

1 Deployment-invariant probability of detection characterization for aerial LiDAR methane
2 detection

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16 **Abstract**

17 Accurate detection sensitivity characterization of remote methane monitoring technologies is critical for
18 designing, implementing, and auditing effective emissions monitoring and mitigation programs. Several
19 research groups have developed test methods based on single/double-blind controlled release protocols
20 and regression-based data analysis techniques to create probability of detection (PoD) models for
21 characterizing remote sensor detection sensitivities. The previously created methods and models
22 account for some of the important factors that affect detection sensitivity, such as wind speed, and in
23 the case of Conrad et al. flight altitude. However, these models do not account for other important
24 factors, such as terrain albedo, variation in individual sensor performance, or spatial density of the
25 remote sensing measurements. In this paper, we build on the work of Conrad et al. by introducing a gas
26 concentration noise (GCN) model for Gas Mapping LiDAR aerial methane detection technology that,
27 when combined with wind speed at the emission location, accounts for all significant sensor and
28 environmental parameters that affect detection sensitivity for scenarios involving an isolated emission

29 source resulting in a single methane plume. We incorporate the GCN model into Conrad et al.'s PoD
30 model and apply it to several sets of controlled release data acquired across widely varying deployment
31 and environmental conditions to develop PoD models for Bridger Photonics Inc.'s first- and second-
32 generation (GML 2.0) Gas Mapping LiDAR sensors. Finally, we compare controlled release data acquired
33 by GML 2.0 in different geographic regions and terrain cover types, in different wind conditions,
34 deployed on different aircraft types, and with different flight parameters. Results show that the GML 2.0
35 PoD model remains valid regardless of the location or conditions under which the sensors are deployed,
36 and the aircraft and flight parameters used for deployment. Based on PoD measurements in 12
37 production basins across North America, the average 90% PoD emission rate for sites measured by GML
38 2.0 in 2023 was 1.27 kg/h.

39 Keywords: LiDAR, remote sensing, methane, emissions monitoring, detection sensitivity, probability of
40 detection, gas mapping

41 1 Introduction

42 Reducing methane emissions to the atmosphere, especially from oil and gas infrastructure, is understood
43 to be among the most cost-effective approaches available for mitigating the near-term effects of climate
44 change.¹ However, recent efforts to quantify methane emissions, while subject to high levels of
45 uncertainty, suggest that emissions remain stubbornly high.² Many studies have measured elevated
46 methane emissions for oil and gas production areas, using regional aircraft,^{3,4} tower,⁵ and satellite
47 methods,⁶ but these approaches cannot easily attribute emissions to individual operations and sources
48 as required to guide emissions mitigation efforts. The need for source-resolved data has spurred
49 extensive aerial measurement campaigns using technologies that can attribute emissions to individual
50 equipment groups or equipment units. Total emissions estimates from source-resolved measurement
51 efforts are subject to uncertainty due to the difficulty of monitoring the vast oil and gas infrastructure
52 with sufficient detection sensitivity, sample size, frequency, and quantification accuracy to reliably

53 inventory emissions. Recent efforts to evaluate the performance of existing monitoring technologies^{7,8}
54 and understand the distribution of emissions from oil and gas infrastructure^{9,10} indicate that existing
55 technologies can produce accurate emissions inventories (i.e. low-uncertainty) if monitoring campaigns
56 are designed with sufficient detection sensitivity, sample size, monitoring frequency, quantification
57 accuracy, and emission source resolution and attribution. Reliable, measurement-based emissions
58 inventories are expected to form the basis for measurement, monitoring, reporting, and verification
59 (MMRV) programs to benchmark emissions and track mitigations as companies and governments pursue
60 their stated emissions reduction targets.¹¹ Emissions inventories with lower uncertainties will allow
61 stakeholders to confidently resolve smaller changes in emissions behavior enabling operators to validate
62 the efficacy of mitigation efforts, such as leak detection and repair (LDAR) programs and improvements
63 to equipment designs, facility designs, and operational processes.

64 Characterizing the detection sensitivity of a monitoring technology is challenging because the probability
65 of detecting (PoD) an emission source can vary strongly with many environmental conditions and, in the
66 case of aerial technologies, with the flight parameters of the sensor deployment. Until recently, the
67 detection sensitivity of remote monitoring technologies was generally characterized by the minimum
68 detection limit (MDL), which defines the smallest emission rate observed by the monitoring technology.
69 Recent testing of airborne sensors has shown that environmental parameters have a strong effect on the
70 probability of detection for a given emissions source,^{7,8,12} such that the emission rate corresponding to,
71 for example, 90% PoD for a technology can be more than an order of magnitude greater than a cited
72 MDL that may be derived from different deployment parameters or favorable environmental
73 conditions.^{13,14} Moreover, even for fixed operational and environmental conditions, the probability of
74 detection has been shown to vary rapidly as a function of emission rate, such that a small change in
75 emission rate can lead to drastic changes in the PoD for a particular emission source. These discrepancies
76 between MDL and the true detection sensitivity of monitoring technologies have led to significant

77 confusion in recent attempts to characterize emission rate distributions and intermittency of emission
78 sources from oil and gas infrastructure.

79 More recent efforts to characterize detection sensitivity have used measurements of the wind speed
80 near the controlled release location and a linear predictor logistic inverse link (LP-LIL) model to compute
81 the PoD as a function of the wind speed normalized emission rate.^{7,12,15} This method represented a
82 significant improvement over MDL characterization by quantifying the dependence of emission rate PoD
83 on wind speed. However, it provides limited flexibility in modeling the functional form of the wind speed
84 dependence, assuming a linear relationship which results in unrealistically good detection sensitivity
85 predictions for small emissions as wind speed approaches zero. It also provides no mechanism for
86 modeling other parameters that affect PoD, such as flight altitude, flight speed, terrain albedo, and
87 others. Bell et al. used the LP-LIL method to characterize the dependence of flight altitude by conducting
88 controlled release (CR) tests at different flight altitudes and fitting separate LP-LIL models to the
89 corresponding data sets.⁷ This work showed the 90% PoD emission rate for a fixed wind speed increased
90 by 65% when the flight altitude was increased by just 35%. Since the LP-LIL analysis lacks the ability to
91 model flight altitude as a continuous variable it was not possible to determine the functional form of the
92 PoD dependence on flight altitude using this analysis. Moreover, the other parameters that affect PoD
93 were not considered, meaning the results are only valid for areas with similar conditions to the
94 controlled release test site at the time the study was performed when they are scanned with equivalent
95 sensor and flight parameters.

96 Conrad et al. improved upon the LP-LIL analysis by developing a method that allows generalized
97 relationships between the emission rate and variables that affect PoD using a maximum likelihood fit
98 method.⁸ This approach provides a statistically robust method to compare the fit performance of various
99 candidate models and to determine coefficient values that minimize information loss of the models
100 relative to a set of controlled release data points. They applied this method to GML controlled release

101 data and found strong dependence of PoD on flight altitude ' h ' with an optimized functional form of $h^{-2.44}$, very close to the theoretically expected functional form of $h^{-2.5}$, due to the combined effects of
102 $h^{-2.44}$, very close to the theoretically expected functional form of $h^{-2.5}$, due to the combined effects of
103 diminishing light collection and reduced measurement spatial density with increasing flight altitude.
104 In this paper we present a noise model for aerial LiDAR methane concentration measurements that
105 distills the myriad parameters that affect detection sensitivity into a single parameter called gas
106 concentration noise. We then develop a generalized PoD model with three input variables - emission
107 rate, wind speed, and gas concentration noise – that is demonstrated to remain valid across a wide range
108 of environmental conditions, deployment configurations, and sensor performance. This provides users of
109 GML data robust estimates of detection sensitivity for every flyover pass and potential emission source
110 location within the scanned infrastructure. These improvements to detection sensitivity characterization
111 are expected to benefit a wide variety of applications in methane emissions monitoring including 1)
112 auditable compliance with the detection sensitivity requirements set forth in new US and Canadian
113 methane emissions regulations; 2) the ability to reliably estimate the measured and unmeasured
114 portions of the methane emission distribution from individual monitoring campaigns; 3) development of
115 accurate measurement-based methane emissions inventories and emission factors; and 4) certification
116 of differentiated gas through dependable methane loss rate measurements.

117 2 Methods

118 2.1 Data acquisition

119 Data from controlled release testing campaigns conducted in four locations across North America were
120 analyzed to produce deployment invariant PoD models for GML sensor versions 1 and 2 (GML 1.0 and
121 GML 2.0). Table 1 lists the details for each controlled release test with a row for each unique single-blind
122 testing location including the GML sensor version, test date range, aircraft type, flight speed, flight
123 altitude, controlled release height, ground cover type, and ground cover reflectivity at the LiDAR

124 methane measurement wavelength of 1651 nm. Descriptions of the GML 1.0 sensor can be found
 125 elsewhere in the literature.^{8,16} Specifications for the GML 2.0 sensor are similar to GML 1.0 except that
 126 GML 2.0 acquires LiDAR path-integrated methane concentration measurements at a 33% faster rate for
 127 denser spatial distribution of LiDAR measurements and, as concluded in this study, achieves
 128 approximately 35% improved detection sensitivity performance relative to GML 1.0 under equivalent
 129 conditions primarily due to improvements in the LiDAR transceiver efficiency and the detection
 130 electronics' noise properties.

131 GML was deployed for each controlled release test using the typical contracted flight service provider
 132 companies and aircraft types that are used for commercial deployment in each test region and industry
 133 segment. For example, the campaigns in Midland, TX, Bozeman, MT, and Wonowon, BC were deployed
 134 on Cessna 172 fixed wing aircraft using typical flight parameters for production sector monitoring scans.
 135 The campaign in Los Angeles, CA was deployed on a Bell 206 helicopter using typical flight parameters
 136 for distribution sector monitoring scans (see Table 1). The organization that administered the controlled
 137 release tests in each region is listed in Table 1 using the following abbreviations: Colorado State
 138 University (CSU), Southern California Gas Company (SoCalGas, SCG), Bridger Photonics (BP), and Carleton
 139 University (CU).

140 **Table 1. Details of the controlled release experiments conducted for this work.**

GML version	Testing org	General Location	Aircraft type	Terrain type/reflectivity	Test type	Test Dates	Flyover # [total, valid, missed]	Release height (m)	Flight altitude (m)	Flight speed (mph)
1.0	CSU	Midland, TX	Cessna 172	sage desert/0.28	single-blind	October 4-8, 2021	420,411,158	3.3	141-223	90-120
2.0	SCG	Los Angeles, CA	Bell 206	pavement/0.19	single-blind	July 25, 2022 – July 26, 2022	93,82,43	0.5	142-176	60-90
2.0	SCG	Los Angeles, CA	Bell 206	road mix/0.2	single-blind	August 5, 2022 – December 16, 2022	282,235,142	0.5	140-191	60-90
2.0	BP	Bozeman, MT	Cessna 172	prairie grass/(0.22-0.41) snow/(0.08-0.12)	single-blind	September 7, 2022 – August 31, 2023	284,242,45	1.85	130-226	90-120
2.0	BP	Bozeman, MT	Cessna 172	prairie grass/(0.27-0.45) snow/0.14	single-blind	February 10, 2023 – May 19, 2023	217,197,47	1.85	142-239	90-120
2.0	BP	Bozeman, MT	Cessna 172	prairie grass/0.38	single-blind	August 16, 2023	18,18,4	1.85	86-229	90-120
2.0	BP	Bozeman, MT	Cessna 172	prairie grass/(0.26-0.29)	single-blind	August 16, 2023 – November 15, 2023	234,233,56	1.85	125-236	90-120
2.0	CU	Wonowon, BC	Cessna 172	operational production pads - road mix and grass/(0.17-0.39)	single-blind and double-blind	September 11, 2023 – September 14, 2023	529,515,71	1.15-1.19	205-279	90-120

141

142 For all single-blind testing campaigns the release point and anemometer were positioned away from
143 topography, structures, and foliage that might disturb local wind flow to ensure the gas flow speed at the
144 controlled release location was well characterized by the anemometer measurements. The controlled
145 release point was positioned at a height above ground level (AGL) corresponding to the typical height of
146 methane emissions from infrastructure being scanned in that region and industry segment. For all
147 experiments, the anemometer(s) were positioned at least 1.85 m above ground level to avoid
148 measurement resolution and accuracy limitations. A minimum time delay between changes to the mass
149 flow controller rate and the subsequent flyover passes was enforced to allow adequate time for the
150 methane plume to develop at each flow rate. Since the release locations were typically positioned near
151 the center of the GML scan swath, the plume length was required to be two-thirds of the swath width to
152 ensure the entire length of the plume within the scan swath corresponded to the new steady-state
153 release rate being issued by the mass flow controller. The time threshold was based on the wind speed
154 at the plume height (u) and the scan swath width, which depended on flight altitude with both GML
155 sensor versions having a 32° field of view (FOV) resulting in a flight altitude (h) dependent scan swath
156 width $s = 2\tan(FOV/2)h$. An example computation of the minimum plume development time is as
157 follows: given a flight altitude of 700' (213 m) AGL and a wind speed of 1 m/s the minimum delay for
158 adequate plume development is calculated by $\Delta t_{min} = 2s/3u = 82$ s. A controlled release flyover
159 experiment with these parameters will only be considered valid if the time between the emission rate
160 change and the flyover pass is greater than or equal to 82 s. For the flyover measurements presented
161 here, the minimum plume development time varied from 7.3 s to 367 s with an average value of 53 s.
162 The equipment and protocols used for the CSU controlled release test are described in Ref. [7]. For the
163 tests administered by SoCalGas in Los Angeles, CA methane from compressed cylinders (distribution
164 system gas, 93.7% methane fraction) was released through Alicat MCP-50SLPM-D mass flow controllers
165 with rated accuracy of $\pm 0.8\%$ of reading and $\pm 0.2\%$ of full scale at a height of 0.5 m AGL. A Young model

166 81000 ultrasonic anemometer (resolution 0.01 m/s and 0.1°, accuracy $\pm 1\% \pm 0.05$ m/s and $\pm 2^\circ$) was
167 positioned within 20 feet of the controlled release points at a height of 3 m AGL to record the local wind
168 speed and direction. For the tests conducted in Bozeman, MT, administered by Bridger, methane from
169 compressed cylinders (General Distributing, 93% methane fraction) was released through Alicat MCR-
170 250SLPM-D/5M mass flow controllers with rated accuracy of $\pm 0.8\%$ of reading and $\pm 0.2\%$ of full scale at
171 a height of 1.85 m above ground level. Two Young model 91500 ultrasonic anemometers (resolution
172 0.01 m/s and 0.1°, accuracy $\pm 2\% \pm 0.3$ m/s (0-30 m/s) and $\pm 2^\circ$) were positioned within 20 feet of the
173 controlled release points at a height of 1.85 m above ground level to record the local wind speed and
174 direction.

175 Controlled methane releases tests conducted in Wonowon, BC were administered by CU Energy and
176 Emissions Research Lab from within active and inactive oil and gas facilities using similar equipment and
177 protocols to those described in Ref. [8]. Experiments were completed over four-days during September
178 11-14, 2023, and included releases from 20 different oil and gas production sites in Northern British
179 Columbia. The sites ranged in complexity from single wellheads to a large gas processing plant, providing
180 a diversity of infield measurement conditions with varying ground cover and reflectivity, on-site
181 infrastructure, wind conditions, and the potential for interaction with additional on-site sources. Release
182 test sites were selected and grouped into five sets of four locations balancing elevation changes and site
183 spacing to maximizing the number of measurement laps per test circuit while allowing sufficient time
184 between laps (approximately 3 to 4 minutes) for the ground team to change the release rates and/or
185 release locations. In total, seven test circuits were flown with the number of flight passes ranging from
186 11 to 19 in each circuit. At each site, methane from compressed gas cylinders (>99% purity) was released
187 vertically at 1 or 2 distinct locations simultaneously from small (rates of ~ 0.1 -2.9 kg/h) and large (rates of
188 ~ 1.6 -64 kg/h) calibrated Bronkhorst thermal mass flow controllers at heights between 1.15 m and
189 1.19 m AGL, respectively. A custom heated regulator and liquid-gas heat exchanger system was used to

190 manage Joule-Thomson cooling effects. This ensured the gas temperature prior to entering the mass
191 flow controllers was suitable to maintain each controller's rated accuracy. After each mass flow
192 controller, methane passed through approximately 20 m of flexible tubing allowing the release point to
193 be moved between laps. Finally, local wind speed at 3 m AGL was measured using an ultrasonic
194 anemometer (Anemoment TriSonica mini) with a rated accuracy of ± 0.2 m/s over a range of 0-
195 10 m/s. Release rate data, as well as local wind speed data were recorded using GPS-synchronized data
196 loggers to ensure data could be subsequently time-matched with aerial detection data during post
197 processing. Controlled releases with measured 3 m wind speeds less than 0.5 m/s were discarded from
198 the analysis to avoid high relative anemometer error, which filtered 14 flyover passes from the analyzed
199 data set.

200 Wind speed at the plume location was estimated by mapping the observed wind speed u , measured at
201 the anemometer location and height above ground level, to the plume height above ground level using a
202 logarithmic wind profile equation:

$$203 \quad u(z_2) = u(z_1) \frac{\ln((z_2-d)/z_0)}{\ln((z_1-d)/z_0)},$$

204 where $u(z_1)$ is the wind speed measured by the anemometer, z_2 is the measured plume height, $d = 0.066$
205 m, and $z_0 = 0.01$ m. The emitter height is used as a proxy for the plume height to map wind speed from
206 measured height to plume height in cases where the release height and anemometer height differ.

207 During typical GML scans the plume height for detected releases is estimated using a technique that
208 compares the LiDAR gas concentration measurements of the plume from multiple view angles described
209 in Ref [17].

210 Prior to each controlled release test, the group administering the test created a matrix of controlled
211 release rates to be issued during the test. The release rates were withheld from Bridger's data processing
212 team until after the detection analysis for the test was completed and Bridger had issued reports to the

213 administering organization. In the case of the CU controlled release tests, Bridger was not aware that
214 two releases were happening simultaneously nor that the locations of small-scale releases were
215 changing and therefore effectively double-blind. The matrix of controlled release rates for the CSU, SCG,
216 and BP tests were designed to cover the probability of detection interval from <5% to >99% PoD. Bridger
217 supplied each testing group with the best available PoD characterization prior to the test to define the
218 emission rate intervals based on the flight parameters used for each test and the environmental
219 conditions observed during the test. For all tests, Bridger deployed GML flyovers of the test site before
220 the test to determine the terrain reflectivity near the release location. Terrain reflectivity was estimated
221 from GML measurements using the LiDAR equations presented in Supplemental Material S1. For the
222 tests administered by CSU, SCG, and BP, release rate intervals were defined for wind speed intervals 0-
223 5 mph, 5-10 mph, and 10-20 mph to account for the wind speed dependence on the detection
224 probability. Additional sets of emission rates for the three wind speed intervals were defined for
225 different flight altitudes for the CSU and BP tests. The SCG tests were limited to a single altitude. For the
226 Wonowon, BC test, administered by CU, flights were nominally conducted at a single flight altitude,
227 although hilly topography led to significant variation in the actual altitude of individual flyover passes.
228 Release rates for the CU test were selected to ensure good sample coverage near the nominal detection
229 limit of the instrument. This was accomplished by inverting Bridger's preliminary PoD model for GML 2.0
230 and evaluating the emission rate corresponding to a random PoD between 0.5% and 99.5% given local,
231 anemometer-measured, average wind speed during the preceding minute and a random GCN between
232 18 and 28 ppm-m (based on preliminary LiDAR measurements). Controlled releases spanned 0.13–
233 64.3 kg/h, with 19 additional zero releases performed to evaluate the frequency of false positives of
234 which there were none.

235 2.2 Data processing

236 Data acquired during GML controlled release tests were processed using Bridger’s standard work
237 practice. First, the raw sensor data is processed using an automated routine to determine the locations
238 and extents of regions of elevated methane concentration (i.e., methane plumes, also referred to as
239 detections) relative to ambient methane concentration levels, using the statistical algorithm described in
240 Ref. [18]. Automated detection processing used the same algorithm and calibration parameters as
241 commercial GML scans, thereby eliminating the possibility of human bias or subjectivity in the
242 identification of detection events. After the automated processing step, Bridger’s data processors use a
243 custom software application to assign emission source locations to the methane detection plume
244 images. First, the underlying topographic LiDAR point cloud data, colored by the signal-to-noise ratio of
245 the path-integrated gas concentration for each LiDAR measurement point is analyzed to determine if a
246 source location can be assigned. The threshold for assigning an emitter to a detected methane plume is
247 three or more adjacent LiDAR points with path-integrated gas concentration signal-to-noise ratio
248 exceeding a fixed threshold that is identical to the threshold used for commercial operations. Assigned
249 emission locations within 5 m of the controlled release location are marked as *detections* and flyover
250 passes without corresponding emission locations are marked as *misses*. Next, the plume height is
251 determined for detections as described previously. The detections and misses along with flyover
252 timestamp, source location, and wind speed at the measured plume height are compiled into a report
253 that is issued to the testing organization and used for subsequent PoD analysis.

254 2.3 Gas concentration noise model

255 Gas Mapping LiDAR uses wavelength modulation spectroscopy to perform path-integrated methane
256 concentration measurements along the path traversed by the transmitted LiDAR beam between the
257 airborne sensor and the remote topographic target that backscatters the LiDAR beam. Lock-in detection

258 is performed on the received LiDAR signal at harmonics of the modulation frequency, similar to Ref [19].

259 A mathematical description of the LiDAR equations for the first three harmonic components of the
260 received signal is presented in Supplementary Material S1. The path-integrated methane concentration
261 (C_{PI}) for each LiDAR measurement is calculated by:

$$262 \quad C_{PI} = \frac{mP_{2f}}{2P_{1f}} \gamma \left(T, p, \frac{mP_{2f}}{P_{1f}}, \eta, \xi \right),$$

263 where:

264

265 P_{1f} = first harmonic of the received optical power

266 P_{2f} = second harmonic of the received optical power

267 m = the intensity modulation depth that relates P_{1f} to the total average received laser power

268 γ = coefficient that relates the ratio $\frac{mP_{2f}}{2P_{1f}}$ to the path-integrated concentration

269 η = laser modulation parameters of the transmitted LiDAR beam

270 ξ = methane spectroscopic parameters

271 T = atmospheric temperature

272 p = atmospheric pressure

273

274 A gas concentration noise (GCN) value can be computed for each LiDAR measurement using a calibrated
275 noise model. The GCN model includes contributions from incoherent noise sources (n_{in}) and coherent
276 noise sources (n_{cn}) and has the following functional form:

$$277 \quad GCN = \sqrt{n_{in}^2 + n_{cn}^2},$$

278 where n_{in} includes shot noise, Johnson noise, and relative intensity noise and n_{cn} includes speckle
279 noise^{20,21}. The equation used to calculate the incoherent noise contribution to each LiDAR measurement
280 is:

281
$$n_{in} = \frac{m\gamma NEP}{2P_{1f}\sqrt{\Delta t}},$$

282 where:

283 NEP = noise equivalent power without considering coherent noise, and

284 Δt = LiDAR gas concentration measurement duration.

285 A mathematical description of NEP is presented in Supplementary Material S2. The detection system NEP
286 and the LiDAR transmitter/receiver speckle noise (n_{cn}) are calibrated during sensor manufacturing and
287 validated through flight testing. Flight test validation is performed by computing the observed gas
288 concentration noise using statistical analysis of sets of measured LiDAR gas concentration values and
289 comparing them against the GCN values produced for each individual LiDAR gas concentration
290 measurement by the model. A description of the calibration process, with example flight test data is
291 presented in Supplementary Material S3. To account for detection sensitivity effects related to the spatial
292 distribution and density of individual LiDAR measurements, and to create gas plume imagery, the geo-
293 registered gas concentration and GCN values are averaged onto a raster grid with 2 m pixel size using the
294 weighted average described in Supplementary Material S4. Due to the near conical scan pattern of GML
295 LiDAR measurements, there is significant variation in spatial density of gas concentration measurements
296 as a function of position across the GML scan swath, with the center of the scan swath having the lowest
297 spatial density, and the highest density near the edges. Since the majority of flyover passes presented in
298 this work targeted the controlled release locations near the center of the scan swath, the associated GCN
299 values represent the worst possible detection sensitivity when compared to otherwise equivalent test
300 conditions with emission sources positioned randomly within the GML scan swath.

301 2.4 Probability of detection model

302 The emission rate probability of detection model is adapted from Ref [8] to use raster pixel gas
303 concentration noise with local wind speed and the emission rate issued during the controlled release

304 test. Similar to Ref [8], PoD is modeled using the Bernoulli distribution with a ‘predictor’ function $g(x;\beta)$
305 with coefficients β and variables $x = [u, n, Q]$, where u , n , and Q represent wind speed, (raster pixel
306 GCN)/1000, and emission rate, respectively. The model relates the predictor function to PoD using an
307 ‘inverse link’ function $F(g(x;\beta);\alpha)$ with coefficients α , such that the probability of GML detecting an
308 emission x with rate Q at a location with wind speed u , and $n = (\text{raster pixel GCN})/1000$ is given by,

309
$$POD \stackrel{\text{def}}{=} F(g(x; \beta); \alpha).$$

310 Values of the inverse link coefficients α are held constant such that the mean and variance for the
311 corresponding inverse link function probability distributions are equal to one. Maximum likelihood
312 estimation (MLE) is used to optimize the coefficient values for candidate pairs of predictor and inverse
313 link functions by minimizing the negative logarithm of the likelihood function (NLLF) for the Bernoulli
314 distribution. The Akaike Information Criterion (AIC) is used to compare the MLE optimization outcomes
315 for various functional forms to determine the predictor and inverse link functions that best represent the
316 controlled release data while penalizing excessive model coefficients.²² To arrive at the best PoD model
317 we tested the predictor and inverse link functions described in Ref [8] and shown in Supplementary
318 Material S5 Table 2.

319 3 Results

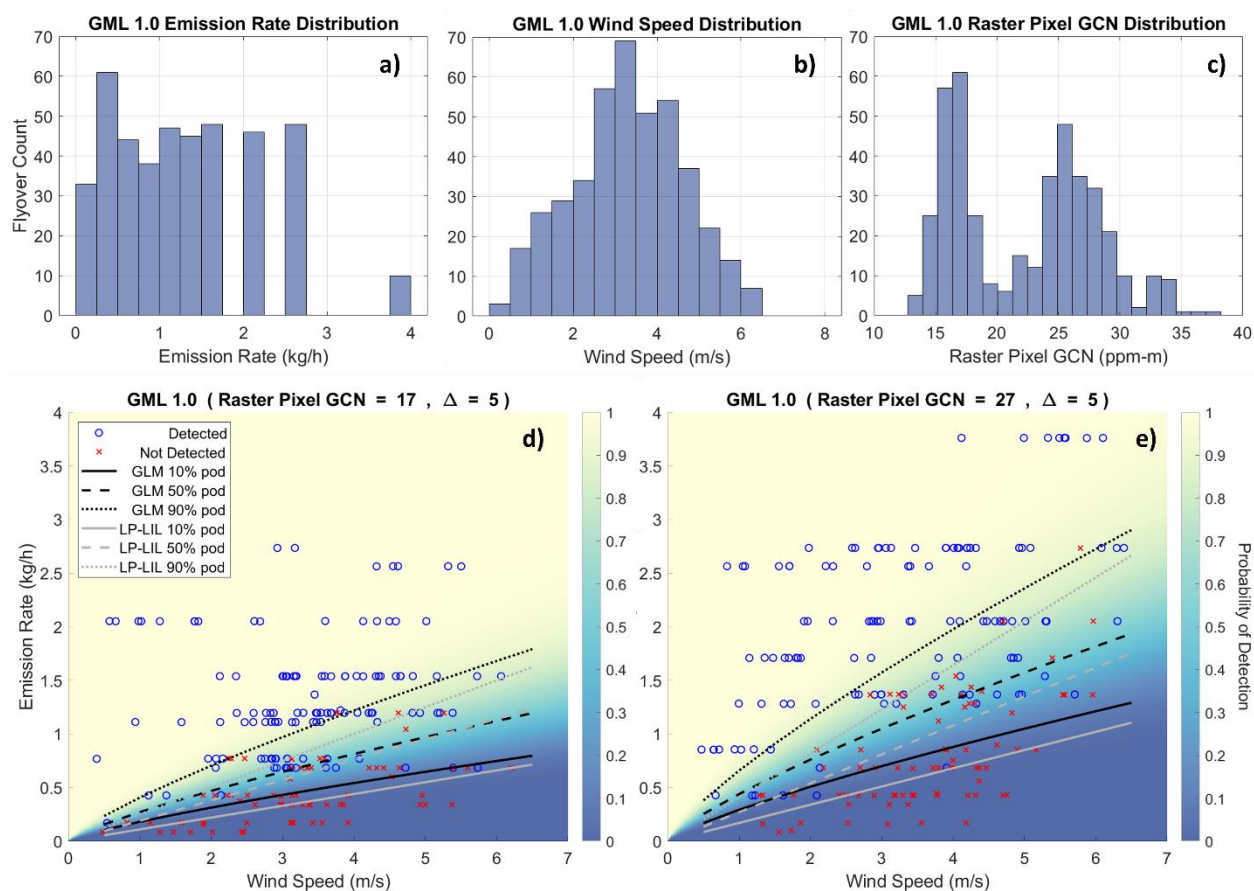
320 3.1 GML 1.0

321 The single-blind controlled release test administered by CSU in Midland, TX in early October 2021
322 represents the most comprehensive data set for characterizing GML 1.0 detection sensitivity. The test
323 consisted of 420 flyover passes with emission rates below 4 kg/h, 411 of which met the criteria for a
324 valid experiment (e.g., emission location within LiDAR swath coverage, mass flow controller and
325 anemometer equipment functioning properly, and adequate plume development time before flyover
326 pass). Of the 411 valid flyover passes, 253 of the releases were detected and 158 were missed, as shown

327 in Table 1. Histograms of the test conditions including emission rates, wind speeds, and raster pixel GCN
328 values for each flyover pass are shown in Figure 1 a-c. Wind speed conditions during the test varied
329 between 0.4 m/s and 6.4 m/s with an average speed of 3.36 m/s. Raster pixel GCN values ranged from
330 13.1 ppm-m to 46.5 ppm-m and had two distinct peaks at 17 ppm-m and 27 ppm-m corresponding to
331 the two flight altitudes targeted during the test, 500' (152 m) and 675' (206 m) AGL, respectively. The
332 detection data and optimized PoD model based on the MLE fits and AIC comparison are shown in Figure
333 1 d-e with blue circles indicating detections and red x's indicating misses. Figure 1 d) shows results for
334 the 500' (152 m) AGL flyover passes. Slices of the optimized PoD model for 10%, 50%, and 90% PoD at a
335 raster pixel GCN of 17 ppm-m are plotted in black with detection data for the raster pixel GCN interval
336 12-22 ppm-m. Similarly, Figure 1 e) shows results for the 675' (206 m) AGL flyover passes. Data is shown
337 for the raster pixel GCN interval 22-32 ppm-m and slices of the optimized PoD model for 10%, 50%, and
338 90% PoD at a raster pixel GCN of 27 ppm-m are plotted in black. For each candidate model, the relative
339 likelihood of minimizing information loss is shown in Supplementary Material S5 and the top two
340 performing models are highlighted in green and yellow, respectively. The optimized PoD model for this
341 controlled release data set consisted of the p4 predictor function and the Log Normal inverse link
342 function with coefficient values listed in Supplementary Material S5, Table 3.

343 The optimized PoD model can be used to estimate the average detection sensitivity for GML 1.0 facility
344 flyover passes in the Permian basin. Bridger does not have a pre-computed record of raster pixel GCN for
345 GML 1.0 flights because that information was not integrated into the GML data platform until near the
346 end of GML 1.0 commercial deployments. However, if we assume the controlled release test location is
347 representative of terrain albedo in the greater Permian Basin, and that the average wind speed in
348 Midland, TX, at the average emitter height as measured by GML (4.7 m) is 3.3 m/s, the optimized PoD
349 model indicates the basin average 90% PoD emission rates for GML 1.0 at 500' (152 m) and 675' (206 m)
350 AGL flight altitudes are 1.03 kg/h and 1.60 kg/h, respectively.

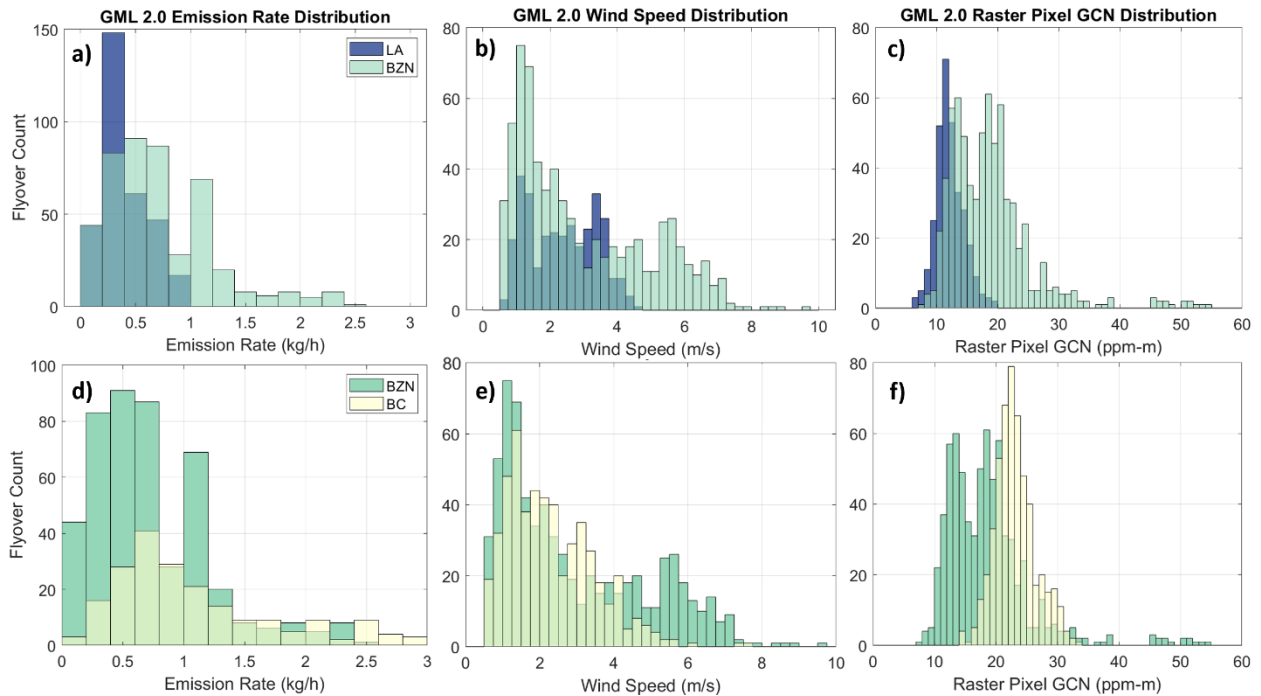
351 The lines for 10%, 50%, and 90% PoD based on the LP-LIL model developed in Ref [7] are plotted in gray
 352 in Figure 1 d-e for comparison. A visual inspection of the detection data with the two PoD models reveals
 353 noticeable discrepancies between the LP-LIL PoD predictions and the flyover data in several portions of
 354 the wind speed and raster pixel GCN parameter space. Compared to the generalized PoD model
 355 presented here, the LP-LIL model, which is based on wind-speed-normalized emission rates, tends to
 356 overestimate the detection sensitivity of the GML measurements, especially at low wind speeds.
 357 Specifically, the 90% PoD detection sensitivity estimated by the LP-LIL model is 50% lower than the
 358 generalized model at 1 m/s and 30% lower at 3 m/s wind speed, respectively.



359
 360 **Figure 1. GML 1.0 controlled release data and PoD model results. The top row shows histograms for a) emissions rates, b)**
 361 **wind speeds, and c) raster pixel GCNs for the detection sensitivity portion of the controlled release test. Detection results**
 362 **corresponding to 500' AGL flyover passes d) and 675' AGL flyover passes e) are shown with blue circles indicating detections**
 363 **and red x's indicating misses. The presented detection data is filtered by raster pixel GCN intervals 12-22 ppm-m d) and 22-32**
 364 **ppm-m e) and slices of the optimized PoD model for 10%, 50%, and 90% PoD are plotted in black for a raster pixel GCN of 17**
 365 **ppm-m d) and 27 ppm-m e). PoD lines for the 500' and 675' AGL flyover passes for 10%, 50%, and 90% PoD based on the LP-**
 366 **LIL model developed in Ref. 7 are plotted in gray for comparison.**

367 3.1.1 GML 2.0

368 Characterization of GML 2.0 PoD is based on three separate single-blind controlled release test
369 campaigns. The first was administered by SoCalGas at two locations in the Los Angeles basin (LA) starting
370 on July 25, 2022, and ending on December 16, 2022. The second test campaign was administered by
371 Bridger Photonics' GML fleet management team at four locations near Bozeman, MT (BZN) and took
372 place between September 7, 2022, and November 15, 2023. The third test was administered by Carleton
373 University's (CU) Energy and Emissions Research Laboratory on 20 oil and gas production facilities near
374 Wonowon, BC and took place between September 11, 2023, and September 14, 2023. The LA test
375 consisted of 375 total flyover passes, 317 of which met the criteria for a valid experiment. Of the 317
376 valid flyover passes 132 of the releases were detected and 185 were missed. The BZN test consisted of
377 753 total flyover passes, of which 690 were valid, 538 were detected, and 152 were missed. The CU test
378 consisted of 529 total flyover passes, of which 515 were valid, 444 were detected, and 71 were missed.
379 Histograms of the test conditions including emission rates, wind speeds, and raster pixel GCN values are
380 shown in Figure 3 a-d with the blue bars representing the LA test, mint representing BZN, and yellow
381 representing BC. Wind speed conditions during the LA test varied between 0.5 m/s and 4.5 m/s with an
382 average speed of 2.4 m/s. Raster pixel GCN values ranged from 6.0 ppm-m to 19.8 ppm-m with a single
383 peak near the distribution mean of 12.1 ppm-m, corresponding to an average flyover altitude of 518'
384 (158 m). Wind speed for the BZN test varied between 0.5 m/s and 9.7 m/s with an average speed of 2.9
385 m/s. Raster pixel GCN values ranged from 7.6 ppm-m to 54.5 ppm-m with two distinct peaks at 13 ppm-
386 m and 19 ppm-m corresponding to the two flight altitudes targeted during the test, 500' (152 m) and
387 700' (213 m), respectively. Wind speed for the BC test with varied between 0.6 m/s and 7.6 m/s with an
388 average speed of 2.3 m/s. Raster pixel GCN values ranged from 14.2 ppm-m to 33.4 ppm-m with a single
389 peak near the distribution mean of 23.1 ppm-m, corresponding to an average flyover altitude of 722'
390 (220 m).

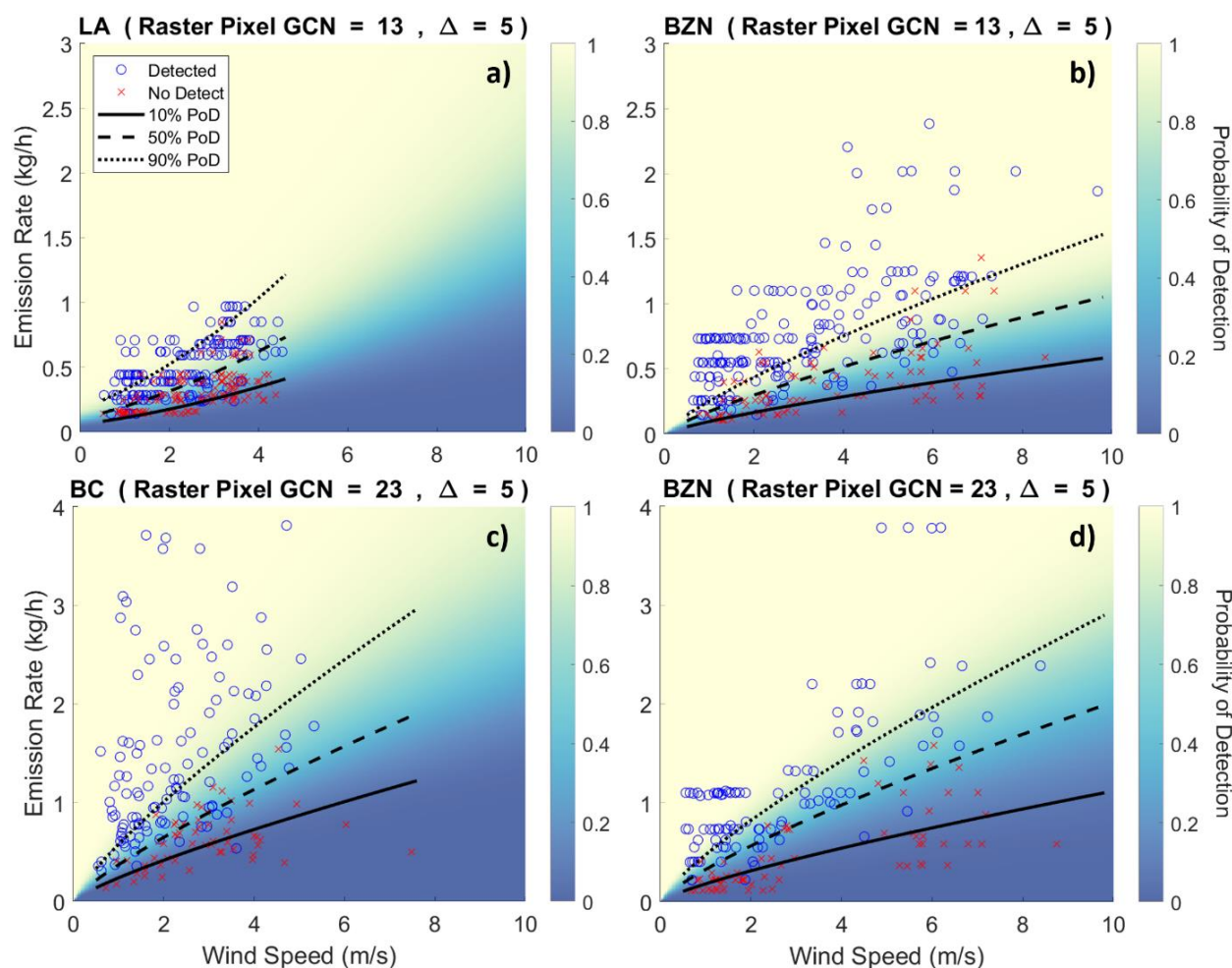


391

392 **Figure 2. GML 2.0 PoD model results for controlled release test performed in Los Angeles, CA, Bozeman, MT, and Wonowon,**
 393 **BC. Histograms show the emissions rates a) & d), wind speeds b) & e), and raster pixel GCNs c) & f) issued for each flyover**
 394 **pass for the detection sensitivity portion of the test.**

395 Detection data and optimized PoD models for each controlled release test region are shown in Figure 4
 396 a)-d). Results for individual flyover tests are plotted with blue circles indicating detections and red x's
 397 indicating misses. All figures show black lines representing slices of the region-specific optimized PoD
 398 models for 10%, 50%, and 90% PoD. Figure 3 a) & b) show the LA and BZN data for the raster pixel GCN
 399 interval of 8-18 ppm-m and model slices computed at raster pixel GCN = 13 ppm-m, corresponding to
 400 the value of maximum raster pixel GCN overlap between the LA and BZN data sets. Figure 3 c) & d) show
 401 the BC and BZN data for the raster pixel GCN interval of 18-28 ppm-m and model slices computed at
 402 raster pixel GCN = 23 ppm-m, corresponding to the value of maximum raster pixel GCN overlap between
 403 the BC and BZN data sets. For the BC data set, only flyover passes corresponding to emission rates below
 404 3.5 kg/h are plotted in Figure 3 c) to allow clear visualization of the optimized PoD curves. The complete
 405 data set is shown in Supplementary Material S6, Figure 9.

406 The performance of the candidate models for each data set and the functional form and coefficient
 407 values for the optimal models are shown in Supplementary Material S5, Table 3. The optimized PoD
 408 model for the LA data set consists of the p2 predictor function and the Burr inverse link function. The
 409 second-best performing predictor model was p3 with the Burr inverse link function, which had a 0.9512
 410 probability of minimizing information loss relative to the optimal model. The small difference in
 411 performance between the p2 and p3 indicates weak preference for the p2 model, which exhibits a
 412 positive second derivative of the emission rate versus wind speed for surfaces of constant PoD. This is

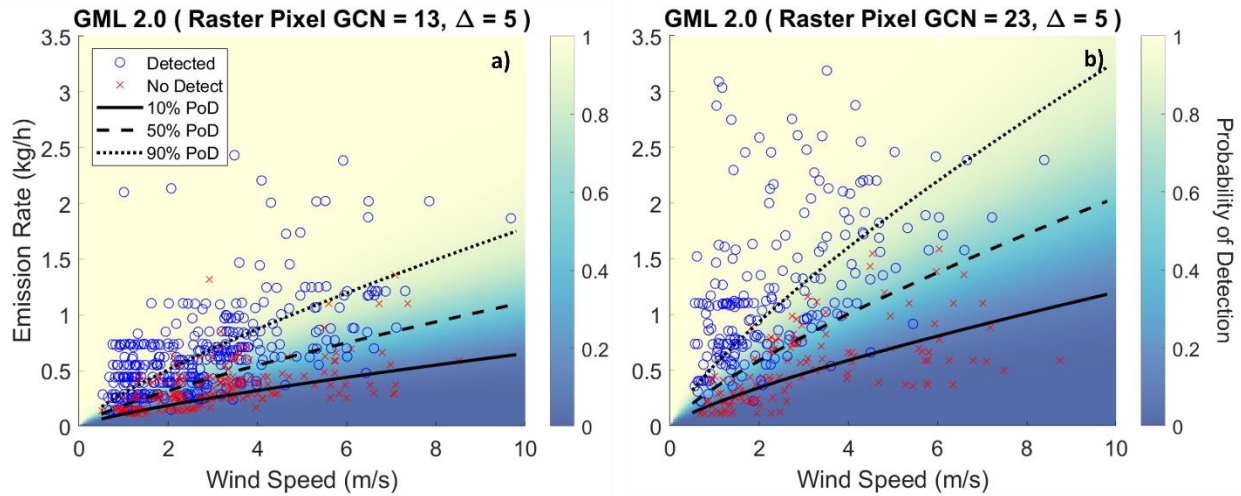


413
 414 **Figure 3. Detection results for GML 2.0 controlled release tests in the three regions – a) LA , b) & d) BZN, and c) BC. Flyover**
 415 **passes with blue circles indicate detections and red x's indicate misses. a) & b) show data for raster pixel GCN interval 8-18**
 416 **ppm-m and c) & d) show data for raster pixel GCN interval 18-28 ppm-m. All figures show the optimized PoD model for 10%,**
 417 **50%, and 90% PoD plotted in black at a raster pixel GCN of 13 ppm-m [a) & b)] and 23 ppm-m [c) & d)].**

418 likely due to the limited range of wind speed conditions sampled in the LA controlled release tests. The
419 optimized PoD model for the BZN data set consists of the p4 predictor function and the Weibull inverse
420 link function. The GML 2.0 BZN data set has a significant preference for the p4 model, likely due to the
421 larger range of wind speed and raster pixel GCN conditions sampled in the BZN controlled release tests.
422 The optimized PoD model for the BC data set also consists of the p4 predictor function and the Log
423 Normal inverse link function.

424 The optimized PoD model for the combination of GML 2.0 controlled release data sets acquired in LA,
425 BZN, and BC are shown in Figure 4 for the low a) and high b) modes of the raster pixel GCN range. The
426 optimized PoD model for the combined data set consists of the p4 predictor function and the Burr
427 inverse link function. Preference for the p4 predictor function in the combined data set is likely due to
428 the larger range of wind speed and raster pixel GCN conditions sampled in the BZN and BC controlled
429 release tests that suppresses the super-linear trend of the p2 model at high wind speeds seen in the LA
430 model.

431 Supplementary Material S5, Figure 8 a) and b) show the combined model plotted with the optimized PoD
432 models for the LA, BZN, and BC data sets. Figure 8 c) and d) show the fractional difference between the
433 combined GML 2.0 PoD model and the individual LA, BZN, and BC PoD models. Agreement within 30%
434 fractional difference is observed between the individual and combined PoD models for both raster pixel
435 GCN slices, all PoD thresholds, and for most of the wind speed range. The slightly better detection
436 sensitivity performance for the BZN tests may be attributed to a combination of hardware improvements
437 that reduced the noise for GML 2.0 sensors constructed after sensors were deployed for the LA and BC
438 measurement campaigns and a transition to slightly more conservative selection of n_{in} and n_{cn} values
439 from flight test data during calibration.

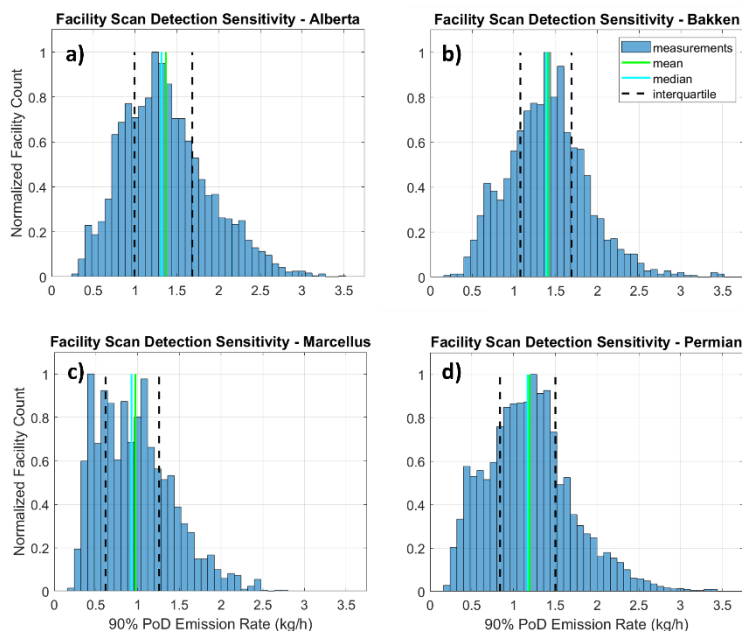


440

441 **Figure 4. GML 2.0 PoD model from combination of LA, BZN, and BC data sets. a) Detection data and model results for the low**
 442 **end of the raster pixel GCN spectrum with data interval 8-18 ppm-m and model raster pixel GCN = 13 ppm-m. Detection data**
 443 **and model results for the high end of the raster pixel GCN spectrum with data interval 18-28 ppm-m and model raster pixel**
 444 **GCN = 23 ppm-m.**

445 For GML 2.0 sensors, the raster pixel GCN values are computed as a standard output during automated
 446 data processing such that the optimized PoD models can be used to estimate the detection sensitivity for
 447 any GML scan data. For example, facility-level detection sensitivity estimates can be determined by
 448 inputting the average raster pixel GCN across the facility and the wind speed at the average emission
 449 source height above ground level into the model. Figure 5 shows facility-level detection sensitivity
 450 estimates for 2023 GML 2.0 scans in several regions across North America using the combined PoD
 451 model. Figure 5 a)-d) show histograms of 90% PoD facility-level detection sensitivity distributions for site
 452 scans in four oil and gas production regions. Figure 5 (right) shows a table containing the mean, median,
 453 and interquartile range for sites in 12 production regions and the Southern California Gas Company
 454 distribution infrastructure based on the 90% PoD detection sensitivity distributions. The best production
 455 sector facility-level detection sensitivity performance was achieved in the San Joaquin Valley (0.35 kg/h
 456 @ 90% PoD) where the GML sensor was deployed on a helicopter flying at 500' AGL and 70 mph, and the
 457 average wind speed at the average emission source height (2.3 m) was 1.7 m/s. The poorest upstream
 458 facility-level detection sensitivity performance was in Saskatchewan (1.804 kg/h @ 90% PoD) where GML

459 was deployed on a fixed wing aircraft flying at 750' AGL and 100 mph and the average wind speed at the
 460 average emission source height (5.6 m) was 3.8 m/s. Since raster pixel GCN data is available for all areas



Basin	Mean	Median	Interquartile
Alberta	1.370	1.315	[0.993,1.684]
Anadarko	1.713	1.588	[1.233,2.183]
Bakken	1.407	1.389	[1.078,1.692]
British Columbia	1.326	1.323	[0.986,1.636]
Denver Julesburg	0.982	0.819	[0.670,1.547]
Eagle Ford	1.586	1.553	[1.337,1.809]
Haynesville	1.540	1.602	[1.326,1.957]
Los Angeles	0.339 [†] , 0.524 [‡] , 0.669 ^{††}	N/A	N/A
Marcellus	0.974	0.927	[0.617,1.257]
Permian	1.191	1.168	[0.838,1.502]
San Joaquin Valley	0.353	0.353	[0.261,0.433]
Saskatchewan	1.804	1.772	[1.443,2.139]
Uinta	0.985	0.964	[0.856,1.106]

Emission source height above ground level: 0.2m - ground leaks[†],
 1.0m - meter leaks[‡], 5.0m - roof top leaks^{††}

461

462 **Figure 5. (Left) Histograms of the facility-level detection sensitivity for 2023 scans in select regions. Histograms are**
 463 **normalized to the bin with maximum counts and are plotted with the distribution mean, median, and interquartile. (Right)**
 464 **Detection sensitivity statistics (mean, median, and interquartile) for 2023 GML scans in basins/provinces across North**
 465 **America.**

466 covered by a GML scan, the detection sensitivity estimates can be computed for any region of interest.

467 For example, emission source height information, obtained from monitoring data and/or infrastructure

468 dimensional data, can be used in conjunction with raster pixel GCN and local wind speed data to

469 compute facility- and equipment-level PoD estimates.

470 4 Discussion

471 The generalized PoD model presented in this work provides a practical method to generate localized

472 detection sensitivity estimates for aerial LiDAR methane monitoring scans in scenarios involving

473 geospatially isolated methane emission sources in relatively simple topography, structures, and foliage,

474 typical of the production and transmission sectors of the natural gas supply chain. The model achieves

475 this by distilling many significant factors affecting detection sensitivity into three measurable input
476 variables, wind speed, gas concentration noise, and emission rate. The GCN consists of two physical
477 sensing parameters, calibrated through flight testing, that enable the computation of a concentration
478 noise value for each LiDAR path-integrated gas concentration measurement. Sets of scattered LiDAR
479 concentration measurements are averaged onto a raster grid to provide a common spatial scale for
480 comparing GCN values in different locations. Using a moderate number of controlled release
481 experiments, covering a broad range of measurement conditions, the detection sensitivity performance
482 of GML scans has been characterized in a several geographic regions and a wide variety of weather
483 conditions, terrain albedo, and deployment configurations. Good agreement is observed between the
484 derived PoD model and measurement results in these varied measurement conditions.

485 The optimized PoD model provides an auditable record of detection sensitivity performance with
486 applications for several tasks in remote methane emissions monitoring. Examples include documenting
487 and validating detection sensitivity performance for regulatory compliance scans, such as the EPA new
488 source performance standards (e.g under OOOOb), and leveraging the PoD for detections, facility scans,
489 and equipment scans to estimate the number and size of emissions that were present during GML scans
490 but were unmeasured due to the statistical nature of emissions detection. The latter application will be
491 critical for developing accurate methane emissions inventories and emissions intensities for tracking and
492 informing emissions mitigations efforts and strategies.⁹

493 Pathways exist to further improve detection sensitivity estimates using higher accuracy and specificity
494 wind speed and raster pixel GCN inputs that better match the actual conditions in the immediate vicinity
495 of a methane emission source. For example, the accuracy of raster pixel GCN estimates will improve as
496 the analysis area becomes more localized due to reduced likelihood of variations in terrain albedo,
497 received optical power for LiDAR measurements, and LiDAR spatial point density within the analysis
498 region. Accurate raster pixel GCN estimates will require maintaining a valid calibration of the GCN model

499 for each deployed sensor, which can be accomplished through routine analysis of the scan data using the
500 GCN model fit method described in Supplementary Material S3. While the facility-level detection
501 sensitivity estimates shown in Figure 5, leveraging remote wind model data, represent a significant
502 advancement in detection sensitivity characterization for aerial methane monitoring, the method will
503 also become more accurate with high temporal resolution local wind speed information, such as
504 anemometer measurements derived from local meteorological stations. Implementation of better wind
505 measurement networks will benefit all types of methane emissions monitoring technologies. Finally,
506 even more precise knowledge of the emission source height, which can be obtained from operator
507 feedback from leak response activities, will enable better mapping of the observed wind speed to the
508 wind speed at the emission source location.

509 Further testing will be required to characterize the detection sensitivity performance for scans of
510 complex upstream oil and gas production sites, due to effects such as 1) spatially overlapping plumes
511 from multiple, closely-spaced emission sources, 2) occlusion of the LiDAR measurement beam from the
512 emission source location by facility equipment, 3) large and high-spatial-frequency variations in albedo
513 near emission source locations, and/or 4) complex local wind fields that may deviate from local
514 anemometer measurements. Additional opportunities to extend the scope of detection sensitivity
515 characterization include spatially extended or distributed emission sources and scenes with large
516 structures, complicated topography, and/or large and dense foliage. PoD characterization of complex
517 measurement scenarios will benefit from analysis of topographic LiDAR data to determine portions of
518 the measurement scene that are occluded for a given scan and to enable better wind speed estimates
519 through fluid dynamics modeling. Characterization will also be necessary to create reliable PoD estimates
520 for elevated temperature exhaust gas such as lit flares and compressor exhaust emissions. The current
521 PoD model assumes all methane plumes are at ambient temperature. However, since the optical
522 absorption of methane at 1651 nm decreases with increasing gas temperature, it is anticipated that

523 there will be a detection sensitivity dependence on gas temperature, which is not captured in the
524 formalism presented here. While GML routinely detects and quantifies hot emission sources, further
525 research and characterization of a method to account for high-temperature gas (e.g. exhaust plumes) is
526 warranted to ensure accurate PoD estimates for these emission sources.

527 Finally, the approach presented here provides a template for detection sensitivity analysis of remote
528 methane emissions monitoring data which can be generally applied to any emissions detection solution.
529 Specifically, the proposed method (a) characterizes the practical response of the solution's sensor
530 system, using internally accessible estimates of noise or other factors, (b) combines sensor performance
531 with key situational variables such as wind speed and emission rate, and (c) deploys a robust statistical
532 model for probability of detection. Development of similar models for multiple methods would improve
533 confidence in, and acceptance of, advanced emissions detection solutions for regulatory and voluntary
534 reporting programs.

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548 Author Contributions

549 **MT**: detection algorithm and gas concentration noise model development, extension of pod model to
550 include gas concentration noise, controlled release experiment design, software, data analysis,
551 visualization, writing – manuscript development, review, and editing, **AK**: detection algorithm and gas
552 concentration noise model development, software, **DA, CD**: data curation, data analysis, software,
553 visualization, writing – review, and editing **BC, DT, MJ**: pod model development, controlled release
554 experiment design, data analysis, writing – review and editing, **JB**: gas concentration noise model
555 calibration method development, writing – review and editing, **PR, WK, ACB, DZ**: writing – review and
556 editing, **JA, TP**: software, **DY, BK**: data processing, writing – review and editing, **EN, ER, OIE**: single-blind
557 controlled release experiment design and independent execution, data analysis, writing – review and
558 editing.

559 Conflict of Interest

560 Bridger Photonics, Inc. profits from sales of Gas Mapping LiDAR methane emissions monitoring services.

561 Supplementary Material

562 SI – Wavelength modulation spectroscopy LiDAR equations

563
564 The LiDAR equations describing the signal power in the first three harmonics of the signal received by
565 the sensor are calculated as follows:

$$566 \quad P_{DC} = \frac{A\chi\rho}{R^2} S_{DC} ,$$

$$567 \quad P_{1f} = \frac{A\chi\rho}{R^2} S_{DC} m ,$$

$$568 \quad P_{2f} = \frac{A\chi\rho}{R^2} S_{DC} \frac{2C_{PI}}{\gamma} .$$

569 Where:

570 S_{DC} = DC component of the LiDAR beam transmitted power,

571 P_{DC} = DC component of the LiDAR received power,

572 A = area of the receiver,

573 χ = receiver optics collection efficiency,

574 ρ = terrain reflectivity per steradian,

575 R = distance from sensor to terrain, and

576 C_{PI} = path-integrated concentration.

577
578 *S2 – LiDAR receiver noise equivalent power*

579
580 The photodetector NEP can be derived by computing the signal (i_s) and noise photocurrent (i_n) using the
581 photodetector responsivity, the LiDAR equations, and the equations for photodetection noise processes.

582 The second harmonic signal photocurrent (RMS) for the gas concentration measurement is given by:

583
$$i_s = \frac{1}{\sqrt{2}} s P_{DC} \frac{2C_{PI}}{\gamma},$$

584 where s is the photodetector responsivity. The photodetector noise for a measurement duration Δt is
585 given by:

586
$$i_n = \sqrt{\left[s^2 (RIN) P_{DC}^2 + 2e(sP_{DC} + sP_{amb} + i_{dark}) + \frac{4k_B T}{R_p} \right] \frac{1}{2\Delta t}} = \frac{sNEP}{\sqrt{2\Delta t}},$$

587 where:

588 s = photodetector responsivity,

589 RIN = relative intensity noise on the transmitted LiDAR beam,

590 e = electron charge,

591 P_{amb} = ambient light power illuminating photodetector,

592 i_{dark} = photodetector dark current,

593 k_B = Boltzmann constant,

594 T = photodetector temperature, and

595 R_p = photodetector parallel resistance.

596 The photodetector noise is defined as a unity signal-to-noise (SNR), such that:

597
$$\frac{i_s}{i_n} = 1 = \frac{2P_{DC}C_{PI}\sqrt{\Delta t}}{\gamma NEP}.$$

598 The path-integrated concentration detection limit due to photodetector noise is then given by:

599

$$C_{PI} = n_{in} = \frac{\gamma NEP}{2P_{DC}\sqrt{\Delta t}} = \frac{\gamma m NEP}{2P_{1f}\sqrt{\Delta t}}$$

600

601 S3 – Gas concentration noise model calibration

602

603 The gas concentration noise model parameters for a GML sensor are determined by fitting the model to

604 flight test data, as shown in Figure 6. Given a candidate set of GCN model parameters (detector NEP and

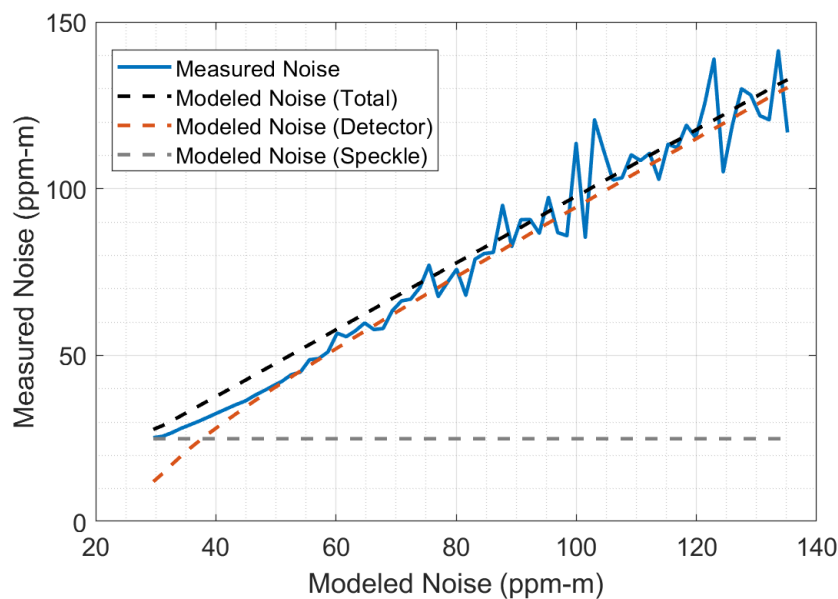
605 speckle noise), the GCN value is computed for each LiDAR measurement using equation 2. The LiDAR gas

606 concentration measurements are then binned by GCN and the standard deviation of gas concentration

607 measurements in each bin is computed to represent the measured gas concentration noise. A least

608 squares optimizer is applied to the modeled and measured GCN values to determine the set of noise

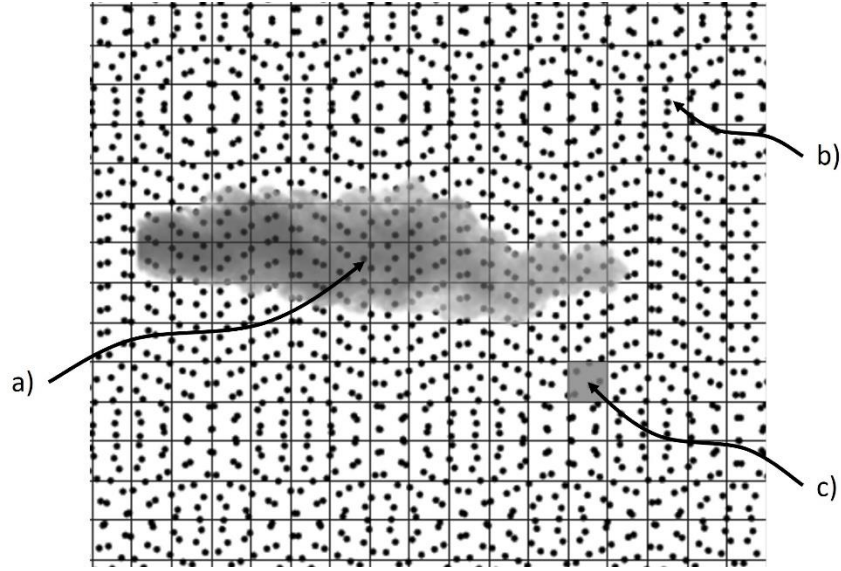
609 parameters (NEP and n_{ci}) that minimizes the squared error between the model and the measurement.



610

611 **Figure 6. Example fit of noise model to determine the gas concentration noise**
612 **model parameters for a GML 2.0 sensor from flight test data.**

613



616
 617 **Figure 7. Example of a gas plume a) scanned with GML LiDAR measurements b),**
 618 **which are interpolated onto a regular grid of pixels c).**

619 An example of the spatial distribution of LiDAR points and raster grid are shown in Figure 7. The
 620 raster image path-integrated gas concentration pixel values are computed by a weighted
 621 average of the individual LiDAR path-integrated gas concentration values within each pixel,
 622 where the weighting factors for the individual path-integrated gas concentration LiDAR
 623 measurements are derived from the GCN values. Specifically, the pixel path-integrated gas
 624 concentration and GCN values are computed using the following equations:

625

$$626 \quad c_j = \frac{\sum w_i c_i}{\sum w_i},$$

627

$$628 \quad w_i = \frac{1}{GCN_i^2},$$

629

$$630 \quad n_j = \frac{1}{\sqrt{\sum w_i}}.$$

631 where:
 632 j = the j^{th} pixel in the raster grid,
 633 i = the i^{th} LiDAR point in the j^{th} raster pixel,
 634

- 635 c_i = the P-Concentration for the i^{th} LiDAR point in the j^{th} raster pixel,
636 c_j = the weighted average P-Concentration for the j^{th} raster pixel,
637 w_i = the weighting factor for the i^{th} LiDAR point in the j^{th} raster pixel, and
638 n_j = the weighted average GCN for the j^{th} raster pixel.
639

640 S5 – Optimized probability of detection models

641
642 Candidate predictor and inverse link functions are shown in Table 2. The pair of predictor and
643 inverse link functions that best represent the controlled release detection data is determined
644 during the PoD model optimization process. Optimized PoD models for both GML sensor
645 versions and each controlled release test region are shown in
646 Table 3. The combined GML 2.0 PoD model represents the combination of GML 2.0 controlled
647 release test data sets from all controlled release test regions. The functional form for each PoD
648 Model as a function of emission rate Q , wind speed u , and $n = (\text{raster pixel GCN})/1000$ is
649 presented. The inverse PoD

650 **Table 2. List of candidate predictor functions and inverse link functions tested for the statistical**
651 **analysis.**

Predictor Functions	Inverse Link Functions
P1: $g(x; \beta) = \frac{\beta_1(Q + \beta_5)^{\beta_2}}{n^{\beta_3}u^{\beta_4}}$	Log Normal: $F(g; \alpha) = \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{\ln(g) - \alpha_1}{\sqrt{2}\alpha_2} \right) \right]$
P2: $g(x; \beta) = \frac{\beta_1 Q^{\beta_2}}{(n)^{\beta_3}(u + \beta_5)^{\beta_4}}$	Log Logistic: $F(g; \alpha) = \frac{1}{1 + (g/\alpha_1)^{-\alpha_2}}$
P3: $g(x; \beta) = \frac{\beta_1 Q^{\beta_2}}{(n + \beta_5)^{\beta_3}(u)^{\beta_4}}$	Fréchet: $F(g; \alpha) = e^{-(g/\alpha_1)^{-\alpha_2}}$
P4: $g(x; \beta) = \frac{\beta_1 Q^{\beta_2}}{n^{\beta_3}u^{\beta_4}}$	Burr: $F(g; \alpha) = 1 - (1 + g^{\alpha_1})^{-\alpha_2}$
	Weibull: $F(g; \alpha) = 1 - e^{-(g/\alpha_1)^{\alpha_2}}$

652

653 **Table 3. Optimized PoD models for the controlled release data sets from each test region and for the combination of all GML**
 654 **2.0 controlled release data sets.**

Location	PoD Model	Inverse PoD Model	Coefficient Values
Midland, TX GML 1.0	$PoD = 0.5 \times \left(1 + \operatorname{erf} \left(\frac{\left(\frac{\beta_1 Q \beta_2}{n \beta_3 u \beta_4} \right) - \alpha_1}{\sqrt{2} \alpha_2} \right) \right)$	$Q = \left(\frac{n \beta_3 u \beta_4}{\beta_1} \exp(\sqrt{2} \alpha_2 \times \operatorname{erfinv}(2PoD - 1) + \alpha_1) \right)^{\frac{1}{\beta_2}}$	$\alpha_1 = -0.3466$ $\alpha_2 = 0.8326$ $\beta_1 = 2.998e-4$ $\beta_2 = 2.6339$ $\beta_3 = 2.7501$ $\beta_4 = 2.0877$
Los Angeles, CA GML 2.0	$PoD = 1 - \left(1 + \left(\frac{\beta_1 Q \beta_2}{n \beta_3 (u + \beta_5) \beta_4} \right)^{\alpha_1} \right)^{-\alpha_2}$	$Q = \left(\frac{1}{\beta_1} \left((1 - PoD)^{-\frac{1}{\alpha_2} - 1} \right)^{\frac{1}{\alpha_1}} n \beta_3 (u + \beta_5) \beta_4 \right)^{\frac{1}{\beta_2}}$	$\alpha_1 = 2.0000$ $\alpha_2 = 1.5000$ $\beta_1 = 0.1897$ $\beta_2 = 1.7999$ $\beta_3 = 1.8917$ $\beta_4 = 3.2289$ $\beta_5 = -2.3488$
Bozeman, MT GML 2.0	$PoD = 1 - e^{-\left(\frac{\beta_1 Q \beta_2}{\alpha_1 n \beta_3 u \beta_4} \right)^{-\alpha_2}}$	$Q = \left(\frac{\alpha_1}{\beta_1} (-\ln(1 - PoD))^{\frac{1}{\alpha_2}} n \beta_3 u \beta_4 \right)^{\frac{1}{\beta_2}}$	$\alpha_1 = 1.0000$ $\alpha_2 = 1.0000$ $\beta_1 = 33.66e-5$ $\beta_2 = 3.1874$ $\beta_3 = 3.5578$ $\beta_4 = 2.5257$
Wonowon, BC GML 2.0	$PoD = 0.5 \times \left(1 + \operatorname{erf} \left(\frac{\left(\frac{\beta_1 Q \beta_2}{n \beta_3 u \beta_4} \right) - \alpha_1}{\sqrt{2} \alpha_2} \right) \right)$	$Q = \left(\frac{n \beta_3 u \beta_4}{\beta_1} \exp(\sqrt{2} \alpha_2 \times \operatorname{erfinv}(2PoD - 1) + \alpha_1) \right)^{\frac{1}{\beta_2}}$	$\alpha_1 = -0.3466$ $\alpha_2 = 0.8326$ $\beta_1 = 0.0201$ $\beta_2 = 2.4088$ $\beta_3 = 1.5777$ $\beta_4 = 1.9428$
Combined GML 2.0	$PoD = 1 - \left(1 + \left(\frac{\beta_1 Q \beta_2}{n \beta_3 u \beta_4} \right)^{\alpha_1} \right)^{-\alpha_2}$	$Q = \left(\frac{1}{\beta_1} \left((1 - PoD)^{-\frac{1}{\alpha_2} - 1} \right)^{\frac{1}{\alpha_1}} n \beta_3 u \beta_4 \right)^{\frac{1}{\beta_2}}$	$\alpha_1 = 2.0000$ $\alpha_2 = 1.5000$ $\beta_1 = 2.41e-3$ $\beta_2 = 1.9505$ $\beta_3 = 2.0836$ $\beta_4 = 1.5185$

655

656 model represents the emission rate detection threshold as a function of probability of detection PoD ,

657 wind speed u , and $n = (\text{raster pixel GCN})/1000$.

658 Optimized PoD model selection is performed by minimizing the negative log likelihood function (NLLF)

659 for each combination of candidate predictor and inverse link functions. The optimized NLLF is used to

660 compute the Akaike Information Criterion (AIC) for each candidate pair, and the AIC is used to compute

661 the relative likelihood of minimizing information loss (RLMIL). The function used to compute this value is

662 as follows:

663
$$RLMIL = \exp\left(\frac{AIC_{min} - AIC_i}{2}\right),$$

664 where AIC_{min} is the minimum AIC value and AIC_i is the AIC value for the i th unique candidate-predictor

665 combination.²² The candidate pair with RLMIL equal to one represents the optimal model and is

666 highlighted in green in

667 Table 4 for each data set. The RLMIL for non-optimal models represents the probability that it is the

668 model that best represents the data.

669
670
671
672

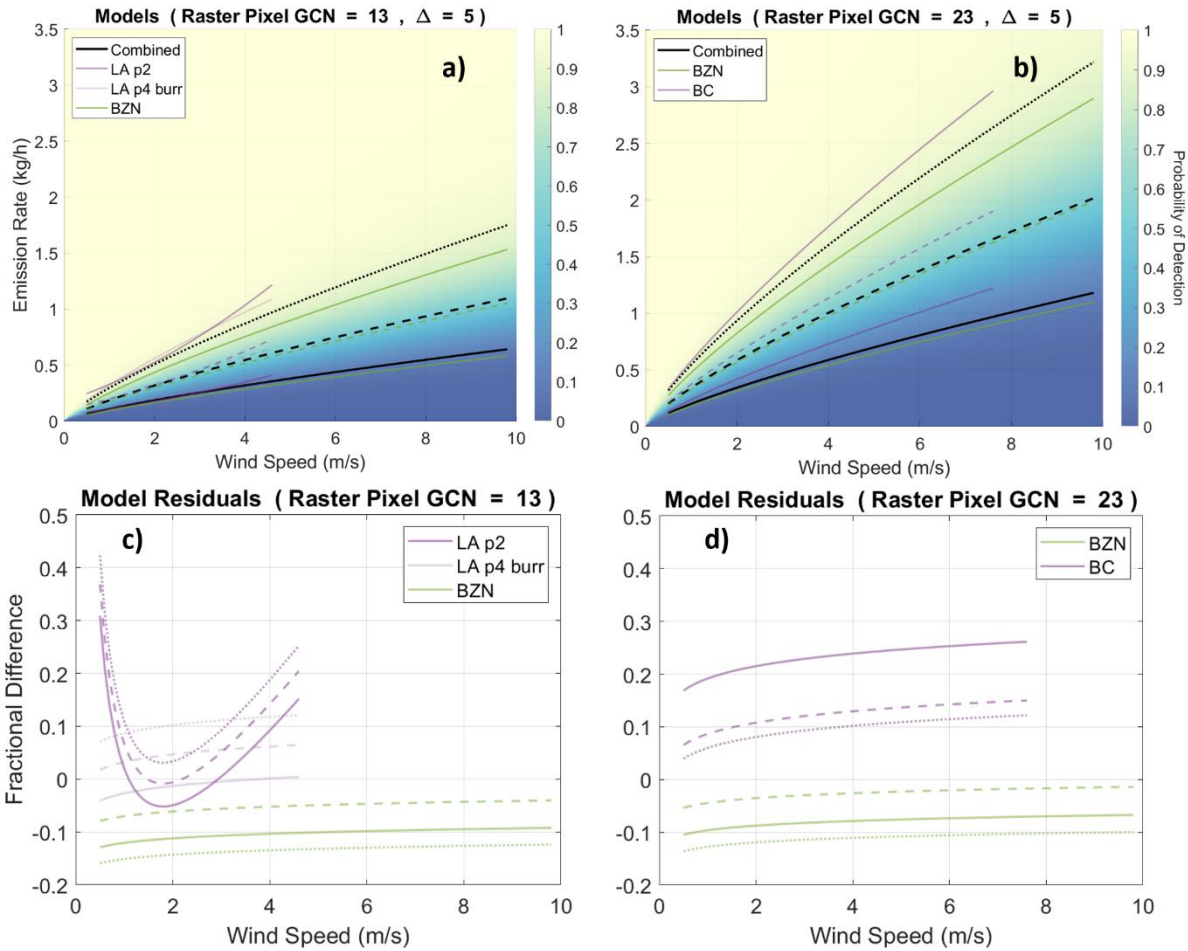
Table 4. Relative likelihood of minimizing information (RLMIL) for candidate PoD models. Optimal PoD model (i.e. combination of predictor and inverse link functions) has RLMIL equal to one and is highlighted in green. The second-best performing model is highlighted in yellow, and the best performing predictor function is highlighted in blue.

Predictor Function	Inverse Link Function	Midland GML 1.0	Los Angeles GML 2.0	Bozeman GML 2.0	British Columbia GML 2.0	Combined GML 2.0
p1	Log Normal	0.3609	0.6385	0.1259	0.3613	0.3534
p1	Log Logistic	0.1497	0.5803	0.0320	0.2696	0.1271
p1	Fréchet	0.0042	0.0342	0.0003	0.0473	0.0000
p1	Burr	0.3113	0.8049	0.1009	0.3422	0.3655
p1	Weibull	0.3131	0.8088	0.3744	0.2457	0.0462
p2	Log Normal	0.5979	0.7621	0.1262	0.5691	0.4094
p2	Log Logistic	0.2698	0.8557	0.0326	0.3937	0.1434
p2	Fréchet	0.0055	0.0473	0.0003	0.0688	0.0000
p2	Burr	0.5930	1.0000	0.1061	0.5260	0.4287
p2	Weibull	0.5191	0.4324	0.4633	0.4334	0.0450
p3	Log Normal	0.6040	0.7596	0.1440	0.4015	0.6062
p3	Log Logistic	0.2820	0.7496	0.0369	0.2876	0.1739
p3	Fréchet	0.0124	0.0542	0.0003	0.0477	0.0000
p3	Burr	0.5266	0.9512	0.1202	0.3783	0.5477
p3	Weibull	0.3321	0.5515	0.5021	0.2900	0.0728
p4	Log Normal	1.0000	0.6621	0.3389	1.000	0.9663
p4	Log Logistic	0.4055	0.6630	0.0878	0.7347	0.3411
p4	Fréchet	0.0117	0.0482	0.0009	0.1234	0.0000
p4	Burr	0.8500	0.7910	0.2749	0.9459	1.0000
p4	Weibull	0.8408	0.4407	1.0000	0.6574	0.0874

673

674 The p4 predictor function is found to best represent the controlled release data in three of the four
675 region-specific data sets and the combined GML 2.0 data set, while the p2 model is found to be optimal
676 for the LA data set. Predictors functions p1-p3 are less likely to minimize information loss because small
677 reductions in the NLLF observed when fitting PoD models to the controlled release data are outweighed
678 by the penalty for including an additional model parameter. Depending on the data set the Log Normal,
679 Burr, and Weibull inverse link functions are found to be optimal, whereas the Log Logistic and Fréchet
680 inverse link functions are less competitive.

681 A comparison of the optimized GML 2.0 PoD models for each data set is shown in Figure 8. The black
682 lines in Figure 8 a) & b) show the optimal PoD model for the combined data set at three PoD levels and



683

684 **Figure 8. a) & b) Comparison of optimized PoD models for individual data sets with combined model at 10%**
 685 **(solid), 50% (dashed), and 90% (dotted) PoD for 13 ppm-m and 23 ppm-m raster pixel GCN values,**
 686 **respectively. c) & d) Residual plots showing fractional differences between optimized PoD model for**
 687 **individual data sets compared to the combined PoD model for 13 ppm-m and 23 ppm-m raster pixel GCN**
 688 **values, respectively.**

689 two raster pixel GCN levels. The purple and green lines show the optimal PoD models for the individual
 690 LA, BZN, and BC data sets, respectively. Figure 8 c) & d) show the fractional differences between the
 691 individual data set models and the combined model. Individual models typically agree with the
 692 combined model to within 20% fractional difference across most of the valid wind speed range and at
 693 both raster pixel GCN values.

694

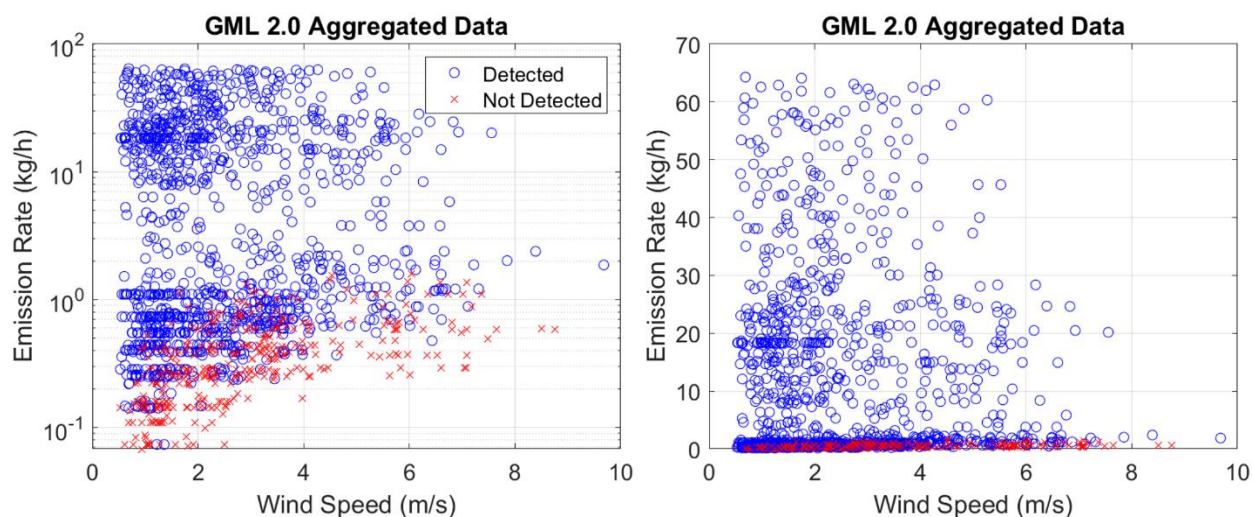
695 S6 – Aggregate controlled release data sets

696

697 The aggregate controlled release data set for all valid GML 2.0 flyover passes described in Table 1 is

698 shown in Figure 9. The aggregate controlled release data set for all valid GML 1.0 flyover passes can be

699 found in Ref. [7].



700

701 **Figure 9. Semi-log plot of emission rate versus wind speed for all valid GML 2.0 flyover passes (left). Linear**
702 **plot of emission rate versus wind speed for all valid GML 2.0 flyover passes (right).**

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