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Numerical simulation of the 2D trajectory of a non-buoyant fluid parcel under the influence of inertial oscillation

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Abstract

The trajectory of non-buoyant fluid parcels under the influence of inertial oscillations is a pivotal phenomenon in geophysical fluid dynamics, impacting processes such as tracer transport, pollutant dispersion, and the dynamics of marine organisms. This study presents a comprehensive numerical investigation of the two-dimensional trajectory of a non-buoyant fluid parcel subjected to inertial oscillations, complemented by abrupt external forcing events.

The simulations were implemented using multiple open-source, code-based general programming languages, including Fortran, Python, GNU Octave, R, and Julia. By running 1,000 iterations in each environment, we rigorously evaluated the computational performance and accuracy of these languages in tackling this idealized problem. The results, visualized through static plots and an animation generated using the Matplotlib library, capture the oscillatory trajectories

and the influence of rotational effects, validating the numerical models' ability to represent the fundamental physics governing fluid motion.
Furthermore, a robust statistical analysis compared the execution times across the programming environments. The Kruskal-Wallis test and Dunn's post-hoc test with Bonferroni correction reveal that Fortran exhibits significantly faster execution times compared to the other environments, highlighting its suitability for computationally intensive simulations in geophysical fluid dynamics. This study provides valuable insights into selecting appropriate computational tools and contributes to educational resources for teaching idealized fluid dynamics models, laying the foundation for more sophisticated hierarchical models applicable to ocean circulation, atmospheric dispersion, and biological transport influenced by oscillating currents.

Keywords: Inertial oscillations; Fluid parcel trajectories; Geophysical fluid dynamics; Open-source programming languages; Idealized models

1 Introduction

The trajectory of neutrally buoyant fluid parcels in oscillating flows is a phenomenon of profound significance within the realm of geophysical fluid dynamics (GFD), encompassing both oceanic and atmospheric contexts. Inertial oscillations, arising from the intricate interplay between the Earth's rotation and fluid motions, exert a profound influence on the behavior of these fluid parcels [1]. Accurately predicting the transport and dispersion of tracers such as pollutants, nutrients, and biological organisms in the oceans and atmosphere hinges on a comprehensive understanding of the trajectories of non-buoyant particles subjected to such oscillating flows [2].

Geophysical fluids, including the vast oceans and the Earth's atmosphere, are characterized by their immense spatial scales, intricate dynamics, and the pervasive influence of rotational effects [3–5]. The Earth's rotation, manifesting through the Coriolis force, introduces a unique set of challenges in precisely modeling fluid motions. Inertial oscillations, also referred to as inertial waves or inertial currents, are a direct manifestation of this rotational influence, causing fluid parcels to oscillate about their mean trajectory due to the conservation of absolute angular momentum. This phenomenon has far-reaching implications for the transport and mixing processes within geophysical fluid systems.

While analytical solutions can provide valuable insights into the behavior of fluid parcels under inertial oscillations, their applicability is often limited to highly simplified cases or requires restrictive assumptions, such as neglecting nonlinear effects or assuming constant background flows. As a result, numerical simulations have emerged as an indispensable tool for exploring the complex dynamics of these systems [6–8], particularly in two-dimensional (2D) scenarios where the effects of vertical motions are neglected. These idealized 2D models offer a crucial starting point for developing intuition and understanding the fundamental physics governing fluid parcel trajectories under the influence of inertial oscillations. Such parsimonious modeling efforts, which strip away unnecessary complexities, are vital for gaining insights into the core dynamics and informing more sophisticated hierarchical models.

In this study, we present a comprehensive numerical investigation of the 2D trajectory of a non-buoyant fluid parcel subjected to inertial oscillations, a problem of profound relevance in GFD. To ensure a thorough and robust analysis, we have implemented the simulation using multiple open-source, code-based general programming languages, including Fortran 95, Python, GNU Octave, R, and Julia. The choice of these open-source languages not only promotes transparency and reproducibility but also leverages the power of their extensive ecosystems and user communities, facilitating collaborative development and knowledge sharing [9–11].

By running each implementation for 1,000 iterations, we aim to identify the most efficient and reliable approach for simulating this phenomenon, providing valuable insights into the selection of appropriate computational tools for similar numerical simulations in the field of GFD. The use of general-purpose programming languages offers flexibility, scalability, and the ability to integrate with a wide range of libraries and tools, enabling seamless integration of visualization and analysis components.

Furthermore, we developed separate code modules specifically designed for visualizing the simulated trajectories using the powerful Matplotlib library in Python. These visualization tools enable the generation of animated file, offering an intuitive and dynamic representation of the particle's motion over time. Such visualizations play a crucial role in aiding the interpretation and communication of the results, facilitating a deeper understanding of the underlying physics governing the trajectories of fluid parcels under inertial oscillations.

The objectives of this paper are multifaceted. Firstly, we aim to provide a comprehensive and detailed description of the numerical methods employed in simulating the 2D trajectory of a non-buoyant fluid parcel under inertial oscillations. Secondly, we seek to evaluate the computational performance and accuracy of various open-source programming languages in tackling this problem, thereby contributing to the ongoing discourse on the selection of appropriate computational tools for numerical simulations in GFD. Finally, we strive to contribute to the educational resources available for teaching and learning about idealized models in fluid dynamics, fostering a deeper understanding of the fundamental principles governing fluid parcel trajectories in rotating systems among students and researchers alike.

By addressing these objectives through a parsimonious modeling approach [12] and leveraging the power of open-source, code-based general programming languages, we aim to fill a crucial research gap by providing a comprehensive analysis of the numerical simulation of this idealized problem. The insights gleaned from this study will not only offer invaluable guidance on the selection of appropriate computational tools for similar numerical simulations in GFD but also lay the foundation for developing more sophisticated hierarchical models [13]. The findings have the potential to inform and advance a wide range of applications, from ocean circulation modeling [14] to atmospheric dispersion studies [15], and from the tracking of marine debris to the understanding of the transport of biological organisms influenced by oscillating currents [16]. Moreover, the idealized model and accompanying visualizations serve as powerful educational resources, equipping students and researchers with the tools necessary to grasp the intricate interplay between fluid dynamics and rotational effects, ultimately fostering a deeper appreciation for the fundamental principles that govern the behavior of fluid parcels in rotating systems.

2 Methods

2.1 Mathematical Derivation

To define the motion of a fluid parcel influenced by inertial oscillations in two dimensions, the first step was to derive the Navier-Stokes equations specifically for a 2D scenario. Applying Newton's second law to fluid dynamics, the Navier-Stokes equation accounts for the conservation of momentum. It integrates the effects of external forces, pressure gradients, and viscous forces within the fluid [17, 18]:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \mu\nabla^2\mathbf{u} + \mathbf{f} \tag{1}$$

In this context, ρ defines the fluid density, and $\mathbf{u} \equiv (u, v)$ represents the fluid velocity vector, where u and v are the velocity components in the x and y directions, respectively. Furthermore, p denotes the pressure, μ signifies the dynamic viscosity, and $\mathbf{f}$ refers to the body forces per unit volume. In the case of 2D flow, the Navier-Stokes equations can be separated into two components. One component represents the momentum equation in the $x-$direction and the other in the $y-$direction:

$$\begin{aligned} \rho\left(\tfrac{\partial u}{\partial t} + u\tfrac{\partial u}{\partial x} + v\tfrac{\partial u}{\partial y}\right) &= -\tfrac{\partial p}{\partial x} + \mu\left(\tfrac{\partial^2 u}{\partial x^2} + \tfrac{\partial^2 u}{\partial y^2}\right) + f_x \\ \rho\left(\tfrac{\partial v}{\partial t} + u\tfrac{\partial v}{\partial x} + v\tfrac{\partial v}{\partial y}\right) &= -\tfrac{\partial p}{\partial y} + \mu\left(\tfrac{\partial^2 v}{\partial x^2} + \tfrac{\partial^2 v}{\partial y^2}\right) + f_y \end{aligned} \tag{2}$$

To simplify our derivation of the Navier-Stokes equations for practical GFD applications [3], several key assumptions were typically made. The fluid is treated as a Newtonian fluid, where the stress is linearly proportional to the strain rate. Under the continuum hypothesis, fluids are considered as continuous media. The assumption of incompressibility holds that the fluid density remains constant. Additionally, the fluid's viscosity is considered constant, unaffected by changes in pressure, temperature, or velocity. The no-slip boundary condition is applied, meaning that fluid particles at solid boundaries exhibit no relative motion. Fluid properties are assumed isotropic, uniform in all directions. Generally, external forces other than body forces like gravity are ignored unless specifically stated.

To further refine our analysis of 2D inertial oscillation equations derived from the three-dimensional Navier-Stokes equation, we incorporated additional considerations about the flow dynamics and the assumptions under which we operated. We retained the assumption that the flow is predominantly horizontal ($w = 0$) with no vertical gradients ($\partial(.)/\partial z$ terms are zero), which is suitable for scenarios where flow dynamics are limited to a thin layer, such as near the ocean surface or in the atmosphere away from significant topographical or frontal influences [e. g. 19–22]. In this context, we extended the geostrophic balance assumption by considering environments where pressure gradient forces are negligible compared to the Coriolis and external forcing terms. This may occur in conditions with strong rotational effects or in areas where the pressure field is relatively uniform, minimizing the impact of pressure gradients on momentum balance.

Additionally, we considered external forcings that can influence the dynamics of the flow. We introduced forcings defined as:

$$\begin{aligned} f_x &= \frac{\partial u_f}{\partial t} \\ f_y &= \frac{\partial v_f}{\partial t} \end{aligned} \tag{3}$$

In this case, u_f represents a uniform external forcing in space, such as wind stress or other steady external influences. This assumption allows us to focus on the effect of external forcings on the fluid without the complications introduced by spatial variability.

In our simplified model, we ignored viscosity due to its minimal impact compared to inertial terms, and we continued to assume that the flow is irrotational, which simplifies the momentum equations. The Navier-Stokes equations then reduce primarily to expressions influenced by inertial effects and the Coriolis force, introduced through the Coriolis parameter f, twice the angular velocity of Earth's rotation [3]:

$$\begin{aligned} \frac{\partial u}{\partial t} &= -fv + f_x \\ \frac{\partial v}{\partial t} &= fu + f_y \end{aligned} \tag{4}$$

These equations govern the inertial oscillations in a rotating reference frame, simplified to highlight the oscillatory behavior of fluid parcels. We were particularly interested in predicting the pathway of a non-buoyant fluid parcel, which simplifies our assumptions further by neglecting buoyancy effects [3]. The pathway of these fluid parcels is then given by:

$$\begin{aligned} \frac{dx}{dt} &= U_0 + u \\ \frac{dy}{dt} &= V_0 + v \end{aligned} \tag{5}$$

In this context, U_0 and V_0 represent the ambient uniform flow, and u and v the velocity perturbations due to inertial oscillations. This model is highly simplified, and deriving this rigorously would involve linearizing the Navier-Stokes equations and possibly introducing adjustments for external forcing terms and considering geostrophic balance if large-scale geophysical flows are being examined. This approach gives us a useful framework for understanding and predicting the movement of non-buoyant fluid parcels under specific geophysical conditions.

2.2 Numerical Experiments

The semi-implicit approach and the local rotation method are two numerical techniques used to predict the trajectory of non-buoyant fluid parcels in a rotating fluid system, specifically addressing inertial oscillation problems. These methods offer different perspectives and computational strategies to model the influence of the Coriolis force and other relevant factors.

Starting with the semi-implicit approach, we used the following numerical scheme to predict the trajectory of the fluid parcel as described in the equation 4:

$$\begin{aligned} u^{n+1} &= \tfrac{(1-\beta)u^n+\alpha v^n}{1+\beta} \\ v^{n+1} &= \tfrac{(1-\beta)v^n-\alpha u^n}{1+\beta} \end{aligned} \tag{6}$$

Here, u^n and v^n are the velocities at the current time step n, and u^{n+1} and v^{n+1} are the velocities at the next time step $n+1$. The parameters α and β are defined as $\alpha = \Delta t f$ and $\beta = \frac{1}{4}\alpha$, respectively. This semi-implicit scheme efficiently integrates the inertial oscillation equations over time, providing accurate predictions of the fluid parcel's velocity components (u, v). To predict the x and y coordinates of a non-buoyant fluid parcel using this approach, we discretized the kinematic equation (equation 5) with finite differences:

$$\begin{aligned} \Delta x &= \tfrac{\alpha v^n}{1+\beta}\Delta t \\ \Delta y &= \tfrac{-\alpha u^n}{1+\beta}\Delta t \end{aligned} \tag{7}$$

Moving on to the local rotation method, we simulated the Coriolis force using a local rotation or velocity vector

$$\begin{aligned} u^{n+1} &= \cos(\theta)u^n + \sin(\theta)v^n \\ v^{n+1} &= \cos(\theta)v^n - \sin(\theta)u^n \end{aligned} \tag{8}$$

In this context, θ is determined based on the time step Δt and the Coriolis parameter f, given by $\theta = 2\sin^{-1}\left(\frac{1}{2}\Delta t f\right)$. For small $\Delta t|f|$, θ approximates $\Delta t f$. This method effectively captures the Coriolis effect by rotating the velocity components, aiding in predicting the trajectory of fluid parcels in a rotating system. By incorporating the finite difference approximation to the equation 5, we obtained:

$$\begin{aligned} \Delta x &= (\cos(\theta) - 1)\, u^n \Delta t + \sin(\theta) v^n \Delta t \\ \Delta y &= \cos(\theta) v^n \Delta t - (\sin(\theta) + 1)\, u^n \Delta t \end{aligned} \tag{9}$$

By discretizing these equations and incorporating the velocity updates from semi-implicit and/or local rotation schemes, we were able to iteratively predict the x and y coordinates of fluid parcels over time. In that study, we aimed to use these numerical schemes to predict the trajectory of a non-buoyant fluid parcel floating with ambient uniform flow (U_0, V_0), which is sometimes influenced by abrupt wind events. In that simulation, we modeled the ambient flow as a uniform northeastward flow with values of $U_0 = 5$ cm/s and $V_0 = 5$ cm/s. The total simulation time is 6 days with $\Delta t = 4320$ seconds, approximately 1.2 days. There were three abrupt events that changed the relative flow speed and direction ($\Delta u_f, \Delta v_f$). These changes were explained in Table 1 as follows.

Table 1: Velocity disturbance parameters.

time (days)	Δu_f (cm / s)	Δv_f (cm / s)
1	10	0
2	10	0
4	0	10

To tackle this issue, we explored various numerical schemes within several open-source computing environments highly regarded in earth science and specifically GFD. Initially, we applied these schemes using Fortran, a language with a long history in solving such problems [e. g. 23–26]. Fortran remains widely used today, especially in general circulation models (GCMs) [27]. For instance, the 1995 version of Fortran has been pivotal in many classic problem-solving scenarios within this domain [28, 29].

Another avenue we pursued was implementing the solutions in Python, leveraging the NumPy library [30]. Python has emerged as a dominant force in programming due to its versatility and extensive libraries like NumPy, which are invaluable for numerical simulations and statistical computations in earth sciences [e. g. 31–35]. Its modern features make it a go-to choice for researchers and practitioners alike. Moreover, we explored GNU Octave, an open-source platform akin to MATLAB® but freely available. Its syntax, resembling MATLAB®'s, appeals to those familiar with MATLAB® but seeking a cost-effective solution. This makes it a viable option for GFD modelers and researchers interested in numerical computations [e. g. 36–38].

Additionally, we delved into Julia, a computing environment gaining traction among GFD modelers. Julia's ability to match Fortran's speed while maintaining Python's ease of understanding has attracted attention. Many are considering transitioning from Fortran to Julia for ocean models due to this combination of speed and user-friendliness [e. g. 37, 39–43]. Lastly, we conducted numerical calculations in R, a popular choice within the atmospheric and oceanic sciences communities [e. g. 44–47]. R's strengths in data analysis and visualization make it a valuable tool for researchers working with numerical data in these fields.

Following the numerical calculations in various computing environments, we preserved the resulting simulation data in a structured format as a text file. This data storage method ensures that the data remains accessible and can be easily manipulated for further analysis or visualization. For visualization purposes, we turned to the Matplotlib library [48] within the Python computing environment. Matplotlib offers a robust suite of tools for creating static plots and animations, making it a preferred choice among researchers and practitioners in data visualization. The process involved writing a separate code specifically for plotting and animating the simulation results.

Once the plotting and animation code was executed, we generated two distinct outputs: a static plot saved in Portable Network Graphics (`.png`) format and an animation saved in Graphics Interchange Format (`.gif`). These formats are widely supported and compatible with various platforms, ensuring the accessibility and usability of the visualized data across different systems and applications. The static plot provides a snapshot of the simulation results, while the animation offers a dynamic representation, capturing temporal variations and trends within the data.

2.3 Statistical Analysis

Our next steps was to employ a Python code to rigorously evaluate and compare the execution times of code across diverse programming languages, aiding in informed language selection for scientific tasks. We began by compiling a Fortran code using the gfortran compiler, recording the compilation time for insights into overhead. Once compiled, the Python code executed each numerical solver for 1,000 times, ensuring robust statistical sampling. Crucial metrics like execution time, return codes, standard output, and errors were captured using the Subprocess library. Code was executed based on file extensions: Fortran via the compiled executable, and others (Python, Julia, MATLAB, and R) using their respective interpreters or runtimes. Data for each run included code details, execution time, return codes for error handling, and output/error messages. The resulting dataset was structured into a Pandas DataFrame [49] and exported to comma-separated values (`.csv`) for analysis.

For statistical comparison, we applied the Kruskal-Wallis (KW) test , a non-parametric method suitable for comparing multiple independent groups [50]. The Kruskal-Wallis test evaluates the null hypothesis that the medians of all groups are equal, indicating no significant difference in performance among programming languages. This test produces a test statistic H along with a p-value:

$$H = \frac{12}{N(N+1)} \sum_{j=1}^{k} \frac{R_j^2}{n_j} - 3(N+1) \tag{10}$$

In this case, N is the total number of observations, k is the number of groups, n_j is the number of observations in the jth group, and R_j is the sum of ranks for the jth group. The degrees of freedom for the Kruskal-Wallis test is $df = k - 1$.

If the p-value from the KW test is below a pre-defined significance level $\alpha = 0.05$, we proceeded with Dunn's post-hoc test. Dunn's test is used for pairwise comparisons between groups to identify which groups exhibit statistically significant differences in performance [51].

Dunn's test statistic for pairwise comparisons between groups i and j is given by:

$$Z_{ij} = \frac{|R_i - R_j| - (N(N+1))/12}{\sqrt{N(N+1)(N+2)/12}} \tag{11}$$

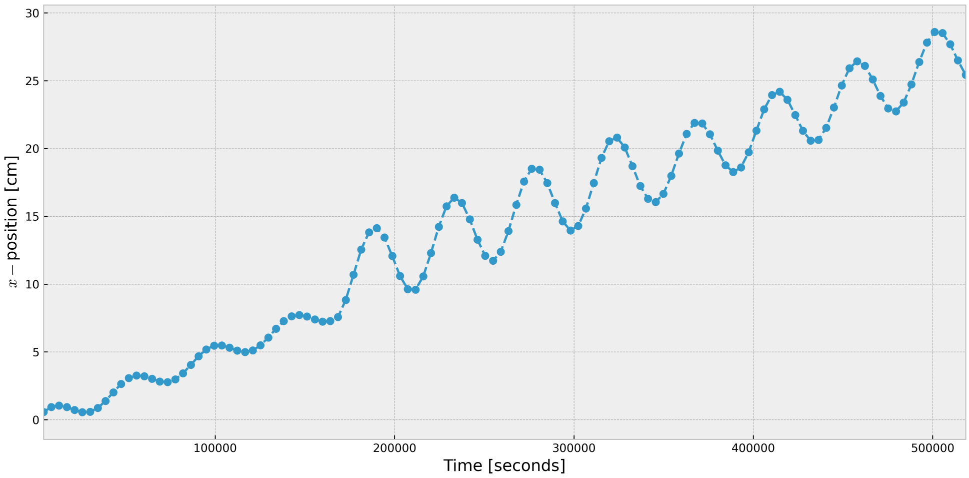
The critical value for Dunn's test is obtained using the Bonferroni adjustment, where the significance level α is divided by the number of pairwise comparisons m to control for multiple testing:

$$\alpha_{\text{adjusted}} = \frac{\alpha}{m} \tag{12}$$

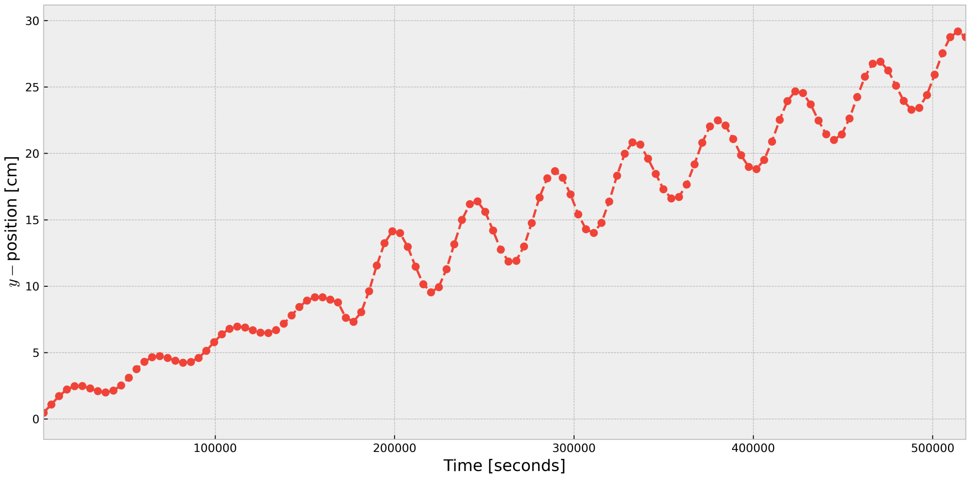
Pairwise comparisons with $|Z_{ij}|$ exceeding the adjusted critical value indicate statistically significant differences between the corresponding groups. We performed these calculations automatically using the statistics module in the SciPy [52] and the scikit-posthoc [53] libraries in the Python computing environment. This rigorous statistical approach ensured reliable insights into the computational performance of multiple computing environments for simulating a 2D fluid parcel trajectory over 1,000 iterations.

3 Results and Discussion

Simulations modeled the trajectory of a non-buoyant fluid parcel under the combined influence of inertial oscillations induced by the Earth's rotation, a uniform ambient northeasterly flow over six days, and abrupt disturbance events on days one, two, and four. Figures 1a and 1b depict oscillatory trajectories plotted against time. Both exhibit increasing amplitude due to Coriolis effects, but with slight path deviations attributable to numerical precision.



(a)



(b)

Fig. 1: Temporal evolution of a non-buoyant fluid parcel undergoing inertial oscillation and abrupt forcing events in (a) x and (b) y directions.

Figure 2 reveals a spiraling cyclical trajectory forming expanding loops - the expected inertial oscillation pattern. However, distinct perturbations are evident, likely caused by the simulated disturbance events capable of amplifying or damping the oscillations, with profound impacts on parcel transport and dispersion. The increasing oscillation amplitudes accurately capture the conservation of absolute angular momentum as the Coriolis force deflects the parcel from its mean path. Superimposed ambient flow adds an advective component, further complexifying the trajectory. Observed trajectory patterns validate the numerical models' ability to represent the fundamental rotational dynamics governing fluid motion in this idealized scenario. While implementation differences were minor here, they highlight the importance of numerical accuracy and algorithm design for faithfully representing intricate fluid behavior, which could be amplified under more complex conditions. Because this paper is static, we were not displaying the trajectory animation from Fig. 2. If readers are interested in obtaining it, they can visit our GitHub page as mentioned in the Acknowledgments section.

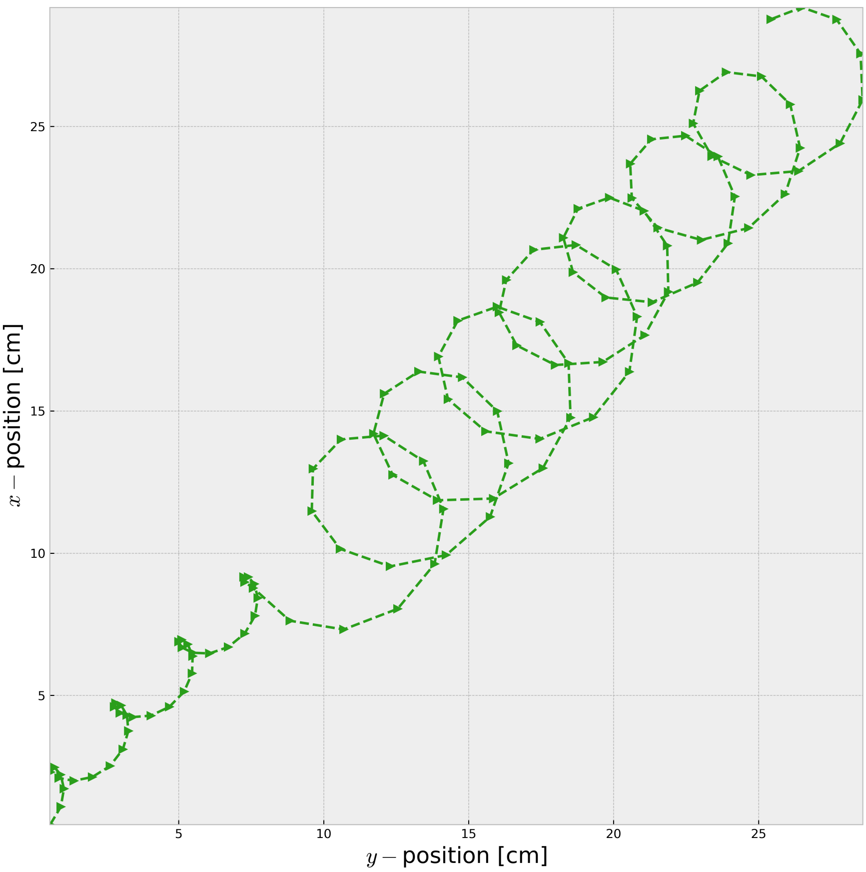


Fig. 2: The trajectory of a fluid element propelled within an ambient flow and undergoing inertial oscillation.

Figure 3 examines the execution times of various computing environments for simulating the trajectory of a fluid parcel under inertial oscillations in a two-dimensional geophysical fluid system. 1,000 simulations were run in each environment: Fortran, Python, Julia, GNU Octave, and R. The execution time is measured in seconds.

The results reveal Fortran as the clear leader in terms of both speed and consistency. The boxplots visually demonstrate this by clustering the majority of data points around the median for Fortran. Statistical data reinforces this observation. The mean execution time (0.01 seconds) for Fortran closely aligns with the median (0.01 seconds), indicating a symmetrical distribution with minimal outliers impacting the mean. Furthermore, the exceptionally low standard deviation of 0.001 seconds for Fortran underscores the remarkable consistency achieved in execution times within this environment.

In contrast, Python, R, Julia, and GNU Octave exhibit greater variability in execution times, as evidenced by the larger interquartile ranges (IQRs) and the presence of outliers. Among these environments, Python delivers a median execution time of 0.47 seconds, which is faster than both R (1.01 seconds) and GNU Octave (0.52 seconds). Additionally, Python boasts a relatively small IQR of 0.064 seconds. However, Julia lags behind considerably with a median execution time of 3.8 seconds and a larger IQR of 1.14 seconds. This signifies that half of the simulations in Julia took between 3.12 and 4.26 seconds to complete. The standard deviations further substantiate this variability. Julia exhibits the largest standard deviation (0.593 seconds) compared to Python (0.046 seconds) and GNU Octave (0.090 seconds). The presence of outliers in Julia (one at 6.86 seconds) and Octave (one at 1.69 seconds) reinforces this observation.

The choice of the most suitable environment may depend on other factors beyond just execution time. These factors include ease of use and coding expertise required for each language, availability of libraries or functionalities specific to the scientific domain, and memory usage and scalability for larger datasets. It would be beneficial to investigate the reasons behind the slower and more variable execution times observed in Julia, Python, and GNU Octave. One possibility is that the code implementations in these environments might be less optimized compared to Fortran. Further analysis could involve code profiling to identify performance bottlenecks in Julia, Python, and GNU Octave.

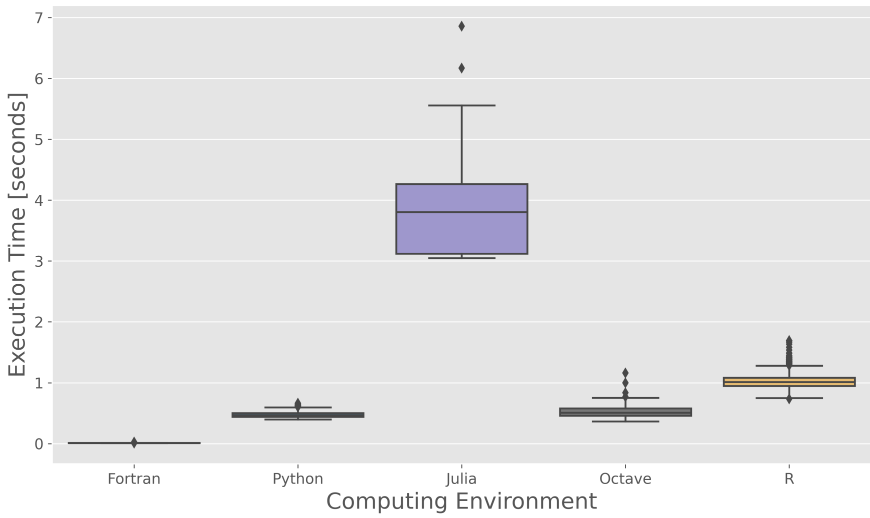


Fig. 3: Box plots comparing the execution times of simulating the 2D trajectory of a non-buoyant fluid parcel under inertial oscillations using different open-source programming languages (Fortran, Python, Julia, GNU Octave, and R) over 1,000 iterations.

Statistical analysis was employed to rigorously evaluate the execution time variations across five computing environments: Fortran, Julia, GNU Octave, Python, and R. The KW test, a non-parametric method for comparing medians across multiple groups, was conducted. The test yielded a statistically significant result (p-value = 0.000, test statistic = 4577.973), rejecting the null hypothesis that the medians of all environments were equal. This confirms the initial observations from the boxplots, suggesting at least one environment exhibits a median execution time demonstrably different from the others.

To identify environments with statistically distinct medians, Dunn's post-hoc test with Bonferroni correction was utilized. This correction accounts for multiple comparisons and minimizes the likelihood of false positives. The results corroborated Fortran's exceptional performance. Execution times in Fortran were statistically different from all other environments (Julia, Octave, Python, and R) at a significance level of $\alpha = 0.05$ (all p-values were 0.000). This translates to a significantly faster median execution time for Fortran compared to the other environments. Furthermore, pairwise comparisons among Julia, GNU Octave, Python, and R revealed statistically significant differences in their median execution times as well (all p-values were 0.000) (Fig. 4). However, the specific order of their performance remains undetermined from the this test.

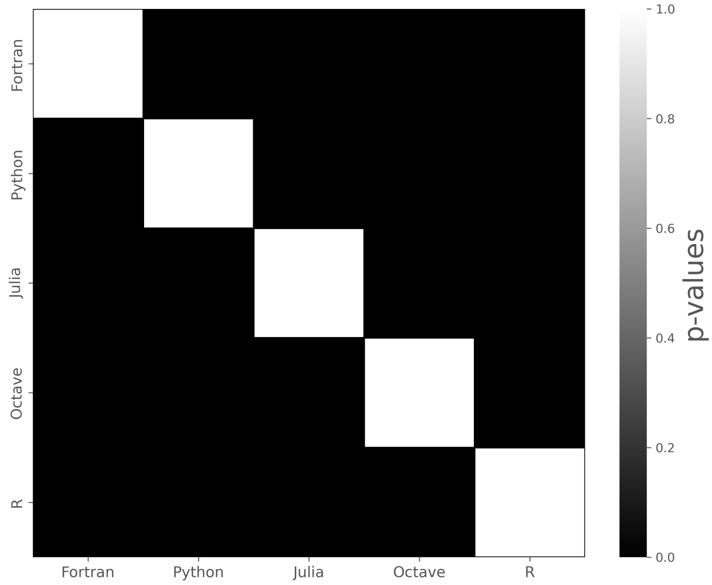


Fig. 4: Heatmap of p-values from Dunn's test (Bonferroni adjusted) comparing the computational performance of programming languages (Fortran, Python, Julia, GNU Octave, and R) in simulating a 2D fluid parcel trajectory over 1,000 iterations. Black shades indicate statistically significant performance differences.

This statistical analysis strongly supports the initial observations. Fortran emerged as the clear leader in terms of execution speed, boasting a median time demonstrably faster than all other environments. Julia, GNU Octave, Python, and R exhibited greater variability in execution times and statistically significant differences in their medians compared to each other. Further investigation, potentially focusing on code optimization within these environments, could be insightful in elucidating the reasons behind these variations.

4 Conclusion

The numerical simulation of the trajectory of a non-buoyant fluid parcel under inertial oscillation in a two-dimensional geophysical fluid system provides valuable insights into the fundamental physics governing fluid parcel transport and dispersion. By leveraging open-source programming languages, this study not only contributes to the reproducibility and transparency of scientific research but also facilitates collaborative knowledge sharing within the GFD communities.

The evaluation of computational performance across multiple programming environments, including Fortran, Python, Julia, GNU Octave, and R, revealed Fortran as the most efficient choice for simulating this idealized scenario. Statistical analysis confirmed that Fortran exhibited significantly faster execution times compared to the other environments, highlighting its suitability for computationally intensive numerical simulations in GFD. However, the selection of an appropriate programming language should also consider factors such as coding expertise, availability of specialized libraries, and scalability requirements.

Ultimately, this study serves as a foundation for developing more sophisticated hierarchical models that capture the intricate dynamics of fluid parcel trajectories in rotating systems. The insights gained from this idealized model have the potential to inform a wide range of applications.

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