Title: Estimating microphysical properties of ice clouds for climate modeling and remote sensing applications

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Estimating microphysical properties of ice clouds for climate modeling and remote sensing applications

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Technical note

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Abstract

Ice clouds pose crucial challenges in climate model simulations and remote sensing retrievals due to complicated mechanisms of ice cloud formation that span from micro scale to planetary wave scale. Despite numerous attempts to parameterize these processes, many questions remain unanswered. This technical note provides a summary of the most common and recent studies on ice clouds and compiles important equations in a simple yet applicable manner to relate key microphysical properties of cirrus and ice clouds such as ice particle effective diameter, ice water path, cloud optical depth, particle number concentration, fall velocity, and particle size distribution. Additionally, it introduces relationships between ice particle mass (m) and dimension (D), as well as ice particle projected area (A) and D, which are essential for parameterizing the diverse shapes of ice particles in ice clouds. The equations provided in this technical note are valuable for the accurate representation of cirrus and ice clouds in remote sensing and climate modeling.

Cloud optical depth

Assuming ice water path (IWP) and ice particle effective diameter (D_e) are determined, ice cloud optical depth (τ_i) is calculated as suggested by Wang et al. (2011):

$$\tau_i = \frac{3Q_e \mathrm{IWP}}{2\rho_i D_e},$$

where ρ_i is pure ice density (=0.917 g cm^{-3}) and Q_e is bulk average extinction efficiency (=2).

Number concentration

In addition to ice water content (IWC) and D_e, assumptions on ice particle size distribution (PSD), particle mass (m)-maximum dimension (D) relationship, and particle projected area (A)-D relationship are required to estimate ice cloud particle number concentration (N_i). Here, we assume a gamma PSD, which is very common in climate modeling and remote sensing, and can be described following Eq. (8) in Erfani and Mitchell (2016):

$$n(D) \propto D^{\nu} exp\,(-\lambda D),$$

where $n(D)$ is ice particle number density, ν is the dispersion parameter, and λ is the slope parameter, which is computed from D_e based on Eq. (1) in Mitchell et al. (2020) by inverting Eq. (14) in Erfani and Mitchell (2016):

$$\lambda = \left[\frac{2\rho_i \gamma \Gamma(\delta + \nu + 1) D_e}{3\alpha\, \Gamma(\beta + \nu + 1)}\right]^{\frac{1}{\delta - \beta}},$$

where Γ indicates the gamma function, α and β refers to the coefficients of the m-D power law, and γ and δ are the coefficients of the A-D power law, following Erfani and Mitchell (2016) and Erfani and Mitchell (2017):

$$m = \alpha D^{\beta},$$

$$A = \gamma D^{\delta}.$$

The estimation of m-D and A-D coefficients will be discussed later. N_i is then estimated by inverting Eq. (12) in Erfani and Mitchell (2016):

$$N_i = \frac{\Gamma(\nu + 1)\ \mathrm{IWC}\ \lambda^{\beta}}{\alpha\, \Gamma(\beta + \nu + 1)}.$$

m-D and *A-D* relationships

Numerous studies estimated α and β for various ice particle shapes (habits) (Mitchell, 1996; Erfani and Mitchell, 2017). It is more useful for climate modeling and remote sensing to determine those coefficients based on ice cloud type (synoptic or anvil cirrus) and temperature range because ice clouds consist of a combination of different habits with varying concentrations (Brown and Francis, 1995; Erfani and Mitchell, 2016; Lawson et al., 2019). Additionally, most studies assumed constant coefficients with particle size, however, a single m-D power law cannot represent the whole spectrum of particle size. To overcome this, Erfani and Mitchell (2016) developed m-D polynomial fits in log space:

$$\ln(m) = a_o + a_1 \ln(D) + a_2[\ln(D)]^2,$$

where a_o, a_1, and a_o are constant parameters for each cloud type and temperature range. These polynomial fits can easily be reduced to *m*-*D* power laws with α and β varying with size:

$$\beta = a_1 + 2a_2 \ln(D),$$

$$\alpha = \frac{\exp\{a_o + a_1 \ln(D) + a_2[\ln(D)]^2\}}{D^\beta}.$$

Depending on the application, these coefficients are calculated for the PSD dimension of interest such as mean dimension and median number concentration dimension, which will be discussed later. Table 1 and Figure 1 summarize a few *m*-*D* relationships.

Table 1. Examples of *m*-*D* relationships for ice particles.

Name	Reference	Relationship	Coefficients (cgs units)	Comments
BF95	Brown and Francis (1995)	$m = \alpha D^\beta$	α=0.002938 β=1.9	▪ For ice particles at the surface ▪ Valid for $D > 100\ \mu m$
EM16 polynomial	Erfani and Mitchell (2016)	$\ln(m) = a_o + a_1 \ln(D) + a_2[\ln(D)]^2$	a_o=-6.72924 a_1=1.17421 a_2=-0.15980	▪ For synoptic (in-situ) cirrus clouds ▪ Based on polynomial fit in log space ▪ -40 °C $< T < -20$ °C
EM16 power	Lawson et al. (2019)	$m = \alpha D^\beta$	α=0.003777 β=2.1857	▪ For synoptic (in-situ) cirrus clouds ▪ Power law fit to data from Erfani and Mitchell (2016) ▪ Valid for $D > 25\ \mu m$ and -40 °C $< T < -20$ °C
L19	Lawson et al. (2019)	$m = \alpha D^\beta$	α=0.0073325 β=2.39	▪ For synoptic (in-situ) cirrus clouds ▪ Valid for $D > 15\ \mu m$

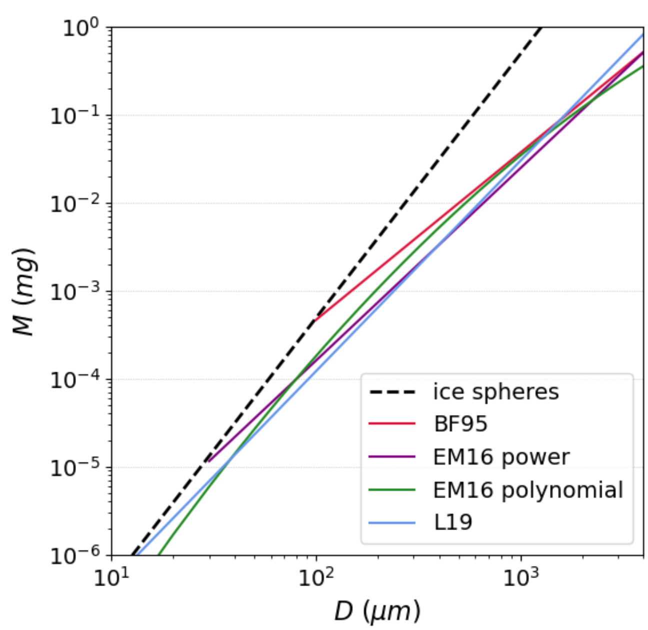

Figure 1. Relationships between ice particle mass (*m*) and ice particle maximum dimension (*D*) based on a few studies, summarized in Table 1.

A number of studies determined γ and δ for *A*-*D* relationships based on ice habit or cirrus cloud type and temperature range. Table 2 and Figure 2 summarize a few *A*-*D* relationships.

Table 2. Examples of *A*-*D* relationships for ice particles.

Name	Reference	Relationship	Coefficients (cgs units)	Comments
M96 hexagonal	Mitchell (1996)	$A = \gamma D^{\delta}$	γ=0.5 δ=2.0	▪ For ice particles at the surface ▪ Based on hexagonal plate habit
M96 graupel	Mitchell (1996)	$A = \gamma D^{\delta}$	γ=0.65 δ=2.0	▪ For ice particles at the surface ▪ Based on lump graupel habit
F98	Francis et al. (1998)	$A = \gamma D^{\delta}$	γ= 0.026240 δ=1.26667	▪ For ice particles at the surface ▪ Valid for $D > 100\ \mu m$
EM16 polynomial	Erfani and Mitchell (2016)	$\ln(A) = b_o + b_1 \ln(D) + b_2[\ln(D)]^2$	b_o=-2.46356 b_1=1.25892 b_2=-0.07845	▪ For synoptic (in-situ) cirrus clouds ▪ Based on polynomial fit in log space ▪ $-40\ ^{\circ}\mathrm{C} < T < -20\ ^{\circ}\mathrm{C}$

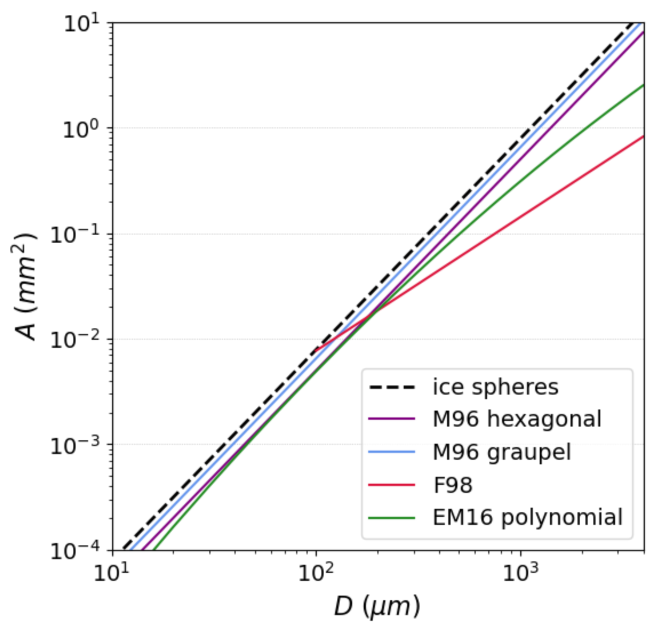

Figure 2. Relationships between ice particle projected area (A) and ice particle maximum dimension (D) based on a few studies, summarized in Table 2.

Fall velocity

Many models use ice particle fall velocity (V)-D power law based on Locatelli and Hobbs (1974):

$$V = aD^{b},$$

where a and b are constant with D and vary with different ice particle shapes. However, this relationship cannot represent a gradual change in ice particle V as particles grow. Mitchell and Heymsfield (2005) developed a method to derive a and b for different flow regimes and particle sizes by using the Best number (X)-Reynolds number (Re) power law:

$$Re = A_x X^{B_x},$$

where A_x and B_x are determined using Eqs. (6) and (7) in Mitchell and Heymsfield (2005), and Eqs. (9)-(12) in that study can be used to compute a and b.

D_e from IWC and T

Sun and Rikus (1999) employed multiple field campaigns and developed simple expressions to determine D_e from IWC and T:

$$c = 45.8966(\text{IWC})^{0.2214},$$

$$d = 0.7957(\text{IWC})^{0.2535},$$

$$D_e = c + d(T + 190),$$

where T is in °C, D_e is in μm, and IWC is in $g\ m^{-3}$. Although D_e is a relatively weak function of T, it is sensitive to IWC (Figure 3).

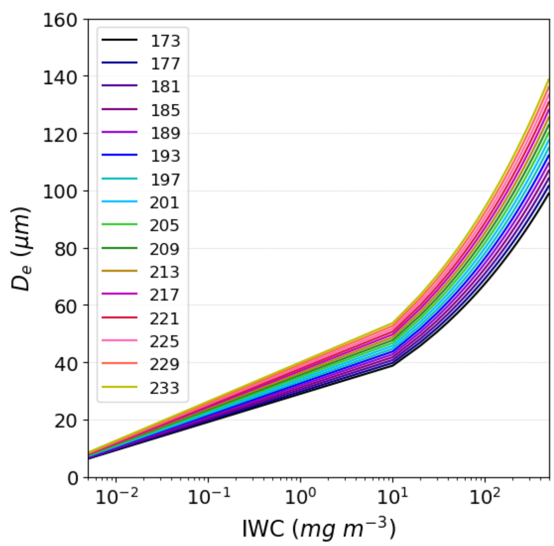

Figure 3. D_e versus IWC for different T regimes based on Sun and Rikus (1999).

Dimension of interest

For climate modeling purposes, it is useful to select the dimension of interest based on the microphysical characteristics. For ice nucleation calculations, the mean dimension or median number concentration dimension (D_N) are useful. If the derivations are most relevant to IWC, the median mass dimension (D_m) is most useful. Similarly, the median area dimension (D_A) or the median radar reflectivity dimension (D_Z) might be of interest (Erfani and Mitchell, 2016):

$$D_N = \frac{\nu + 0.67}{\lambda}$$

$$D_m = \frac{\beta + \nu + 0.67}{\lambda}$$

$$D_A = \frac{\delta + \nu + 0.67}{\lambda}$$

$$D_Z = \frac{2\beta + \nu + 0.67}{\lambda}$$

Other useful relationships

$$\mathrm{IWC} = \int_0^\infty m(D)n(D)dD = \int_0^\infty \alpha D^\beta n(D)dD = \frac{\alpha\Gamma(\beta + \nu + 1)}{\lambda^{\beta+\nu+1}}$$

$$\lambda = \left[\frac{\alpha\Gamma(\beta + \nu + 1)N_i}{\Gamma(\nu + 1)\mathrm{IWC}}\right]^{\frac{1}{\beta}}$$

$$D_e = \frac{3\alpha\Gamma(\beta + \nu + 1)}{2\rho_i\gamma\Gamma(\delta + \nu + 1)}\lambda^{\delta-\beta}$$

Recommendations

- Check out Wood (2006) for useful relationships for liquid clouds.
- See Erfani and Mitchell (2016) for more discussions on *m-D* and *A-D* relationships and their importance in ice microphysical properties.
- Erfani and Mitchell (2017) provided a new *m-D* for rimed ice particles based on observations, and an approach to parameterize the riming process in models.
- For a novel approach to retrieving CALIPSO satellite ice particle number concentration, check out Mitchell et al. (2018).
- See Lawson et al. (2019) for a review of cirrus cloud particle shape, including various *m-D* relationships.
- Mitchell et al. (2020) provide calculations of ice cloud properties for use in climate models.

Data Availability Statement

The parameters and relationships, used to create plots in this technical note, are explicitly provided in tables and equations with accurate citations to previous studies.

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