How much is enough? Uncertainty aware sample mass determination of coarse-grained soils for particle size analyses

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Abstract

Determining particle size distributions (PSD) of soils is a basic first step in many geotechnical analyses and guidance is given in different national standards. For ambiguous reasons, the recommended required minimum sample mass (m_{min}) for the PSD-analyses of soils with a main component of gravel or greater is always based on equations including the soil's maximum grain diameter (D_{max}). We claim that the recommended m_{min} is overestimated in many cases as D_{max} does not represent the relevant large soil fraction but only the PSD's uppermost outlier. Furthermore, sampling confidence is not considered in the recommended m_{min} and thus it is not clear why certain sample masses should even be necessary. We conducted Monte-Carlo simulation-based sieve analyses of coarse-grained soils and developed a new, practically applicable equation to determine m_{min} based on $d90$ that also includes explicit consideration of sampling confidence. Real sieve tests performed on three different sands and gravels corroborate the theoretical results and show that substantially lower sample masses yield PSDs with only marginal differences to PSDs from samples according to the standards. While the results are promising, they open up for new research questions about which geotechnical application requires which soil sampling confidence.

List of notations

C_u	coefficient of uniformity
C_c	coefficient of curvature
d_{min}	minimum particle diameter for simulated soil fraction
d_{max}	maximum particle diameter for simulated soil fraction
dXX	particle diameter at XX percent of a sieve curve
D_{max}	estimated maximum grain diameter of soil
KS	Kolmogorov-Smirnov statistic used as error metric between two sieve curves
KS_{med}	median of multiple KS values
KS_{p95}	95^{th} percentile of multiple KS values
m_{min}	required minimum soil sample mass
$m_{available}$	available soil sample mass
S_0	sorting coefficient
U	Uniform distribution
ε	error exponent to control desired soil sampling confidence
ρ	particle density

Keywords

Soil classification; Soil characterization; Particle Size Distribution; Uncertainty, Confidence

1. Introduction

A reliable particle size distribution (PSD) analysis is key in geotechnical front-end engineering design and imperative for engineering geological soil characterization and classification. For instance, preliminary design of offshore structures relies on the PSDs as the percentages of fines content, or $d10$ are key to estimate soil behaviour to loading, e.g. drainage conditions, cyclic response, consolidation, etc (see Andersen and Schjetne (2013); Andersen (2015)). Tailings dams are another application where reliable PSDs are crucial for material characterization and modelling (Liu et al., 2024) and to determine if the dam's composition complies with regulations in all depths. Extraterrestrial geotechnics is a more exotic field where PSDs are required to get a preliminary idea of the ground conditions on other planets for potential human settlements, however, the potential for large scale regolith sampling is very limited and thus determined PSDs may be questionable (Quinteros et al., 2024).

Methodologically the PSD is determined through sieving and/or sedimentation, and standards such as ISO 17892-4 (Standard Norge, 2017) or ASTM D6913/D6913M – 17 (D18 Committee, 2017) provide guidelines for the testing procedure. The first step to determine a PSD, is to take a soil sample and for that the standards suggest sampling of certain amounts of soil (mass). In principle, the goal of the sampling is to take a sample that is large enough to sufficiently represent the soil's PSD but simultaneously not too large to avoid excessive efforts with sample transportation, handling and testing. The mentioned standards, for example, address this problem by determining the required minimum soil sample mass (m_{min}) as a function of the soil's estimated maximum grain diameter (D_{max}). As also pointed out by Zhang et al. (2017), the origin and scientific justification for this procedure is unknown, despite widespread adoption. Equally unknown is the desired sampling confidence that the different guidelines seek to achieve, thus leading the sampling operator to the question, "How much is enough?". This is of particular relevance in coarse grained soils (i.e. ≥ sand acc. to ISO 14688 (2019)) where the suggested soil sample masses easily exceed tens of kilograms of soil, while it is unclear why such large masses should be required.

Even though PSD determination is a fundamental step in soil-mechanics, the literature on soil sampling for PSD determination is remarkably sparse and the above mentioned Zhang et al. (2017) is the only other known study that investigated this problem. From a statistical point of view, using D_{max} as the decisive criterium to determine m_{min} implies that m_{min} depends on the extreme particle sizes of the PSD, resp. on the rightmost point of the distribution. We hypothesize that today's standards overestimate the required sample size in many cases and that D_{max} is a conservative criterium to determine m_{min}. This often forces practitioners who deal with coarse grained soils to act outside the standard framework without being aware of what the consequences of smaller sample masses are. Furthermore, we see it as problematic that the recommendations for m_{min} are made without the indication of an achievable sampling confidence.

This paper therefore investigates the issue of sample mass determination for coarse grained soils and proposes a new criterium to determine m_{min} that is easily applicable in practice as it is just an equation with estimated input values. The new criterium is developed through Monte-Carlo simulation of virtual sieve tests and allows to explicitly set a desired level of confidence. To provide a baseline, the sampling confidence of today's standards is back calculated within the simulations. The approach i) allows one to take samples according to a desired level of confidence that is to be achieved; ii) provides the possibility to assess the uncertainty that needs to be expected if one has a sample mass that is < m_{min}; iii) reduces the required m_{min} for many soils and especially for those where D_{max} comes from single large grains.

2. Background

ISO 17892-4 (2017) defines that m_{min} [kg] depends solely on D_{max} [mm], for soils with a D_{max} > 20 mm and is to be derived from eq. 1.

$$m_{min} = \left(\frac{D_{max}}{10}\right)^2$$

eq. 1

The ASTM D6913/D6913M – 17 (2017) standard also defines m_{min} in dependence of D_{max}, for a D_{max} > 9.5 mm. m_{min} is "based on the mass of an individual spherical shaped particle, at the given sieve, multiplied by 100 then 1.2 (factor to account uncertainty) and finally rounded to a convenient number." For soils with a D_{max} > 76.2 mm, the same applies "except 1.2 factor is omitted". ASTM D6913/D6913M – 17 only gives this instruction and no equation, so eq. 2 was reconstructed based on that explanation. ρ in eq. 2 denotes the particle density of the grains which is also not directly specified in the standard but based on the therein given values for m_{min}, it can be back calculated that a ρ of 3.016 g/cm^3 must have been applied.

$$m_{min} = \frac{4}{3} * \pi * \left(\frac{D_{max}}{2}\right)^3 * \rho * 100 * 1.2$$

eq. 2

Based on these equations, both standards require minimum sample sizes in the range of hundreds of kilograms for soils with a D_{max} larger than 5-10 centimetres which is unpracticable and often impossible to achieve in terms of sampling, availability and sievability in the laboratory. Figure 1 shows the required m_{min} for the mentioned standards for up to a maximum particle size of 300 mm diameter where ISO 17892-4 would require almost 1000 kg of soil and ASTM D6913/D6913M – 17 more than 1200 kg.

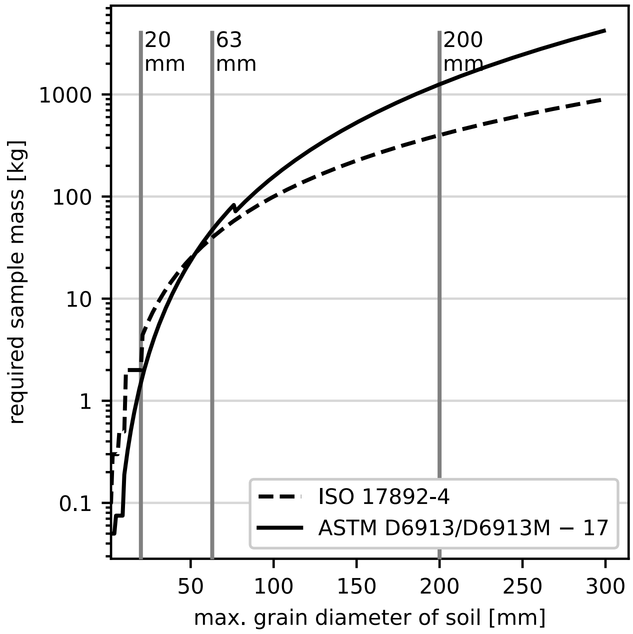

Figure 1: Minimum required sample masses as defined in ISO 17892-4 and ASTM D6913/D6913M – 17. Steps in the plot result from hard defined sample masses in the standards.

Similar recommendations for the required sample mass are given in other standards (e.g. Eurocode 1997:2007, AASHTO T2, Australian Standard AS 1141.11, DJS 112-4:2015, etc.) and guidelines by Ministries or Departments of Transportation (MTs or DOTs). Most recommendations of the minimum sample mass for sieving for soils, range from silt to cobble sizes. In General, standards from Ontario, Canada recommend similar minimal masses, but lower than the European counterpart.

3. Development of new minimum sample mass criterium

In this study, we propose an alternative way of determining m_{min} that is first theoretically developed through Monte-Carlo simulations using virtual sieve tests and then underpinned with experimental results from real sieve tests. The Python source code, the simulation- and experimental results are available in the Github repository in the supplementary information of the paper.

3.1. Monte-Carlo Simulations

To theoretically investigate this problem, virtual sieve tests were conducted on randomly generated coarse-grained soils. The basic idea is that first a "ground-truth" coarse grained soil is generated and then samples with different masses are taken from this underlying soil to investigate how large the error between the samples' PSDs and the "ground-truth" soil's PSD is. The soil generation of the Monte-Carlo simulation was set up with the goal to generate a wide variety of PSDs including poorly graded-, well graded- and gap graded coarse grained soils. One limitation of the simulation is that all grains are spherical which slightly reduces its realism (Kaviani-Hamedani et al., 2024), (see also section 6). The soils are created by the following process:

- Step 1: Randomly generate between 1 and 5 percentages of soil fractions (e.g. a soil may have 30% fraction A, 20% fraction B and 50% fraction C).
- Step 2: For each fraction, randomly set the minimum- (d_{min}) and maximum particle diameters (d_{max}) between 1 mm and 200 mm. These boundaries are sampled from $\exp\Big(U\big(\ln(1), \ln(200)\big)\Big)$ where U is a uniform distribution with upper and lower boundaries.
- Step 3: For each fraction, individual particle diameters are generated by sampling from a beta distribution that gives numbers between 0 and 1 and then scaled to d_{min} and d_{max}. The beta distribution's parameters alpha and beta parameters are uniformly, randomly set between 1 and 4 for each sample.

This sampling procedure was developed as PSDs, sampled from single distributions such as lognormal or exponential, did not show the desired large variability. Furthermore, this sample generation process is an attempt to more closely mimic real soils that consist of multiple soil fractions dependent on the geological history. In Figure 2, 100 of 1000 exemplary sieve curves are shown to visualize the diversity of PSDs that were generated. The sieve curves are coloured according to the sorting coefficient (S_0, see eq. 5 in Table 1).

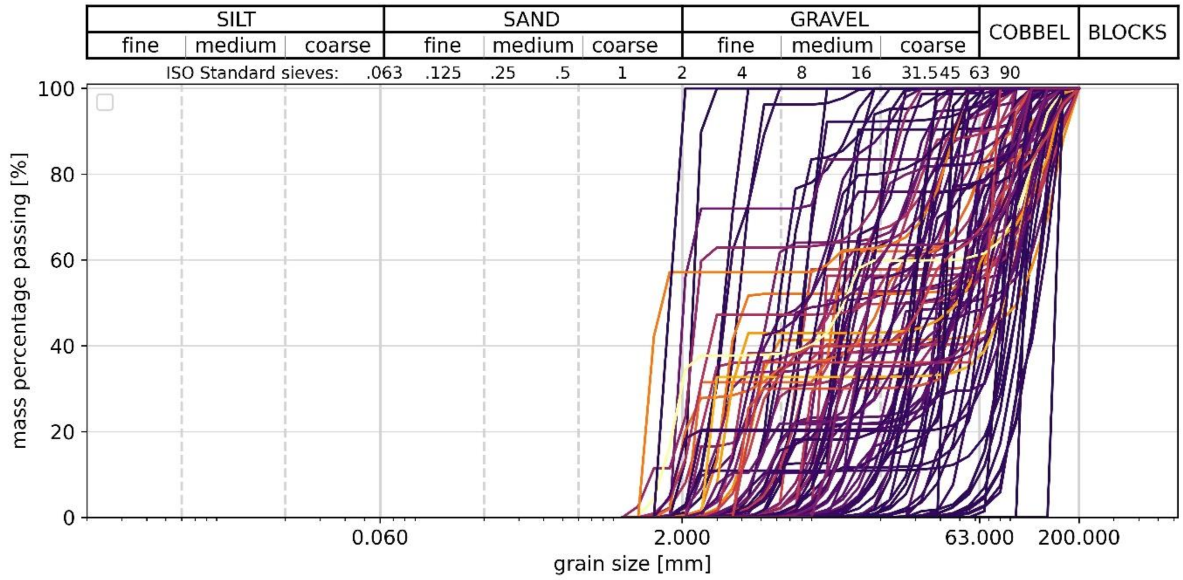

Figure 2: 100 exemplary sieve curves of samples that were generated for the Monte-Carlo simulation. Sieve curves are coloured according to the sorting coefficient (S_0).

To quantify the difference/error between the PSD of the underlying soil and a sample's PSD, the Kolmogorov-Smirnov statistic (KS) was chosen. KS denotes the maximum vertical distance between two cumulative density functions which in this case means the maximum mass percentage difference between two sieve curves. Thus, KS – herein – has the unit of mass percent and the minimum and maximum of 0 or 100 would be reached if a sample's sieve curve either has a perfect fit or complete misfit with respect to the underlying soil. For example, let $X = \{100, 95, 70, 20, 10, 5\}$ and $Y = \{100, 90, 50, 15, 7.5, 5\}$ be the mass percent passing sieves of mesh sizes 90-, 63-, 45-, 31.5-, 16- and 8 mm. KS is then computed as $KS = \max(|X - Y|)$ and would be 20% in this example (Figure 3). KS is seen as a well-suited error metric in this case as the goal for the soil sampling is to find a sample mass whose sieve curve fits as well as possible to the sieve curve of the underlying soil.

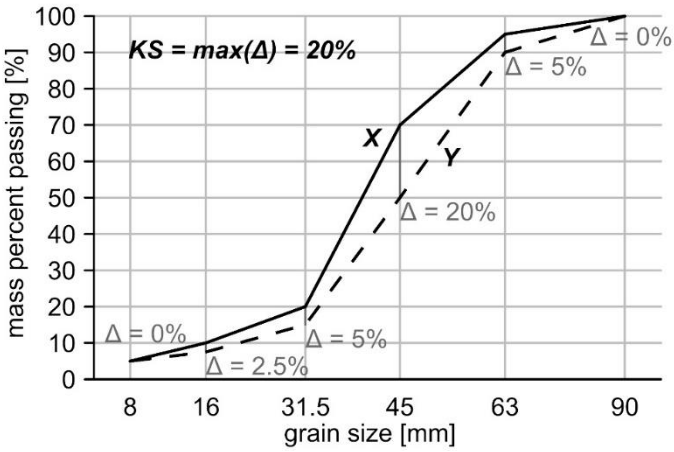

Figure 3: Example of how the Kolmogorov-Smirnov statistic (KS) quantifies the difference between two sieve curves X (solid) and Y (dashed).

Figure 4 shows an example where a soil was generated and multiple samples with decreasing masses were taken. The highest sample mass was determined according to eq. 1 (ISO 17892-4) and the subsequent samples are 75%, 50%, 25%, 10%, 5% and 1% fractions of the recommended sample mass. The lowest sample mass results in the highest KS with respect to the underlying soil (i.e. highest error). Note, however, there is not a consistently decreasing KS observable above that which will be explained in the next section.

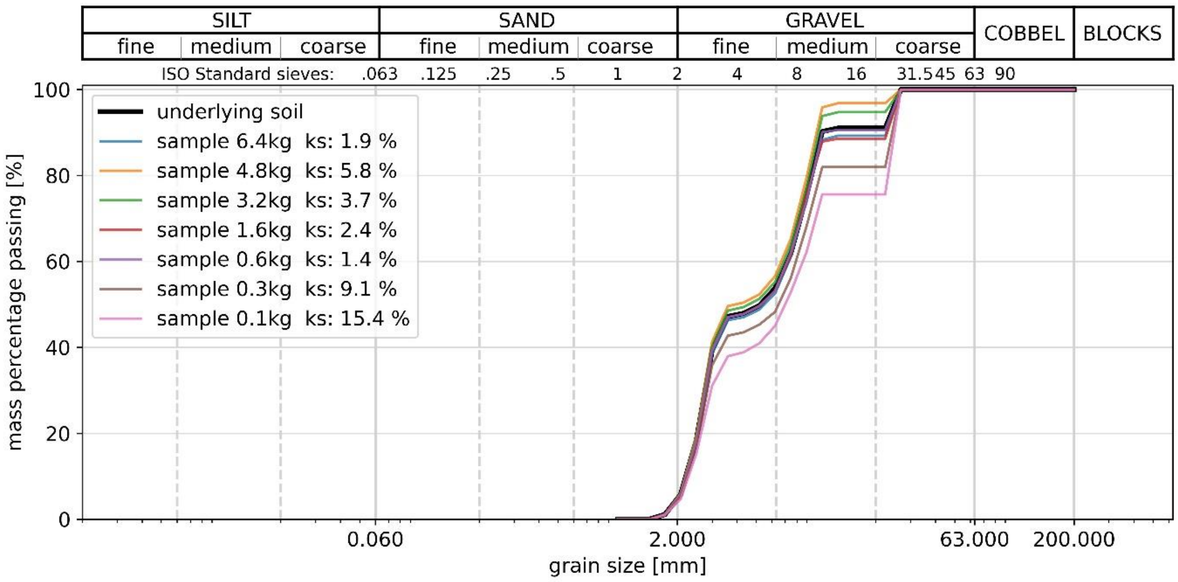

Figure 4: One example of a generated soil, where multiple samples with decreasing sample masses were taken and the Kolmogorov-Smirnov statistic computed for each of them.

For each simulation, the parameters given in Table 1 were recorded. A multitude of parameters was recorded to facilitate comprehensive Monte-Carlo simulation analyses afterwards.

Table 1: Parameters that are recorded for each simulated sample.

Parameter	**Description |**
C_u [-]	Coefficient of uniformity $C_u = \frac{d60}{d10}$ *eq. 3*
C_c [-]	Coefficient of curvature $C_c = \frac{d30^2}{d60*d10}$ *eq. 4*
S_0 [-]	Sorting coefficient $S_0 = \frac{d75}{d25}$ *eq. 5*
USCS soil classes	Soil classification according to the unified soil classification system (ASTM).
D_{max} [mm]	Maximum particle diameter of underlying soil.
total masses [kg]	Total mass of generated underlying soil.
req. mass ks_p95 <= 10 [kg]	Required mass to achieve a KS_{p95} of ≤ 10% in a "bottom up" approach (see section 3.2).
X.X mm sieve [m%]	Mass percent soil passing a sieve of mesh size X.X mm. Mesh sizes increase logarithmically from 1 to 200 mm in 30 steps. This large number of virtual mesh sizes was chosen to get higher resolution sieve curves than it would be possible with standard mesh sizes.
dXX [%]	Particle diameters at 10, 12, 20, 25, 30, 40, 50, 60, 70, 75, 90 and 90 % of the sieve curve of the underlying soil.
ISO req. mass [kg]	Required sample mass acc. to ISO 17892-4 (2017).
ASTM req. mass [kg]	Required sample mass acc. to ASTM D6913/D6913M – 17 (2017).
const req. mass [kg]	Constant sampling mass of 10kg as a reference.
new X.X req. mass [kg]	Required sample mass acc. to eq. 6 with an ε = X.X. X.X ranges from 1.0 to 2.5 in steps of 0.1
ISO ks [%]	KS between a sample's sieve curve that was taken acc. to ISO 17892-4 (2017) and the underlying soil's sieve curve.
ASTM ks [%]	KS between a sample's sieve curve that was taken acc. to ASTM D6913/D6913M – 17 (2017) and the underlying soil's sieve curve.
const ks [%]	KS between the sieve curve of a sample with constant mass = 10 kg and the underlying soil's sieve curve.
new X.X ks [%]	KS between a sample's sieve curve that was taken acc. to eq. 6 and the sieve curve of the underlying soil with an ε = X.X. X.X ranges from 1.0 to 2.5 in steps of 0.1.

3.2. Bottom-up determination of required sample mass

One of the goals of the simulation was to "experimentally" determine the required sample mass by generating a soil and then taking samples with progressively increasing masses until a defined

KS threshold is reached. As individual samples with the same or only slightly differing masses may show a significant variability of KS (see Figure 4) each sampling was repeated 20 times as a trade-off between computational efficiency and representative results. The large fluctuation in repeated sampling with same masses originates from the chance whether or not individual large grains that significantly influence the resulting PSD are being sampled. The KS threshold was set so that the sample mass is seen as sufficient if the p95 percentile (i.e. 95% of values are lower than this) of the KSs of the 20 repeated samples is ≤ 10 mass %. In other words, if 19 of the 20 samples achieve a KS ≤ 10 mass %, the sample mass is sufficient. Note that this threshold has no general geotechnical meaning and was only set to have a threshold to "experimentally" determine a required sample mass to qualitatively investigate the relationship between sample mass and sampling confidence.

3.3. Insights from the Monte-Carlo Simulations

The Monte-Carlo simulations were used to i) find out the sampling confidence / error that results from determining m_{min} according to ISO and ASTM and ii) to develop a new approach for m_{min} determination that reduces the required sample mass and also explicitly considers the sampling confidence. To this end, 1000 simulations were made and it was observed that the ISO recommendation (eq. 1) achieves a median KS (KS_{med}) of 3.0% and a p95 percentile of KS (KS_{p95}) of 7.7%. This means that 95% of samples taken according to ISO have a KS < 7.7% with respect to the underlying soil. Due to the higher required sample masses, the ASTM recommendation (eq. 2) achieves lower KS statistics of a KS_{med} of 2.2% and a KS_{p95} of 5.3%. A violin plot of the ISO- and ASTM- recommended sample masses and the achieved KS values for all 1000 simulations is given in Figure 5.

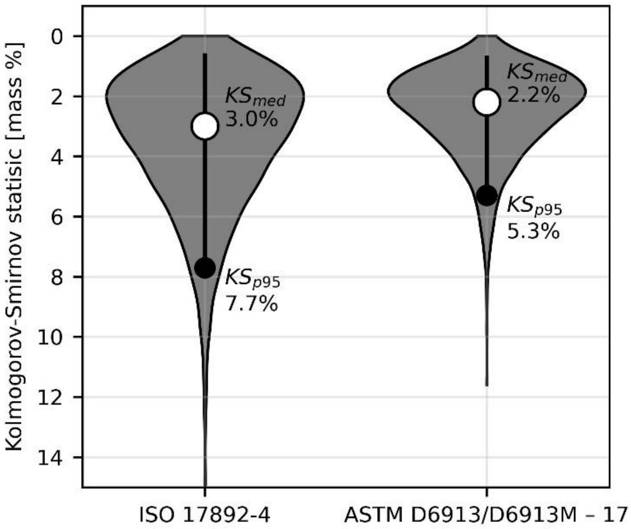

Figure 5: Violin plots of the Kolmogorov Smirnov statistic for sample masses taken according to ISO and ASTM standards.

The “bottom up” determination of required sample mass (see section 3.2) allows to investigate the relationship between the experimentally determined required sample mass to achieve a certain error and other investigated parameters. This study’s original hypothesis was that the required sample mass to achieve a certain error must be dependent on the grading of the soil rather than solely on D_{max}. Figure 6 was made to verify if grading can be used to complement the selection of m_{min}, and the following insights are gathered from this:

- There indeed is a relationship between grading and required sample mass as samples with a high S_0 (i.e. well graded) also require larger sample masses. However, the figure also shows that there are samples with a low S_0 that require a large sample mass and thus this hypothesis was rejected.
- Figure 6 furthermore shows that the functions from the standards (esp. ISO) do not always overestimate the required sample mass but rather describe the upper limit of the required sample mass. Thus, it can be qualitatively confirmed that there is a relationship between a soil's grain size and the required sample mass to reach a certain sampling confidence.

- Lastly, Figure 6 shows that there are many samples that have a comparably large D_{max} but require sample masses several times smaller than suggested by the standards. It is thus shown that the standards do overestimate the required sample mass in many cases.

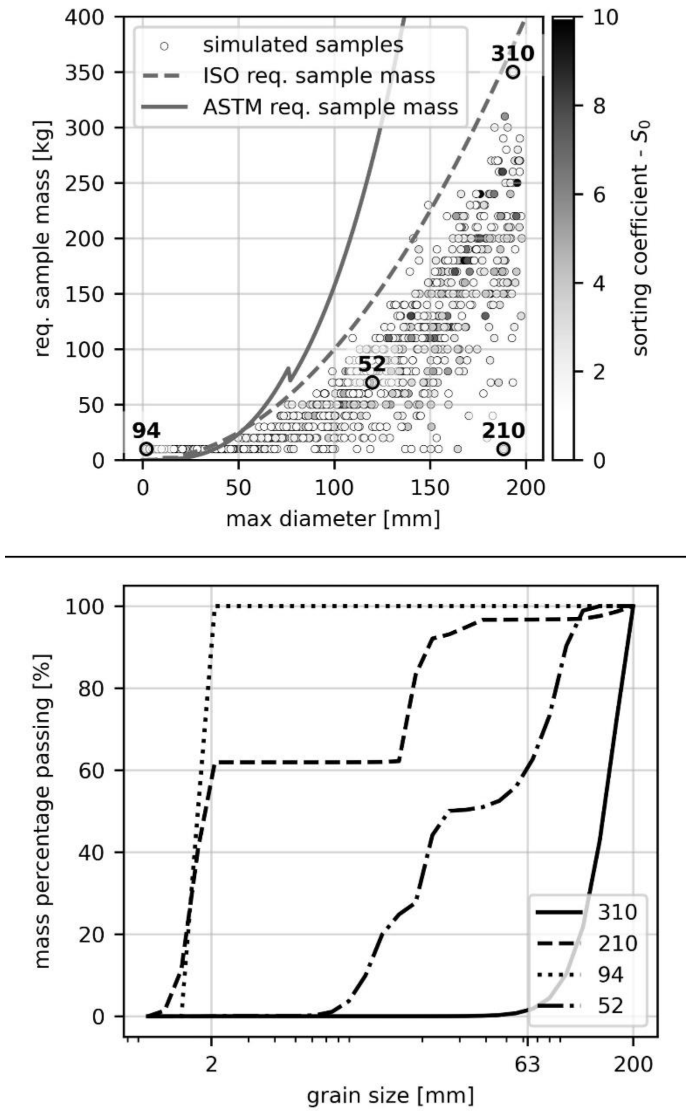

Figure 6: Top: Relationship between a soil's maximum particle diameter and the required sample mass. Required sample masses acc. to ISO and ASTM are also shown for reference. Bottom: Exemplary sieve curves from the top figure, marked with sample "ID" (see data in the supplementary information).

Based on these insights, the simulations were investigated with respect to which grain parameter achieves a high correlation with the required sample mass. It was found that $d90$ (i.e. the grain size where 90% of a soil's mass are smaller than this diameter) has a stronger correlation with the required sample mass than D_{max}. Visualizing the simulations as $d90$ vs. required sample mass and colouring the data points according to the maximum grain diameter (Figure 7) shows that soils with a large $d90$ also require large sample masses for representative sampling. The same exemplary PSDs as in Figure 6 are marked in Figure 7. Note for example that samples 210 and 94 have vastly different D_{max} but very similar $d90$. In general, can it be seen that there are several soils with a low $d90$ that still have a large maximum particle diameter, but they do not require large sample masses for representative sampling. We thus conclude that the relationship between grain size and required sample mass as implied by the standards is qualitatively correct, but D_{max} is an ill-suited criterium as it represents the rightmost point of a soil's PSD which is often an outlier in case of course grained soils. D_{max}, therefore, does not represent a soil's significant large particle sizes. $d90$ – which is not a PSD's extreme value - on the other hand, is not sensitive to outliers and shows a more robust relationship with the required sample mass.

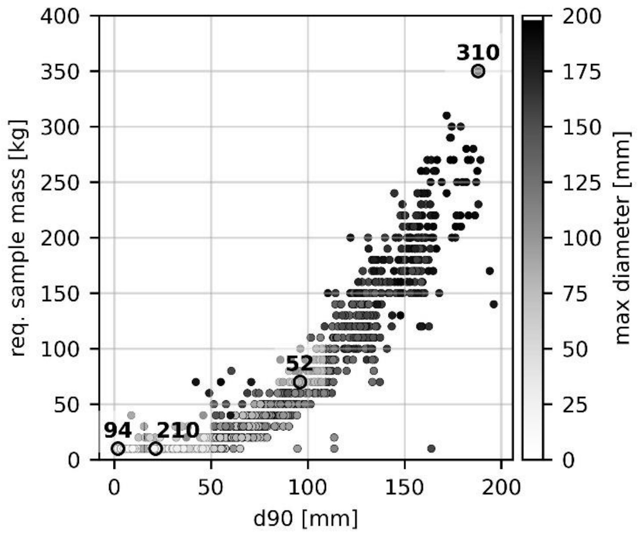

Figure 7: Relationship between a soil's $d90$, the required sample mass and the maximum particle diameter. The same PSDs as shown in Figure 6 (bottom) are marked.

3.4. Proposed criterium for minimum required mass

Based on the insights from the Monte-Carlo simulations, a new criterium to determine m_{min} for coarse grained soils was developed. Based on eq. 1, D_{max} was replaced with $d90$ and a dedicated error-exponent ε that gives control over the maximum error that one wants to achieve with the taken sample mass was introduced (eq. 6).

$$m_{min} = \left(\frac{d90}{10}\right)^{\varepsilon}$$

eq. 6

This new criterium was included in the Monte-Carlo simulation to determine the KS that are achievable with different ε by repeated sampling from one soil with different required masses (see parameters "new X.X req. mass [kg]" and "new X.X ks [%]" in Table 1). As KS_{med} and KS_{p95} of the current standards were determined in Figure 5 in section 3.3, we determined these errors for different ε on a range from 1 to incl. 2.5 (Figure 8, top). 2.5 was set as the upper limit as this yields sample masses larger than the ASTM standard. Based on this, the relationships between the achievable KS_{p95} and KS_{med} and ε was assessed and is shown in (Figure 8, bottom). These relationships can be described with the exponential functions of eq. 7 and eq. 8.

$$KS_{p95} = 123.65 * e^{-1.29*\varepsilon}$$

eq. 7

$$KS_{med} = 34.24 * e^{-1.11*\varepsilon}$$

eq. 8

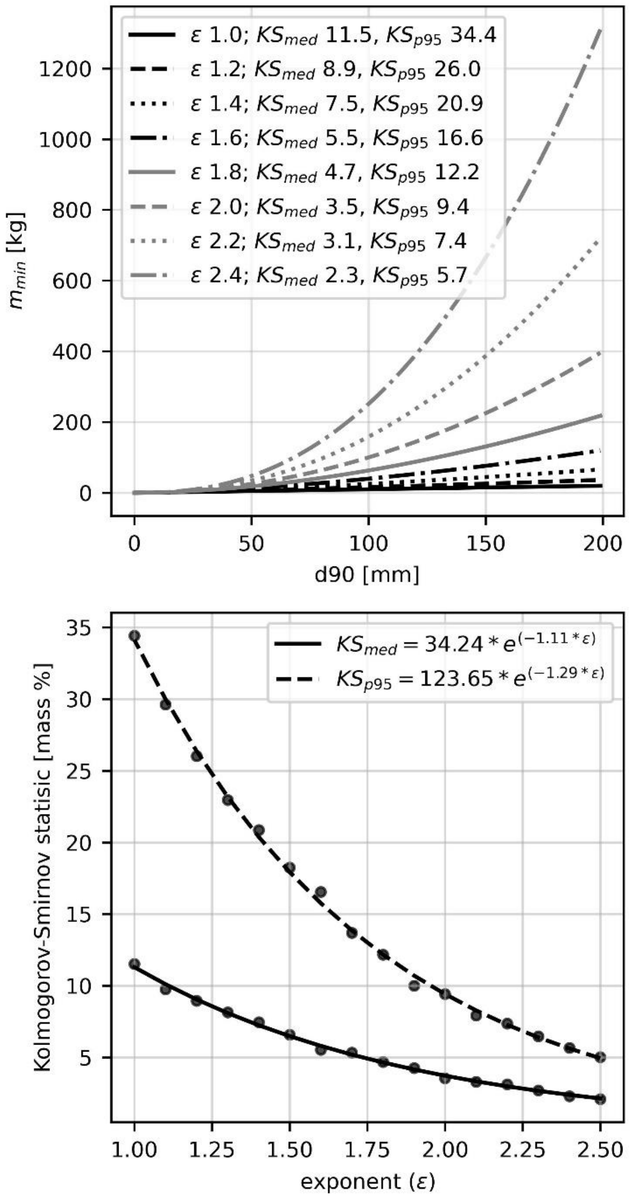

Figure 8: Top: The new criterium to determine the minimum sample mass (m_{min}) with different error exponents (ε). Bottom: The assessed KS_{p95} and KS_{med} vs. different error exponents ε.

Solving eq. 7 for ε and substituting for ε in eq. 6, finally gives the new recommended equation to determine $m_{\min}$ in a sampling confidence-aware manner in eq. 9.

$$m_{\min} = \left(\frac{d90}{10}\right)^{\frac{\ln(KS_{p95}) - \ln(123.65)}{-1.29}} \qquad \text{eq. 9}$$

This equation allows one to determine the minimum required sample mass, given an estimated $d90$ of the soil and a desired sampling confidence in mass percent (KS_{p95}). The $m_{\min}$ will in 95%

of cases be a sample mass that is sufficient to satisfy the desired error threshold. An example application of the equations is given at the end of section 3.5. Just as the d_{max} that is required by other standards is only a field estimation based on a given sample, $d90$ can also be estimated in the field as the maximum relevant grain size excluding obvious large outliers in the soil. Defining desirable PSD errors for different geotechnical applications is not in the scope of this study and should be investigated with dedicated research as mentioned in section 6.

As fine-grained soils were not considered in the simulation and sands and fine-gravels only represent the lower boundary of the Monte-Carlo simulation, the same criteria as specified in the ISO standard should be applied for soils with a D_{max} ≤ 20 mm. Furthermore, in cases where the estimated D_{max} > 20 mm but the estimated $d90$ < 10 mm, 1 kg of sample mass should be used. Otherwise, eq. 9 is to be used.

3.5. Comparison to standards and further usage

Figure 5 shows that ISO and ASTM achieve KS_{p95} of 7.7% and 5.3 % respectively. Using these values in eq. 9 allows to directly compare the required sample masses from the new criterium to the previous standards (Figure 9). On average, across all simulated samples, the new criterium requires ca. 3.5 times lower sample masses than the ISO standard and ca. 5.4 times lower sample masses than the ASTM to achieve similar sampling confidences. In extreme cases, however, the required sample masses according to the new criterium to reach the same sampling confidence are up to 650 times lower than the ISO and up to 1200 times lower than the ASTM. The multitude of samples below the dashed lines in Figure 9 show that these are no single cases and underline the above-mentioned problem that the current standards determine the required sample mass based on outliers. In Figure 9 top, it can also be seen that above ISO required sample mass of ca. 150 kg (i.e. soils with a D_{max} > ~120 mm, see eq. 1), the new criterium to determine $m_{\min}$ mostly leads to larger sample masses than the ISO standard, aside from the aforementioned outliers, where ISO vastly overestimates the required sample masses. This can be an indication that the ISO standard is in fact unconservative for very coarse-grained soils that contain a significant amount of large grains and not just outliers.

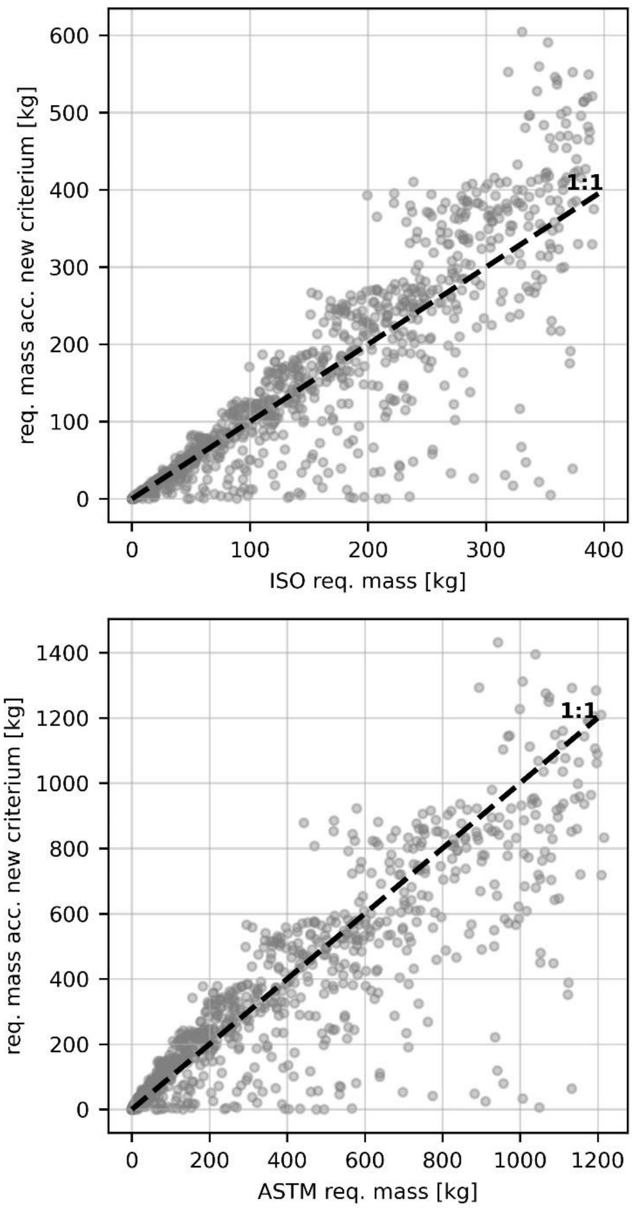

Figure 9: Comparison between sample masses acc. to ISO (top) and ASTM (bottom) to the new criterium at equal level of confidence. Dashed lines indicate lines of 1:1 equal mass in the plots. Datapoints are 50% transparent.

Lastly it must be acknowledged that there are cases where the available sample mass is smaller than the desired sample mass and acquiring more sample is unviable. Today, operators either avoid sampling all together in these cases or have to do sampling outside the standards' framework and thus are not aware of the error that they may or may not experience through this undersampling. We recommend also taking samples to determine a PSD in these cases, but the operator should be aware of the expectable error that the sampling is subjected to. In this case m_{min} in eq. 6 can be substituted with the available sample mass ($m_{available}$) and then the equation solved for ε, thus giving eq. 10.

$$\varepsilon = \frac{ln(m_{available})}{\ln(d90) - \ln{(10)}} \qquad \text{eq. 10}$$

By using the determined ε in eq. 7 and eq. 8 or Figure 8 bottom, one can find which KS_{med} and KS_{p95} is to be expected given the available sample mass. The consequence of knowing the error that must be expected given the available sample mass is that the subsequent geotechnical analysis can consider this uncertainty by setting a higher focus on probabilistic analyses, adjusting how conservative approaches are or considering different plausible scenarios.

The theoretical part of the paper is finished with an example: For example, one wants to determine the PSD of a coarse-grained fluviatile soil with an estimated D_{max} of 150 mm (there are some cobbles), but also an estimated $d90$ of 80 mm. According to eq. 1 from ISO 17892-4 the required m_{min} is 225 kg of soil (see eq. 11) and it is not clear why so much soil would be required. In contrast to that, the new eq. 9 allows setting a desired maximum error / sampling confidence (KS_{p95}) of e.g. 10 % and based on the estimated $d90$ one can then estimate the required sample mass to be ~58 kg with explicit consideration of that desired sampling confidence (see eq. 12). If the total available soil sample mass is however only 20 kg, for example, then eq. 10 can be used to determine the error exponent ε (see eq. 13) which would be 1.44. Substituting that into eq. 7 reveals that in that particular soil, one needs to expect that the determined PSD has error of up to ~20% with respect to the real soil's PSD if only 20 kg of soil sample are available (see eq. 14).

$$m_{min}[kg] = 225 = \left(\frac{150}{10}\right)^2 \qquad \text{eq. 11}$$

$$m_{\min}[kg] = 58 = \left(\frac{80}{10}\right)^{\frac{\ln(10)-\ln(123.65)}{-1.29}} \quad \textit{eq. 12}$$

$$\varepsilon = 1.44 = \frac{ln(20)}{\ln(80) - \ln(10)} \quad \textit{eq. 13}$$

$$KS_{p95}[m\%] = 19.3 = 123.65 * e^{-1.29*1.44} \quad \textit{eq. 14}$$

4. Experimental underpinning

4.1. Experimental program and tested soils

Several sieve analyses were performed in the laboratory to practically test the hypotheses presented in the previous chapter. The goal of the sieve analyses was to investigate if it is also practically the case that significantly lower sample masses than recommended by the standards yield sufficient PSDs. Three different soils were used, namely a (A) medium to fine sand, (B) a medium to fine gravel and (C) a sandy, medium to coarse gravel. Different test programs were conducted for each soil:

- Soil A: A medium to fine sand from the Isle of Rum in Scotland was used to investigate how far one can go with reducing the ISO recommended sample mass even below the considered size of the Monte-Carlo analyses. With an estimated D_{max} of 4 mm, an ISO 17892-4 recommended (dry) sample mass of 200 g was taken from one large sample. Further samples with 100 g, 75 g, 50 g, and 5 g were also taken and PSDs determined for all of them.
- Soil B: A medium to fine fluviatile gravel was collected from the river Akerselva in Oslo in Nydalen. The D_{max} is estimated to be 30 mm, thus the ISO required sample mass is 9 kg of soil (eq. 1) which was used for one sieve test. The estimated $d90$, however, is around 8 mm and thus < 10 mm. Therefore, the new recommendation of 1 kg sample mass was tested (see end

of section 3.4). To also include an extreme case, one more sieve analysis with 300 g of sample was done.

- Soil C: An artificial, pre-sieved, sandy, medium to coarse gravel from Austria with a known D_{max} of 70 mm was used for soil C. One sieve test with a sample mass of 50 kg according to ISO was done and one with a 2.5 times lower sample mass of 20 kg.

4.2. Experimental results

Table 2 gives an overview of the experimental results and Figure 10 shows the sieve curves for the different soils.

Table 2: Overview of the experimental results.

Test	**Sample mass [g]**	**d10 [mm]**	**d30 [mm]**	**d60 [mm]**	**Cc**	**Cu**	**KS to ISO [mass %]**
Soil A	200	0.081	0.148	0.236	2.90	1.14	-
Soil A1	100	0.085	0.145	0.227	2.66	1.08	3.32
Soil A2	75	0.090	0.158	0.242	2.69	1.14	3.97
Soil A3	50	0.082	0.140	0.227	2.74	1.05	3.18
Soil A4	5	0.079	0.137	0.230	2.91	1.03	3.88
Soil B	9000	0.599	1.557	3.615	1.12	6.04	-
Soil B1	1000	0.608	1.536	3.527	1.10	5.80	1.34
Soil B2	300	0.553	1.245	2.782	1.08	5.03	12.34
Soil C	50000	0.369	3.694	14.167	2.610	38.387	-
Soil C1	20000	0.563	5.152	18.451	2.557	32.802	7.8

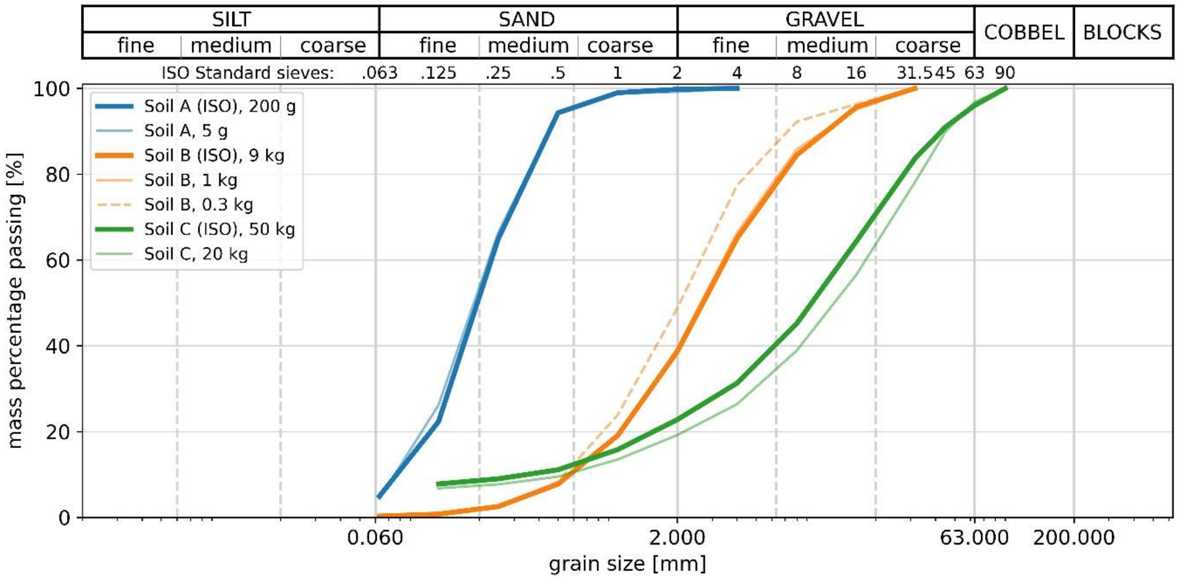

Figure 10: Sieve curves of the conducted lab tests to investigate how different sample masses influence practical results. For each soil, the sieve curve with a sample mass acc. to ISO 17892-4 and the sieve curve based on the smallest sample mass is shown.

For all soils, no remarkable discrepancy can be observed between the PSDs obtained using different amounts of sample mass. While this study aims at coarse grained soils with the main grain size being gravel or larger, soil A demonstrates that lower sample masses can also give sufficient results for sands. In soil A, even a 40 times lower sample mass than what would be required by ISO 17892-4 only yields a KS of 3.88%. For Soil B, a mass 9 times lower than the suggested by ISO (i.e. the mass as recommended herein) shows a KS of 1.34% only. A test with a 30 times lower sample mass (300 g) was also conducted on Soil B and results in a KS of 12.34% with respect to the ISO recommended of 9000 g. This more substantial deviation results from a low sample mass which is also not recommended and the test was done for demonstration purposes only to show what happens in substantially lower sample masses in coarse grained soils. In case of Soil C, the error between the PSD resulting from the ISO recommended sample mass of 50 kg and a test with a 2.5 times lower sample mass yielded a KS of 7.8%. While the effort of doing a sieve test with 20 kg instead of 50 kg of sample mass is significantly lower, the resulting difference in the PSD is small and still leads to the same characterization of the soil as a sandy, medium to coarse gravel.

Table 2 shows that also the differences between the parameters that describe the sieve curves' geometry are small and Figure 11 visualizes the difference in C_c and C_u between tests with a sample mass according to ISO and tests with a lower sample mass. In all cases, the values become slightly lower with decreasing sample masses. Nevertheless, the total differences are small and would not change a soil's classification based on C_c and C_u.

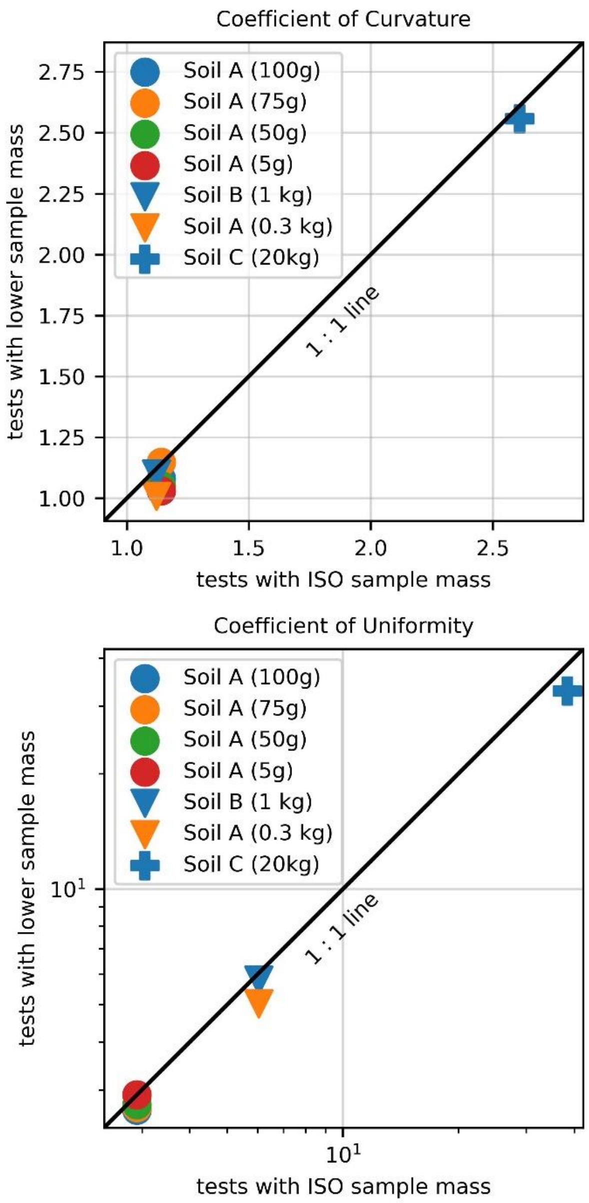

Figure 11: C_c and C_u differences for tests with a sample mass acc. to ISO and tests with a lower sample mass.

5. Discussion

The proposed new method for m_{min} determination leads to a reduction of the required sample masses for coarse grained soil, is easily applicable in practice and also permits to take samples under explicit consideration of the sampling confidence. The practicality and explicit accounting for sampling confidence are improvements over the method for sample mass determination that Zhang et al. (2017) propose which is a theoretical one and requires an intricate computational procedure.

The proposed new methodology is based on simulations of laboratory sieve tests, but practical laboratory sieve tests on real soils corroborate the theoretical results. Nevertheless, the simulation includes some simplifying assumptions such as perfectly spherical grains which might influence the result, especially for very coarse grain sizes that in reality seldomly are perfectly spherical. Studies such as Kaviani-Hamedani et al. (2024) address this issue, but in large scale simulation of sieve tests, explicitly including non-spherical grains might heavily impact the computational performance and thus render large scale Monte-Carlo simulations infeasible.

The simulation of individual and discrete grains and the subsequent explicit sampling from these grains is on the one hand seen as a benefit of this study as it is the most realistic way of simulating sieve tests, on the other hand it is very computationally demanding as especially memory limits are reached fast the smaller the grain sizes become. Besides the main goal to investigate coarse grain sizes, the lower grain size boundary of 1 mm in this study is related to computational limitations of this approach. To conduct simulated PSD analyses starting from clay sizes, would require a different simulation concept, that is rather based on statistical distributions than on individual grains.

6. Conclusions and Outlook

A new method to determine the minimum required sample mass for PSD assessments was proposed. The new method explicitly considers sampling confidence which is an improvement on the one hand but on the other opens up for a plethora of new research questions related to "How much is enough for application X?". As given in the introduction, PSDs are not only fundamental for general purpose soil characterization but also feed directly into different geotechnical engineering applications. These may, however, tolerate different sampling errors depending on the downstream usage of a PSD and derived parameters such as $d10$, $d60$, C_u, C_c, etc. Speculating about required confidences of soil sampling for different geotechnical applications is out of the scope of this study and future research related to this topic is highly encouraged to provide a sound decision base for sampling confidences.

Another point for investigation is how reliable parameters like D_{max} and $d90$ can be estimated by operators in the field. This would require studies about human cognitive biases as they were recently done in other fields of geotechnics (Elmo and Stead, 2021; Skretting et al., 2023) and eventually the use of image processing technology for PSD-pre-assessment (Ferrer et al., 2021) could be considered. Due to the required level of technological proficiency, it is however not expected that image processing techniques will replace estimations of PSDs in practice in the near future and approaches like the one proposed herein will remain relevant.

Supplementary information

The code for the Monte-Carlo Simulations and the results of the real laboratory tests can be found in the following Github repository: https://github.com/norwegian-geotechnical-institute/sieve_analyses/releases/tag/v1.0.0

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