3-DOF Simulation of Hypothetical Crew Vehicle Launch into Low Mars Orbit

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Note: this is a non-peer reviewed preprint submitted to EarthArXiv.

Abstract

A major challenge in space exploration has been the return of a vehicle from the surface of Mars to Earth. This problem has proven to be quite difficult, with current plans to return small rock samples turning out to be too expensive and time consuming[1]. If humans are ever to explore the surface of the red planet, a practical way to launch a crew from the surface of Mars into an orbit from which they can continue their return journey to Earth must be developed. This simulation is intended to explore the requirements for such a mission.

I – Introduction

NASA is currently engaged in international efforts to develop a crewed mission to Mars[2]. Over the past 50 years, dozens of spacecraft launched by several different countries have made it to Mars[3], but so far none have even attempted to return. The Mars Sample Return program was meant to be the first instance of a spacecraft returning from the surface of Mars, intended to collect samples gathered by the Perseverance rover for analysis on Earth. However, this program was scrapped by NASA after their plan was determined to be too expensive and time consuming[1].

If a crewed mission is ever to be sent to Mars, it is absolutely imperative that reliable technology and procedures be developed to ensure its safe return. The first step in a return journey would be a launch from the surface of Mars into low Mars orbit (LMO) via a Mars ascent vehicle (MAV).

There are some assumptions which can be made about the MAV to optimize both the vehicle's performance and the complexity of calculations made in this project. The first is that the MAV will launch from somewhere on Mars's equator into a circular, prograde orbit with zero inclination. This causes the vehicle to launch with some horizontal velocity due to Mars's rotation, which reduces fuel consumption. Additionally, assuming that the planet rotates clockwise about the z-axis in a Mars-centered inertial frame, maneuvers out of plane with the x-y plane do not need to be considered. This further reduces fuel consumption and allows for the use of a 3-DOF simulation along the x- and y-axes rather than a 6-DOF simulation along the x- y- and z-axes. The second assumption is that the MAV's only purpose is to transport a crew from the surface to Mars orbit, and not to return the crew all the way to Earth. This reduces the weight requirements and significantly simplifies design considerations.

II – Vehicle Design Considerations

Payload Mass

The crew vehicle considered for this simulation is a modified version of the Orion spacecraft. The standard Orion spacecraft consists of a crew capsule and service module. The crew capsule is 11 feet (3.4 m) in length, 16.5 feet (5 m) in diameter, with a weight of approximately 22,000 pounds at liftoff, and the service module is 15.7 (4.8 m) feet in length, 16.5 (5 m) feet in diameter and weighs about 34,000 pounds at liftoff[4]. The adapter which connects the spacecraft to the launch vehicle weighs about 2,000 pounds.

In this simulation, the crew vehicle is meant only to transport a crew from the surface of Mars into Mars orbit and is not meant to return the crew all the way back to Earth, similar to the Lunar Module used in the Apollo program which simply returned the crew to lunar orbit for rendezvous with the Command and Service Module. Therefore, it is assumed that the vehicle will not have a heatshield, which would reduce the vehicle weight by about 1,000 pounds[5], or any structures associated with a heatshield, which would reduce the weight by about another 3,000 pounds[6]. It is also Under these assumptions, the total mass of the payload to be launched into orbit is 54,000 pounds, or about 24,500 kg.

Launch Vehicle Design

The launch vehicle is designed to have a total Δv of 5,000 m/s split into two stages, one with 3,000 m/s of Δv and one with 2,000 m/s of Δv.

The specifications for the MAV's propulsion system are based on the RL-10B-2 engine. Fueled by LOX/LH_2, with an I_{sp} of 465.5s and a thrust of 110.1 kN[7], this engine was selected due to its high I_{sp} and because it has been reliably used for many years in various rocket upper stages. The exhaust velocity for the engines is[8]:

$$v_e = g_e I_{sp} = 9.806 * 465.5 = 4565\ m/s$$

And the mass flow rate per engine is:

$$\dot{m} = \frac{T}{u_e} = \frac{110100}{4565} = 24.12\ kg/s$$

The mass requirements for the upper stage with 2,000 m/s of Δv are computed first. The ratio of the payload mass to the initial total mass of the stage is 0.57[8]. With a payload mass of 24,500 kg, the initial mass of the second stage is:

$$M_{0,2} = \frac{M_{L,2}}{0.57} = \frac{24500}{0.57} = 42{,}982\ kg$$

The required propellant mass is:

$$M_{p,2} = M_{0,2}\left(1 - e^{\frac{-\Delta v}{v_e}}\right) = 42982\left(1 - e^{\frac{-2000}{4565}}\right) = 15248\ kg$$

The second stage is powered by 2 engines, which leads to a mass flow rate of 48.24 kg/s and a maximum burn time of 316 seconds.

The same procedure is used to compute the mass requirements for the first stage, which has a payload mass to initial mass ratio of 0.43:

$$M_{0,1} = \frac{M_{L,1}}{0.43} = \frac{42982}{0.43} = 99{,}958\ kg$$

$$M_{p,1} = M_{0,1}\left(1 - e^{\frac{-\Delta v}{v_e}}\right) = 99958\left(1 - e^{\frac{-3000}{4565}}\right) = 48{,}148\ kg$$

The first stage is powered by 6 engines, which leads to a mass flow rate of 144.72 kg/s and a burn time of 332.7 seconds.

III – Vehicle Equations of Motion

The acceleration of the vehicle due to gravity is determined by Newton's law of universal gravitation:

$$\vec{a}_{grav.} = \frac{\mu_{Mars}\vec{r}}{r^3}$$

Where μ_{Mars} is 4.28283e13 m^3/s^2 [9], $\vec{r}$ is the position vector in meters, and *r* is the magnitude of the position vector.

The acceleration of the vehicle due to thrust is:

$$\vec{a}_{thrust} = \frac{Thrust}{M}\hat{t}$$

Where *M* is the vehicle's mass at any given moment and $\hat{t}$ is the vehicle steering vector.

The effects of the Martian atmosphere are not considered in this simulation for reasons that will be discussed later.

The overall acceleration of the vehicle in the inertial frame is:

$$\vec{a}_{total} = \vec{a}_{grav.} + \vec{a}_{thrust}$$

The vehicle's velocity $\vec{v}$ in the inertial frame is the time integral of $\vec{a}$, and the position $\vec{p}$ in the inertial frame is the time integral of $\vec{v}$. These values are computed using MATLABs built in ode45 function, where the state vector $\boldsymbol{x}$ and its time derivative $\dot{\boldsymbol{x}}$ are:

$$\boldsymbol{x} = \begin{bmatrix} r_x \\ r_y \\ r_z \\ v_x \\ v_y \\ v_z \\ m \end{bmatrix}, \dot{\boldsymbol{x}} = \begin{bmatrix} v_x \\ v_y \\ v_z \\ a_x \\ a_y \\ a_z \\ \dot{m} \end{bmatrix}$$

The initial conditions at first stage ignition are:

$$\boldsymbol{x}_{01} = \begin{bmatrix} 0 \\ r_{Mars} \\ 0 \\ v_{surf} \\ 0 \\ 0 \\ M_{0,1} \end{bmatrix}$$

Where r_{Mars} is the equatorial radius of Mars, 3396.19 km[9], and v_{surf} is the speed of Mars's surface in the inertial frame due to the planet's rotation, given by:

$$v_{surf} = \frac{2\pi r_{Mars}}{88560} = 241\ m/s$$

Where 88560 seconds is the length of time it takes Mars to make one full rotation (1 sol)[10].

The angle between the position vector in the inertial frame and the y-axis is:

$$\theta = \tan^{-1}\frac{r_x}{r_y}$$

The position of the vehicle relative to the launch site, which rotates with Mars, is calculated as:

$$\alpha = \frac{2\pi}{88560}$$

$$\vec{r}_{gr} = \begin{bmatrix} r\sin(\theta - \alpha t) \\ r\cos(\theta - \alpha t) \\ 0 \end{bmatrix}$$

Where α is the angular velocity of Mars and t is the time since launch.

The surface-relative velocity of the vehicle is computed as:

$$\vec{v}_{surf} = \begin{bmatrix} v_{surf}\cos\theta \\ v_{surf}\sin\theta \\ 0 \end{bmatrix}$$

$$\vec{v}_{gr} = \vec{v} - \vec{v}_{surf}$$

Where $\vec{v}_{surf}$ is the velocity vector of the point on the surface directly below the vehicle.

First Stage Steering

The steering rules for the first stage are broken up into three phases. The first phase is from liftoff until the vehicle reaches an altitude of 100 meters, where the vehicle will point directly upwards. The second phase starts at an altitude of 100 meters, where the vehicle will pitch over to a predetermined angle from vertical. The third phase begins when the angle between ground-relative velocity vector and local vertical is greater than or equal to the predetermined pitch angle from the previous phase. The steering vector for the first stage is determined by:

$$alt \; = r - r_{Mars}$$

$$\hat{r} = \frac{\vec{r}}{r}, \hat{v}_{gr} = \frac{\vec{v}_{gr}}{v_{gr}}$$

$$\gamma = \cos^{-1}(\hat{r} \cdot \hat{v}_{gr})$$

$$\hat{t} = \begin{cases} [0\ 1\ 0]^T \; if \; alt < 100\, m \\ [\sin(pitch)\cos(pitch)\ 0]^T \; if \; alt \geq 100\, m \;\&\; \gamma < pitch \\ \hat{v}_{gr} \; if \; alt \geq 100\, m \;\; \gamma \geq pitch \end{cases}$$

Once the third phase of steering begins, the vehicle will follow a trajectory by which it gains altitude to escape the Martian atmosphere while gradually arcing over under the influence of gravity, allowing it to build up horizontal speed.

After 332.7 seconds, the first stage burns out, and the position and velocity in the inertial frame are recorded as:

$$\vec{r}_{MECO} = \begin{bmatrix} x_{MECO} \\ y_{MECO} \\ 0 \end{bmatrix}, \vec{v}_{MECO} = \begin{bmatrix} \dot{x}_{MECO} \\ \dot{y}_{MECO} \\ 0 \end{bmatrix}$$

Second Stage Steering

The initial conditions at second stage ignition are:

$$\boldsymbol{x}_{02} = \begin{bmatrix} x_{MECO} \\ y_{MECO} \\ 0 \\ \dot{x}_{MECO} \\ \dot{y}_{MECO} \\ 0 \\ M_{0,2} \end{bmatrix}$$

The steering rules for the second stage are based on the guidance algorithm used for the Ariane 4 launcher[11]. The algorithm begins by finding the current altitude, horizontal speed, and vertical speed, using the current position and velocity in the inertial frame:

$$alt = r - r_{Mars}$$

$$\gamma = \cos^{-1}(\hat{r} \cdot \hat{v})$$

$$u = v \sin \gamma$$

$$w = v \cos \gamma$$

Next, the desired altitude and horizontal velocity relative to the surface are computed:

$$r_{des} = r_{Mars} + alt_{des}$$

$$v_{des} = \sqrt{\frac{\mu_{Mars}}{r_{des}}}$$

Then, the specific stage time is computed[12]:

$$\tau = \frac{v_e}{\frac{Thrust}{M_{0,2}}}$$

The difference between the current state and the desired state is:

$$Du = v_{des} - u$$

$$Dw = -w$$

$$Dr = alt_{des} - alt$$

Note that because the vehicle is launching into a circular orbit, the desired vertical velocity will always be zero.

The required steering vector is computed by:

$$q = \frac{Du}{v_e}$$

$$p = 1 - e^{-q}$$

$$Dt = p(\tau - t)$$

$$C = \begin{cases} \dfrac{q - p}{q - p - 0.5qp} \; if \; Dt \geq 30 \\ 0 \; if \; Dt < 30 \end{cases}$$

$$p_1 = \frac{v_e Dt}{\tau - t}$$

$$g_{Mars} = \frac{-\mu_{Mars}}{r_{Mars}^2}$$

$$p_2 = Dw - g_{Mars} Dt + C(\frac{Dr}{Dt} - \frac{w}{2})$$

$$\chi = \tan^{-1}\frac{p_2}{p_1}$$

$$\hat{t} = \begin{bmatrix} \cos(\theta - \chi) \\ -\sin(\theta - \chi) \\ 0 \end{bmatrix}$$

Gain *C* is set to zero as the vehicle enters terminal guidance 30 seconds before orbit injection to avoid any aggressive pitch maneuvers before second stage cutoff.

The second stage engine cuts off when the semi-major axis of the current trajectory is equal to or greater than the desired orbital radius:

$$\varepsilon = \frac{v^2}{2} - \frac{\mu_{Mars}}{r}$$

$$Semi\ major\ axis = \frac{-\mu_{Mars}}{2\varepsilon}$$

IV – Results & Discussion

The target orbit for this simulation is circular, with an altitude of 200 km. After repeated trial and error, the optimal initial pitch angle for this vehicle to reach the target trajectory was found to be 4.35°.

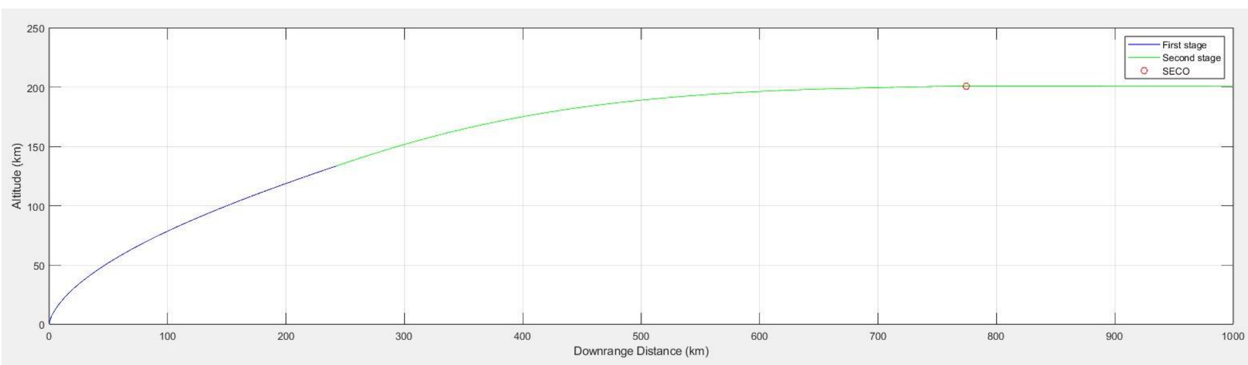

Figure 1: Altitude vs. Downrange Distance

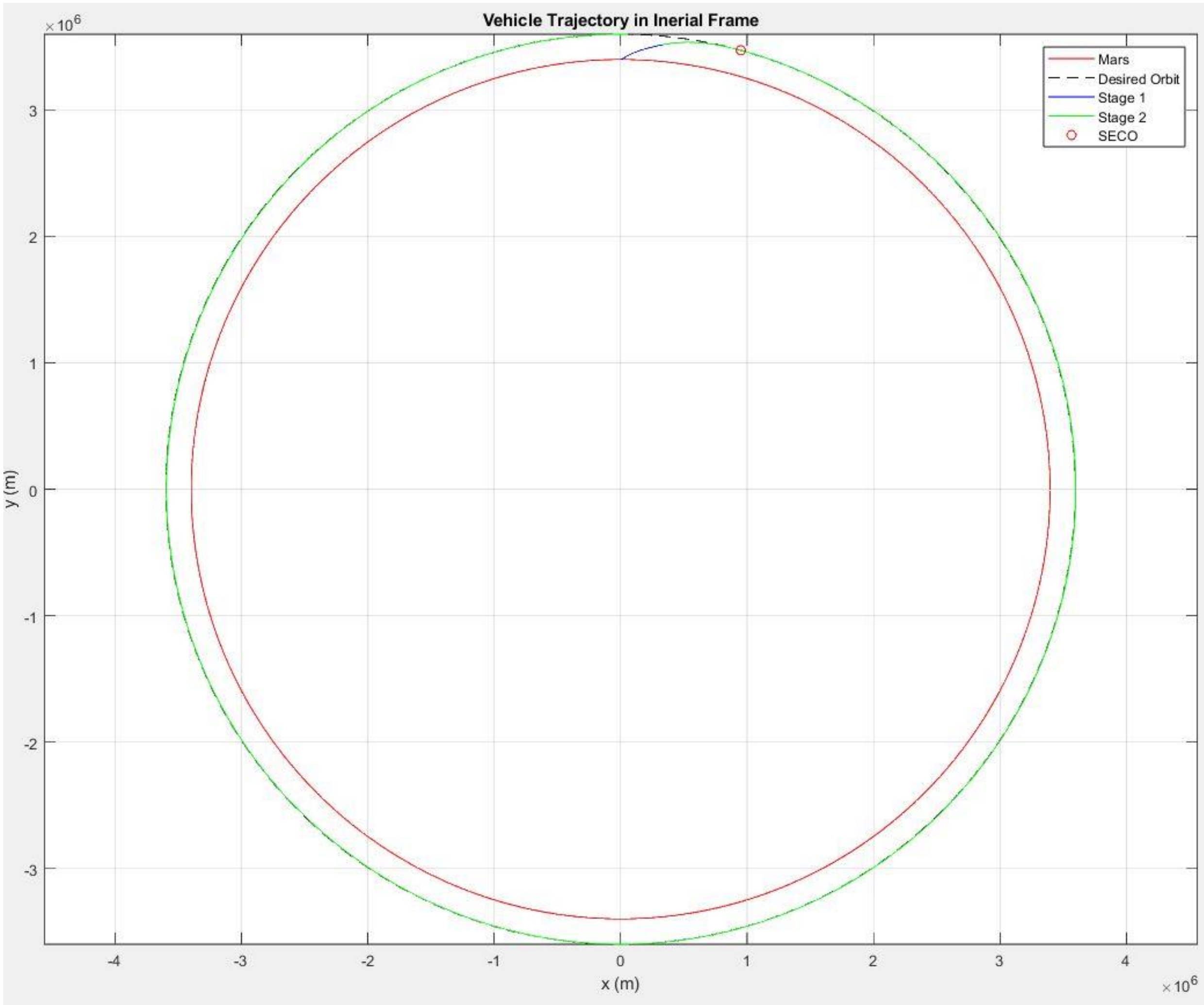

Figure 2: Vehicle Trajectory in inertial frame, over the course of about 6500 seconds

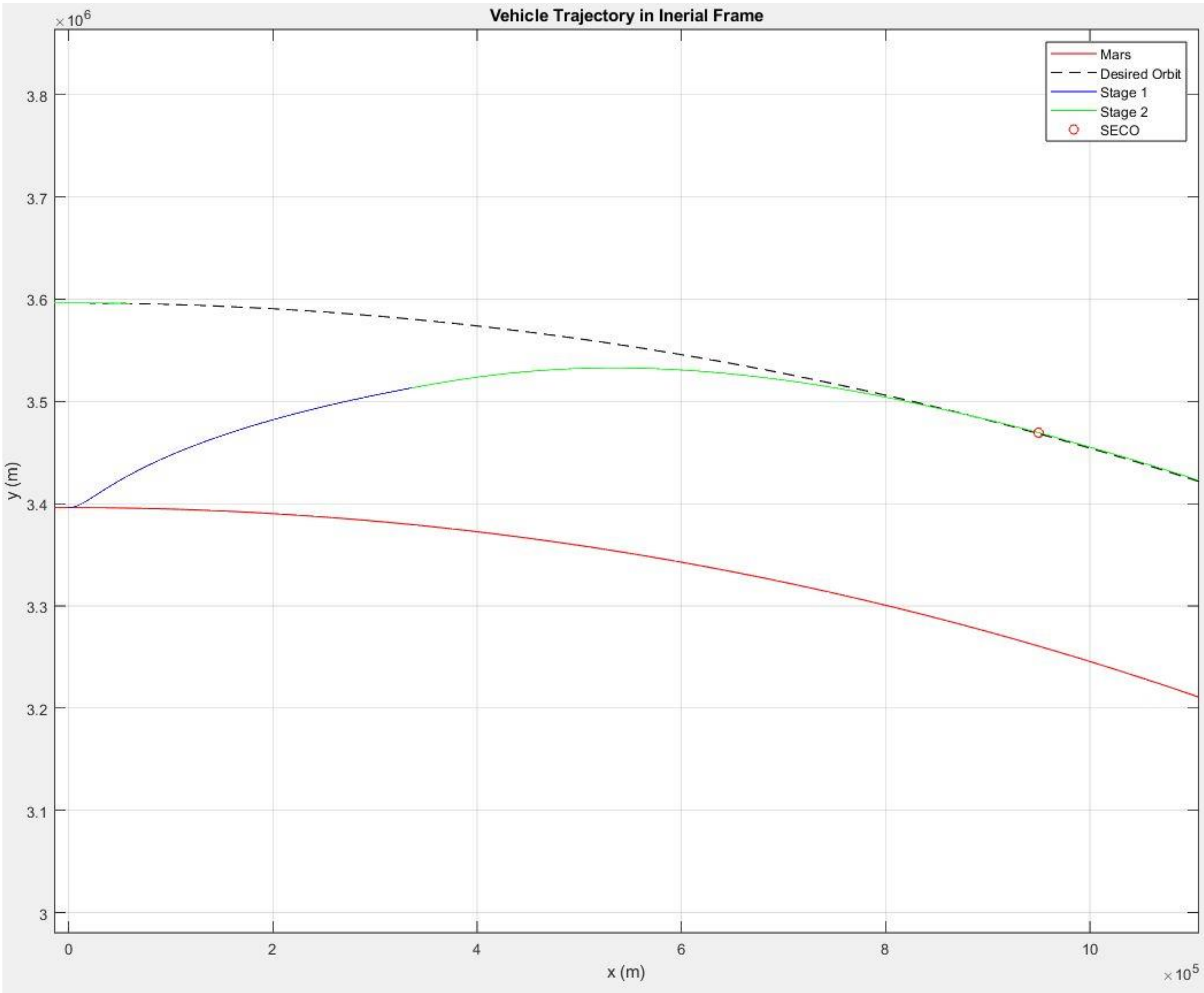

Figure 3: Vehicle Trajectory in inertial frame, zoomed in.

The vehicle achieves orbit in 546.2 seconds having expended 4,250 m/s of Δv. Upon second stage cutoff, the vehicle is within 1 m/s of its target velocity and within 1 km of its target altitude, with an apoapsis of 203 km and a periapsis of 197 km. The steering algorithm adapted from Ariane 4's guidance program guides the vehicle to a trajectory extremely close to the desired orbit.

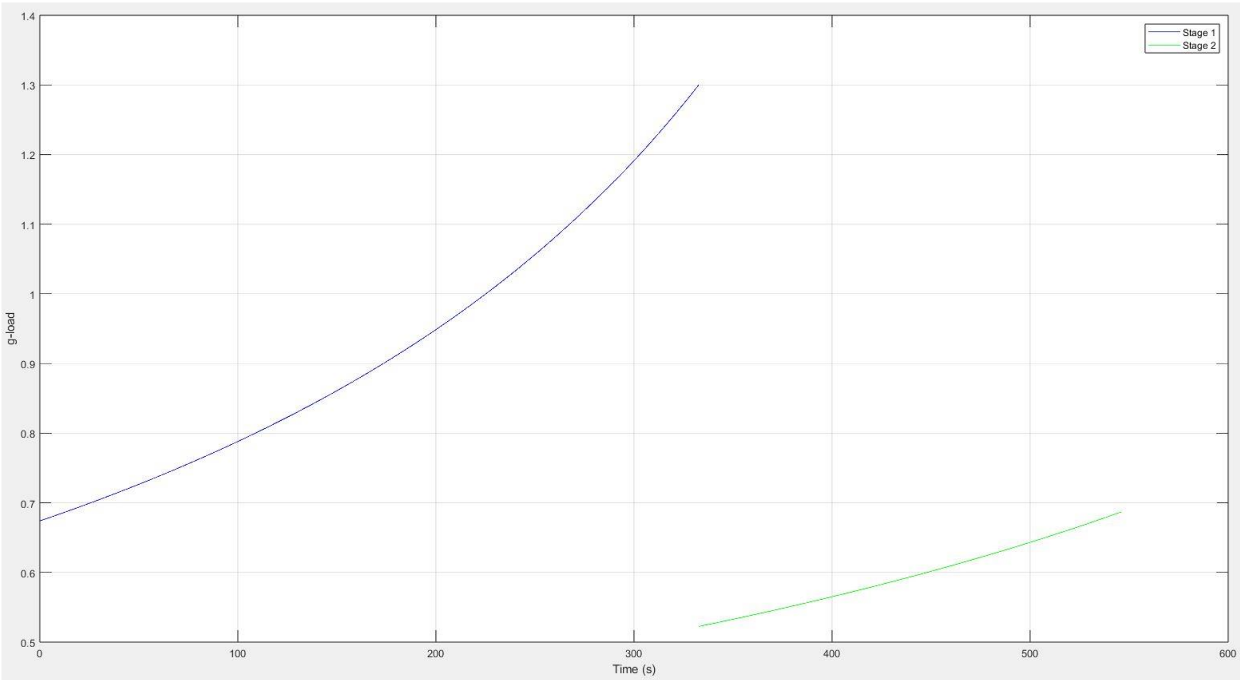

Figure 4: g-loading over time

As can be seen in Figure 4, the highest g-load the vehicle experiences during launch is about 1.3 g. As the peak acceleration is little more than what is experienced due to Earth's gravity, it is safe to assume that the vehicle remains within tolerance for both crew wellbeing and the vehicle's structural limits.

Discussion on Aerodynamic Effects

The Martian atmosphere can be modeled with the following equations [13]:

$$T = \begin{cases} -31 - 0.000998(alt), if\ altitude < 7000\ m \\ -23.4 - 0.00222(alt), if\ altitude > 7000\ m \end{cases}$$

$$p = 0.699e^{-0.00009(alt)}$$

$$\rho = \frac{p}{0.1921(T + 273.1)}$$

The drag the vehicle would experience in the Martian atmosphere is a function of the surface area presented to the windstream, the drag coefficient, and the dynamic pressure [14], which is in turn a function of the vehicle's ground-relative speed and the density of the atmosphere at the vehicle's altitude[14].

$$Q = \frac{\rho v_{gr}^2}{2}$$

$$D = QSC_D$$

The maximum dynamic pressure the vehicle would have experienced on ascent can be estimated by plugging the velocity and position over time for the simulated ascent (which ignores drag) into the formula for dynamic pressure (note that the atmosphere is assumed to be a vacuum above 100 km):

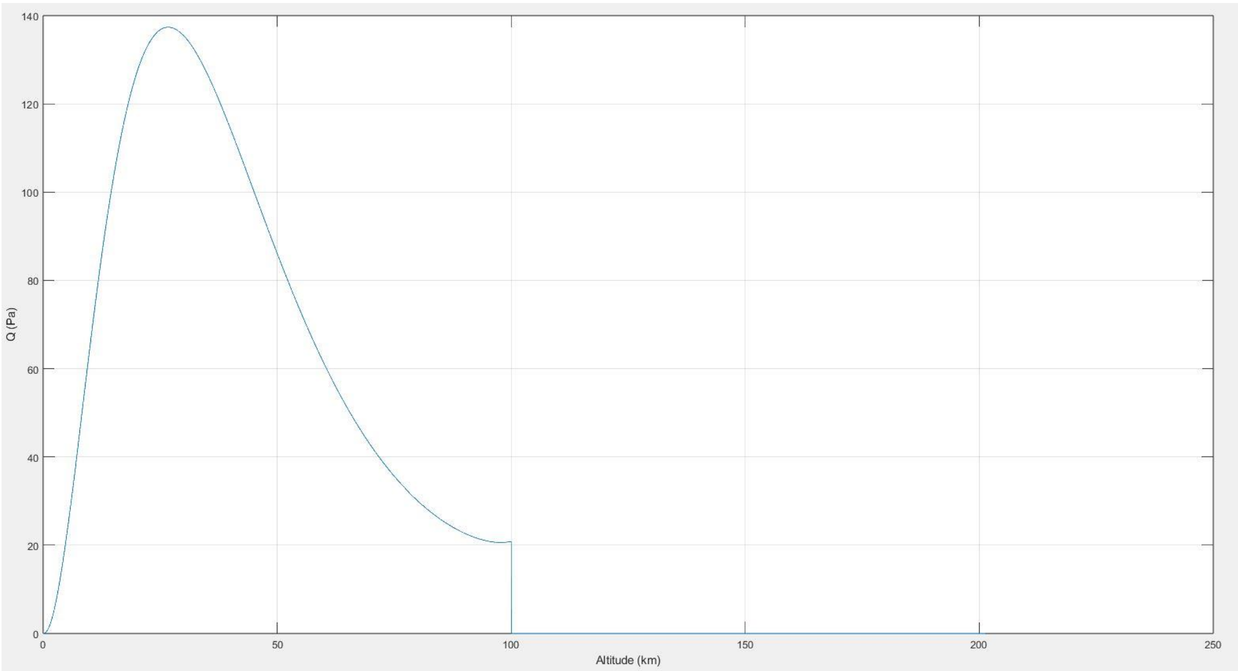

Figure 5: Dynamic pressure vs altitude.

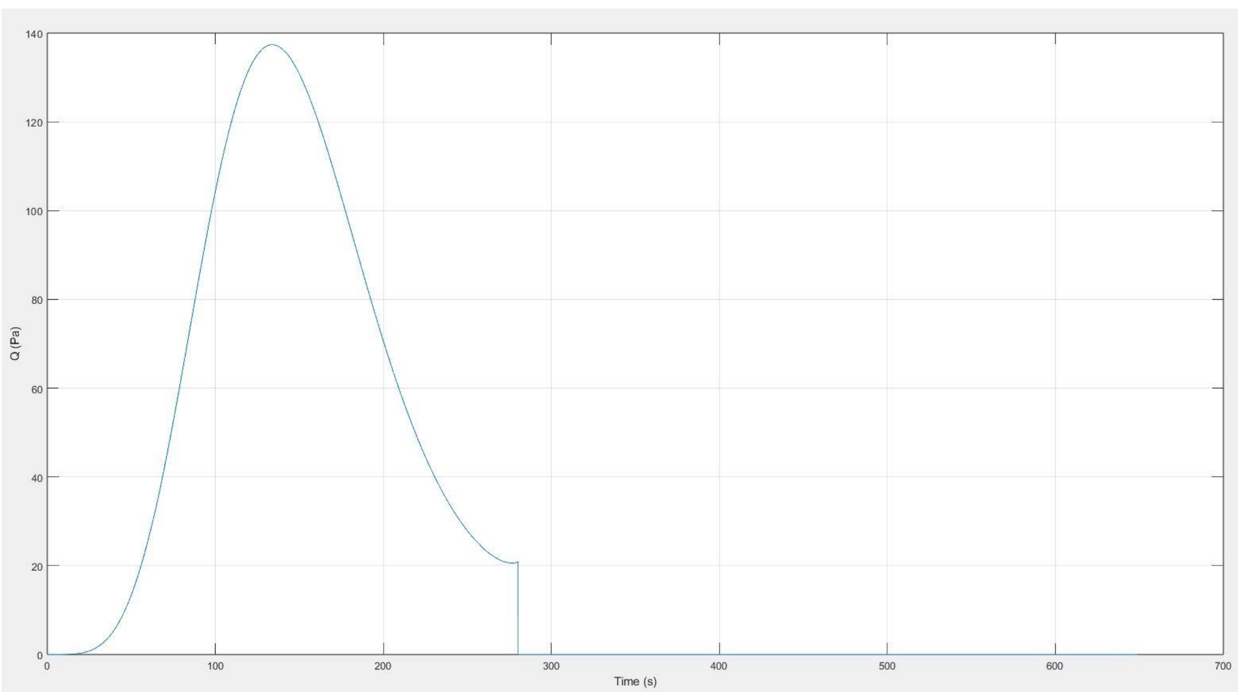

Figure 6: Dynamic pressure vs. time.

As can be seen in Figures 5 and 6, the vehicle would experience a max-Q of 137 Pa at an altitude of 27 km, 134 seconds into flight. Based on the vehicle dimensions discussed in section II, the surface area *S* is assumed to be about 20 square meters. It can be safely assumed that the drag coefficient is somewhere between 0.1 and 1.2 [14]. From this, the estimated maximum possible value for drag force is calculated to be about 3,300 N. The minimum force of thrust during launch is 220,200 N, and the minimum force exerted by gravity is about 100,000 N, both 2 orders of magnitude greater than the estimated drag force. Therefore, it is assumed for the purposes of this simulation that air resistance is negligible.

V – Future Work

In this simulation, it was assumed that the vehicle's attitude matched exactly with the attitude demanded by the guidance program at all times. In future simulations, the vehicle dynamics will be more thoroughly included, with the guidance program controlling attitude via engine gimballing.

Despite the high accuracy of the guidance program in this simulation, it is not the most fuel-efficient way to orbit. The guidance algorithm will be optimized in future programs to utilize more efficient methods such as linear tangent steering.

The effects of altering the propulsion system, such as using an engine fueled by methane/LOX or a hybrid propellant will be investigated.

Although aerodynamic effects were considered to be negligible in this simulation, they will be included in future simulations once the aerodynamic properties of the vehicle can be more precisely determined.

Acknowledgements

Special thanks to Dr. Robert Melton, who provided me with advice in the early stages of this project.

Citations

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Appendix (MATLAB code)

```
clc; close all; clear all;

%% Launch Simulation

R_Mars = 3396.19*1000; %m

mu_Mars = 4.28283e13; %m^3/s^2

c_Mars = 2*pi*R_Mars; % Mars circumference, m.

vrot_Mars_surf = c_Mars/88560; % Surface velocity due to Mars's rotation, m/s;

ang_rate_Mars = 2*pi/88560; % Angular velocity of Mars surface, rad/s.

%% Stage 1 specs

M01 = 99958; % Initial mass for stage 1 in kg

Mb1 = 51810; % Burnout mass;

R1 = M01/Mb1; % Mass ratio.

I_sp = 465.5; % Specific impulse, s.

ue = 9.806*I_sp; % Exhaust velocity, m/s.
```

```
dv1 = ue*log(R1); % Delta-v of first stage.

t_b1 = 332.7; % Burn time, s.

%% Stage 2 specs

M02 = 42982; % Mass of the vehicle just after stage sep.

t_b2 = 316; % Maximum burn time, s.

x01 = [0 R_Mars 0 vrot_Mars_surf 0 0 M01]; %[rx ry rz vx vy vz m]

global pitch_angle

pitch_angle = deg2rad(input('Enter pitch angle in degrees: '));

tol=1e-12;

options = odeset('AbsTol', tol, 'RelTol', tol);

[t1, x1] = ode45(@(t, x) stage1(t, x), 0:0.1:t_b1, x01, options);

r1 = [x1(:,1), x1(:,2), x1(:,3)];

v1 = [x1(:,4), x1(:,5), x1(:,6)];

m1 = x1(:,7);
```

```
r_MECO = r1(length(r1), :);

x_MECO = r_MECO(1);

y_MECO = r_MECO(2);

z_MECO = r_MECO(3);

v_MECO = v1(length(v1), :);

xdot_MECO = v_MECO(1);

ydot_MECO = v_MECO(2);

zdot_MECO = v_MECO(3);

x02 = [x_MECO y_MECO z_MECO xdot_MECO ydot_MECO zdot_MECO M02];

[t2, x2] = ode45(@(t, x) stage2(t, x), 0:0.1:t_b2, x02, options);

r2 = [x2(:,1), x2(:,2), x2(:,3)];

v2 = [x2(:,4), x2(:,5), x2(:,6)];

m2 = x2(:,7);

dm2 = zeros(1,length(m2) - 1);
```

```
for i = 1:length(m2)-1
    dm2(i) = m2(i+1)-m2(i);
end

i_SECO = length(find(dm2));

m_SECO = m2(length(m2));

t_SECO = i_SECO/10;

r_SECO = [x2(i_SECO, 1) x2(i_SECO, 2) x2(i_SECO, 3)];

v_SECO = [x2(i_SECO, 4) x2(i_SECO, 5) x2(i_SECO, 6)];

dv2 = ue*log(M02/m_SECO);

dv_TOTAL = dv1 + dv2;

r_end = r2(length(r2), :);

x_SECO = r_SECO(1);

y_SECO = r_SECO(2);

z_SECO = r_SECO(3);

v_end = v2(length(v2), :);
```

```
xdot_SECO = v_SECO(1);

ydot_SECO = v_SECO(2);

zdot_SECO = v_SECO(3);

energy = norm(v_SECO)^2/2 - mu_Mars/norm(r_SECO);

a = -mu_Mars/(2*energy);

h = norm(cross(r_SECO, v_SECO));

e = sqrt(1 + 2*h^2*energy/mu_Mars^2);

rp = a*(1 - e);

ra = a*(1 + e);

fprintf('e = %d.\n', e);
fprintf('Periapsis altitude = %f km.\n', (rp - R_Mars)/1000);
fprintf('Apoapsis altitude = %f km.\n', (ra - R_Mars)/1000);
fprintf('Burnout mass: %f kg.\n', m_SECO);
fprintf('Cutoff time: %f s.\n', t_SECO + t_b1);
fprintf('Delta-v used: %f m/s.\n', dv_TOTAL);
fprintf('Cutoff altitude = %f km.\n', (norm(r_SECO)-R_Mars)/1000);
fprintf('Speed at cutoff = %f m/s.\n', norm(v_SECO));

r = vertcat(r1, r2);
```

```
v = vertcat(v1, v2);

m = vertcat(m1, m2);

a1 = (660600./m1)./9.806;

a2 = (220200./m2)./9.806;

t = vertcat(t1, t2 + t_b1);

figure
plot(t1(:), a1, '-b', t2(1:i_SECO) + t_b1, a2(1:i_SECO), '-g');
legend('Stage 1', 'Stage 2');
xlabel('Time (s)');
ylabel('g-load');
grid on

alt = zeros(1, length(r));
launchsite_ang = zeros(1, length(r));
vehicle_ang = zeros(1, length(r));
downrange_angle = zeros(1, length(r));
downrange_dist = zeros(1, length(r));
launchsite_vx = zeros(1, length(r));
launchsite_vy = zeros(1, length(r));
launchsite_vz = zeros(1, length(r));
v_grx = zeros(1, length(r));
v_gry = zeros(1, length(r));
```

```
v_grz = zeros(1, length(r));

v_gr = zeros(1, length(r));

Q = zeros(1, length(r));

m = vertcat(m1, m2);


t = vertcat(t1, t2 + t_b1);

for i = 1:length(r) % Convert trajectory from inertial frame to altitude and ground-relative
downrange distance and velocity.

  alt(i) = norm(r(i,:)) - R_Mars;

  launchsite_ang(i) = ang_rate_Mars*t(i);

  vehicle_ang(i) = atan2(r(i, 1), r(i, 2));

  downrange_angle(i) = vehicle_ang(i) - launchsite_ang(i);

  downrange_dist(i) = c_Mars * downrange_angle(i)/(2*pi);

  launchsite_vx(i) = vrot_Mars_surf*cos(launchsite_ang(i));

  launchsite_vy(i) = -vrot_Mars_surf*sin(launchsite_ang(i));

  v_grx(i) = v(i, 1) - launchsite_vx(i);

  v_gry(i) = v(1, 2) - launchsite_vy(i);

  v_gr(i) = norm([v_grx(i) v_gry(i) v_grz(i)]');


   if alt(i) < 7000

  T(i) = -31 - 0.000998*alt(i);


  p(i) = 0.699*exp(-0.00009*alt(i));


  rho(i) = p(i)/(0.1921 * (T(i) + 273.1));

  elseif alt(i) >= 7000

  T(i) = -23.4 - 0.00222*alt(i);


  p(i) = 0.699*exp(-0.00009*alt(i));
```

```
    rho(i) = p(i)/(0.1921 * (T(i) + 273.1));
    end

    if T(i) <= -273.1
        T(i) = -273.1;
    end

    if alt(i) >= 100000
        rho(i) = 0;
    end

    Q(i) = 0.5*rho(i)*v_gr(i)^2;
end

figure
plot(downrange_dist(1:t_b1*10)/1000, alt(1:t_b1*10)/1000,'-b', downrange_dist(t_b1*10 + 1:length(downrange_dist))/1000, alt(t_b1*10 + 1:length(downrange_dist))/1000, '-g', downrange_dist(i_SECO + t_b1*10)/1000, alt(i_SECO + t_b1*10)/1000, 'or');
xlabel('Downrange Distance (km)');
ylabel('Altitude (km)');
legend('First stage', 'Second stage', 'SECO');
axis equal
grid on
ylim([0 250])
xlim([0 1000])

figure
plot(alt/1000, Q);
```

```
xlabel('Altitude (km)');
ylabel('Q (Pa)');
grid on

figure
plot(t, Q);
xlabel('Time (s)');
ylabel('Q (Pa)');
grid on

r_des = R_Mars + 200000;

v_des = sqrt(mu_Mars/r_des);

fprintf('Required v = %f.\n', norm(v_des));

angle = 0:0.005:2*pi; % Mars data for plotting

x_mars = R_Mars*cos(angle);

y_mars = R_Mars*sin(angle);

x_orb = r_des*cos(angle);

y_orb = r_des*sin(angle);

figure
plot(x_mars, y_mars, '-r', x_orb, y_orb, '--k', x1(:,1), x1(:,2), '-b', x2(:,1), x2(:,2),'-g', r_SECO(1),
r_SECO(2), 'or');
```

```
title('Vehicle Trajectory in Inerial Frame');
legend('Mars', 'Desired Orbit', 'Stage 1', 'Stage 2', 'SECO');
xlabel('x (m)');
ylabel('y (m)')
axis equal
grid on

altitude = 0:200000;

T = zeros(1, 200001);
p = zeros(1, 200001);
rho = zeros(1, 200001);

for i = 1:200001
    if i < 7000
    T(i) = -31 - 0.000998*altitude(i);

    p(i) = 0.699*exp(-0.00009*altitude(i));

    rho(i) = p(i)/(0.1921 * (T(i) + 273.1));
    elseif i >= 7000
    T(i) = -23.4 - 0.00222*altitude(i);

    p(i) = 0.699*exp(-0.00009*altitude(i));

    rho(i) = p(i)/(0.1921 * (T(i) + 273.1));
    end
```

```
    if T(i) <= -273.1
        T(i) = -273.1;



    end

    if i >= 100000
        rho(i) = 0;
    end
end

figure
plot(altitude, T);
xlabel('Altitude (m)');
ylabel('Temperature (°C)');
grid on

figure
plot(altitude, p);
xlabel('Altitude (m)');
ylabel('Pressure (KPa)');
grid on

figure
plot(altitude, rho);
xlabel('Altitude (m)');
ylabel('Density (kg/m^3)')
grid on;
```

```
function xdot = stage1(t, x) % Equations of motion for stage 1, powered by 6 RL-10B-2 engines.

global pitch_angle

%% Stage 1 specs

M0 = 99958; %kg

n = 6; % Number of engines.

I_sp = 465.5; % s.

ue = 9.806*I_sp; % Exhaust velocity, m/s.

t_b = 332.7; % Burn time, s.

%% Mars specs

R_Mars = 3396.19*1000; %m

mu_Mars = 4.28283e13; %m^3/s^2

g_Mars = -mu_Mars/R_Mars^2; % Gravitational acceleration at Mars surface, m/s^2.

c_Mars = 2*pi*R_Mars; % Mars circumference, m.

vrot_Mars_surf = c_Mars/88560; % Surface velocity due to Mars's rotation, m/s;
```

```
ang_rate_Mars = 2*pi/88560; % Angular velocity of Mars surface, rad/s.

%% Vehicle position

% Inertial position

rx = x(1);

ry = x(2);

rz = x(3);

r = [rx ry rz]';

rhat = r/norm(r);

theta = atan2(rx, ry); % Downrange angle.

% Ground-relative position

theta_gr = theta - ang_rate_Mars*t; % Ground relative downrange angle.

rx_gr = norm(r)*sin(theta_gr);

ry_gr = norm(r)*cos(theta_gr);

rz_gr = rz;
```

```
r_gr = [rx_gr ry_gr rz_gr]'; % Position relative to launch site.

%% Vehicle velocity

% Inertial velocity

vx = x(4);

vy = x(5);

vz = x(6);

v = [vx vy vz]';

% Ground-relative velocity

v_surf_x = vrot_Mars_surf*cos(theta); % Surface horizontal velocity in inertial frame.

v_surf_y = -vrot_Mars_surf*sin(theta); % Surfave vertical velocity in inertial frame.

v_surf_z = 0;

v_surf = [v_surf_x v_surf_y v_surf_z]'; % Inertial surface velocity at point directly below
vehicle.

v_gr = v - v_surf; % Ground-relative velocity.

%% Vehicle mass
```

```
m = x(7);

alt = norm(r) - R_Mars;

%% Atmospheric condtitions (source: https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/mars-atmosphere-equation-metric/)

if alt < 7000
    T = -31 - 0.000998*alt;

    p = 0.699*exp(-0.00009*alt);

    rho = p/(0.1921 * (T + 273.1));
elseif alt >= 7000
    T = -23.4 - 0.00222*alt;

    p = 0.699*exp(-0.00009*alt);

    rho = p/(0.1921 * (T + 273.1));
end

if T <= -273.1
    T = -273.1;
end

if alt >= 100000
    rho = 0;
end
```

```
Q = 0.5*rho*norm(v_gr)^2; % Dynmaic pressure.

thrust = 110100*n; %N

mdot = -thrust/ue;

if norm(v) == 0 % Vehicle is pointing up initially.
   v_gr_hat = [0 1 0]';
elseif norm(v) > 0
   v_gr_hat = v_gr/norm(v_gr); % Ground relative velocity unit vector.
end

gamma = acos(dot(rhat, v_gr_hat)); % Flight path angle, ground relative.

a_grav = -mu_Mars*r/norm(r)^3;

if gamma >= pitch_angle
   that = v_gr_hat; % Gravity turn

   elseif alt < 100

      that = [0 1 0]';   % Vertical climb

   elseif alt >= 100

      that = [sin(pitch_angle), cos(pitch_angle), 0]'; % Pitchover
```

```
end

%% Vehicle acceleration

a_thrust = thrust*that/m;

acc = a_grav + a_thrust;

ax = acc(1);

ay = acc(2);

az = acc(3);

xdot = [vx vy vz ax ay az mdot]';

end

function xdot = stage2(t, x) % Equations of motion for stage 2, powered by 2 RL-10B-2 engines.

%% Stage 2 specs

M0 = 42982; %kg

n = 2; % Number of engines.

t_b = 316; % Maximum burn time, s.
```

```
I_sp = 465.5; % s.

ue = 9.806*I_sp; % Exhaust velocity, m/s.

%% Mars specs

R_Mars = 3396.19*1000; %m

mu_Mars = 4.28283e13; %m^3/s^2

g_Mars = -mu_Mars/R_Mars^2; % Gravitational acceleration at Mars surface, m/s^2.

%% Vehicle position

rx = x(1);

ry = x(2);

rz = x(3);

r = [rx ry rz]';

rhat = r/norm(r); % Position unit vector.

theta = atan2(rx, ry); % Downrange angle.

alt = norm(r) - R_Mars;
```

```
%% Vehicle velocity

vx = x(4);

vy = x(5);

vz = x(6);

v = [vx vy vz]';

vhat = v/norm(v); % Velocity unit vector.

gamma = acos(dot(rhat, vhat)); % Flight path angle relative to local horizon.

u = norm(v)*sin(gamma); % Horizontal velocity relative to local horizon.

w = norm(v)*cos(gamma); % Vertical velocity relative to local horizon.

%% Vehicle mass

m = x(7);

%% Desired orbit

alt_des = 200000; % m

r_des = R_Mars + alt_des;
```

```
v_des = sqrt(mu_Mars/r_des);

%% Current orbit

energy = norm(v)^2/2 - mu_Mars/norm(r); % Orbital energy.

a = -mu_Mars/(2*energy); % Semi-major axis.

h = norm(cross(r,v)); % Angular momentum.

e = sqrt(1 + 2*h^2*energy/mu_Mars^2); % Eccentricity.

rp = a*(1 - e); % Periapsis.

ra = a*(1 + e); % Apoapsis.

%% Guidance equations (Flat-Mars, for a circular orbit)

tau = ue/(110100*n/M0); % Stage specific time.

Du = v_des - u; % Horizontal velocity-to-go.

Dw = 0 - w; %Vertical velocity to go.

Dr = alt_des - alt; % Altitude to go.

q = Du/ue;
```

```
p = (1 - exp(-q));

Dt = p*(tau - t); % Time-to-go to LMO injection.

if Dt < 30
    C = 0; % Gain reduction at 30s to injection for terminal guidance.
else
    C = (q - p)/(q - p - 0.5*q*p);
end

p1 = ue*Dt/(tau - t); % Horizontal component of thrust.

p2 = Dw - g_Mars*Dt + C*(Dr/Dt - w/2); % Vertical component of thrust.

chi = atan2(p2, p1); % Required pitch angle above local horizon.

that = [cos(theta - chi), -sin(theta - chi), 0]'; % Thrust unit vector, inertial frame.

if alt < 7000
    T = -31 - 0.000998*alt;

    p = 0.699*exp(-0.00009*alt);

    rho = p/(0.1921 * (T + 273.1));
elseif alt >= 7000
    T = -23.4 - 0.00222*alt;

    p = 0.699*exp(-0.00009*alt);
```

```
    rho = p/(0.1921 * (T + 273.1));
end

if T <= -273.1
    T = -273.1;
end

if alt >= 100000
    rho = 0;
end

%% Cutoff criteria

if a >= r_des || t >= 316
    thrust = 0;
else
    thrust = 110100*n;
end

%% Fuel consupmtion

if thrust == 0
    mdot = 0;
else
    mdot = -thrust/ue;
end
```

```
a_thrust = thrust*that/m;

a_grav = -mu_Mars*r/norm(r)^3;

acc = a_grav + a_thrust;

ax = acc(1);

ay = acc(2);

az = acc(3);

xdot = [vx vy vz ax ay az mdot]';
end
```

Sample output:

```
Enter pitch angle in degrees: 4.35
e = 1.025471e-03.
Periapsis altitude = 195.939803 km.
Apoapsis altitude = 203.314619 km.
Burnout mass: 32684.019409 kg.
Cutoff time: 546.200000 s.
Delta-v used: 4250.012582 m/s.
Cutoff altitude = 200.918212 km.
Speed at cutoff = 3449.933236 m/s.
Required v = 3450.993200.
>>
```