Uncertainty aware sample mass determination of coarse-grained soils for particle size analyses

Georg H. Erharter[1] [ORCID: 0000-0002-7793-9994]*, Santiago Quinteros[1] [ORCID: 0000-0002-4895-1580], Diana Cordeiro[1] [ORCID: 0000-0002-9242-4147], Matthias Rebhan[2] [ORCID: 0000-0002-0638-6202], Franz Tschuchnigg[2] [ORCID: 0000-0002-4279-7703]

1) Norwegian Geotechnical Institute, Sandakerveien 140, Oslo, Norway
2) Graz University of Technology, Rechbauerstraße 12, Graz, Austria

* corresponding author: georg.erharter@ngi.no

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Abstract

Determining particle size distributions (PSD) of soils is a basic first step in many geotechnical analyses and guidance is given in different national standards. For ambiguous reasons, the recommended minimum sample mass (m_{min}) for the PSD-analyses of soils with a main component of gravel or greater is based on equations including the soil's maximum grain diameter (D_{max}). We claim that the recommended m_{min} is overestimated as D_{max} does not represent the relevant large soil fraction but only the PSD's uppermost outlier. Furthermore, the recommended m_{min} is not based on a specific sampling confidence (i.e. how closely does the sample's PSD need to approximate the soil's PSD?) and thus it is not clear why the m_{min} should even be necessary. We conducted Monte-Carlo simulation-based sieve analyses of coarse-grained soils and developed a new, practically applicable framework to determine m_{min} based on D_{90} that also includes explicit consideration of sampling confidence. A survey was conducted that shows that there is no significant difference in how well operators are able to assess parameters like D_{90} or D_{max}. Real sieve tests performed on three different sands and gravels corroborate the theoretical results and show that substantially lower sample masses yield PSDs with only marginal differences to PSDs from samples according to the standards. While the results are promising, they open up for new research questions about which geotechnical application requires which soil sampling confidence.

List of notations

C_u	coefficient of uniformity
C_c	coefficient of curvature
D_{min}	estimated minimum grain diameter of soil
D_{max}	estimated maximum grain diameter of soil
D_{xx}	grain diameter at xx percent of a sieve curve
KS	Kolmogorov-Smirnov statistic used as error metric between two sieve curves
KS_{med}	median of multiple KS values
KS_{p95}	95th percentile of multiple KS values
$m_{available}$	available soil sample mass
m_{min}	required minimum soil sample mass
S_0	sorting coefficient
U	Uniform distribution
ε	error exponent to control desired soil sampling confidence
ρ	grain density

Definitions and conventions

- Soil: A volume of granular material in the ground that is too large to analyze as a whole. This definition only applies in the herein given context of soil sampling for grain size distribution determination. For other definitions of soil see e.g. EN ISO 14688-1.
- Sample: a portion of a larger soil volume, taken to represent its characteristics (based on e.g. O'Toole (2015)). The term "specimen" is not used herein due to ambiguous definitions where a specimen may either be a subset of a sample or the other way round.
- Grading and sorting are two equivalent terminologies to describe the shape of a sieve curve. In this work, we consistently use "grading" where "well graded" ≈ "poorly sorted" and "poorly graded" ≈ "well sorted".
- Uncertainty communicating language is given in accordance with Erharter et al. (2024).

Keywords

Soil classification; Soil characterization; Grain Size Distribution; Uncertainty, Survey, Confidence

1. Introduction

A reliable particle size distribution (PSD) analysis is key in geotechnical front-end engineering design and imperative for engineering geological soil characterization and classification. For instance, preliminary design of offshore structures relies on PSDs as the percentages of fines content, or D_{10} are key to estimate soil behavior to loading, e.g. drainage conditions, cyclic response, consolidation, etc. (see Andersen (2015); Andersen and Schjetne (2013)). In tailings dams reliable PSDs are crucial for material characterization and modelling (Liu et al., 2024) and to determine if the dam's composition complies with regulations in all depths. Extraterrestrial geotechnics is a more exotic field where PSDs are required for preliminary ground investigations for potential human settlements (Quinteros et al., 2024).

The first step to determine a PSD is to take a test sample from the soil. Several significant error sources such as the sampling technique or the choice of the sample mass are entailed in this process (Rawle, 2015). Readers are referred to works like Gerlach et al. (2002); Gerlach et al. (2003) or Dubé et al. (2021) for information about sampling techniques such as riffle splitting or fractional shoveling. With respect to the sample mass, the primary goal is to take a sample that is sufficiently large to be representative for whichever characteristic of the soil that one is interested in (Al-Rumaithi and Al-Sherrawi, 2020; Dubé et al., 2021; Pitard, 2019). It must be noted, however, that it makes a difference for the practical sampling if, for example, an investigation's goal is a soil's chemical composition that permits crushing of large grains, or an investigation's goal is the actual soil PSD that does not allow that. The former case is relevant in the context of mining, metallurgy and environmental studies and is historically addressed by Gy's sampling theory (Gy, 2012). The latter case is relevant in engineering geological investigations in the context of geotechnical engineering projects.

The present paper is exclusively concerned with the sample mass determination to assess a soil's PSD for engineering geological soil characterization. In that context, achieving the best possible representation of a soil is also the goal, but practical problems that come with too large samples such as transport difficulties, storage capacity limitations or uneconomic testing efforts must also be considered. In contrast to the above-mentioned applications, the literature on the sample mass determination for engineering geological soil characterization is remarkably sparse and Zhang et al. (2017) and the recent publication of Jia et al. (2024) are few exceptions.

Methods of engineering geological investigation such as soil sampling for PSD determination are regulated and codified through different national and international standards. These methods are related to sieving, sampling techniques, sampling of aggregates, reducing sample sizes, alternative grain size determination through images, sample size estimates and sampling probability: ISO 17892-4 (2017), ASTM D6913/D6913M (2017), ASTM C136/C136M (2020), ASTM C702 (2018), ASTM E1382 (2023), ASTM D75 (2019), ASTM D3665 (2024), ASTM E105 (2021), ASTM E122 (2022), ASTM E141 (2023). Besides ISO and ASTM standards, other relevant ones are AASHTO T2, Australian Standard AS 1141.11, DJS 112-4:2015. Standards from Ontario, Canada recommend similar minimal masses, but lower than the European counterpart.

These standards recommend to determine the required minimum sample mass (m_{min}) as a function of the soil's estimated maximum grain diameter (D_{max}). As also pointed out by Zhang et al. (2017), the origin and scientific justification for this procedure is unknown, despite widespread adoption. This is of particular relevance in coarse-grained soils (i.e. ≥ sand acc. to ISO 14688 (2019)) where the suggested sample masses easily exceed tens of kilograms if one follows the recommendations. Equally unknown is the desired sampling confidence that the different guidelines seek to achieve. Gale and Hoare (1992) also addressed the topic of soil sample mass determination and give a recommendation based on D_{max}. But as others, i) they do not justify why a D_{max} based approach is adopted and ii) they aim for “reliable” grain size analyses but do not specify what reliable means in terms of how close the soil is approximated.

From a statistical point of view, using D_{max} as the decisive criterium to determine m_{min} implies that m_{min} depends on the extreme large grain sizes of the PSD, resp. on the rightmost point of the distribution. We hypothesize that today's standards overestimate the required sample mass in many cases and that D_{max} is a conservative criterium to determine m_{min}. This often forces practitioners who deal with coarse-grained soils to act outside the standard framework without being aware of what the consequences of smaller sample masses are. Furthermore, it is problematic that the recommendations for m_{min} are made without the indication of a desired sampling confidence.

This paper investigates the issue of sample mass determination for coarse-grained soils and proposes a new criterium to determine m_{min} that is easily applicable in practice as it is just an equation with estimated input values. We show through a dedicated survey that the inputs that are required for our criterium can be as well estimated as those recommended by today’s standards. The new criterium is developed through Monte-Carlo simulation of virtual sieve tests and allows one to explicitly set a desired level of confidence. The Monte-Carlo simulation simulates real laboratory tests as closely as possible with only minor assumptions such as spherical grain shapes. To provide a baseline, the sampling confidence of today's standards is back calculated within the simulations. The approach i) allows one to take samples according to a desired level of confidence that is to be achieved; ii) provides the possibility to assess the uncertainty that needs to be expected if one has a sample mass that is < m_{min}; iii) reduces the required m_{min} for many soils and especially for those where D_{max} comes from single large grains.

2. Background

In this section, extended information about the sample mass recommendations from ISO 17892-4 and ASTM ASTM D6913/D6913M is given as they explicitly give recommendations for soil characterization. The rest of this paper also directly refers to these two standards. Other standards that were mentioned in the introduction are thematically connected to this work, but are not directly relevant as they address other issues such as aggregates for concrete.

ISO 17892-4 (2017) defines that m_{min} [kg] depends solely on D_{max} [mm], for soils with a D_{max} > 20 mm. m_{min} according to this standard is to be derived from eq. 1.

$$m_{min} = \left(\frac{D_{max}}{10}\right)^2$$

eq. 1

The ASTM D6913/D6913M (2017) standard also defines m_{min} in dependence of D_{max}, for a D_{max} > 9.5 mm. m_{min} is "based on the mass of an individual spherical shaped grains, at the given sieve, multiplied by 100 then 1.2 (factor to account uncertainty) and finally rounded to a convenient number." For soils with a D_{max} > 76.2 mm, the same applies "except 1.2 factor is omitted". ASTM D6913/D6913M only gives this instruction and no equation, so eq. 2 was reconstructed based on that explanation. ρ in eq. 2 denotes the grain density which is also not directly specified in the standard but based on the therein given values for m_{min}, it can be back calculated that a ρ of 3.016 g/cm^3 must have been applied.

$$m_{min} = \frac{4}{3} * \pi * \left(\frac{D_{max}}{2}\right)^3 * \rho * 100 * 1.2$$

eq. 2

Based on these equations, both standards require minimum sample masses in the range of hundreds of kilograms for soils with a D_{max} larger than 5-10 centimeters which is unpracticable and often impossible to achieve in terms of practical sampling, availability and sievability in the laboratory. Figure 1 shows the required m_{min} for the mentioned standards for up to a maximum grain size of 300 mm diameter where ISO 17892-4 would require a sample with a mass of 900 kg and ASTM D6913/D6913M more than 1200 kg.

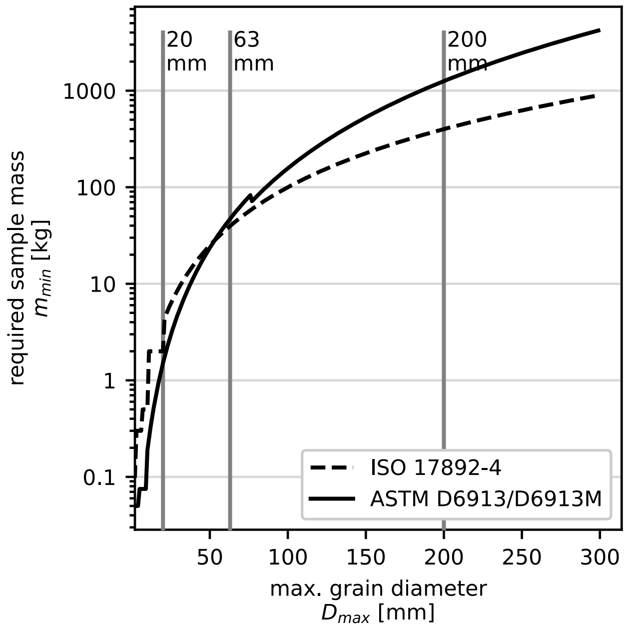

Figure 1: Minimum required sample masses as defined in ISO 17892-4 and ASTM D6913/D6913M. Steps in the plot result from fixed sample masses and conditions in the standards.

3. Development of new minimum sample mass criterium

In this study, we propose an alternative way of determining m_{min}. We first investigate the sample mass determination problem theoretically with Monte-Carlo simulations using virtual sieve tests and then underpin it with experimental results from real sieve tests. The Python source code, the simulation- and experimental results are available in the Github repository in the supplementary information of the paper.

3.1. Monte-Carlo Simulations

To theoretically investigate this problem, virtual sieve tests were conducted on generated coarse-grained soils. The basic idea is that first a "ground-truth" coarse-grained soil is generated and then samples with different masses are taken from this soil to investigate how large the error between the samples' PSDs and the soil's PSD is. The Monte-Carlo simulation was set up with the goal to generate a wide variety of PSDs including poorly graded-, well graded- and gap graded coarse-grained soils to reflect many possible geological scenarios. All grains are modelled as spherical which slightly reduces its realism (Kaviani-Hamedani et al., 2024), (see also section 6).

The soils are generated by the following process:

- Step 1: The simulation should include well- to poorly graded sediments. While poorly graded sediments can be modelled with a single statistical distribution (e.g. normal as done by Jia et al. (2024), lognormal or exponential) , well graded ones are compositions of multiple distributions due to different depositional environments. To account for this in the simulation, the first step is to randomly generate between 1 and 5 percentages of soil distributions (e.g. a soil may consist 100% of one distribution, or, for example, 30% of distribution A, 20% of distribution B and 50% of distribution C).
- Step 2: For each distribution, randomly set the minimum- and maximum grain diameters between 1 and 200 mm. The minimum diameter must be smaller than the maximum. These diameters are sampled from a uniform distribution U with $\ln(1)$ and $\ln(200)$ being the lower and upper limits of the distribution. The logarithm is taken to avoid oversampling of large diameters. The logarithmic values are then scaled back between 1 and 200 mm by calculating their exponential. This gives: lower-/upper limit = $\exp\left(U\left(\ln(1),\ \ln(200)\right)\right)$.
- Step 3: For each distribution, individual grain diameters are generated by sampling from a beta distribution that gives numbers between 0 and 1 and then scaling the output to the minimum and maximum grain diameters that were chosen in the previous step. The beta distribution’s parameters alpha and beta parameters are uniformly, randomly set between 1 and 4 for each sample.

This sample generation process is an attempt to mimic real soils that may consist of one or several soil distributions dependent on the geological history. In Figure 2, 100 exemplary sieve curves are shown to visualize the diversity of PSDs that were generated. The sieve curves are colored according to the sorting coefficient (S_0, see eq. 5 in Table 1).

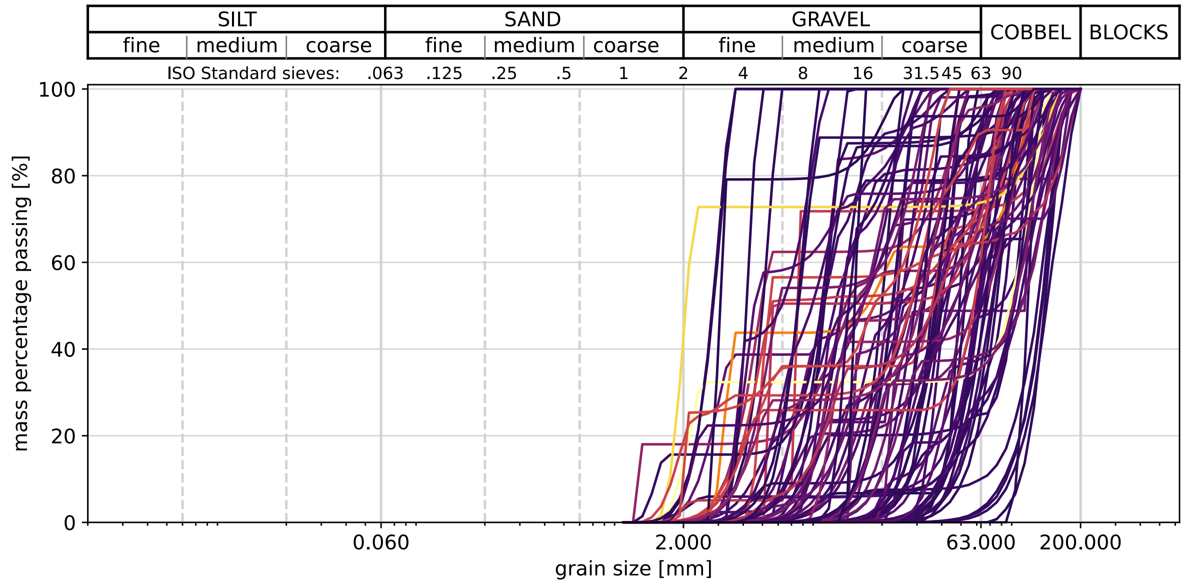

Figure 2: 100 exemplary sieve curves of samples that were generated for the Monte-Carlo simulation. Sieve curves are colored according to the sorting coefficient (S_0): dark purple = 1, yellow = 7.

To quantify the difference/error between the PSD of the soil and a sample's PSD, the Kolmogorov-Smirnov statistic (KS) was chosen. KS denotes the maximum vertical distance between two cumulative density functions which in this case means the maximum mass percentage difference between two sieve curves. Thus, KS – herein – has the unit of mass percent and the minimum and maximum of 0 or 100 would be reached if a sample's sieve curve either has a perfect fit or complete misfit with respect to the soil. For example, let $X = \{100, 95, 70, 20, 10, 5\}$ and $Y = \{100, 90, 50, 15, 7.5, 5\}$ be the mass percent passing sieves of mesh sizes 90-, 63-, 45-, 31.5-, 16- and 8 mm. KS is then computed as $KS = \max\left(|X - Y|\right)$ and would be 20% in this example (Figure 3). KS is seen as a well-suited error metric for this task as the goal for the soil sampling is to find a sample mass whose sieve curve fits as well as possible to the sieve curve of the soil.

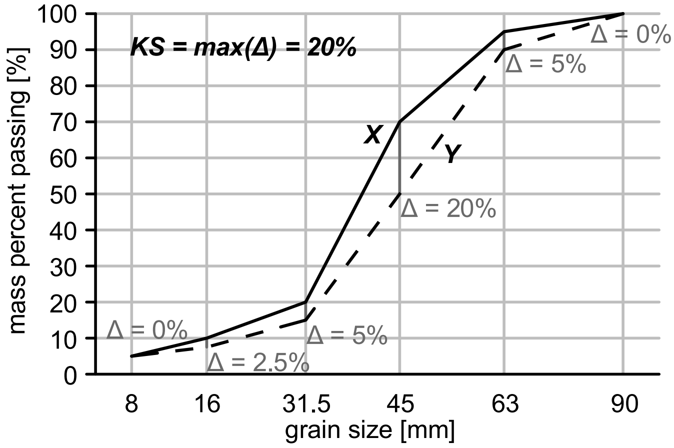

Figure 3: Example of how the Kolmogorov-Smirnov statistic (KS) quantifies the difference between two sieve curves X (solid) and Y (dashed).

Figure 4 shows an example where a soil was generated and multiple samples with decreasing masses were taken. The highest sample mass was determined according to eq. 1 (ISO 17892-4) and the subsequent samples are 75%, 50%, 25%, 10%, 5% and 1% fractions of the recommended sample mass. The lowest sample mass results in the highest KS with respect to the soil (i.e. highest error). Note, however, that KS is not consistently increasing with decreasing sample size which will be explained in the next section.

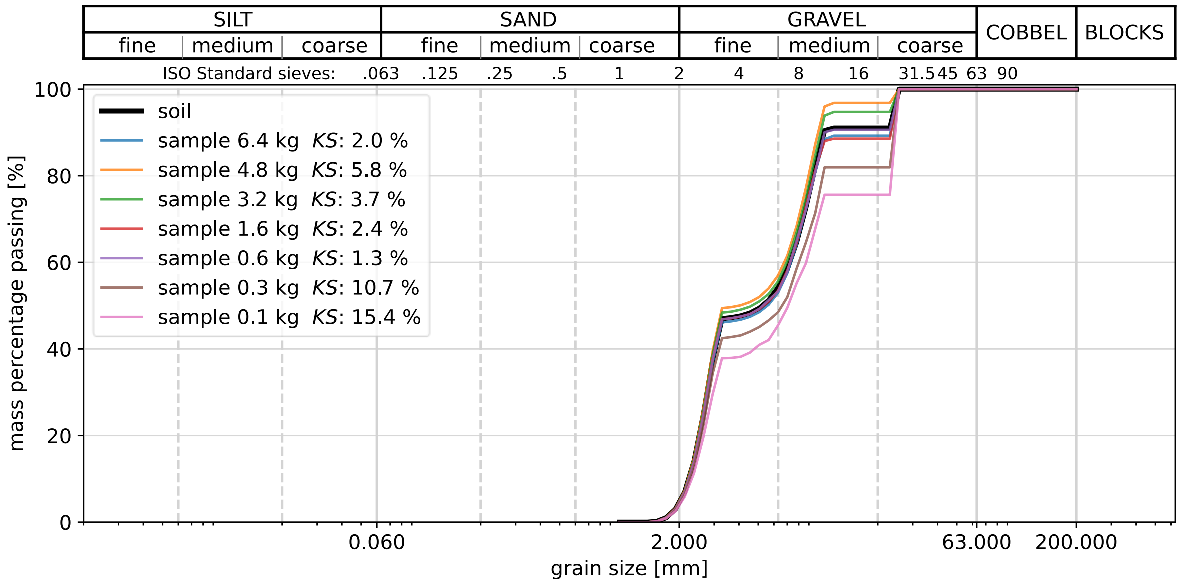

Figure 4: One example of a generated soil, where multiple samples with decreasing sample masses were taken and the Kolmogorov-Smirnov statistic computed for each of them.

For each simulation, the parameters given in Table 1 were recorded. A multitude of parameters was recorded to facilitate comprehensive Monte-Carlo simulation analyses afterwards.

Table 1: Parameters that are recorded for each simulated sample.

Parameter	**Description |**
ID	A unique id of the simulation for later identification.
C_u [-]	Coefficient of uniformity $C_u = \frac{D_{60}}{D_{10}}$ *eq. 3*
C_c [-]	Coefficient of curvature $C_c = \frac{{D_{30}}^2}{D_{60} * D_{10}}$ *eq. 4*
S_0 [-]	Sorting coefficient $S_0 = \sqrt{\frac{D_{75}}{D_{25}}}$ *eq. 5*
USCS soil classes	Soil classification according to the unified soil classification system (ASTM D 2487 – 06, 2006).
D_{min} [mm]	Minimum grain diameter of soil.
D_{max} [mm]	Maximum grain diameter of soil.
total masses [kg]	Total mass of generated underlying soil.
req. mass ks_p95 <= 10 [kg]	Required mass to achieve a KS_{p95} of ≤ 10% in a "bottom up" approach (see section 3.2).
X.X mm sieve [m%]	Mass percent soil passing a sieve of mesh size X.X mm. Mesh sizes increase logarithmically from 1 to 200 mm in 50 steps. This large number of virtual mesh sizes was chosen to get higher resolution sieve curves than would be possible with standard mesh sizes.
D_{xx} [mm]	Grain diameters at 10, 12, 20, 25, 30, 40, 50, 60, 70, 75, 80 and 90 mass % of the soil from a cumulative density function.
ISO req. mass [kg]	Required sample mass acc. to ISO 17892-4 (2017).
ASTM req. mass [kg]	Required sample mass acc. to ASTM D6913/D6913M (2017).
const req. mass [kg]	Constant sampling mass of 10kg as a reference.
new X.X req. mass [kg]	Required sample mass acc. to eq. 6 with an ε = X.X. X.X ranges from 1.0 to 2.5 in steps of 0.1
ISO ks [%]	KS between a sample's sieve curve that was taken acc. to ISO 17892-4 and the underlying soil's sieve curve.
ASTM ks [%]	KS between a sample's sieve curve that was taken acc. to ASTM D6913/D6913M and the underlying soil's sieve curve.
const ks [%]	KS between the sieve curve of a sample with constant mass = 10 kg and the underlying soil's sieve curve.
new X.X ks [%]	KS between a sample's sieve curve that was taken acc. to eq. 6 and the sieve curve of the underlying soil with an ε = X.X. X.X ranges from 1.0 to 2.5 in steps of 0.1.

3.2. Bottom-up determination of required sample mass

One of the goals of the simulation was to experimentally determine the required sample mass by generating a soil and then taking samples with progressively increasing masses until a defined KS threshold is reached. As individual samples with the same or only slightly differing masses may show a significant variability of KS (see Figure 4) each sampling was repeated 20 times as a trade-off between computational efficiency and representative results. The large fluctuation in repeated sampling with same masses originates from the chance whether or not individual large grains that significantly influence the resulting PSD are being sampled. The KS threshold was set so that the sample mass is seen as sufficient if the p95 percentile (i.e. 95% of values are lower than this) of the KSs of the 20 repeated samples is ≤ 10 mass %. In other words, if 19 of the 20 samples achieve a KS ≤ 10 mass %, the sample mass is sufficient. Note that this threshold has no general geotechnical meaning and was only set to have a threshold to experimentally determine a required sample mass to qualitatively investigate the relationship between sample mass, sampling confidence and further parameters such as D_{max} or D_{90}.

3.3. Insights from the Monte-Carlo Simulations

The Monte-Carlo simulations were used to i) investigate the sampling confidence / error that results from determining m_{min} according to ISO and ASTM and ii) to develop a new approach for m_{min} determination that reduces the required sample mass and explicitly considers the sampling confidence. To this end, 1200 simulations were made and it was observed that the ISO recommendation (eq. 1) achieves a median KS (KS_{med}) of 3.5% and a p95 percentile of KS (KS_{p95}) of 8.0%. This means that 95% of samples taken according to ISO have a KS < 8.0% to the soil. Due to the higher required sample masses, the ASTM recommendation (eq. 2) achieves lower KS error of a KS_{med} of 2.6% and a KS_{p95} of 5.7%. A violin plot of the ISO- and ASTM- recommended sample masses and the achieved KS errors for all 1200 simulations is given in Figure 5.

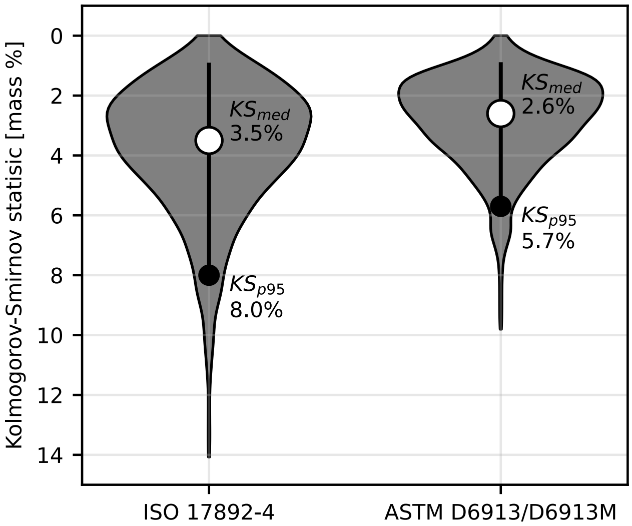

Figure 5: Violin plots of the Kolmogorov Smirnov error for samples taken according to ISO and ASTM standards.

The "bottom up" determination of required sample mass (see section 3.2) allows to investigate the relationship between the experimentally determined required sample mass to achieve a certain error and other parameters that describe the samples. This study's original hypothesis was that the required sample mass to achieve a certain error must be dependent on the grading of the soil rather than solely on D_{max}. Figure 6 was made to verify if grading can be used to complement the selection of m_{min}. The following insights are gathered from this:

- There is a relationship between grading and required sample mass as samples with a high S_0 (i.e. well graded) also require larger sample masses. However, the figure also shows that there are samples with a low S_0 that require a large sample mass and thus this hypothesis was rejected (high confidence).
- The recommendations from the standards (esp. ISO) do not always overestimate the required sample mass but rather describe the upper limit of the required sample mass. Thus, it can be qualitatively confirmed that there is a relationship between a soil's large grain sizes and the required sample mass to reach a certain sampling confidence (high confidence).

- It is observed that there are samples that have a comparably large D_{max} but require sample masses several times smaller than suggested by the standards. It is thus shown that the standards overestimate the required sample mass in several- but not in all cases (high confidence).

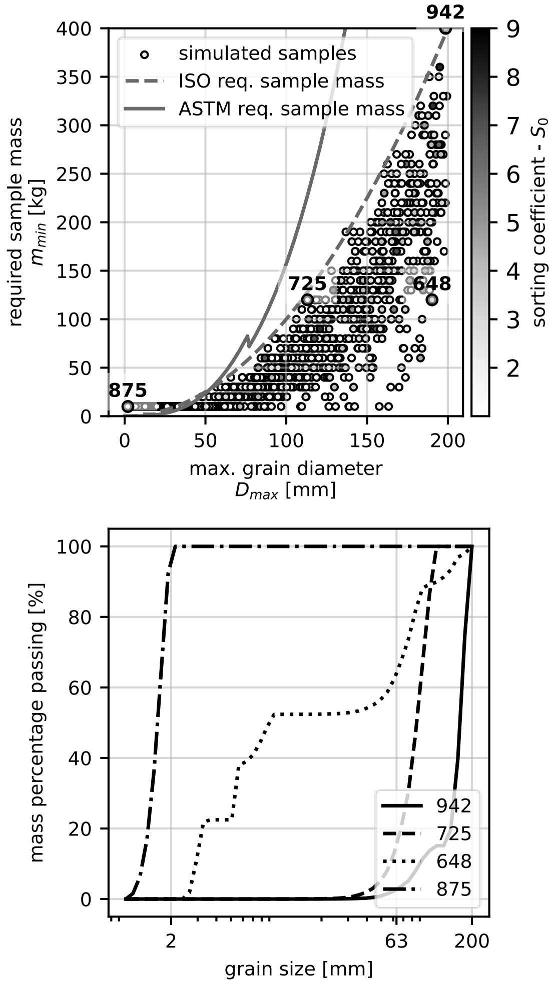

Figure 6: Top: Relationship between a soil's maximum grain diameter (x-axis) and the required sample mass (y-axis). The datapoint color indicates the soil's sorting coefficient (S_0). Theoretically, required sample masses acc. to ISO and ASTM are also shown for reference. Bottom: Exemplary sieve curves from the top figure, marked with sample "ID" (see data in the supplementary information).

Based on these insights, we investigated the correlation between different parameters that describe a sieve curve's geometry and the required sample mass. We used Pearson's correlation coefficient where values of 1 and -1 indicate very strong positive and very strong negative correlations respectively and 0 indicates very weak correlation. The results are shown in Table 2.

Table 2: Correlation analyses between parameters that describe a sieve curve's geometry and the required sample mass that was determined in the Monte Carlo simulation.

Parameter	Correlation with required sample mass
C_u [-]	0.25
C_c [-]	0.09
S_0 [-]	0.25
D_{min} [mm]	0.12
D_{10} [mm]	0.36
D_{20} [mm]	0.42
D_{30} [mm]	0.48
D_{40} [mm]	0.56
D_{50} [mm]	0.62
D_{60} [mm]	0.72
D_{70} [mm]	0.80
D_{80} [mm]	0.87
D_{90} [mm]	0.91
D_{max} [mm]	0.84

This analysis showed that the currently used parameter to determine the required sample mass - D_{max} – only achieves a correlation of 0.84 with it. A slightly stronger correlation of 0.87 is achieved by d_{80} and the strongest correlation of 0.91 by D_{90} (i.e. the grain size where 90% of a sample's mass has a smaller diameter).

Visualizing the simulations as D_{90} vs. required sample mass and coloring the data points according to the maximum grain diameter (Figure 7) shows that soils with a large D_{90} also require large sample masses for representative sampling. The same exemplary PSDs as in Figure 6 are marked in Figure 7. Note for example that samples 648 and 725 have very different D_{max} but similar D_{90}. In general, can it be seen that there are several soils with a low D_{90} that still have a large maximum grain diameter, but they do not require large sample masses for representative sampling. We thus conclude with high confidence that the relationship between grain size and required sample mass as implied by the standards is qualitatively correct, but D_{max} is an ill-suited criterium as it represents the rightmost point of a soil's PSD which is often an outlier in coarse-grained soils. Consequently, D_{max} does not represent a soil's significant large grain sizes and is affected by outliers. D_{90} – which is not a PSD's extreme value – on the other hand, is not sensitive to outliers and shows a more robust relationship with the required sample mass.

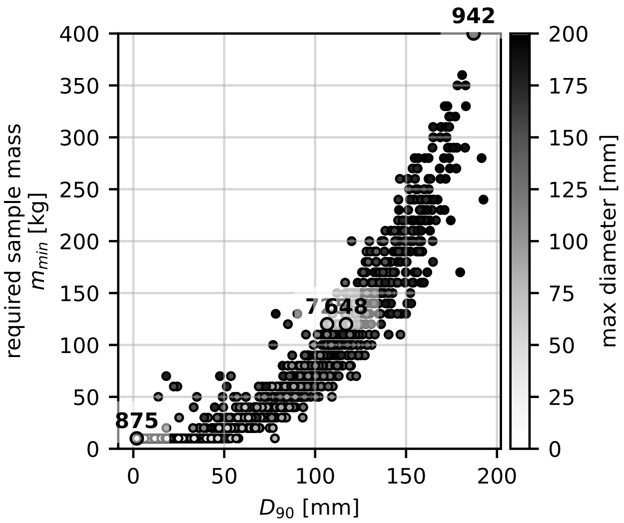

Figure 7: Relationship between a soil's D_{90} (x-axis), the required sample mass (y-axis) and the maximum grain diameter (datapoint color). The same PSDs as shown in Figure 6 (bottom) are marked.

3.4. Proposed criterium for minimum required mass

Based on the insights from the Monte-Carlo simulations, a new criterium to determine m_{min} for coarse-grained soils was developed. The theoretical framework is presented in this chapter and an exemplary application is given in Appendix 1. Based on eq. 1, D_{max} was replaced with D_{90} and a dedicated error-exponent ε that gives control over the maximum error that one wants to achieve with the taken sample mass was introduced (eq. 6).

$$m_{min} = \left(\frac{D_{90}}{10}\right)^{\varepsilon} \qquad \text{eq. 6}$$

This new criterium was included in the Monte-Carlo simulation to determine the KS errors that are achievable with different ε by repeated sampling from one soil with different masses (see parameters "new X.X req. mass [kg]" and "new X.X ks [%]" in Table 1). As KS_{med} and KS_{p95} of the current standards were determined in Figure 5 (section 3.3), we determined these errors for different ε on a range from 1 to incl. 2.5 (Figure 8, top). 2.5 was set as the upper limit as this yields sample masses larger than the ASTM standard. Based on this, the relationships between the achievable KS_{p95} error and ε, respectively KS_{med} error and ε was assessed and is shown in Figure 8, bottom. These relationships can be described with the exponential functions of eq. 7 and eq. 8.

$$KS_{p95} = 118.11 * e^{-1.24*\varepsilon} \quad \text{eq. 7}$$

$$KS_{med} = 37.38 * e^{-1.09*\varepsilon} \quad \text{eq. 8}$$

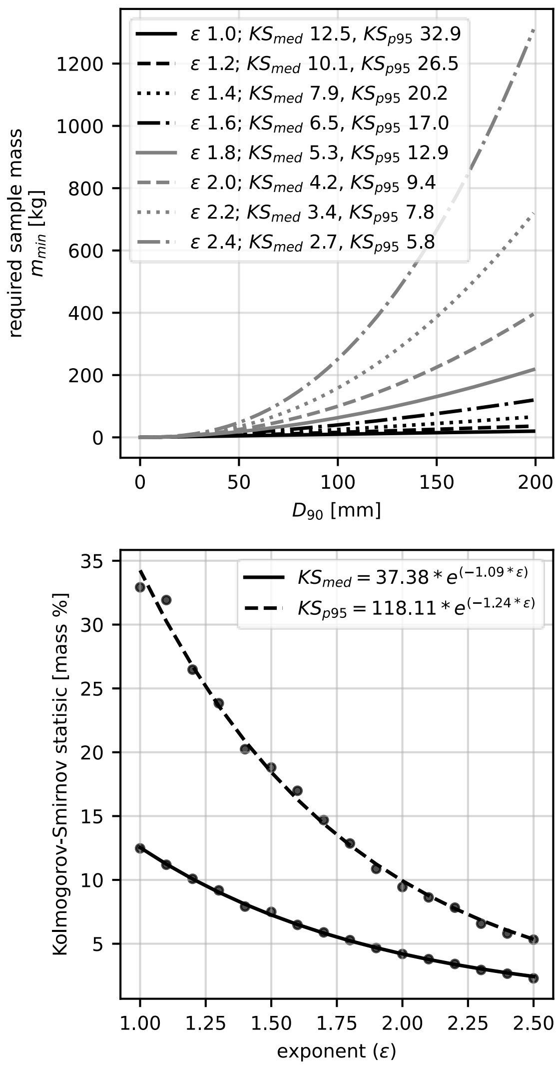

Figure 8: Top: The new criterium to determine the minimum sample mass (m_{min}) with different error exponents (ε). Bottom: The assessed KS_{p95} and KS_{med} vs. different error exponents ε.

Solving eq. 7 for ε and substituting for ε in eq. 6, finally gives the new recommended equation to determine $m_{\min}$ in a sampling confidence-aware manner in eq. 9.

$$m_{\min} = \left(\frac{D_{90}}{10}\right)^{\frac{\ln(KS_{p95}) - \ln{(118.11)}}{-1.24}}$$

eq. 9

This equation allows one to determine the minimum required sample mass, given an estimated D_{90} of the soil and a desired sampling confidence in mass percent (KS_{p95}). The $m_{\min}$ will in 95% of cases be a sample mass that is sufficient to satisfy the desired error threshold.

A decisive question that comes up in this context is how reliably an operator can come up with a field estimate of a sample's D_{90} vs. a field determination of a sample's D_{max} as it is required in today's standards. First, one must acknowledge that both parameters can only be estimated as the full soil body under investigation is never observable. Secondly, a dedicated survey that investigates whether operators achieve a higher performance in estimating one parameter over the other showed that there is no significant reason to believe that. The capability to estimate parameters is equally well / poor for all D-values. Gap graded soils may be the only exception here, where it can be seen that the D-value closest to the gap has a significant variation but the characterization of gap-graded soils constitutes a research problem on its own and cannot be addressed herein. The full survey results can be found in Appendix 2. We recommend estimating D_{90} in the field as *the maximum relevant grain size of the soil excluding obvious large outliers*.

Defining desirable PSD errors for different geotechnical applications is not in the scope of this study and should be investigated with dedicated research (see discussion). As fine-grained soils were not considered in the simulation and sands and fine-gravels only represent the lower boundary of the Monte-Carlo simulation, the same criteria as specified in the ISO standard should be applied for soils with a D_{max} ≤ 20 mm. Furthermore, in cases where the estimated D_{max} > 20 mm but the estimated D_{90} < 10 mm, 1 kg of sample mass should be used. Otherwise, eq. 9 is to be used.

3.5. Comparison to standards and further usage

Figure 5 shows that ISO and ASTM achieve KS_{p95} of 8.0% and 5.7 % respectively. Using these values in eq. 9 allows to directly compare the required sample masses from the new criterium to the previous standards (Figure 9). On average, across all simulated samples, the new criterium requires ca. 4 times lower sample masses than the ISO standard and ca. 9 times lower sample masses than the ASTM to achieve similar sampling confidences. In extreme cases, however, the required sample masses according to the new criterium are several thousand times lower than the ISO or ASTM standards while reaching the same sampling confidence.

In Figure 9 top, it can be seen that for the majority of samples the new criterium to determine m_{min} yields a larger sample masses than the ISO standard. While this is only a theoretical result, it indicates that the ISO standard is unconservative and inconsistent when it comes to recommending required sample masses for very coarse-grained soils that contain a significant amount of large grains (low confidence). Nevertheless, in both cases of ISO and ASTM, it can be seen that there are samples that are far below the 1:1 line in Figure 9 thus suggesting with high confidence that the newly proposed criterium is more precise for sample mass determination and not sensitive to outliers.

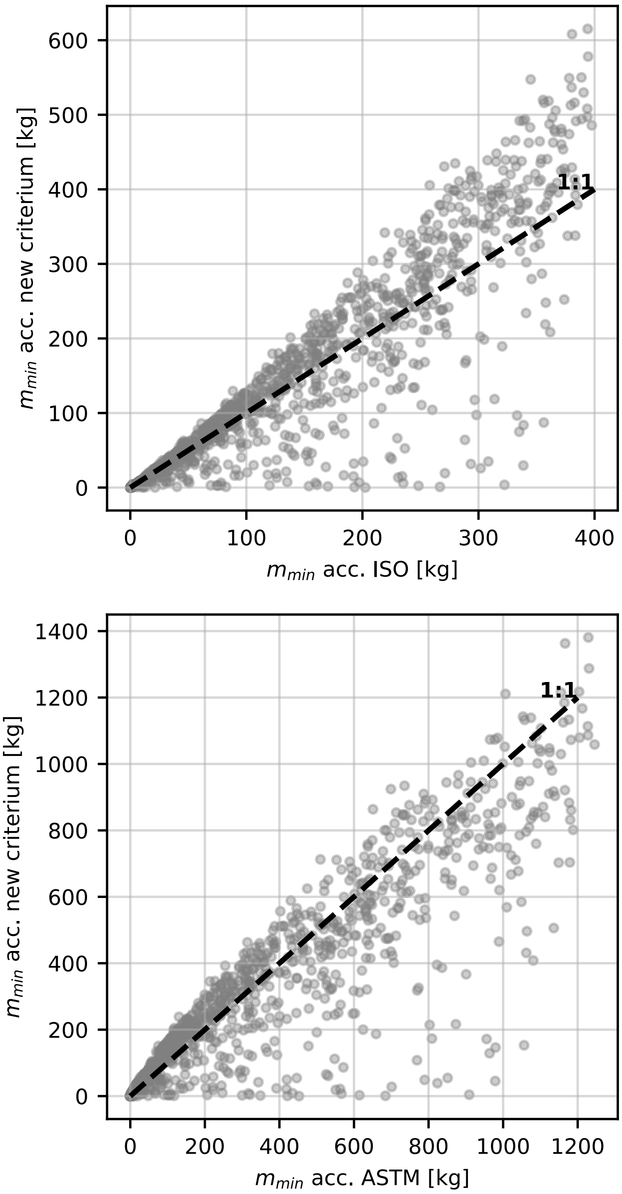

Figure 9: Comparison between sample masses acc. to ISO (top) and ASTM (bottom) to the new criterium at equal confidence levels. Dashed lines indicate lines of 1:1 equal mass in the plots. Datapoints are 50% transparent.

Lastly it must be acknowledged that there are cases where the available sample mass is smaller than the desired / required sample mass and acquiring more sample is unviable. Today, operators either avoid sampling all together in these cases or must do sampling outside the standards' framework. Thus they are not aware of the error that they may or may not introduce through this undersampling. We recommend also taking samples to determine a PSD in these cases, but the operator should be aware of the expectable error that the sampling is subjected to. In this case

m_{min} in eq. 6 can be substituted with the available sample mass ($m_{available}$) and then the equation solved for ε, thus giving eq. 10.

$$\varepsilon = \frac{ln(m_{available})}{\ln(D_{90}) - \ln{(10)}} \qquad \text{eq. 10}$$

By using the determined ε in eq. 7 and eq. 8 or Figure 8 bottom, one can find which KS_{med} and KS_{p95} is to be expected given the available sample mass. The consequence of knowing the error that must be expected given the available sample mass is that the subsequent geotechnical analysis can consider this uncertainty by setting a higher focus on probabilistic analyses, adjusting how conservative approaches are or considering different plausible scenarios.

4. Experimental underpinning

4.1. Experimental program and tested soils

Several sieve analyses were performed in the laboratory to practically test the hypotheses presented in the previous chapter. The goal of the sieve analyses was to investigate if it is also practically the case that significantly lower sample masses than recommended by the standards yield sufficient PSDs. Three different soils were used, namely a (A) medium to fine sand, (B) a medium to fine gravel and (C) a sandy, medium to coarse gravel. Different test programs were conducted for each soil:

- Soil A: A medium to fine sand from the Isle of Rum in Scotland (United Kingdom) was used to investigate how far one can go with reducing the ISO recommended sample mass even below the considered size of the Monte-Carlo analyses. With an estimated D_{max} of 4 mm, an ISO 17892-4 recommended (dry) sample mass of 200 g was taken from one large sample. Further samples with 100 g, 75 g, 50 g, and 5 g were also taken and PSDs determined for all of them.

- Soil B: A medium to fine fluviatile gravel was collected from the river Akerselva in Nydalen, Oslo (Norway). The D_{max} is estimated to be 30 mm, thus the ISO required sample mass is 9 kg of soil (eq. 1) which was used for one sieve test. The estimated D_{90}, however, is around 8 mm and thus < 10 mm. Therefore, the new recommendation of 1 kg sample mass was tested (see end of section 3.4). To also include an extreme case, one more sieve analysis with 300 g of sample was done.
- Soil C: An artificial, pre-sieved, sandy, medium to coarse gravel from Austria with a known D_{max} of 70 mm was used for soil C. One sieve test with a sample mass of 50 kg according to ISO was done and one with a 2.5 times lower sample mass of 20 kg.

4.2. Experimental results

Table 3 gives an overview of the experimental results and Figure 10 shows the sieve curves for the different soils.

Table 3: Overview of the experimental results.

Test	Sample mass [g]	D_{10} [mm]	D_{30} [mm]	D_{60} [mm]	C_c	C_u	KS to ISO [mass %]
Soil A (ISO)	200	0.081	0.148	0.236	2.90	1.14	-
Soil A1	100	0.085	0.145	0.227	2.66	1.08	3.32
Soil A2	75	0.090	0.158	0.242	2.69	1.14	3.97
Soil A3	50	0.082	0.140	0.227	2.74	1.05	3.18
Soil A4	5	0.079	0.137	0.230	2.91	1.03	3.88
Soil B (ISO)	9000	0.599	1.557	3.615	1.12	6.04	-
Soil B1	1000	0.608	1.536	3.527	1.10	5.80	1.34
Soil B2	300	0.553	1.245	2.782	1.08	5.03	12.34
Soil C (ISO)	50000	0.369	3.694	14.167	2.610	38.387	-
Soil C1	20000	0.563	5.152	18.451	2.557	32.802	7.8

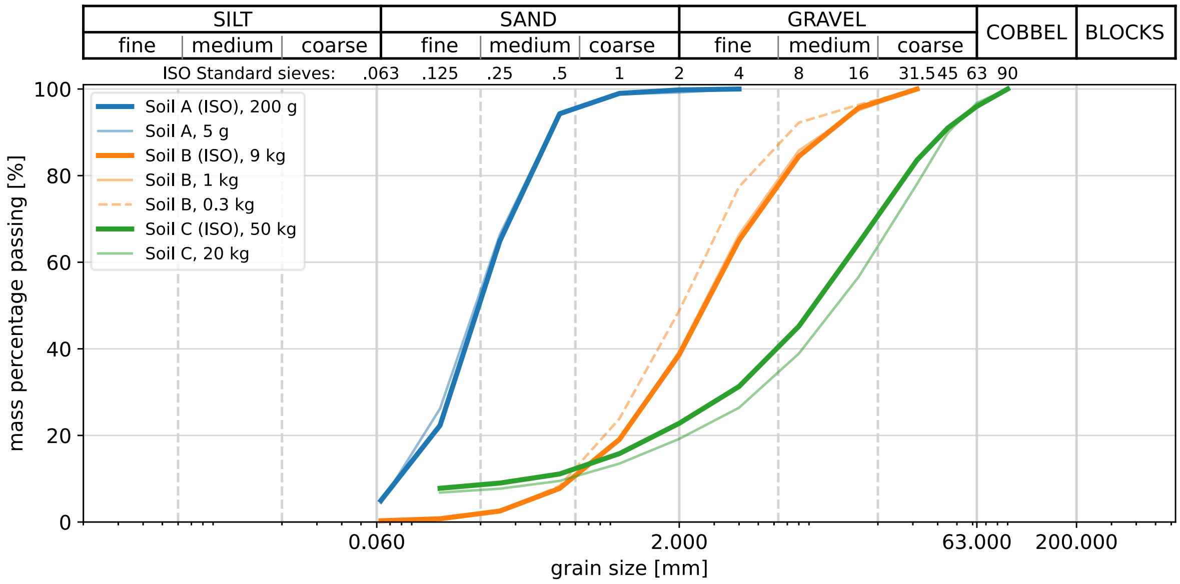

Figure 10: Sieve curves of the conducted lab tests to investigate how different sample masses influence practical results. For each soil, the sieve curve with a sample mass acc. to ISO 17892-4 and the sieve curve based on the smallest sample mass is shown.

For all soils, no remarkable discrepancy can be observed between the PSDs obtained using different amounts of sample mass. While this study aims at coarse-grained soils with the main grain size being gravel or larger, soil A demonstrates that lower sample masses can also give sufficient results for sands. In soil A, even a 40 times lower sample mass than what would be required by ISO 17892-4 only yields a KS of 3.88%. For Soil B, a mass 9 times lower than the suggested by ISO (i.e. the mass as recommended herein) shows a KS of 1.34% only. A test with a 30 times lower sample mass (300 g) was also conducted on Soil B and results in a KS of 12.34% with respect to the ISO recommended of 9000 g. This more substantial deviation results from a low sample mass which is also not recommended, and the test was done for demonstration purposes only to show what happens in substantially lower sample masses in coarse-grained soils. In case of Soil C, the error between the PSD resulting from the ISO recommended sample mass of 50 kg and a test with a 2.5 times lower sample mass yielded a KS of 7.8%. While the effort of doing a sieve test with 20 kg instead of 50 kg of sample mass is significantly lower, the resulting difference in the PSD is small and still leads to the same characterization of the soil as a sandy, medium to coarse gravel.

Table 3 shows that also the differences between the parameters that describe the sieve curves' geometry are small and Figure 11 visualizes the difference in C_c and C_u between tests with a sample mass according to ISO and tests with a lower sample mass. In all cases, the values become slightly lower with decreasing sample masses. Nevertheless, the total differences are small and would not change a soil's classification based on C_c and C_u.

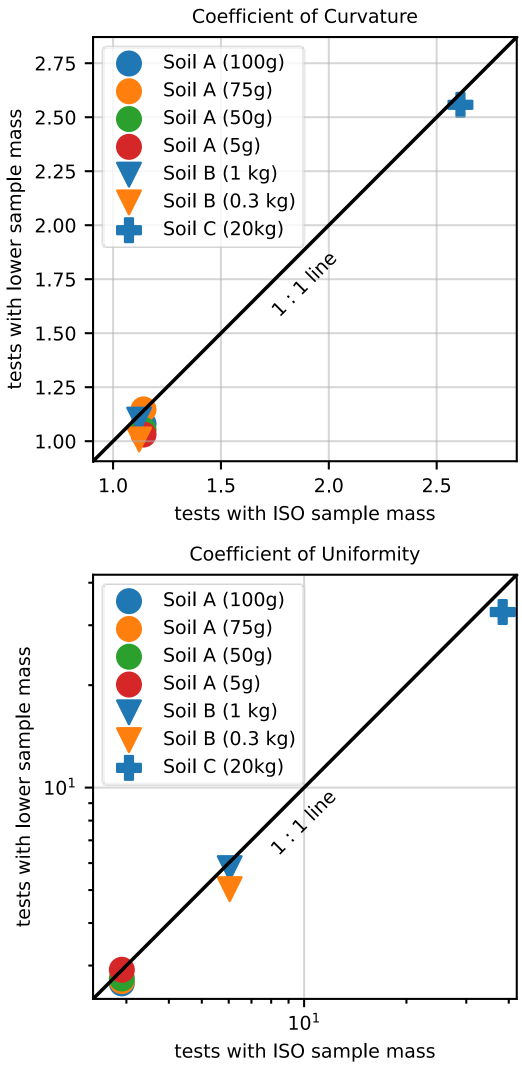

Figure 11: C_c and C_u differences for tests with a sample mass acc. to ISO and tests with a lower sample mass.

5. Discussion

The proposed new method for m_{min} determination leads to more precise recommendations for the required sample masses for coarse-grained soils including a significant reduction of the required sample mass for few cases. It is easily applicable in practice and also permits to take samples under explicit consideration of the sampling confidence. The practicality, outlier awareness, explicit accounting for sampling confidence and consideration of a wide range of soil types are improvements over previously proposed methods for sample mass determination (Gale and Hoare, 1992; Jia et al., 2024; Zhang et al., 2017).

The proposed new methodology is based on simulations of laboratory sieve tests, but practical laboratory sieve tests on real soils corroborate the theoretical results thus new recommendations and conclusions are made with very high confidence. Nevertheless, the simulation includes some simplifying assumptions such as perfectly spherical grains which might influence the result, especially for very coarse grain sizes that seldomly are perfectly spherical. Studies such as Kaviani-Hamedani et al. (2024) address this issue, but in large scale simulation of sieve tests, explicitly including non-spherical grains heavily impacts the computational performance and thus renders large scale Monte-Carlo simulations infeasible, today.

The simulation of individual and discrete grains and the subsequent explicit sampling from these grains is on the one hand seen as a benefit of this study as it is the most realistic way of simulating sieve tests, on the other hand it is computationally very demanding as especially memory limits are reached fast the smaller the grain sizes become. Besides the main goal to investigate coarse grain sizes, the lower grain size boundary of 1 mm in this study is related to computational limitations of this approach. To conduct simulated PSD analyses starting from clay sizes, would require a different simulation concept, that is rather based on statistical distributions than on individual grains.

6. Conclusions and Outlook

A new method to determine the minimum required sample mass for PSD assessments was proposed. The new method explicitly considers sampling confidence, which is an improvement on the one hand but on the other opens up for a plethora of new research questions related to "How much is enough for application X?". As given in the introduction, PSDs are not only fundamental for general purpose soil characterization but also feed directly into different geotechnical engineering applications. These may, however, tolerate different sampling errors depending on the downstream usage of a PSD and derived parameters such as D_{10}, D_{60}, C_u, C_c, etc. Speculating about required confidences of soil sampling for different geotechnical applications is out of the scope of this study and future research related to this topic is highly encouraged to provide a sound decision base for sampling confidences.

The conducted survey to investigate how reliable parameters like D_{max} and D_{90} can be estimated by operators in the field showed that there is no significant difference for visual assessments (medium confidence). More surveys like this and similar ones (Elmo and Stead, 2021; Skretting et al., 2023) are required to get a quantitative understanding of the cognitive biases and human uncertainty that is involved in engineering geological and geotechnical observations. Further surveys like this are encouraged where the survey scope could be extended by the use of real soil samples instead of generic visualizations. However, in the case of PSD determination of coarse-grained soils the use of image processing technology for PSD-pre-assessment (Ferrer et al., 2021) could be considered. Nevertheless, due to the required level of technological proficiency and eventually also soft- and hardware cost, it is not expected that image processing techniques will replace estimations of PSDs in practice in the near future and approaches like the one proposed herein will remain relevant.

Supplementary information

The code for the Monte-Carlo Simulations and the results of the real laboratory tests can be found in the following Github repository: https://github.com/norwegian-geotechnical-institute/sieve_analyses/releases/tag/v2.0.0

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Appendix

Appendix 1 – Application example

For example, one wants to determine the PSD of a coarse-grained fluviatile soil with an estimated D_{max} of 150 mm (there are some cobbles) and an estimated D_{90} of 80 mm. According to eq. 1 from ISO 17892-4 the required m_{min} is 225 kg of soil (eq. 11) and it is not clear why so much soil would be required. In contrast to that, the new eq. 9 allows setting a desired maximum error / sampling confidence (KS_{p95}) of e.g. 10 %. Based on the estimated D_{90} one can then estimate the required sample mass to be ~63 kg with explicit consideration of that desired sampling confidence (eq. 12). If the total available soil sample mass would, however, only be 20 kg, then eq. 10 can be used to determine the error exponent ε (eq. 13) which is 1.44. Substituting this into eq. 7 reveals that in this particular soil, one needs to expect that the determined PSD has an error of up to ~20% with respect to the real soil's PSD if only 20 kg of soil sample are available (eq. 14).

$$m_{min}[kg] = 225 = \left(\frac{150}{10}\right)^2 \qquad \textit{eq. 11}$$

$$m_{\min}[kg] = 63 = \left(\frac{80}{10}\right)^{\frac{\ln(10)-\ln(118.11)}{-1.24}} \qquad \textit{eq. 12}$$

$$\varepsilon = 1.44 = \frac{\ln(20)}{\ln(80) - \ln(10)} \qquad \textit{eq. 13}$$

$$KS_{p95}[m\%] = 19.8 = 118.11 * e^{-1.24*1.44} \qquad \textit{eq. 14}$$

Appendix 2 - Grain size distribution characterization survey

A survey was conducted to investigate how well operators can visually estimate different parameters that describe the geometry of a sieve curve. The survey was done using Microsoft Forms and responses that were submitted between the start of the survey on 25th of November 2024 until its end on the 9th of December 2024 were included in this analysis.

The following metadata was collected from each participant:

- Name
- Email Address
- Main area of expertise, where participants could choose one of the following answers: Geotechnical engineering, Engineering Geology, Sedimentology, Hydrogeology, Quaternary geology, other (to be specified).
- Current main field of work, where participants could choose one of the following answers: Academia, Industry (consulting, contractors, technology development,…), Other
- Years of experience post master, where participants could choose one of the following answers: 0-5, 5-10, 10-20, 20-30, >30, None (still student or not from this field).

After collecting this information, the participants were presented with a series of four synthetic sediment samples that were generated with the code framework of this project that is provided in the Supplementary information of the paper. Each sample shows spherical black grains in a 500 by 500 mm large field on white ground. A measuring scale is given on the border of the field with 50 mm spaced ticks and some reference grains are given below the sample with sizes between Ø=100 to Ø=2mm. The samples are shown in Figure A 1 to Figure A 4.

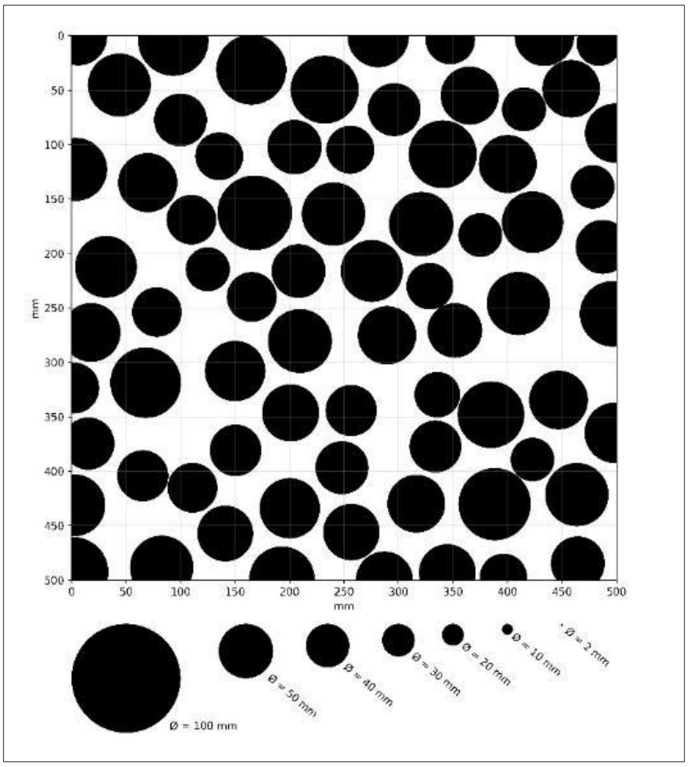


Figure A 1: Sample 1.

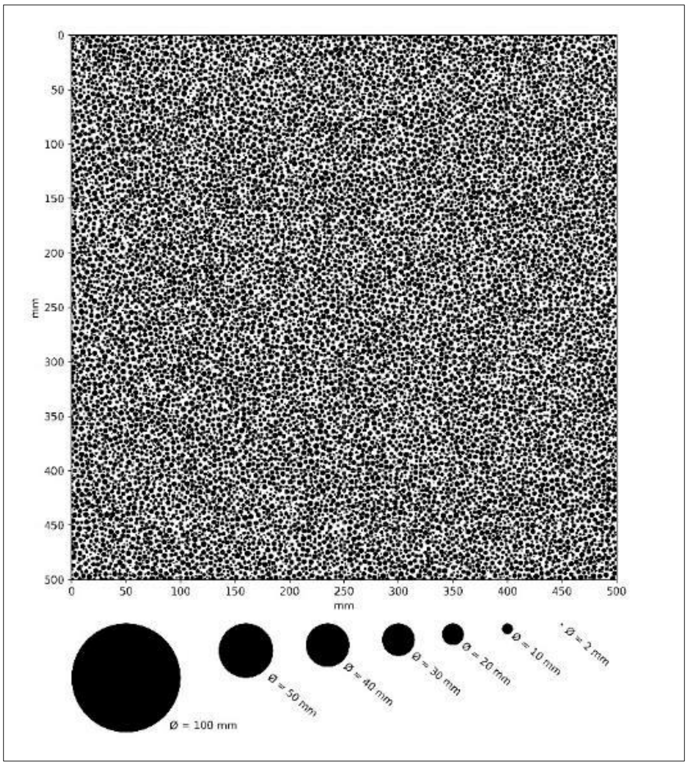


Figure A 2: Sample 2.

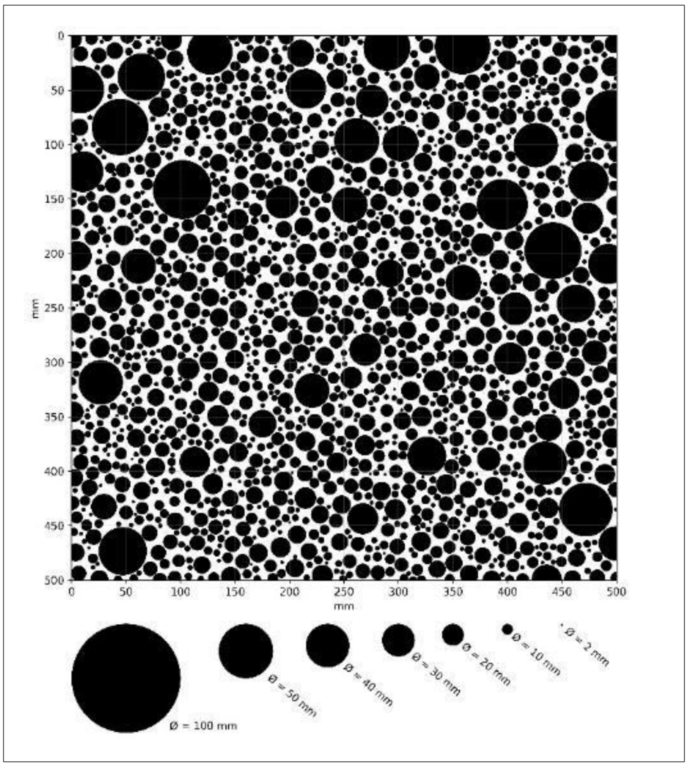


Figure A 3: Sample 3.

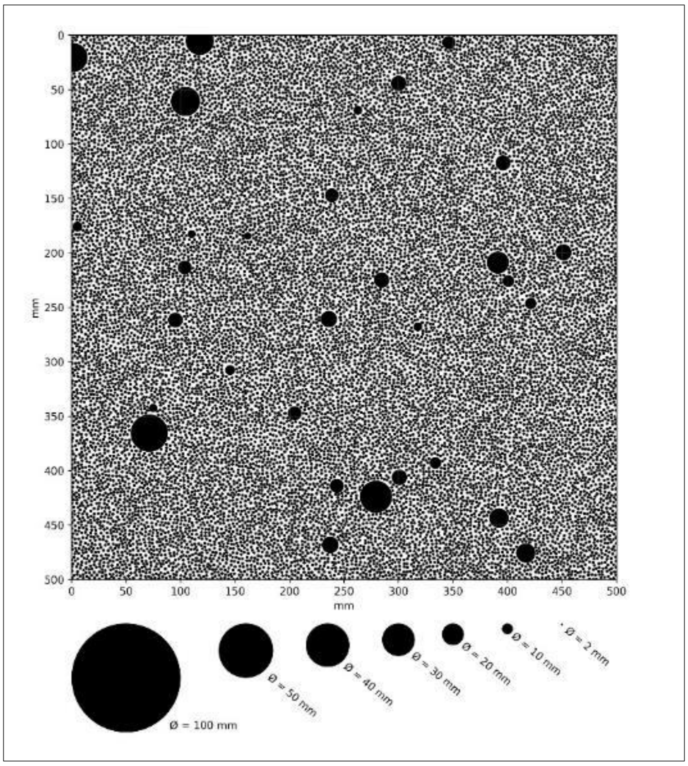


Figure A 4: Sample 4.

For each sample, the participants were asked to estimate the D_{min}, D_{10}, D_{30}, D_{50}, D_{60}, D_{90} and D_{max}. The participants were told not to be too precise and to take not more than 3 minutes per sample. A total number of 95 responses were collected. From these 95 responses, 14 had to be completely removed because the participants gave consistently not credible responses that indicated a misunderstanding of the survey (e.g. always the same number, decreasing grain sizes from D_{min} to D_{max}, etc.). Furthermore, single results for samples had to be removed for similar reasons but it can be observed that there are more erroneous submissions for sample 1 than for the others, thus indicating that some participants needed the first sample to get used to the task. After response cleaning, a total of 71, 81, 80 and 80 responses were left for the samples 1-4 respectively. A visualization of the collected participant metainformation is shown in Figure A 5.

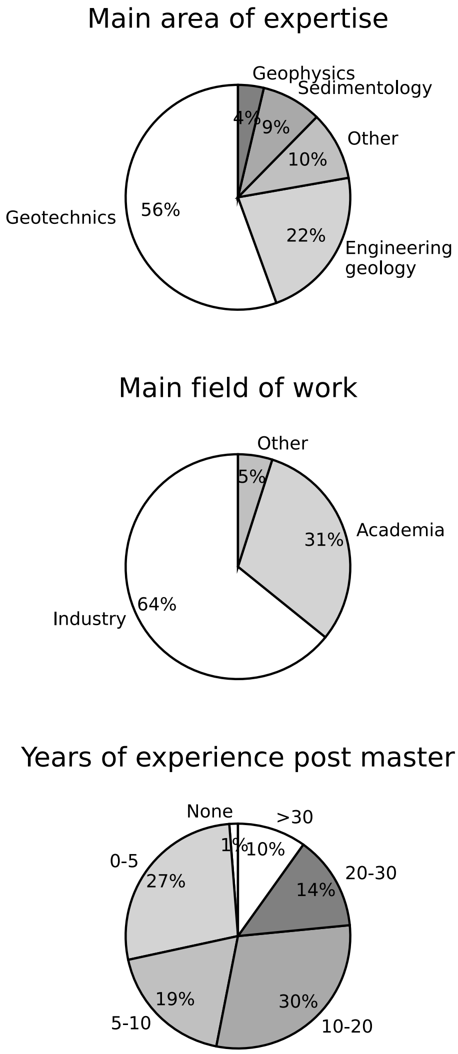
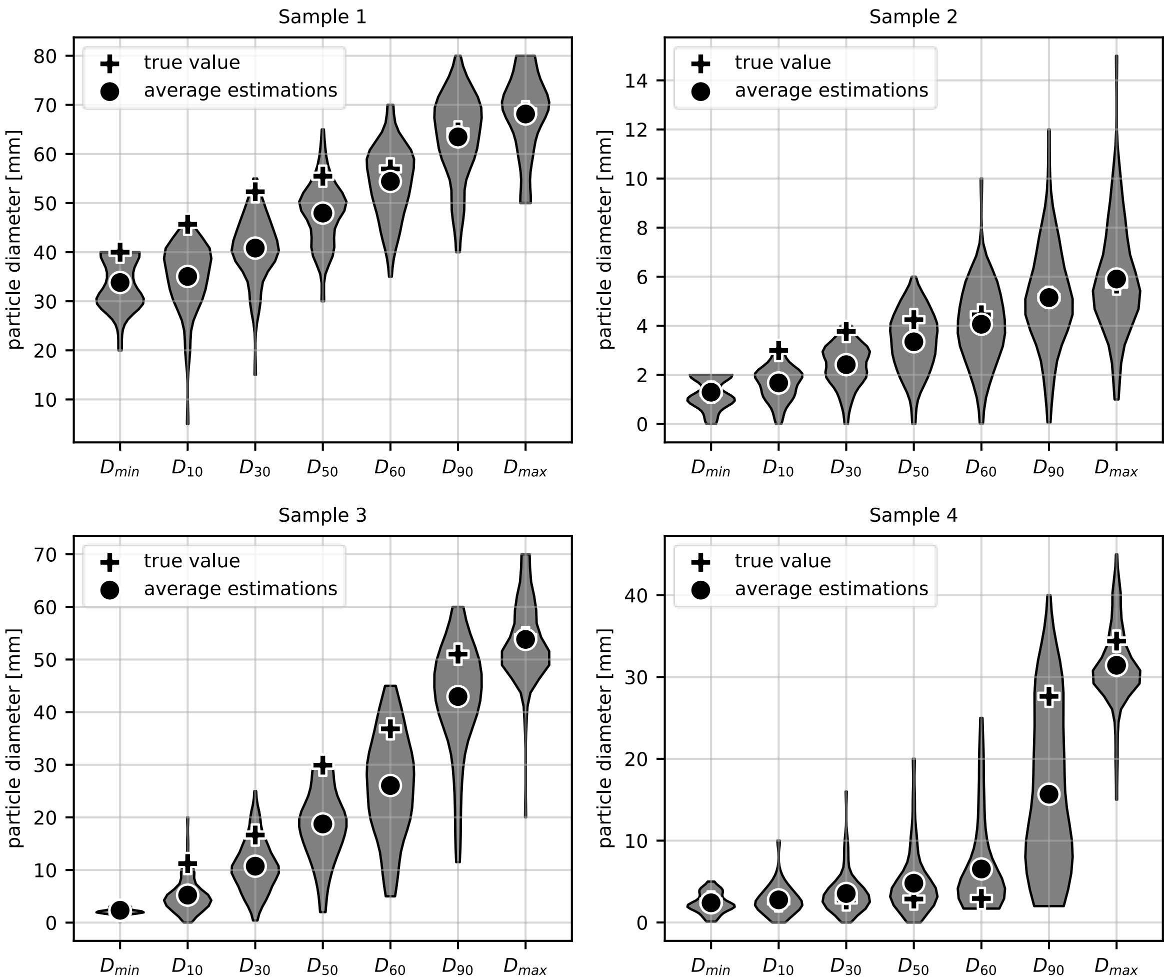


Figure A 5: Statistics of the metainformation that was collected from the participants in the survey.

A visualization of the participants' responses in relation to the true values (assessed based on the simulated grain distribution) for every sample is given in figure Figure A 6. While the average estimated parameters are close to the true values, it can be seen that all parameters show substantial variability. There are no generally observable trends, and it is not observable that the D_{max} is, for example, significantly easier to assess than other D-values. The only exception is sample 4 which has a pronounced gap graded distribution, and it is visible that participants alternate between assigning the D_{90} to the small or the large grain sizes. Analyzing these results also must consider the logarithmic scale of the problem where e.g. overestimating the size of a 4 mm grain by 100% is less severe than overestimating the size of a 40 mm grain by 50%.



608

609 *Figure A 6: Results of the survey. The distribution and bandwidth of participants' responses is shown with grey violin*
610 *plots.*

611 Lastly, the participant assessed values were used to compute C_u and C_c for the samples and their
612 respective distribution based on the participants feedback variability (Figure A 7). It can be seen
613 that the variability for these computed values is substantial but it also must be considered that
614 these are calculated values and not directly estimated values. The ground truth values for the
615 parameters under investigation of the survey are given in Table A 1.

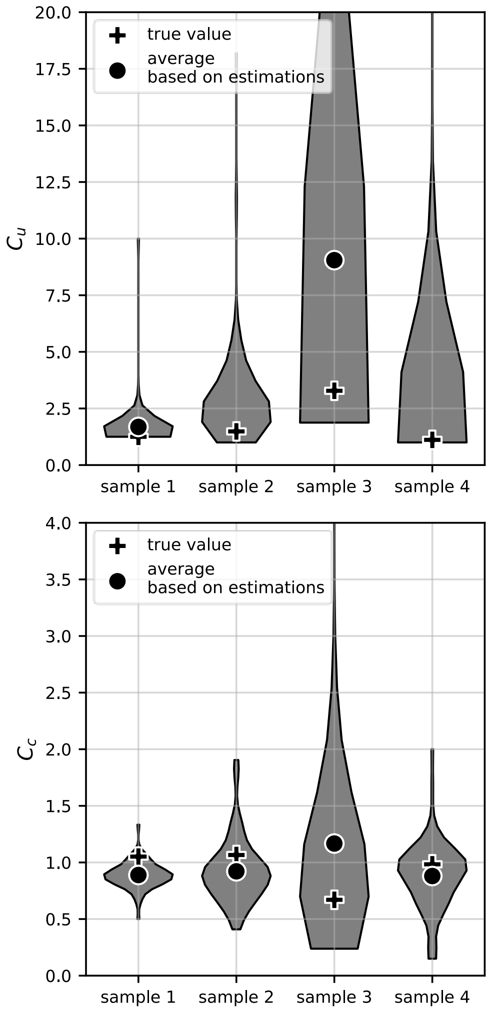


Figure A 7: Variability of C_u and C_c computed from the participants responses.

Table A 1: Ground truth values for the parameters of the survey.

Sample	D_{min} **[mm]**	D_{10} **[mm]**	D_{30} **[mm]**	D_{50} **[mm]**	D_{60} **[mm]**	D_{90} **[mm]**	D_{max} **[mm]**	C_u	C_c
1	40.0	45.7	52.3	55.5	57.0	64.5	68.6	1.2	1.1
2	1.3	3.0	3.8	4.2	4.4	5.2	5.7	1.5	1.1
3	2.3	11.2	16.6	29.9	36.8	51.0	54.2	3.3	0.7
4	2.5	2.6	2.8	2.9	2.9	27.6	34.4	1.1	1.0