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Key Points:

- SSH eddies overestimate coherent core and fail to reveal more than half of Lagrangian eddies
- The leakage of SSH eddies reach more than 50% over the lifetime
- About half of SSH eddies are not significantly different from random ocean pieces in material coherence

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14 Abstract

15 An idealized eddy-resolving ocean basin, closely resembling the North Pacific ocean, is simu-
 16 lated using MITgcm. We identify rotationally coherent Lagrangian vortices (RCLVs) and sea surface
 17 height (SSH) eddies based on the Lagrangian and Eulerian framework, respectively. General sta-
 18 tistical results show that RCLVs have a much smaller coherent core than SSH eddies with the ratio
 19 of radius is about 0.5. RCLVs are often enclosed by SSHA contours, but SSH eddy identification
 20 method fails to detect more than half of RCLVs. Based on their locations, two types of eddies are
 21 classified into three categories: overlapping RCLVs and SSH eddies, non-overlapping SSH eddies,
 22 and non-overlapping RCLVs. Using Lagrangian particles, we examine the processes of leakage and
 23 intrusion around SSH eddies. For overlapping SSH eddies, over the lifetime, the material coherent
 24 core only accounts for about 25%, and about 50% of initial water leak from eddy interior. The
 25 remaining 25% of water can still remain inside the boundary, but only in the form of filaments
 26 outside the coherent core. For non-overlapping SSH eddies, more water leakage (about 60%) occurs
 27 at a faster rate. Following the number and radius of real SSH eddies, fixed circles and moving circles
 28 are randomly selected to diagnose the material flux around these circles. We find that the leakage and
 29 intrusion trends of moving circles are quite similar to that of non-overlapping SSH eddies, suggesting
 30 that the material coherence properties of non-overlapping SSH eddies are not significantly different
 31 from random pieces of ocean with the same size.

32 Plain Language Summary

33 Material coherent transport is widely estimated based on the Eulerian methods, but the organ-
 34 ization of Eulerian eddies determines Eulerian eddies can not keep material coherence during the
 35 lifespan. In this study, we examine the relationship between eddy detection methods and material
 36 coherent transport, and quantitatively estimate the Eulerian eddy leakiness in material transport. We
 37 compare the statistical features of RCLVs and SSH eddies identified from a high-resolution idealized
 38 model based on two different frameworks. We find that the Eulerian method fails to detect many
 39 Lagrangian structures and overestimate the real coherent core. Besides, using Lagrangian particles,
 40 we quantify the material flux across SSH eddies boundaries. Our results suggest that SSH eddies are
 41 far from coherent structures, with the leakage of initial water reach at least 50%. We also find that
 42 one type of SSH eddies are not significantly different from random pieces of ocean with the same
 43 size in material coherence properties. Our findings suggest that using Eulerian eddy boundaries to
 44 estimate coherent transport may be misleading.

45 1 Introduction

46 Mesoscale eddies with horizontal spatial scales broadly between tens and a few hundred kilo-
 47 meters are ubiquitous structures in the ocean (*Fu et al. [2010]*), transporting and redistributing mass,
 48 heat, salt, potential vorticity, and other biochemical tracers throughout the ocean. This transport is
 49 believed to have significant impacts on the large-scale ocean circulation, marine ecosystems, and
 50 long-term earth climate (e.g. *Jayne and Marotzke [2001]*; *Dong et al. [2014]*; *Zhang et al. [2014]*;
 51 *Gaube et al. [2015]*; *Griffies et al. [2015]*; *Kouketsu et al. [2016]*). Understanding the degree of
 52 coherent eddy transport is a key issue for more accurate parameterization of mesoscale transport
 53 in coarse-resolution climate models. Here “coherent eddy transport” refers to the water trapping
 54 process, in which material transport barriers prevent effective water exchange between the eddy in-
 55 terior and background flows over a finite time interval (*Frenger et al. [2015]*; *Haller and Beron-Vera*
 56 *[2013]*; *Haller [2015]*). However, a survey of previous studies reveals that estimates of coherent
 57 eddy transport based on different identification methods vary dramatically. In this paper, we aim to
 58 clarify the relationship between eddy identification methods and material coherent transport. Also,
 59 we seek to examine the extent to which the eddies detected by different methods maintain coherence
 60 over their lifetimes.

61 A number of methods have been employed to identify coherent eddies. We review these
 62 methods in two categories: Eulerian methods, based on instantaneous information of the flow

63 field, and Lagrangian methods, based on water parcel trajectories over a finite time interval. The
64 fundamental concept of Eulerian methods is to identify the eddy boundary through anomaly contours
65 of some instantaneous Eulerian field, and then track these boundaries on the basis of their geographic
66 locations at a proximate time. The Eulerian fields used to define the ocean eddy boundaries include
67 OkuboWeiss (OW) parameter (*Okubo* [1970]; *Weiss* [1991]; *Isern-Fontanet et al.* [2003]), sea surface
68 height (SSH; *Chelton et al.* [2011]; *Faghmous et al.* [2015]), velocity streamlines (*Nencioli et al.*
69 [2010]), sea surface temperature (*Dong et al.* [2011]; *Hausmann and Czaja* [2012]), and potential
70 vorticity (*Zhang et al.* [2014]). Due to their relative computational simplicity, the Eulerian methods
71 have been more widely used in the oceanography community, especially the eddy census of *Chelton*
72 *et al.* [2011] derived from satellite altimetry.

73 Eulerian eddy detection has many valuable applications. However, several shortcomings of
74 Eulerian detection methods (discussed by *Haller* [2015] and *Peacock et al.* [2015]) must be considered
75 in the context of eddy transport. First, the Eulerian methods are not objective; material transport
76 should not depend on the choice of reference frame, but Eulerian eddies may change dramatically
77 when the reference frame changes (e.g. via translation or rotation). Second, the boundary of the
78 Eulerian eddy is not material; the contours in scalar field do not represent persistent material
79 barriers that forbid material exchange. That means the water inside Eulerian eddy boundaries is not
80 necessarily the same fluid at different times. In addition, the choice of numerous parameters and
81 thresholds is another inevitable drawback of Eulerian methods. In contrast, Lagrangian coherent
82 structures derived from Lagrangian fluid motion are a natural choice for assessing coherent eddy
83 transport, since they are material by construction. Recently, several different methods, including
84 finite-time Lyapunov exponents (FTLE; *Beron-Vera et al.* [2008]), finite-scale Lyapunov exponent
85 (FSLE; *d'Óvidio et al.* [2009]), geodesic transport barriers (*Haller and Beron-Vera* [2013]; *Beron-*
86 *Vera et al.* [2013, 2015]), and Lagrangian-averaged vorticity deviation (LAVD; *Haller et al.* [2016]),
87 have been introduced to reveal materially coherent ocean eddies.

88 Among numerous studies to estimate material coherent transport based on Eulerian identification
89 methods, the following two have received considerable attention due to their global scope and
90 surprising results. Under the framework of Eulerian eddy detection, *Dong et al.* [2014] combined
91 Argo profile data and a velocity-based Eulerian eddy dataset to estimate horizontal heat and salt
92 transports due to coherent eddy translation. They concluded that most of the eddy transport in the
93 ocean is due to individual eddy movements. *Zhang et al.* [2014] defined the eddy boundary via
94 the outermost closed potential vorticity contour and conducted a similar calculation for eddy mass
95 transport. They found that the zonal westward mass flux induced by eddies in the subtropical gyres
96 can reach about 30 Sv, which is comparable to the wind-driven transport.

97 However, several works using Lagrangian frameworks have presented quite conflicting results.
98 For instance, *Abernathey and Haller* [2018] adopted the Lagrangian-averaged vorticity deviation
99 method of *Haller et al.* [2016] to detect eddies in the eastern Pacific and quantified the contribution
100 of coherent transport to total transport. They found that the coherent meridional transport accounted
101 for less than 1% of the total flux, and the incoherent part, including chaotic stirring and filamentation
102 outside of eddy cores, was the dominant mechanism for eddy transport. Similarly, the work of
103 *Beron-Vera et al.* [2013], *Froyland et al.* [2015]) and *Wang et al.* [2015, 2016] examined the ability
104 of Agulhas rings to coherently carry Agulhas leakage water using Lagrangian methods, finding
105 considerably smaller transport estimates than previous Eulerian-based methods. Most recently,
106 *Cetina-Heredia et al.* [2019] quantified the time that mesoscale eddies along the southeast coast of
107 Australia retain water and pointed out the change in eddy shape relates to water exchange between
108 the eddy interior and the background flow. Though Eulerian eddies can temporarily trap and
109 transport water parcels and tracers (*Gaube et al.* [2015]; *Condie and Condie* [2016]), under the
110 effects of stretching and filamentation, such eddies may lose coherence and experience leakage and
111 deformation in the turbulent flow (*Haller* [2005]; *Early et al.* [2011]). The Lagrangian frameworks
112 suggest that the coherent material transport is very limited in the ocean, and that Eulerian-based
113 estimates of transport with strong assumptions about material coherence need revision. A systematic
114 quantification of the Eulerian eddy leakiness over their full lifespan has, to our knowledge, not yet
115 been attempted.

116 The main goal of this study is to answer the following key issues: (1) What are the differences
 117 between Eulerian and Lagrangian eddies in terms of coherent transport? (2) How leaky are Eulerian
 118 eddies over their lifetime? We choose rotationally coherent Lagrangian vortices (RCLVs) and sea
 119 surface height (SSH) eddies as the representative of Lagrangian eddies and Eulerian eddies in this
 120 study. The paper is organized as follows. In Section 2, we present a brief description of the idealized
 121 eddy-resolving model and two different coherent eddy identification methods. In Section 3, we
 122 compare general statistical features of Lagrangian eddies to those of Eulerian eddies. In Section 4,
 123 we examine the material flux across SSH eddy boundaries. Section 5 describes the material flux for
 124 random-circle eddy model, as a baseline for comparison against the SSH eddies. Discussion and
 125 conclusions are shown in Section 6.

126 2 Methods

127 2.1 Description of idealized model

128 Our goal is to create an eddy-resolving simulation which produces materially coherent eddies
 129 with characteristics and spatial variability broadly similar to those of the real ocean. Because a
 130 past study focused on the Pacific basin *Abernathy and Haller* [2018], here we create an idealized
 131 Pacific-like scenario. The Massachusetts Institute of Technology general circulation model (MITgcm;
 132 *Marshall et al.* [1997a,b]) is employed to construct an eddy-resolving idealized model with a 0.1
 133 degree resolution. This resolution is adopted by many state-of-the-art global climate models (e.g.
 134 GFDL CM2.6) and it allows the model to resolve the deformation radius with least 2 grid points at
 135 about 40 ° N. North of this latitude is must be considered eddy permitting (*Hallberg* [2013]). The
 136 model domain (shown in Fig. 1d) is a rectangular basin spanning 144 degrees in longitude and 60
 137 degrees in latitude. It is bounded by simple land masses along the western and eastern boundary,
 138 closely approximating the topography of the North Pacific, which is motivated by the model of *Cox*
 139 [1985]. A no-slip condition is imposed at four solid boundaries. The basin bottom is flat and 4000 m
 140 deep, except for a continental shelf along the western boundary, obeying an exponential profile from
 141 200 m at the land to 4000 m in 4 degrees to the east of the land. Vertically, there are 29 unevenly
 142 spaced layers, ranging from 10 m at the surface to 500 m at the bottom.

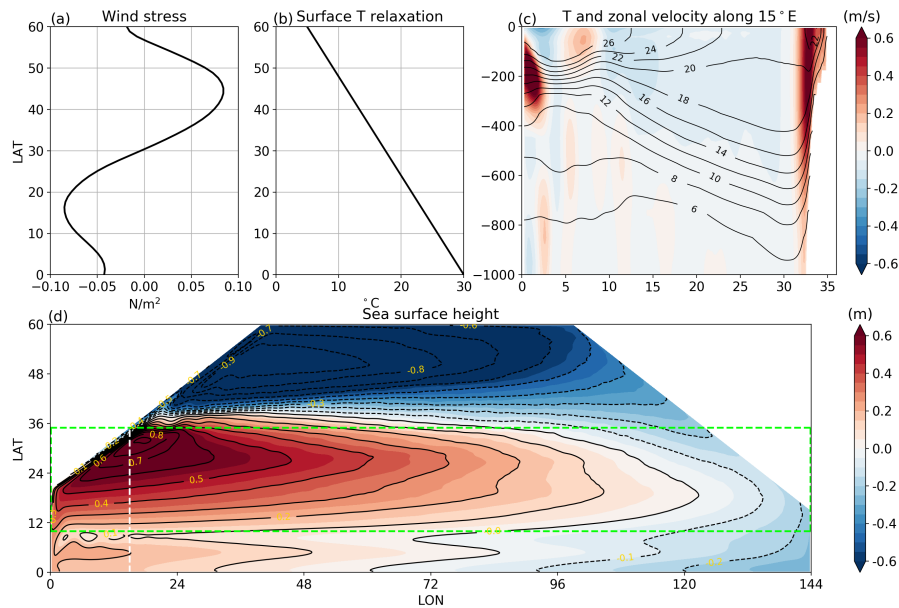
143 The surface forcing in the model consists of zonally uniform zonal wind stress and surface
 144 temperature relaxation. The wind stress profile is based on zonally averaged zonal wind stress over
 145 the North Pacific from QuikSCAT (*Risien and Chelton* [2008]). The surface temperature relaxation
 146 forcing is described as a linear function of latitude only, with the value decrease from 30°C at the
 147 equator to 5°C at 60° N. The relaxation time scale is 1 month. The distributions of wind stress
 148 and surface temperature forcing are shown in Figure 1 (a) and (b). In order to maintain the vertical
 149 stratification in the tropical region, we introduce a narrow sponge layer along the southern boundary,
 150 in which the temperature is relaxed to prescribed stratification profile from World Ocean Atlas 2013
 151 (<https://www.nodc.noaa.gov/OC5/woa13/woa-info.html>) with a time scale of one month. For the sake
 152 of model simplicity, a linear equation of state is adopted and the salinity equation is turned off. The
 153 nonlocal K-profile parametrization (KPP; *Large et al.* [1994]) is used to parameterize vertical mixing
 154 in the surface boundary layer, and the Modified Leith Viscosity (*Fox-Kemper and Menemenlis* [2008])
 155 is adopted for numerical viscosity. Full details about the physical and numerical parameters used in
 156 the model and a reproducible configuration are available at https://github.com/liutongya/Box_model.

157 The model run consists of three stages. In the first stage, a coarse grid model with a 1 degree
 158 resolution is integrated for 300 years, whereupon a near steady state circulation is obtained indicated
 159 by the mean kinetic energy. Then the 0.1 degree model is initialized at this point via interpolation and
 160 integrated for 100 years to generate the circulation brimming with eddies. Finally, the Lagrangian
 161 particle advection and eddy identification is conducted on the basis of the eddy-resolving solution
 162 for another 5 years.

163 Climatological averages are calculated over the last 50 years and are used to assess this idealized
 164 solution via comparisons with previous literature and observations of gyre structure, vertical strati-
 165 fication, and eddy kinetic energy (EKE) distribution. As shown in Figure 1 (d), the uniform zonal

166 wind stress generates a double gyre circulation, with a strong concentrated eastward flow (like the
 167 Kuroshio extension) between the subtropical and subpolar gyres. The pattern of sea surface height
 168 (SSH) is quite comparable to mean dynamic topography derived from satellite and in situ data (see
 169 Figure 3 in *Rio et al.* [2011]). The latitude-depth section of temperature along 15° E (approximately
 170 equivalent to 137° E in real ocean) is shown in Figure 1 (c, black contours), together with zonal
 171 velocity expressed by colors. The model captures the main features of thermal structures and banded
 172 flows in tropical and subtropical regions reflected by hydrographic surveys in *Qiu and Chen* [2010]
 173 (Figure 2). That is, the main thermocline (taking the 10° C isotherm) deepens strongly with latitude
 174 in the subtropical interior, and then upraises suddenly at about 32° N. In addition, the comparison
 175 of EKE distributions between AVISO data and model result indicates the model properly reproduces
 176 two eddy bands in the subtropical circulation, with one band located in Kuroshio extension region
 177 and another one located in the North Equatorial Current (NEC) region. A supplementary animation
 178 of daily SST in the final year is available at <https://vimeo.com/341264533>.

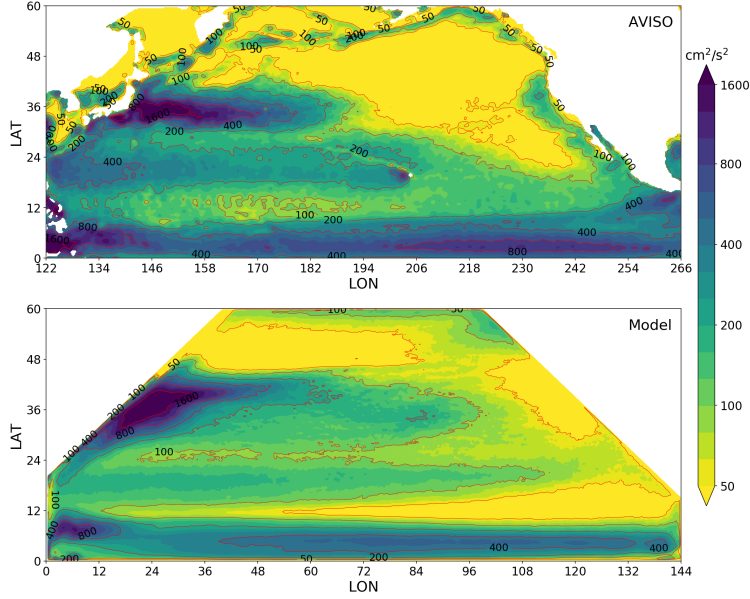
179 Overall, the idealized model well simulates the general wind-driven circulation with strong
 180 eddy activity in the upper ocean. This simulation is a computationally efficient way to probe eddy
 181 dynamics at a resolution similar to contemporary state-of-the-art high-resolution global climate
 182 models. Based on the results of *Sinha et al.* [2019], we expect that higher resolution would probably
 183 lead to fewer coherent eddies, due to the emergence of submesoscale structures at mesoscale eddy
 184 boundaries, but we do not pursue the question of resolution dependence here.



185 **Figure 1.** Model forcing fields and climatological results. (a) wind stress, (b) relaxation temperature, (c)
 186 temperature (black contours) and zonal velocity (colors) profile along 15° E, and (d) sea surface height. The
 187 white dashed line is the profile for (c). The green dashed line box is the region for eddy identification.

189 2.2 Identification of Lagrangian and Eulerian eddies

190 In the final 5 years, the Lagrangian particles are initialized with uniform spacing of $1/80^\circ$ on
 191 the surface covering the whole domain, with more than 55 million points. As described in *Haller*
 192 *et al.* [2016] and *Abernathey and Haller* [2018], a dense mesh of Lagrangian particles, much higher
 193 resolution than the model grid itself, is necessary to reflect the fine structure of transport barriers and
 194 properly identify Lagrangian eddies. The particles can be advected for 90 days by 2D surface flow



188

Figure 2. Comparison of EKE fields from (upper) AVISO and (bottom) model.

195

in "online mode" till the next initialization. In the following analysis, we use 20 non-overlapping 90-day time intervals in total for identifying eddies. In this paper our focus is on the near-surface horizontal flow, and consequently our particles are advected by the surface flow only (top model vertical level). Such trajectories are thus most representative of buoyant material such as marine debris or pollutants. However, due to the relatively low Rossby number and consequently weak divergence in the 2D flow, these trajectories are reasonably close to full 3D trajectories. (This was confirmed by full 3D advection tests.) A future study in preparation will examine the 3D structure of material eddies in this simulation.

203

Our definition of a Lagrangian eddy, one of many possible choices, is based on the concept of coherent rotation. Over its lifetime, all water parcels along a coherent eddy boundary should have the same average angular speed when rotating around the eddy core, in analogy to rigid-body rotation,. Based on this physical concept, *Haller et al.* [2016] proposed that the material boundary of a rotationally coherent Lagrangian vortex (RCLV, also called Lagrangian eddy) can be found by identifying the outermost closed contours of the Lagrangian-averaged vorticity deviation (LAVD, adopted by *Abernathey and Haller* [2018]; *Tarshish et al.* [2018]; *Sinha et al.* [2019]).

210

In a two dimensional flow, given a finite time interval (t_0, t_1) , LAVD is defined as the average of the vorticity deviation along the Lagrangian particle trajectory, that is,

211

$$LAVD_{t_0}^{t_1}(x_0, y_0) = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} |\zeta'[X(x_0, y_0, t), Y(x_0, y_0, t), t]| dt. \quad (1)$$

212

Where (X, Y) denotes the position for the particle initially located in point (x_0, y_0) and ζ' indicates the instantaneous relative vorticity deviation from the spatial average over the whole domain. LAVD reflects the average magnitude of local rotation for each Lagrangian particle over the time interval. The Lagrangian view makes it essentially different from Eulerian detection methods, for instance, defining coherent eddies by instantaneous vorticity contours. The local maximum of the LAVD field (corresponding to the strongest local rotation) represents the Lagrangian vortex center, and the vortex boundary is the outermost closed LAVD curve encircling the center.

218

219 Any closed curve of the LAVD field is a plausible, objective choice for the Lagrangian eddy
 220 boundary. To constrain our choice of boundary, we employ two additional heuristics. Convexity
 221 deficiency (CD), defined by *Haller et al.* [2016], describes how strongly the boundary curve departs
 222 from a convex curve. Coherency index (CI), introduced by *Tarshish et al.* [2018], measures how much
 223 the particles within the eddy disperse away from the eddy center. Starting from a local maximum of
 224 LAVD, we search outward for the closed LAVD contour using a bisection search algorithm, until we
 225 obtain the outermost contour satisfying both CD and CI thresholds. In this study, we choose threshold
 226 values of $CD=0.1$ and $CI=-2$, which are reasonable conditions for RCLV detection according to the
 227 sensitivity analysis in *Tarshish et al.* [2018]. We have integrated the LAVD detection method into
 228 an open source Python package named `floater` available at <https://github.com/rabernat/floater>.

229 The method we use to detect and track SSH eddies is described in *Faghmous et al.* [2015]. This
 230 geometric eddy identification method is very similar to that proposed by *Chelton et al.* [2011] and
 231 detects mesoscale eddies based on closed contours of SSH anomaly (SSHA) data. Similar to *Chelton*
 232 *et al.* [2011], they published a global daily mesoscale ocean eddy map over a period of 1993-2004
 233 from the AVISO dataset. However, *Faghmous et al.* [2015] developed an open source MATLAB
 234 code which can be applied to reproduce their detection algorithm and apply it to new scenarios.
 235 The algorithm is available at <https://github.com/jfaghm/OceanEddies>. We used this code to detect
 236 Eulerian eddies in our model.

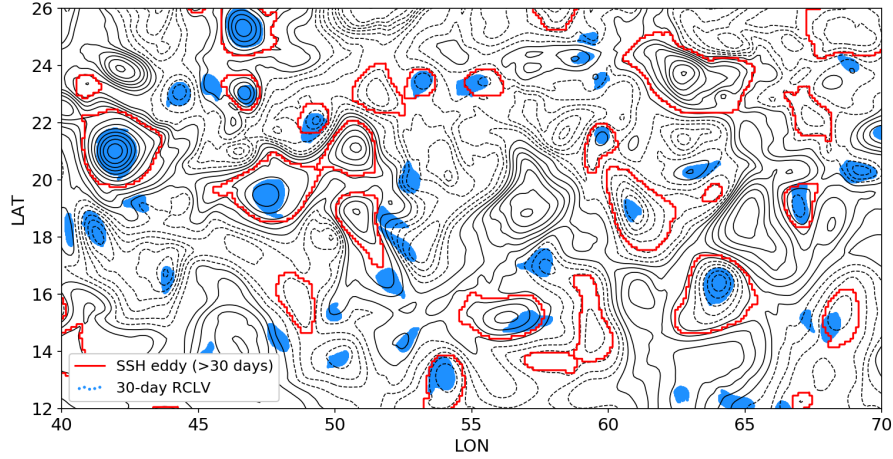
237 3 Census of Lagrangian vortices and Eulerian eddies

238 3.1 Quantity statistics of eddies

239 In the subtropical region (green dashed line box shown in Figure 1d), the LAVD method and
 240 SSH contour method are applied to identify RCLVs and SSH eddies in 20 90-day time intervals.
 241 Compared to SSH eddies, the most peculiar thing about RCLVs is that they are identified on specific
 242 time intervals. In this case, we initialize the detection at the first day of each 90-day time interval and
 243 then identify 30-, and 90-day RCLVs from that initialization date. For SSH eddies, the identification
 244 is carried out continuously in 5 years and the complete SSH eddy dataset is obtained. To compare
 245 RCLVs and SSH eddies in the same time interval, we extract SSH eddies from the complete dataset,
 246 by the condition that the SSH eddy has existed on the RCLV initialization time and can last for more
 247 than 30 and 90 days from that date. The eddies with the radius less than 25 km are excluded.

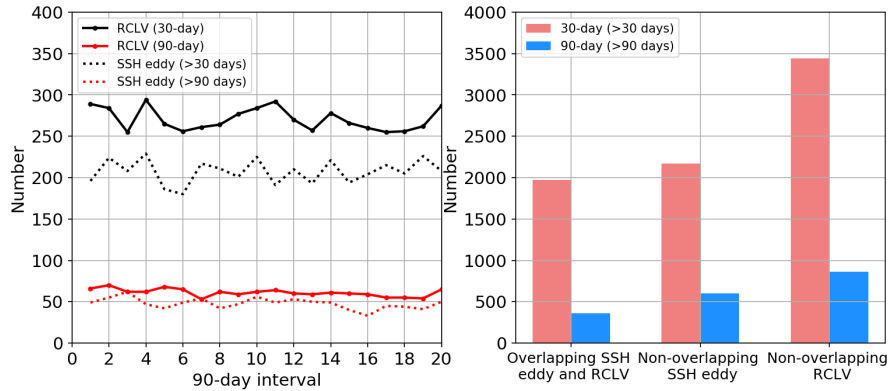
248 In Figure 3, we randomly choose a small region (40° - 70° E & 12° - 26° N) to show the initial
 249 locations of detected 30-day RCLVs and SSH eddies with lifespans longer than 30 days, with the
 250 SSHA field superimposed. Based on whether there is a RCLV whose center is located within the
 251 boundary of a SSH eddy, all eddies can be classified into three categories: overlapping RCLVs and
 252 SSH eddies, non-overlapping SSH eddies, and non-overlapping RCLVs. For overlapping RCLVs
 253 and SSH eddies, one notable character is that the RCLVs have a much smaller area than that of the
 254 corresponding SSH eddy and are enclosed by SSHA contours, which is consistent with the results
 255 of *Abernathy and Haller* [2018]. For non-overlapping SSH eddies, though they are identified and
 256 tracked by Eulerian detection algorithm, they do not maintain material coherence over the detection
 257 time interval. For example, an Eulerian eddy with with a large SSHA amplitude (such as the one
 258 located in about 63° E & 24° N) appears to be a eddy, but our result in section 4 illustrate that this
 259 kind of eddy may not be a materially coherent structure. Conversely, the non-overlapping RCLVs
 260 are the Lagrangian structures that SSH contour algorithm fails to identify. Among non-overlapping
 261 RCLVs, there are a fraction of vortices (about 25% of total RCLVs) that are not located nearby the
 262 local peak of SSHA. They are quite different from the classic geostrophic eddy centered on an SSHA
 263 extremum, but they are indeed materially coherent as indicated by LAVD method. Here we mainly
 264 focus on examining the leakiness of SSH eddies, so we will not discuss the details of these special
 265 RCLVs in this paper. The statistics presented below confirm that these results are typical rather than
 266 anomalous.

271 The statistics for the total numbers of RCLVs and SSH eddies are shown in Figure 4. We
 272 identify 5412 RCLVs and 4144 SSH eddies (extracted from the complete dataset) with lifetimes ≥ 30



267 **Figure 3.** The initial locations of 30-day RCLVs and SSH eddies (>30 days) in a randomly selected time
 268 interval and region. The blue dots present particles within each RCLV boundary. The red lines denote SSH
 269 eddy boundaries. The SSH contours are shown in black solid and dashed lines, with the contour interval of 1
 270 cm.

273 days in subtropical region over five years. The numbers of 30-day RCLVs in each 90-day interval
 274 fluctuates between 250 and 300, The longer-lived vortices are rare, with about 70 in each interval for
 275 90-day RCLVs. A similar ratio of >90 days eddies to >30 day eddies (1/5) is found in SSH eddies.
 276 Another interesting observation is that the total number of SSH eddies is slightly less than that of
 277 RCLVs, since non-overlapping RCLVs are the most numerous type.



278 **Figure 4.** RCLVs and SSH eddies number statistics. (left) Time series of RCLVs and SSH eddies number in
 279 20 90-day intervals. (right) Total numbers of overlapping SSH eddies and RCLVs, non-overlapping SSH eddies,
 280 and non-overlapping RCLVs. The colors correspond to the eddy lifespans in both two panels.

281 3.2 Radius, propagation speed, and zonal coherent transport

282 The equivalent radius R of RCLVs and SSH eddies is converted from the eddy horizontal area
 283 A by the formula $R = \sqrt{A/\pi}$. We also calculate the first Rossby radius of deformation L_d via

$$L_d = \frac{1}{f\pi} \int_{-H}^0 N(z) dz, \quad (2)$$

284 where, f is the Coriolis parameter for earth rotation and latitude, H is the water depth, and $N(z)$ is
 285 the buoyancy frequency obtained from model vertical stratification.

286 The top two panels in Figure 5 show the comparison of radius for 30-day RCLVs and SSH
 287 eddies with lifetimes longer than 30 days. A box-and-whisker plot represents statistics of all eddies
 288 in 2.5° latitude bins. With the increase of Coriolis parameter, the first Rossby deformation radius
 289 (purple dashed lines) decreases from about 95 km at 10° N to about 25 km at 35° N, which is similar
 290 to the distribution in *Chelton et al.* [1998]. Comparing with SSH eddies, the mean RCLV radius
 291 (35.5 km) is approximately about half of the mean SSH eddy radius (72.4 km). In contrast with
 292 *Abernathey and Haller* [2018], no significant latitude dependence or relationship with deformation
 293 radius is visible in either RCLVs or SSH eddies. The SSH eddy radii of about 60 km in the low
 294 latitudes (about 10° N) are smaller than the observation (more than 100 km). The relative lower
 295 eddy activity in the region around 10° N (Figure 2) indicates our model may fail to generate realistic
 296 eddies in the tropics, likely caused by biases in the model background state. Most of our focus in
 297 what follows is on the midlatitudes.

298 The zonal propagation speed c of RCLVs and SSH eddies, derived from the zonal displacement
 299 over the lifetime, is shown in the bottom two panels of Figure 5. We observe RCLVs and SSH eddies
 300 have similar westward propagation speeds at each latitude in the subtropical region, nearly consistent
 301 with the phase speed of nondispersive long-wave Rossby wave (also captured by the result based on
 302 AVISO data in *Abernathey and Haller* [2018]) except for the low latitude region. The phase speed
 303 c_p is calculated by

$$c_p = \bar{U} - \beta L_d^2, \quad (3)$$

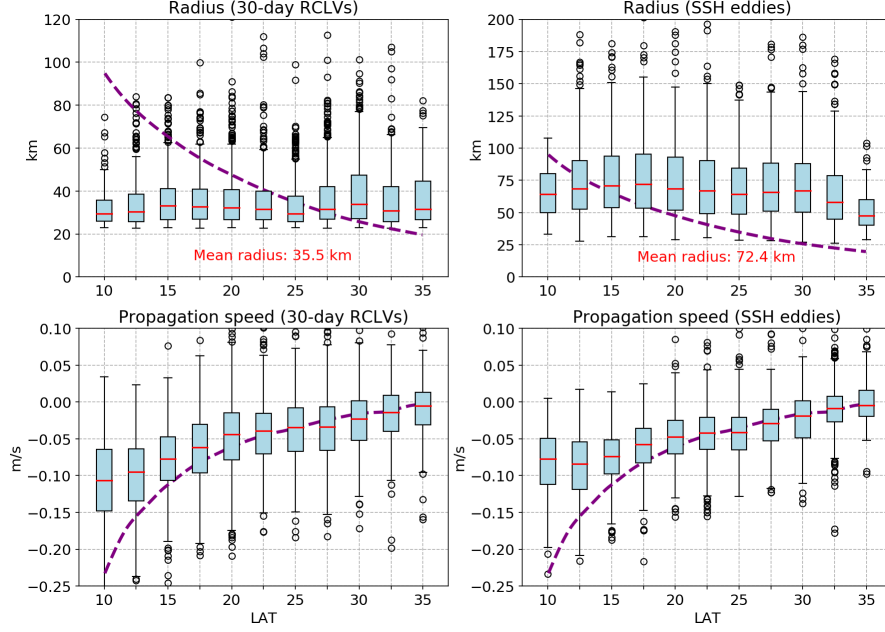
304 where, \bar{U} is the depth-averaged zonal flow (*Klocker and Marshall* [2014]).

311 Having obtained radius and zonal propagation speed of eddies, we now can estimate the advective
 312 material transport by coherent eddy movement following the methods proposed by *Dong et al.* [2014]
 313 and *Zhang et al.* [2014] These methods assume Eulerian eddy is materially coherent, trapping its
 314 interior water parcels and translating them uniformly without water exchange with the surrounding
 315 environment. (This is true by construction for RCLVs.) Here we temporarily assume SSH eddies
 316 can be materially coherent in order to make this calculation, despite evidence to the contrary. The
 317 vertical depth and structure of coherent eddies is still an open issue; therefore, to avoid artificially
 318 introducing the depth of eddies, we define cross-longitude transport by $Q = cL_{yz} = 2Rc$, with units
 319 of m^2/s . Figure 6 shows the zonally averaged zonal transport per degree latitude averaged over 5
 320 years for 30- and 90-day eddies. The coherent transport of RCLVs is uniformly smaller than that
 321 of SSH eddies at each latitude, since the coherent core of RCLVs is only half of SSH eddy radius,
 322 applying to both 30- and 90-day eddies. If the constant vertical depth 500 m is used to compute
 323 volume transport, the peak value of the zonal advective transport for 30-day RCLVs is about 0.3 Sv,
 324 much weaker than the estimate larger than 1 Sv in the Pacific ocean by *Zhang et al.* [2014] (see their
 325 Figure 3).

326 These statistical results suggest that, the RCLVs and SSH eddies move at the similar speed, and
 327 the zonal coherent transport by RCLVs is around half of the transport by SSH eddies due to large
 328 coherent core defined by SSH contours. However, this is not the case; the next section shows that
 329 the SSH eddies are far from materially coherent structures.

332 4 Material Flux Across Eddy Boundaries

333 The RCLVs are materially coherent by construction, but the material coherence properties of
 334 the SSH eddies require investigation. Having described the statistics of Lagrangian vortices and
 335 Eulerian eddies, we now focus on the core issue of our study: how leaky are the Eulerian eddies in
 336 terms of material transport?



305 **Figure 5.** (top) Radius and (bottom) propagation speed statistics of (left) 30-day RCLVs and (right) >30 days
 306 SSH eddies. A box-and-whisker plot shows statistics of all eddies in 2.5 bins. The red line represents the
 307 median. The blue box covers 25th-75th percentiles of the distribution. The black whiskers indicate 5th-95th
 308 percentiles. The black circle symbols represent the outliers beyond 5th-95th percentiles. The purple dashed
 309 line in top two panels is zonally averaged first baroclinic Rossby radius of deformation in kilometers. Mean
 310 radii of RCLVs and SSH eddies (labeled in red text) are 35.5 km and 72.4 km, respectively.

337 4.1 Reynolds transport theorem

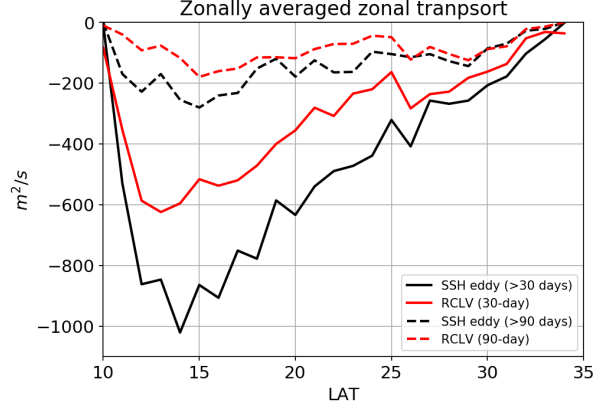
338 For a conserved scalar ϕ (e.g. conservative temperature for heat transfer) without source and
 339 sink in a Boussinesq flow, the conservation equation is given by,

$$\frac{\partial \phi}{\partial t} + \vec{u} \cdot \nabla \phi = 0. \quad (4)$$

340 Here, \vec{u} is the velocity vector field. Consider a 2-D mesoscale eddy system and a coinciding control
 341 surface Ω with a control boundary $\partial\Omega$, according to Reynolds transport theorem (*Reynolds et al.*
 342 [1903]), the time derivative of integrated ϕ over the eddy system is determined by the time rate of
 343 change of ϕ within the control eddy surface and the net rate of flux of ϕ through the moving eddy
 344 boundary,

$$\frac{d}{dt} \int_{\Omega} \phi dA = \int_{\Omega} \frac{\partial \phi}{\partial t} dA + \oint_{\partial\Omega} \phi \vec{u}_b \cdot \hat{n} dS. \quad (5)$$

345 Where, dA and dS are area and line elements, u_b is the velocity of eddy boundary, and \hat{n} is the
 346 outward-pointing unit normal vector. The definition of a material region is that the boundary
 347 velocity \vec{u}_b equals the fluid velocity \vec{u} . Combining Gauss's theorem and (3), (4) Reynold's transport
 348 theorem for a material eddy becomes



330 **Figure 6.** Zonally averaged zonal transport of RCLVs and SSH eddies (assuming SSH eddies could keep
 331 coherence).

$$\frac{d}{dt} \int_{\Omega} \phi dA = \int_{\Omega} \frac{\partial \phi}{\partial t} dA + \oint_{\partial \Omega} \phi \vec{u} \cdot \hat{n} dS = \int_{\Omega} \left(\frac{\partial \phi}{\partial t} + \vec{u} \cdot \nabla \phi \right) dA = 0. \quad (6)$$

349 In words, when the eddy boundary is a material boundary, the total amount of ϕ substance within
 350 the eddy is conserved, allowing the eddy to trap and translate its contents over large distances.

351 Generally, the velocity of Eulerian eddy boundaries (e.g. SSH contours in *Chelton et al.* [2011],
 352 PV contours in *Zhang et al.* [2014], and contours of the stream function field in *Nencioli et al.*
 353 [2010]) does not coincide with the fluid velocity because of the non-material detection method,
 354 which indicate the non-zero material flux across these boundaries. The question we address below
 355 is, *how much material leaks across the Eulerian eddy boundaries?*

356 4.2 Overlapping RCLVs and SSH eddies

357 The material flux across the eddy boundary can be understood as two processes: leakage and
 358 intrusion. Leakage means the initially inside water parcels escape into the background environment
 359 through the eddy boundary, and intrusion stands for the background water parcels entering into the
 360 eddy interior. Due to the practical difficulty of obtaining the eddy boundary velocity normal vector
 361 and conducting the integration along the curved boundary, we don't diagnose the material flux across
 362 the eddy boundary directly by (5). As a substitute, we approximate it via quantifying the number of
 363 Lagrangian moving across the eddy boundary. To do this, we employ the same Lagrangian particle
 364 experiments that were used to identify the RCLVs, described in the previous section.

365 In this section we consider the leakage across Eulerian eddies which overlap with RCLVs, 1973
 366 eddies in total. For each eddy, the numbers of initially inside particles $N_{in}(t)$ and background particles
 367 $N_{out}(t)$ surrounded by SSH contour boundaries are calculated every day during the eddy lifetime.
 368 In addition, a normalized parameter describing the change of amplitude at the eddy center point is
 369 defined by, $SSHA_{norm} = (\eta_1 - \eta_0) / \overline{\eta(t)}$, where η_0 , η_1 , and $\overline{\eta(t)}$ represent initial, final, and mean
 370 amplitude over the eddy lifetime, respectively. Base on the $SSHA_{norm}$, these eddies are classified into
 371 three categories: growing eddy ($SSHA_{norm} > 0.25$), steady eddy ($-0.25 \leq SSHA_{norm} \leq 0.25$),
 372 and decaying eddy ($SSHA_{norm} < -0.25$). Each category accounts for about 30% of the overall
 373 sample, and the numbers are labeled in the legend of Figure 8.

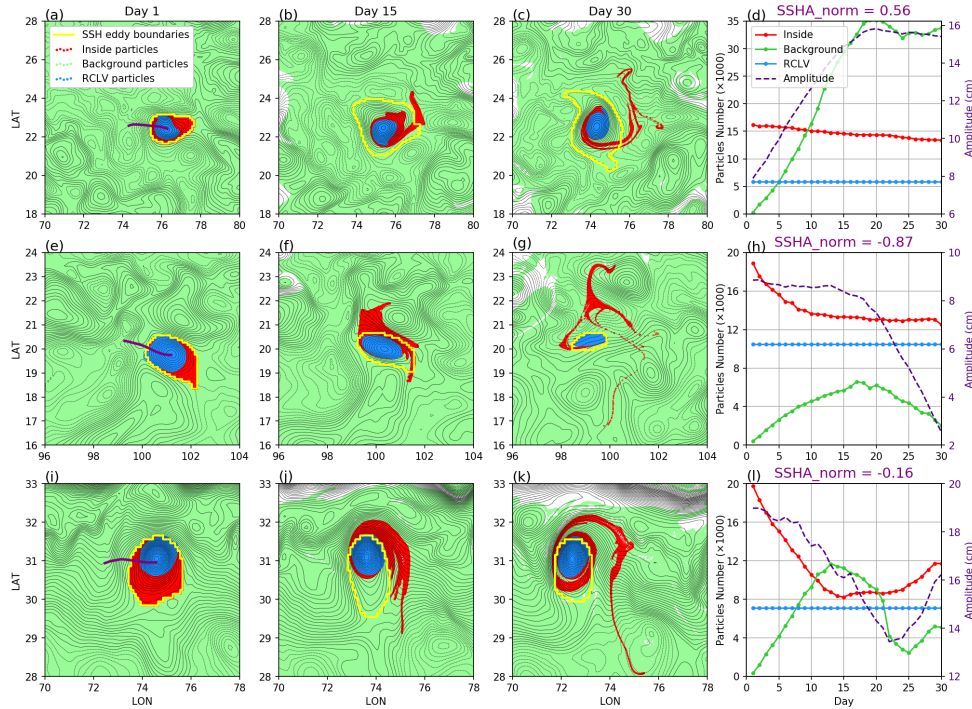
374 Figure 7 depicts the processes of leakage and intrusion for three pairs of 30-day overlapping
 375 RCLVs and SSH eddies selected randomly, including the movement of particles and time series of
 376 particle numbers. The trends of SSH amplitude indicate they belong to growing eddy, decaying eddy,

377 and steady eddy, respectively. These three SSH eddies all move westward and contain a coherent
 378 core (RCLV) trapping initial water inside its boundary in 30 days. Unlike the coherent transport by
 379 RCLVs, part of the initial water inside the SSH eddies rapidly escapes from the eddy interior and
 380 joins in the background environment, then moves along with the local current. Simultaneously, the
 381 background water can intrude into SSH eddies. Among these three cases, the leakage ratio of the
 382 initial water reach about 20%, 33%, and 40%, respectively. The strongest intrusion occurs in the
 383 growing eddy, with about 34,000 Lagrangian particles from background flow intruding into it. This
 384 is because the eddy is expanding and more and more background water becomes enclosed by the
 385 outermost closed SSH contour. In contrast, the intrusion in the decaying eddy is relatively weak.
 386 To measure the rate of leakage and intrusion, the crossover point of leakage and intrusion trends is
 387 considered. The appearance of the crossover day means the volume of intruded water exceeds that
 388 of initial eddy water retained inside the SSH eddy. A quick (slow) crossover day corresponds to
 389 a strong (weak) material flux across boundaries. In this example, for both the growing and steady
 390 eddy, the crossover day is about 10th day. The decaying eddy, in contrast, has no crossover day
 391 because the intrusion is weak, due to the continuously decreasing eddy volume. Besides the coherent
 392 RCLV core, there appear to be some initial particles enclosed by SSH eddy boundaries, but not
 393 recognized as part of the Lagrangian coherent structure. The LAVD algorithm doesn't include this
 394 structure as part of RCLV, because it rotates differently from the main coherent core, existing in the
 395 form of filaments. A supplementary animation to show these three leaky SSH eddies is available at
 396 <https://vimeo.com/341157321>.

397 This material flux across SSH eddy boundaries from these random cases illustrates that the water
 398 surrounded by SSH contours at neighboring time isn't necessarily the same fluid. Since leaky part
 399 doesn't contribute to the coherent transport, the transport estimate based on SSH eddy boundaries
 400 may strongly overestimate the real coherent material transport, as represented by the RCLV core.

408 In the following statistical analysis, the leakage and intrusion are normalized by the initial
 409 particle numbers inside SSH eddies, which are given by $\frac{N_{in}(t)}{N_{in}^0}$ and $\frac{N_{out}(t)}{N_{out}^0}$, and their mean trends
 410 are described as $\frac{\overline{N_{in}(t)}}{N_{in}^0}$ and $\frac{\overline{N_{out}(t)}}{N_{out}^0}$. Figure 8 shows the normalized leakage trend and intrusion
 411 trend for all 30-day overlapping SSH eddies. The mean leakage ratio of all eddies is about 50%,
 412 which indicates roughly half of water initially enclosed by the SSH contour isn't coherently carried
 413 by the moving SSH eddy by the end of the detection period. The degree of leakage and intrusion
 414 for different categories of eddies are distinctive. Growing eddies and decaying eddies corresponds
 415 to weaker (40%) and stronger (60%) leakage of initial water, respectively. And the contrary trait
 416 is observed in the process of intrusion. The large mean value and standard deviation (red line and
 417 error bars in middle panel) for intrusion trend is from the growing eddy, whose normalized intruded
 418 particle number may be very large if the eddy area expands for several times than initial size. For
 419 steady eddies, the behavior of the leakage curve is close to that of growing eddies, with the leaky
 420 water accounting for about 40%. Besides the leaky part, the truly coherent structure (RCLV) accounts
 421 for about 25% of initially inside water, and its material boundary ensure the volume of this part of
 422 water remains constant during the lifetime (see dashed blue line in the right panel). The additional
 423 25% or retained water evolves as a filament structure but still remain inside the SSH eddy boundary.
 424 The crossover time of averaged leakage and intrusion is 20 days.

425 We also estimate the material flux for the 359 longer-lived 90-day eddies. The results, shown
 426 in Figure 9, display similar behaviors to those of 30-day eddies, including the trends of leakage and
 427 intrusion for three categories of eddies. The average leakage ratio at the final time is about 50%
 428 as well. The crossover day of leakage and intrusion occurs on about 23th day. After the crossover
 429 day, the rate of leakage and intrusion become obviously slow, because mostly only the coherent core
 430 remains. Figure 10 shows histograms of the crossover day for 30- and 90-day SSH eddies. Grouped
 431 by eddy growth stage, for both shorter-lived and longer-lived eddies, the crossover day occurs in the
 432 sequence growing eddies, steady eddies, and decaying eddies. About 50% decaying 30-day eddies
 433 have no crossover day due to the weak intrusion. Distinct features are not observed when we group
 434 eddies as cyclonic and anticyclonic (not shown).



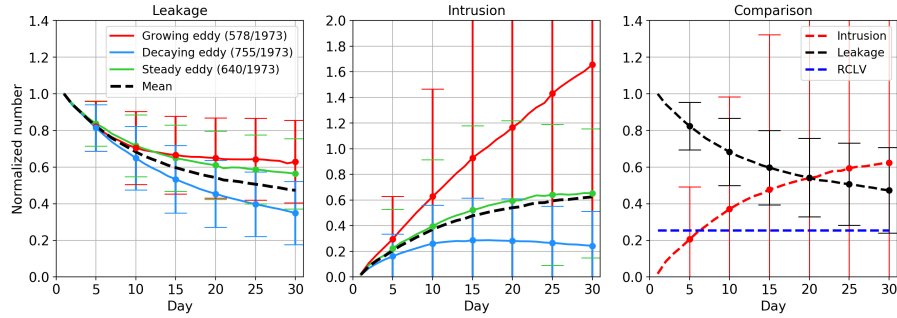
401 **Figure 7.** (the first three columns) The particle locations around three randomly selected overlapping eddies
 402 over 30 days. (fourth column) Time series of particle numbers for these three eddies. The growing eddy, the
 403 decaying eddy, and the steady eddy is shown in (a-d), (e-h), and (i-l), respectively. The red dots represent the
 404 initial particles inside SSH eddy boundaries (initial particles). The green dots represent the initial particles
 405 outside SSH boundaries (background particles). The blue dots indicate RCLVs particles. The yellow lines are
 406 SSH boundaries. The center points trajectories of SSH eddies are superimposed in purple lines. The normalized
 407 SSHA of three eddies are labeled in (d, h, and l).

443 4.3 Non-overlapping SSH eddies

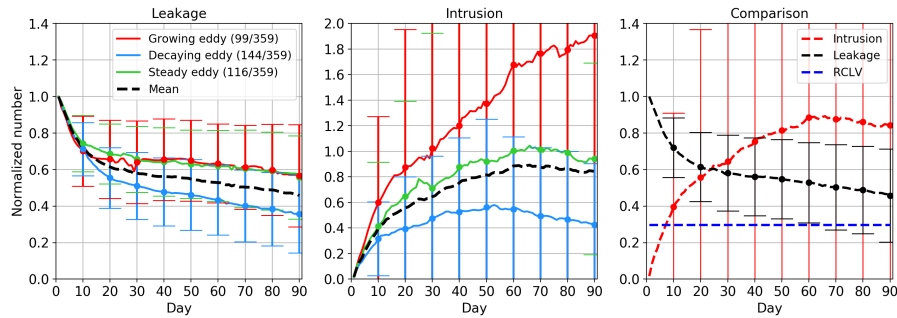
444 The above examples give the impression that SSH eddies contain a coherent material core;
 445 however, we have so far only examined overlapping SSH and RCLV eddies. We now turn to the
 446 numerous non-overlapping SSH eddies, which account for roughly half of SSH eddy samples.

447 These SSH eddies can be detected by the SSHA contour method, but there is no Lagrangian
 448 coherent structure inside these SSH eddies. Thus we anticipate stronger exchange with the back-
 449 ground. Figure 11 shows the movement of particles in 30 days around three randomly selected
 450 non-overlapping SSH eddies. For the growing eddy (top panels), after 30 days the SSH eddy is
 451 almost filled with intruded water, along with more than 90% initial water leaking out the boundary.
 452 Some particles still remain in the eddy interior for decaying eddy (middle panels) and steady eddy
 453 (bottom panels), but evidently these do not meet the requisite criteria for RCLV detection. The
 454 statistical results are shown in Figure 12. On average, the leakage for non-overlapping eddies reaches
 455 60%, stronger than that of overlapping SSH eddies with a coherent core. In the same way, the intru-
 456 sion process is also stronger compared to overlapping SSH eddies. And we note that the crossover
 457 day of leakage and intrusion is the 12th day, revealing that the material flux occurs at a faster rate.

458 Tightening the contour threshold will not enable non-overlapping SSH eddies keep the material
 459 coherence properties because of the strong leakage and intrusion process, which suggests that the



435 **Figure 8.** (left) Normalized leakage trend, (middle) intrusion trend, and (right) the comparison of leakage
 436 and intrusion for 30-day overlapping SSH eddies. The numbers of three categories of eddies are labeled in the
 437 legend in the left panel. The error bar indicates one standard deviation of the mean over all samples on that day.
 438 The invariable blue dashed line in the right panel indicates normalized RCLV particle number.



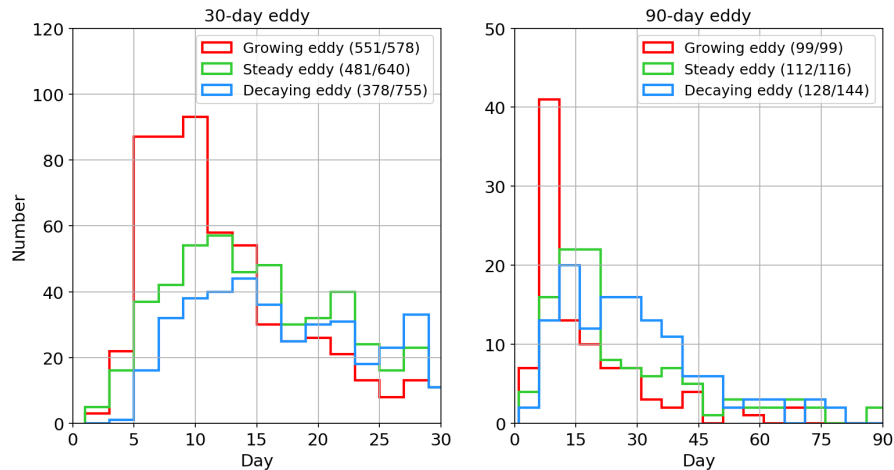
439 **Figure 9.** Same as Figure 8, but for 90-day overlapping eddies.

460 eddy coherence properties depend on whether the organization method is material, rather than the
 461 choice of thresholds or parameters. In other words, the water exchange across SSH eddy boundaries
 462 over the eddy whole lifetime shows the SSH eddies are far from materially coherent structures.

476 5 Random-Circle Eddy Model

477 The analysis above showed that the non-overlapping SSH eddies were significantly leakier than
 478 the ones that overlapped with RCLVs. But is there still some inherent material coherence to these
 479 SSH eddies that the RCLV detection algorithm fails to identify? In this section, we calculate the
 480 degree of leakage and intrusion around completely artificial eddies defined by randomly generated
 481 circles. This allows us to assess the differences in the material coherence properties between SSH
 482 eddies and arbitrary patches of water.

483 For a fair comparison, we generate a dataset of artificial eddies based on the statistical features
 484 of SSH eddies. The number distribution with radius of overlapping SSH eddies is displayed in Figure
 485 14 (left), which suggests that more than 70% of overlapping SSH eddies range from about 50 km to
 486 90 km in radius. Here, the SSH eddies with radius larger than 170 km are excluded because of their
 487 infrequent occurrence. Following this number and radius distribution, we structure the same amount
 488 of circles in the subtropical region for the following random eddy experiments. The initial locations
 489 of these circles are randomly selected in the subtropical basin during a random 90-day time interval.
 490 The random choice of the initial position and occurrence time distributes these 1969 circles roughly
 491 uniformly over the study region and time interval.



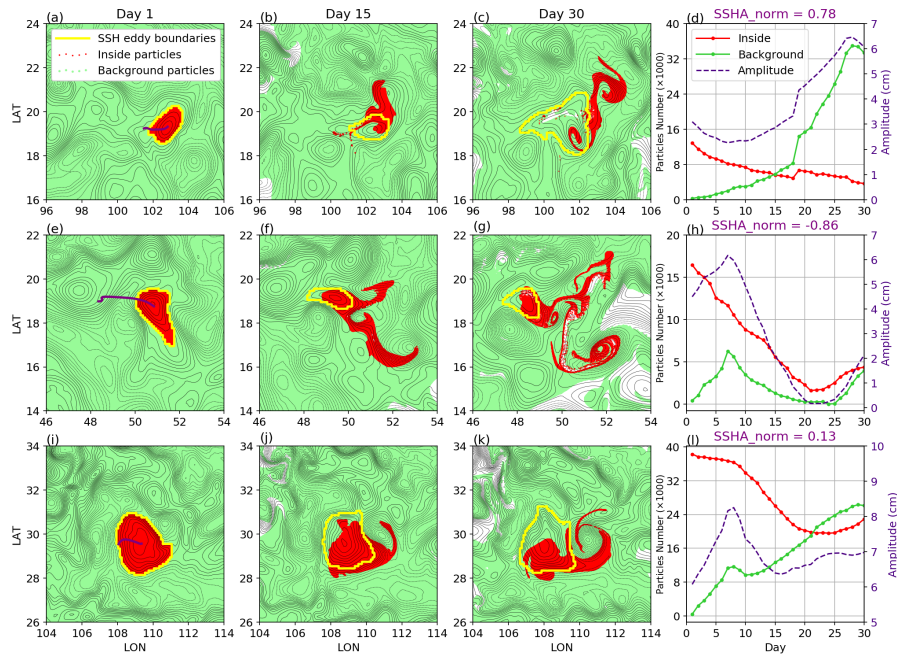
440 **Figure 10.** Histograms of the crossover day for (left) 30-day and (right) 90-day SSH eddies. Growing eddies,
 441 steady eddies, and decaying eddies are indicated by red, green, and blue lines, respectively. The total number
 442 of each group and the number of eddies with the crossover day are labeled in the legends.

492 We set up two simple scenarios for the random-circle eddy model. In scenario 1, the positions
 493 of the artificial circles are fixed at their initial positions. In scenario 2, these circles are advected by
 494 the mean velocity of the center point over 30 days. The radius of the random-circle remains constant
 495 for both two cases in the experiment, without the growth or decay of size. A quantification of leakage
 496 and intrusion across boundaries is conducted for the fixed-circle model and moving-circle model.

497 Figure 13 shows the process of material flux across artificial boundaries for two randomly
 498 selected circles from two different scenarios. For the fixed circle (top panels), which is initially
 499 overlaid on several closed SSH contours, the initial water parcels inside the circle boundary leak
 500 into the background flow in a fast rate, with the particle number decreasing above 80%, from about
 501 19000 to about 3000 in 30 days. Figure 13 (a-c) shows that some initial particles surrounded by the
 502 boundary on the 30th day do not always remain inside the circle, but rather reenter into the circle
 503 carried by an eddy-like structure after a quick leakage. For the moving circle (bottom panels), which
 504 propagates westward following the mean flow, about 30% initial particles remain after 30 days. The
 505 crossover time is 12 days, which indicate the leakage rate is slower comparing with the fixed case (7
 506 days). These two cases suggest that, even though there is no the coherent structure, initial particles
 507 defined by artificial boundaries still have the possibility to remain inside these boundaries under
 508 effects of local background flow field.

509 Statistical results of normalized leakage and intrusion from fixed circles model and moving
 510 circles model are shown in Figure 14. After 30 days, there are about 18% and 32% initial water
 511 surrounded by these artificial boundaries for fixed and moving cases, respectively. The standard
 512 deviation of the intrusion for random-circles is close to that of the leakage, in contrast to the real
 513 SSH eddies. The reason is that the size of the random circle is constant, so there is no large
 514 normalization value of intrusion which is common in real eddies accompanying with the eddy size
 515 expansion. In particular, the leakage and intrusion trends of moving circles are quite similar to those
 516 of non-overlapping SSH eddies, and their crossover times are both 12 days.

517 Figure 15 displays the bar diagram for the ratio of leakage and intrusion after 30 days from
 518 5 categories of eddies (circles), including RCLVs, overlapping SSH eddies, non-overlapping SSH
 519 eddies, fixed circles, and moving circles. RCLVs by construction have no leakage and intrusion
 520 during the lifetime because their boundaries are material. Among the other 4 cases, the leakage in



463 **Figure 11.** Same as Figure 7, but for three randomly selected non-overlapping SSH eddies. The top, middle,
 464 and bottom panels correspond to a growing eddy, a decaying eddy, and a steady eddy, respectively.

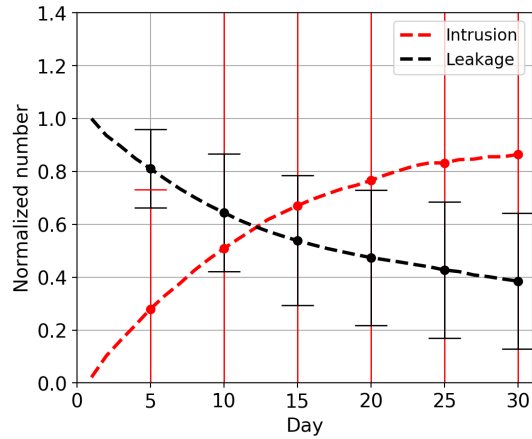
521 30 days for SSH eddies with the coherent core (overlapping SSH eddies) is the weakest, but it still
 522 exceeds 50% of initial water parcels.

523 These findings suggest that, though there is a slight difference in the magnitude of leakage and
 524 intrusion ratio, the material coherence properties of non-overlapping SSH eddies (about half of SSH
 525 eddy samples) are not significantly different from randomly selected pieces of ocean with the same
 526 size.

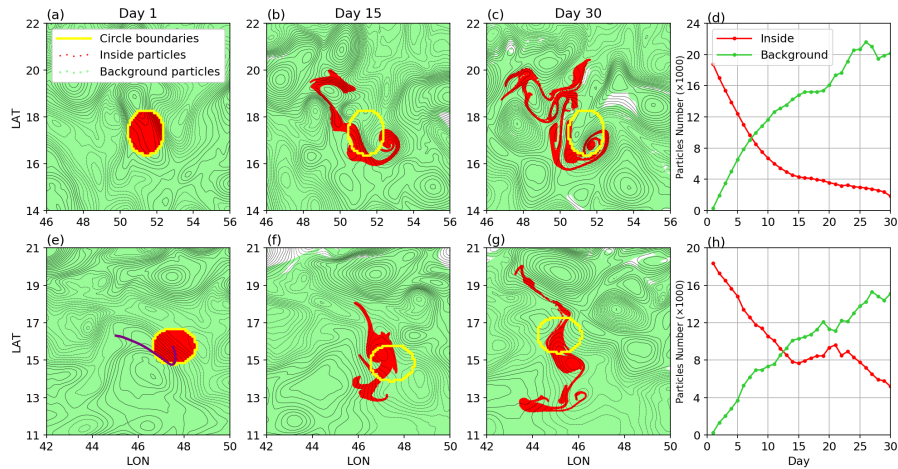
527 6 Discussion and Conclusions

528 Existing coherent eddy identification methods can be classified into two general categories:
 529 Eulerian (e.g., *Chelton et al.* [2011], *Nencioli et al.* [2010], *Faghmous et al.* [2015]) and Lagrangian
 530 (e.g., *Beron-Vera et al.* [2013], *Haller et al.* [2016]). A wealth of literature has reported some
 531 provocative results about coherent eddy transport based on Eulerian eddy boundaries, especially
 532 the surprising zonal eddy mass transport (30Sv) in *Zhang et al.* [2014]. However, several works
 533 (e.g. *Wang et al.* [2015], *Abernathey and Haller* [2018]) using Lagrangian frameworks indicate
 534 these Eulerian methods strongly overestimate the degree of eddy material transport. In this study,
 535 we explored the relationship between eddy detection methods and coherent material transport and
 536 quantitatively calculated the leakiness of Eulerian eddies.

537 An idealized eddy-resolving ocean basin, closely resembling the North Pacific ocean, was
 538 simulated using MITgcm. More than 55 million Lagrangian particles were introduced into the
 539 model and advected by 2D surface flow in the final 5 years. We identified Lagrangian coherent
 540 eddies (also called RCLVs) in 30- and 90-day intervals by the method *Haller et al.* [2016]. In the
 541 corresponding time interval, SSH eddies were detected from SSHA fields through the approach
 542 proposed by *Faghmous et al.* [2015].



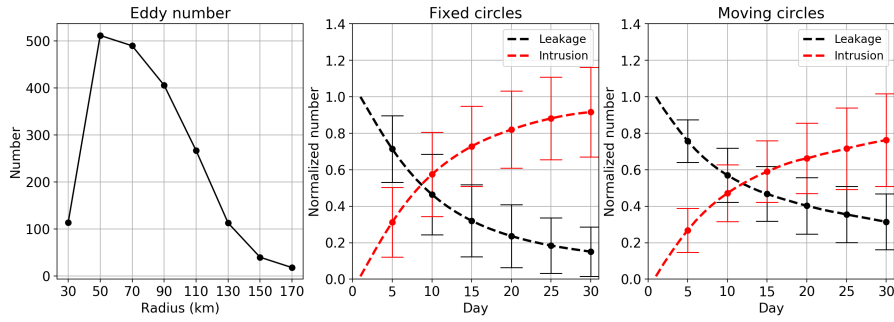
465 **Figure 12.** The averaged leakage and intrusion trend for non-overlapping SSH eddies in 30 days. The error
 466 bar indicates one standard deviation of the average over all eddies on the same day.



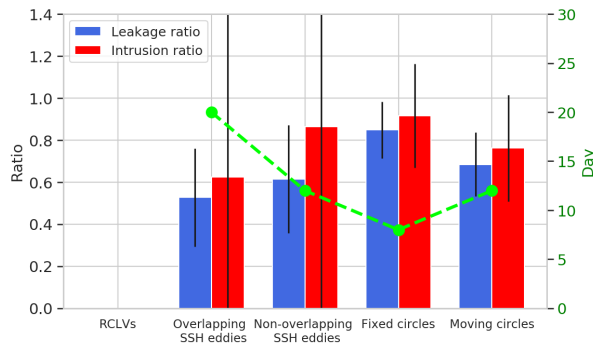
467 **Figure 13.** Same as Figure 7, but for randomly selected (top panels) a fixed circle and (bottom panels) a
 468 moving circle.

543 General statistical features of RCLVs were compared to those of SSH eddies. We found that
 544 RCLVs have a much smaller coherent core than SSH eddies and are often enclosed by SSHA contours.
 545 Specially, about 25% of RCLVs were found to be not located near a local peak of SSHA. We also
 546 found that mean radius of all RCLVs (35.5 km) is approximately about half of the mean SSH eddy
 547 radius (72.4 km). Both RCLVs and SSH eddies had similar westward propagation speeds with
 548 the latitude. Assuming SSH eddies could maintain coherence over their whole lifetimes, the zonal
 549 coherent transport by RCLVs was around half of the transport by SSH eddies.

550 The principal focus and innovation of this paper was the calculation of material flux across
 551 Eulerian eddy boundaries. The eddies were classified into three categories: overlapping RCLVs and
 552 SSH eddies, non-overlapping SSH eddies, and non-overlapping RCLVs. Using Lagrangian particles,
 553 we examined the processes of leakage and intrusion around overlapping and non-overlapping SSH
 554 eddies. For 30-day overlapping SSH eddies, over their lifetimes, the materially coherent part is only
 555 about 25% of the total volume (recognized as RCLVs), while about 50% of initial water escapes into



469 **Figure 14.** (left) The SSH eddy number distribution with radius. (middle and right) The averaged leakage
 470 and intrusion trend for randomly selected fixed and moving circles in 30 days. The error bar indicates one
 471 standard deviation of the average over all circles on the same day.



472 **Figure 15.** Bar diagram showing the ratio of leakage and intrusion after 30 days for RCLVs, overlapping SSH
 473 eddies, non-overlapping SSH eddies, fixed circles, and moving circles. The black error bar indicates the one
 474 standard deviation. The crossover day of leakage and intrusion is presented by the green dashed line. Specially,
 475 RCLVs have zero leakage and intrusion for their coherent property.

556 the background flow. The remaining 25% of water can still stay inside the SSH eddy boundary, but
 557 in the form of filaments outside the coherent core. After about 20 days, the intruded water exceeds
 558 the amount of initial water in SSH eddies, which provides an evidence that there is strong water
 559 exchange around SSH eddies. Strong leakage and intrusion are also found in 90-day overlapping
 560 SSH eddies. For non-overlapping SSH eddies, more water leakage (about 60%) occurs at a faster
 561 rate.

562 Following the number and radius of real SSH eddies, fixed circles and moving circles were
 563 selected in random regions and time interval to provide a baseline against which to compare the
 564 SSH eddies. After 30 days, there was still about 18% and 35% of initial water surrounded by these
 565 artificial boundaries for fixed and moving cases. In particular, the leakage and intrusion trends of
 566 moving circles were quite similar to that of non-overlapping SSH eddies. These findings suggests
 567 that the material coherence properties of roughly 50% of SSH eddies are not significantly different
 568 from randomly selected pieces of ocean with the same size.

569 The final conclusions from this study are clear: if one wishes to estimate material transport, a
 570 Lagrangian eddy detection algorithm is best, since Eulerian eddies are highly leaky. While Eulerian
 571 eddies may contain a materially coherent core, this core is identified well by the RCLV algorithm.

572 But there are just as many Eulerian eddies with no materially coherent core. We see no obvious way
573 to separate these two cases *a priori*, i.e. without also doing Lagrangian eddy detection.

574 However, the fact is that running a model with millions of Lagrangian particles and conducting
575 the identification remains a heavy computational task. The recent publication of an efficient La-
576 grangian Coherent Structure software package may mitigate this somewhat (`CoherentStructures.jl`;
577 *Karrasch and Schilling* [2019]). We are also in the process of compiling a global RCLV database
578 based on satellite altimetry, which should provide useful for future transport estimates.

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