

1 **Tornado damage ratings estimated with cumulative logistic regression***

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ABSTRACT

9 Empirical studies have led to improvements in evaluating and quantifying
10 the tornado threat. However more work is needed to put the research onto a
11 solid statistical foundation. Here the authors begin to build this foundation
12 by introducing and then demonstrating a statistical model to estimate dam-
13 age rating probabilities. A goal is to alert researchers to available statistical
14 technology for improving severe weather warnings. The model is cumulative
15 logistic regression and the parameters are determined using Bayesian infer-
16 ence. The model is demonstrated by estimating damage rating probabilities
17 from values of known environmental factors on days with many tornadoes in
18 the United States. Controlling for distance-to-nearest town/city, which serves
19 as a proxy variable for damage target density, the model quantifies the chance
20 that a particular tornado will be assigned any damage rating given specific
21 environmental conditions. Under otherwise average conditions the model es-
22 timates a 65% chance that a tornado occurring in a city or town will be rated
23 EF0 when bulk shear is weak (10 m s^{-1}). This probability drops to 38% when
24 the bulk shear is strong (40 m s^{-1}). The model quantifies the corresponding
25 *increases* in the chance of the same tornado receiving higher damage ratings.
26 Quantifying changes to the probability distribution on the ordered damage
27 rating categories is a natural application of cumulative logistic regression.

28 **1. Introduction**

29 Advances in evaluating and quantifying the tornado threat have recently been made. These ad-
30 vances come from a better understanding of relationships between near-storm regional-scale envi-
31 ronmental conditions and the resulting mode of convection [see Smith et al. (2012) and Thompson
32 et al. (2012) for a review of the literature on this topic] and from careful statistical analysis of
33 relationships between radar-based rotational signals at the storm scale and the probability of spe-
34 cific damage rating categories (Smith et al., 2015; Thompson et al., 2017). Cohen et al. (2018)
35 investigated multivariate models as a way to combine various environmental and storm-scale fac-
36 tors influencing the probability of specific enhanced Fujita (EF) ratings. While this approach is an
37 improvement over earlier bi-variate methods (e.g., box-and-whisker plots), more work is required
38 to put the research onto a solid statistical foundation.

39 The purpose of the present study is to introduce a statistical model to estimate a per-tornado
40 damage rating (and associated uncertainties) directly and to demonstrate features of the model
41 by using it to estimate damage ratings with environmental factors on days with many tornadoes.
42 The aim differs from earlier studies in that the sole focus is on methodology. The goal here is to
43 make researchers aware of modern statistical technology that can be leveraged to help them more
44 effectively improve severe weather prediction.

45 Mathematically the approach we take is similar to that outlined in Cohen et al. (2018), who fit
46 a linear regression to wind speeds corresponding to midpoints of the EF rating intervals. But our
47 approach differs in that we fit a cumulative logistic regression model to the recorded highest EF
48 rating directly. Statistically the approach we take is similar to the approach used in Thompson
49 et al. (2017) who estimated conditional empirical probabilities of EF ratings by binning various
50 indicators from WSR-88D radar. But our approach differs in that we use a multivariate model

51 and we include estimates of uncertainty on the model output. In short, our approach is unique in
52 that we use damage ratings directly as ordered categorical outcomes and we provide estimates of
53 uncertainty on estimated probabilities.

54 While our focus in this paper is solely methodological, the application might have some op-
55 erational relevance. This is because environmental ingredients needed to produce an outbreak of
56 severe convective weather are well known and can be leveraged to make predictions. Considerable
57 skill exists in outlining areas under greatest risk of severe weather on a given day. Outbreaks have
58 large variation in terms of tornado frequency and intensity with much of this variability resulting
59 from the convective mode (Smith et al., 2012; Thompson et al., 2012; Smith et al., 2015). Given
60 a forecast of a severe weather outbreak will conditions favor many violent tornadoes? Dynamical
61 models provide forecast guidance through products like updraft helicity swaths and models that
62 allow for convection can anticipate the convective mode to some degree. But statistical models
63 trained on thousands of tornadoes occurring across dozens of outbreaks can provide a baseline cli-
64 matology for this risk. The U.S. Storm Prediction Center (SPC) currently uses long-run frequency
65 of two or more tornadoes and long-run frequency of at least one strong (EF2–EF5) tornado as
66 climatology.

67 In the above sense the present paper is similar to a recent study that employs a model to estimate
68 the probability of at least one significant (EF2+) tornado on days with at least one tornado-warned
69 supercell (Togstad et al., 2011). But it differs in a couple of key ways. First, in demonstrating
70 the approach, we condition our model on the occurrence of a tornado ‘outbreak’ (at least ten
71 tornadoes occurring within a relatively small area) rather than on the occurrence of a tornado
72 warning. Second, we use cumulative frequency distribution by EF rating as the outcome variable
73 rather than relative frequency of at least one EF2+ tornado. The paper is outlined as follows. The

74 mathematics of cumulative logistic regression are given in §2. The data used to demonstrate the
 75 model is described in §3. Model results are presented in §4 and a summary is given in §5.

76 **2. Cumulative logistic regression**

77 The goal of this research is to put current knowledge about severe convective storms onto a solid
 78 statistical foundation. The purpose is to introduce cumulative logistic regression model as a way to
 79 estimate damage rating probabilities directly and to demonstrate its features by using it to estimate
 80 damage rating probabilities from large-scale environmental variables. We begin with a description
 81 of the model in the context of estimating damage ratings from environmental variables.

82 Let $\Pr(T_i \leq k)$ be the probability that tornado T_i has a maximum EF rating less than or equal to
 83 k , where $k = 0, \dots, 5$. Then the log-cumulative odds (cumulative logit) is defined as

$$\alpha_k = \log \frac{\Pr(T_i \leq k)}{1 - \Pr(T_i \leq k)}, \quad (1)$$

84 where α_k ('intercept' parameter) has a unique value for each EF rating. Note that the cumulative
 85 logit for the highest EF rating (EF5) is infinity since $\log(\frac{1}{1-1}) = \infty$. So for $K = 6$ possible EF
 86 ratings, we have $K - 1 = 5$ intercepts that need to be determined.

87 A rating EF_i is assigned to tornado T_i using an *Ordered* distribution, which is a categorical
 88 distribution that takes a vector of probabilities ($\mathbf{p} = \{p_0, p_1, p_2, p_3, p_4\}$) one for each EF rating
 89 below EF5. Each probability value p_k in the vector is defined by its link to the intercept parameter
 90 value α_k . To include a predictor variable in the model we define the log-cumulative odds as the
 91 sum of α_k and a linear model term ($\beta_j x_{ij}$), where x_{ij} is the value of a population-level or group-
 92 level variable j (e.g., distance-to-nearest-city/town for a population-level variable and month for
 93 a group-level variable) associated with tornado T_i and, where β_j is the coefficient (or coefficient
 94 vector for group-level variables) associated with that variable. We determine the α_k 's and β_j 's

95 using Bayesian inference so the model includes prior distributions on these parameters. We put a
96 flat normal distribution prior on the α_k 's and a flat student- t distribution prior on the β_j 's.

97 Following the notation of McElreath (2015), mathematically, we write the model as

$$\text{EF}_i \sim \text{Ordered}(\mathbf{p})$$

$$\text{logit}(p_k) = \alpha_k - \phi_i$$

$$\phi_i = \beta_1 x_{i1} + \beta_2 x_{i2} + \dots + \beta_J x_{iJ} \quad (2)$$

$$\alpha_k \sim \text{Normal}(0, 10)$$

$$\beta_j \sim \text{Student}_t(7, 0, 10)$$

98 The model gives the correct ordering of the EF ratings while allowing for changes in the likeli-
99 hood for each tornado based on associated environmental conditions and other factors. The nega-
100 tive sign ensures that as the log-cumulative odds of every EF rating below the highest *decreases*,
101 the probability mass shifts *upwards* toward higher EF ratings (McElreath, 2015).

102 3. Data

103 We illustrate the utility of cumulative logistic regression for estimating EF rating categories by
104 fitting the model to a set of data. The data consist of the outcome variable [highest (maximum) per-
105 tornado EF rating], predictor variables (environmental factors and distance to nearest city/town),
106 and grouping variables (month and cluster number). Data are filtered to include only tornadoes
107 occurring on days with at least ten tornadoes over the period 1994–2017 within the contiguous
108 United States. Here we describe the procedure that we used to organize the data and provide
109 summary statistics.

110 First we extract the date, time, genesis location, and maximum EF rating from the tornado record
111 obtained from the Storm Prediction Center. Each row in the record contains information about an
112 individual tornado. The start year of 1994 marks the beginning of extensive use of the WSR-

113 88D radar. There are 29,372 tornadoes over this period of record. We convert the geographic
114 coordinates of the genesis locations to a Lambert conformal conic projection centered on 107° W
115 longitude.

116 Next we assign a cluster number to each tornado based on space-time differences between gen-
117 esis locations. If two tornadoes occur close together in space and time, they are assigned the same
118 cluster number (see Fig. 1 for an example of a tornado cluster). Clustering stops when the differ-
119 ence between individual tornadoes and an existing cluster exceeds 50K seconds (~ 14 hours). The
120 differences have units of time because we divide the spatial distance by 15 m s^{-1} . Details of the
121 procedure along with a comparison to a subjective grouping are detailed in Schroder and Elsner
122 (2018). Finally, we filter the tornadoes to include only those occurring as part of clusters with at
123 least ten tornadoes within a single convective day (12 UTC to 12 UTC). This filtering results in
124 16,501 tornadoes in 742 clusters with the majority of the clusters occurring during April, May, and
125 June (Fig. 2).

126 Next we extract environmental variables from the North American Regional Reanalysis (NARR)
127 obtained from the National Center for Environmental Predictions (NCEP) North American Re-
128 gional Reanalysis (NARR) from the National Center for Atmospheric Research (NCAR) (National
129 Centers for Environmental Prediction). Variables are available on a 32.4 km grid and are a blend of
130 modelled and observed data. We use the files that contain environmental data for each day ranging
131 from 12 UTC to 12 UTC in three-hour increments. Variables considered include the 180 to 0 hPa
132 above ground level (AGL) CAPE and CIN (layer 375, 376), the 0 to 3000 m AGL helicity (layer
133 323), and the 0 to 6000 m AGL u and v components of storm motion (layer 324, 325). Addi-
134 tionally, we compute total storm motion as the square root of the sum of the velocity components
135 squared and bulk shear as the square root of the sum of the squared differences between the u and
136 v winds for the 1000 hPa and 500 hPa levels. We consider these variables because they are well

137 known to be associated with tornado activity (Cheng et al., 2016). For each tornado cluster, we
138 find the closest three-hour time *before* the appearance of the first tornado in the cluster and use the
139 environmental variables from that time. We pick a time before the event starts to have a sample
140 of conditions prior to the appearance of any tornado. We join the environmental variables at the
141 cluster level with the data at the tornado level.

142 Finally, for each tornado, we compute the distance between the genesis location and the nearest
143 city/town. Population values are based on the 2010 U.S. Census data [obtained from Steiner (2019)
144 and accessed through the `USAboundaries` package (Mullen and Bratt, 2018) in R] and range from
145 a few hundred people to more the eight million people. The distance between a tornado and the
146 nearest city/town serves as a proxy for the potential number of damage targets. All else being
147 equal a tornado occurring within a city or town will have a greater opportunity to impact a damage
148 target, on average, than one that occurs in a rural area. With distance-to-nearest-city/town as a
149 predictor variable we are able to quantify how the potential number of damage targets shifts the
150 distributions for example from EF0 to EF1 and from EF1 to EF2. We know that the chance of
151 getting an EF4+ tornado in the data set increases with the number of targets, but we don't know
152 by how much relative to an EF3. Table 1 list the predictor variables along with the associated
153 extremes and average values. Averages are computed over all tornadoes. We remove tornadoes
154 occurring during the May 30, 2003 cluster since the maximum helicity value for this cluster had
155 an erroneously high value.

156 4. Results

157 We begin with a histogram of maximum EF rating per tornado (highest rating given to the
158 tornado as recorded in the SPC data). As expected, the histogram (Fig. 3) shows that the vast
159 majority of tornadoes that occur as part of a big cluster are rated EF0 or EF1 with far fewer rated

160 EF4 or EF5. But relative to all tornadoes the distribution of tornadoes in big clusters favors higher
161 ratings. For example, 3.5% of tornadoes occurring in big clusters are rated EF3 compared to 2.3%
162 of all tornadoes. And .08% of tornadoes occurring in big clusters are rated EF5 compared to .05%
163 of all tornadoes.

164 Next, we describe this histogram on the log-cumulative-odds scale by constructing the odds of a
165 cumulative probability and then taking logarithms. The logit function is the logarithm of the odds
166 (log-odds) so the cumulative logit is log-cumulative-odds. Both the logit and the cumulative logit
167 constrain the probabilities to lie in the interval between 0 and 1. Predictor variables are added
168 on the cumulative logit scale (Eq. 2). The link function takes care of converting the parameter
169 estimates on these variables to the proper probability scale (McElreath, 2015). We compute the
170 cumulative probabilities from the histogram, which are the discrete proportions of tornadoes by
171 each EF rating. We then compute the series of intercept parameters to re-describe the histogram
172 in terms of log-cumulative odds (Eq. 1). Each intercept is on the log-cumulative-odds scale and
173 stands in for the cumulative probability associated with each EF rating (Fig. 4). The discrete
174 probability for each EF rating $\Pr(T_i = k)$ is the successive difference between the elements of the
175 vector of cumulative probabilities. These probabilities are the likelihoods that are conditioned on
176 the values of the predictor variables and combined with the priors to complete the model (Eq. 2).

177 Posterior distributions on the model parameters are obtained using the Stan computational en-
178 gine (Carpenter et al., 2017) accessed through the brms package (Bürkner, 2017). Mildly informa-
179 tive conservative priors are specified to improve convergence of the sampler and to guard against
180 over-fitting. To improve the efficiency of the sampler, predictor variables are scaled by subtracting
181 their respective means and dividing by their respective standard deviations. The environmental
182 variables and year are included as population-level effects (fixed effects). The month of the cluster
183 and the unique cluster identification number are included as group-level effects (random effects).

184 The model reproduces the distribution of tornadoes by EF rating category as expected (Table 2).
185 It slightly under estimates the proportion of EF0 tornadoes and slightly over estimates the number
186 of EF1 tornadoes and EF3 tornadoes, but overall the proportions from the model match the data
187 very well. Signs on the fixed-effect coefficients (Table 3) are consistent with expectations based on
188 physical reasoning derived from the current understanding of how environmental factors influence
189 tornado activity (Smith et al., 2012; Thompson et al., 2017). The coefficient on cluster year is
190 positive indicating a trend toward higher rated tornadoes as discussed in Elsner et al. (2018). The
191 coefficient on distance-to-nearest-city/town is negative as expected. The closer a tornado occurs
192 to a city/town, the greater the chance it will get rated at the next higher EF rating relative to the
193 same tornado occurring in a rural area. The largest effect occurs with bulk shear. The sign on the
194 coefficient indicates that greater shear results in a better chance of a higher EF rating as we would
195 expect from physical reasoning.

196 Coefficients on the fixed effects and on the month random effect are plotted in Fig. 5. Magnitude
197 of the departure from zero indicates the importance of the variable to the model for estimating
198 damage ratings as discussed above. The monthly variation in the distribution of tornadoes by EF
199 rating is an important model component with May and June having a significantly lower proportion
200 of most damaging tornadoes after accounting for the fixed effects. January and November have a
201 larger than average proportion of most damaging tornadoes.

202 To get an idea how much a particular variable statistically influences the distribution of EF rat-
203 ings while holding the other variables constant we examine marginal effects (Fig. 6). A variable's
204 marginal effect is computed by holding the other variables at their respective mean values. Con-
205 sider the marginal effect of bulk shear. For tornadoes occurring in environments of low shear (less
206 than 10 m s^{-1}) the model estimates the probability that a tornado gets rated EF0 at nearly 75%.
207 This probability drops to 40% for tornadoes occurring in environments of high shear (greater than

208 40 m s⁻¹). There are compensating increases in the chance of EF1 and higher ratings across the
209 range of bulk shear values. Further, we see that CAPE and helicity have less of an effect on the
210 probability distribution of EF ratings compared to bulk shear (posterior means on the respective
211 coefficients are farther from the zero line). We also quantify the trend toward higher EF ratings
212 and the relative changes over time depending on where the tornado occurs (near a city/town or
213 outside a city/town; Fig. 7).

214 Importantly we can use the model to get an estimate of the probability distributions for any
215 *particular* set of predictor values. Since the model uses Bayesian inference, we get *posterior*
216 *predictive samples* of the EF probability distribution for any set of values. As an example, we
217 show the posterior predictive samples across a range of bulk shear values setting the variables to
218 their respective averages except distance-to-nearest-city/town, which we set to zero (Fig. 8). Bulk
219 shear is illustrated because it has the largest influence on the outcome (distribution of EF ratings)
220 as noted above. Individual samples (100 of them) of the cumulative proportion of tornadoes for
221 different EF ratings are shown.

222 When bulk shear is 10 m s⁻¹ the posterior mean relative percentage of an EF0 tornado is 65%
223 [(56%, 79%), interquartile range (IQR)] but when bulk shear is 40 m s⁻¹ the posterior mean
224 relative percentage of an EF0 tornado drops to 38% [(25%, 50%), IQR]. This decrease in percent
225 is compensated by increases in the relative percentage of tornadoes rated higher. For example,
226 when bulk shear is 10 m s⁻¹ the posterior mean relative percentage of an EF3 tornado is 2.0%
227 [(0.9%, 2.5%), IQR] but when bulk shear is 40 m s⁻¹ the posterior mean relative percentage of an
228 EF3 tornado rises to 6.1% [(2.9%, 8.2%), IQR]. This quantification of the effect of bulk shear on
229 EF ratings is possible with a cumulative logistic regression model.

230 We can use the model in a similar way to quantify a well-known (but not well quantified) EF
231 rating bias. We find that under average environmental conditions when a tornado occurs near the

232 center of a city or town 47% [(31%, 59%), IQR] of the time it will get rated EF0. This compares
233 with 56% [(41%, 70%), IQR] of the time when the same tornado occurs 50 km from the center.
234 This increase in the percentage of EF0 tornadoes going from city to rural areas is compensated
235 by corresponding decreases in percentages of tornadoes getting rated higher. For example, the
236 chance that a tornado gets rated as EF3 or higher is 5% in the city compared with 3.6% at a
237 distance of 50 km from the city/town and only 2.4% at a distance of 100 km from the city/town.
238 This quantification of an EF rating bias is possible with a cumulative logistic model. By including
239 an interaction between year and distance to nearest city/town in the model we determine that this
240 bias is not diminishing over time. This differs from the decreasing population bias on the tornado
241 reports as documented and quantified elsewhere (Elsner et al., 2013; Jagger et al., 2015).

242 **5. Summary**

243 We introduced the cumulative logistic regression model to estimate damage rating probabilities
244 directly and we demonstrated features of the model by using it to estimate probabilities from en-
245 vironmental variables for tornadoes occurring in large clusters (ten or more tornadoes). Model
246 parameters were determined by Bayesian inference using the method of Hamiltonian Monte Carlo
247 with the Stan language. Stan code was generated from R through the brms package. The flexibility
248 of this approach makes it straight forward to adjust the model to estimate other outbreak charac-
249 teristics (e.g., overall number of tornadoes) and to include domain-specific knowledge. Results
250 show that the chance of higher damage ratings can be explained statistically by increasing values
251 of bulk shear, CAPE, and helicity by decreasing values of distance to nearest city/town.

252 Coefficients on the environmental variables are consistent with expectations based on physical
253 reasoning derived from the current understanding of how environmental factors influence tornado
254 activity. There is a trend toward higher rated tornadoes with time as inferred in Elsner et al.

255 (2018). The closer a tornado occurs to a city/town, the greater the chance it will get rated at the
256 next higher EF rating. Bulk shear has the strongest relationship to damage rating proportions.
257 Under otherwise average conditions, the model estimates a 65% [(53%, 78%), IQR] chance that
258 any tornado occurring near a city or town will be rated EF0 when the bulk shear is weak (10 m s^{-1}).
259 This probability drops to 38% [(26%, 50%), IQR] when the bulk shear is strong (40 m s^{-1}) but
260 with compensating *increases* in the chance of *higher* ratings. This quantification is only possible
261 with a cumulative logistic regression.

262 This study makes the case that cumulative logistic regression is the right tool for quantifying the
263 combined role environmental factors play on the distribution of tornadoes by EF rating. It might be
264 tempting to fit a simpler model to these data as was done in Cohen et al. (2018) who suggested that
265 simulated tornado wind speeds from their model can be scaled within the context of the damage
266 ratings. But it is unclear how this can be done while preserving the relative frequency of ratings
267 given that the model residuals are assumed to be described by a normal distribution centered about
268 the conditional mean wind speed. Cumulative logistic regression makes no such assumption and
269 estimates probabilities directly.

270 Finally, although the results from applying the model for demonstration purposes are consistent
271 with past research on this topic, there are limitations to the inferences that can be made with them.
272 In particular, our exclusive focus on days with at least ten tornadoes is a type of selection bias
273 meaning that the sample of data used to fit the model does not represent the population of all
274 tornadoes, which limits what we can say in general about the effect of convective environments
275 on the probability of a particular EF rating. Further, no attempt was made to assess model skill
276 in the context of its potential value in actual forecast situations. At a minimum a cross-validation
277 exercise [see Elsner and Schmertmann (1994)] would be needed.

278 **Acknowledgments**

279 The code and data to produce all the figures and results of this paper are available at [https:](https://github.com/jelsner/EF-dist)
280 [//github.com/jelsner/EF-dist](https://github.com/jelsner/EF-dist).

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333 estimated proportions are from a cumulative logistic regression model. 20

334 **Table 3.** Estimated coefficients on the population-level effects. Abbreviations in the sub-
335 scripts refer to the variables listed in Table 1 21

Variable Name	Abbreviation	Minimum	Maximum	Average
Year	YR	1994	2017	2006
Nearest distance to city/town (km)	D	.019	137	19.2
Convective available potential energy (J kg^{-1})	CAPE	0	6530	2134
Helicity ($\text{m}^2 \text{s}^{-2}$)	HLCY	23	1027	350
Bulk shear (m s^{-1})	BS	5.7	45.5	28.5
Convective inhibition (J kg^{-1})	CIN	-651	0	-176

336 TABLE 1. Variables used in the model to estimate damage ratings. The values are based on 16,483 tornadoes
337 in 741 clusters.

Damage Rating	Observed	Estimated
EF0	.5325	.5220
EF1	.3183	.3489
EF2	.1057	.0945
EF3	.0348	.0279
EF4	.0079	.0061
EF5	.0008	.0006

338 TABLE 2. Observed and estimated proportions of tornadoes by EF damage rating. The estimated proportions
339 are from a cumulative logistic regression model.

Coefficient	Estimate	Error	95% UI
β_{YR}	0.10	0.03	(0.03, 0.16)
β_D	-0.13	0.02	(-0.17, -0.09)
β_{CAPE}	0.10	0.04	(0.02, 0.18)
β_{HLCY}	0.12	0.05	(0.03, 0.21)
β_{BS}	0.28	0.04	(0.20, 0.38)

340 TABLE 3. Estimated coefficients on the population-level effects. Abbreviations in the subscripts refer to the
341 variables listed in Table 1

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361 distance-to-nearest-city/town whose value is set to zero. 30

FIG. 1. Tornado locations (origin) during one tornado cluster used in this study.

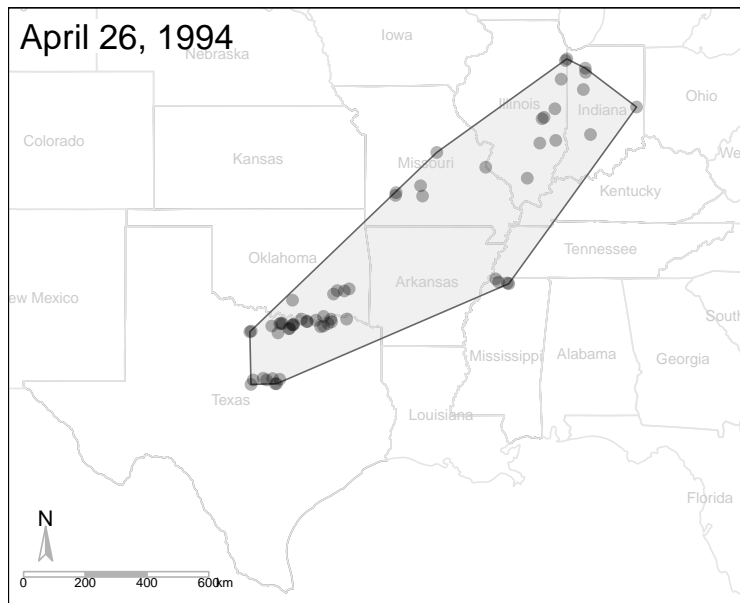
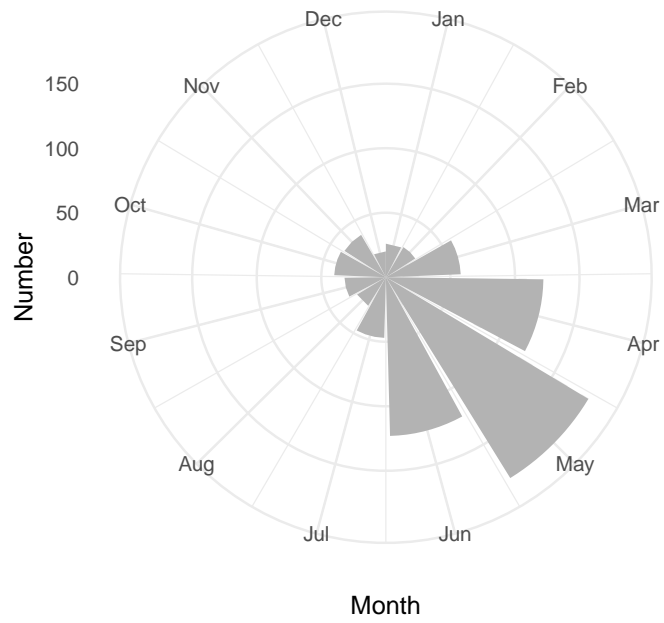
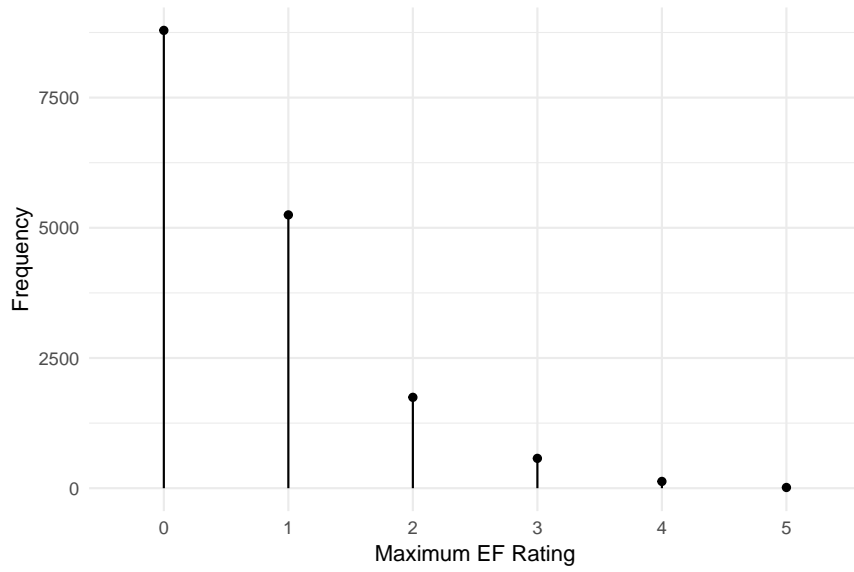


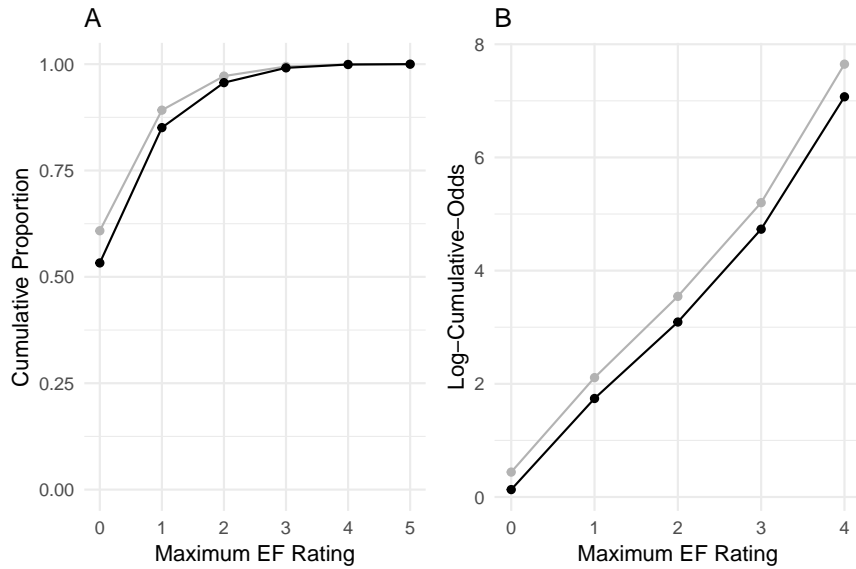
FIG. 2. Monthly frequency of tornado clusters (convective days with at least ten tornadoes), 1994–2017.



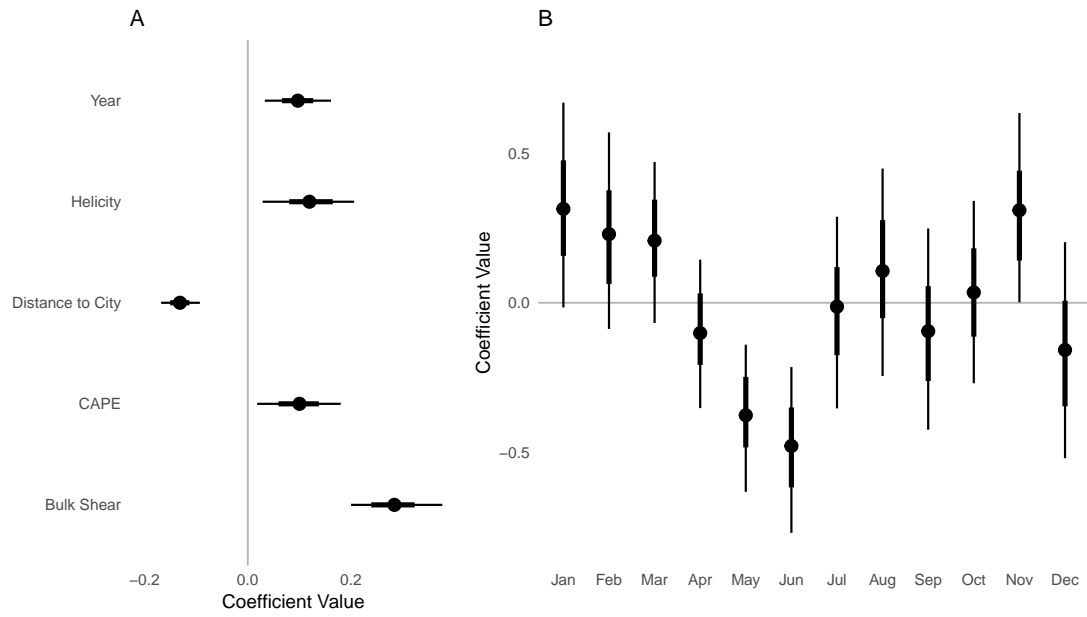
362 FIG. 3. Histogram of tornadoes by maximum EF rating. Only tornadoes occurring in big clusters are consid-
363 ered (see text).



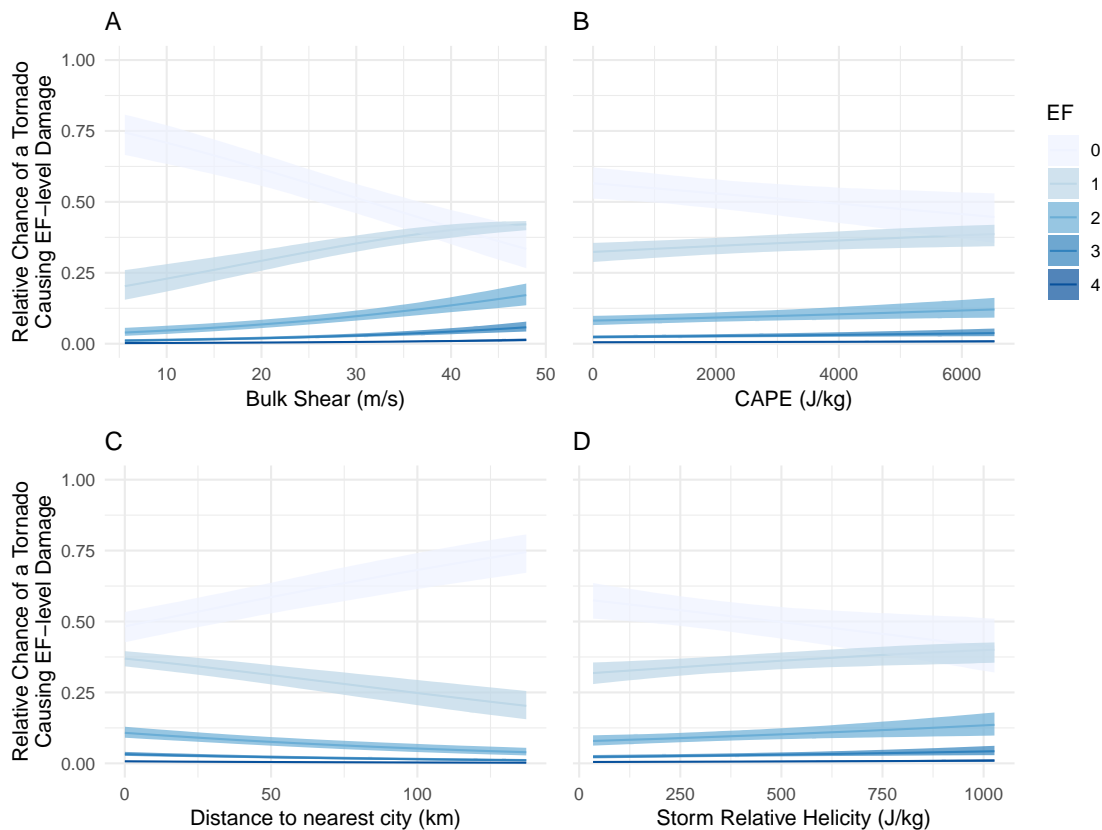
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365 tornadoes (gray) and for tornadoes occurring in big clusters (black). Note the cumulative logit for the EF5 rating
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