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Ultramafic Melt Viscosity: A Model

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ABSTRACT

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3 A non-Arrhenian model for the Newtonian viscosity (η) of ultramafic melts is presented. The
4 model predicts the viscosity of ultramafic melts as a function of temperature (T), pressure (P),
5 H₂O content and for a range of melt compositions ($70 < Mg\# < 100$). The calibration consists of
6 63 viscosity measurements at ambient pressure for 20 individual melt compositions and 5 high-P
7 measurements on a single melt composition, all drawn from the literature. The data span 14
8 orders of magnitude of η (10^{-2} to $10^{11.8}$ Pa s), a T range of 880 to 2700K, pressures from 1 atm to
9 25 GPa, and include measurements on hydrous melts containing 0.2 to 4.4 wt. % H₂O. The T -
10 dependence of viscosity is modelled with the VFT equation [$\log \eta = A + B/(T(K) - C)$] whereby
11 A is assumed to be a common, high- T limit for these melt compositions (*i.e.* $\log \eta_{\infty} = -5.4$). The
12 pressure and composition effects are parameterised in terms of 5 adjustable parameters in
13 expanded forms of B and C . The viscosity model is continuous across T - P -composition space
14 and can predict ancillary transport properties including glass transition temperatures (T_g) and
15 melt fragility (m). Melt viscosity decreases markedly with increasing H₂O content but increases
16 significantly with increasing pressure and decreasing $Mg\#$ (*i.e.* higher Fe-content). We show
17 strong systematic decreases in T_g and m with increasing H₂O content whereas an increase in P
18 causes a rise in T_g and decrease in m . The predictive capacity of this model for ultramafic melt
19 viscosity makes it pertinent to the fields of volcanology, geophysics, petrology, and the material
20 sciences. Moreover, it provides constraints on models of magma oceans on terrestrial planets
21 and, the evolution of planetary atmospheres via magmatic degassing on exoplanets.

22

23 **Keywords:** Viscosity, Melts, Ultramafic, Peridotite, Pyrolite, Komatiite, Model, Temperature,
24 Pressure, Volcanic, Magmatic, Magma Ocean

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26

27 **1. Introduction**

28 Terrestrial magmatism and volcanism have involved ultramafic silicate melts throughout earth
29 history, via the generation of komatiite, kimberlite, and other less common alkaline melts (e.g.,
30 Arndt, 2003). Primordial Earth is believed to have been host to a deep, mafic to ultramafic,
31 magma ocean that facilitated core formation and early differentiation and crystallization of
32 Earth's interior (e.g. Fiquet et al., 2010; de Vries et al., 2016; Sun et al., 2020; Bajgain et al.,
33 2022). Reasonable estimates of the processes controlling the accumulation and differentiation of
34 terrestrial planets, including 1000s of newly discovered exoplanets, suggest the near-ubiquitous
35 presence of ultramafic melts at some stage in the geological history of planets (Putirka and Xu,
36 2021).

37 Despite the crucial role they play in planetary differentiation and degassing, ultramafic
38 melts are under-investigated relative to other terrestrial silicate melts (cf. Xie et al., 2021; Russell
39 et al., 2022). The viscosity of ultramafic melts, as a function of temperature (T), pressure (P), and
40 water content (X_{H_2O}), is, as a result, poorly constrained (DiGenova et al., 2023; See Supplement).
41 The prediction of viscosity for ultramafic melts at terrestrial P - T - X_{H_2O} conditions is an important
42 component to modelling 1) timescales of crystallization and degassing of magma oceans (e.g.,
43 Bajgain et al., 2022), 2) fragmentation conditions driving explosive eruption of low viscosity
44 magmas (Moss et al., 2011; Jones et al., 2022), and 3) the efficiency of differentiation processes
45 (e.g., convection and cooling) in lithosphere-hosted magma reservoirs.

46 Here, we have compiled and employed the available experimental data to develop a
47 predictive model for the temperature(T)-pressure(P)-composition(X) dependence of viscosity for
48 anhydrous and hydrous ultramafic melts. The model reproduces the original data ($T \sim 880$ -
49 2800K; $P \leq 25$ GPa) to within experimental error and predicts the rheological behaviour of

50 ultramafic melts, including the glass transition temperature (T_g) and melt fragility (m), as a
51 function of magnesium number (i.e. $Mg\#$) and H_2O content, temperatures, and pressures up to
52 160 GPa. The model provides a robust means of exploring volcanic, magmatic and mantle
53 processes involving anhydrous and hydrous ultramafic melts including pyrolite, peridotite, and
54 komatiite.

55

56 **2. Data Compilation**

57 Our compilation of viscosity measurements for ultramafic melts together with the
58 corresponding melt compositions and their literature sources can be found as Supplementary
59 Material. The compiled dataset includes five base melt compositions including peridotite
60 (Dingwell et al., 2004; DiGenova et al., 2023), pyrolite (Casas et al., 2023), Fe-free and high-Ca
61 peridotite (DiGenova et al., 2023) and oxidized or reduced equivalents. Melt compositions are
62 reported in terms of the five major oxide components of ultramafic chemistry SiO_2 - Al_2O_3 - FeO -
63 MgO - CaO (wt. %) as well as several minor oxide components (e.g. TiO_2 , Na_2O , etc.). The
64 compositional ranges of the melts used to calibrate the model include (in wt. %): SiO_2 , 40-51;
65 Al_2O_3 , 3-6.6; FeO_T , 0-16; MgO , 25-41; and CaO , 2-17 and the melts have magnesium numbers
66 ($Mg\#$ calculated as mol. % $MgO/[MgO+FeO_T]$) of 74-99 (average of 88; Fig. 1). The
67 compilation also includes measurements on peridotite melts with four different water contents
68 from 0.23 to 4.4 wt. % H_2O (DiGenova et al., 2023). We have restricted our experimental
69 database to include highly depolymerized, multicomponent silicate melts whose NBO/T (Mysen
70 et al., 1982; Mysen, 1988) range from 1.9 to 3.5 and SM (structural modifier parameter) values
71 vary from 47 to 64 (Fig. 1D). SM indices are calculated after Giordano and Dingwell (2003) as
72 the mol. % summation of CaO , MgO , MnO , 0.5 FeO_T , Na_2O , and K_2O .

73 The experimental measurements used to calibrate the viscosity model comprise a total of
74 68 pairs of viscosity–temperature data (Fig. 1A), including 9 made by concentric cylinder
75 viscometry and 10 by micropenetration dilatometry. The data were directly taken from the
76 publications, without modifications. The database also includes 44 estimates of melt viscosity
77 made from conventional (N=37) or flash (N= 7) differential scanning calorimetry (DSC)
78 experiments using the shift factor (SF) method (Scherer, 1984). DiGenova et al. (2023) opted to
79 employ a chemically invariant shift factor concept (SF onset = 11.20; SF peak = 9.84), which is
80 commonly used in the technical glass community (e.g., Al-Mukadam et al., 2020). Most
81 commonly, those melts are relatively fragile, and one can assume a compositional independence
82 for the shift factor as a first order approximation. However, Gottsmann et al. (2002) showed, for
83 geological melts, a significant compositional dependence of the shift factor. Based on that work,
84 Dingwell et al. (2004) derived a SF peak of 9.65 for peridotite melt compositions and this value
85 was recently used successfully by Casas et al. (2023) for pyrolite melt compositions. On that
86 basis, we have elected to use a SF peak of 9.65 and an adjusted SF onset of 11.01 to convert
87 DSC data (i.e. T_g peak, T_g onset) from DiGenova et al.'s (2023) study of anhydrous peridotite
88 melts to equivalent values of melt viscosity.

89 The hydrous samples of DiGenova et al. (2023) were synthesised at high-temperature and
90 high-pressure (Fig. 1). However, the resulting experimental data were obtained on supercooled
91 liquids, which had undergone relaxation at 1 atm. Those data, therefore, do not preserve
92 information on viscosity at elevated pressure and do not inform on any potential pressure
93 dependence of water speciation or its effect on viscosity. If water speciation is pressure
94 dependent our pressure-dependent model (see below) would not capture its effect(s).

95 Lastly, the compilation includes five high-pressure measurements of anhydrous melt
96 viscosity using *in situ* falling sphere viscometry (Xie et al., 2021). Values of melt viscosity range
97 from $10^{-1.8}$ to $10^{11.8}$ Pa s over the temperature range of 622 to 2500°C and a pressure range of 1
98 atm to 25 GPa (Fig. 1A). Although our compiled dataset is comprehensive, it must be considered
99 sparse and not extensive enough to explore all possible dependencies - such as the effects of
100 pressure on redox or H₂O speciation. Furthermore, it must be emphasized that the pressure
101 dependence we model is based on physical experiments performed on a single anhydrous (i.e.
102 not hydrous) melt composition (Xie et al., 2021). Future datasets will provide a means to, both
103 test and refine, our model.

104

105 **3. Model Development**

106 We have elected to use the Vogel-Fulcher-Tammann (VFT, Eq. 1) function:

$$107 \quad \log \eta = A + \frac{B}{T - C} \quad (1)$$

108 to account for the T-dependence of viscosity of the silicate melts (Fulcher, 1925). The VFT
109 function fits viscosity data well over large ranges of temperature and composition, is purely
110 empirical, and has only three adjustable parameters (e.g., Richet, 1984; Russell et al., 2003;
111 2022).

112 Based on the compiled measurements of viscosity, the largest effect on viscosity, after
113 temperature, is dissolved H₂O content which, as observed for most silicate melts, decreases melt
114 viscosity by up to 4 orders of magnitude (e.g., Hess and Dingwell, 1996; Schulze et al., 1996;
115 Giordano et al., 2008). Iron content has a subordinate effect wherein, relative to average
116 peridotite (FeO_T ~8-9 wt. %), viscosity increases or decreases by ~1 log unit at lower FeO_T (0-1
117 wt. %) and higher FeO_T (~15 wt. %) contents, respectively (Di Genova et al., 2023). Fe redox

118 variations generate minor variations in the viscosity of these melts that are close to measurement
119 uncertainties (DiGenova et al., 2023; Casas et al., 2023). Lastly, scrutiny of the limited high
120 pressure data set (7-25 GPa; Xie et al., 2021) suggests an increase in viscosity of about 0.1 log
121 units per GPa. On this basis, our parameterization accommodates the compositional effects of
122 H₂O and FeO_T, as well as the effects of pressure but does not consider the recently inferred
123 effects of iron redox state.

124 We have fit the VFT function to the $T(K)$ –log η dataset assuming that all melts converge
125 to a common, but unconstrained, constant representing the high-T limit to melt viscosity (i.e. A ;
126 Russell et al., 2003; Persikov and Bukhtiyarov, 2009). The concept of a high-T limit to silicate
127 melt viscosity is difficult to test directly because it requires observations at extreme
128 temperatures. However, the value of A (constant or not) must be less than any of our physical
129 measurements of melt viscosity (e.g., $\ll 10^{-1}$ Pa s for peridotitic melt). The constant A implies
130 that at super-liquidus temperatures all silicate melts become highly disordered liquids, regardless
131 of their structural arrangement at lower temperatures, and converge to a common, lower
132 viscosity limit. At these temperatures melt viscosity becomes diffusion-driven and shows an
133 Arrhenius-type T-dependence (e.g., Bottinga et al., 1995; Le Losq and Neuville, 2017).

134 In our model, therefore, each melt composition shares a common value of A but has
135 unique values of B and C reflecting the effects of other variables (i.e. composition, pressure). We
136 have expanded the terms B and C to account for two compositional variations within ultramafic
137 melts and for pressure. The term B is expanded as a function of pressure:

$$138 \quad B = b_0 + b_1 (P - 0.0001) \quad (2)$$

139 P is the pressure in GPa. B is treated as independent of major element composition for the
140 restricted range of melt compositions we consider in this model. The parameter C accounts for

141 both variations in major element concentrations using the magnesium number (i.e. $Mg\# =$
 142 $MgO/[MgO + FeO_T]$ mol %), as well as H_2O content:

$$143 \quad C = c_0 + c_1 Mg\# + c_2 X_{H_2O}^{0.5} \quad (3)$$

144 where X_{H_2O} is the mole fraction of dissolved water. The parameter C is treated as independent of
 145 pressure.

146 The optimal solution was obtained by χ^2 minimization of the function weighted to the
 147 experimental uncertainties reported for each viscosity measurement (i.e. $\log \eta_i \pm \sigma_i$):

$$148 \quad \chi^2 = \sum_{i=1}^N \left[\left(\frac{\log \eta_i - \left(A + \frac{b_0 + b_1 (P_i - 0.0001)}{T_i - (c_0 + c_1 Mg\#_i + c_2 X_{H_2O,i})} \right)}{\sigma_i} \right)^2 \right] \quad (4)$$

149 and calibrated against 68 (i.e. N) viscosity measurements. The parameter A is predetermined (A
 150 $= -5.4$) by taking the average of values derived from fitting all of the melts individually. The
 151 optimization of Eq. 4 solves for 5 parameters for the VFT-based temperature-dependent viscosity
 152 model, including b_{0-1} , and c_{0-2} . The model values with their associated uncertainties (1σ) and the
 153 covariance matrix are listed in Table 1. The magnitudes and nature of covariances between the
 154 model parameters are summarized in the Supplementary Material.

155 Two other models were considered and rejected. A model where the C term was solely a
 156 function of $Mg\#$ (i.e. not including H_2O) was initially developed. However although it fit the
 157 experimental data well, that model predicted unreasonable (i.e. aphysical) values of melt
 158 fragility. We initially considered a model where B was also a function of H_2O content however,
 159 the associated model parameter had a 1σ confidence limit that included zero. Dropping that term
 160 and reducing the adjustable parameters (from 6 to 5) made no appreciable difference to the
 161 quality of the fit. A sample calculation for the viscosity for a hydrous peridotite melt at pressure
 162 is included as Supplementary Material.

163 4. Results

164 4.1 Model Parameters

165 The model reproduces the data well (Fig. 2A,B) and the Root Mean Square Error
166 (RMSE) for the optimization is 0.21 log units. The average misfit for the anhydrous sample
167 measurements is 0.17 log units and the maximum is 0.72 log units for a single sample (S34F0,
168 Hi-Ca peridotite; DiGenova et al., 2023). The average misfit for measurements of hydrous melts
169 (N= 7) is 0.30 log units and the maximum deviation is 0.66 log units for a single measurement
170 (S38F5W1; DiGenova et al., 2023). The model reproduces the high-pressure data (N=5) well
171 with an average misfit of 0.05 log units. The model VFT functions for each melt composition are
172 well behaved and show a systematic variation with H₂O content (Fig. 2B); hydrous melts have
173 lower viscosity and are more Arrhenian (e.g., Giordano et al., 2008), similar to the case for
174 hydrous rhyolites (Hess and Dingwell, 1996). The model curves for high pressure ultramafic
175 melts (Fig. 2B; green symbols) show an increase in viscosity and become more Arrhenian with
176 pressure. The effect of increasing iron content, expressed as decreasing *Mg#*, is to cause a
177 subordinate increase in melt viscosity.

178 The model high-temperature limit to melt viscosity (i.e. *A*) is -5.4 which is slightly lower
179 than theoretical expected limits (i.e. -4.5 to -5; Angell, 1985) but like that found for other
180 multicomponent silicate melts (i.e. Russell et al., 2003; Giordano et al., 2008; Li et al., 2020).
181 The *B* term is related to activation energy and has a model value of 5558.3 K for anhydrous
182 ultramafic melts at ambient pressure (*bo*; Table 1). The effect of pressure is to increase *B* at a
183 rate of ~77.5 K per GPa (Fig. 2C). The base value of *C* for anhydrous melt, defined by *c₀*, is
184 422.9 K (Table 1) which increases weakly with increasing *Mg#* (i.e. *c₁*) and decreases strongly as

185 a function of $X_{H_2O}^{0.5}$ (Fig. 2D). The values of C converge to between 22 K and 103 K as X_{H_2O}
186 approaches 1.0 for melts having $Mg\#$ s of 70 to 100, respectively (Fig. 2D).

187 The effects of H_2O on melt viscosity are pronounced (Fig. 3A) causing significant and
188 continuous decreases in viscosity with increased H_2O content. As observed in other silicate melt
189 systems (e.g., Hess and Dingwell, 1996; Schulze et al., 1996; Giordano et al., 2008), the effects
190 of H_2O on melt viscosity are greatest at lower H_2O contents and decrease with increased H_2O
191 content. These hydrous ultramafic melts exhibit a more Arrhenian-like temperature dependence
192 than their anhydrous counterparts, a feature also exhibited by hydrous calcalkaline rhyolite melts
193 (Hess and Dingwell, 1996). The viscosity of ultramafic melts increases with pressure (Fig. 3B)
194 and the predicted increase in melt viscosity with pressure is most pronounced at lower
195 temperatures. The corresponding curves for hydrous ultramafic melts are displaced to lower
196 viscosity (Fig. 3B) but the relative effects of pressure on the model viscosity are the same. This
197 results because of our assumption that B is linearly dependent on pressure and C being
198 independent of pressure.

199

200 *4.2 Transport Properties: Tg_{12} and m*

201 Important attributes of this model for ultramafic melt viscosity are: i) it is based solely of
202 results of high-P-T physical experimentation, ii) it accurately reproduces the original data to
203 within experimental error, iii) it uses a minimum number of adjustable parameters ($N=7$), iii) it is
204 continuous in composition (i.e. H_2O , $Mg\#$), pressure, and temperature space, and iv) it
205 independently predicts other transport properties including glass transition temperatures (Tg_{12})
206 and melt fragility (m).

207 We take the glass transition temperature (Tg_{12}) as the temperature (K) at which melt
 208 viscosity reaches a value of 10^{12} Pa s. Values of Tg_{12} are calculated from the parameters A , B and
 209 C predicted as a function of melt composition and pressure (Table 1):

$$210 \quad Tg_{12} = \frac{B}{12-A} + C. \quad (5)$$

211 Our model independently reproduces the Tg_{12} - X_{H2O} relationship (1 atm) described by DiGenova
 212 et al. (2023) which used a Gordon-Taylor expansion constrained to match the Tg_{12} value of
 213 water, similar to the approach of Weidendorfer et al. (2023) for hydrous carbonates. Glass
 214 transition temperatures decrease nonlinearly with increased H_2O content and increase linearly
 215 with pressure (Fig. 3C). Anhydrous ultramafic melts with an $Mg\#$ of 88 have a Tg_{12} of 980 K,
 216 decrease continuously with water content, and extrapolate to a value of 390 K at $X_{H2O} = 1$. At a
 217 pressure of 25 GPa, Tg_{12} for an anhydrous melt increases to 1090 K and extrapolates with
 218 increasing water content to its limit at 501 K.

219 Melt fragility (m) is the measure of how rapidly viscous flow properties change with
 220 temperature as melts approach Tg_{12} (Angell, 1985). Fragility values discriminate between strong
 221 liquids (low m) having near-Arrhenian behaviour *versus* fragile melts (high m) which exhibit
 222 non-Arrhenian T-dependence (Angell, 1985). Here, we use the steepness index (m) as an
 223 estimate of melt fragility which for the VFT function can be calculated as (see Russell et al.,
 224 2022 and references therein):

$$225 \quad m = \frac{B}{Tg_{12} \left(1 - C/Tg_{12}\right)^2}. \quad (6)$$

226 The fragility of ultramafic melts decreases nonlinearly with increasing H_2O content and with
 227 increasing pressure (Fig. 3D). An anhydrous melt with an $Mg\#$ of 88 has a fragility of ~ 52 at
 228 ambient pressure *vs.* ~ 44 at 25 GPa. However, the rate of decrease in fragility with increasing
 229 X_{H2O} is independent of pressure.

230 For all systems where A is assumed to be a constant, melt fragility is limited to values $>$
231 $[12-A]$ (e.g., Russell et al., 2017; 2022) which, for this parameterization, implies a lower limit to
232 fragility of 17.4 (Fig. 3D). Hydrous ultramafic melts at ambient pressure and at high pressure
233 (i.e. 25 GPa) show decreasing fragility with increased water content and extrapolate to fragilities
234 of ~ 31 and ~ 27 at X_{H_2O} of 1, respectively. These extrapolated values are reasonably close to
235 experimental estimates of fragility for low ($m = 14$) and high ($m \sim 20-25$) density water (Amann-
236 Winkel et al. 2013). In contrast, the DiGenova et al (2023) model has a theoretical fragility limit
237 of 14.9 (i.e. $A \sim -2.9$) but extrapolates to negative (nonphysical) values of m at high water
238 contents (Fig. 3D).

239 Pressure and H_2O content have competing and opposing effects on melt viscosity (Fig.
240 4A). Under isothermal conditions for a melt at 2300°C, increased H_2O content ($X_{H_2O} = 0 - 0.35$)
241 causes ~ 1 log unit decrease in melt viscosity at pressures of 0 – 30 GPa, whereas rising pressure
242 causes ~ 1 log unit increase in viscosity. Values of Tg_{12} contoured for X_{H_2O} and P show Tg_{12} to
243 decrease by 200 – 300K for X_{H_2O} of 0 to 0.35 and to increase ~ 100 K over a pressure range of 30
244 GPa for a fixed H_2O content (Fig. 4B). In contrast to Tg_{12} , fragility decreases in response to
245 increased water contents and increased pressure; values of m for anhydrous melts decrease $\sim 40\%$
246 for $X_{H_2O} \sim 0.35$ at all pressures (Fig. 3D, 4C).

247 The relationships between Tg_{12} and m as a function of pressure and H_2O content are
248 summarized in Figure 5 for an ultramafic melt with an Mg# of 88. At constant pressure (Fig. 5,
249 dashed lines for 5 to 30 GPa), Tg_{12} and m decrease with increased H_2O content, whereas at
250 constant water content (Fig. 5, blue lines), Tg_{12} increases with pressure whilst m decreases with
251 pressure. We have also compared our predictions for anhydrous peridotite to model values of
252 Tg_{12} and m for anhydrous diopside ($NBO/T = 2$) as a function of pressure (Fig. 5, heavy black

253 line). Li et al. (2020) developed a pressure dependent model for predicting the viscosity of
254 anhydrous melts in the system Albite-Anorthite-Diopside. Their model predicts a pressure-
255 dependent trend in Tg_{12-m} values for diopside melt that parallels the values we independently
256 predict for anhydrous ultramafic melts (Fig. 5). Both models predict an increase in Tg_{12} with
257 pressure and a concomitant decrease in fragility whereas for more polymerized melts, albite and
258 anorthite (i.e. NBO/T =0), Li et al. (2020) predicted an increase in melt fragility with pressure.

259

260 *4.3 Filling a Gap in Knowledge*

261 Our model is a means of predicting the viscosity of ultramafic melts over a wide range of
262 geological conditions ($T, P, Mg\#, X_{H_2O}$). These melt compositions are pertinent to many
263 magmatic, volcanic, and tectonic processes. As discussed by DiGenova et al. (2023) previously
264 published models for silicate melt viscosity fail to reproduce measurements on ultramafic melts
265 (see Supplement Material for full discussion). In that regard, our model fills a gap in knowledge
266 and is a means to explore the properties and behaviour of these melts within a variety of
267 terrestrial and extra-terrestrial environments. We use $Mg\#$ as a proxy for compositional
268 variations in ultramafic melts and this supports the model's use for melts having MgO contents in
269 excess of 20 wt. % and over a range of Mg:Fe ratios (i.e. $Mg\#$ 70-100). This includes melt
270 compositions spanning pyrolite, peridotite and komatiites.

271 Two models that account for the effects of pressure on melt viscosity (Persikov and
272 Bukhtiyarov, 2009; Duan, 2014) fail to predict the viscosity of ultramafic melts at ambient or
273 high-pressure (see Supplementary Material). Our calibration for pressure relies on the most
274 recent, but limited (N=5), dataset of Xie et al. (2021) who used in-situ falling sphere viscometry
275 to measure peridotite melt viscosity at 7 to 25 GPa. Our model accurately reproduces both the

276 high- and low-pressure datasets as well as reproducing the pressure dependence that is implicit in
277 the high-P data to within error. This can be illustrated by fitting the Xie et al. (2021) dataset to an
278 Arrhenian model:

$$279 \quad \log \eta = A_x + \frac{Bo_x + Bp_x \times P}{T(K)} \quad (7)$$

280 where A_x (-6.36), Bo_x (10053.8), and Bp_x (105.5) are adjustable parameters and Bp_x provides a
281 linear dependence on pressure (P in GPa). As would be expected, the simple Arrhenian model
282 (Eq. 7) reproduces the original data well (Fig. 6A; dashed lines). The isothermal viscosity curves
283 predicted as a function of P , at the temperature of each experiment, intersect each datapoint (i.e.
284 $\log \eta : P$). Our model also assumes a linear pressure dependence for B (see Table 1; $b_2 = 84.2$)
285 and isothermal curves from our model (Fig. 6A; solid lines) reproduce the Xie et al. (2021) data
286 equally well. Furthermore our model extrapolates slightly better to the 1 atm data of Dingwell et
287 al. (2004).

288 We elected not to calibrate our model using the high-pressure measurements of ultramafic
289 melt viscosity reported by Liebske et al. (2005) and Brown (2012). We made this decision
290 because of the significantly larger scatter in their data (as previously observed by Xie et al.,
291 2021; Huang et al., 2024) when compared to the more precise ambient-pressure measurements
292 ($\pm 0.08 - 0.25 \log_{10} \eta$) and the more coherent character of the data from Xie et al. (2021). Their
293 data, however, do provide an independent test of our model.

294 Liebske et al. (2005) used in-situ falling sphere experiments to provide measurements of
295 viscosity for peridotite melts at temperatures of 1750 – 2250 °C and pressures of 2 – 13 GPa
296 (Fig. 6). The experiments are identified as being in Cell A and B or in Cell C. The latter set-up
297 (Cell C) featured a slightly different geometry (i.e. shorter travel distance) which resulted in
298 fewer snapshots of the falling spheres (pers comm.; C. Liebske, April 2024; Xie et al., 2021) and,

299 thus, less precision (see Kono, 2018; Ashley et al., 2024) and more potential scatter (Fig. 6B).
300 Our model predicts most of their data to within 0.25 log units and all data to within 0.5 log units.

301 The high-pressure viscosity data of Brown (2012) are for two different komatiite melts at
302 temperatures of ~1570 – 2200K and pressures of 1 – 10.8 GPa. Our model reproduces most of
303 the viscosity data for the Barberton komatiite to within 0.25 log units but fails to reproduce the
304 Gorgona komatiite data (Fig. 6B). This discrepancy between model and data is because the
305 Gorgona komatiite lies outside the compositional range of our calibration. Although it has an
306 Mg# of 74 (within our model range), the MgO content is too low (< 18 wt. %) and outside of our
307 model range (25 < MgO wt. % < 41).

308

309 **5. Discussion**

310 *5.1 Pressure dependence of viscosity for depolymerized melts ($NBO/T > 2$)*

311 A benefit of predictive models calibrated on observations or direct physical measurements
312 is that they can be interrogated for additional insights. This model, for example, allows for the
313 calculation and independent prediction of ancillary properties of silicate melts, such as glass
314 transition temperatures and melt fragility. These values can be compared against values
315 measured directly by calorimetric or spectroscopic methods, respectively (e.g., Di Genova et al.,
316 2023). Robust models calibrated on high quality data and fit to a minimum number of adjustable
317 parameters commonly allow for reliable extrapolation beyond the original data. This provides
318 insights into parameter spaces that have yet to be explored experimentally.

319 The effect of pressure on melt viscosity is important for constraining the properties of
320 mantle and crustal melts and for modelling their origins, transport, and the processes that govern
321 their thermochemical evolution. An in-depth discussion of the effects of pressure on silicate melt

322 viscosity, including structural effects on melt viscosity, is afforded by the review papers of
323 Sakamaki and Ohtani (2022) and Kono (2018). Our model is consistent with the viscosity of
324 ultramafic silicate melts being linearly dependent on pressure (Fig. 6C-D); the model reproduces
325 the (limited) available high-pressure data and extrapolates to reasonable values beyond the
326 original database. Several MD simulations have suggested, however, that fragile, depolymerised
327 silicate melts, including basalt (i.e. Bajgain et al., 2022) and peridotite (i.e. Huang et al., 2024),
328 show non-linear variations in melt viscosity with increasing pressure. At present the
329 experimental data do not support a higher order treatment for pressure which is best tested by
330 additional physical experimentation. Furthermore, we have no idea if extrapolation of the
331 model's pressure effect at undercooled conditions is sound; for example, we have no data that
332 inform on the effect(s) of pressure at temperatures close to T_g . The pressure dependence of
333 viscosity for fragile melts could be very different near T_g due to temperature-driven changes in
334 melt structure.

335 The experimental results and model proposed by Liebske et al. (2005) bear additional
336 comment because of its implications for the pressure dependence of melt viscosity. Their
337 analysis of the experimental data led them to suggest that there was a maximum in viscosity at
338 ~7–8 GPa followed by a steady decrease with pressure (see Fig. 3 in Liebske et al., 2005). On
339 that basis they proposed a VFT-based model which expressed B as a 3rd order polynomial in P .
340 Their model results in a maximum viscosity at 7–10 GPa and then a monotonic decrease to
341 unrealistic values which precludes extrapolation beyond their original data (grey lines; Fig. 6C-
342 D). However, we see no suggestion of a maximum in melt viscosity in the experimental datasets
343 nor in our model (black lines; Fig. 6C-D). The maximum in viscosity and the negative P-
344 dependence proposed by Liebske et al. (2005) is mainly driven by data derived from Cell C

345 which show more scatter (Fig. 6B) and may be less precise (see Ashley et al., 2024). Clearly,
346 additional experiments designed to inform on this issue are needed in lieu of further speculation
347 or discussion at this time.

348 There is general consensus on the behaviour of viscosity with increasing pressure for
349 polymerised melts ($NBO/T < 1$) where a negative pressure dependence can occur between 1 and
350 13 GPa before increasing with additional pressure (Wang et al., 2014; Sakamaki and Ohtani,
351 2022). For depolymerised melts with an $NBO/T > 2$, viscosity is weakly dependent on P and
352 generally increases (monotonously) with pressure at isothermal temperatures (see Wang et al.,
353 2014; Sakamaki and Ohtani, 2022; Xie et al., 2021). Recent experimental work on simple
354 depolymerised melts, including: i) Spice et al. (2015) on Fe_2SiO_4 melts ($NBO/T = 4$) and ii)
355 Cochain et al. (2017) on $MgSiO_3$ (NBO/T 1.75-1.96) and $CaSiO_3$ ($NBO/T = 1.92$ -2.12) melts,
356 have suggested a weak negative pressure dependence up to 13 GPa. In contrast, the molecular
357 dynamic simulation studies of melt viscosity by Zhang et al. (2010; $MgSiO_3$ and $CaSiO_3$ melts)
358 and Sun et al. (2018; Fe_2SiO_4), and simulations of Mg_2SiO_4 melt viscosity by Adjaoud et al.
359 (2008) and Drewitt et al. (2022) argue for a continuous increase in viscosity with increasing P.
360 The most recent MD simulation data by Dufils et al. (2018) and Huang et al. (2024) on
361 multicomponent ultramafic melts ($NBO/T > 2$), including komatiite, peridotite, and pyrolite, also
362 show a continuous increase in viscosity with pressure. These latter results are fully consistent
363 with our experimentally constrained predictive model.

364

365 *5.2 Mantle melts and viscosity*

366 A unique strength of this model is to predict the viscosity of mantle melts at mantle
367 temperature-pressure conditions. The viscosity values calculated for an anhydrous and hydrous
368 (5 mol% H_2O) ultramafic peridotitic melt ($Mg\# \sim 88$) are plotted as a function of the temperature-

369 pressure conditions defined by the liquidus for a fertile mantle composition (KLB-1; Fig. 7).
370 Melt viscosity at the surface is $\sim 1 - 1.5$ Pa s and shows a slight decrease (~ 0.5 log units) with
371 increasing T and P to the base of the lithosphere after which viscosity remains nearly constant
372 (Fig. 7B). Whilst the effect of pressure is to increase viscosity, the corresponding rise in
373 temperature with depth compensates. The pattern is the same for anhydrous and hydrous melts; a
374 5 mol% H₂O content simply reduces melt viscosity by ~ 0.5 log units. For comparison, we
375 calculated the melt viscosity at the same mantle liquidus temperatures but without accounting for
376 the effects of pressure (i.e. b_l set to 0; Table 1). Ignoring the effect of pressure on the viscosity of
377 ultramafic melts would erroneously suggest a near linear continuous decrease in viscosity (Fig.
378 7B; ~ 2 orders of magnitude over 30 GPa).

379 The Arrhenian model of Shaw (1972) has commonly been used to calculate the viscosity of
380 the magma ocean. Shaw's model is quite robust for silicate melts (anhydrous and hydrous) at
381 high temperatures where melts show Arrhenian behaviour (Russell et al., 2022) although, here, it
382 predicts viscosity values at surface temperatures ~ 1.5 log units higher than the present model.
383 The Shaw (1972) model does not account for pressure and, therefore, also predicts a linear
384 decrease in viscosity as a function of mantle temperature that is too high at upper mantle
385 conditions and too low below the transition zone. More importantly it predicts a negative
386 gradient in viscosity that could significantly impact calculated values of melt mobility (i.e. ratios
387 of η/ρ) at high mantle pressures.

388 We have extrapolated our model well beyond the calibration dataset (see caption Fig. 7C)
389 to predict melt viscosity at depths and pressures (0–150 GPa) found in the early Earth's magma
390 oceans (Fig. 7C). We have adopted the P-T array of Huang et al. (2023; modified from Fiquet et
391 al., 2010) to represent the temperature distribution with depth in the magma ocean. The change

392 in temperature with depth is normalized and plotted as $[1 - T/T_{150\text{GPa}}]$ where $T_{150\text{GPa}}$ is the
393 maximum temperature located at 150 GPa (dashed line; Fig. 7C).

394 Our model has constant values of A and C (for a fixed H_2O content), and the B parameter
395 rises from 5558 K to 17,957 K over 160 GPa. The model values of viscosity for anhydrous and
396 hydrous (5 mol%) peridotitic melts are calculated along the P-T array and show a pronounced
397 (0.5-1 log units) decrease to a pressure of ~ 40 GPa before increasing steadily at a lower rate. The
398 complex pattern in melt viscosity reflects the interplay between temperature and pressure effects
399 on viscosity dictated by the shape of the adopted P-T array in the magma ocean. The steep rise in
400 temperature in the shallow mantle drives the melt to a low extreme but once the temperature
401 gradient shallows the effect of pressure begins to dominate driving the melt viscosity back to
402 higher values.

403 For comparison we show two model viscosity curves derived from MD simulations and
404 using the same P-T array. Huang et al. (2024) fit VFT-based polynomial equations to a series of
405 MD simulations of peridotitic melt viscosity at temperatures of 2200, 3000, 4000 and 6000K
406 over the pressure range of 1–159 GPa. They proposed two separate equations for the P-T
407 dependence of viscosity for the temperature intervals 2200 – 3000 K and 4000 – 6000K, which
408 are connected by a dashed line in Figure 7C. The two model expressions have constant values of
409 C but pressure dependent expressions for A and B . In the lower temperature equation, values A
410 and B vary with pressure up to 160 GPa as -5.93 to -27.3, and 748 to 62,916, respectively. The
411 higher temperature expression implies ranges of A and B of -7.1 to -10.3, and 1414 to 28,726,
412 respectively.

413 We used the dataset derived from MD simulations of peridotite (PHN1611) melt viscosity
414 reported by Dulfis et al. (2018) to create a pressure dependent VFT-based model (i.e., $\log \eta = -$

415 $2.39 + [523.7 + 22.04 P(\text{GPa})]/[T(\text{K}) - 1557]$). This simple model reproduces the MD dataset and
416 has an RMSE of 0.025 and an average and maximum misfit of 0.019 and 0.05 log units,
417 respectively. The model for the MD simulations of Dulfis et al. (2018) has constant A and C
418 parameters, whilst B rises from 524 to 4030 over 160 GPa. Our VFT-fit to their data is used to
419 plot another MD-based viscosity curve for the early Earth magma ocean as a function of the
420 peridotite P-T liquidus (Fig. 7C).

421 The model curves for the two MD datasets (i.e. Huang et al., 2024; Dulfis et al., 2018)
422 show a similar decrease in viscosity within the upper mantle due to the steep temperature
423 gradient. The model curves define viscosity minima in the early Earth molten mantle at 11 and
424 34 GPa, respectively (*versus* ~40 GPa for our model). At pressures greater than 1 GPa, the two
425 MD models predict a total range in (anhydrous) melt viscosity of ~0.3-0.4 log units whereas our
426 model for anhydrous and hydrous peridotitic melts predicts a total range in viscosity of 0.8 log
427 units. The Huang et al. model agrees reasonably well with ours in two areas: i) ~ 25 GPa where
428 we have data (heavy line segments), and ii) at depth (high-T and P) where the MD simulations
429 are optimal, and our model is extrapolated well past the calibration data. The largest deviation is
430 found at ~40-50 GPa which coincides with temperatures between Huang's two predictive
431 equations and pressures just outside of our calibration (Fig. 7C). The Dulfis et al. (2018) MD-
432 based model is nearly parallel to the Huang et al. model but predicts viscosities ~0.25-0.5 log
433 units higher along the early Earth peridotite liquidus curve.

434

435 **6. Conclusions**

436 Beyond the Earth and terrestrial planets lies the rapidly growing realm of discovery of
437 exoplanets. These objects number in their thousands already with no end in sight of their
438 growing number. One of the major data sets being accumulated on these planetary bodies is that

439 of the chemistry, including identification of gas species, in their planetary exospheres (e.g., Heng
440 and Showman, 2015). Models that are being developed for atmospheric chemistry of exoplanets
441 must in future rely on a more robust set of models for the processes of planetary degassing and
442 these processes in turn will rely for many exoplanets on the understanding of the behavior of
443 ultramafic melts at depth or at the surface of the exoplanet (both at variable pressures). With the
444 model presented here we hope to have contributed to a path towards the modelling of ultramafic
445 planetary liquids and their degassing behavior.

446

447 **CrediT authorship contribution statement**

448 **J.K. Russell:** Conceptualization, modelling, data analysis, writing. **K-U Hess:** Data compilation,
449 literature review, model evaluation and comparison, writing. **D.B. Dingwell:** Data assessment,
450 implications, writing, reviewing.

451

452 **Declaration of competing interest**

453 The authors declare they have no competing financial interests or personal relationships
454 that could have appeared to influence the work reported herein.

455

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463

464 **Appendix A. Supplementary material**

465 Supplementary material related to this article can be found on-line at x-xxxxx.

466

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592

Figure Captions

594 **Figure 1.** Compiled experimental database of ultramafic melts used to calibrate predictive
 595 model. (A) Data plotted as $\log \eta$ (Pa s) vs. $10000/T(K)$ where symbols denote measurements of
 596 anhydrous (grey), hydrous (blue) and anhydrous high-pressure (green) melts. Data of Liebske et
 597 al. (2005; crosses) were not used in the calibration. Compositional range of compiled ultramafic
 598 melts expressed as (B) Al_2O_3 vs. $SiO_2/(MgO+FeO)$; (C) Mg# vs. H_2O content; and (D) SM vs.
 599 NBO/T indices.

600
 601 **Figure 2.** Results of model optimization. (A) Comparison of predicted values of $\log \eta$ (Pa s) to
 602 measured values. Dashed lines denote ± 0.5 log units. (B) Model VFT curves (solid lines)
 603 calculated (see Table 1) for each melt composition and compared to data (see Supplementary
 604 Materials). VFT curves for high-pressure data are calculated at their experimental pressures (i.e.
 605 Xie et al., 2021). (C) Model values of B as a function of pressure (1 atm to 25 GPa). Note B is
 606 independent of H_2O content and Mg#. (D) Model values of C as a function of X_{H_2O} for ultramafic
 607 melts having Mg#'s from 70 to 100. Solid line is for a melt with Mg# of 88. Note C is
 608 independent of pressure.

609
 610 **Figure 3.** Model predictions for an ultramafic melt (Mg# = 88). (A) Temperature-dependent
 611 curves of melt viscosity for H_2O contents of 0 (anhydrous) to 30 mol%. (B) The effect of
 612 pressure on model VFT curves for an anhydrous and hydrous (5 mol%) ultramafic melt. (C)
 613 Values of Tg_{12} (K) (i.e. $\eta = 10^{12}$ Pa s) predicted as a function of X_{H_2O} (solid black line) at 1 atm;
 614 dashed black lines show increase in Tg_{12} with increased pressure. Model 1 atm. values from
 615 DiGenova et al. (2023) are shown by red dashed line. Values for anhydrous and hydrous melts in
 616 compiled dataset are shown as grey and blue coloured symbols, respectively. (D) Values of
 617 fragility predicted as a function of X_{H_2O} (solid black line); dashed black lines show decrease in
 618 melt fragility with increasing pressure. Red dashed line is 1 atm. model of DiGenova et al.
 619 (2023). Grey shaded line denotes the lower limit in melt fragility predicated by a constant value
 620 of A (i.e. -5.4; Russell et al., 2017; 2022).

621
 622 **Figure 4.** Contour maps of effects of pressure (P) and H_2O content (X_{H_2O}) on transport properties
 623 of a fixed ultramafic melt composition (Mg# = 88). (A) Contours of isothermal (2300 °C) melt
 624 viscosity decrease with H_2O content and increase with P . (B) Contour plot showing increase in
 625 Tg_{12} values with increasing P and a decrease with increasing X_{H_2O} . (C) Fragility (m) contours
 626 show decrease from ~ 50 to ~ 32 with increasing X_{H_2O} and P .

627
 628 **Figure 5.** Graphical summary of variations in Tg_{12} (K) and melt fragility (m) as a function of
 629 H_2O content (0-30 mol%) and P (0-25 GPa). Predicted Tg_{12} (K) increases with rising P and
 630 decreases strongly with increased H_2O . Melt fragility (m) decreases with increased H_2O content
 631 and increased P . Solid black line is for anhydrous diopside melt as predicted by model of Li et al.
 632 (2021) and shows a similar rise in Tg_{12} (K) and decrease in melt fragility (m) with increased P (0-
 633 25 GPa).

634
 635 **Figure 6.** P -dependence of predicted and observed viscosity of peridotite melts. (A) Isothermal
 636 values of viscosity predicted by this model (Table 1; solid lines) and values predicted by an
 637 Arrhenian model (dashed lines) fitted to the original high P measurements of Xie et al. (2021;

638 black symbols; see text). (B) Comparison of high P data sets not used to calibrate the model
639 *versus* the values predicted by our model; data of Xie et al. (2021) were used to calibrate the
640 effects of pressure and are shown here for comparison. Non-calibration high P datasets include
641 experiments of Liebske et al. (2005) and Brown (2012). Dashed lines indicate ± 0.25 log units.
642 (C) Isothermal values (2000K and 2800K) of viscosity predicted for a range of P (0 - 30GPa).
643 Model lines are for this work (solid black; blue lines for 5 mol% H₂O), model of Liebske et al.
644 (2005; grey lines), and Dingwell et al. (2004; 1 atm model). Shaded fields indicate the range of
645 data used to calibrate the two models: i) Liebske et al. (2005; yellow, solid outline) and ii) this
646 model (green, dashed outline). (D) Model values of Tg_{12} (K) for a range of P (0 - 30GPa) as
647 discussed in (C).
648

649 **Figure 7.** Predicted viscosity of ultramafic melts at mantle conditions. (A) Mantle T distributions
650 to 30 GPa in modern Earth for fertile mantle composition (KLB-1; modified from Grove and
651 Parman, 2004) at solidus (S) and liquidus (L) conditions. Thin lines show rise in predicted Tg_{12}
652 (K) values with P for anhydrous (black) and hydrous (5 mol%, blue) melt. (B) Heavy lines
653 denote melt viscosity calculated at KLB-1 liquidus T and P (Panel A). Dashed lines are melt
654 viscosities where the effects of P are not accounted for. Thin solid lines are anhydrous and
655 hydrous viscosity values predicted by Shaw (1972). (C) Melt viscosity for an early Earth magma
656 ocean at liquidus conditions and deep mantle pressures (P-T array from Fiquet et al., 2010;
657 Huang et al., 2023). (1) Viscosity calculated with this model for anhydrous and hydrous
658 peridotitic melt. (2) Viscosity of peridotitic melt predicted by Huang et al. (2023); dashed line is
659 extrapolation between 2 functions for different temperature regimes. (3) Model curve for
660 peridotite melt viscosity-based MD simulations reported by Dulfis et al. (2018; see text). Heavy
661 line segments (1, 2, and 3) indicate range of pressures over which models were calibrated.
662 Dashed black line denotes mantle temperature scaled as $\Delta T = 1 - T/T_{150\text{GPa}}$.
663

Table 1. Model parameters for VFT-based temperature-dependent viscosity of anhydrous and hydrous ultramafic melts¹.

$\log \eta = A + [b_0 + b_1 (P-0.0001)] / [T - (c_0 + c_1 \text{ Mg\#} + c_2 X_{\text{H}_2\text{O}}^{0.5})]$								
Parameters	Values	$\pm \sigma$						
A	-5.4	0.2						
Covariance Matrix								
	b ₀	b ₁	c ₀	c ₁	c ₂			
b ₀	5558.3	106	b ₀	11163				
b ₁	77.49	12	b ₁	-495	147			
c ₀	422.93	9.2	c ₀	-663	30	84		
c ₁	2.69	0.1	c ₁	-0.30	0.01	-0.49	0.01	
c ₂	-589.4	13	c ₂	-634	29	42	-0.06	175

¹Model optimization has MSE of 0.045 and RMSE of 0.21

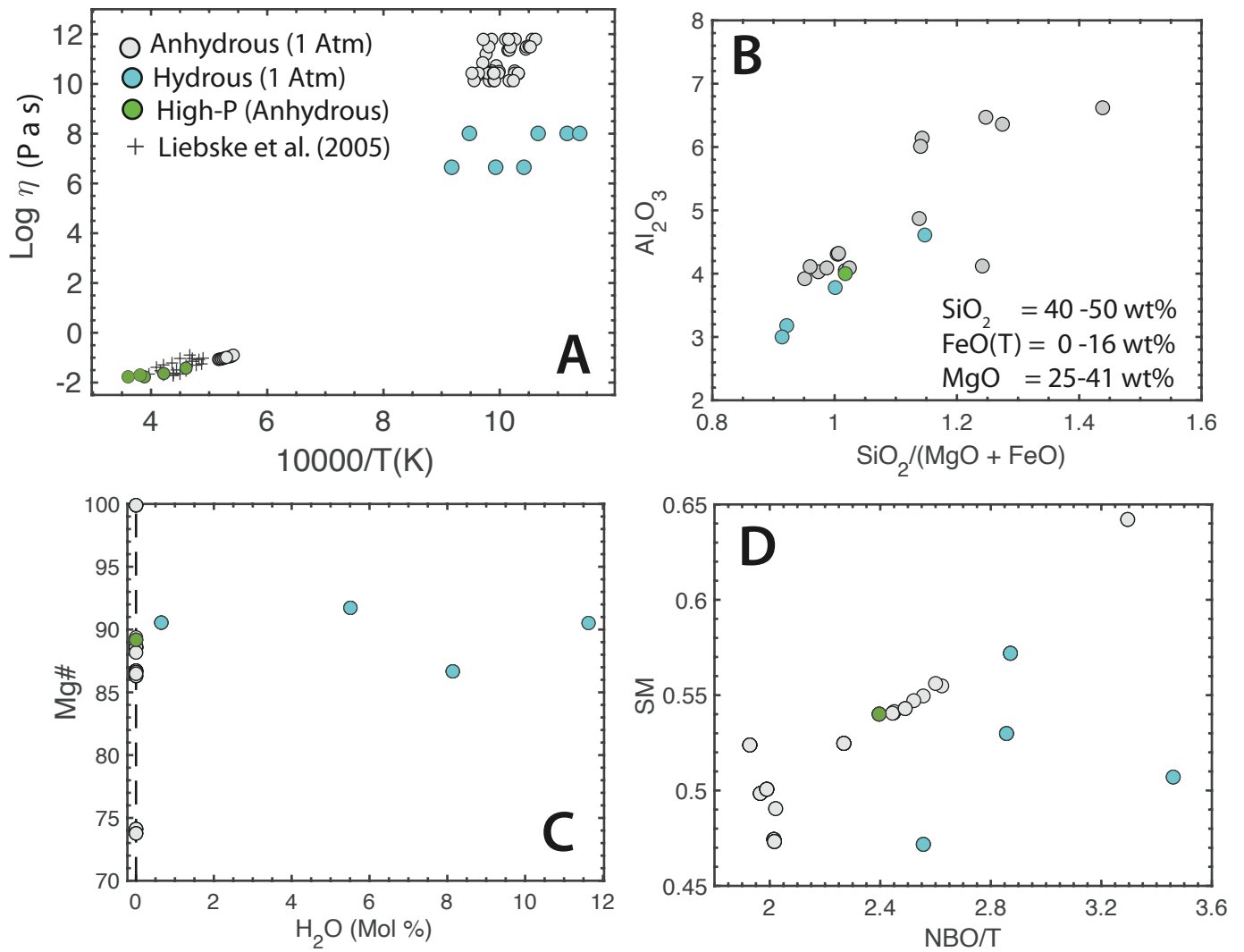


Figure 1. Ultramafic melt viscosity, 2024)

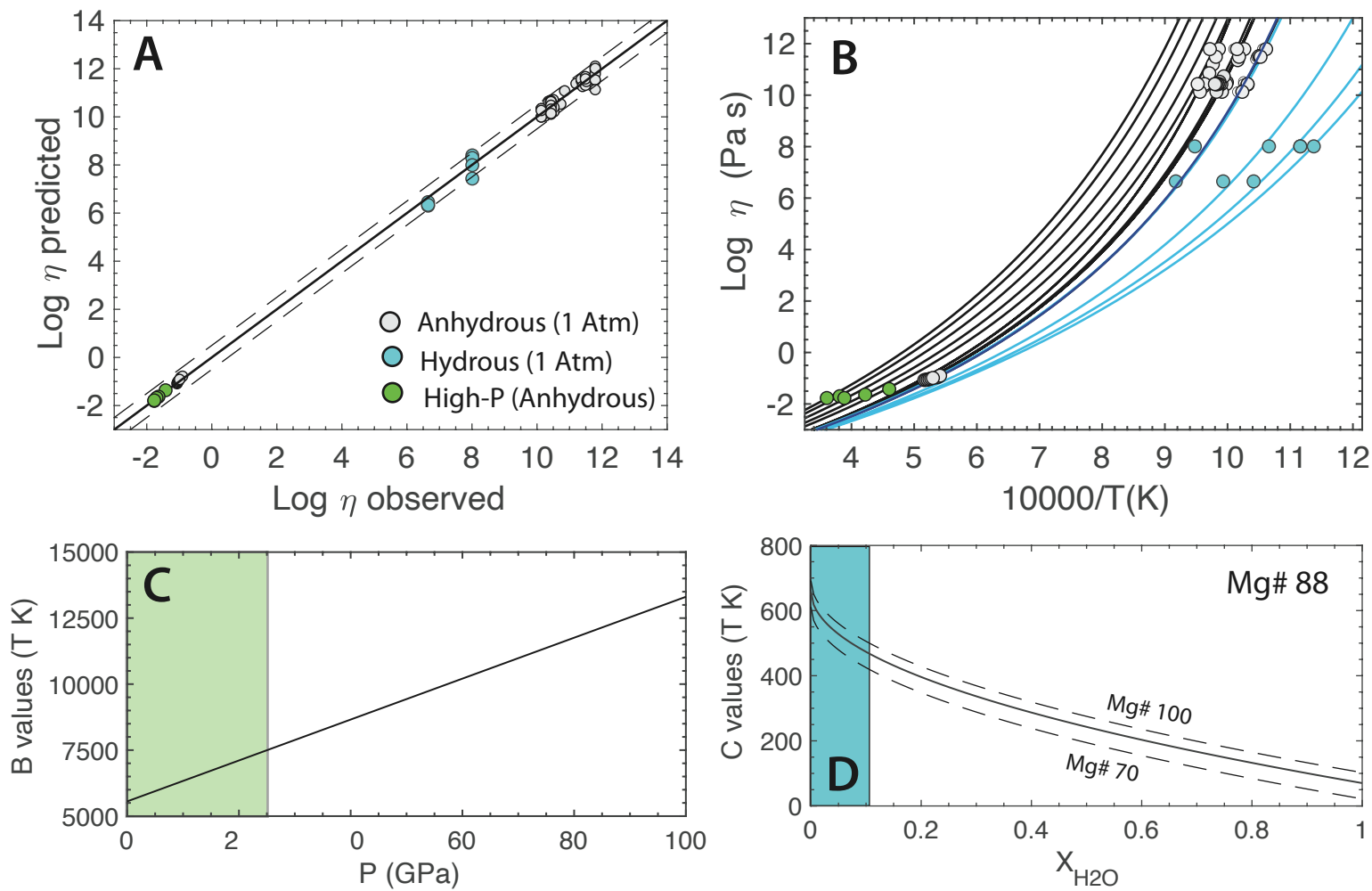


Figure 2. Ultramafic melt viscosity, 2024

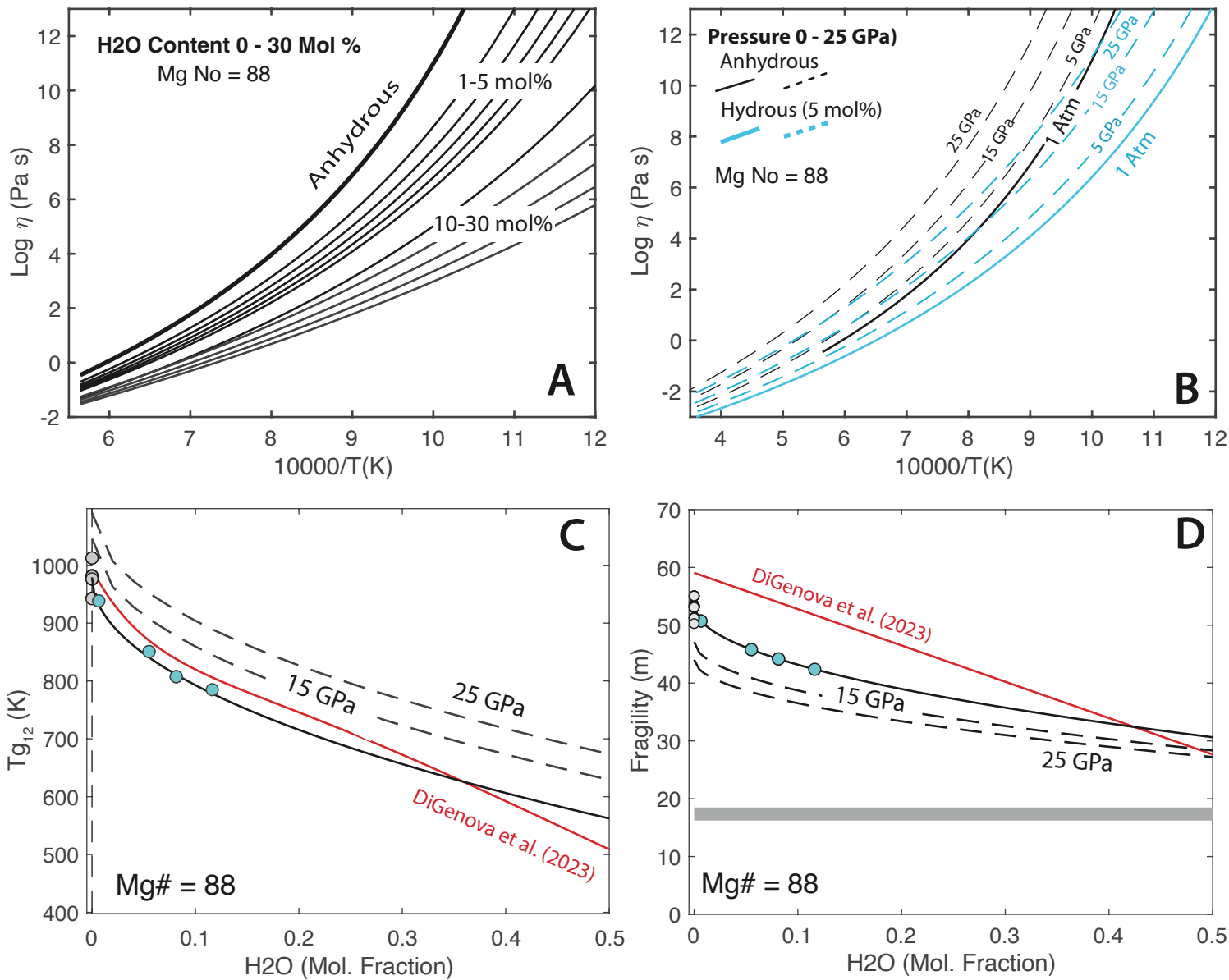


Figure 3. Ultramafic melt viscosity, (2024)

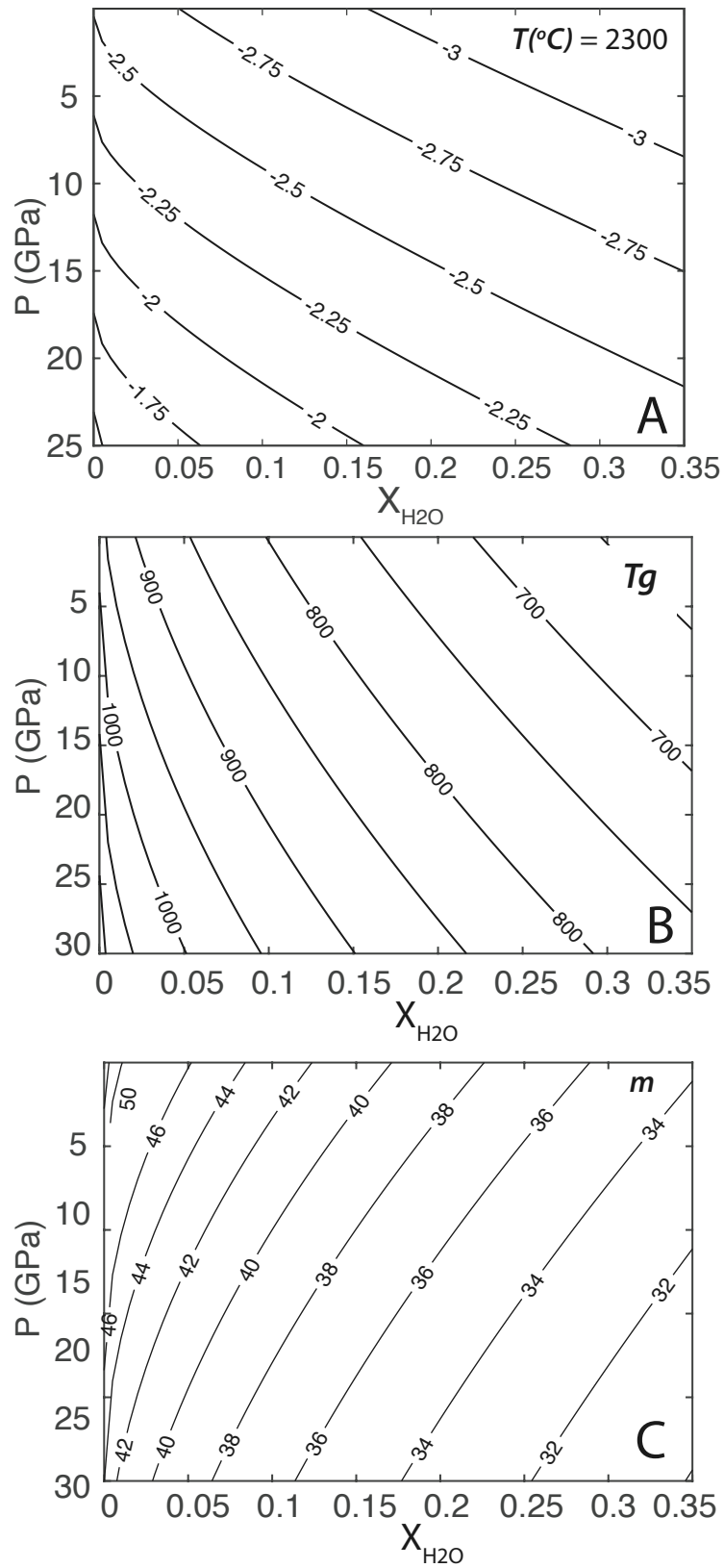


Figure 4. Ultramafic melt viscosity, 2024

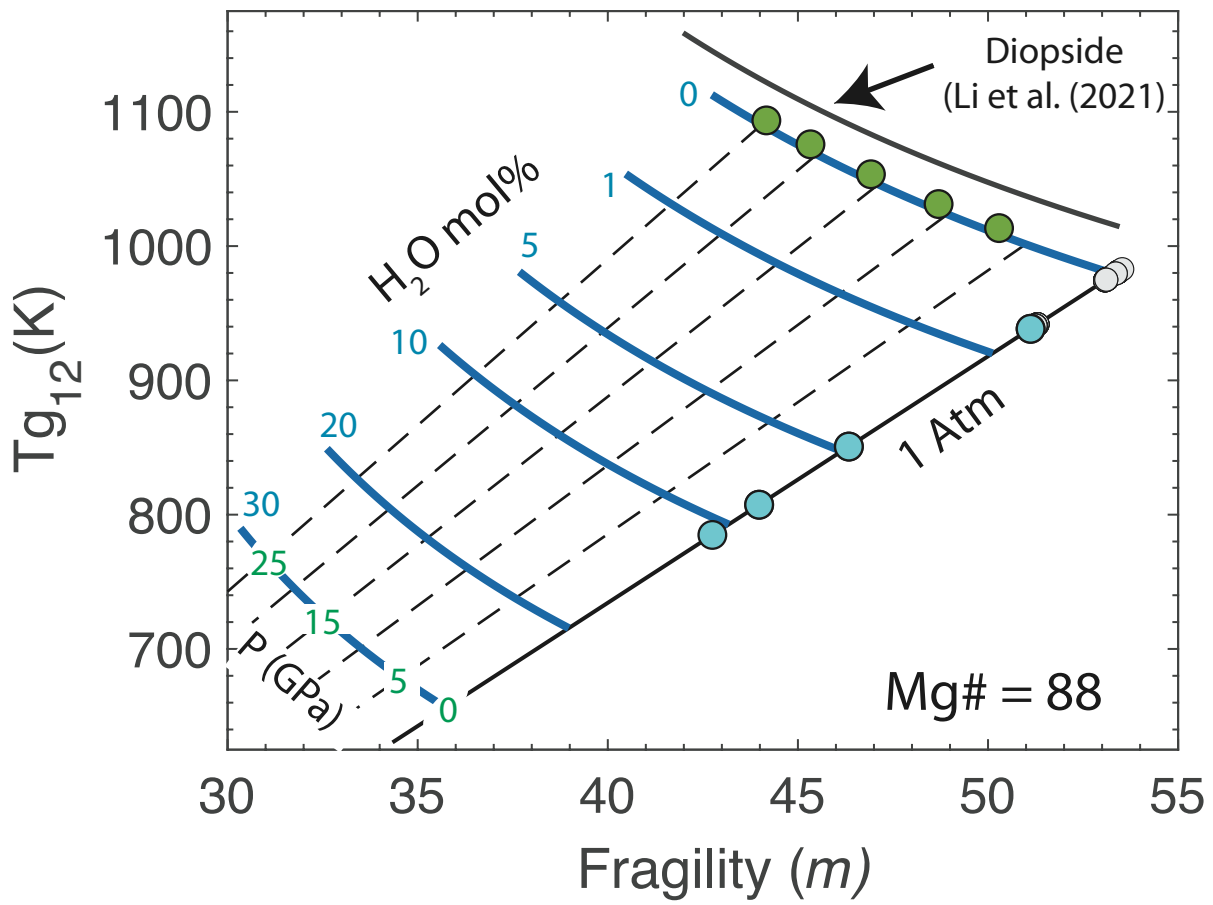


Figure 5. Ultramafic melt viscosity, 2024

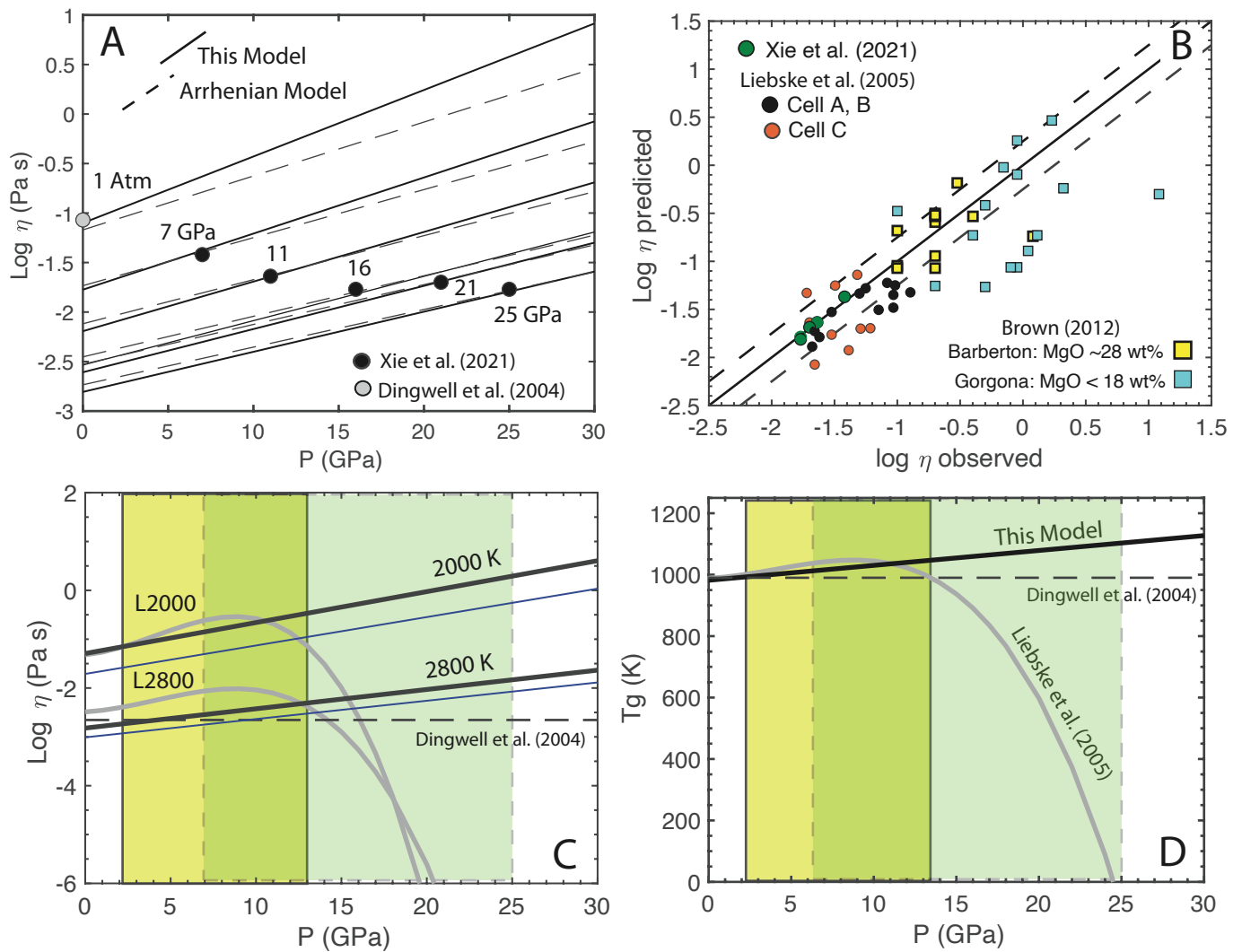


Figure 6. Ultramafic melt viscosity, 2024

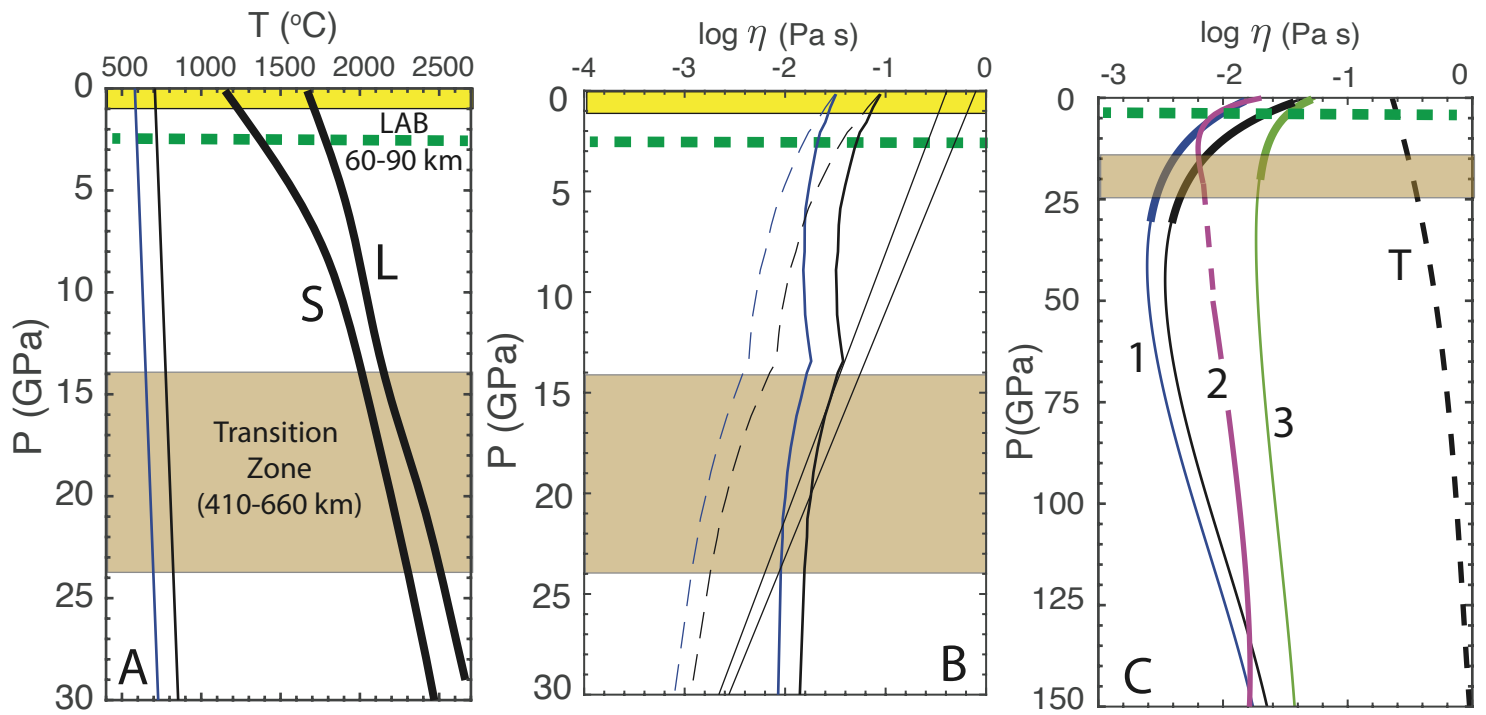


Figure 7. Ultramafic melt viscosity, 2024

Table S1. Compositions of peridotite, pyrolite and related melts used for measurements of melt viscosity.

Sample Method ¹	Dingwell et al. (2004)				Casas et al. (2023; Samples BAPYR-)					DiGenova et al. (2023)											Xie et al.	
	Peridotite		Pyrolite	19-LMU1	211-LMU07-09	211-LMU10-12	211-LMU13-15	19-LMU6	211-LMU16-19	S44F0	S44F6+	S44F6	S44F12+	S44F12	S40F6	S45F7	S34F0	S38F5W1	S40F5W6	S43F7W8	S39F6W12	Peridotite
	CC & DSC	CC	MP	MP	MP	MP	MP	MP	MP	MP & DSC	MP & DSC	MP & DSC	MP & DSC	MP & DSC	DSC	DSC	DSC	FDSC	FDSC	FDSC	FDSC	FSV
SiO ₂	45.83	46.89	46.73	46.02	45.52	46.39	46.87	45.73	50.80	48.56	48.11	46.41	46.50	46.57	51.30	39.99	44.49	45.25	46.58	42.97	46.60	46.60
TiO ₂	0.18	-	-	-	-	-	-	-	0.03	0.01	0.01	0.03	0.03	0.02	0.02	0.02	0.20	0.21	0.32	0.14	-	-
Cr ₂ O ₃	0.36	-	-	-	-	-	-	-	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.21	0.21	0.24	0.19	-	-
Al ₂ O ₃	4.87	4.31	4.31	4.03	3.92	4.09	4.32	4.11	6.62	6.36	6.47	6.14	6.01	4.05	4.12	4.09	3.18	3.78	4.61	3.00	4.00	4.00
FeO(T)	8.63	8.71	8.62	8.83	8.99	8.68	8.70	8.33	0.07	8.23	8.51	15.58	15.82	8.83	9.00	0.07	7.57	6.26	8.73	7.40	8.70	8.70
MnO	-	-	-	-	-	-	-	-	0.01	0.01	0.01	0.01	0.03	0.01	0.01	0.00	0.15	0.13	0.17	0.09	-	-
MgO	31.63	37.91	37.91	38.45	38.88	38.32	37.88	39.31	35.25	29.87	30.07	25.02	24.96	36.94	32.31	38.98	40.70	38.95	31.86	39.61	37.10	37.10
CaO	6.37	2.16	2.14	2.37	2.41	2.41	2.15	2.43	7.13	6.88	6.73	6.73	6.58	3.52	3.16	16.82	3.01	2.92	4.25	2.00	3.60	3.60
Na ₂ O	0.32	-	-	-	-	-	-	-	0.02	0.03	0.02	0.00	0.02	0.02	0.01	0.01	0.21	0.23	0.28	0.16	-	-
K ₂ O	0.00	-	-	-	-	-	-	-	0.02	0.03	0.03	0.03	0.03	0.02	0.04	0.00	0.04	0.06	0.06	0.02	-	-
P ₂ O ₅	-	-	-	-	-	-	-	-	0.05	0.00	0.03	0.05	0.03	0.03	0.03	0.03	0.00	0.00	0.00	0.00	-	-
H ₂ O	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.23	2.00	2.90	4.44	-	-
Total	98.19	99.98	99.71	99.70	99.72	99.89	99.92	99.91	100	100	100	100	100	100	100	100	100	100	100	100	100.00	100.00
Fe ³⁺ /ΣFe	-	-	0.29	0.27	0.25	0.09	0.06	0.07	0.00	0.54	0.50	0.38	0.31	0.32	0.34	0.00	0.00	0.00	0.00	0.00	-	-
Fe ₂ O ₃	-	-	2.82	2.67	2.54	0.92	0.64	0.61	0.00	4.94	4.73	6.58	5.45	3.14	3.40	0.00	0.00	0.00	0.00	0.00	-	-
Mg No.	86.7	88.6	88.7	88.6	88.5	88.7	88.6	89.4	99.9	86.6	86.3	74.1	73.8	88.2	86.5	99.9	90.5	91.7	86.7	90.5	88.4	88.4
NBO/T	2.27	2.45	2.45	2.56	2.62	2.52	2.44	2.60	1.93	1.97	1.99	2.01	2.02	2.49	2.02	3.30	2.87	2.86	2.56	3.46	2.50	2.50
SM	52.5	54.1	54.1	55.0	55.5	54.7	54.1	55.6	52.4	49.8	50.1	47.4	47.3	54.3	49.1	64.2	57.2	53.0	47.2	50.7	54.4	54.4

¹Measurement techniques include: concentric cylinder viscometry (CC), micro-penetration (MP), conventional differential scanning calorimetry (DSC), flash differential scanning calorimetry (FDSC), and falling sphere viscometry (FSV).

Table S2. Compilation of all experimental data, and their sources, used to calibrate the temperature-dependent model for anhydrous and hydrous ultramafic melts.

Source	Composition	Label	Method ¹	T(°C)	log η (Pa s)	σ
Dingwell et al. (2004)	Peridotitic	OPL1	CC	1593.79	-0.96657	0.08
		OPL2	CC	1583.95	-0.9431	0.08
		OPL3	CC	1574.10	-0.8996	0.08
		OPL4	DSC	732.9	10.73	0.25
		OPL5	DSC	739.9	10.52	0.25
		OPL6	DSC	739.9	10.43	0.25
		OPL7	DSC	743.9	10.25	0.25
		OPL8	DSC	744.9	10.13	0.25
Casas et al. (2023)	Pyrolite (in Air)	BAPYR	CC	1664.0	-1.07	0.08
		BAPYR	CC	1654.0	-1.06	0.08
		BAPYR	CC	1644.0	-1.05	0.08
		BAPYR	CC	1634.0	-1.03	0.08
		BAPYR	CC	1624.0	-1.01	0.08
	Pyrolite (reduced fO ₂)	BAPYR	CC	1614.0	-0.99	0.08
		19-LMU1	DSC	740.0	10.43	0.08
		211-LMU07-09	DSC	743.0	10.43	0.08
		211-LMU10-12	DSC	742.0	10.43	0.08
		211-LMU13-15	DSC	745.0	10.43	0.08
DiGenova et al. 2023	Peridotite (Fe-free)	19-LMU6	DSC	744.0	10.43	0.08
		211-LMU16-19	DSC	748.0	10.43	0.08
		S44F0	MP	750.9	11.20	0.15
		S44F0	MP	765.9	10.35	0.15
		S44F0	MP	756.9	10.85	0.15
		S44F0	DSC	740.9	11.79	0.15
		S44F0	DSC	745.9	11.49	0.15
		S44F0	DSC	764.9	10.43	0.15
		S44F0	DSC	772.9	10.13	0.15
		S44F6	MP	710.9	11.36	0.15
	Peridotite (in Air)	S44F6	MP	727.9	10.50	0.15
		S44F6	DSC	700.9	11.79	0.15
		S44F6	DSC	708.9	11.49	0.15
		S44F6	DSC	728.9	10.43	0.15
		S44F6	DSC	735.9	10.13	0.15
		S44F12	MP	678.9	11.52	0.15
		S44F12	MP	695.9	10.41	0.15
		S44F12	DSC	668.9	11.79	0.15
		S44F12	DSC	676.9	11.49	0.15
		S44F12	DSC	695.9	10.43	0.15
S44F12	DSC	703.9	10.13	0.15		
DiGenova et al. 2023		S40F6	DSC	716.9	11.79	0.15
		S40F6	DSC	737.9	10.43	0.15
		S45F7	DSC	711.9	11.79	0.15
		S45F7	DSC	736.9	10.43	0.15
		S44F6+	MP	712.9	11.36	0.15
		S44F6+	DSC	702.9	11.79	0.15
		S44F6+	DSC	709.9	11.49	0.15
		S44F6+	DSC	729.9	10.43	0.15
S44F6+	DSC	736.9	10.13	0.15		

	Peridotite (Oxidized)	S44F12+	MP	683.9	11.40	0.15
		S44F12+	MP	701.9	10.52	0.15
		S44F12+	DSC	673.9	11.79	0.15
		S44F12+	DSC	682.9	11.49	0.15
		S44F12+	DSC	701.9	10.43	0.15
		S44F12+	DSC	710.9	10.13	0.15
	Peridotite (Fe-free/ High Ca)	S34F0	DSC	755.9	11.79	0.15
	Peridotite (Fe-free)	S34F0	DSC	776.9	10.43	0.15
		S38F5W1-0.65	FDSC	781.9	8.01	0.15
		S38F5W1-0.65	FDSC	816.9	6.65	0.15
		S40F5W6-5.51	FDSC	664.9	8.01	0.15
	Peridotite (Hydrous)	S40F5W6-5.51	FDSC	733.9	6.65	0.15
		S43F7W8-8.13	FDSC	622.9	8.01	0.15
		S43F7W8-8.13	FDSC	686.9	6.65	0.15
		S39F6W12-11.62	FDSC	605.9	8.01	0.15
		S3301-7GPa	FSV	1900	-1.420	0.05
		S3296-11GPa	FSV	2097	-1.638	0.05
Xie t al. (2021)	Peridotite (High-P)	S3298-16GPa	FSV	2300	-1.770	0.05
		S3299-21GPa	FSV	2350	-1.699	0.05
		S3376-25GPa	FSV	2500	-1.770	0.05

¹Measurement techniques include: concentric cylinder viscometry (CC), micro-penetration (MP), conventional differential scanning calorimetry (DSC), flash differential scanning calorimetry (FDSC), and falling sphere viscometry (FSV).

Table S3. Example calculation of VFT parameters for predicting T -dependent melt viscosity [$\log_{10} \eta = A + B/(T(K) - C)$] for a hydrous peridotitic melt at elevated pressure.

$\log \eta = A + [b_0 + b_1 (P-0.0001)] / [T - (c_0 + c_1 \text{ Mg\#} + c_2 X_{\text{H}_2\text{O}}^{0.5})]$					
Parameters	Values	Oxide	Wt %	Mol %	$X_{\text{H}_2\text{O}}$
A	-5.4	SiO ₂	45.83	40.32	0.029
		TiO ₂	0.18	0.12	Mg#
b ₀	5558.3	Al ₂ O ₃	4.87	2.52	86.7
b ₁	77.5	FeO(T)	8.63	6.35	B Term
c ₀	422.93	MnO	0.00	0.00	5752.0
c ₁	2.69	MgO	31.63	41.48	C Term
c ₂	-589.39	CaO	6.37	6.00	555.3
		Na ₂ O	0.32	0.27	Tg K (~10¹² Pa s)
		K ₂ O	0.00	0.00	886
P(GPa)	2.5	H ₂ O	1.00	2.93	Fragility (m)
		Total	98.83	100.00	46.6

Supplement 4: Other models for multicomponent melt viscosity.

Other models for predicting multicomponent silicate melt viscosity at geological conditions do not accurately reproduce the experimental data for ultramafic viscosity melts. The model of Shaw (1972) does best at high-T where ultramafic melts are most Arrhenian-like but fails to reproduce the low-temperature data (anhydrous and hydrous) (Fig. 1A). The multicomponent non-Arrhenian models (Hui and Zhang, 2007; Giordano et al 2008) reproduce the experimental data better on average but do not account for pressure. They also reproduce the high-temperature data better than the low-T data.

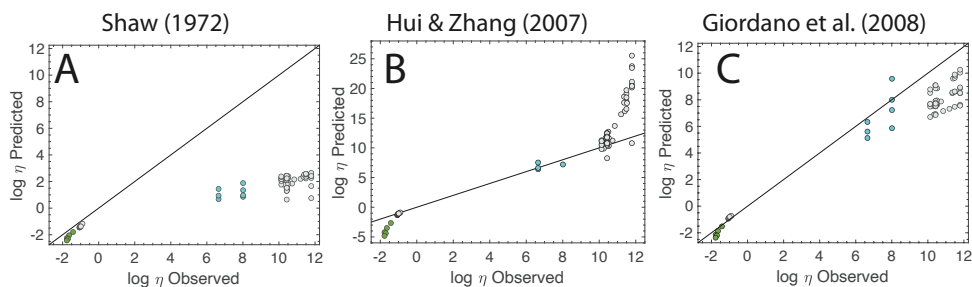


Figure 1. Comparison of measured values of viscosity for ultramafic melts (See Tables S1 & S2) to values predicted by: (A) Arrhenian melt model of Shaw (1972); and non-Arrhenian models of (B) Hui and Zhang (2007) and (C) Giordano et al. (2008). Symbols denote anhydrous (Grey), hydrous (blue) and high-P melts (as in Fig. 1 in text).

There are two multicomponent Arrhenian models that do account for pressure. Neither reproduces the high-pressure data well even at high-T (Fig. 2). The Duan (2014) model underestimates the viscosity of high-T melts at elevated P and overshoots the viscosity of 1-atm, high-T melts. Because the model is Arrhenian, it undershoots the low-T anhydrous and hydrous data (Fig. 2A). The model of Persikov and Bukhtiyarov (2009) accounts for pressure but overshoots the high-P melt viscosity data (Fig. 2B).

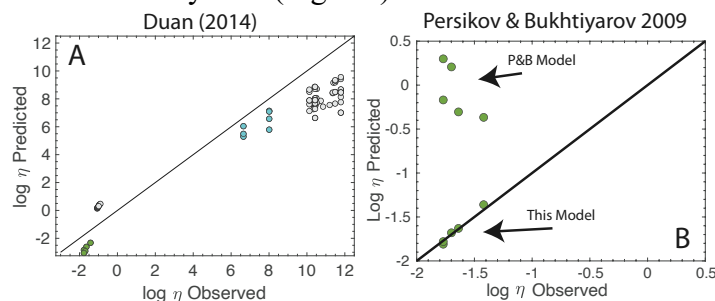


Figure 2. Comparison of measured values of viscosity for ultramafic melts to values predicted by: (A) Arrhenian P-dependent viscosity model of Duan (2014); symbols as in Fig. 1. (B) Comparison of high-P viscosity data (Xie et al. 2021) to values predicted by Persikov and Bukhtiyarov (2009) and the model presented here.

Reference

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Supplement 5: Analysis of Covariance.

The form of the VFT function is non-linear with respect to the unknown parameters (Eq. 1; A , B and C ; see text) and is solved by conventional iterative methods (e.g., Press et al. 1986). One attribute of using the χ^2 merit function is that, rather than considering a single solution that coincides with the minimum residuals, we can map a solution region at a specific confidence level (e.g., 1σ ; Press et al. 1986). This allows delineation of the full range of parameter values that can be considered equally valid descriptors of the experimental data at the specified confidence level (e.g., Russell et al. 2002). Furthermore, the confidence limits accurately portray the magnitude and nature of covariances between model parameters.

Russell et al. (2002, 2003) showed that the nonlinear character of non-Arrhenian models ensures strong numerical correlations between, and even nonunique estimates of, model parameters. One result of the strong covariances between model parameters is that wide ranges of values can be used to describe individual datasets. This is true even where the data are numerous, well-measured and span a wide range of temperatures and viscosities. Stated another way, there is a substantial range of model values which, when combined in a nonarbitrary way, can accurately reproduce the experimental data.

In our model for the temperature, pressure and compositional dependence of viscosity for ultramafic melts we have adopted an independently fixed value for A (-5.4; see text), representing the high-temperature limit to melt viscosity. Our optimization for the values of the 5 adjustable parameters (b_0 , b_1 , c_0 , c_1 , c_2) was based on minimization of the χ^2 function (Eq. 4; see text). These parameters have optimal values and statistical uncertainties reflecting the quality and distribution of the data. The parameters also have covariances which dictate the non-arbitrary way in which the permitted range of parameter values can be combined to reproduce the original data.

In Figure 1, we illustrate these concepts explicitly by displaying the covariances between several of the parameters for which there are significant covariances (Table 1; see text). The 1σ confidence envelopes on the optimal 5 parameter solutions define 5-D ellipsoids. The 2-D ellipses plotted in Fig. 1 approximate those confidence envelopes on two parameters where the other parameters are fixed at their optimal values. These ellipses are planes through the 5-D ellipsoid that contain the solution and are normal to the fixed parameter space. The confidence envelopes are computed by mapping boundaries of constant χ^2 around the optimal solution in the manner described fully by Press et al. (1986). The methods used to compute the confidence ellipses and applied to models for melt viscosity are fully described in Russell et al. (2002), and Giordano et al. (2015).

Firstly, all parameters have variances that are significantly less than their optimal value and, thus, do not allow for a zero value. This means that each parameter is statistically relevant to the model used to capture the data. Two parameter pairs show weak negative covariation, c_0 vs. b_0 and b_0 vs. b_1 and one pair shows a weak positive correlation (c_0 vs. c_2). The parameters c_2 and b_1 show no pronounced covariation. Overall the covariations between model parameters are weak implying that they are more or less independent

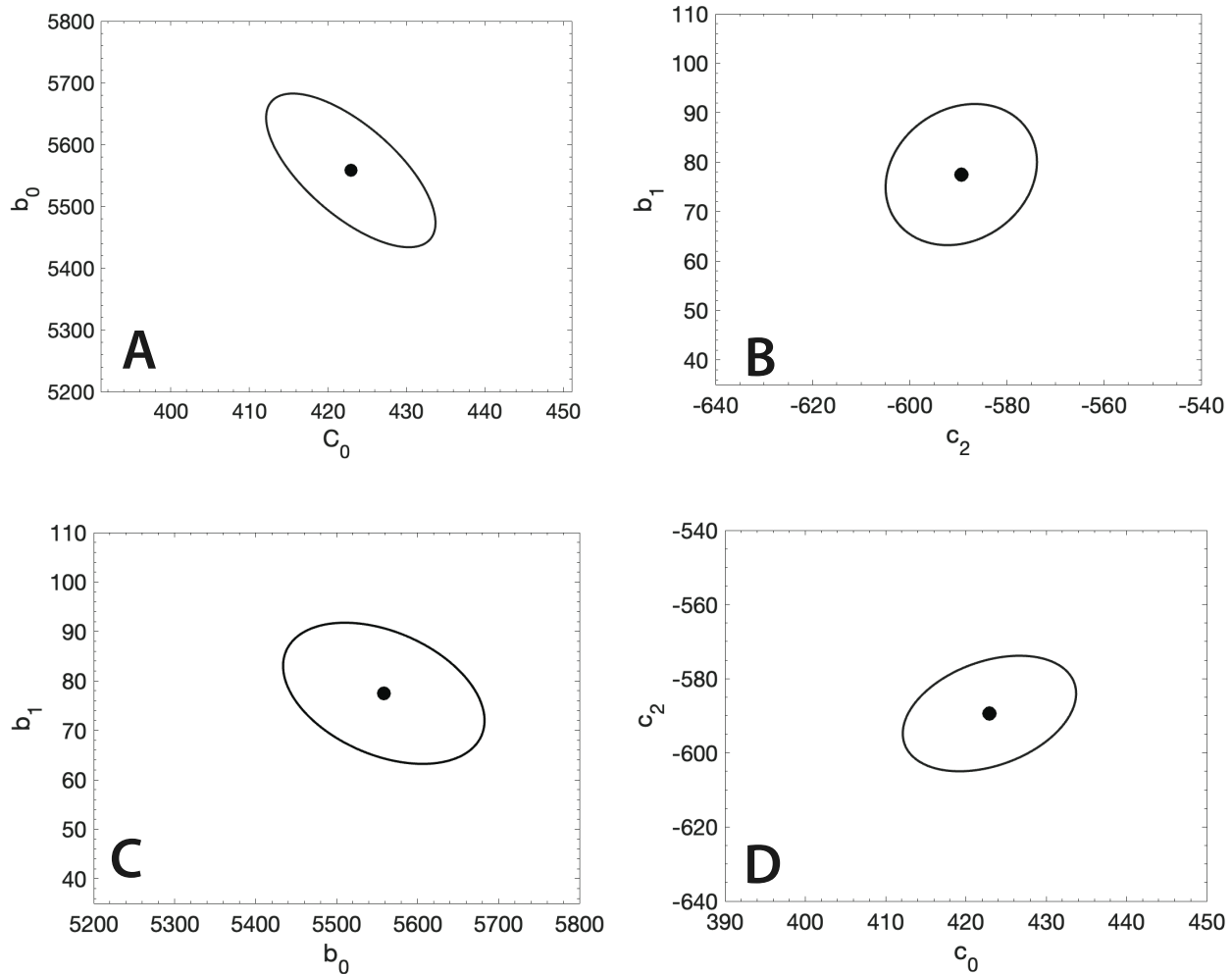


Figure 1. The two-dimensional projected 1σ confidence ellipses (solid lines) centered on the optimal solution for the adjustable parameters b_0 , b_1 , c_0 , c_1 , c_2 . Confidence ellipses are drawn for pairs of parameters where the other three parameters are held constant at their optimal values. The plots show the magnitude and nature of correlation between: (A) c_0 and b_0 , (B) c_2 and b_1 , (C) b_0 and b_1 , and (D) c_0 and c_2 . The axes in each diagram are scaled to ~ 3 standard deviations for each parameter to compare the relative magnitudes of covariance.

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