 78 & Bostock, 2017) due to the same source, and identifies the *similarities* among them
 79 through a formal analysis. Subsequently, it associates the similarities to the spectrum
 80 of s , and the dissimilarities to g . For the success of FBD, we require that the receivers
 81 span a wide range of azimuth-angles and distances with respect to the rupture. In
 82 recent years, large numbers of seismometers have been deployed, so this requirement
 83 can easily be satisfied.

84 2 Redshift in an Earthquake Spectrum

85 Our primary goal is the robust estimation of the earthquake source spectrum
 86 using the aforementioned factorization of the teleseismic waveforms. In this section,
 87 we first assume a kinematic source model for a fault that is vertical. Then, we associate
 88 the parameters of this source model to the features e.g., redshift or Doppler shift in
 89 the estimated source spectrum.

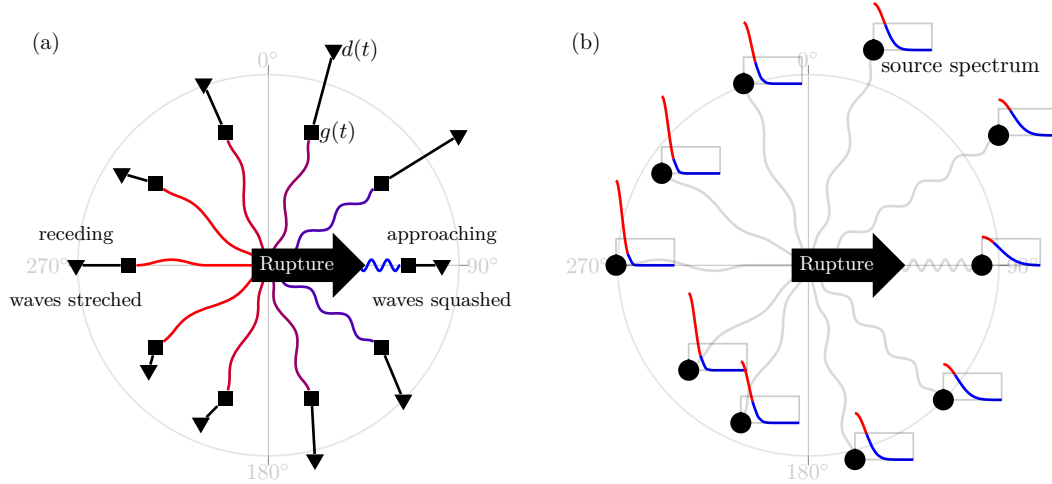


Figure 1. Schematic of waves emitted due to a rupture propagating from west (azimuth $\theta = 270^\circ$) to east (90°). a) Blue waves emitted towards the east are shortened, while the red waves traveling towards the west are lengthened. These waves undergo complex scattering (squares) before they reach the receivers (triangles), resulting in a challenging source-spectrum estimation problem. b) FBD factorizes the measurements for effectively removing the squares and directly estimating the source spectra (red-blue graphs) at the (circles) receivers. The variability of the normalized source spectrum with θ can be used to infer the kinematic rupture parameters.

We set a cylindrical coordinate system with origin O , radius r , azimuth θ , and height z . The fault plane extends from $r = 0$ to L along the radial line $\theta = 90^\circ$, and from $z = 0$ to $H \ll L$ along the cylindrical axis. A unidirectional rupture starts at the hypocenter, located at O , and propagates along the radial line. The kinematic rupture model, explained in Appendix A, is simplified using the Fraunhofer approximation to represent the waves recorded at (r, θ) on the surface $z = 0$ as:

$$d(t; r, \theta) \approx s(t; \theta) *_t g(t; r, \theta). \quad (1)$$

90 Here, the path effects are denoted by a convolution operation (eq. A7) in time with
 91 a function $g(t; r, \theta)$, which corresponds to the response due to impulsive force couples
 92 acting at the hypocenter. The apparent source pulse emitted in the direction of
 93 azimuth θ is given by the function:

$$s(t; \theta) = \begin{cases} \frac{c_r}{|\gamma|} w\left(\frac{tc_r}{\gamma}\right) & \text{when } 0 < \frac{tc_r}{\gamma} < L; \\ 0 & \text{otherwise,} \end{cases} \quad \text{where } \gamma = 1 - \frac{c_r}{c} \sin \theta. \quad (2)$$

94 In the above equation, γ roughly varies between 0 and 2, owing to the common obser-
 95 vation that rupture speed c_r is comparable to wave velocity c . The function w depends
 96 on H and represents the distribution of stress drop along the radial line of the fault.
 97 (Note that we have substituted $\psi = \theta - 90^\circ$ in eq. A5 — this substitution is only valid
 98 for the waves that depart from the fault along radial lines — so in section 4.3, we
 99 primarily analyze the surface waves emitted from steeply-dipping faults.)

100 The source model in eq. 2 is less restrictive compared to a model that regards
 101 the fault as a stationary point source i.e., it also incorporates the seismic wavelength
 102 λ that is comparable to L . However, as in eq. A4, it requires that the receivers are
 103 located at large distances such that $r \gg 2L^2/\lambda$. Accordingly, in section 4.3, we analyze
 104 the above-mentioned surface waves in the long-period seismograms:

- 105 • recorded at teleseismic distances with $r > 1600$ km i.e., epicentral distance
- 106 greater than 15° ;
- 107 • that contain dominant frequencies less than 0.1 Hz — as a result $\lambda \gtrsim 40$ km;
- 108 • from intermediate-magnitude ($6.0 < Mw < 6.5$) earthquakes typically with
- 109 $L \approx 60$ km.

110 In eq. 2, it can be noted that the source function w is scaled depending on

- 111 1. the speed c_r of the rupture propagation;
- 112 2. the direction θ relative to the rupture propagation;
- 113 3. and the velocity c of the propagating waves in the source region.

114 Therefore, if the rupture is approaching ($\theta = 90^\circ \Rightarrow |\gamma| \ll 1$) a station then the
 115 source function w is shortened as depicted in Fig. 1a. Accordingly, as a result of the
 116 scaling property of the Fourier transform, its amplitude spectrum is lengthened over
 117 the frequency ω , as shown in the Fig. 1b. On the other hand, if the rupture is receding
 118 ($\theta = 270^\circ \Rightarrow \gamma = 1 + c_r/c \approx 2$) from a station then the source function w is lengthened
 119 in time, resulting in a shortened-frequency amplitude spectrum of the source. This
 120 causes an apparent shift in the corner frequency, which is considered in the Haskell
 121 fault model (Madariaga, 2015).

122 Unfortunately, the time-scaled source pulse i.e., the apparent source pulse s is
 123 affected by the properties of the subsurface that the signal propagates through before
 124 reaching these stations. Such effects prevent us from directly observing the apparent
 125 source pulse at the stations. In the following sections, we will present a factorization
 126 of the records d of an earthquake to eliminate the path effects, as depicted in Fig. 1.

127 3 Focused Blind Deconvolution

FBD requires that the multiple receivers span a wide range of azimuth-angles
 θ and distances r relative to the rupture. For such a set of receivers, a temporal-
 index window $t \in \{T_1, \dots, T_2\}$, relative to the origin time of the earthquake, has to
 be applied in order to roughly isolate either the body waves or the surface waves.
 Depending upon the temporal window, FBD outputs an estimate $|\hat{S}|$ of the source
 spectrum $|S|$ (of either P- or S-waves) as a function of θ . We consider many azimuthal
 bins Θ , each with n receivers, such that the variability of each $|\hat{S}|_{\theta \in \Theta}$ can be ignored.
 Therefore, we have $s(t; \theta) \approx s(t) \forall \theta \in \Theta$, resulting in a single-input multiple-output
 model

$$d_i(t) = s *_t g_i. \quad (3)$$

128 Here, the subscript $i \in \{1, 2, \dots, n\}$ denotes an index of a receiver that records the
 129 ground motion $d_i(t)$, or else a spatial location where the Green's function $g(\cdot, t)$
 130 is evaluated as $g_i(t)$. We denote a vector of records by $[d_i] : \{T_1, \dots, T_2\} \rightarrow \mathbb{R}^n$, and a
 131 vector of Green's functions by $[g_i] : \{T_1, \dots, T_2\} \rightarrow \mathbb{R}^n$. The duration length of each
 132 element of $[d_i]$ and $[g_i]$ is given by $T_2 - T_1 + 1$, which was chosen so that the d_i may
 133 each contain an identical source pulse $s : \{0, \dots, T\} \rightarrow \mathbb{R}$. It is important to note that
 134 the FBD results are insensitive to the choice of the duration length $T + 1$ of s — as
 135 long as the length is long enough to capture the source effects.

136 In every Θ , the intention is to factorize (i.e., blindly deconvolve) the ground
 137 motion $[d_i]$ in eq. 3 into the path effects $[g_i]$ and the source s , with *much fewer and*
 138 *simpler assumptions* about these factors, compared to those made in conventional
 139 methods. A suitable algorithmic approach, related to multichannel blind deconvolution
 140 (BD), is a least-squares fit of $[d_i]$ to jointly optimize two unknown functions $[g_i]$ and
 141 s . The joint optimization can be suitably carried out using alternating minimization
 142 (Ayers & Dainty, 1988; Sroubek & Milanfar, 2012): in one cycle, we fix one function

143 and optimize the other, and then fix the other and optimize the first. Several cycles
 144 are expected to be performed to reach convergence. However, it is well known that BD
 145 is not solvable, due to non-uniqueness, without making assumptions on at least one
 146 of the two unknowns. These assumptions determine the admissible trade-off between
 147 $[g_i]$ and s during the optimization.

148 Accordingly, we employ focused blind deconvolution (FBD), which first reduces
 149 the trade-off in BD by considering a least-squares fitting of *interferometric* or *cross-*
 150 *correlated* records, instead of the raw records. And second, it determines all the
 151 remaining trade-off (except for a scalar) by associating the *dissimilarities* among the
 152 multiple records to $[g_i]$, while attributing similarities to s . Our examples below demon-
 153 strate that these associations are valid as long as the receivers are placed at dissimilar
 154 locations i.e., their separation distances are much larger than the wavelength.

155 FBD is presented in detail by Bharadwaj et al. (2019), we discuss the underlying
 156 principles below. An illustrative numerical experiment, along with an open-source
 157 software package, is presented in the supplementary material. One import aspect of
 158 FBD is the following reformulation that is simpler to solve, due to the reduced trade-
 159 off, as it only estimates the unknown source auto-correlation and interferometric path
 160 effects.

161 **Definition 1** (IBD: Interferometric Blind Deconvolution). The interferometric record
 162 between i th and j th receivers is given by

$$d_{ij}(t) = \{d_i \otimes d_j\}(t) = \underbrace{\{s \otimes s\}}_{s_a} *_t \underbrace{\{g_i \otimes g_j\}}_{g_{ij}},$$

163 where $\{u \otimes v\}(t) = u(\cdot) *_t v$ defines temporal cross-correlation. IBD aims for a least-
 164 squares fitting of an $(n+1)n/2$ -vector, denoted by $[d_{11}, d_{12}, \dots, d_{22}, d_{23}, \dots, d_{nn}]$ or sim-
 165 ply $[d_{ij}]$, of the unique interferometric records between every possible receiver pair:

$$(\hat{s}_a, [\hat{g}_{ij}]) = \arg \min_{s_a, [g_{ij}]} \sum_{k=1}^n \sum_{l=k}^n \sum_{t=-T_2+T_1}^{T_2-T_1} \{d_{kl}(t) - \{s_a * g_{kl}\}(t)\}^2. \quad (4)$$

166 Along the similar lines of BD, it jointly optimizes two functions, namely the interfero-
 167 metric Green's function $[g_{ij}]: \{-T_2+T_1, \dots, T_2-T_1\} \rightarrow \mathbb{R}^{(n+1)n/2}$ and the auto-correlated
 168 source function $s_a: \{-T, \dots, T\} \rightarrow \mathbb{R}$.

169 The motivation behind dealing with $[d_{ij}]$ is that the cross-correlation operation
 170 discards the phase information from the Fourier representation of the source. There-
 171 fore, the admissible trade-off between the path effects $[g_{ij}]$ and the source s_a is reduced,
 172 compared to trade-off between $[g_i]$ and s in BD. The remaining trade-off, pertaining
 173 to the amplitude spectrum of the source, is determined in FBD by regularizing with a
 174 focusing functional:

$$J = \sum_{k=1}^n \sum_{t=-T_2+T_1}^{T_2-T_1} t^2 g_{kk}(t)^2.$$

175 FBD minimizes J i.e., the energy of the auto-correlated Green's functions g_{ii} multi-
 176 plied by the lag time to result in a solution where the g_i are heuristically as white
 177 (in the frequency domain) as possible. As shown by Bharadwaj et al. (2019), simul-
 178 taneously maximizing the whiteness of any g_i promotes its dissimilarity from all the
 179 $g_{j \neq i}$. Therefore, for the success of FBD, it is important that the true g_i are sufficiently
 180 dissimilar. For instance, in the limit that the true g_i are equal to each other, FBD
 181 just outputs the temporal Kronecker $\delta(t)$ for the g_i , making the s equal the d_i . In
 182 our experiments, we ensure that the "sufficiently dissimilar" requirement is satisfied
 183 by choosing receivers separated by distance r much larger than the wavelength. Note

184 that, for a given receiver configuration, the width $|\Theta|$ of each azimuthal bin Θ deter-
 185 mines the range of r ; we choose each $|\Theta|$ sufficiently large such that receivers span a
 186 wide range of r .

187 Now, after estimating source auto-correlation in every Θ , the next step is to
 188 normalize them such that $\hat{s}_a(0) = 1$. Then, the Fourier representation of \hat{s}_a can be
 189 used to construct the normalized source spectrum. For every Θ , the duration of the
 190 apparent source time function is given by the time necessary for the envelope of \hat{s}_a ,
 191 denoted by $E(\hat{s}_a)$, to decrease below a chosen threshold. The envelope operator E
 192 computes the absolute value of the analytic representation of a real-valued signal. The
 193 final trivial step is to combine the outputs together over all Θ to form the estimated
 194 source properties over the entire interval of θ .

195 4 Applications

196 For a given earthquake, FBD estimates the apparent source auto-correlation
 197 $\hat{s}_a(t; \theta)$, and its zero-phase Fourier representation i.e., the apparent power spectrum
 198 $|\hat{S}(\omega; \theta)|^2$. The benefits of this methodology include:

- 199 1. at any given azimuth θ , the time duration of the apparent source pulse can be
 200 determined using that of \hat{s}_a ;
- 201 2. $|\hat{S}(\omega; \theta)|$ can be inspected for spectral attributes associated with source charac-
 202 teristics e.g., its closeness to a unilateral rupture;
- 203 3. more generally, $|\hat{S}(\omega; \theta)|$ can be used as input to finite-fault inversion to directly
 204 infer the rupture parameters, without being affected by the uncertainties in the
 205 subsurface models;
- 206 4. assuming that multiple earthquakes share identical path effects, the variation of
 207 $|\hat{S}(\omega; \theta)|$ among these earthquakes provides an accurate relative magnitude of
 208 each earthquake.

209 Now, we demonstrate the first two benefits, while leaving the others for future research.

210 4.1 Redshift Attribute

211 Redshift is a spectral attribute of an almost unilaterally propagating rupture. It
 212 is related to the frequency-scaling of the source spectrum as discussed in section 2.
 213 For a given earthquake spectrum and a choice of two different frequency bands, red
 214 and blue for low- and high-frequency bands respectively, we:

- 215 1. compute the spectral energy of $|\hat{S}|$ in the bands as a function of the azimuth θ ,
 216 resulting in a spectral-energy vs azimuth plot;
- 217 2. and inspect if the energy in the red band is dominant in a particular direc-
 218 tion, corresponding to a dominant blue energy in the opposite direction; this
 219 characteristic of the source-spectrum variation is referred to as redshift.

220 Inspecting the FBD estimated (normalized) source spectra $|\hat{S}(\omega; \theta)|$ for redshift will
 221 help us identify unilateral ruptures from those that are more complex. The redshift
 222 attribute is quantifiable using the wide-band ambiguity function (Weiss, 1994; Sibul
 223 & Ziomek, 1981), which we also leave for future research.

224 4.2 Synthetic Experiment

225 We now present a 2-D numerical experiment that demonstrates the benefits
 226 of FBD for rupture characterization. We record both the horizontal- and vertical-
 227 component displacement due to a unilaterally propagating rupture along $\theta = 90^\circ$. As

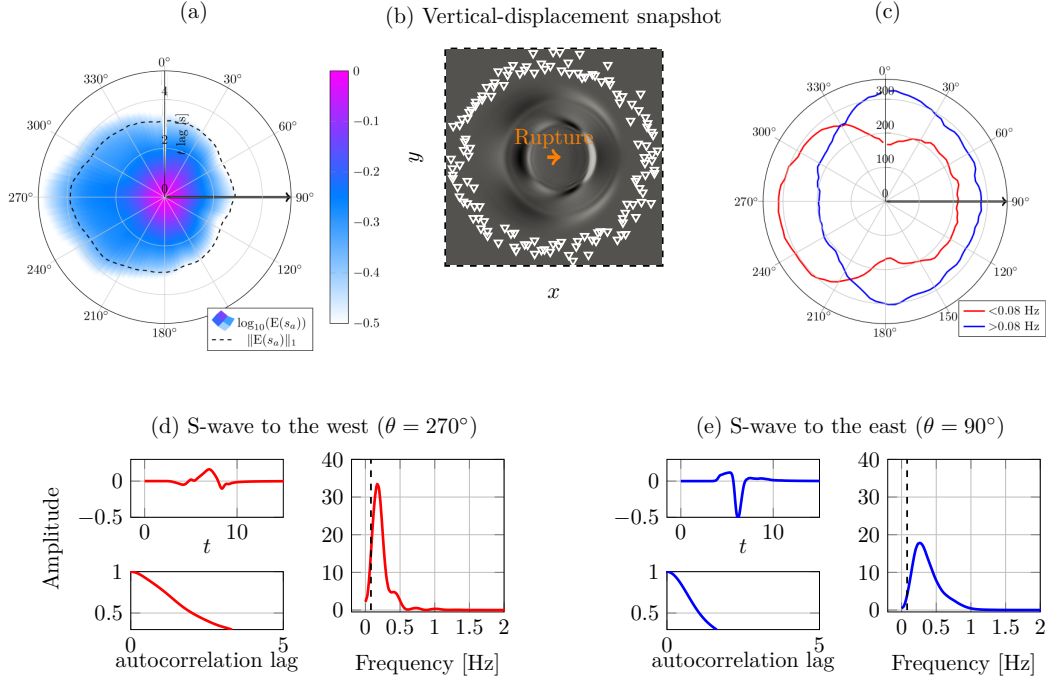


Figure 2. A synthetic experiment. a) The envelope of the FBD-estimated auto-correlated source pulse i.e., $E(\hat{s}_a(t; \theta))$ is plotted as a function of the lag time and azimuth. b) Vertical displacement due to a rupture, colored in gray-scale as a function of the horizontal x and vertical y spatial coordinates, before the P and S waves are scattered by the boundaries (dashed lines) of the medium. Some of the receiver positions are marked by white triangles. c) The energy of the FBD-estimated source spectrum, in both the low (red) and high (blue) frequency bands, is plotted to depict the redshift. In order to validate the FBD results, the direct S-wave pulse on the opposite sides of the rupture is plotted in (d) and (e), respectively. The dashed vertical line separates the two frequency bands of (c).

228 depicted in Fig. 2b, roughly 100 receivers surround the source and span a range of
 229 distances r from 15 to 32 km. The waves are modeled using an elastic finite-element
 230 solver (C. Meng & Wang, 2018) in a homogeneous spatial domain with both x and
 231 y from -32 to 32 km. We deliberately set reflective, instead of absorbing, boundary
 232 conditions to create complex path-specific effects due to multiple scattering. This 2-D
 233 experiment only involves the scattered P and S waves, but similar experiments can
 234 also be performed using surface waves, which are considered later.

235 We employ FBD to estimate $\hat{s}_a(t; \theta)$ from the full-wavefield records — the en-
 236 velope of \hat{s}_a is plotted in Fig. 2a with lag time $t > 0$ on the radial axis and θ
 237 on the azimuthal axis. We isolated the S-wave pulses from the records at 90° and 270° ,
 238 respectively, and plotted them in Figs. 2d and 2e. Observe that the difference in their
 239 durations, as depicted by the envelopes of the auto-correlated pulses, is consistent with
 240 our FBD result. Similarly, we plotted the normalized spectra of the S-wave pulses in
 241 these plots to observe that the pulse at 270° has dominant low frequencies compared to
 242 that at 90° . Again, this is consistent with FBD-estimated spectral energy vs azimuth
 243 plot in Fig. 2c.

244 4.3 Application to Recorded Earthquakes

245 We now use FBD in the source-spectrum analysis of two earthquakes with mag-
 246 nitude $M_w \leq 6.5$. In recent years, a large number of seismometers have been deployed,
 247 which facilitate the capture of the source pulse at a wide range of azimuths θ and
 248 distances r , making FBD application feasible. With regard to the source model dis-
 249 cussed in the previous sections, we only consider strike-slip earthquakes that ruptured
 250 almost-vertical faults at shallow depths of ≈ 15 km. The earthquake locations and
 251 moment-tensor solutions (Dziewonski et al., 1981; Hanka & Kind, 1994) are plotted in
 252 the Fig. 3. Additional information about these earthquakes is provided in the suppl-
 253 ementary material. For each earthquake, we have downloaded long-period records, with
 254 1 Hz sampling rate, from 20 supported international data centers that can be accessed
 255 via the toolbox obspyDMT (Hosseini & Sigloch, 2017). Only stations with epicentral
 256 distance greater than 15° were selected, as plotted in the Figs. 3a and 3d. At each
 257 seismic station, we utilize multiple components of the recorded displacement, which
 258 primarily contain the first-arriving surface waves, termed as R1 (Rayleigh) and G1
 259 (Love) waves, that are the largest-amplitude arrivals.

260 The pre-processing of the records is relatively simple, performed using the toolbox
 261 ObsPY (Beyreuther et al., 2010). We first remove the noisy records, and demean
 262 the records we keep. We then perform an important step i.e., instrument correction,
 263 without which we notice that the instrument response contaminates the FBD-extracted
 264 similarity among the records. Again, note that we associate the similarity among the
 265 records with the source effects; therefore, it is important that there is no *artificial*
 266 similarity in the recorded spectra due the instrument response of the seismometers.
 267 Finally, the records were windowed for a duration ≈ 6750 s following the origin time,
 268 resulting in $d(t; r, \theta)$ as input to FBD.

269 4.3.1 Nicobar (08 November 2015) $M_w=6.5$

270 This strike-slip earthquake ruptured a known fault in a region SE of the An-
 271 daman Island (see supplementary material). The teleseismic stations that were uti-
 272 lized in the FBD analysis are plotted in Fig. 3a. The estimated apparent source pulse
 273 auto-correlation \hat{s}_a , plotted in Fig. 3b, indicates that the source duration is ≈ 15 s
 274 longer in the NW compared to the SE direction. In the spectral energy vs azimuth
 275 plot, the spectral energy is computed in three different frequency bands as plotted
 276 in Fig. 3c. These results, similar to those in Figs. 2b and 2c, indicate a unilateral
 277 rupture propagation, along the SE trend. Accordingly in Fig. 4, the source spectrum
 278 exhibits frequency scaling, with higher corner frequency in the direction of the rupture
 279 propagation, and vice versa. The rupture propagation is consistent with one of the
 280 two possible strike directions indicated by the moment tensor in the Fig. 3a.

281 4.3.2 California (04 July 2019) $M_w=6.4$

282 This is a foreshock of the $M_w = 7.1$ July 5 mainshock in the 2019 Ridgecrest
 283 sequence that occurred as the result of shallow strike slip faulting in the crust of the
 284 North America plate. The FBD analysis of the July 5 mainshock (USGS, 2019b)
 285 is presented in the supplementary material. Compared to the mainshock, the FBD-
 286 extracted spectrum for this earthquake indicates a relatively simple rupturing, with
 287 dominant propagation towards SW. That is: 1) a shorter source duration is noticed in
 288 the SW direction relative to NE, as shown by the \hat{s}_a plot in Fig. 3e; 2) the stations in the
 289 SE direction record dominant high frequencies — as is evident from the spectral-energy
 290 vs azimuth plot in Fig. 3f. These results are consistent with the direct observations,
 291 which suggest that the event ruptured a previously unnoticed NE-SW trending fault.
 292 Moreover, the aftershocks following this particular event also aligned along the NE-SW

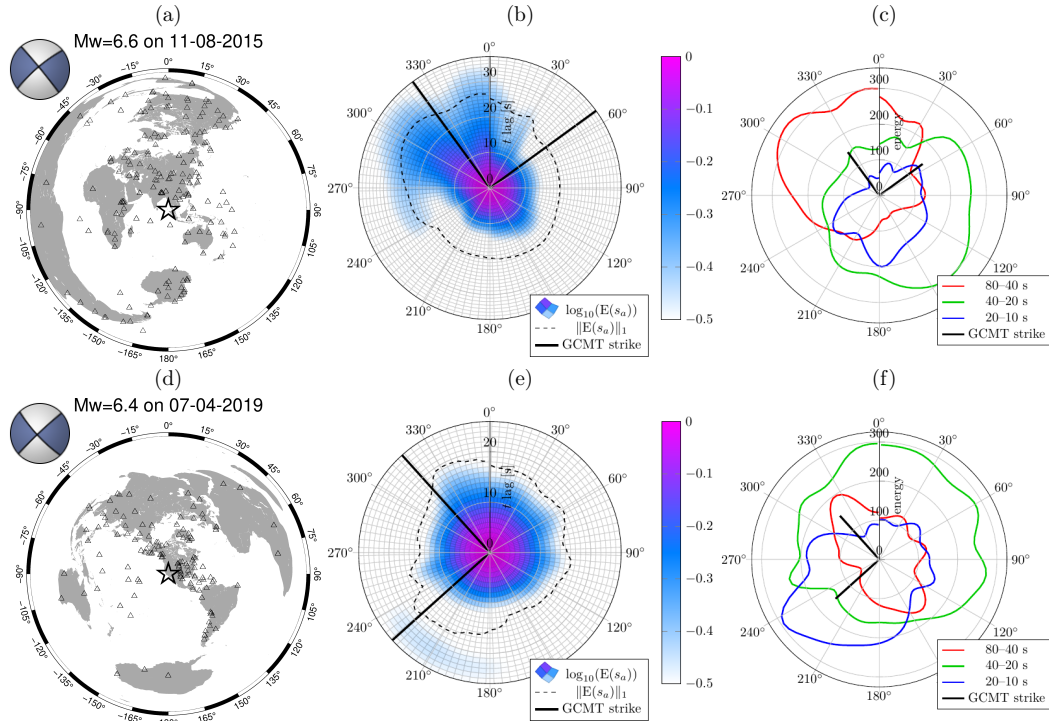


Figure 3. FBD of two recorded strike-slip earthquakes. Left column: event (star) and station (triangles) locations. GEOFON moment tensor solutions are inserted. Middle column: the envelope of the estimated auto-correlated apparent source pulse, $E(\hat{s}_a(t; \theta))$ is plotted along with the two possible strike directions (solid radial lines) — note the variation of the source time duration with azimuth. Right column: the source spectral energy vs azimuth plot in three different frequency bands indicates redshift.

293 trend (USGS, 2019a). Again, note that the rupture propagation is along one of the
 294 two possible strike directions, as indicated by the moment tensor in Fig. 3d.

295 **5 Conclusions**

296 We have demonstrated that focused blind deconvolution (FBD) is a powerful
 297 data-driven tool for factorizing teleseismic records into source and path effects. Instead
 298 of relying on source- or path-related assumptions e.g., the empirical Green’s function,
 299 FBD characterizes an earthquake source by associating it with the similarity among
 300 a multitude of records. However, there is a potential problem with this method: it
 301 may not succeed due to a number of simplifications (like binning and the Fraunhofer
 302 approximation) that were made to arrive at the convolutional model, and there is no
 303 theoretical guarantee that FBD performs a physically meaningful factorization even
 304 for the convolutional model.

305 In our numerical experiments, FBD extracted the earthquake source spectra from
 306 the surface waves of intermediate-magnitude, shallow strike-slip earthquakes. These
 307 spectra are complementary to the ones extracted from other methods using isolated
 308 P-wave arrivals. They were further analyzed to identify unilaterally propagating rup-
 309 tures during the earthquakes; a potential extension is to robustly estimate the rupture
 310 velocity. The FBD results of one of the recent recorded earthquakes that originated in

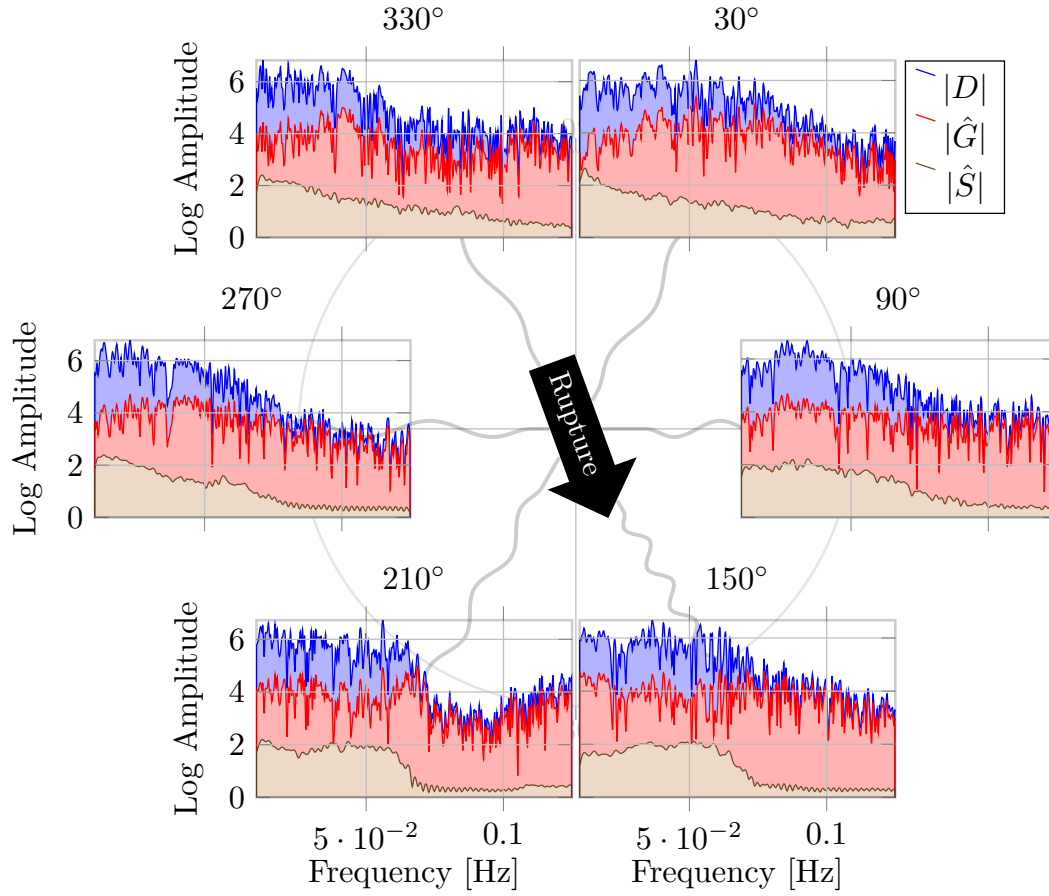


Figure 4. FBD factorizes the recorded spectra $|D|$, due to the Nicobar earthquake, into the source $|\hat{S}|$ and the path $|\hat{G}|$ at multiple azimuths. Note that the source spectrum exhibits frequency scaling, with higher corner frequency in the direction of the rupture propagation (indicated by an arrow) and vice versa.

311 Ridgecrest, California, are consistent with observations from local seismological and
 312 geodetic instrumentation — this showcases the potential of FBD to analyze earth-
 313 quakes without the need of local instrumentation.

314 Appendix A Fraunhofer’s Approximation

315 An active fault surface causing an earthquake can be regarded as a surface dis-
 316 tribution of body forces (Aki & Richards, 2002). The kinematic dislocation model
 317 (Madariaga, 2015) assumes that these equivalent body forces are *activated* in a se-
 318 quence, depending on the parameter(s) that determine the propagation of the slip.
 319 We consider a unidirectional rupture propagation along the length L of a fault plane
 320 Ξ . The fault plane is assumed to be a rectangle that has a small width $H \ll L$. We
 321 denote an infinitesimal surface element at $\xi = (\xi_1, \xi_2)$ on the fault by $d\Xi$, where ξ_1 and
 322 ξ_2 are local two-dimensional coordinates in the length- and width-directions, respec-
 323 tively. In three dimensions, the i th component of the far-field displacement at (\mathbf{x}, t)
 324 due to a displacement discontinuity across a surface element at ξ can be approximated
 325 as:

$$u^i(\mathbf{x}, t; \xi) \approx \sum_{j,k=1}^3 \int \mathcal{G}^{ij,k}(\mathbf{x}, t - \tau; \xi) m^{jk}(\tau; \xi) d\tau, \quad (\text{A1})$$

where m^{jk} denotes the (j, k) th component of the moment density tensor and $\mathcal{G}^{ij,k}$ de-
 notes the k th spatial derivative of the (i, j) th component of the elastodynamic Green’s
 tensor. We now assume an instantaneous slip such that the dependency of the mo-
 ment density tensor on the time τ can be ignored. We also assume that the com-
 ponents of the moment density tensor do not vary relative to each other resulting in
 $m^{jk}(\xi) = h(\xi)m^{jk}(\xi_0)$, where $h(\xi)$ is proportional to the stress drop at ξ and $\xi_0 = (0, 0)$
 is the hypocenter. Rewriting eq. A1 with these assumptions results in:

$$u^i(\mathbf{x}, t; \xi) \approx h(\xi)g^i(\mathbf{x}, t; \xi), \quad \text{where} \quad g^i(\mathbf{x}, t; \xi) = \sum_{j,k} \mathcal{G}^{ij,k}(\mathbf{x}, t; \xi) m^{jk}(\xi_0).$$

326 In this paper, we refer to the terms ‘Green’s function’ and ‘path effects’ with g^i of the
 327 above equation, even though it already includes some directivity effects e.g., due to a
 328 force couple. Also, note that we have dropped the component i (not to be confused
 329 with receiver-label i) because FBD handles all the measured displacement components
 330 identically. Now, consider a constant velocity c_r for the rupture that propagates or
 331 spreads starting from $\xi_1 = 0$ to $\xi_1 = L$. In other words, the slip at the surface element
 332 ξ is *activated* with a delay given by $\tau(\xi) = \xi_1/c_r$. The total far-field displacement d
 333 due to the entire rupture is the sum of contributions from different surface elements:

$$d(\mathbf{x}, t; \Xi) = \int_{\Xi} h(\xi) g(\mathbf{x}, t - \tau(\xi); \xi) d\Xi,$$

334 the contributions being respectively delayed according to $\tau(\xi)$. We now assume that
 335 the dominant seismic wavelength λ that is under consideration significantly exceeds
 336 the width H of the fault, such that $g(\mathbf{x}, t; \xi)$ will be in phase $\forall \xi_2$. Accordingly, we can
 337 rewrite the above equation using another scalar function w as:

$$d(\mathbf{x}, t) = \int_0^L w(\xi_1) g\left(\mathbf{x}, t - \frac{\xi_1}{c_r}; \xi_1\right) d\xi_1. \quad (\text{A2})$$

338 In order to limit the dependency of g on the length coordinate ξ_1 to an overall
 339 translation in time in eq. A2, we make the so-called Fraunhofer approximation, which
 340 only makes an allowance for the far-field phase correction (travel-time difference) be-
 341 tween 0 and ξ_1 . For the part of the wavefield associated with waves having velocity c

342 in the source region, we have

$$g(\mathbf{x}, t; \xi_1) \approx g\left(\mathbf{x}, t - \frac{\xi_1 \cos \psi}{c}; 0\right), \quad (\text{A3})$$

where ψ is the direction, relative to the rupture propagation, in which the waves depart from Ξ . Aki and Richards (2002) showed that this is a valid first-order approximation in a region, where the receivers are located at large distances

$$|\mathbf{x} - \xi_0| \gg \frac{2L^2}{\lambda}. \quad (\text{A4})$$

343 Now, combining eqs. A2 and A3 and dropping the redundant argument 0 of g , we get:

$$d(\mathbf{x}, t) = \int_0^L w(\xi_1) g\left(\mathbf{x}, t - \frac{\xi_1 \gamma}{c_r}\right) d\xi_1, \quad \text{where } \gamma = 1 - \frac{c_r \cos \psi}{c} \quad (\text{A5})$$

344 could be positive, negative or zero. For $\gamma \neq 0$, we now substitute $k = \xi_1 \gamma / c_r$ that
345 belongs to a time interval $\mathbb{T} = \{t \in \mathbb{R} \mid 0 < tc_r / \gamma < L\}$ of length $|\gamma|L/c_r$, to obtain

$$d(\mathbf{x}, t) = \int_{-\infty}^{\infty} s(k; \psi) g(\mathbf{x}, t - k) dk, \quad (\text{A6})$$

346 where the rupture manifests itself in the recorded time as a function commonly known
347 as the apparent source time function (ASTF):

$$s(t; \psi) = \begin{cases} \frac{c_r}{|\gamma|} w\left(\frac{tc_r}{\gamma}\right) & \text{when } \gamma \neq 0 \text{ \& } t \in \mathbb{T}, \\ 0 & \text{otherwise} \end{cases}$$

$$\xrightarrow{\gamma \rightarrow 0} \delta(t) \int_0^L w(x) dx$$

(a corollary of e.g., Stein & Weiss, 2016, Theorem 1.18). Finally, we time-discretize and rewrite eq. A6 as a temporal convolution

$$u *_t v = \sum_{k=-\infty}^{\infty} u(k) v(t - k) \quad (\text{A7})$$

348 between the ASTF and the Green's function g to obtain eq. 1

349 Appendix B Illustration of FBD Using Random Signals

350 Focused Blind Deconvolution (FBD) simultaneously maximizing the whiteness
351 of any g_i promotes its dissimilarity from all the $g_{j \neq i}$. To illustrate this, we consider
352 a numerical experiment with $n = 20$ and plot (only for $i = 1, 2, 3$) the spectra asso-
353 ciated with two possible solutions of Interferometric Blind Deconvolution (eq. 4) in
354 Figs. Appendix Ba and Appendix Bb, respectively. It can be noted that in both cases
355 the observed spectra are satisfied i.e., $|D_i| = |\hat{S}| |\hat{G}_i| \forall i$. The focusing constraint J
356 is designed to choose the (b) solution because the spectra associated with g_i are maxi-
357 mally white. Also, note that this maximally-white solution has maximum dissimilarity
358 among the g_i spectra; as a result, FBD seeks a solution of the ill-posed IBD where the
359 g_i are dissimilar to each other.

360 Appendix C Additional Details of the Earthquakes

361 The locations and moment-tensor solutions of earthquakes presented in the paper
362 are plotted in the Fig. C1, and also listed in Table C1.

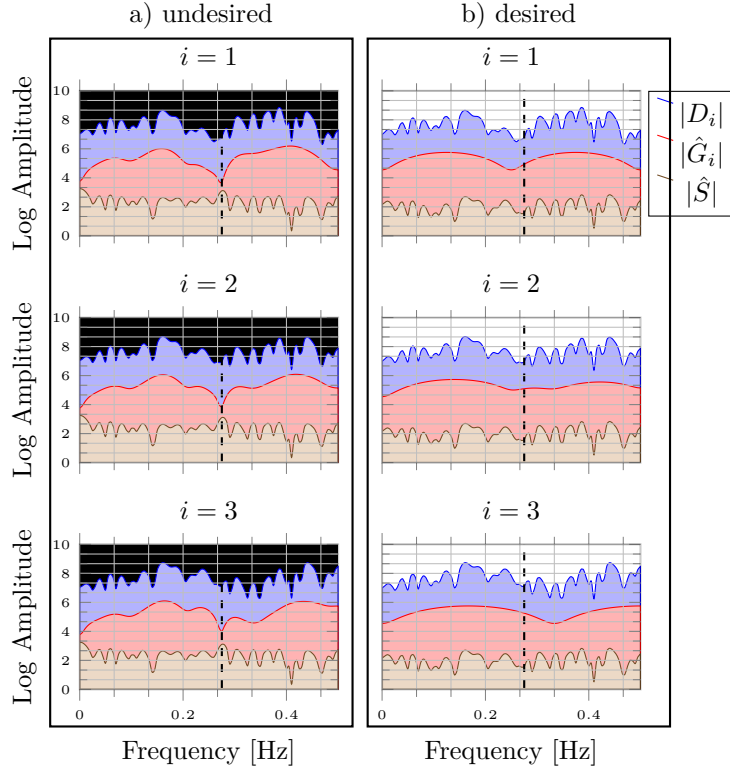


Figure B1. Two possible factorizations of the recorded spectra $|D_i|$, associated with the solutions of the IBD problem in eq. 4, are plotted in (a) and (b). Note that in each of the plots the recorded spectrum is satisfied i.e., $\log |D_i| = \log |\hat{G}_i| + \log |\hat{S}|$ as implied by eq. 3. In factorization (a), the estimated spectra associated with the path effects $|\hat{G}_i|$ are *similar* to each other e.g., they have a common notch at the frequency indicated by the dashed line. Therefore, factorization (a) is undesirable, provided that the receivers are separated by distances much larger than the wavelength. The focusing constraint in FBD obtains the factorization (b) —note that the $|\hat{G}_i|$ in this factorization are not only more *white* but also *dissimilar* to each other. Therefore, in the framework of FBD, the *similarities* in the recorded spectra are identified as source \hat{S} effects.

Table C1. List of earthquakes along with two possible moment-tensor solutions. Courtesy: GEOFON Earthquake Information Service.

Name, Date, Mw	Latitude	Longitude	Depth	Strike ($^\circ$)	Dip ($^\circ$)	Rake ($^\circ$)
Nicobar, 2015-11-08, 6.5	6.79 $^\circ$ N	94.50 $^\circ$ E	15 km	321, 230	87, 82	171, -2
California, 2019-07-04, 6.4	35.69 $^\circ$ N	117.46 $^\circ$ W	14 km	137, 227	82, 87	177, 8

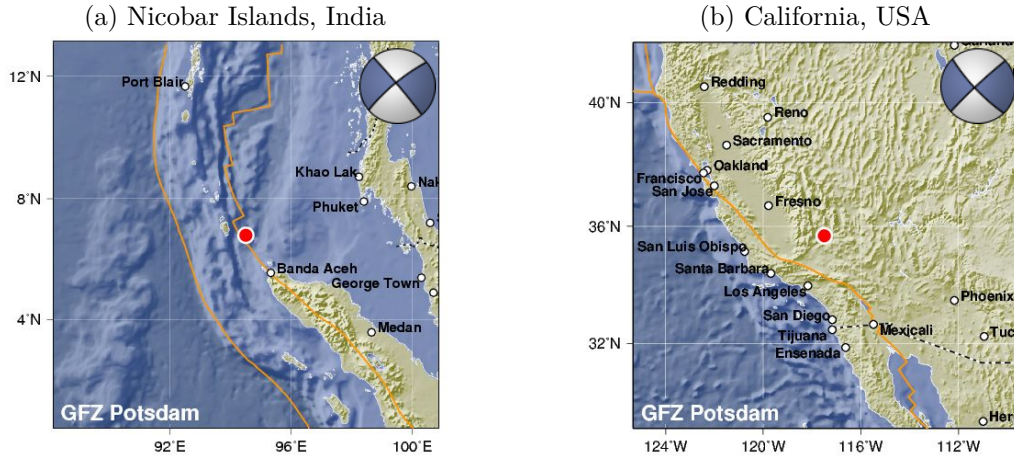


Figure C1. Two strike-slip earthquakes that are analyzed in this paper. Major faults in the region are delineated by orange curves, and moment tensor solutions are inserted. Courtesy: GEOFON Program (Hanka & Kind, 1994), GFZ Potsdam.

Table D1. List of complex earthquakes with two possible moment-tensor solutions. Courtesy: GEOFON Earthquake Information Service.

Name, Date, Mw	Latitude	Longitude	Depth	Strike (°)	Dip (°)	Rake (°)
California, 2019-07-05, 7.1	35.76°N	117.57°W	14 km	140, 233	76, 78	167, 14
Loyalty, 2017-10-31, 6.7	21.64°S	169.21°E	11 km	154, 321	76, 14	93, 77

Appendix D More Complex Earthquakes

We analyzed a wide variety of earthquakes in our research, other than those discussed in this article. Most of them were complex, in the sense that it was difficult to interpret the extracted source spectrum directly via redshift attribute. Therefore, additional spectral attributes have to be defined when continuing this research. Here, we present the source spectra of two slightly complex events, listed in Table D1. The locations of these events and their corresponding stations are plotted in Figs. D1a and D1d, respectively.

We first present the FBD analysis for the July 5 mainshock (USGS, 2019b) in the Ridgecrest sequence. Compared to its foreshock, presented in the main text, the estimated auto-correlated source pulse \hat{s}_a in Fig. D1b is complex. However, there is a minor indication that a dominant rupture mode is propagating towards the NW direction — note the longer source-pulse duration around 160° azimuth. Nevertheless, its corresponding spectral-energy vs azimuth plot in Fig. D1c was too complicated to interpret a unilateral propagation.

Similarly, the FBD analysis of a Mw = 6.7 earthquake to the Southeast of Loyalty Islands is presented in Figs. D1d, D1e and D1f. The apparent source-time function estimated using the SCARDEC method (Vallée & Douet, 2016) indicated a duration of silence of about 5 s during the earthquake. This is consistent with the FBD result in Fig. D1e, where \hat{s}_a exhibits a silence during the rupturing for about the same duration. As a result, we conclude that the earthquake didn't consist of a single rupture propagation with a constant velocity.

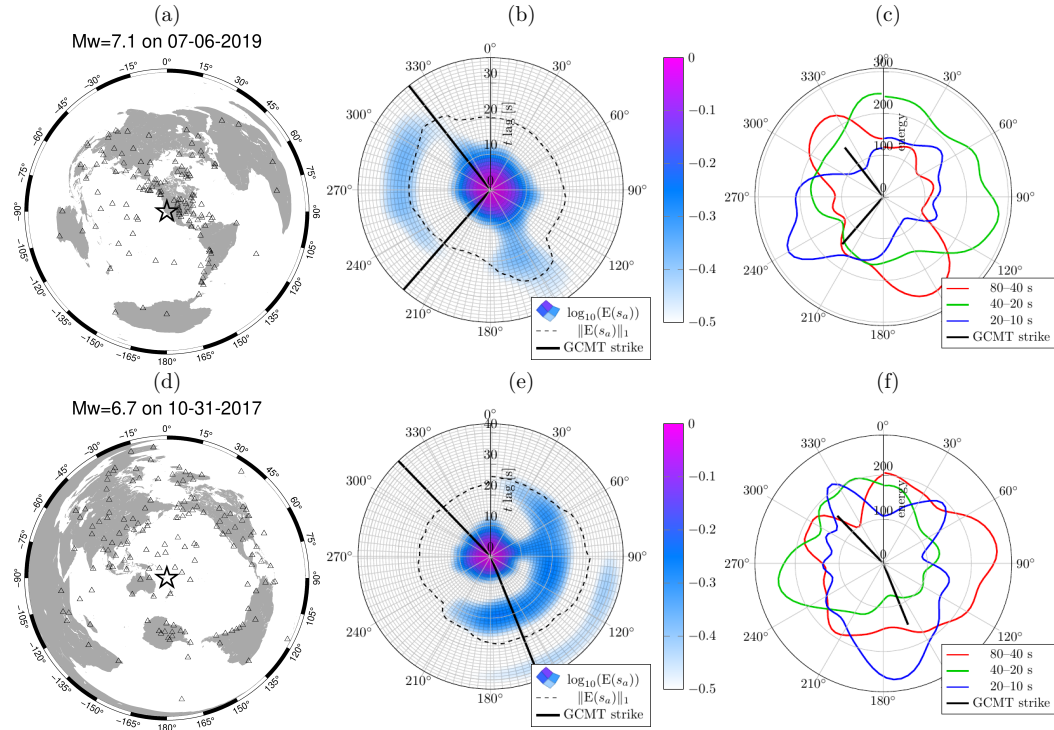


Figure D1. As in Fig. 3, except that the FBD results indicate complex rupturing. Redshift due to a unilateral rupture propagation cannot be identified during the analysis of these earthquakes, which are listed in the Table D1.

Appendix E Software: FocusedBlindDecon.jl

We have made a documented software available to perform focused blind deconvolution through a Julia package: <https://github.com/pawbz/FocusedBlindDecon.jl>.

Acknowledgments

We thank our colleagues Nafi Toksöz, Matt Li, Aurelien Mordret, Tom Herring, Hongjian Fang and Michael Floyd for providing insight and expertise that greatly assisted this research. The material is based upon work assisted by a grant from Equinor. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of Equinor. LD is also supported by AFOSR grant FA9550-17-1-0316. Finally, we are grateful to the organizations who provide and manage data: Global Seismographic Network (GSN) is a cooperative scientific facility operated jointly by the Incorporated Research Institutions for Seismology (IRIS), the United States Geological Survey (USGS), and the National Science Foundation (NSF), under Cooperative Agreement EAR-1261681. Earthquake information was obtained from the GEOFON programme of the GFZ German Research Centre for Geosciences using data from the GEVN partner networks.

References

- Aki, K., & Richards, P. G. (2002). *Quantitative seismology*.
 Ayers, G., & Dainty, J. C. (1988). Iterative blind deconvolution method and its ap-

- 404 plications. *Optics letters*, *13*(7), 547-549.
- 405 Beyreuther, M., Barsch, R., Krischer, L., Megies, T., Behr, Y., & Wassermann,
406 J. (2010, May). ObsPy: A Python Toolbox for Seismology. *Seismological*
407 *Research Letters*, *81*(3), 530-533. doi: 10.1785/gssrl.81.3.530
- 408 Bharadwaj, P., Demanet, L., & Fournier, A. (2019, June). Focused Blind Deconvo-
409 lution. *IEEE Transactions on Signal Processing*, *67*(12), 3168-3180. doi: 10
410 .1109/TSP.2019.2908911
- 411 Clayton, R. W., & Wiggins, R. A. (1976, October). Source shape estimation and
412 deconvolution of teleseismic bodywaves. *Geophysical Journal International*,
413 *47*(1), 151-177. doi: 10.1111/j.1365-246X.1976.tb01267.x
- 414 Dziewonski, A. M., Chou, T.-A., & Woodhouse, J. H. (1981). Determination of
415 earthquake source parameters from waveform data for studies of global and
416 regional seismicity. *Journal of Geophysical Research: Solid Earth*, *86*(B4),
417 2825-2852. doi: 10.1029/JB086iB04p02825
- 418 Gallovic, F., Valentova, L., Ampuero, J.-P., & Gabriel, A.-A. (2019, February).
419 *Bayesian Dynamic Finite-Fault Inversion: 1. Method and Synthetic Test*
420 (Preprint). EarthArXiv. doi: 10.31223/osf.io/tmjv4
- 421 Hanka, W., & Kind, R. (1994, November). The GEOFON Program. *Annals of Geo-*
422 *physics*, *37*(5). doi: 10.4401/ag-4196
- 423 Hartzell, S. H. (1978, January). Earthquake aftershocks as Green's functions. *Geo-*
424 *physical Research Letters*, *5*(1), 1-4. doi: 10.1029/GL005i001p00001
- 425 Heaton, T. H., & Helmberger, D. V. (1977, April). A study of the strong ground
426 motion of the Borrego Mountain, California, earthquake. *Bulletin of the Seis-*
427 *mological Society of America*, *67*(2), 315-330.
- 428 Hosseini, K., & Sigloch, K. (2017, October). ObspyDMT: A Python toolbox for re-
429 trieval and processing large seismological data sets. *Solid Earth*, *8*(5), 1047-
430 1070. doi: 10.5194/se-8-1047-2017
- 431 Kikuchi, M., & Kanamori, H. (1982). Inversion of complex body waves. *Bulletin of*
432 *the Seismological Society of America*, *72*(2), 491-506.
- 433 Lanza, V., Spallarossa, D., Cattaneo, M., Bindi, D., & Augliera, P. (1999, June).
434 Source parameters of small events using constrained deconvolution with em-
435 pirical Green's functions. *Geophysical Journal International*, *137*(3), 651-662.
436 doi: 10.1046/j.1365-246x.1999.00809.x
- 437 Larmat, C., Montagner, J.-P., Fink, M., Capdeville, Y., Tourin, A., & Clévéde,
438 E. (2006). Time-reversal imaging of seismic sources and application to
439 the great Sumatra earthquake. *Geophysical Research Letters*, *33*(19). doi:
440 10.1029/2006GL026336
- 441 Lay, T., Kanamori, H., Ammon, C. J., Hutko, A. R., Furlong, K., & Rivera, L.
442 (2009, November). The 2006-2007 Kuril Islands great earthquake sequence:
443 THE 2006-2007 KURIL ISLANDS EARTHQUAKES. *Journal of Geophysical*
444 *Research: Solid Earth*, *114*(B11). doi: 10.1029/2008JB006280
- 445 Madariaga, R. (2015). Seismic Source Theory. In *Treatise on Geophysics* (p. 51-71).
446 Elsevier. doi: 10.1016/B978-0-444-53802-4.00070-1
- 447 McGuire, J. J. (2004, April). Estimating Finite Source Properties of Small Earth-
448 quake Ruptures. *Bulletin of the Seismological Society of America*, *94*(2), 377-
449 393. doi: 10.1785/0120030091
- 450 Meng, C., & Wang, H. (2018, April). A finite element and finite difference mixed
451 approach for modeling fault rupture and ground motion. *Computers & Geo-*
452 *sciences*, *113*, 54-69. doi: 10.1016/j.cageo.2018.01.015
- 453 Meng, L., Zhang, A., & Yagi, Y. (2016). Improving back projection imaging with
454 a novel physics-based aftershock calibration approach: A case study of the
455 2015 Gorkha earthquake. *Geophysical Research Letters*, *43*(2), 628-636. doi:
456 10.1002/2015GL067034
- 457 Olson, A. H., & Apsel, R. J. (1982, December). Finite faults and inverse theory with
458 applications to the 1979 Imperial Valley earthquake. *Bulletin of the Seismolog-*

- 459 *ical Society of America*, 72(6A), 1969-2001.
- 460 Plourde, A. P., & Bostock, M. G. (2017, June). Multichannel Deconvolution for
461 Earthquake Apparent Source Time Functions. *Bulletin of the Seismological So-*
462 *ciety of America*, ssabull;0120170015v1. doi: 10.1785/0120170015
- 463 Sibul, L., & Ziomek, L. (1981, March). Generalized wideband crossambigu-
464 ity function. In *ICASSP '81. IEEE International Conference on Acous-*
465 *tics, Speech, and Signal Processing* (Vol. 6, p. 1239-1242). doi: 10.1109/
466 ICASSP.1981.1171180
- 467 Somala, S. N., Ampuero, J.-P., & Lapusta, N. (2018, July). Finite-fault source in-
468 version using adjoint methods in 3-D heterogeneous media. *Geophysical Jour-*
469 *nal International*, 214(1), 402-420. doi: 10.1093/gji/ggy148
- 470 Sroubek, F., & Milanfar, P. (2012). Robust multichannel blind deconvolution via
471 fast alternating minimization. *IEEE Transactions on Image Processing*, 21(4),
472 1687-1700.
- 473 Stein, E. M., & Weiss, G. (2016). *Introduction to Fourier Analysis on Euclidean*
474 *Spaces (PMS-32)*. Princeton University Press.
- 475 Tocheport, A., Rivera, L., & Chevrot, S. (2007). A systematic study of source time
476 functions and moment tensors of intermediate and deep earthquakes. *Journal*
477 *of Geophysical Research: Solid Earth*, 112(B7).
- 478 Ulrych, T. J. (1971, August). Application of homomorphic deconvolution to seismol-
479 ogy. *GEOPHYSICS*, 36(4), 650-660. doi: 10.1190/1.1440202
- 480 USGS. (2019a, July). *Ridgecrest M 6.4; 11km SW of Searles Valley, CA.*
481 <https://earthquake.usgs.gov/earthquakes/eventpage/ci38443183/executive>.
- 482 USGS. (2019b, May). *Ridgecrest M 7.1; 11km SW of Searles Valley, CA.*
483 <https://earthquake.usgs.gov/earthquakes/eventpage/ci38457511/executive>.
- 484 Vallée, M., Charléty, J., Ferreira, A. M., Delouis, B., & Vergoz, J. (2011).
485 SCARDEC: A new technique for the rapid determination of seismic moment
486 magnitude, focal mechanism and source time functions for large earthquakes
487 using body-wave deconvolution. *Geophysical Journal International*, 184(1),
488 338-358.
- 489 Vallée, M., & Douet, V. (2016). A new database of source time functions (STFs) ex-
490 tracted from the SCARDEC method. *Physics of the Earth and Planetary Inte-*
491 *riors*, 257, 149-157.
- 492 Weiss, L. G. (1994, January). Wavelets and wideband correlation processing. *IEEE*
493 *Signal Processing Magazine*, 11(1), 13-32. doi: 10.1109/79.252866
- 494 Yin, J., & Denolle, M. A. (2019, May). Relating teleseismic backprojection images
495 to earthquake kinematics. *Geophysical Journal International*, 217(2), 729-747.
496 doi: 10.1093/gji/ggz048