A new unit Weibull distribution

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***Abstract*: A new 2 parameter unit Weibull distribution is defined on the unit interval (0,1). The methodology of deducing its PDF, some of its properties and related functions are discussed. The paper is supplied by many figures illustrating the new distribution and how this can make it illegible to fit a wide range of skewed data.**

***Keywords*:**

Median Based Unit Weibull (MBUW) distribution, new distribution, unit distribution.

Introduction

Waloddi Weibull(1951) was the first to introduce the Weibull distribution. It is one of the famous distributions used to model life data and reliability. It can describe the increasing failure rate cases as well as the decreasing failure rate cases. The exponential distribution is a special case of it, when the shape parameter is one. Rayleigh distribution is another special case of it, when the shape parameter is 2 . It can also describe and explain the life expectancy of the elements entailed in the fatigue derived failure and can also evaluate the electron tube reliability and load handling machines. It is used in many fields like medicine, physics, engineering, biology, and quality control. As the distribution does not represent a bathtub or unimodal shapes, this enforces many researchers to generalize and transform this distribution in the recent decades. To mention some of these researchers:, Singla et al. (2012) elucidated beta generalized weibull, Khan et al. (2017) described in details the transmuted weibull, Xie et al. (2002) explored the modified weibull, Lee et al.(2007) clearly explained the beta weibull, Corderio et al. (2010) demonstrated Kumaraswamy weibull, Silva et al. (2010) expounded beta modified weibull, Mudhokar and Srivastav (1993) expatiated the exponentiated weibull, Zhang and Xie (2011) interpreted truncated weibull, Khan and King (2013) explicated transmuted modified weibull, and Marshall and Olkin (1997) handled the extended weibull.

Many distributions were defined on unit interval by many authors. Some of these distributions are:

1) Johnson S_B distribution (Johnson, 1949).
2) Beta distribution (Eugene et al., 2002).
3) Unit Johnson (S_U) distribution (Gündüz & Korkmaz, 2020).
4) Topp- Leone distribution (Topp & Leone, 1955).
5) Unit Gamma (Consul & Jain, 1971; Grassia, 1977; Mazucheli et al., 2018; Tadikamalla, 1981).
6) Unit Logistic distribution (Tadikamalla & Johnson, 1982).
7) Kumaraswamy distribution (Kumaraswamy, 1980).
8) Unit Burr-III (Modi & Gill, 2020).
9) Unit modified Burr-III (Haq et al., 2023).
10) Unit Burr-XII (Korkmaz & Chesneau, 2021).
11) Unit-Gompertz (Mazucheli, Maringa, et al., 2019).
12) Unit-Lindely (Mazucheli, Menezes, et al., 2019).
13) Unit-Weibull (Mazucheli et al., 2020).
14) Unit Muth distribution (Maya et al., 2024).

Mazucheli et al. (2018) proposed unit Weibull distribution to describe data on the unit interval like the real data he used describing maximum flood level and Petroleum reservoirs data. He proposed a quantile regression model for this unit weibull distribution and found that it sueprexceeded the competing distributions like the beta, Kumaraswamy, Unit Logistic, Simplex, Unit Gamma, Exponentiated Topp-Leone and Extended Arcsine. It has a closed form of the quantile function.

In this paper, another methodology is used to describe a new unit weibull distribution relying on the pdf of the median order statistics of a sample size n=3. The author will discuss the new unit 2 parameter distribution Median based unit Weibull (MBUW) and some of its basic properties.

The paper is arranged into 2 sections. In section 1, the author will explain the methodology of obtaining the new distribution. In section 2, elaboration of its PDF, CDF, Survival function, Hazard function and reversed hazard function will be presented.

Section 1

Methodology:

Derivation of the MBUW Distribution:

Using the pdf of median order statistics of a sample size=3 and parent distribution Weibull, both the scale parameter alpha and shape parameter beta are positive.

$$f_{i:n}(x) = \frac{n!}{(i-1)!\,(n-i)!}\{F(x)\}^{i-1}\{1-F(x)\}^{n-i}f(x), \qquad x > 0$$

$$f_{2:3}(x) = \frac{3!}{(2-1)!\,(3-1)!}\{F(x)\}^{2-1}\{1-F(x)\}^{3-2}f(x), \qquad x>0$$

$$F(x) = 1-e^{-\left(\frac{x}{\alpha}\right)^{\beta}}\ ,\ f(x) = \frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} e^{-\left(\frac{x}{\alpha}\right)^{\beta}}\ , x>0, \qquad \alpha\ \&\ \beta>0$$

$$f_{2:3}(x) = 3!\left\{1-e^{\frac{-x^{\beta}}{\alpha^{\beta}}}\right\}^{2-1}\left\{e^{\frac{-x^{\beta}}{\alpha^{\beta}}}\right\}^{3-2}\frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} e^{-\left(\frac{x}{\alpha}\right)^{\beta}}\ , x>0$$

$$f_{2:3}(x) = \frac{6\ \beta x^{\beta-1}}{\alpha^{\beta}}\left[1-e^{\frac{-x^{\beta}}{\alpha^{\beta}}}\right]\left[e^{\frac{-2x^{\beta}}{\alpha^{\beta}}}\right], \qquad x>0\,, \qquad \alpha\ \&\ \beta>0$$

$$f_{2:3}(x) = \frac{6\ \beta x^{\beta-1}}{\alpha^{\beta}}\left[1-e^{\frac{-x^{\beta}}{\alpha^{\beta}}}\right]\left[e^{\frac{-2x^{\beta}}{\alpha^{\beta}}}\right], \qquad x>0\ , \qquad \alpha\ \&\ \beta>0$$

Using the following transformation:

$$let\ \ y = \ e^{-x^{\beta}}$$

$$-ln(y) \ = \ x^{\beta}$$

$$[-ln(y)]^{\frac{1}{\beta}} \ = \ x$$

$$\frac{dx}{dy} = \frac{1}{\beta}[-ln(y)]^{\frac{1-\beta}{\beta}}\left(\frac{-1}{y}\right)$$

So the new distribution is the Median Based Unit Weibull (MBUW) Distribution.

Section 2

Some of the properties of the new distribution (MBUW):

1- The following is the pdf :

$$f(y) = \frac{6}{\alpha^{\beta}}\left[1 - y^{\frac{1}{\alpha^{\beta}}}\right] y^{\left(\frac{2}{\alpha^{\beta}}-1\right)} \ , \ 0 < y < 1 \, , \qquad \alpha > 0, \qquad \beta > 0$$

2- The following is the CDF:

$$F(y) = 3y^{\frac{2}{\alpha^{\beta}}} - 2y^{\frac{3}{\alpha^{\beta}}} \ , \ 0 < y < 1 \, , \qquad \alpha > 0, \qquad \beta > 0$$

3- The following is the survival function :

$$S(y) = 1 - F(Y) = 1 - \left(3y^{\frac{2}{\alpha^{\beta}}} - 2y^{\frac{3}{\alpha^{\beta}}}\right) \ , \ 0 < y < 1 \ , \alpha > 0 \, , \beta > 0$$

4- The following is the hazard function (hf) and reversed hazard function (rhf) respectively:

$$h(y) = \frac{f(y)}{S(y)} = \frac{\frac{6}{\alpha^{\beta}}\left(1 - y^{\frac{1}{\alpha^{\beta}}}\right) y^{\left(\frac{2}{\alpha^{\beta}}-1\right)}}{1 - \left(3y^{\frac{2}{\alpha^{\beta}}} - 2y^{\frac{3}{\alpha^{\beta}}}\right)} \ , \ 0 < y < 1 \ , \alpha > 0, \beta > 0$$

$$rh(y) = \frac{f(y)}{F(y)} = \frac{\frac{6}{\alpha^{\beta}}\left(1 - y^{\frac{1}{\alpha^{\beta}}}\right) y^{\left(\frac{2}{\alpha^{\beta}}-1\right)}}{3y^{\frac{2}{\alpha^{\beta}}} - 2y^{\frac{3}{\alpha^{\beta}}}} \ , \ 0 < y < 1 \ , \alpha > 0, \beta > 0$$

The following figures, Fig (1-4), show the PDF for different values of alpha (0.5 , 1, 1.5, 2 , 2.5 , 3 , 3.5 , 4) and beta (0.1 , 0.6 , 1.1 . 3.5):

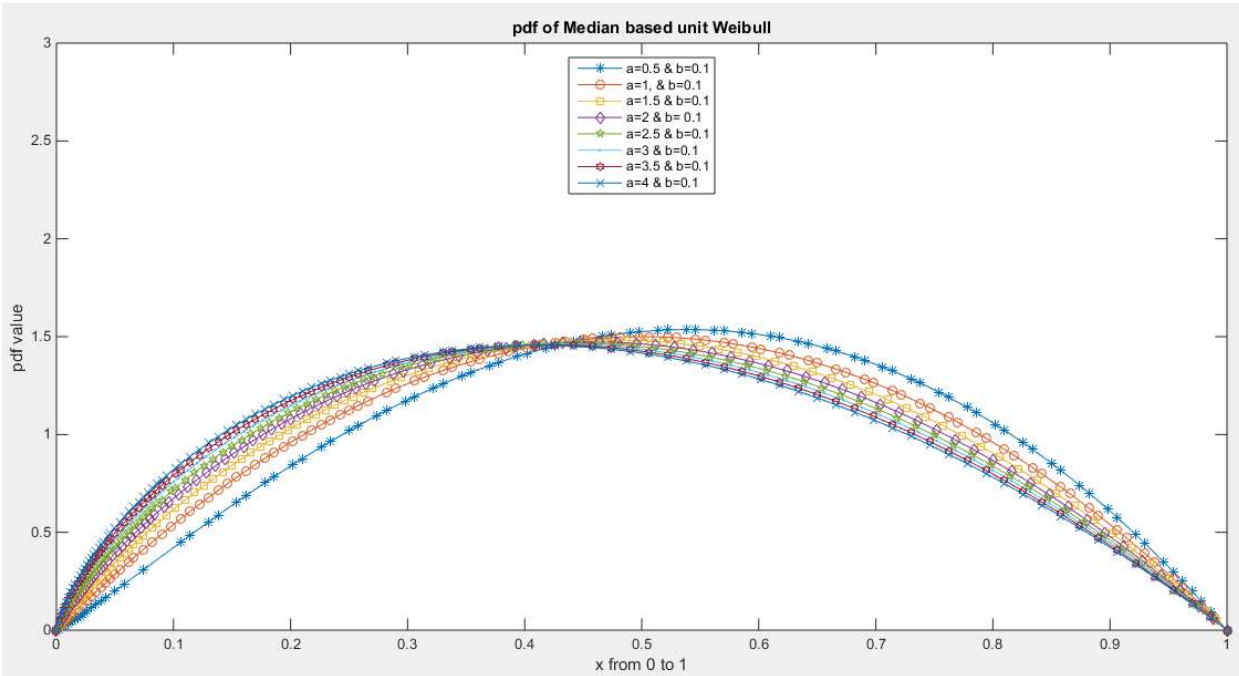

Fig. 1: pdf of Median Based Unit Weibull (MBUW) distribution, alpha (0.5 , 1, 1.5, 2 , 2.5 , 3 , 3.5 , 4) and beta (0.1) .

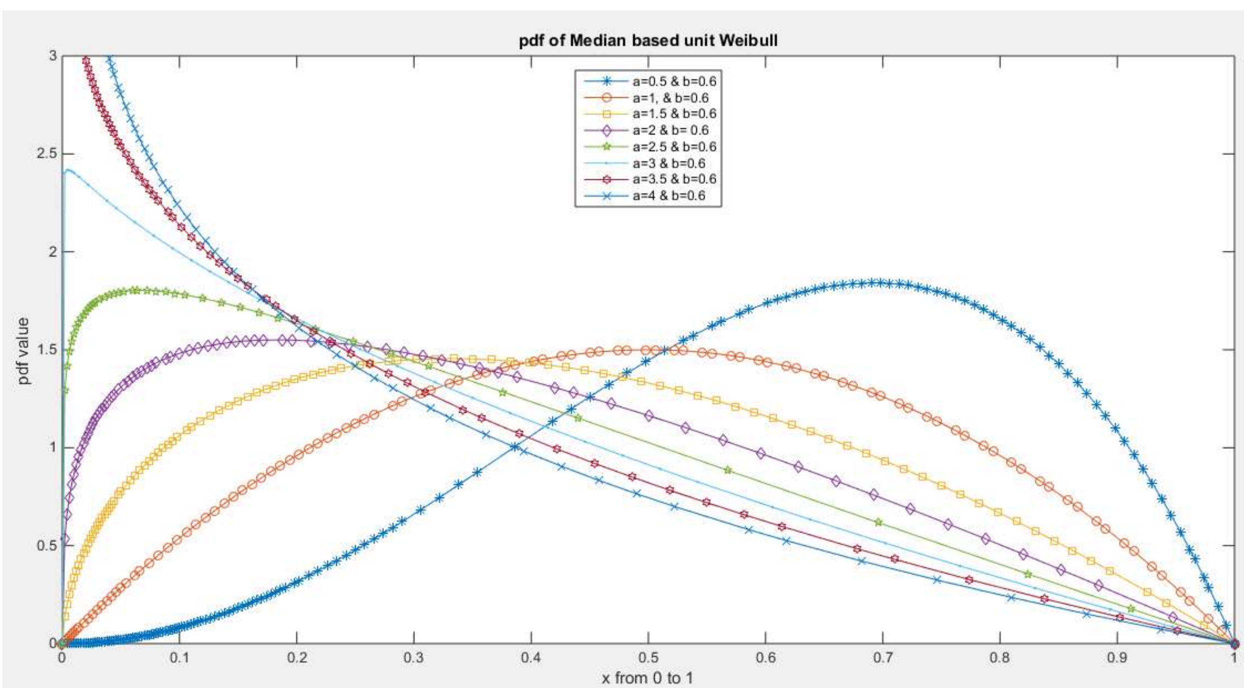

Fig. 2: pdf of Median Based Unit Weibull (MBUW) distribution, alpha (0.5 , 1, 1.5, 2 , 2.5 , 3 , 3.5 , 4) and beta (0.6)

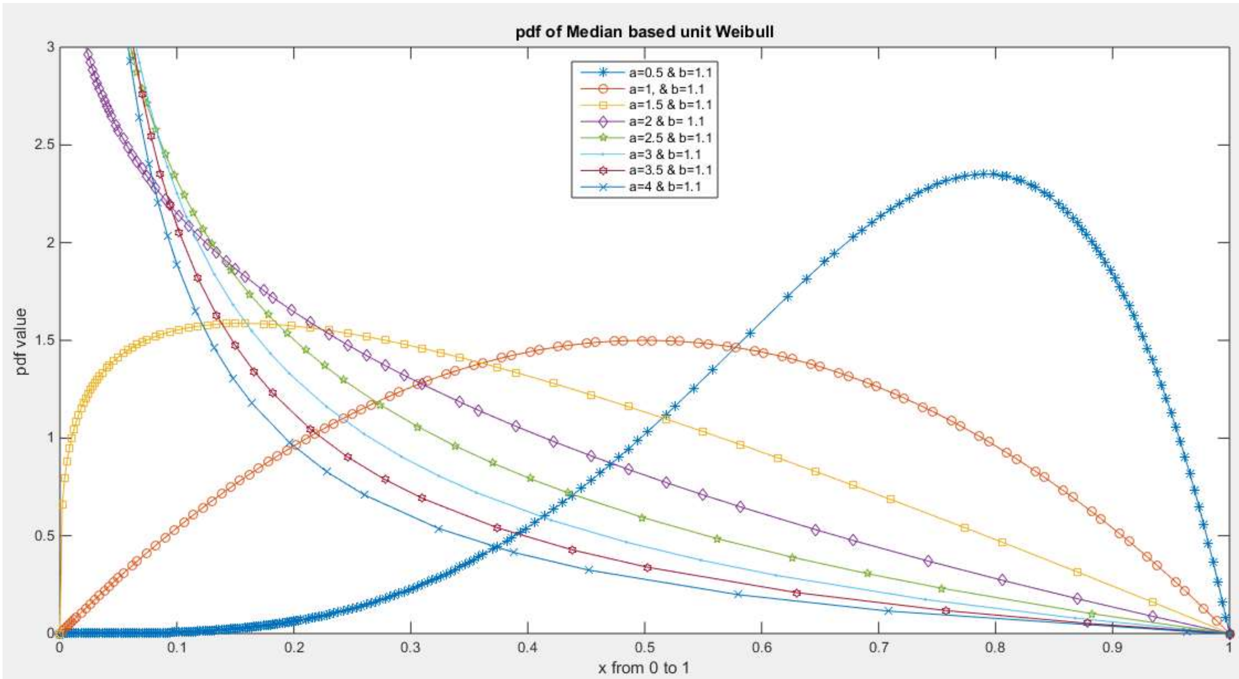

Fig. 3: pdf of Median Based Unit Weibull (MBUW) distribution, alpha (0.5 , 1, 1.5, 2 , 2.5 , 3 , 3.5 , 4) and beta (1.1)

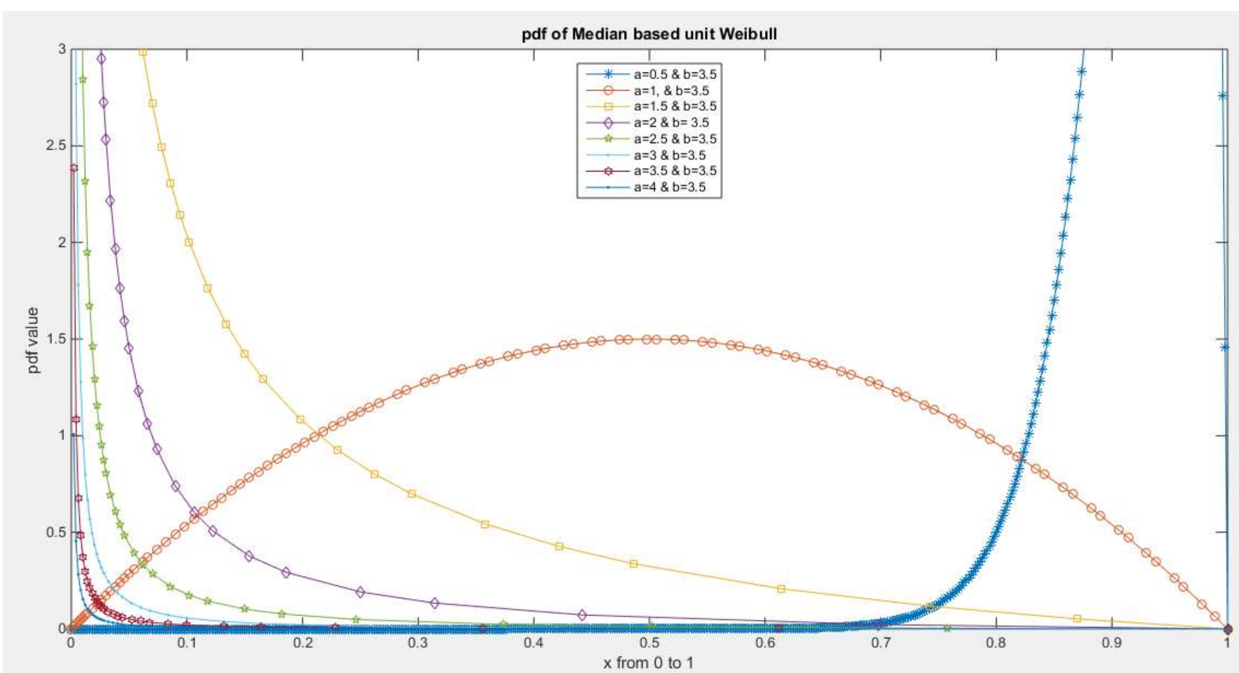

Fig. 4: pdf of Median Based Unit Weibull (MBUW) distribution, alpha (0.5 , 1, 1.5, 2 , 2.5 , 3 , 3.5 , 4) and beta (3.5)

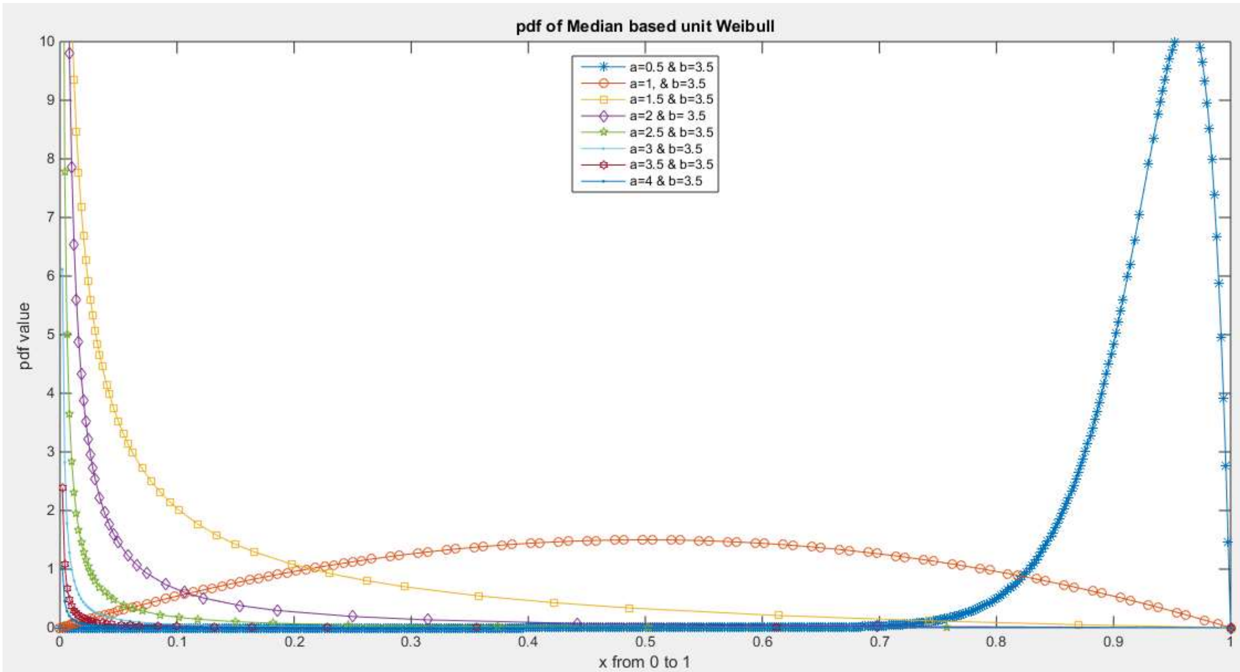

Fig. 5: pdf of Median Based Unit Weibull (MBUW) distribution, alpha (0.5 , 1, 1.5, 2 , 2.5 , 3 , 3.5 , 4) and beta (3.5), changing vertical scale.

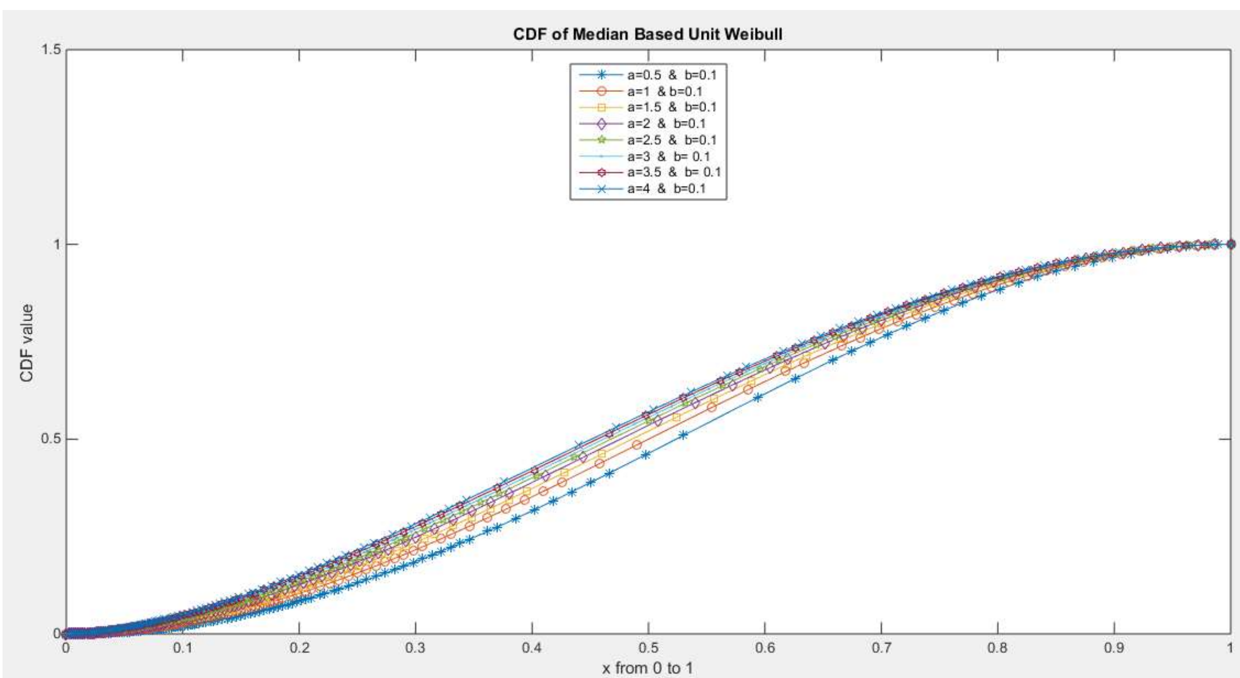

Fig. 6: cdf of Median Based Unit Weibull (MBUW) distribution, alpha (0.5 , 1, 1.5, 2 , 2.5 , 3 , 3.5 , 4) and beta (0.1).

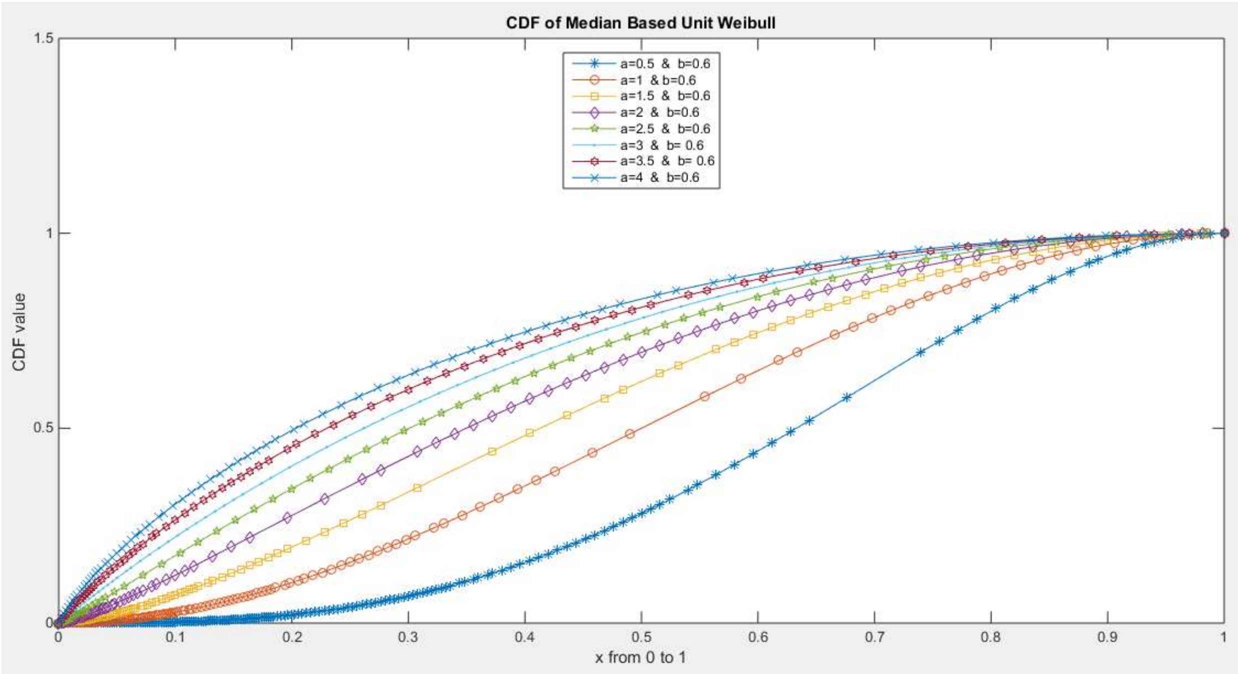

Fig. 7: cdf of Median Based Unit Weibull (MBUW) distribution, alpha (0.5 , 1, 1.5, 2 , 2.5 , 3 , 3.5 , 4) and beta (0.6).

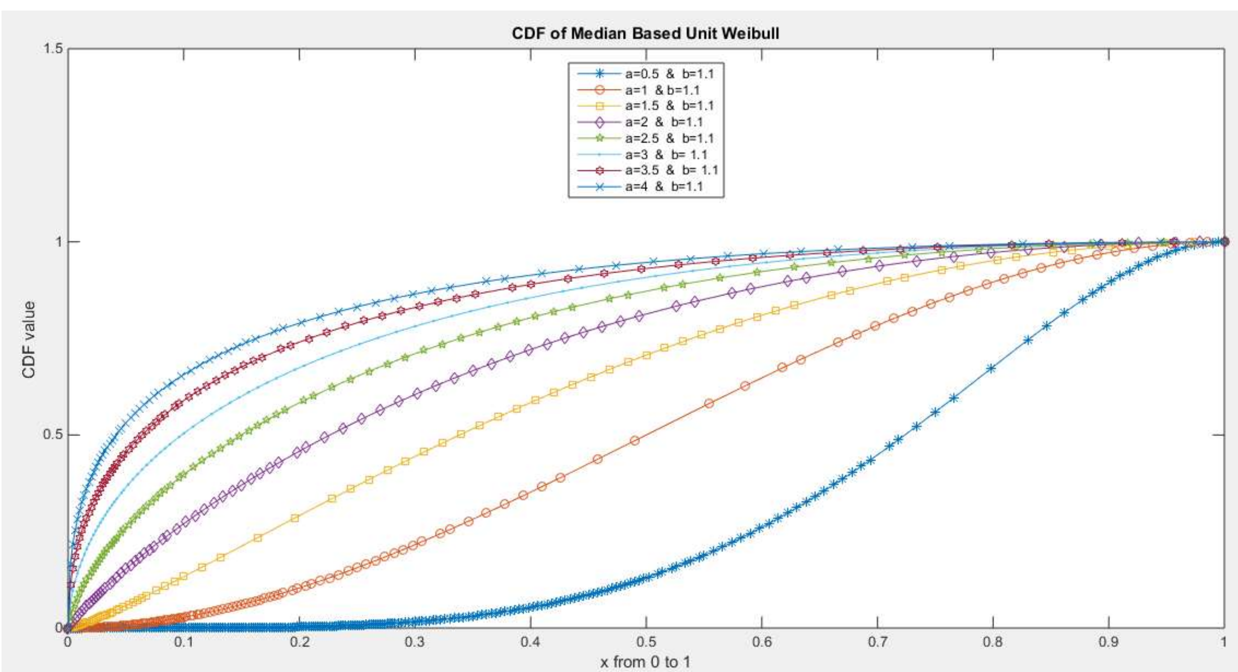

Fig. 8: cdf of Median Based Unit Weibull (MBUW) distribution, alpha (0.5 , 1, 1.5, 2 , 2.5 , 3 , 3.5 , 4)

and beta (1.1).

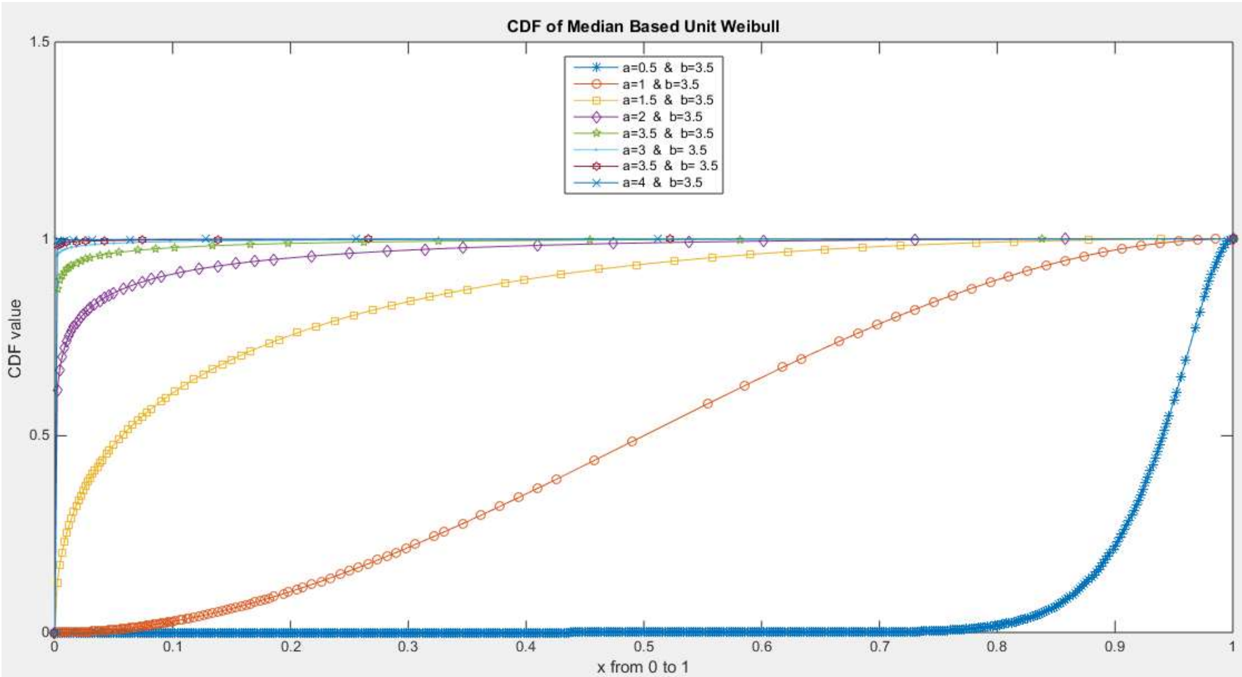

Fig. 9: cdf of Median Based Unit Weibull (MBUW) distribution, alpha (0.5 , 1, 1.5, 2 , 2.5 , 3 , 3.5 , 4) and beta (3.5).

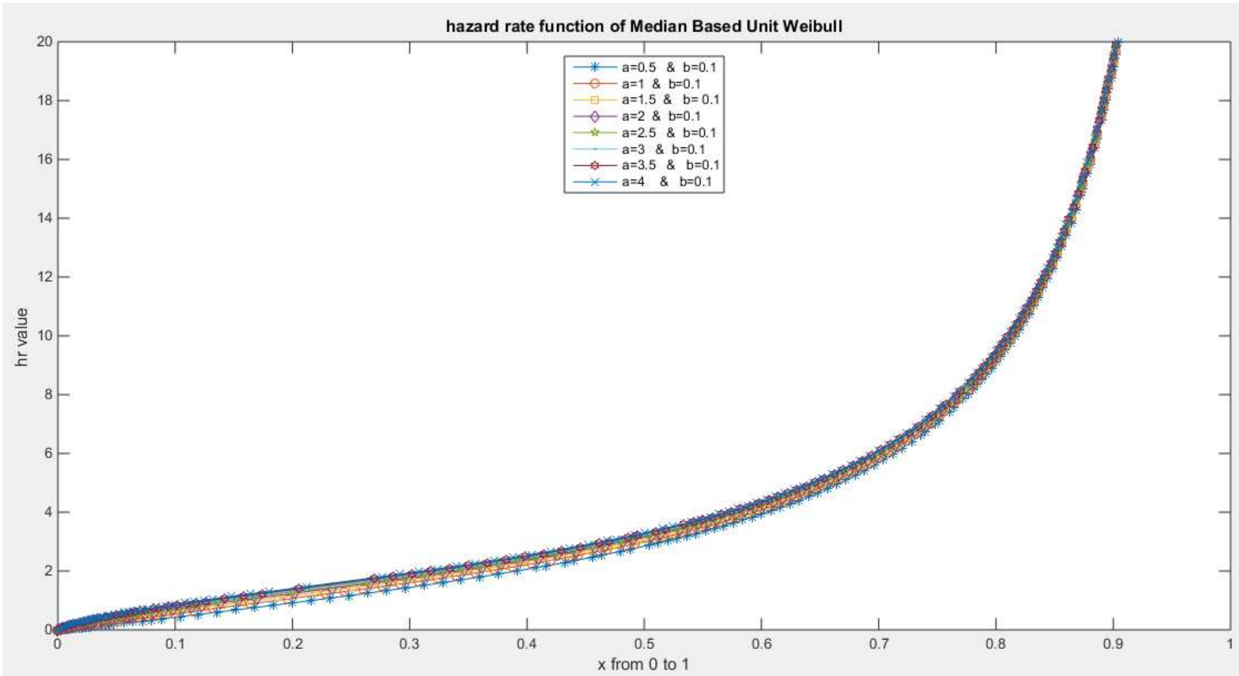

Fig. 10: hazard rate of Median Based Unit Weibull (MBUW) distribution, alpha (0.5 , 1, 1.5, 2 , 2.5 , 3 , 3.5 , 4) and beta (0.1).

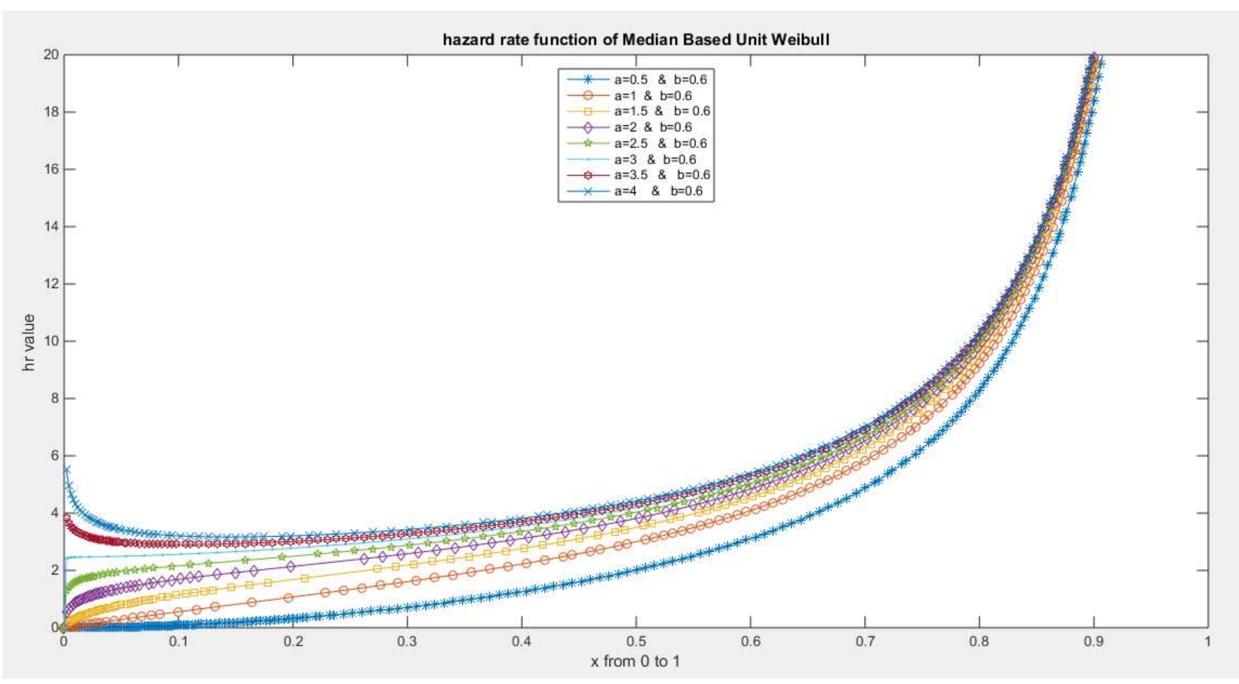

Fig. 11: hazard rate of Median Based Unit Weibull (MBUW) distribution, alpha (0.5 , 1, 1.5, 2 , 2.5 , 3 , 3.5 , 4) and beta (0.6)

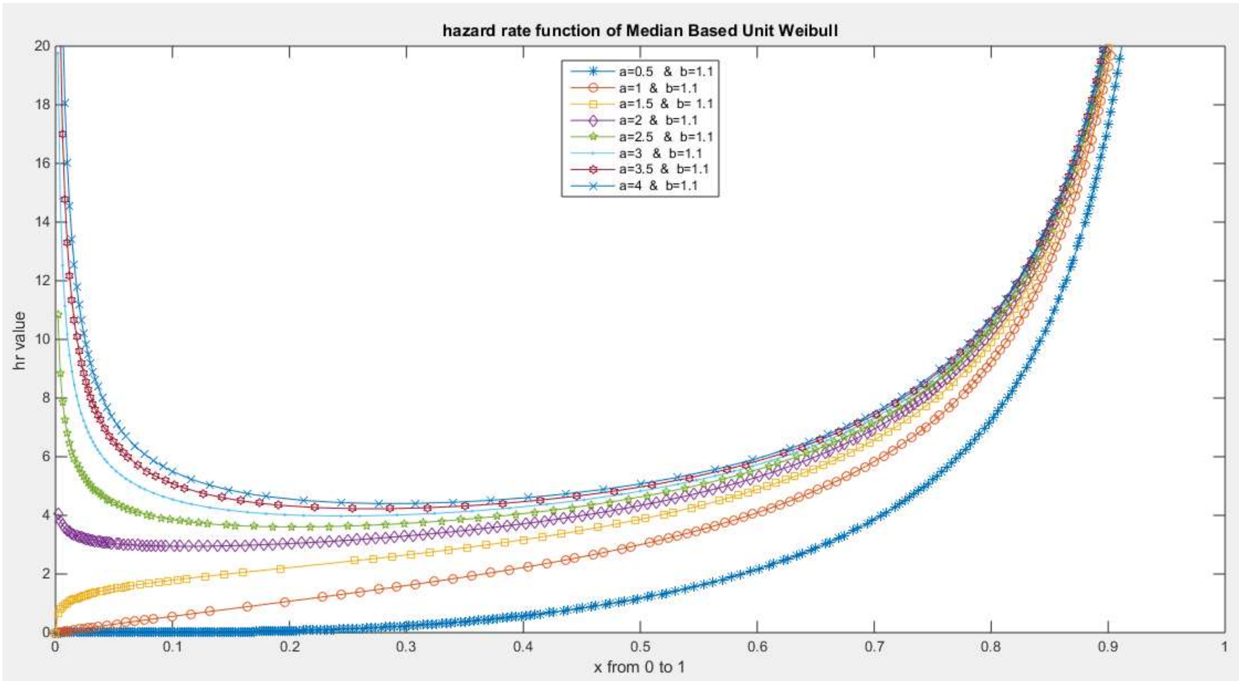

Fig. 12: hazard rate of Median Based Unit Weibull (MBUW) distribution, alpha (0.5 , 1, 1.5, 2 , 2.5 , 3 , 3.5 , 4) and beta (1.1)

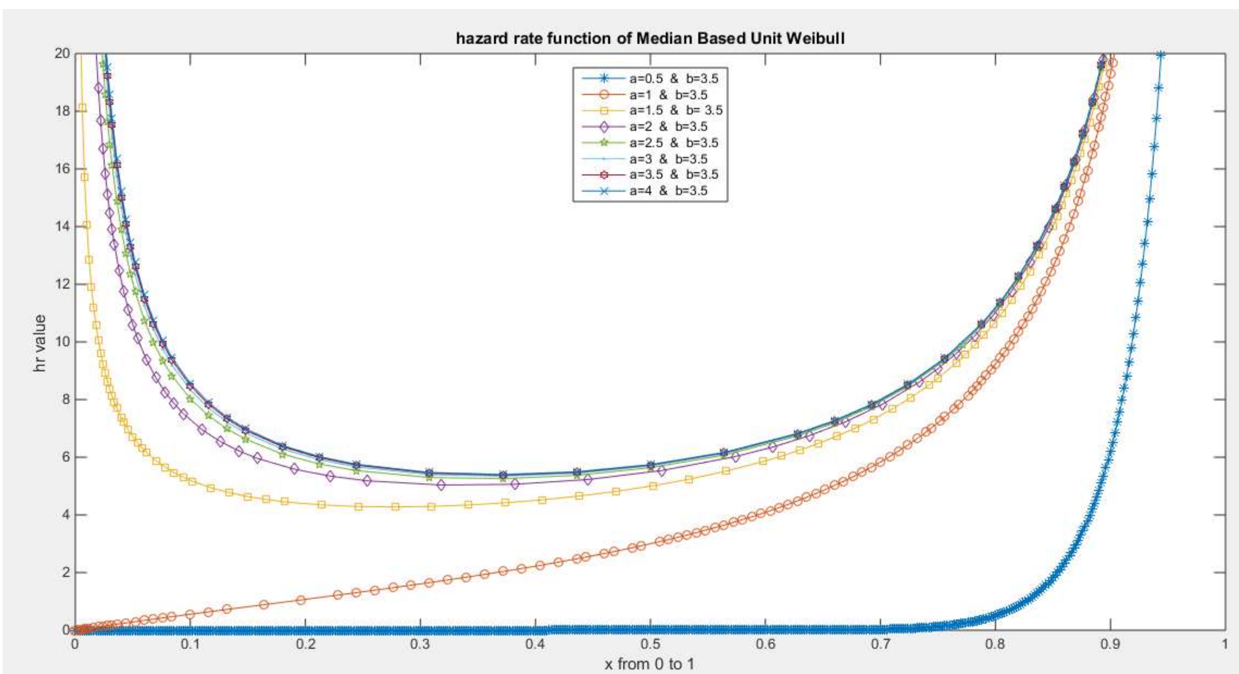

Fig. 13: hazard rate of Median Based Unit Weibull (MBUW) distribution, alpha (0.5 , 1, 1.5, 2 , 2.5 , 3 , 3.5 , 4) and beta (3.5)

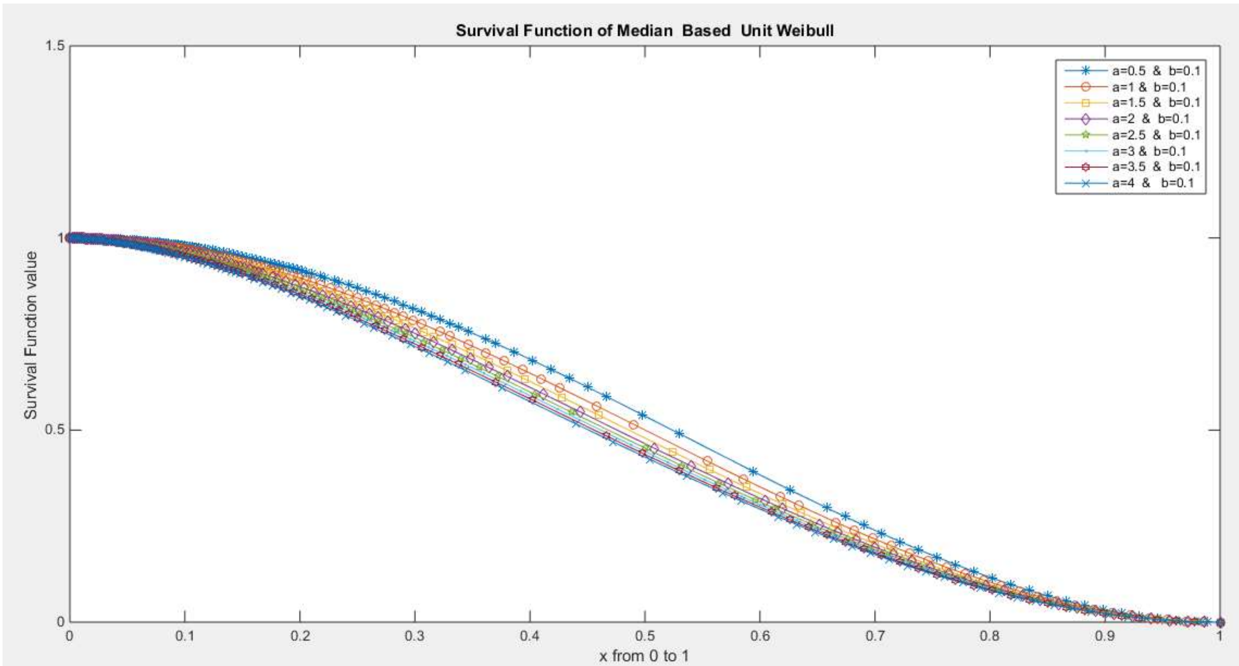

Fig. 14: Survival function of Median Based Unit Weibull (MBUW) distribution, alpha (0.5 , 1, 1.5, 2 , 2.5 , 3 , 3.5 , 4) and beta (0.1)

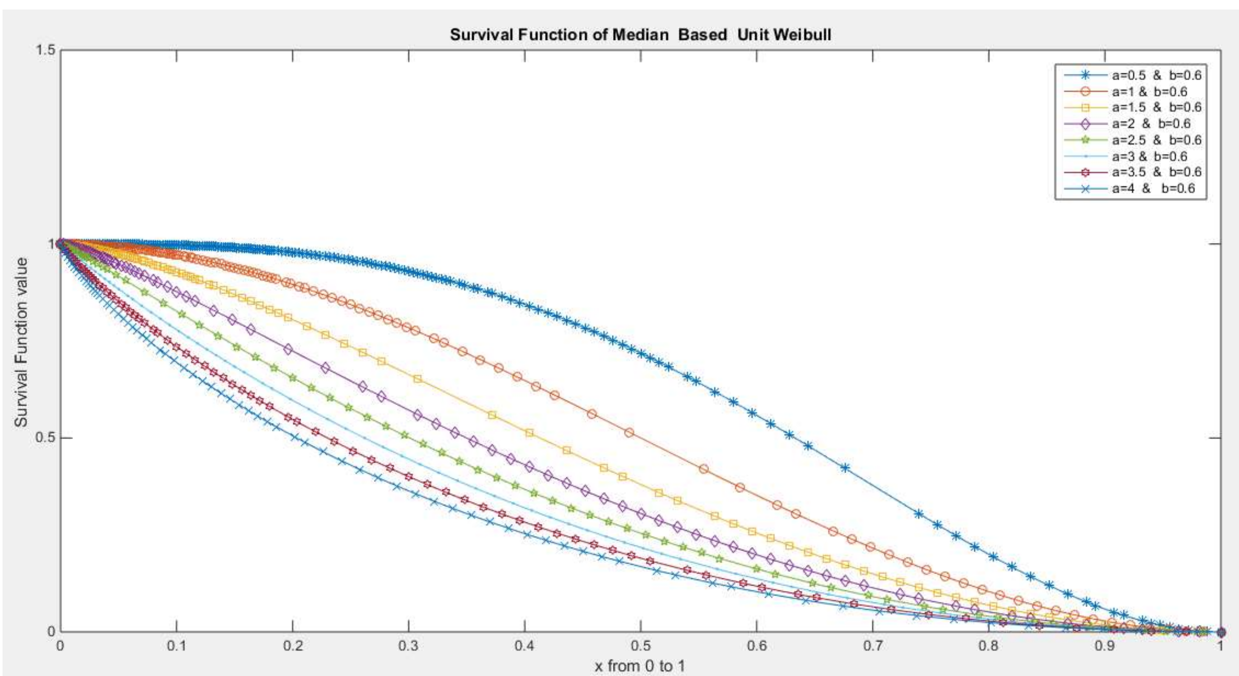

Fig. 15: Survival function of Median Based Unit Weibull (MBUW) distribution, alpha (0.5 , 1, 1.5, 2 , 2.5 , 3 , 3.5 , 4) and beta (0.6)

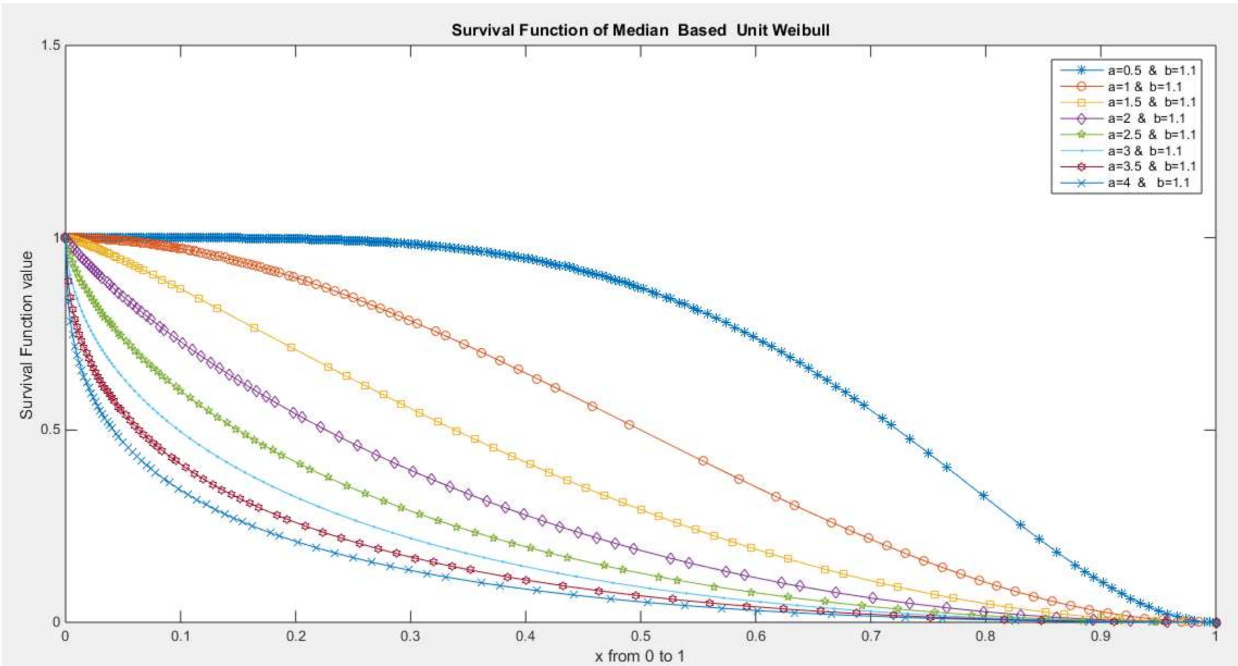

Fig. 16: Survival function of Median Based Unit Weibull (MBUW) distribution, alpha (0.5 , 1, 1.5, 2 , 2.5 , 3 , 3.5 , 4) and beta (1.1)

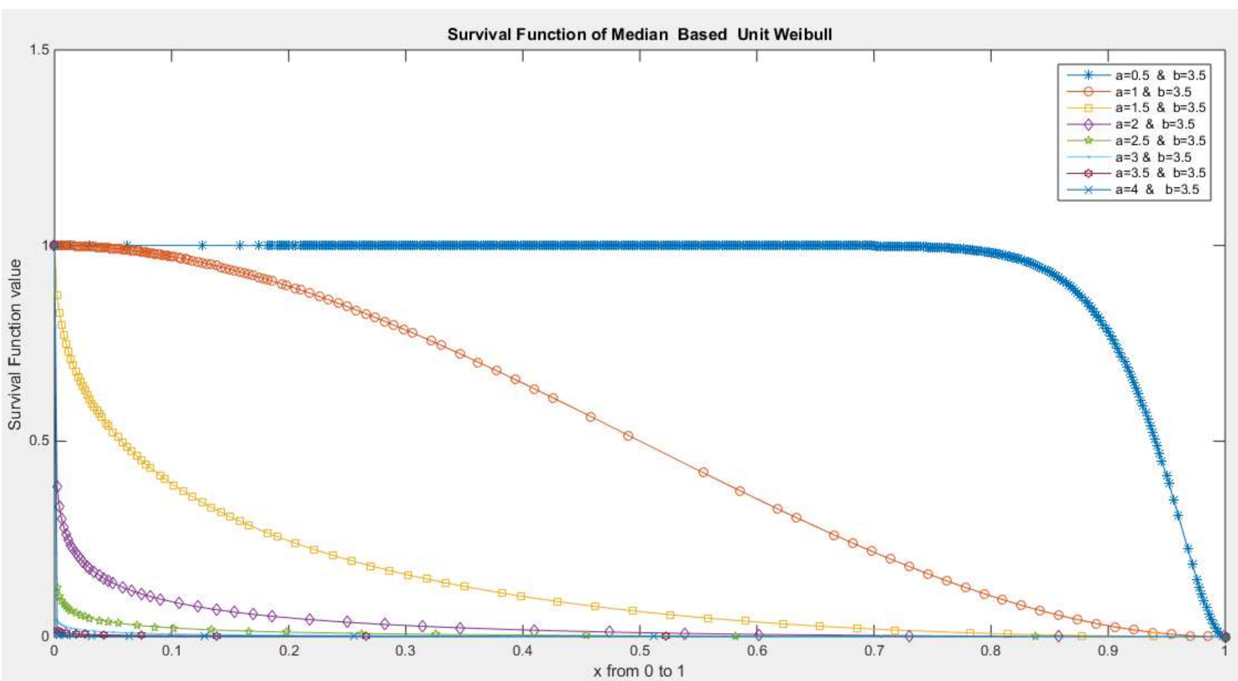

Fig. 17: Survival function of Median Based Unit Weibull (MBUW) distribution, alpha (0.5 , 1, 1.5, 2 , 2.5 , 3 , 3.5 , 4) and beta (3.5)

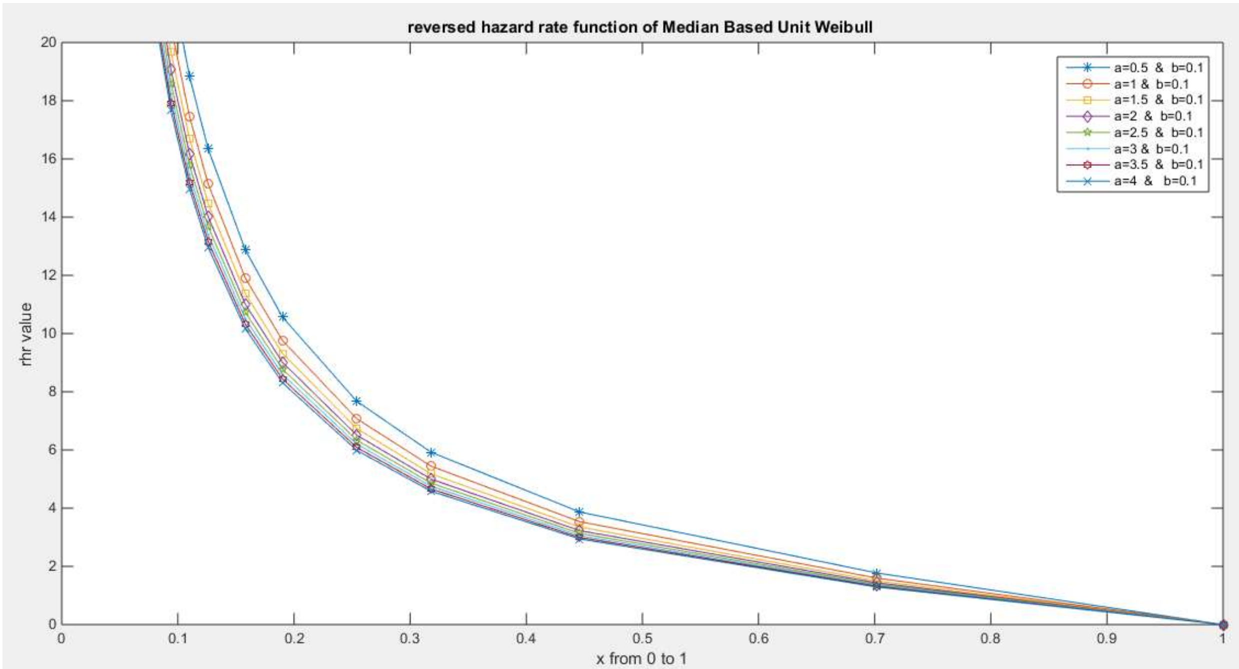

Fig. 18: reversed hazard rate of Median Based Unit Weibull (MBUW) distribution, alpha (0.5 , 1, 1.5, 2 , 2.5 , 3 , 3.5 , 4) and beta (0.1)

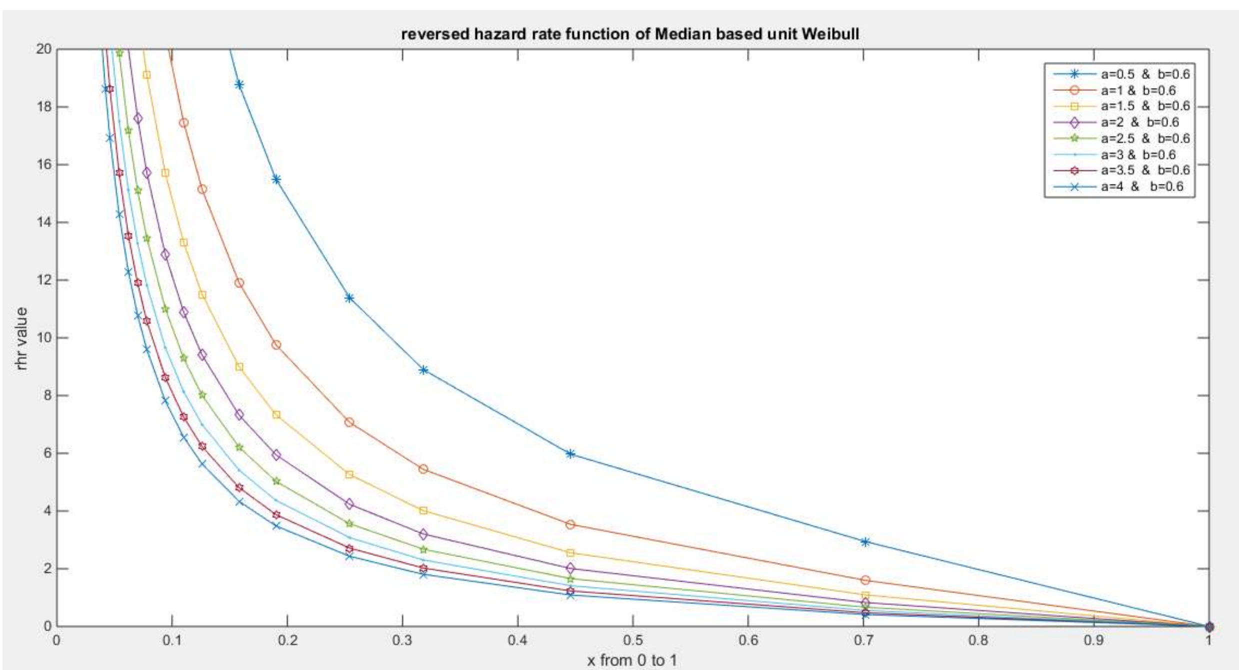

Fig. 19: reversed hazard rate of Median Based Unit Weibull (MBUW) distribution, alpha (0.5 , 1, 1.5, 2 , 2.5 , 3 , 3.5 , 4) and beta (0.6)

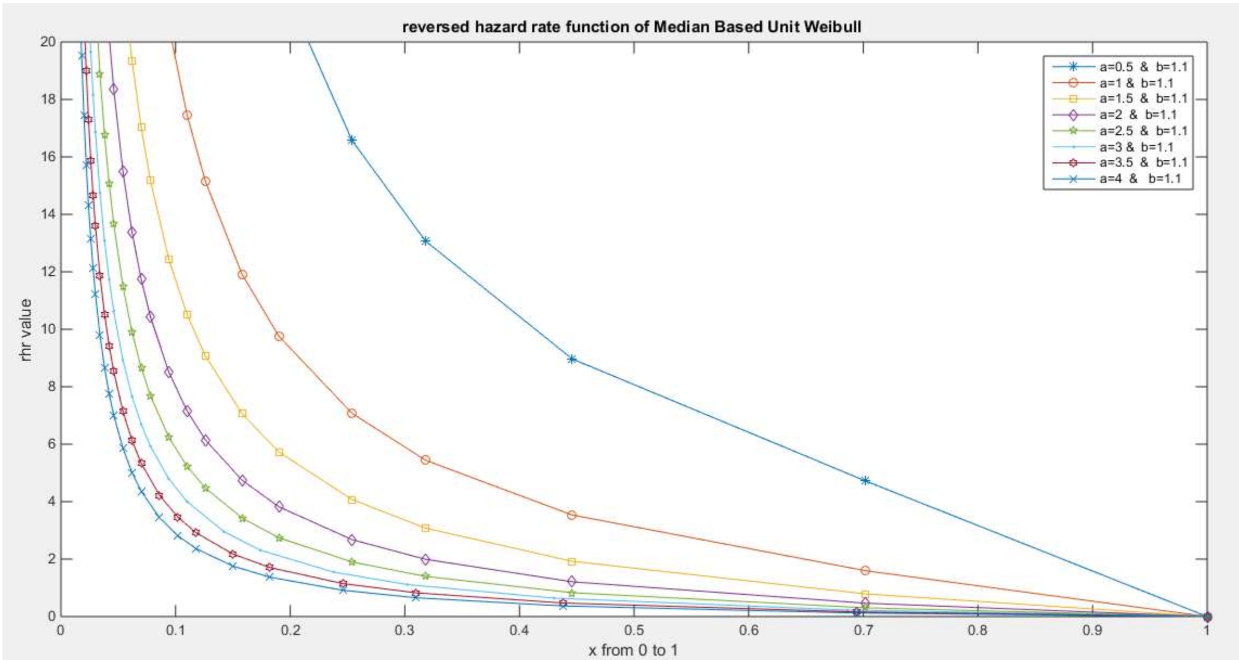

Fig. 20: reversed hazard rate of Median Based Unit Weibull (MBUW) distribution, alpha (0.5 , 1, 1.5, 2 , 2.5 , 3 , 3.5 , 4) and beta (1.1)

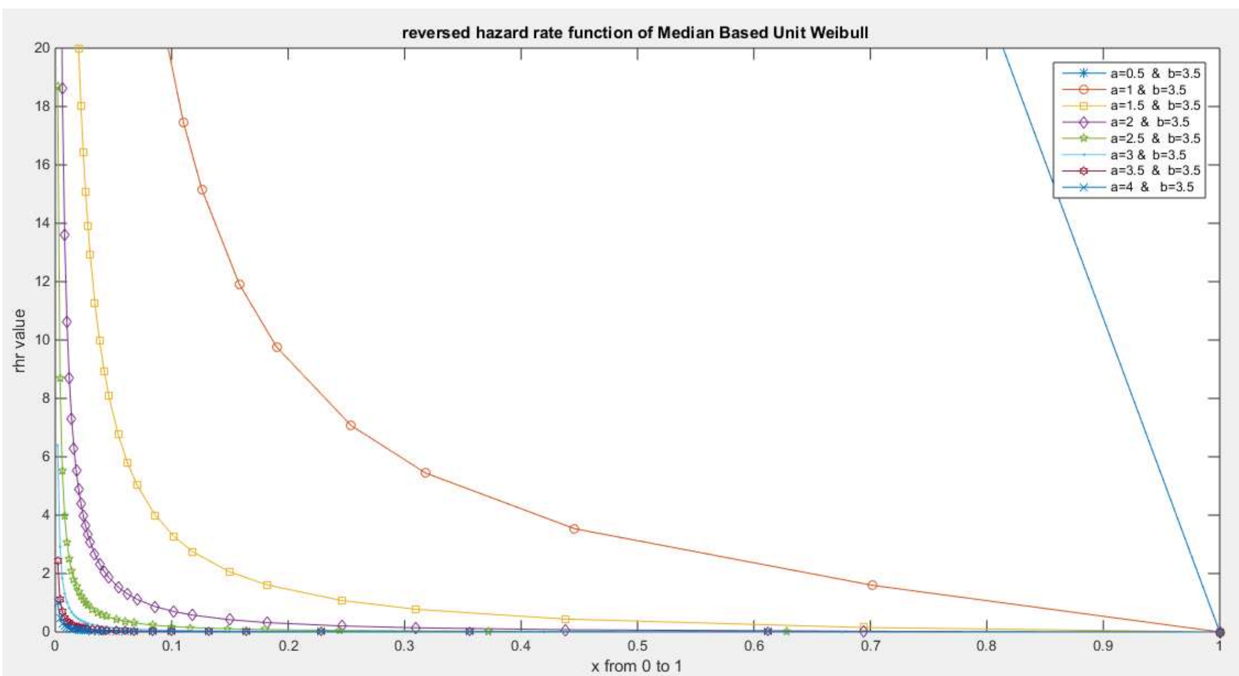

Fig. 21: reversed hazard rate of Median Based Unit Weibull (MBUW) distribution, alpha (0.5 , 1, 1.5, 2 , 2.5 , 3 , 3.5 , 4) and beta (3.5)

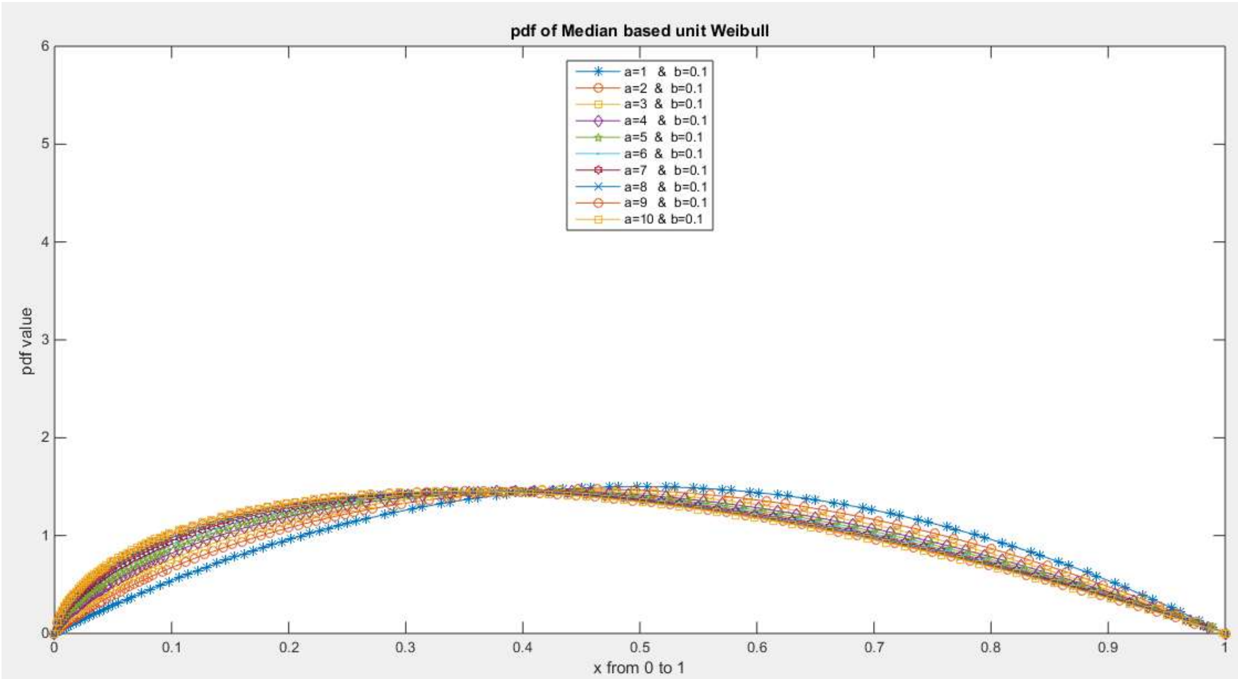

Fig.22 : pdf of Median Based Unit Weibull (MBUW) distribution, alpha (from 1 to 10) and beta (0.1)

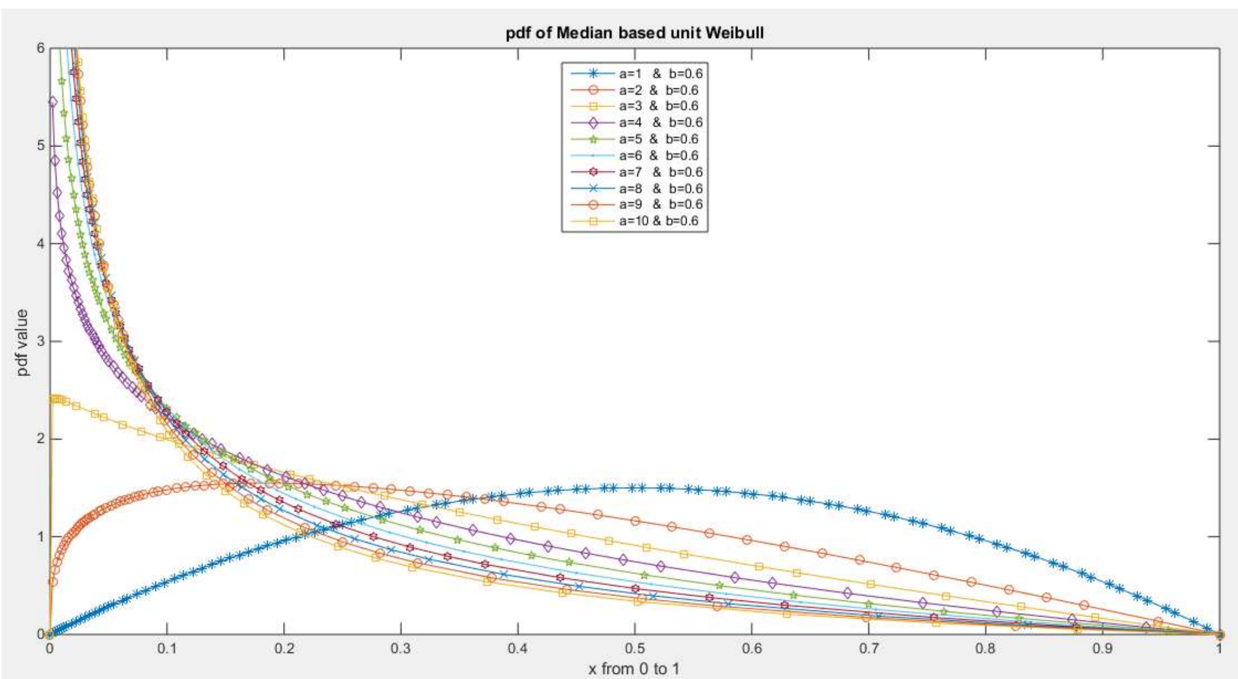

Fig.23 : pdf of Median Based Unit Weibull (MBUW) distribution, alpha (from 1 to 10) and beta (0.6)

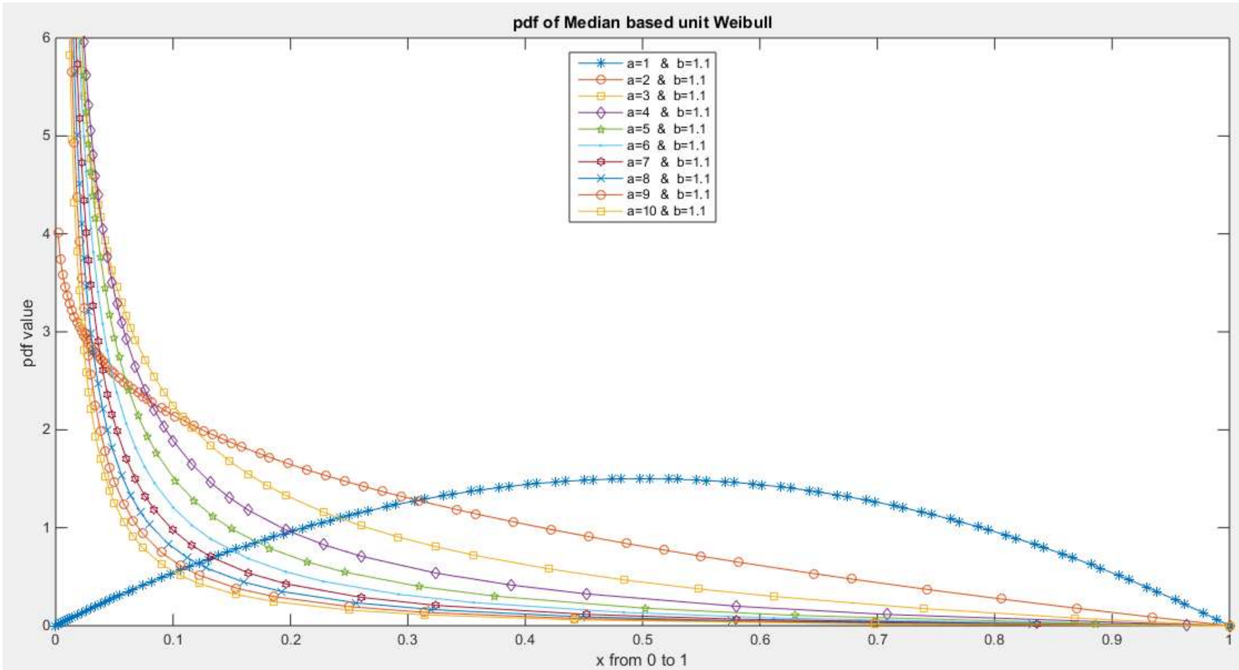

Fig.24 : pdf of Median Based Unit Weibull (MBUW) distribution, alpha (from 1 to 10) and beta (1.1)

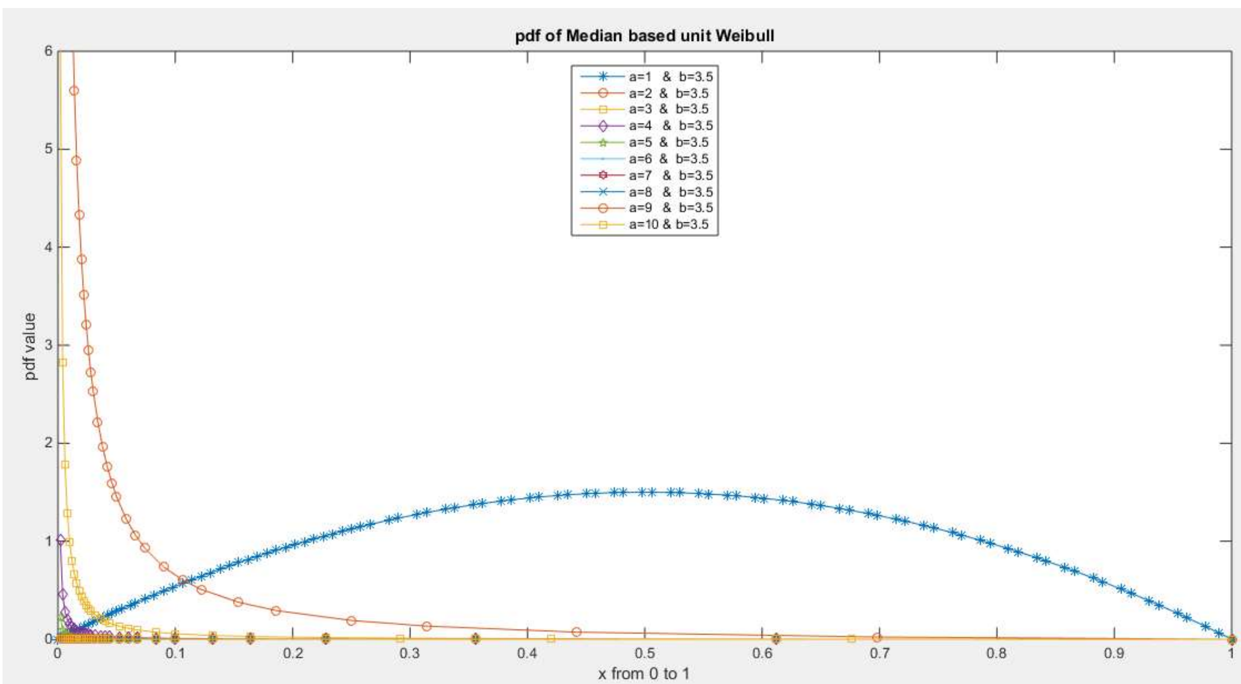

Fig.25 : pdf of Median Based Unit Weibull (MBUW) distribution, alpha (from 1 to 10) and beta (3.5)

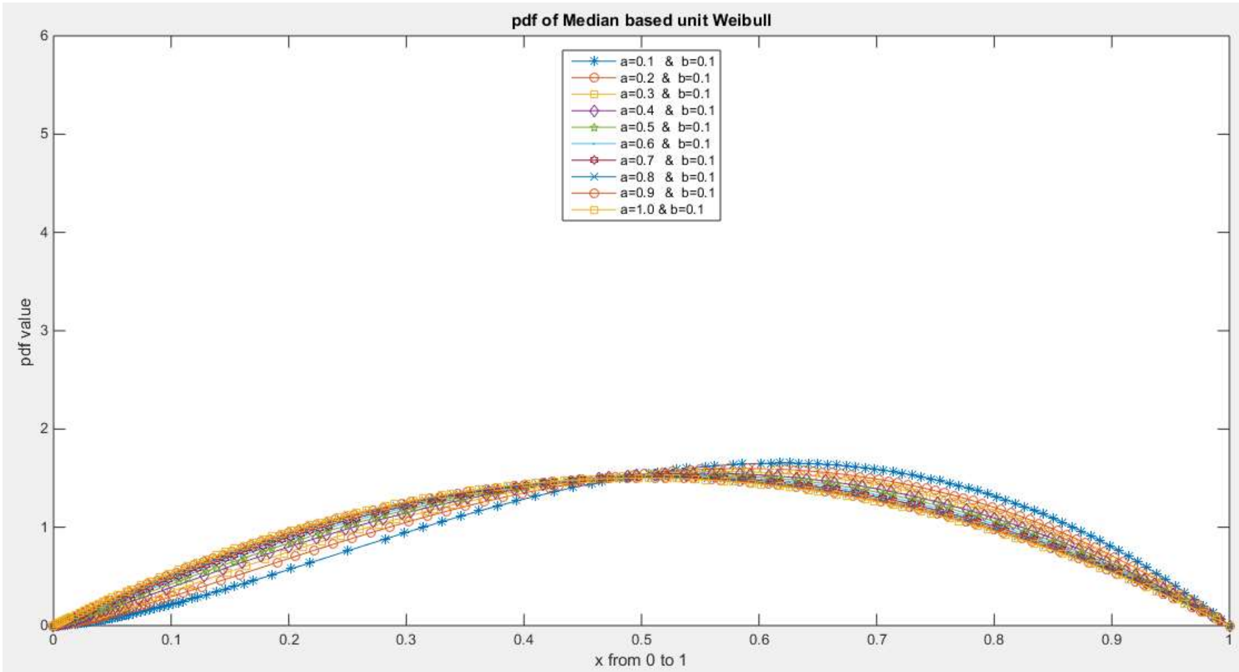

Fig.26 : pdf of Median Based Unit Weibull (MBUW) distribution, alpha (from 0.1 to 1) and beta (0.1)

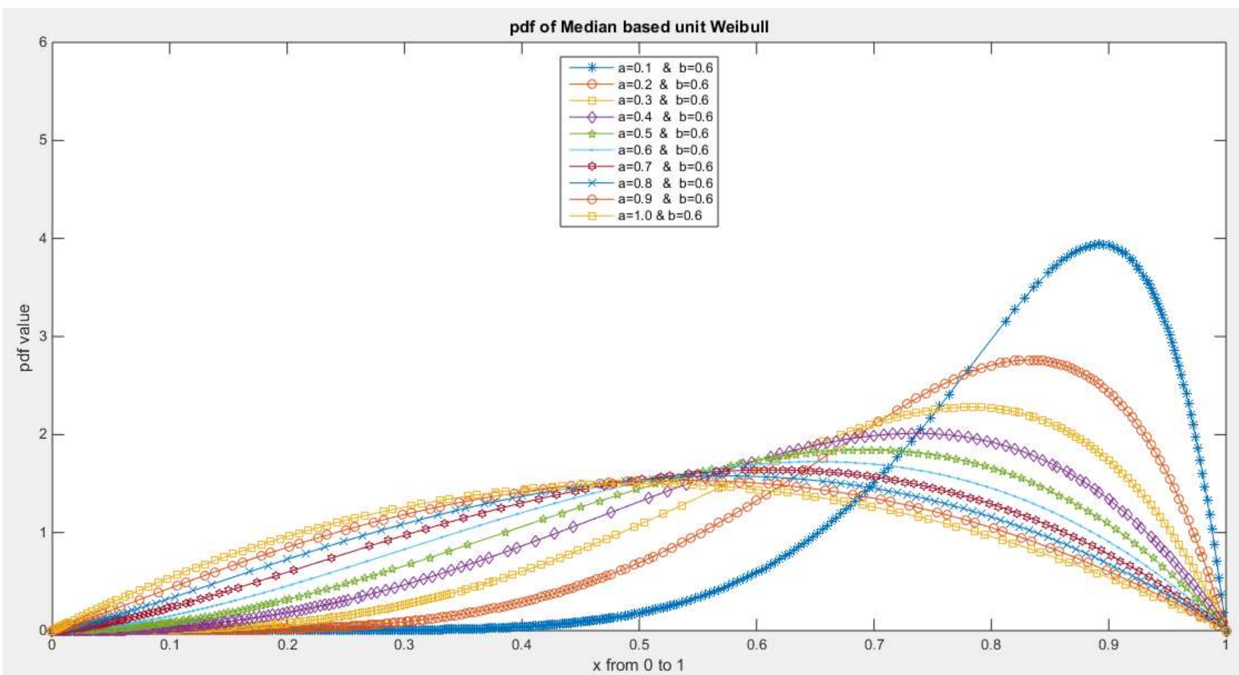

Fig.27 : pdf of Median Based Unit Weibull (MBUW) distribution, alpha (from 0.1 to 1) and beta (0.6)

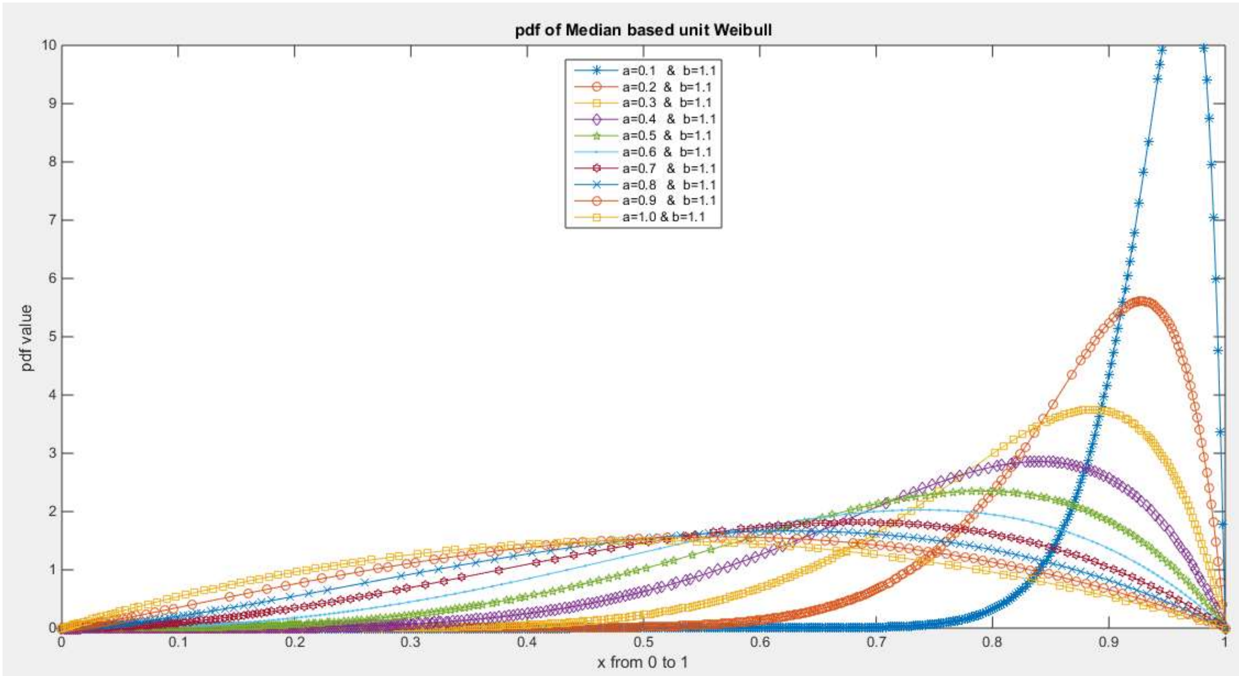

Fig.28 : pdf of Median Based Unit Weibull (MBUW) distribution, alpha (from 0.1 to 1) and beta (1.1)

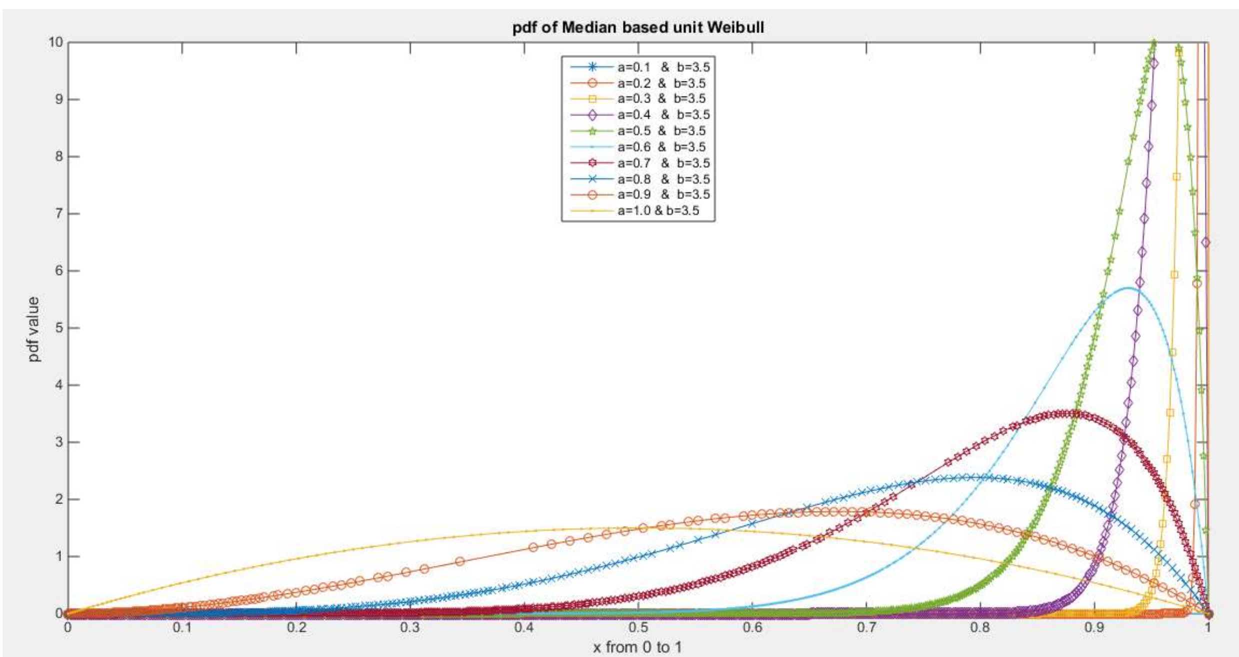

Fig.29 : pdf of Median Based Unit Weibull (MBUW) distribution, alpha (from 0.1 to 1) and beta (3.5)

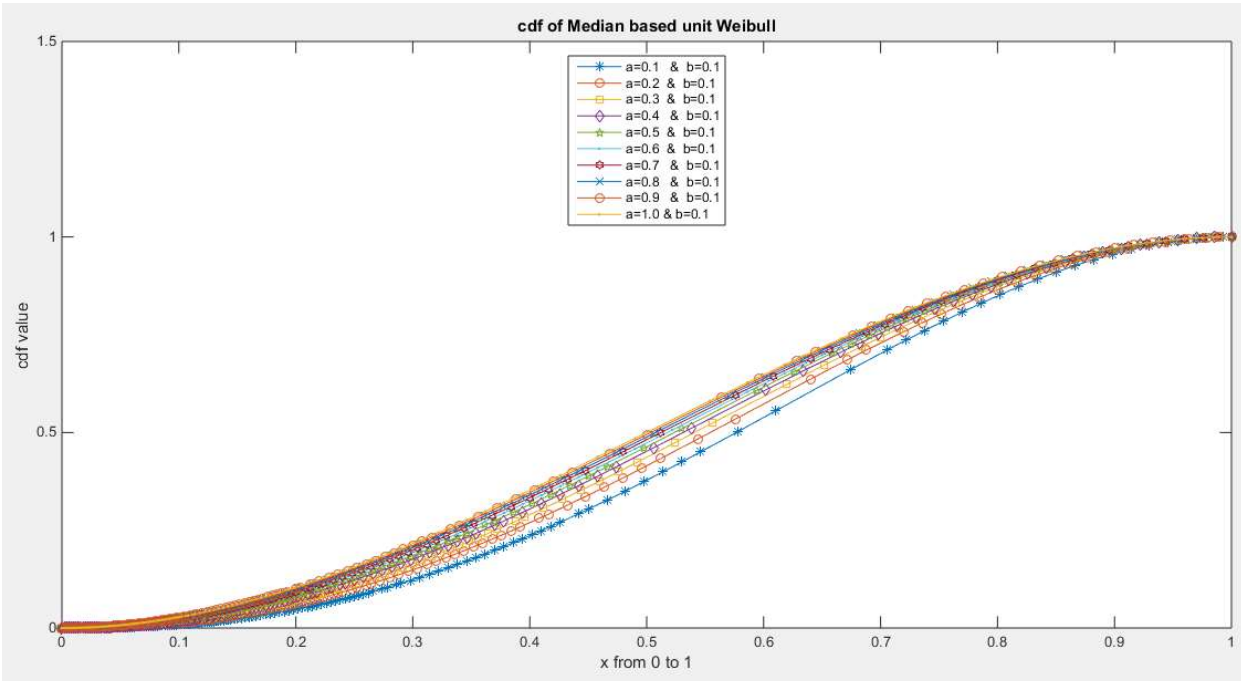

Fig.30 : cdf of Median Based Unit Weibull (MBUW) distribution, alpha (from 0.1 to 1) and beta (0.1)

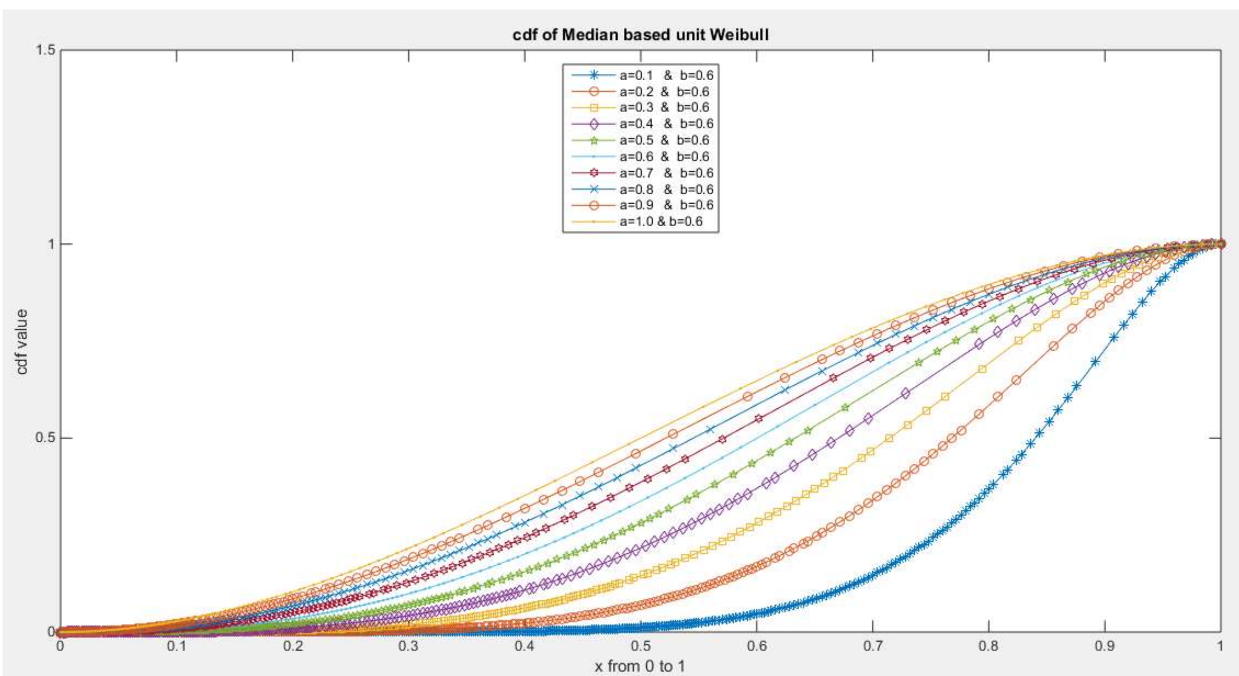

Fig.31 : cdf of Median Based Unit Weibull (MBUW) distribution, alpha (from 0.1 to 1) and beta (0.6)

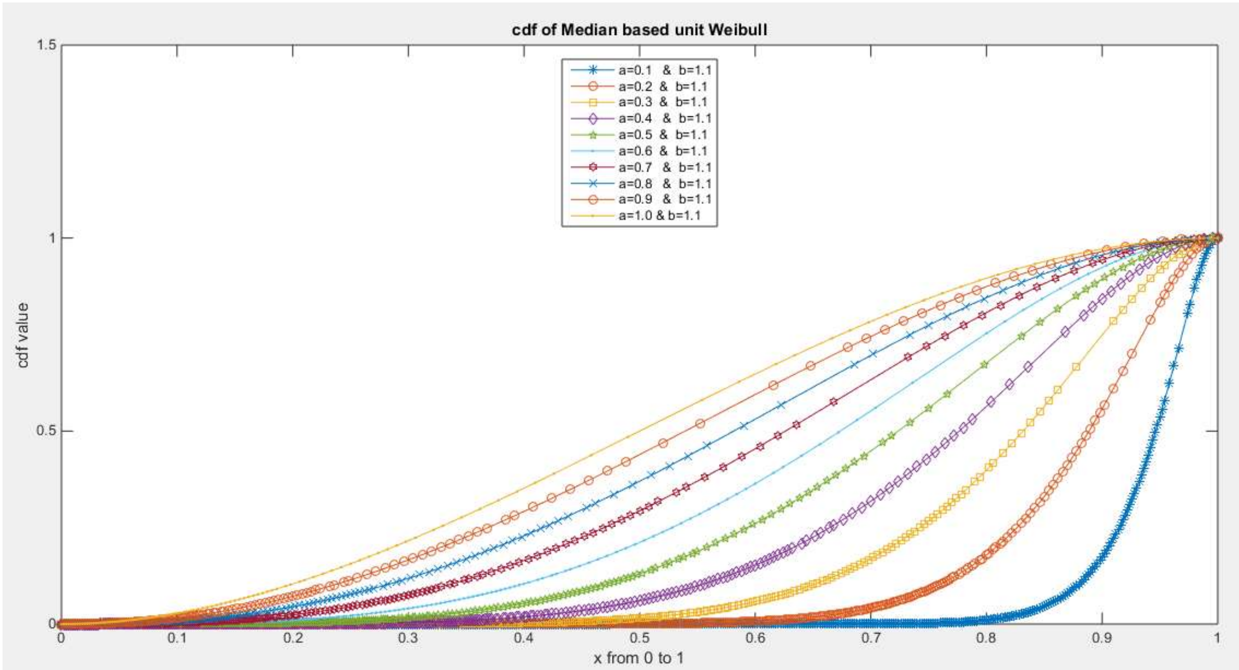

Fig.32 : cdf of Median Based Unit Weibull (MBUW) distribution, alpha (from 0.1 to 1) and beta (1.1)

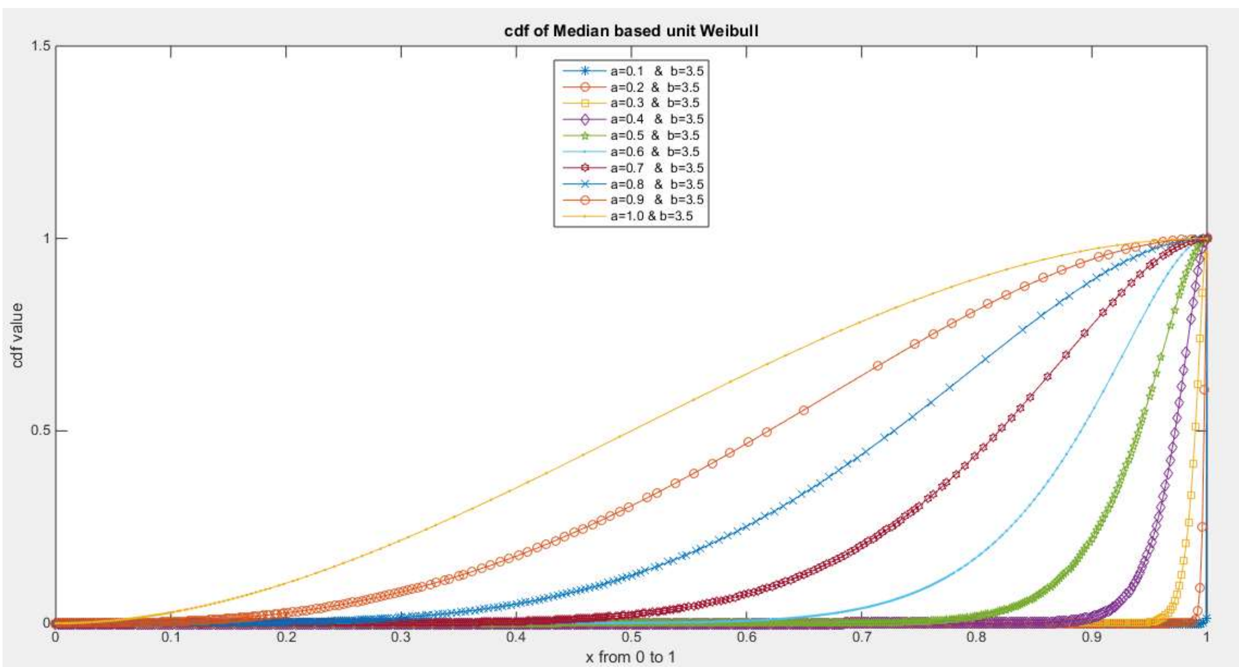

Fig.33 : cdf of Median Based Unit Weibull (MBUW) distribution, alpha (from 0.1 to 1) and beta (3.5)

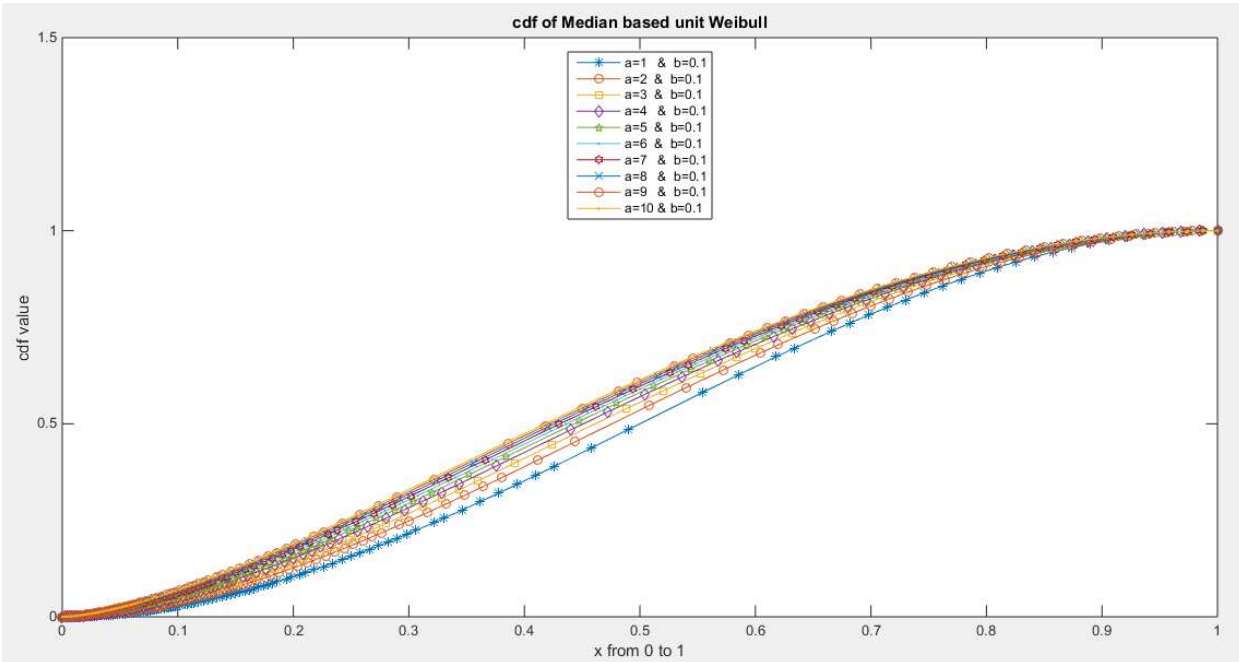

Fig.34 : cdf of Median Based Unit Weibull (MBUW) distribution, alpha (from 1 to 10) and beta (0.1)

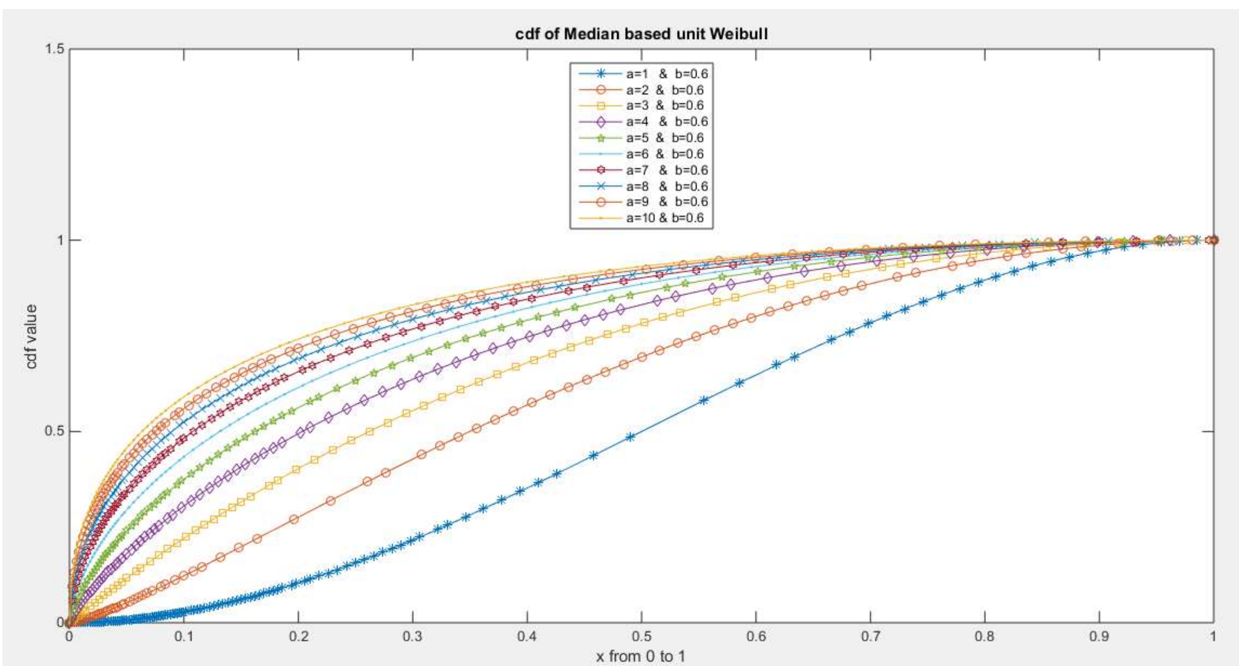

Fig.35 : cdf of Median Based Unit Weibull (MBUW) distribution, alpha (from 1 to 10) and beta (0.6)

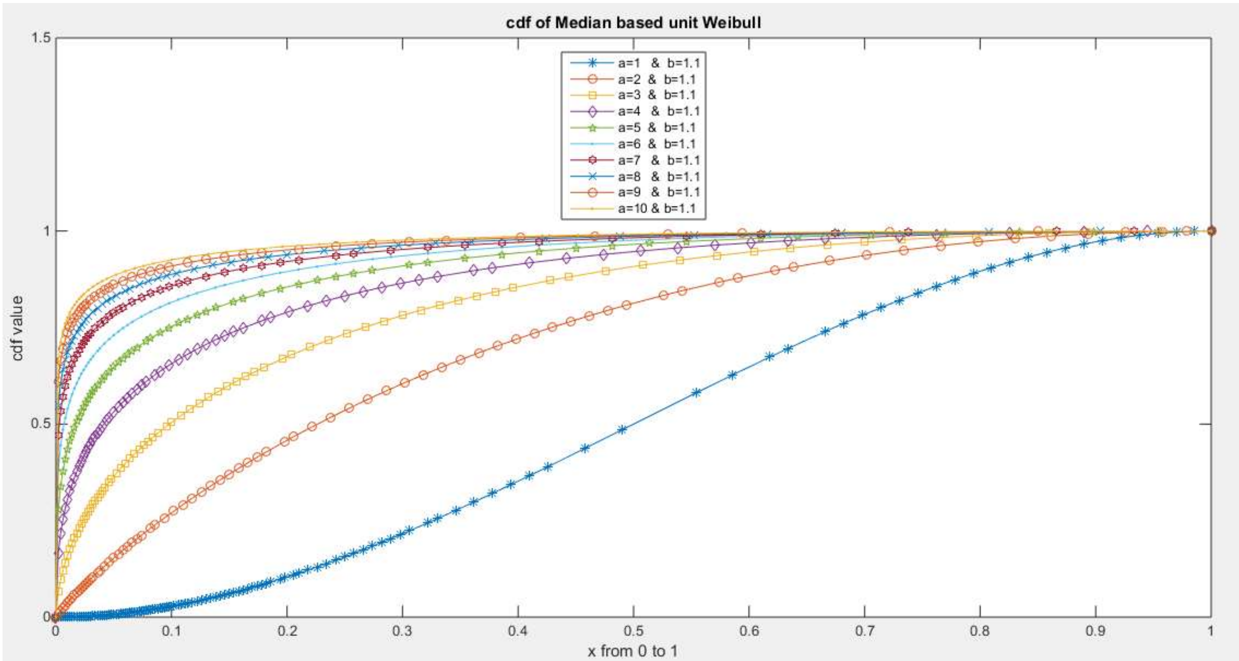

Fig.36 : cdf of Median Based Unit Weibull (MBUW) distribution, alpha (from 1 to 10) and beta (1.1)

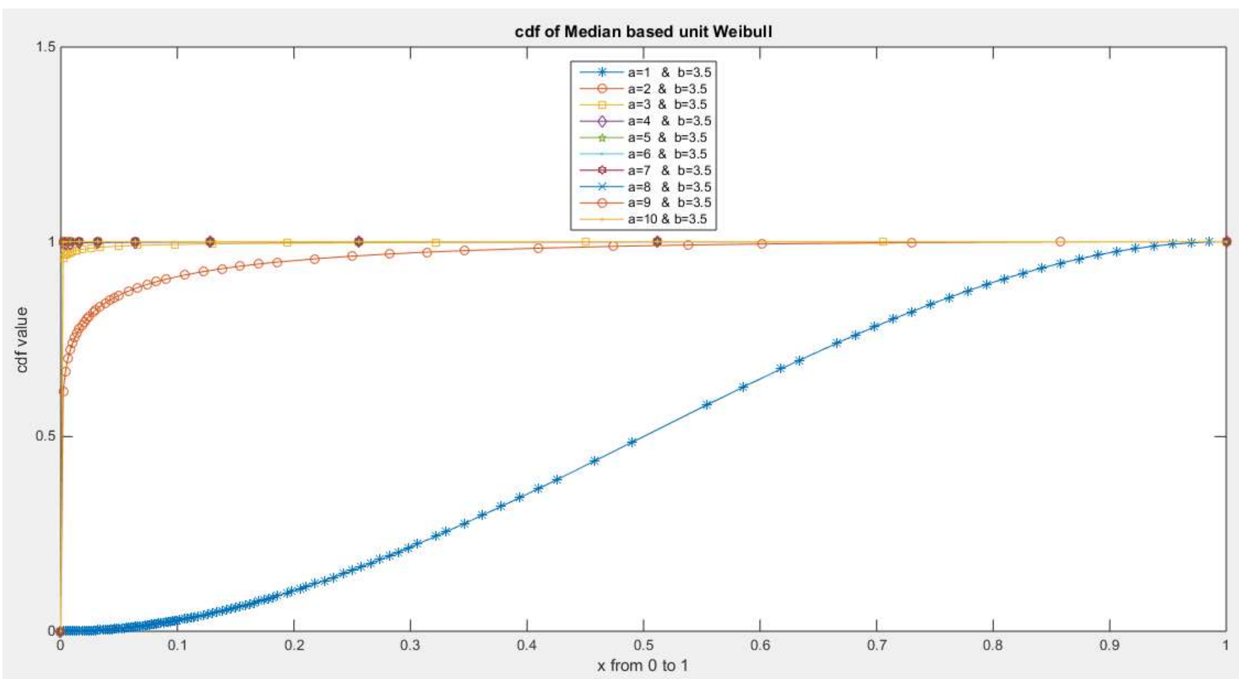

Fig.37 : cdf of Median Based Unit Weibull (MBUW) distribution, alpha (from 1 to 10) and beta (3.5)

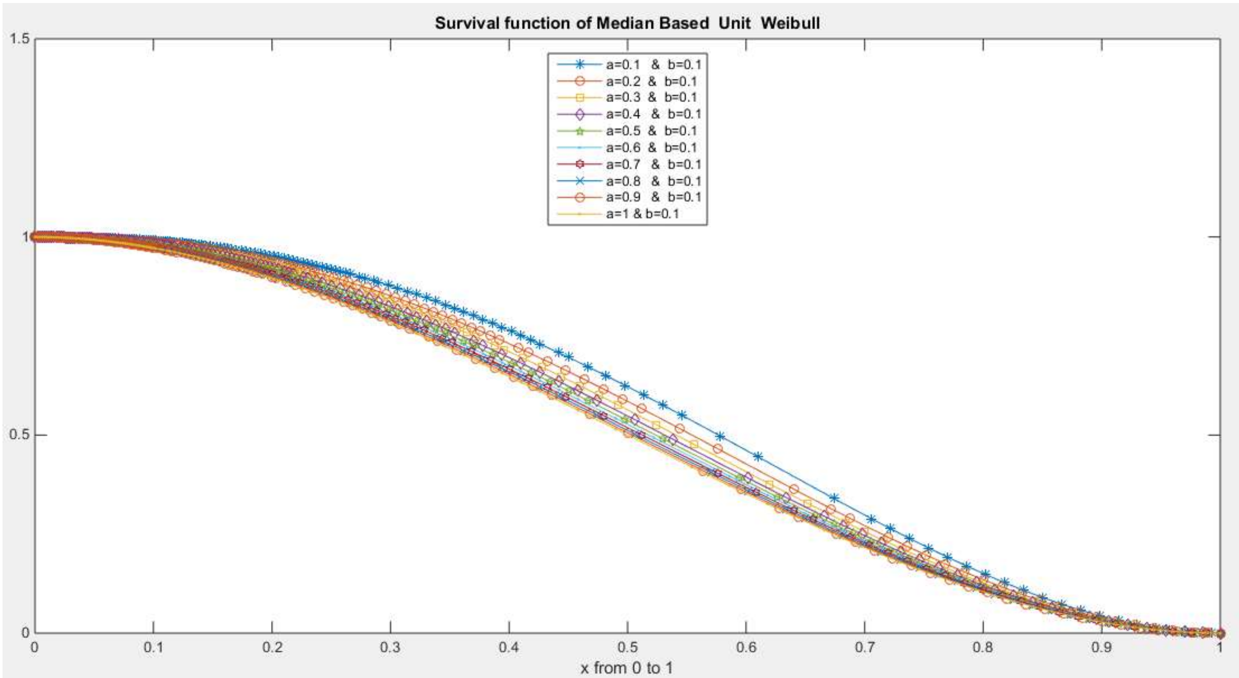

Fig.38 : Sf of Median Based Unit Weibull (MBUW) distribution, alpha (from 0.1 to 1) and beta (0.1)

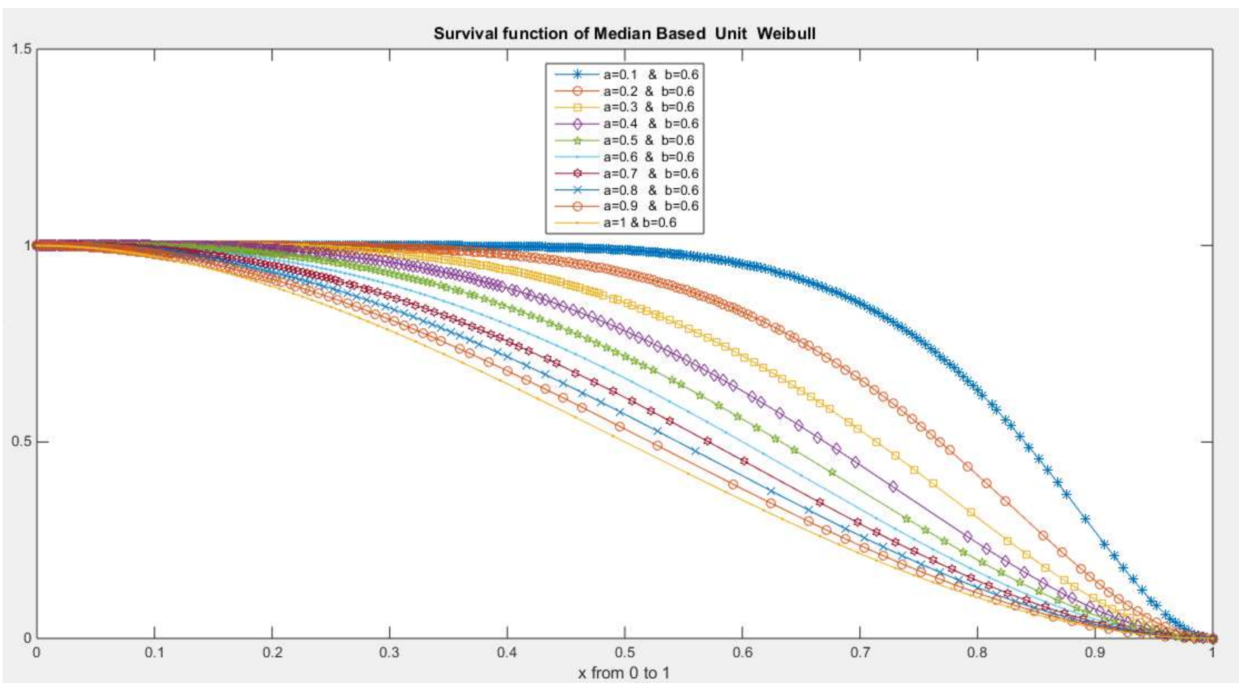

Fig.39 : Sf of Median Based Unit Weibull (MBUW) distribution, alpha (from 0.1 to 1) and beta (0.6)

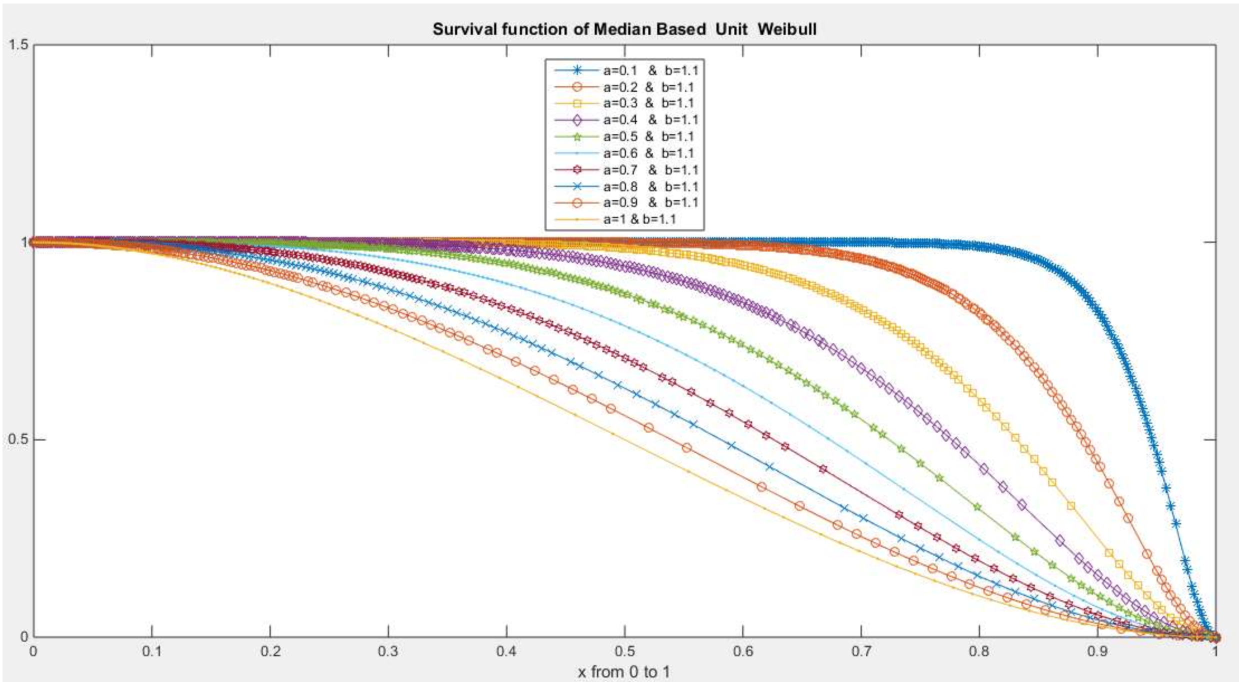

Fig.40 : Sf of Median Based Unit Weibull (MBUW) distribution, alpha (from 0.1 to 1) and beta (1.1)

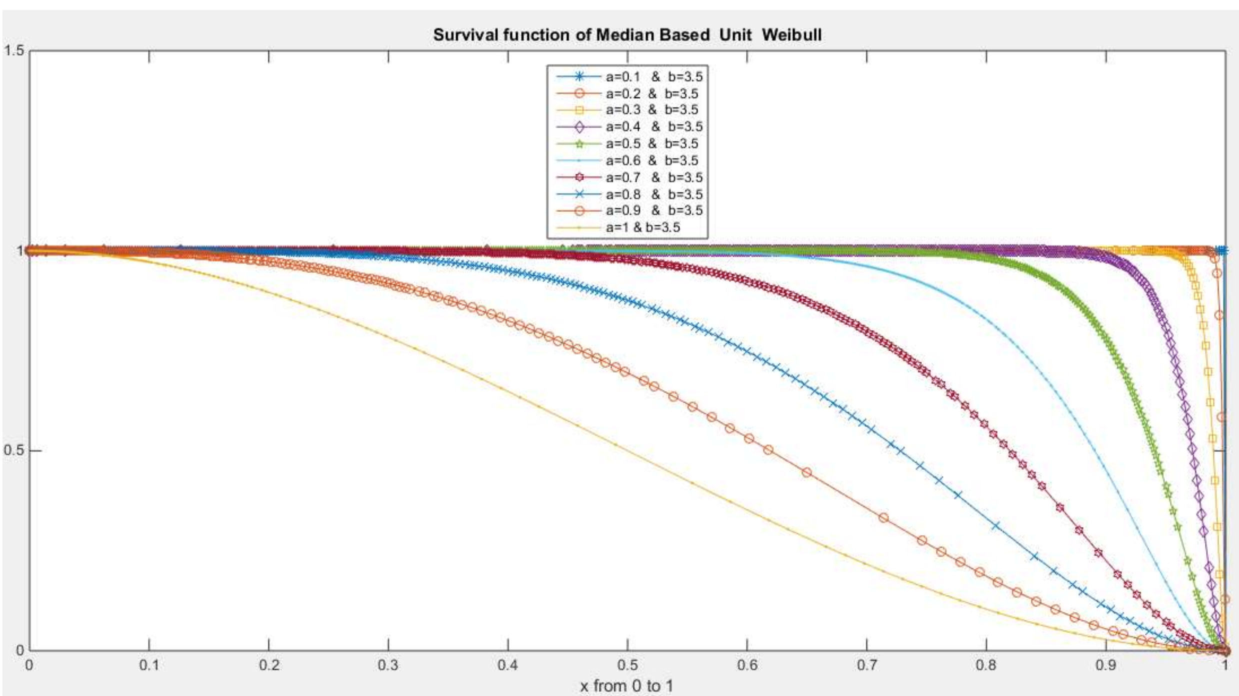

Fig.41 : Sf of Median Based Unit Weibull (MBUW) distribution, alpha (from 0.1 to 1) and beta (3.5)

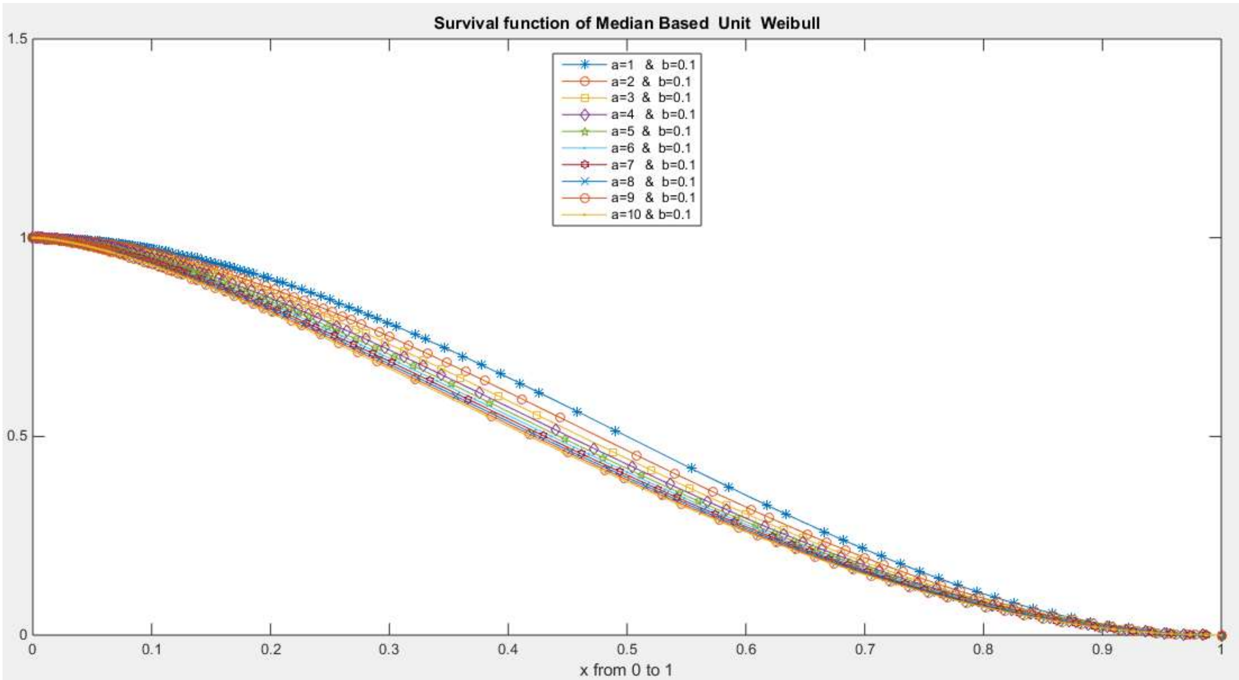

Fig.42 : Sf of Median Based Unit Weibull (MBUW) distribution, alpha (from 1 to 10) and beta (0.1)

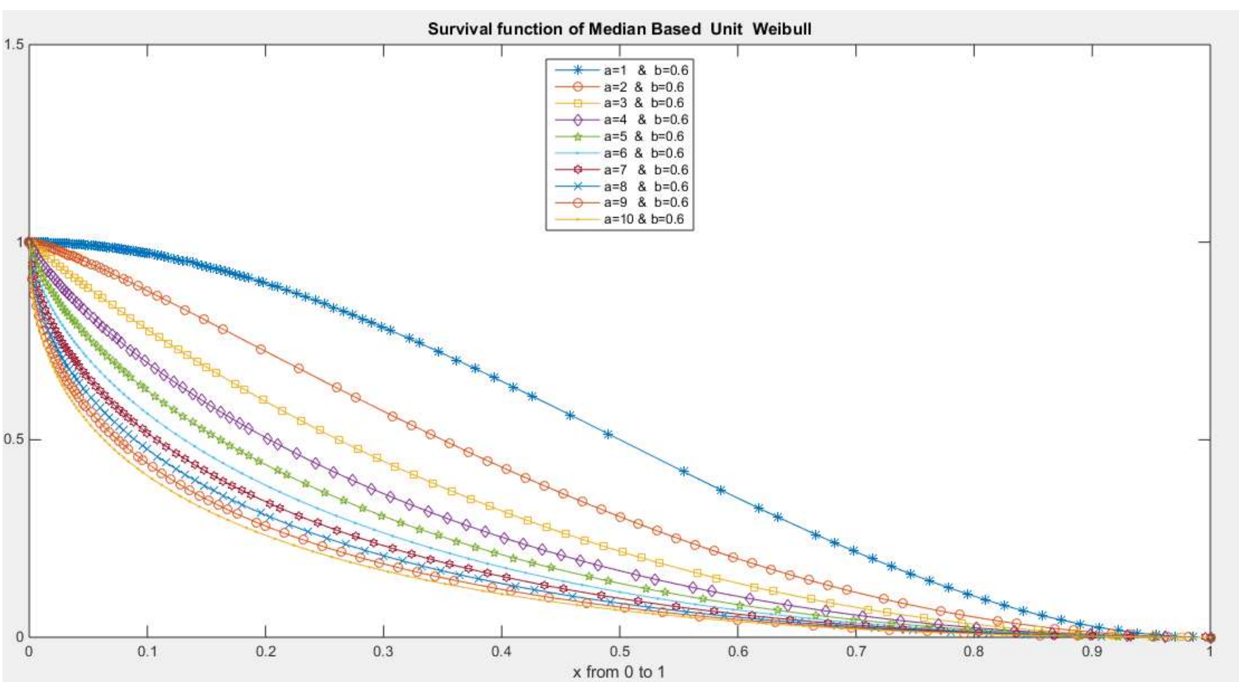

Fig.43 : Sf of Median Based Unit Weibull (MBUW) distribution, alpha (from 1 to 10) and beta (0.6)

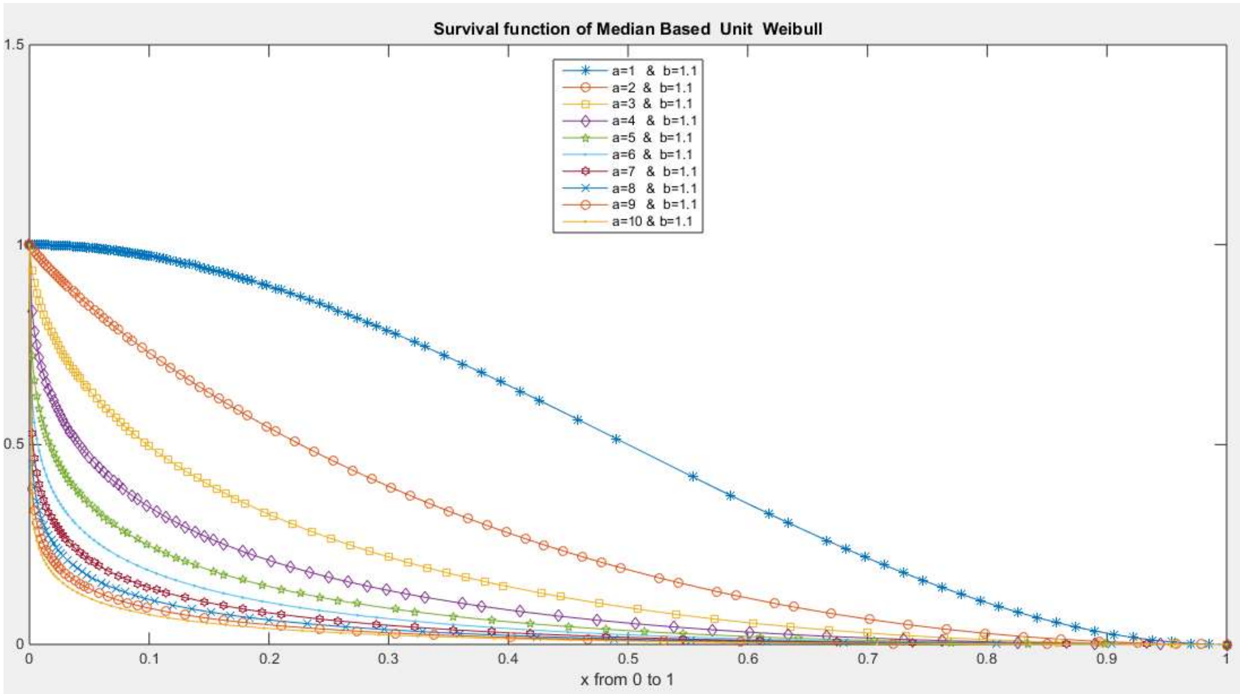

Fig.44 : Sf of Median Based Unit Weibull (MBUW) distribution, alpha (from 1 to 10) and beta (1.1)

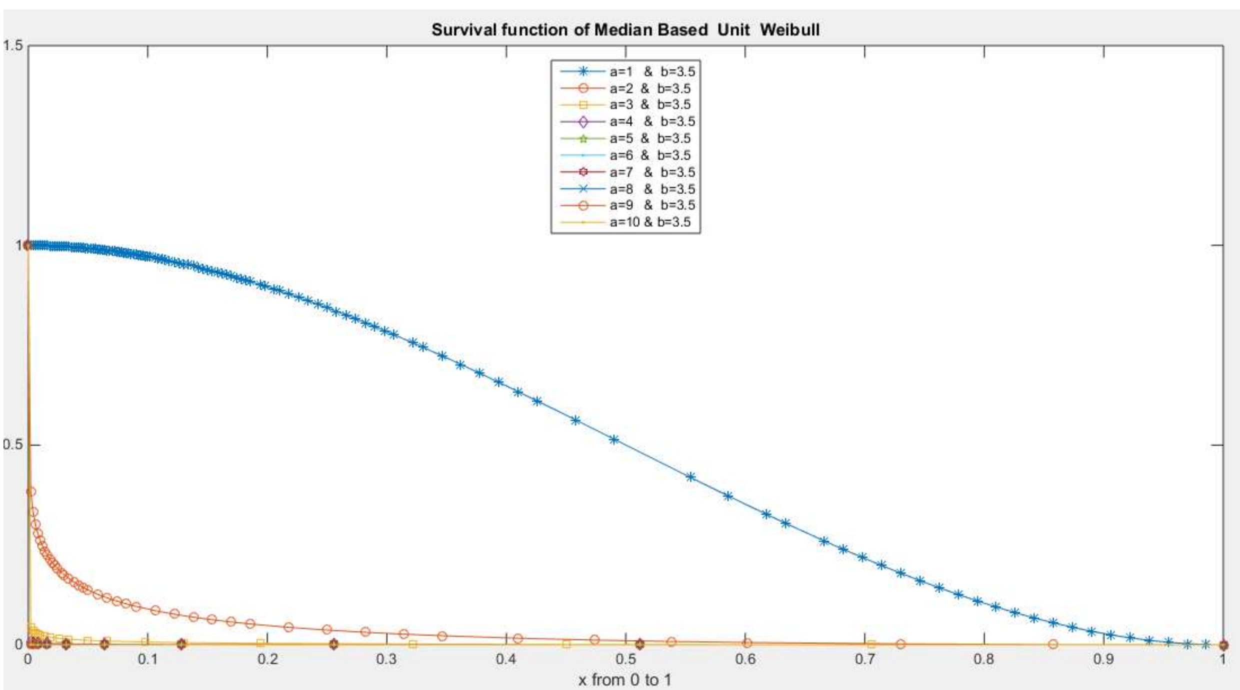

Fig.45 : Sf of Median Based Unit Weibull (MBUW) distribution, alpha (from 1 to 10) and beta (3.5)

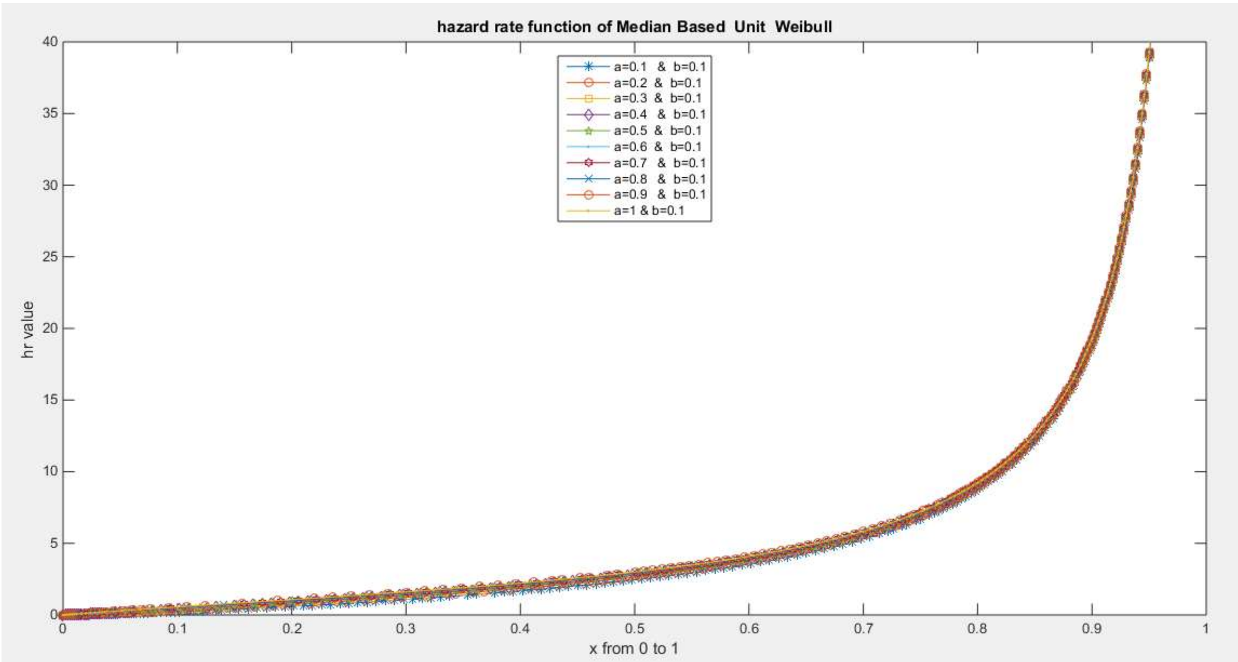

Fig.46 : hr of Median Based Unit Weibull (MBUW) distribution, alpha (from 0.1 to 1) and beta (0.1)

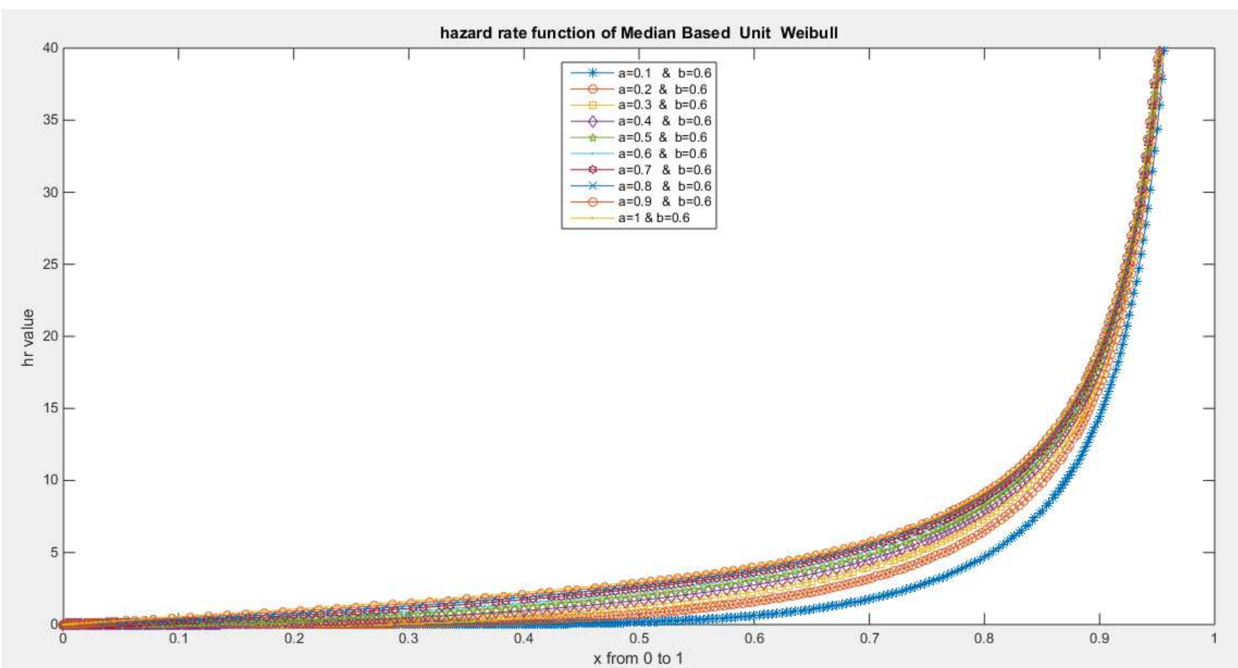

Fig.47 : hr of Median Based Unit Weibull (MBUW) distribution, alpha (from 0.1 to 1) and beta (0.6)

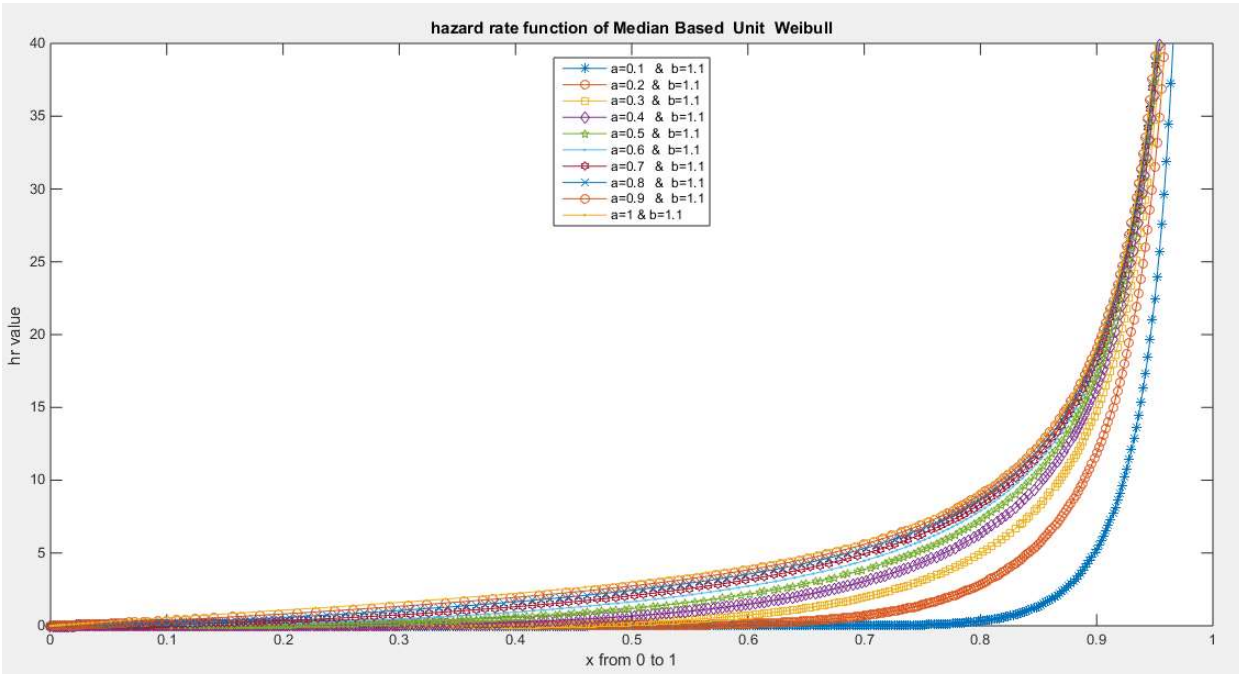

Fig.48 : hr of Median Based Unit Weibull (MBUW) distribution, alpha (from 0.1 to 1) and beta (1.1)

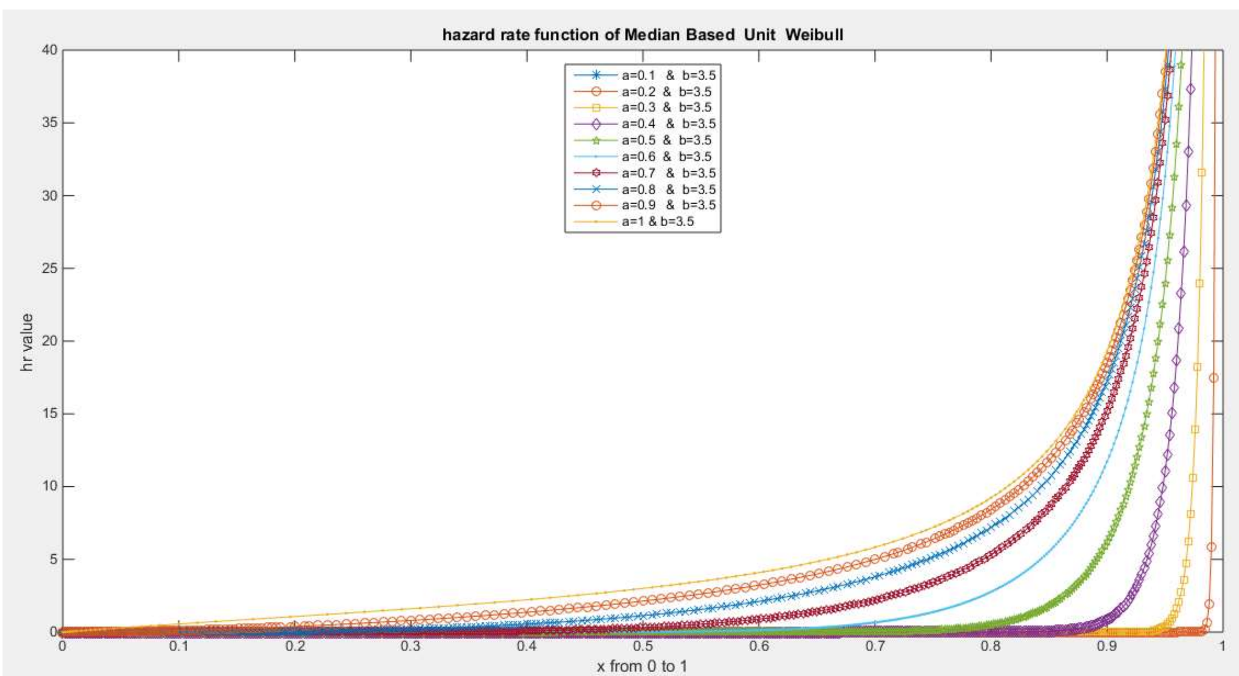

Fig.49 : hr of Median Based Unit Weibull (MBUW) distribution, alpha (from 0.1 to 1) and beta (3.5)

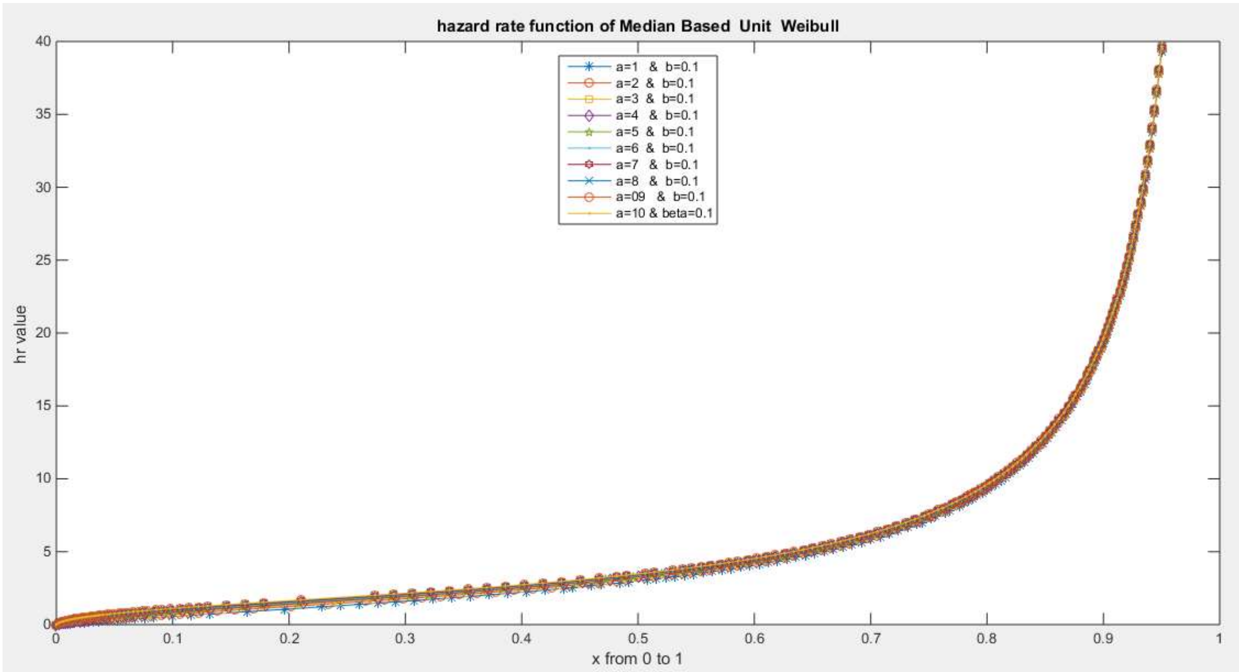

Fig.50 : hr of Median Based Unit Weibull (MBUW) distribution, alpha (from 1 to 10) and beta (0.1)

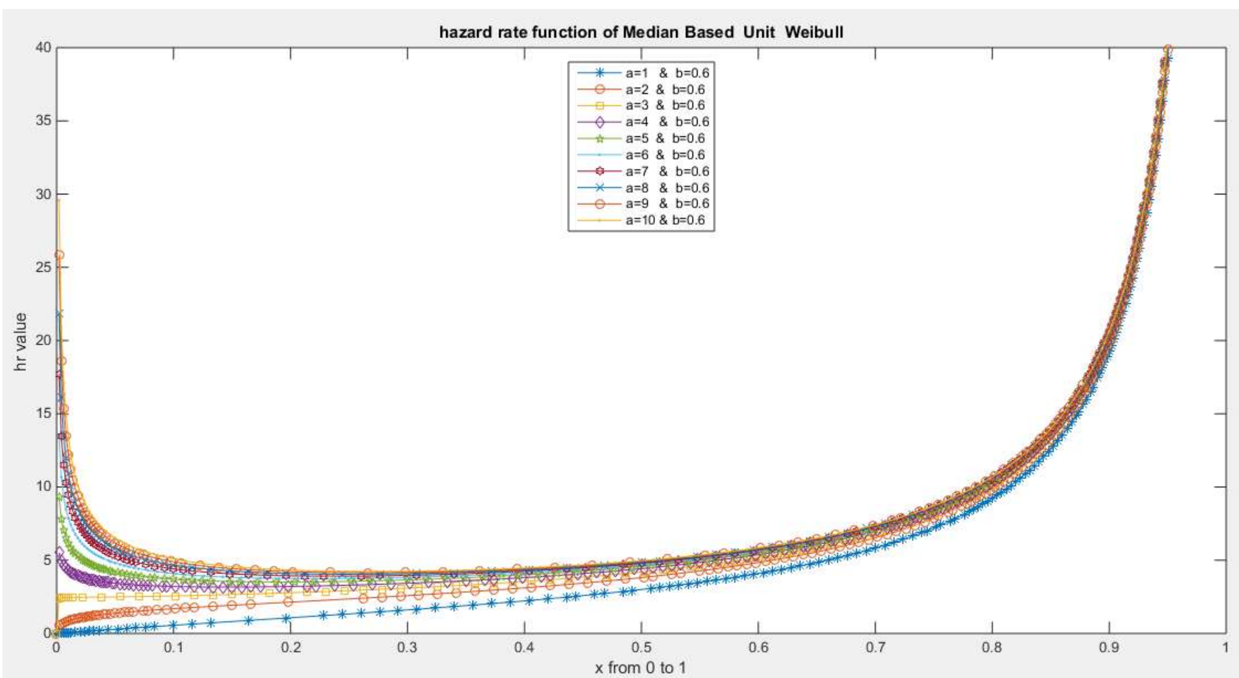

Fig.51 : hr of Median Based Unit Weibull (MBUW) distribution, alpha (from 1 to 10) and beta (0.6)

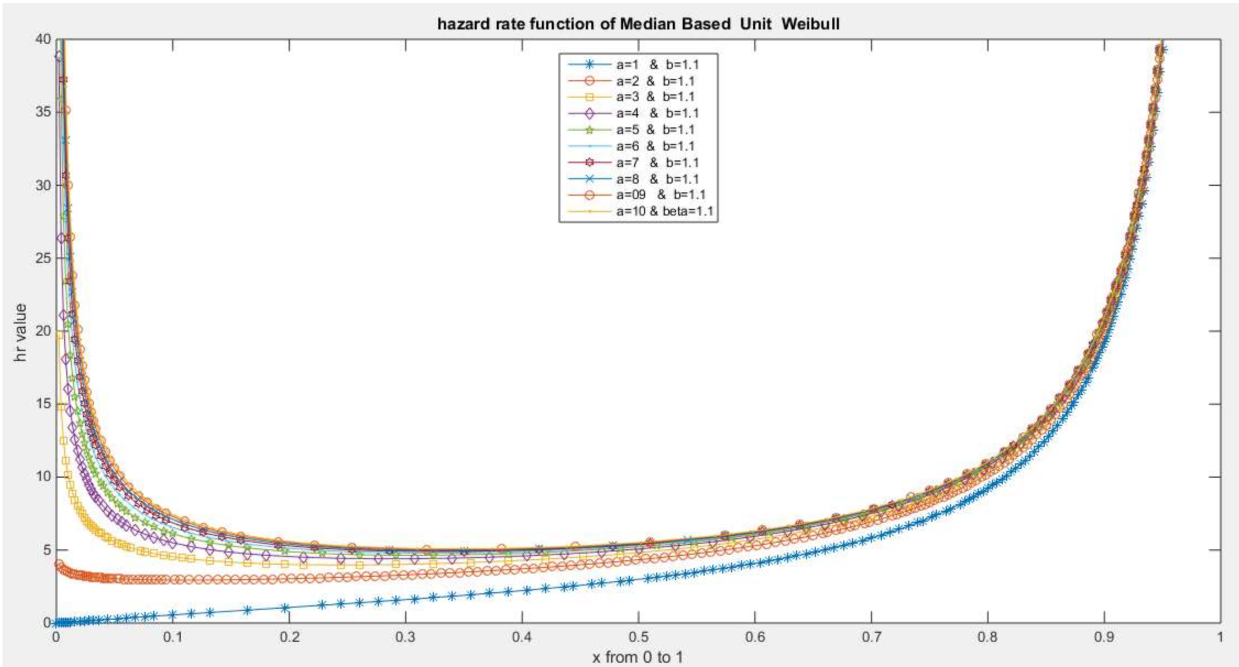

Fig.52 : hr of Median Based Unit Weibull (MBUW) distribution, alpha (from 1 to 10) and beta (1.1)

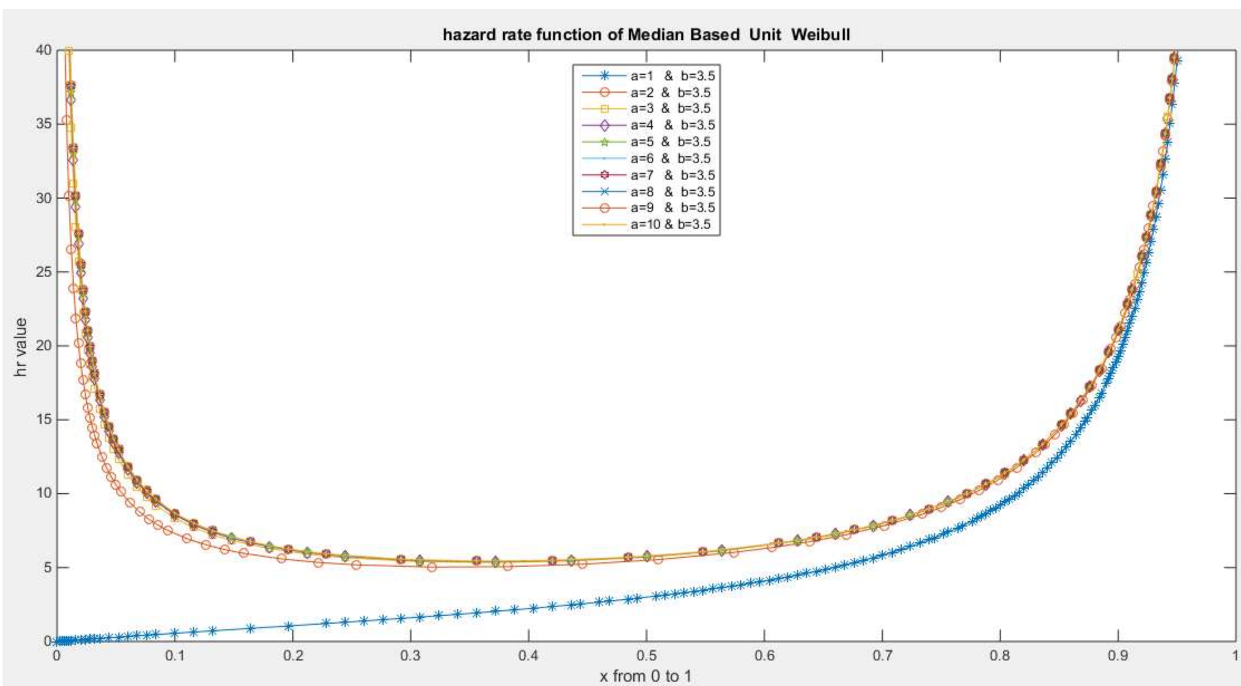

Fig.53 : hr of Median Based Unit Weibull (MBUW) distribution, alpha (from 1 to 10) and beta (3.5)

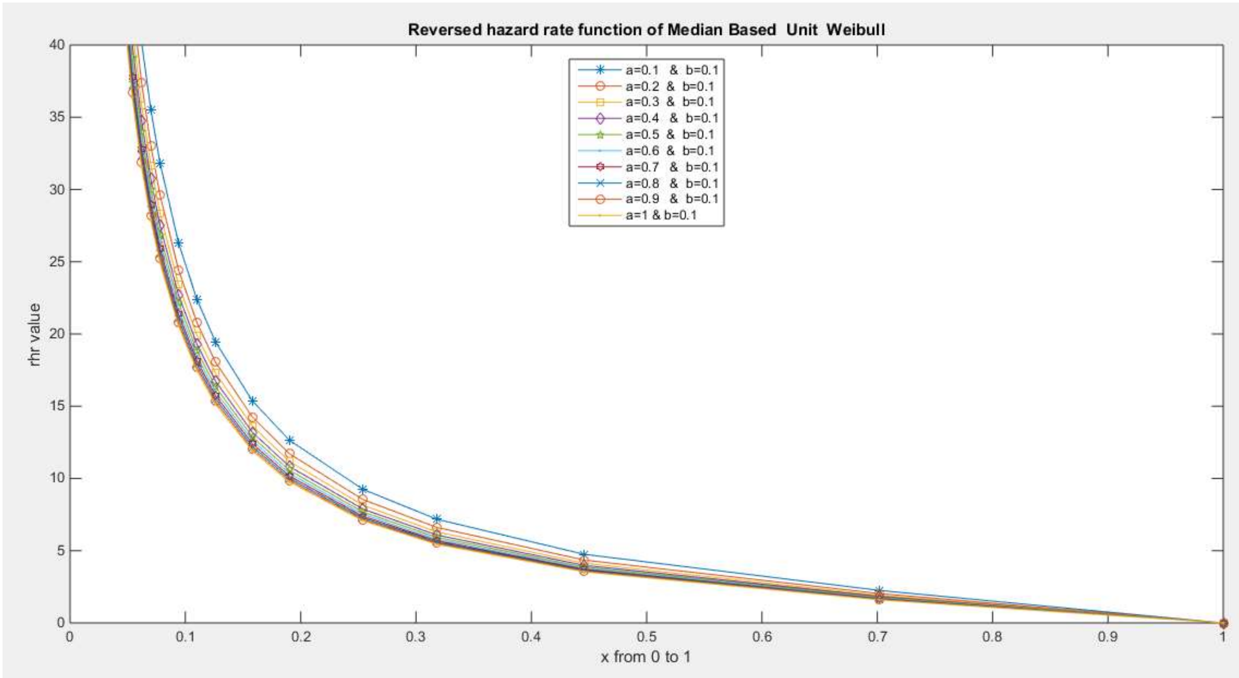

Fig.54 : rhr of Median Based Unit Weibull (MBUW) distribution, alpha (from 0.1 to 1) and beta (0.1)

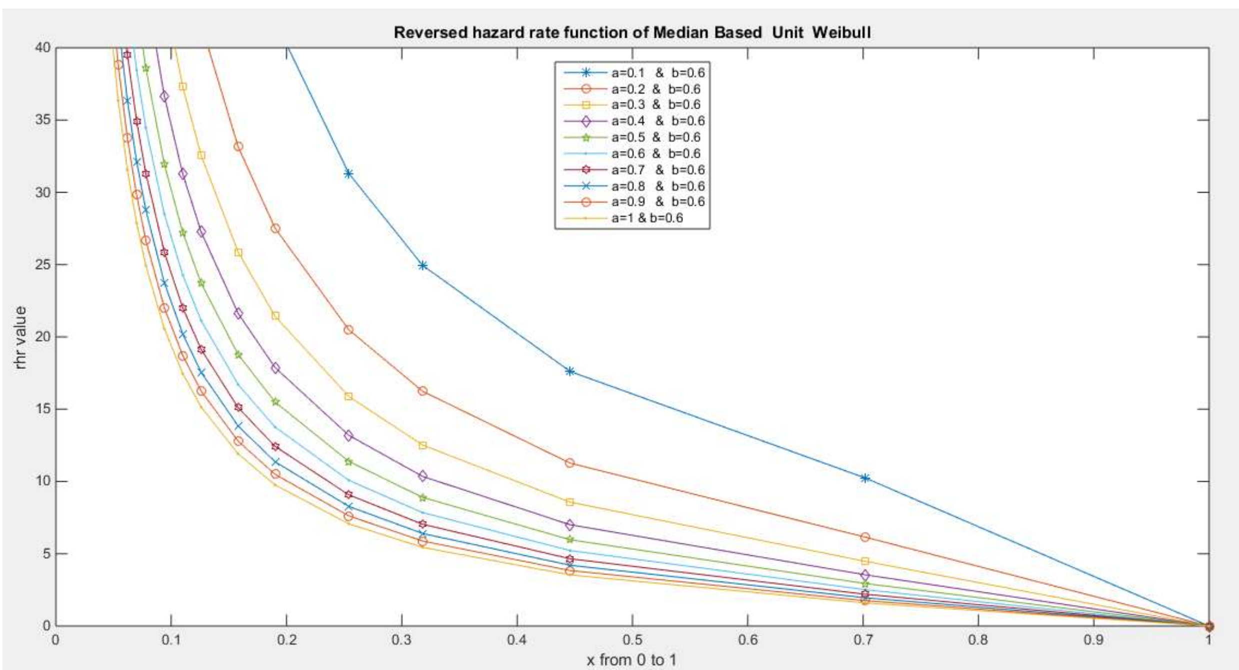

Fig.55: rhr of Median Based Unit Weibull (MBUW) distribution, alpha (from 0.1 to 1) and beta (0.6)

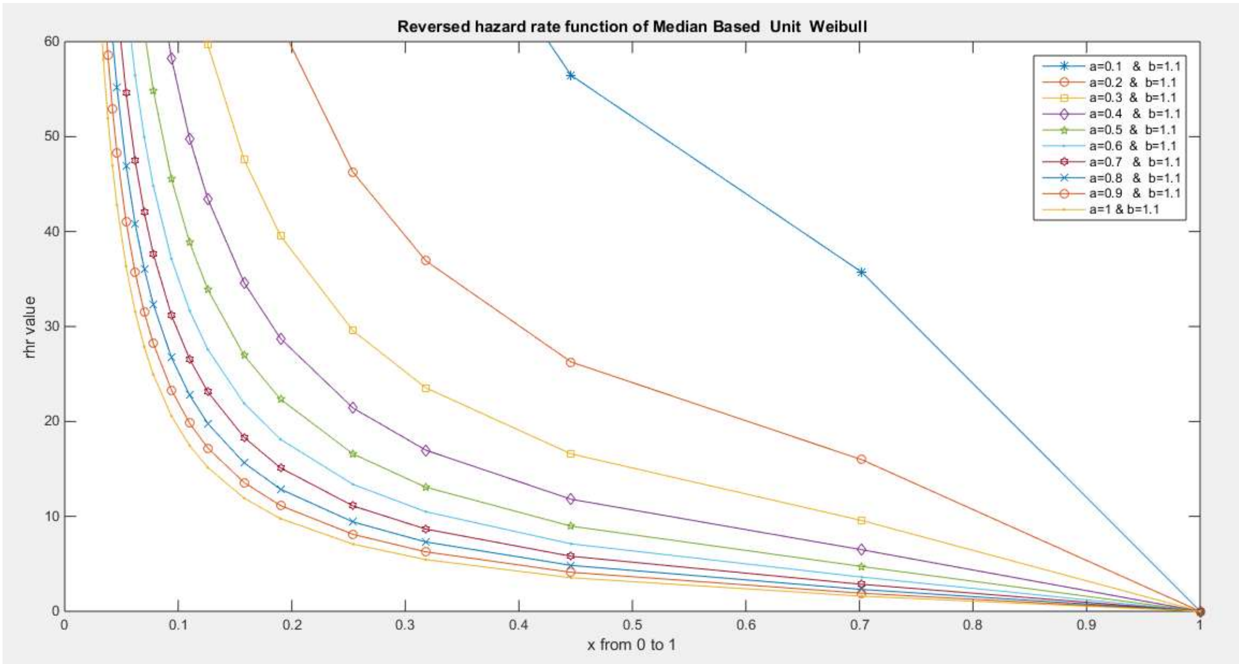

Fig.56 : rhr of Median Based Unit Weibull (MBUW) distribution, alpha (from 0.1 to 1) and beta (1.1)

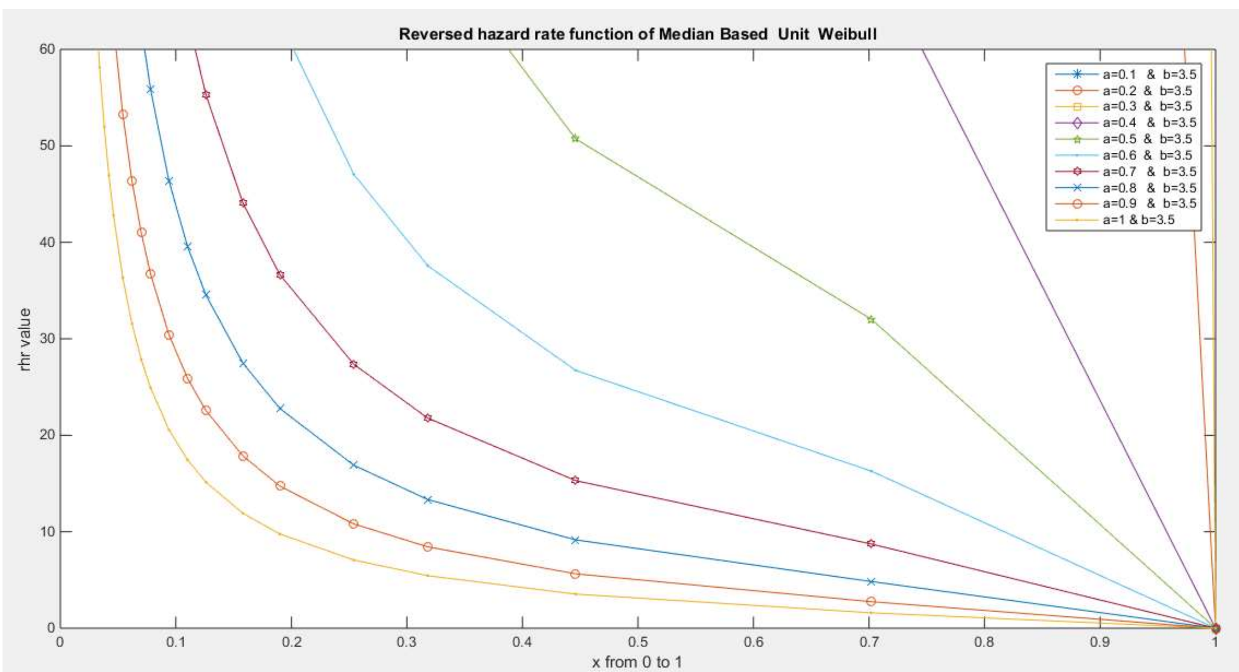

Fig.57 : rhr of Median Based Unit Weibull (MBUW) distribution, alpha (from 0.1 to 1) and beta (3.5)

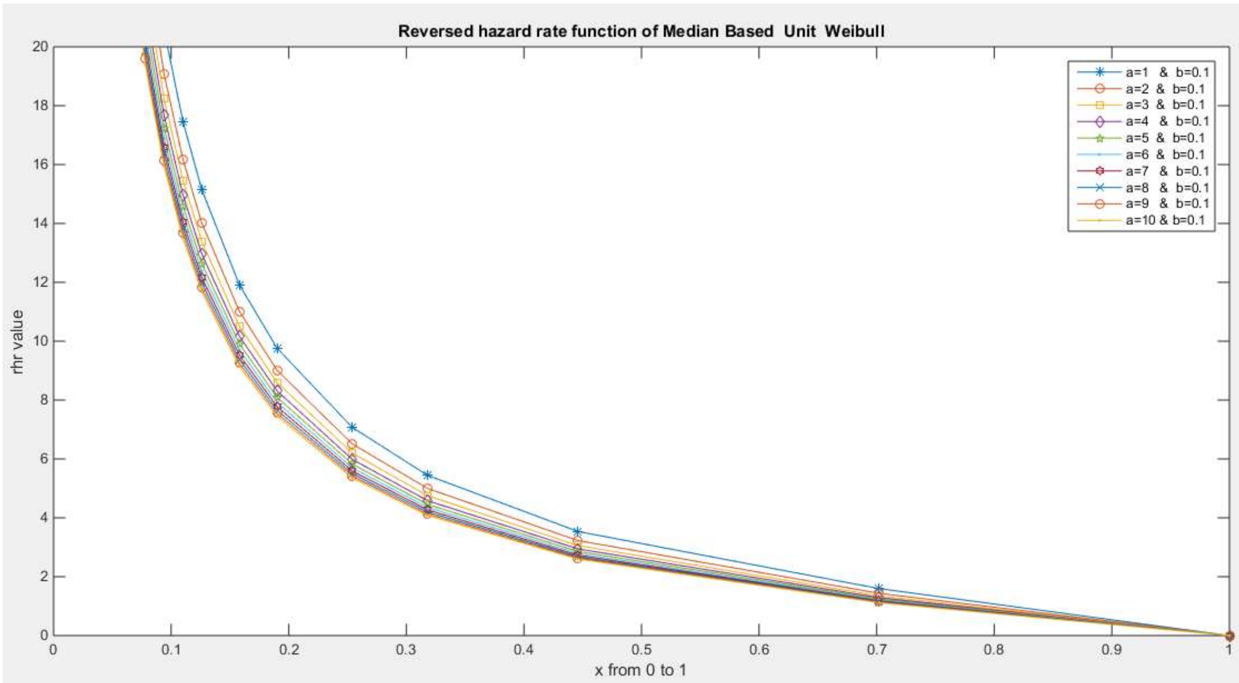

Fig.58 : rhr of Median Based Unit Weibull (MBUW) distribution, alpha (from 1 to 10) and beta (0.1)

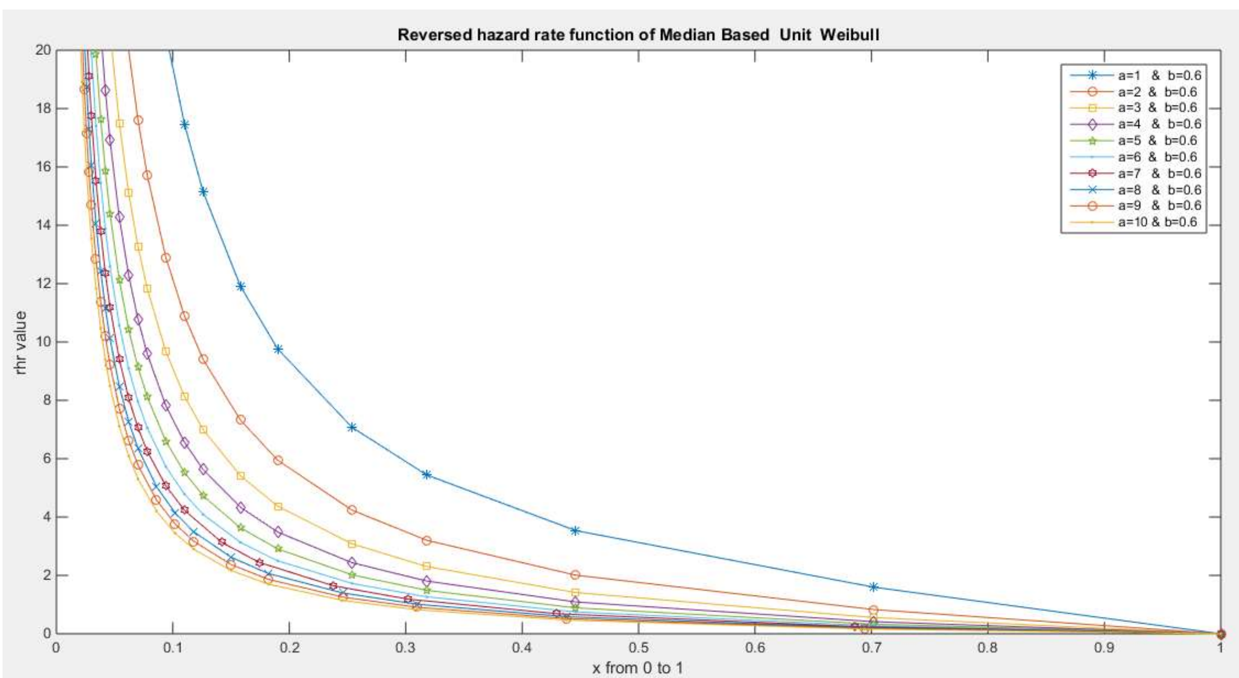

Fig.59 : rhr of Median Based Unit Weibull (MBUW) distribution, alpha (from 1 to 10) and beta (0.6)

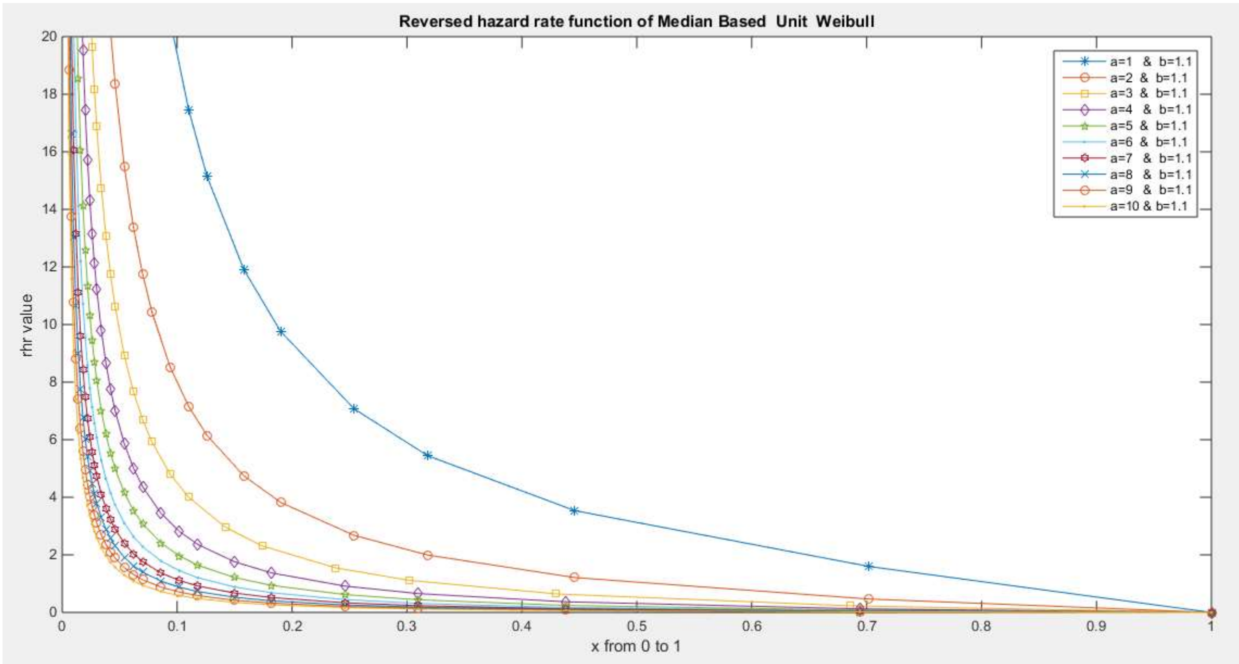

Fig.60 : rhr of Median Based Unit Weibull (MBUW) distribution, alpha (from 1 to 10) and beta (1.1)

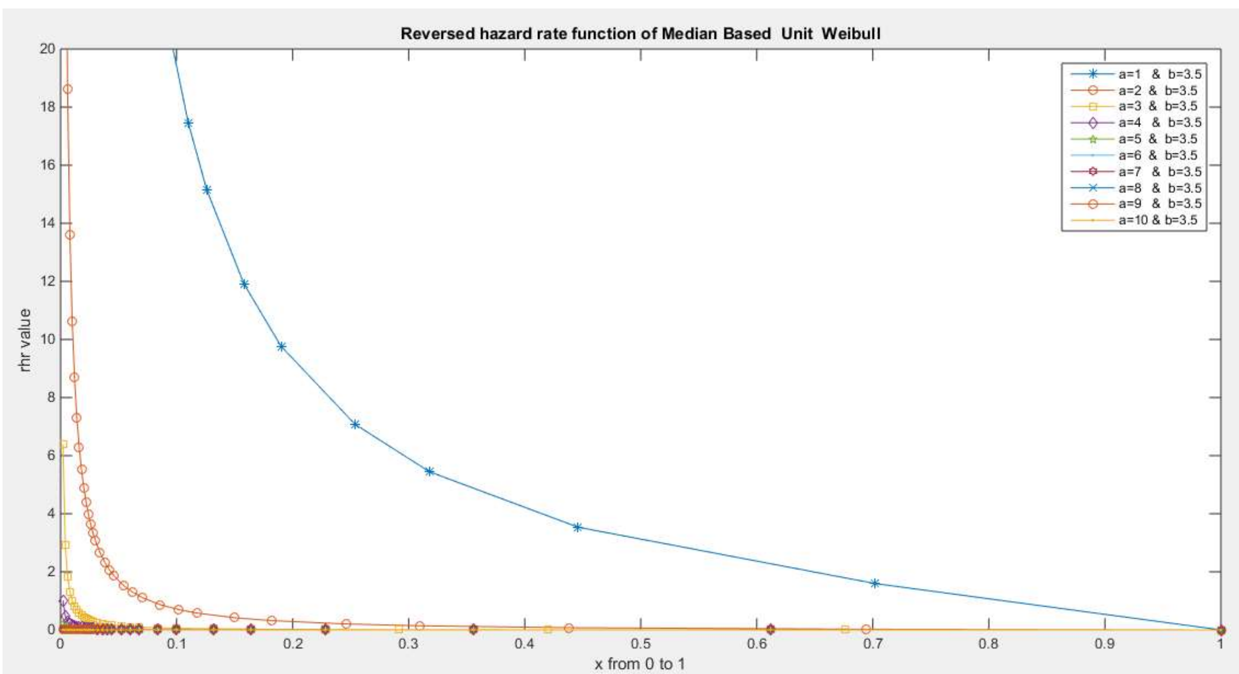

Fig.61 : rhr of Median Based Unit Weibull (MBUW) distribution, alpha (from 1 to 10) and beta (3.5)

5- Quantile Function:

$$u = F(y) = 3y^{\frac{2}{\alpha^\beta}} - 2y^{\frac{3}{\alpha^\beta}} = -2\left(y^{\frac{1}{\alpha^\beta}}\right)^3 + 3\left(y^{\frac{1}{\alpha^\beta}}\right)^2$$

The inverse of the CDF is used to obtain y , the real root of this 3rd polynomial function is :

$$y = F^{-1}(y) = \left\{-.5\left(cos\left[\frac{cos^{-1}(1-2u)}{3}\right] - \sqrt{3}\, sin\left[\frac{cos^{-1}(1-2u)}{3}\right]\right) + .5\right\}^{\alpha\beta}$$

To generate random variable distributed as MBUR:

1- Generate uniform random variable (0,1): $u \sim uniform(0,1)$.

2- Choose alpha and beta levels

3- Substitute the above values of u (0,1) and the chosen alpha and beta in the quantile function, to obtain y distributed as $y \sim MBUW(\alpha, \beta)$

6- r^th Raw Moments:

$$E(y^r) = \frac{6}{(2 + r\alpha^\beta)(3 + r\alpha^\beta)}$$

$$E(y^r) = \int_0^1 y^r \, \frac{6}{\alpha^\beta}\left[1 - y^{\frac{1}{\alpha^\beta}}\right] y^{\left(\frac{2}{\alpha^\beta} - 1\right)} \, dy$$

$$E(y) = \frac{6}{(2 + \alpha^\beta)(3 + \alpha^\beta)}$$

$$E(y^2) = \frac{6}{(2 + 2\alpha^\beta)(3 + 2\alpha^\beta)}$$

$$E(y^3) = \frac{6}{(2 + 3\alpha^\beta)(3 + 3\alpha^\beta)}$$

$$E(y^4) = \frac{6}{(2 + 4\alpha^\beta)(3 + 4\alpha^\beta)}$$

$$\mathrm{var}(y) = E(y^2) - [E(y)]^2$$

$$\mathrm{var}(y) = \frac{78\alpha^{2\beta} + 60\alpha^{3\beta} + 6\alpha^{4\beta}}{(6 + 10\alpha^\beta + 4\alpha^{2\beta})(6 + 5\alpha^\beta + \alpha^{2\beta})^2}$$

<u>7-</u> <u>Coefficient of Skewness:</u>

$$E\,\frac{(y-\mu)^3}{\sigma^3} = \frac{E(y^3) - 3\mu E(y^2) + 3\mu^2 E(y) - \mu^3}{\sigma^3}$$

$$= \frac{E(y^3) - 3\mu[E(y^2) - \mu E(y)] - \mu^3}{\sigma^3} = \frac{E(y^3) - 3\mu[E(y^2) - \mu\mu] - \mu^3}{\sigma^3}$$

$$\text{coefficient of skewness} = \frac{E(y^3) - 3\mu\sigma^3 - \mu^3}{\sigma^3} = \frac{E(y^3) - \mu(3\sigma^2 + \mu^2)}{\sigma^3}$$

<u>8-</u> <u>Coefficient of Kurtosis:</u>

$$E\,\frac{(y-\mu)^4}{\sigma^4} = \frac{E(y^4) - 4\mu E(y^3) + 6\mu^2 E(y^2) - 3\,\mu^4}{\sigma^4}$$

$$= \frac{E(y^4) - 4\mu E(y^3) + 6\mu^2[\sigma^2 + \mu^2] - 3\,\mu^4}{\sigma^4}$$

$$= \frac{E(y^4) - 4\mu E(y^3) + 6\mu^2\sigma^2 + 6\mu^4 - 3\,\mu^4}{\sigma^4}$$

$$= \frac{E(y^4) - 4\mu E(y^3) + 6\mu^2\sigma^2 + 3\mu^4}{\sigma^4} =$$

$$\text{coefficient of Kurtosis} = \frac{E(y^4) - 4\mu E(y^3) + 3\mu^2\left[2\sigma^2 + \mu^4\right]}{\sigma^4}$$

9- Coefficient of Variation :

$$\mathrm{CV} = \frac{S}{\mu}$$

The following Figures illustrate the graphs for the above coefficients

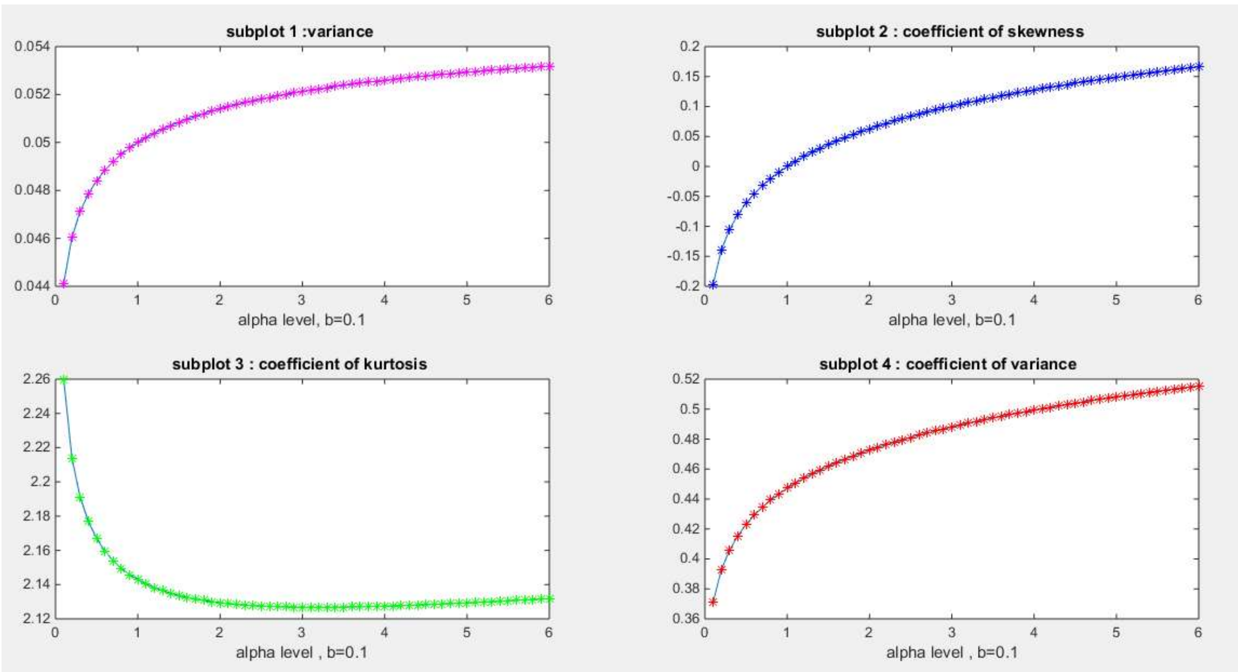

Figure 62: the variance and different coefficients with alpha (from 0.1 to 6) & b=0.1.

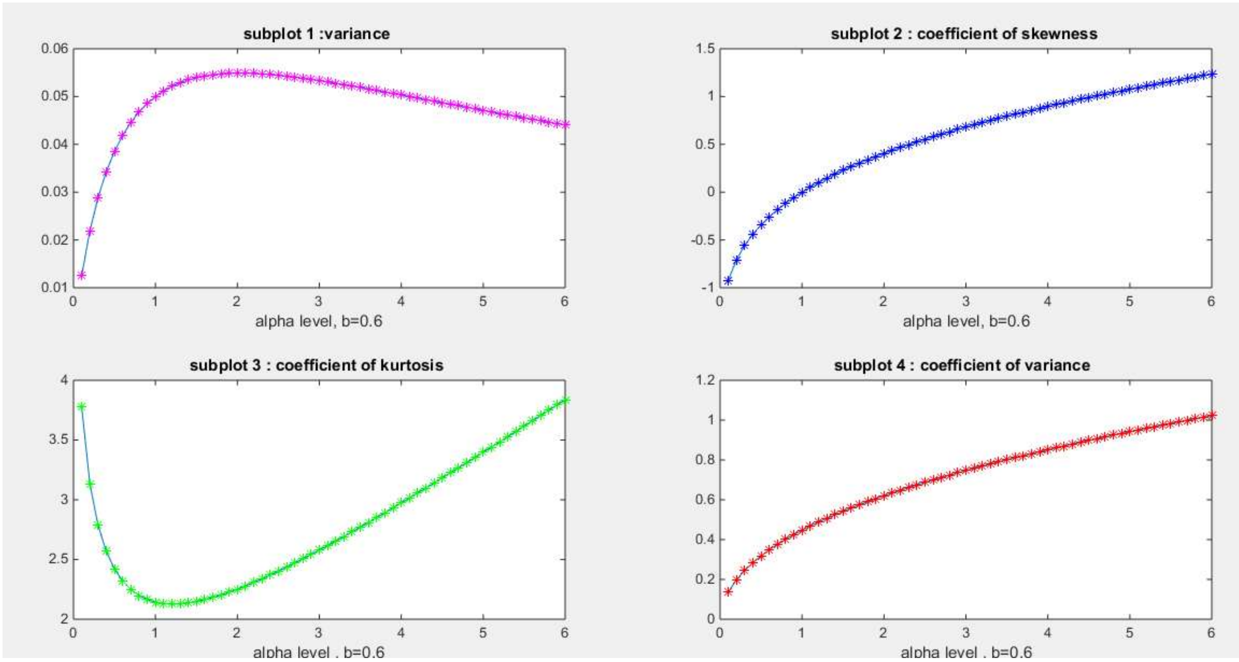

Figure 63: the variance and different coefficients with alpha (from 0.1 to 6) & b=0.6

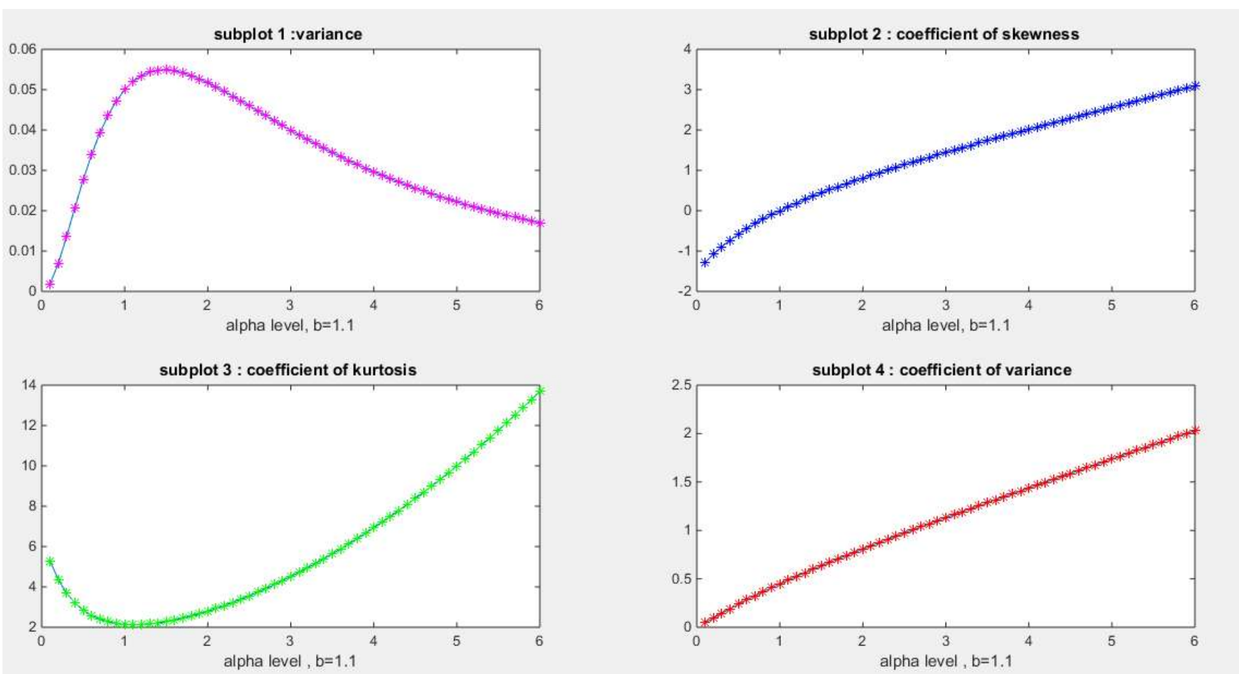

Figure 64: the variance and different coefficients with alpha (from 0.1 to 6) & b=1.1

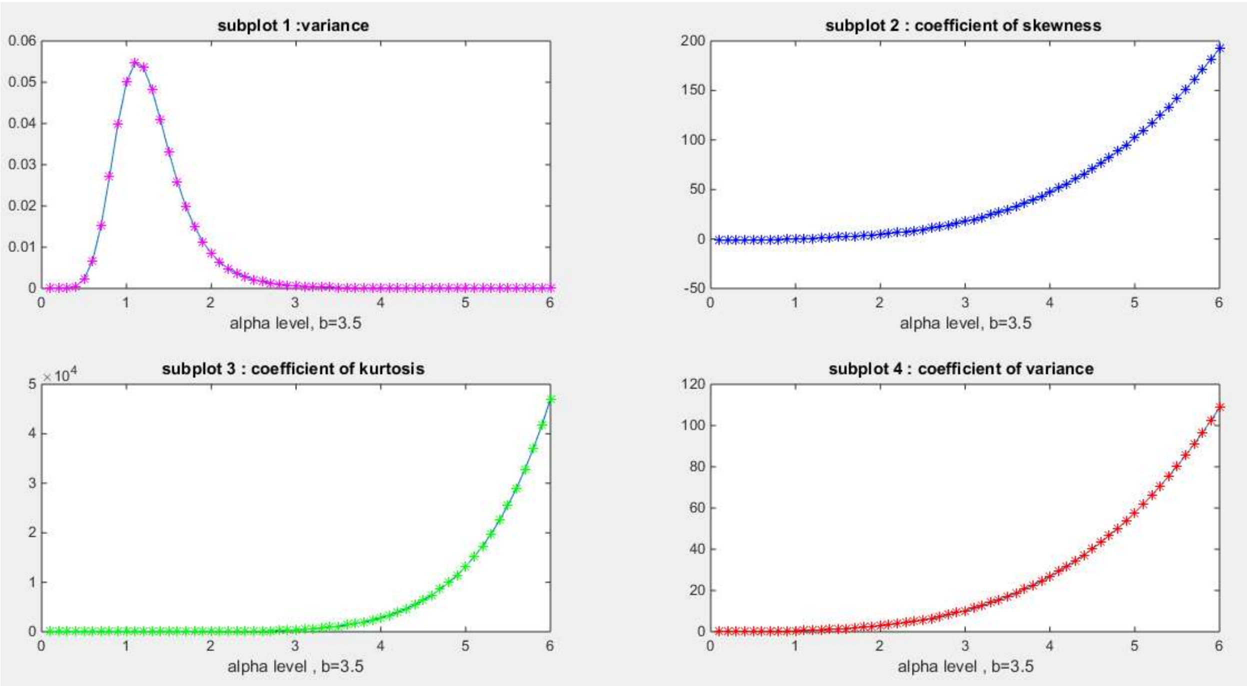

Figure 64: the variance and different coefficients with alpha (from 0.1 to 6) & b=3.5

10-r^th incomplete Moments:

$$E(y^r \mid y < t) = \int_0^t y^r \, \frac{6}{\alpha^\beta}\left[1 - y^{\frac{1}{\alpha^\beta}}\right] y^{\left(\frac{2}{\alpha^\beta}-1\right)} \, dy$$

$$E(y) = \frac{6t^{\frac{2}{\alpha^\beta}+r}}{(2 + r\alpha^\beta)} - \frac{6t^{\frac{3}{\alpha^\beta}+r}}{(3 + r\alpha^\beta)}$$

Conclusion:

This new distribution overcomes some of the weaknesses of the weibull distribution as regard lack of bathtub and unimodal shapes. It is also defined over the unit interval, so it can be used to fit proportions and ratios. It has a well closed form of quantile function and this makes it compatible for parametric quantile regression conditioning on the median or any other quantile rather than conditioning on the mean which is not a good candidate to describe central tendency in such highly skewed distribution.

Future work:

The author is working on methods of estimation of this new distribution and for its applications in regression analysis.

Declarations:

Ethics approval and consent to participate

Not applicable.

Consent for publication

Not applicable

Availability of data and material

Not applicable. Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

Competing interests

The author declares no competing interests of any type.

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Authors' contribution

AI carried the conceptualization by formulating the goals, aims of the research article, formal analysis by applying the statistical, mathematical and computational techniques to synthesize and analyze the hypothetical data, carried the methodology by creating the model, software programming and implementation, supervision, writing, drafting, editing, preparation, and creation of the presenting work.

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