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Title:

The trench pull force: constraints from elasto-plastic bending models

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The trench pull force: constraints from elasto-plastic bending models

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Key Points:

- Unlike isostatic ridge topography, trench topography is non-isostatic, yet its pressure deficit drives the trailing plate.
- Trench pull is a topographic pressure-gradient force: trench depth sets the pressure deficit, and mechanical thickness its equilibration scale.
- Models quantify trench pull at about 2.5 TN m^{-1} for a 3-km trench, comparable to ridge push; strength-model effects are about 10%.

Abstract

The negative buoyancy of subducting slabs is generally regarded as a principal source of plate-driving force, but its mechanical coupling to trailing plates remains uncertain. The conventional slab-pull model represents *slab pull* as a tension-like stress resultant transmitted through the subduction hinge, where it acts as an edge force on the trailing plate. A distinct contribution arises from non-isostatic trench topography: down-bending creates a near-surface pressure deficit relative to an isostatic column in the trailing plate. Because the trench relief is non-isostatic, the column falls outside the standard gravitational potential energy (GPE) framework. Extending that framework requires only retaining the full vertical normal stress that satisfies equilibrium. The trench-pull force is defined here as the change in vertically integrated vertical normal stress between the trench and the first isostatic column, $\Delta\text{GPE}^* = -\Delta\bar{\sigma}_{zz}$. Equilibrium requires the trench-pull force to be balanced jointly by basal shear and a change in the normal-stress-difference resultant, N_D ; the basal term is negligible over the short flexural region, so $\Delta N_D \approx -\Delta\bar{\sigma}_{zz}$. Horizontal gradients of vertical shear stress equilibrate the pressure deficit with depth. Expressing these gradients as an equivalent density recasts the trench-pull force in the familiar density-moment form, with the force determined by the first moment of the combined true and equivalent density difference. Uniform elastic bending predicts an effective moment arm near half the plate thickness. Finite-element models show that this scaling persists across elastic and elasto-plastic plates and variations in loading, background stress and strength distribution. At fixed thickness, trench-pull forces lie within approximately 10% of a common pull-deflection relation; across different thicknesses, they increase nearly in proportion to plate thickness. Applying the scaling to the mechanical thickness of old oceanic lithosphere gives approximately 2.5 TN m^{-1} for a representative trench depth, comparable to ridge push. The combined ridge-to-trench driving force arising from the topographic pressure gradient is approximately 5 TN m^{-1} , sufficient to balance basal drag across an entire oceanic plate for plausible magnitudes of basal shear stress.

Plain Language Summary

Oceanic plates stand high at mid-ocean ridges and bend downward into trenches. The pressure difference associated with ridge topography produces the familiar ridge-push force. This paper shows that trench topography produces another horizontal force, termed trench pull, even though the trench is supported by plate bending rather than by isostasy. Trench depth determines the pressure deficit, while the thickness of the mechanically strong plate determines how far that deficit extends with depth. Numerical bending models show that the resulting force is surprisingly insensitive to the detailed strength distribution within the plate. A typical 3-km trench produces approximately 2.5 trillion newtons per metre of trench, comparable to ridge push. Trench pull may therefore provide an important part of the force that moves oceanic plates towards subduction zones.

1 Introduction

Subducting slabs form the principal convective downwellings of Earth's plate-tectonic convection system, and their negative buoyancy relative to the surrounding mantle is generally regarded as the principal source of the force driving the horizontal motion of the surface plates (Forsyth & Uyeda, 1975; Hager & O'Connell, 1981; Conrad & Lithgow-Bertelloni, 2002; Bercovici et al., 2015). How this vertical buoyancy force produces a horizontal driving force on trailing plates, however, is not well established. The conventional slab-pull model treats the slab and trailing plate as a mechanical *stress guide* (Orowan, 1965; Elsasser, 1969): some fraction of the slab's negative-buoyancy force is transmitted back through the subduction hinge and along the attached surface plate as a tension-

like force (e.g., Davies, 2022). The portion assumed to be available to drive the trailing plate is conventionally termed net slab pull; it is represented locally as a horizontal edge force and globally, on the spherical Earth, as a tangential torque. Although this stress-guide construction provides a route for transmitting stress, the mechanics by which the force changes direction through the hinge remain somewhat opaque (Bird et al., 2008; Bercovici et al., 2015). In this context, tension-like does not imply absolute tension. Lithospheric principal stresses are predominantly compressive (Richter & McKenzie, 1977); the term describes a positive normal-stress-difference resultant, N_D , for which the stress along the plate is less compressive than the vertical stress when integrated through the plate thickness.

Estimates of net slab pull differ widely. Forsyth and Uyeda (1975) inferred that mantle resistance acting directly on a descending slab balances about 90% of its negative buoyancy force. Conrad and Lithgow-Bertelloni (2002) found that observed plate motions were best reproduced when attached upper-mantle slabs supplied about half of the total plate-driving force. In their supporting calculations, transmitting 40–100% of the negative buoyancy force of upper-mantle slabs to the plates as slab pull gives maximum plate-averaged tension-like stresses of 200–500 MPa. In the notation used here, these stresses correspond to $N_D = 12\text{--}30 \text{ TN m}^{-1}$ for a 60-km-thick plate, where N_D is the positive normal-stress-difference resultant defined in Equation (4). Other torque-balance approaches and intraplate-deformation constraints infer much smaller magnitudes for net slab pull: approximately 2.2 TN m^{-1} globally and $0.9\text{--}2.8 \text{ TN m}^{-1}$ along Java and Sumatra (Bird, 1998; Bird et al., 2008, 2026; M. Sandiford et al., 2005). The disagreement therefore does not concern the existence of a large reservoir of slab buoyancy, but the mode and magnitude of its coupling to the trailing plate.

Tension-like transmission is not the only way that slab loading can contribute to the horizontal force balance. Slab loading also bends the trailing plate downward, replacing rock with seawater and creating a pressure deficit proportional to trench depth. Bercovici et al. (2015) provide the broader context: horizontal pressure gradients drive the lateral motion of convective thermal boundary layers, and slab pull is associated with the concentrated low pressure produced where slabs pull away from the surface. Richter and McKenzie (1977) identified the corresponding unresolved problem at trenches: although the pressure deficit might help drive the trailing plate, the stress distribution maintaining the non-isostatic topography was unknown. Consequently, it was “not even clear whether any of the pressure reduction is available to drive the plates.”

Trench deflection itself is described by a mature theory of lithospheric flexure. Classical thin-plate treatments represent slab loading through a vertical shear resultant V and bending moment M (Parsons & Molnar, 1976; Turcotte et al., 1978; Garcia et al., 2019). These resultants describe the loading and deflection, but integration through the plate thickness removes the depth distribution of shear-stress gradients needed to determine how the pressure deficit equilibrates with depth. Davies (1983) proposed that this contribution could be comparable to the vertical shear resultant at the trench, $|V(x_T)|$.

Hence, while previous studies link the pressure deficit beneath trenches—or, more generally, above mantle downwellings—to plate motion, they do not quantify the horizontal force arising specifically from flexural trench topography. This paper refers to the surface pressure anomaly generated by trench topography, $(\rho_m - \rho_w)gw_T = \Delta\rho gw_T$, as the *trench pressure deficit*, where $\Delta\rho = \rho_m - \rho_w$ and $w_T = w(x_T)$ is the plate-top deflection at the trench column relative to the first isostatic column. In the numerical models, x_T is the leftmost vertical line that crosses the full thickness of the deformed plate. The horizontal force contribution associated with this deficit is the *trench-pull force*, following Bird (1998). Figure 1 places it in the force balance of an idealized trailing plate. The isostatic column closest to the trench, x_I , divides the plate into two force-balance intervals. The interval from the trench to x_I isolates the non-isostatic trench contribution; the interval from x_I to the ridge, x_R , isolates the classical isostatic ridge-push con-

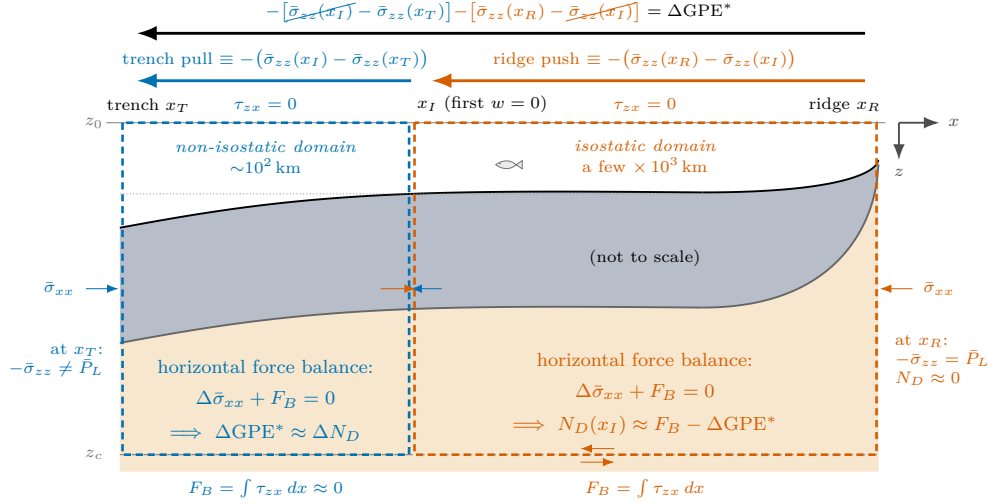


Figure 1. The vertically integrated horizontal force balance for an idealized subducting plate (vertical exaggeration). The first isostatic column, x_I , divides the trailing plate into a non-isostatic trench-side domain (blue) and an isostatic ridge-side domain (orange). This paper addresses the force balance across the non-isostatic domain between the trench, x_T , and x_I . The ridge-side domain between x_I and the ridge, x_R , represents the conventional ridge-push problem associated with the isostatic subsidence of cooling oceanic lithosphere. Because the two domains share the column at x_I , their force balances are additive; the black arrow shows the combined ridge-to-trench topographic pressure-gradient force. Each balance is obtained by integrating the boundary tractions around the corresponding dashed rectangle. The upper boundary is an equipotential level $z_0 = 0$, above the rock–water interface and above any lateral density differences, and carries no horizontal shear traction. The horizontal balance therefore involves the horizontal normal-stress resultants on the vertical boundaries and the basal shear force, F_B . Across the short non-isostatic domain, $L_{TI} = O(10^2 \text{ km})$, basal drag is negligible, reducing the balance to $\Delta\bar{\sigma}_{xx} \approx 0$. Section 2.2 decomposes this resultant as $\Delta\bar{\sigma}_{xx} = \Delta\bar{\sigma}_{zz} + \Delta N_D$, giving $-\Delta\bar{\sigma}_{zz} \equiv \Delta\text{GPE}^* \approx \Delta N_D$. The blue arrow identifies this vertical-normal-stress contribution as the trench-pull force; the orange arrow shows the corresponding ridge-push contribution. The domain labels identify the two force-balance contributions and do not imply that every intervening column is locally isostatic or non-isostatic.

tribution (Lister, 1975; Turcotte & Schubert, 2002; Bercovici et al., 2015). Each balance is obtained by integrating the tractions around its boundary. When the two balances are combined, the force resultants on their shared column cancel and the two contributions add to the total ridge-to-trench balance. This paper addresses the part of that balance associated with the non-isostatic domain between x_T and x_I .

The vertically integrated horizontal force balance developed in Section 2 identifies differences in $\bar{\sigma}_{zz}$ as one contribution to the horizontal balance. Applied to the non-isostatic region between the trench column x_T and the first isostatic column x_I , this gives

$$\text{trench-pull force} \equiv -\Delta\bar{\sigma}_{zz} = -[\bar{\sigma}_{zz}(x_I) - \bar{\sigma}_{zz}(x_T)] \equiv \Delta\text{GPE}^*, \quad (1)$$

where $\bar{\sigma}_{zz}$ is the vertically integrated vertical normal stress. The complete horizontal force balance can be decomposed into contributions whose sum is zero in equilibrium. Equation (1) isolates the contribution associated with the difference in vertically integrated

vertical normal stress; whenever this term is finite, it must be balanced by the remaining contributions. For lithostatic columns, where $-\Delta\bar{\sigma}_{zz} = \Delta\bar{P}_L$, this term reduces to the conventional GPE difference used to calculate ridge push (Lister, 1975; Dahlen, 1981). For non-isostatic columns, horizontal gradients of vertical shear stress contribute to vertical equilibrium. Their distribution with depth determines how the pressure difference returns to zero at the compensation depth and therefore determines its vertical integral. Retaining the full vertical normal stress extends the same force expression to these columns. This GPE-like force contribution is denoted ΔGPE^* , where the asterisk may be read as “GPE-like.” Thin-plate flexure retains the vertical shear resultant but not the depth distribution of its gradient, and therefore cannot determine this contribution to the horizontal force balance.

Section 2 develops the force balance for the non-isostatic trench column, identifies the depth distribution of the horizontal gradient of vertical shear stress as the information required to calculate the trench-pull force, and derives an effective arm near half the plate thickness for a uniform elastic plate at small deflection. Section 3 introduces finite-thickness elastic and elasto-plastic numerical models, and Section 4 tests this scaling across variations in strength structure, loading and background N_D . Section 5 maps plate thickness to the mechanical thickness of oceanic lithosphere, estimates the magnitude of trench pull, and considers its relationship to ridge push, slab pull and the plate-wide topographic pressure gradient.

2 Theory: GPE-like force contributions from non-isostatic columns

This section develops the vertically integrated horizontal force balance for topography supported by isostasy or lithospheric flexure. Vertical normal stress enters the horizontal balance through its contribution to pressure. Applied to the non-isostatic region in Figure 1, the development identifies the depth distribution of shear-stress gradients beneath the trench as the key quantitative unknown. Expressing the shear-stress gradient as an equivalent density then allows the vertical-normal-stress contribution—and hence the same force contribution—to be recovered using the familiar density-based GPE calculation. Uniform elastic and symmetric elasto-plastic plates provide estimates for comparison with the numerical models that follow.

2.1 Scales, coordinates and assumptions

The analysis considers a two-dimensional, plane-strain section normal to the trench, with forces reported per unit trench length. The Cartesian coordinate x is horizontal and z is positive downward from a common horizontal reference level. Stress is negative in compression.

The relevant horizontal scale is the trench–first-isostatic-column distance, $L_{TI} = O(10^2 \text{ km})$ (Caldwell et al., 1976; Hunter & Watts, 2016; Watts, 2001). Because this distance is much smaller than the radius of the Earth, equipotentials are treated as horizontal and Earth curvature is neglected. Flexural topography varies across a short cross-trench length scale set by the thickness of the load-bearing plate, whereas trenches extend much farther along strike. Along-strike support is therefore omitted, providing the basis for the two-dimensional plane-strain approximation.

The analysis assumes static equilibrium, constant vertical gravity and no self-gravitation. Trench relief is treated as flexurally supported, as in classical models of lithospheric bending; mantle-flow-supported dynamic topography is not included. Differences in the full vertical normal stress are assumed to have fully equilibrated by the base of the plate. Oceanic lithosphere instead weakens gradually with depth, and Section 5.2 argues that this equilibration depth is best represented by the mechanical thickness h_m . The general balance retains basal shear explicitly; it is neglected only after its magnitude has

been assessed over the short flexural region (L_{TI}), completing the hydrostatic approximation beneath the plate. The limitations of representing the lithosphere by a plate of finite thickness are considered in Supporting Information Text S3.

2.2 The vertically integrated horizontal force balance

Consider a rectangular region bounded by any two vertical columns x_0 and x_1 . The horizontal tractions on its sides are carried by σ_{xx} , while shear stress acts along its base at z_c . Define $\Delta(\cdot) \equiv (\cdot)(x_1) - (\cdot)(x_0)$, and let an overbar denote a vertical integral from an upper level z_0 , above which density differences between the columns vanish, to the compensation depth z_c , where lateral pressure differences vanish under the hydrostatic assumption. This upper level is adopted as the coordinate origin, $z_0 = 0$; its absolute elevation is immaterial. Because the boundaries follow the Cartesian axes, the horizontal component of static equilibrium reduces to

$$\Delta \bar{\sigma}_{xx} + F_B = 0, \quad F_B \equiv \int_{x_0}^{x_1} \tau_{zx}(x, z_c) dx. \quad (2)$$

Here and throughout, force is per unit length parallel to the trench. Supporting Information Text S1.1 derives Equation (2) directly from the boundary-traction balance.

Under the assumptions made here, Equation (2) is a complete horizontal force balance, but in this form it is not especially diagnostic. The integrated horizontal normal stress $\bar{\sigma}_{xx}$ combines changes in pressure associated with the density distribution and vertical support of a column with changes in the horizontal-vertical normal-stress difference. It therefore conceals how the vertical force balance contributes to the traction acting on a vertical plane. To expose these contributions, first write the horizontal normal stress as the sum of the mean stress σ_m (minus the pressure) and the horizontal deviatoric stress. The same mean stress can be written using the vertical components (i.e., $\sigma_m = \sigma_{zz} - \tau_{zz}$), so

$$\sigma_{xx} = \sigma_m + \tau_{xx} = \sigma_{zz} + (\tau_{xx} - \tau_{zz}). \quad (3)$$

This form isolates the full vertical normal stress—the component constrained by vertical equilibrium—from the remaining horizontal-vertical normal-stress difference. The difference is the same whether written using the full or deviatoric normal stresses, and its depth integral defines the normal-stress-difference resultant,

$$N_D \equiv \int_0^{z_c} (\tau_{xx} - \tau_{zz}) dz = \int_0^{z_c} (\sigma_{xx} - \sigma_{zz}) dz, \quad (4)$$

and hence

$$\bar{\sigma}_{xx} = \bar{\sigma}_{zz} + N_D. \quad (5)$$

A positive N_D does not imply absolute tension. On lithospheric scales the principal stresses are generally compressive; positive N_D means that the column is less compressed horizontally than vertically—a stress state commonly described as extensional because its corresponding deformation or strain-rate regime involves horizontal extension.

Writing

$$\text{GPE}^* \equiv -\bar{\sigma}_{zz} \quad (6)$$

and substituting Eq. (5) into Eq. (2) gives

$$\Delta N_D = \Delta \text{GPE}^* - F_B. \quad (7)$$

Each term in Equation (7) represents a net force or force contribution over the region considered, and any non-zero contribution must be balanced by one or more of the others. For example, a GPE contribution associated with ridge push may be balanced by basal drag, a change in the normal-stress-difference resultant N_D , or both.

Here ΔGPE^* denotes the GPE-like force contribution obtained from the difference in vertically integrated vertical normal stress between two columns; the asterisk may be read as “GPE-like”. A conventional GPE difference can still be calculated from their true-density structures, but when either column is non-isostatic it is no longer proportional to the corresponding horizontal force contribution. The lithostatic stress calculated from density alone omits the contribution of vertical shear-stress gradients and therefore does not satisfy vertical equilibrium. The starred and unstarred quantities are equal when both columns are lithostatic ($-\sigma_{zz} = P_L$).

In either case, a finite force requires the vertical-normal-stress difference to have a non-zero depth integral. Such differences do not require topography, but in ridge push and trench pull they arise from the density contrast created where surface relief exchanges rock with much lower-density water or air. This class of ΔGPE^* force is termed a topographic pressure-gradient force.

2.3 Vertical equilibrium and the depth distribution of stress

The horizontal force balance above isolates the contribution from the vertically integrated vertical normal stress. Evaluating this contribution requires knowing how σ_{zz} varies with depth. Vertical equilibrium shows that σ_{zz} at any depth depends on both the weight above and the accumulated contribution from the horizontal gradient of vertical shear stress:

$$-\sigma_{zz}(z) = \underbrace{\int_0^z \rho g d\xi}_{P_L(z) \text{ lithostatic pressure}} + \underbrace{\int_0^z \tau_{zx,x} d\xi}_{Q(z) \text{ shear function}}, \quad \tau_{zx,x} \equiv \frac{\partial \tau_{zx}}{\partial x}. \quad (8)$$

The lithostatic pressure P_L is the weight of the overburden, and the shear function Q is the depth-integrated contribution from $\tau_{zx,x}$. For brevity, the horizontal gradient of vertical shear stress, $\tau_{zx,x}$, is referred to below as the *shear-stress gradient*. Thus, two columns can differ in vertical normal stress through differences in their density structure, their shear-stress-gradient distribution, or both.

Define the vertical shear resultant through the column as

$$V(x) \equiv \int_0^{z_c} \tau_{zx}(x, z) dz. \quad (9)$$

At the compensation depth, the shear function is therefore

$$Q(z_c) = \int_0^{z_c} \tau_{zx,x} dz = \frac{dV}{dx}, \quad (10)$$

An isostatic column satisfies $Q(z_c) = 0$. This is the condition required for isostasy at the compensation depth; it does not require $Q(z)$, or the shear-stress gradient, to vanish at every depth. Although isostasy is commonly conflated with a lithostatic stress state throughout the column, the latter is the stronger condition (Dahlen, 1981).

Integrating this full vertical normal stress gives the GPE-like force contribution directly. Expressing the shear-stress gradient in density units allows the same calculation to be written in the familiar density-based form. The shear-stress gradient $\tau_{zx,x}$ has dimensions of force per unit volume. Dividing it by g therefore expresses its contribution to vertical equilibrium in density units. Define the equivalent density $\hat{\rho}$ and corrected density ρ^* as

$$\hat{\rho} \equiv \frac{\tau_{zx,x}}{g}, \quad \rho^* \equiv \rho + \hat{\rho}. \quad (11)$$

With these definitions, Equation (8) can be written

$$\begin{aligned} -\sigma_{zz}(z) &= g \int_0^z \rho^*(\xi) d\xi, \\ \text{GPE}^* &\equiv -\bar{\sigma}_{zz} = g \int_0^{z_c} \int_0^z \rho^*(\xi) d\xi dz. \end{aligned} \quad (12)$$

Equation (12) allows GPE^* to be evaluated from the corrected density ρ^* using the familiar density-based GPE expression. The correction to density incorporates the shear-stress gradient in equivalent-density units, so the result is exactly $-\bar{\sigma}_{zz}$ for the vertical normal stress that satisfies equilibrium. The asterisks on ρ^* and GPE^* thus mark the same extension beyond the density-only, lithostatic case. The resulting GPE^* is therefore the vertical-normal-stress contribution that enters the vertically integrated horizontal force balance in Equation (7).

For any two columns whose vertical normal stresses are equal at z_c , Equation (12) requires their corrected-density difference to have zero integral:

$$\int_0^{z_c} \Delta\rho^* dz = 0. \quad (13)$$

Because the corrected-density difference has zero integral, its force can be written as a first moment:

$$\Delta\text{GPE}^* = \int_0^{z_c} \Delta(-\sigma_{zz}) dz = -g \int_0^{z_c} z \Delta\rho^*(z) dz. \quad (14)$$

Equation (14) shows that the force depends on the first moment of the corrected-density difference. Because that difference has zero net integral, its positive and negative parts have equal integrated magnitudes. Its first moment is therefore given exactly by this common magnitude multiplied by the separation between their centres of mass. This separation is referred to as the dipole arm; its formal reduction is given in Supporting Information Text S1.4.

Normalising the force by the common pressure magnitude ΔP of either lobe defines an effective moment arm,

$$a_{\text{eff}} \equiv \frac{\Delta\text{GPE}^*}{\Delta P}, \quad \Delta P = (\rho_m - \rho_w)gw_T = \Delta\rho gw_T \quad \text{for trench pull.} \quad (15)$$

For the two-lobe construction, a_{eff} is the separation between the lobe centres of mass. The same definition applies to ridge push using its corresponding pressure surplus.

The extension to non-isostatic columns can therefore be evaluated in two equivalent forms. The fundamental form retains the full vertical normal stress that satisfies equilibrium and gives the force directly as $\Delta\text{GPE}^* = -\Delta\bar{\sigma}_{zz}$; the effect of the shear-stress gradient is already contained in that stress. Alternatively, the shear-stress gradient can be expressed as an equivalent density and included with the true density in the same first-moment construction used for density-based GPE. The dipole structure follows from vertical equilibrium in either form; the equivalent-density notation makes its relation to the familiar lithostatic case explicit. In either form, the force depends on the depth distribution of the density and shear-stress-gradient differences between the columns. Supporting Information Text S1.3 gives the first-moment expression with these contributions written separately.

2.4 Application to trench pull

The general balance is now applied to the non-isostatic region by setting $x_0 = x_T$ at the trench and $x_1 = x_I$ at the first isostatic column. The left-hand dashed blue box in Figure 1 locates this region within the idealised trailing plate; the adjacent box shows

the isostatic region extending towards the ridge. For the trench construction, the rock–water exchange caused by deflection is taken to be the principal true-density difference between x_T and x_I , and the first isostatic column is approximated as lithostatic throughout, so that $\hat{\rho}_I = 0$. Supporting Information Text S3.1 evaluates density differences omitted from this simplified construction, including those created when deflection vertically offsets otherwise identical thermal and compositional structures at the two columns.

For the vertical normal stresses at the two columns to equilibrate at z_c , the weight removed by replacing rock with water must be balanced by the depth-integrated shear-stress gradient. With $w_T = w(x_T)$ denoting the positive deflection at the trench column,

$$Q_T(z_c) - Q_I(z_c) = (\rho_m - \rho_w)gw_T = \Delta\rho gw_T. \quad (16)$$

This pressure magnitude is the trench pressure deficit. Equation (16) expresses the required balance through the difference in shear function between the two columns.

Under the stated assumptions, the corrected-density closure can be separated into non-overlapping shallow and deep intervals:

$$0 = \int_0^{z_c} \Delta\rho^* dz = \underbrace{\int_0^{w_T} \Delta\rho dz}_{(\rho_m - \rho_w)w_T} + \underbrace{\int_{w_T}^{z_c} \Delta\hat{\rho} dz}_{-(\rho_m - \rho_w)w_T}. \quad (17)$$

Equation (17) describes the difference between the trench column and the first isostatic column. Under the stated approximations, it contains two non-overlapping contributions: the shallow true-density difference created by exchanging rock for water, and the deeper shear-stress-gradient difference expressed in equivalent-density units. Their integrated magnitudes are exactly equal, so the corrected-density difference has zero net integral and the vertical normal stresses equilibrate at z_c . Figure 2a shows these contributions as the two shaded lobes, and panel (b) shows the corresponding vertical-normal-stress difference returning to zero with depth. Equation (17) constrains the integrated magnitude of the deeper lobe, but not its shape. The parabolic form shown in panel (a) follows from the uniform-bending-plate solution developed in Section 2.5; its derivation is given in Supporting Information Text S4.

Figure 2 presents these equivalent descriptions as density differences in panels (a,c) and vertical-normal-stress differences in panel (b). The corresponding ridge-push construction in Figure 2c has the same form, except that both lobes are true-density differences. The shaded lobes in panels (a) and (c) have equal areas but do not appear equal because their horizontal axes are broken; the dots mark their centres of mass. The areas beneath the vertical-normal-stress differences in panel (b) give the trench-pull force, ΔGPE^* , and ridge-push force, ΔGPE . Because both are evaluated relative to the same isostatic column, they are additive contributions to the total ridge-to-trench force balance.

Equation (16) constrains the depth integral of the shear-stress gradient, but not its distribution with depth. Figure 2b shows why the distinction matters: the trench pressure deficit, $\Delta\rho gw_T$, sets the maximum near-surface amplitude of the vertical-normal-stress difference, while its depth of equilibration determines the area beneath the curve. If that maximum amplitude persisted to z_c , as illustrated by the dashed blue line, the enclosed rectangular area would be $(\Delta\rho gw_T)z_c$. This provides an upper limit for the trench-pull force under the assumed equilibration depth. The analysis developed below indicates that equilibration instead occurs through the strong part of the lithosphere. The anomaly therefore acts over a substantially shorter depth scale, giving a much smaller force.

The trench-pull force is $\Delta\text{GPE}^* = -\Delta\bar{\sigma}_{zz}$ between x_T and x_I . Its magnitude follows from vertical equilibrium and the depth distribution of the shear-stress gradient;

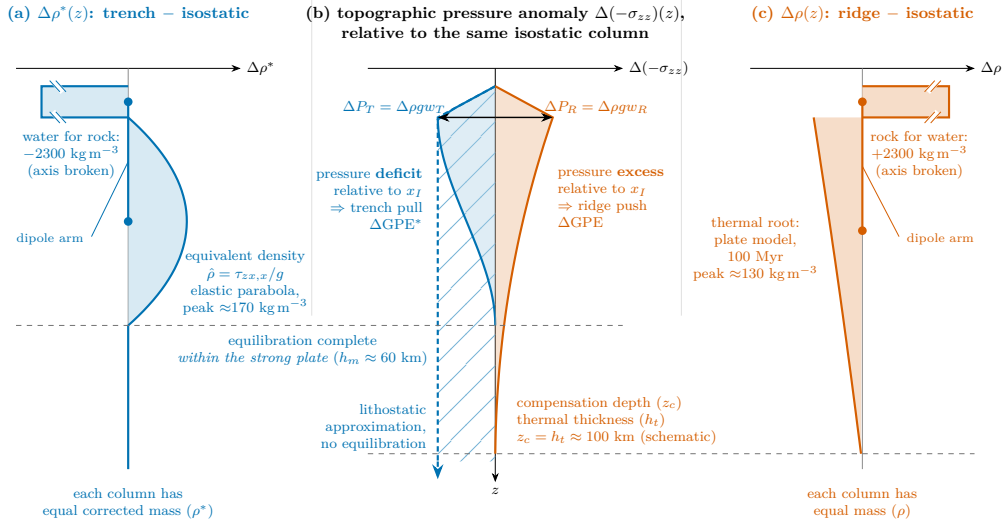


Figure 2. Ridge push and trench pull in the idealized trailing plate introduced in Figure 1. All quantities are expressed relative to the shared first isostatic column, x_I . (a) Corrected-density difference $\Delta\rho^*$ between the trench and isostatic columns, constructed using Equation (11). The shallow true-density lobe represents the replacement of rock by water; the deeper equivalent-density lobe represents the vertical shear-stress gradients through which the trench pressure deficit equilibrates. Under the assumptions adopted here, these non-overlapping lobes have equal area and form a zero-net-mass dipole (Equations (13) and (14)). The true-density difference alone does not integrate to zero because the trench column has a mass deficit relative to x_I . The parabolic equivalent-density profile follows from the uniform-bending solution developed in Section 2.5. (b) Differences in $-\sigma_{zz}$ relative to x_I . The blue and orange areas give the trench-pull force, ΔGPE^* , and ridge-push force, ΔGPE , respectively. The dashed blue line is the lithostatic approximation for the vertical normal stress beneath the trench; because it does not equilibrate, the inferred trench-pull force is unbounded with increasing integration depth. (c) True-density difference $\Delta\rho$ between the ridge and x_I , forming the classical isostatic density-difference dipole. In both dipoles, relief sets the near-surface pressure perturbation, $\Delta P = \Delta\rho g w$, while the separation between the lobe centres of mass defines the dipole arm—the length scale over which that perturbation acts and is converted to a force. The common integration depth is $z_c = h_t \approx 100 \text{ km}$, where the thermal-density difference in the ridge-push construction vanishes. The analysis here infers that the trench pressure deficit equilibrates at the shallower mechanical thickness, $h_m \approx 60 \text{ km}$. Trench pull and ridge push are additive; their sum is the combined blue and orange area in (b) and the total topographic pressure-gradient force shown in Figure 1. The relief interval from 0 to 3 km is enlarged threefold and the density axes are broken across the water–rock blocks; all other depths are to scale.

it does not depend on how the horizontal balance is divided between ΔN_D and F_B . Equation (7) shows that these two terms together balance ΔGPE^* . For scale, a basal shear stress of 1 MPa acting across the approximately 100 km flexural region gives $F_B \approx 0.1 \text{ TN m}^{-1}$, whereas the analysis below gives a trench-pull force of approximately 2.5 TN m^{-1} . Basal shear is therefore a second-order contribution over this interval, and the trench-scale approximation $F_B \approx 0$ is adopted. This completes the hydrostatic approximation beneath the plate but is not required to determine ΔGPE^* . With $F_B \approx 0$,

$$\Delta \bar{\sigma}_{xx} \approx 0, \quad \Delta N_D \approx -\Delta \bar{\sigma}_{zz} = \Delta \text{GPE}^*. \quad (18)$$

Across the flexural region, the change in $\bar{\sigma}_{zz}$ is therefore exchanged for a nearly equal and opposite change in N_D , while $\bar{\sigma}_{xx}$ remains approximately unchanged. Equation (18) shows that the trench-pull force ΔGPE^* adds the positive increment $\Delta N_D \approx \Delta \text{GPE}^* > 0$ between the trench and the first isostatic column; it does not determine the absolute value of N_D at either column. Any resultant transmitted through the slab hinge enters through $N_D(x_T)$, irrespective of whether it is tension-like or compressive, driving or resisting.

2.5 Prediction for a uniform bending plate

The preceding development reduces the trench-pull estimate to one unknown: the centre of mass of the shear-stress-gradient distribution beneath the trench, represented above in equivalent-density units. Topography specifies the centre of mass of the shallow rock–water density lobe, while vertical equilibrium specifies the magnitude of the deeper equivalent-density lobe; the separation between the two centres of mass defines the dipole arm. The uncertainty highlighted by Richter and McKenzie (Richter & McKenzie, 1977) therefore concerns the vertical distribution of the shear-stress gradient beneath a real trench. A uniform elastic plate under simple bending provides the natural starting point, against which the more realistic models developed below can be assessed.

In that solution, the bending stress varies linearly through the plate. Equilibrium requires a parabolic vertical shear stress that vanishes at its upper and lower surfaces, and the horizontal gradient of that shear stress is also symmetric about the mid-plane. Its centre of mass therefore lies at mid-plate. These idealisations are represented schematically in Figure 2; the full derivation is given in Supporting Information Text S4.

This length scale does not depend on the plate remaining wholly elastic. For symmetric elasto-plastic bending, yielding changes the distribution of stress through the plate but preserves its symmetry, so the centre of mass of the shear-stress-gradient distribution remains at the mid-plane. The corresponding elasto-plastic development is also given in Supporting Information Text S4.

Oceanic lithosphere is commonly represented as one mechanically connected load-bearing plate (Chapple & Forsyth, 1979; McNutt & Menard, 1982; Watts, 2001). Its strength generally increases with depth through the brittle regime, passes through a semi-brittle transitional interval, and then decreases as ductile deformation becomes increasingly important (Goetze & Evans, 1979; Kohlstedt et al., 1995). For flexural purposes, this behaviour can be represented by a continuous, broadly quasi-symmetric strength distribution, with the rapid decline in strength defining the mechanical thickness h_m of the plate (McNutt & Menard, 1982; D. Sandiford & Craig, 2023). A centre of mass near the mid-depth of the mechanically strong plate is therefore a useful reference prediction without requiring the lithosphere to remain wholly elastic.

For a deflected plate, the centre of mass of the rock–water lobe lies at $w_T/2$, whereas that of the equivalent-density lobe representing shear support lies at $w_T + h_m/2$. Their separation is therefore $(w_T + h_m)/2$, and Equation (14) becomes

$$\Delta \text{GPE}^* = (\Delta \rho g w_T) \frac{w_T + h_m}{2} \approx (\Delta \rho g w_T) \frac{h_m}{2}. \quad (19)$$

Table 1. Model parameters.

Quantity	Symbol	Value	
plate thickness	h	60 (30–60 in Suite 4)	km
domain length	L	1600	km ($\approx 13\alpha$)
Young’s modulus, Poisson ratio	E, ν	70 GPa, 0.25	
mantle, water density	ρ_m, ρ_w	3300, 1000	kg m ⁻³
foundation stiffness	$k_f = \Delta\rho g$	2.26×10^4	Pa m ⁻¹
trench-load magnitude (reference)	V	4	TN m ⁻¹
Tresca yield stress	σ_Y	150	MPa
plastic moment	$M_p = \sigma_Y h^2/4$	1.35×10^{17}	N
elements (biquadratic)	$n_x \times n_z$	800×48	

The final approximation requires $w_T \ll h_m$. For a uniform elastic plate or a plate with a symmetric elasto-plastic strength profile, the predicted dipole arm is therefore approximately half the mechanical thickness. The numerical models therefore test whether this scaling remains valid under finite trench deflection, yielding, asymmetric strength distributions, variations in the applied vertical shear and background N_D .

3 Model setup and analysis

3.1 Finite-element model formulation

The finite-element models solve two-dimensional static equilibrium, $\nabla \cdot \boldsymbol{\sigma} = 0$, without body forces, for elastic and elasto-plastic plates in plane strain. The reference plate is $h = 60$ km thick and $L = 1600$ km long; Suite 4 varies h from 30 to 60 km (Figure 3; parameters in Table 1). The coordinate x is horizontal, and z is positive downward from $z = 0$ at the plate top to $z = h$ at its base; normal stress is positive in tension. The trench edge carries a prescribed downward vertical shear traction of magnitude V , the upper surface carries a vertical Winkler restoring traction, the base is traction-free and the far end is fixed. The trench traction is parabolic through the plate thickness and vanishes at both surfaces. Model and figure labels quote the positive load magnitude V ; the signed field resultant $V(x) = \int \tau_{zx} dz$ is negative at the trench. The domain is long enough ($L \approx 13\alpha$ for the reference model) that the far boundary has no influence at the trench; the thinner models have still larger L/α . This geometry is the finite-thickness counterpart of the thin-plate models long used to describe trench and outer-rise flexure (Parsons & Molnar, 1976; Caldwell et al., 1976; Turcotte et al., 1978).

Because neither the upper nor lower surface carries a horizontal traction, integration through the thickness requires $\bar{\sigma}_{xx}$ to be the same in every column; horizontal edge loading sets its value. Equation (18) therefore holds exactly in these models, and its recovery to within 0.019% in the reference calculation checks the numerical extraction. Vertical equilibrium likewise requires $dV/dx = \Delta\rho gw$. Neither integrated relation specifies the depth distribution of $\tau_{zx,x}$; resolving that distribution determines the depth over which the trench pressure deficit equilibrates. Here h is the finite geometrical thickness of the model plate; its relationship to the mechanical thickness of oceanic lithosphere is discussed in Section 5.2.

The rock–water density contrast associated with surface deflection enters through the restoring pressure $\Delta\rho gw$, applied at the upper surface as a Winkler foundation of stiffness $k_f = \Delta\rho g$ (Turcotte & Schubert, 2002; Watts, 2001). The model resultants are integrated from the deflected plate top and therefore omit the water wedge between that surface and the sea-level equipotential, which contributes $\frac{1}{2}\Delta\rho gw_T^2 = 0.11$ TN m⁻¹ (4.5% of the reference trench-pull force) to the theoretical sea-level construction; the reported

forces and their comparison with the $h/2$ scaling consistently use the plate-column convention. Subsidiary density differences associated with displacement of the crustal and thermal structure are evaluated separately in Supporting Information Text S3.1. The models are solved with the Ferrite finite-element library (Carlsson et al., 2024) on a structured grid of 800×48 biquadratic quadrilaterals. Total-Lagrangian kinematics accommodate finite rotation, and the trench load is applied incrementally. Implementation details, benchmarks and convergence tests are reported in Supporting Information Text S2.

3.2 Numerical experiments

All models use the same uniform elastic modulus and Poisson ratio; only their yielding behaviour and loading are varied. The numerical experiments comprise four suites. ‘Strength model’ refers here to whether yielding occurs and, where it does, to the yield criterion and its depth dependence. *Suite 1 (strength model at fixed load)* holds the reference trench-load magnitude ($V = 4 \text{ TN m}^{-1}$) and background $N_D = 0$, and compares four strength models: a uniform elastic plate; a uniform in-plane Tresca model that excludes σ_{yy} from the yield condition; and two three-dimensional von Mises (J_2) models under plane-strain kinematics. The latter have either an asymmetric profile that strengthens downward or a symmetric profile with weak surfaces and maximum strength near mid-depth. The initial yield envelopes of all three plastic models have the same plastic moment, $M_p = 1.35 \times 10^{17} \text{ N}$, within the range inferred for $\sim 100\text{-My}$ oceanic lithosphere (Goetze & Evans, 1979; McNutt & Menard, 1982; D. Sandiford & Craig, 2023). None includes time-dependent ductile or creep behaviour; this limitation is considered in Section 5.4. The depth-dependent models include weak linear hardening, used to regularize strain localization in their weak surface layers, and the loaded edge is kept elastic through a local increase in yield strength. The constitutive equations and both modifications are described in Supporting Information Text S2.

Suite 2 (load sweep) fixes the Tresca strength model and varies V from 1 to 4.5 TN m^{-1} . *Suite 3 (background stress sweep)* superposes a uniform background N_D from -3 to $+3 \text{ TN m}^{-1}$ on the reference model. *Suite 4 (thickness at matched deflection)* fixes the Tresca strength model, varies h from 30 to 60 km, and adjusts the load to give the same trench deflection in every model.

3.3 Stress resultants and post-processing

The vertical normal stress is integrated along successive vertical cuts through the deformed plate to obtain $\Delta \text{GPE}^*(x)$. The trench-pull force is its particular difference between x_T and x_I (Equation 1), and the force-based effective arm is $\Delta \text{GPE}^*/(\Delta \rho g w_T)$ for each model. Independently, the signed centre of mass of the shear-stress-gradient distribution is calculated directly from the finite-element field where the net shear-stress gradient is non-zero. It characterises the local depth distribution and gives the tracks and markers in Figures 3(c), 6 and 7; the force-based arm instead relates the integrated trench-pull force directly to the trench pressure deficit. Stress transformation, column selection, integration and verification are described in Supporting Information Texts S2.4 and S2.6.

Figures showing the complete model fields use common plotting conventions. Plate geometry is vertically exaggerated by a factor of 1.2, deflection by a factor of five in the vertical direction only, and horizontal displacement is shown at true scale. The trench line marks x_T , the leftmost vertical line that cuts the full deformed plate thickness and is tangent to the rotated lower corner ($x_T = 0.93 \text{ km}$ in the reference model). Principal-stress crosses show the in-plane Cauchy principal directions; their equal arms scale with the maximum in-plane shear stress $(\sigma_1 - \sigma_2)/2$, reaching 8.5 km in the reference model. Their orientation follows the exaggerated plate, so the axes are perpendicular in the physical plate but not exactly on the page. The signed centre of mass $\int z \hat{\rho} dz / \int \hat{\rho} dz$ is eval-

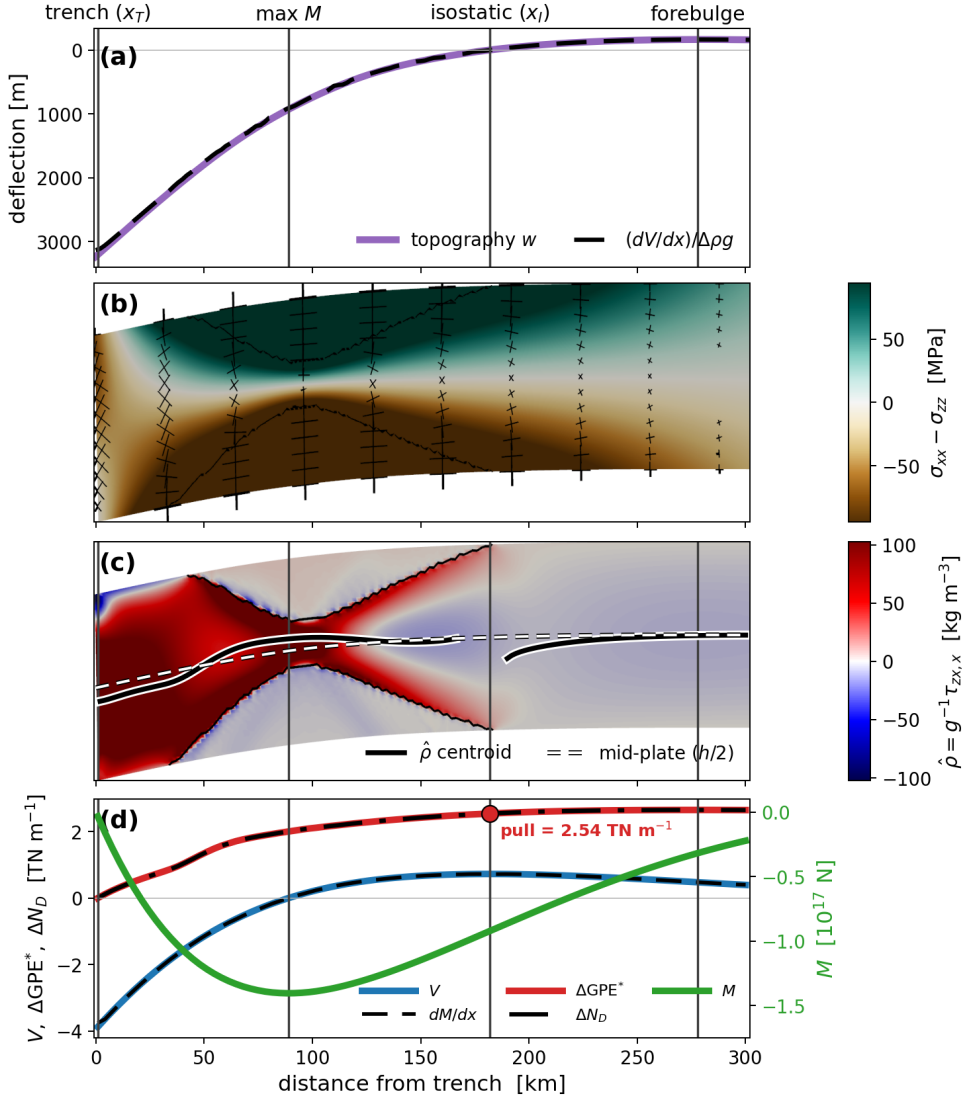


Figure 3. The reference bending model (uniform Tresca, $\sigma_Y = 150$ MPa, $V = 4$ TN m $^{-1}$, $w_T \approx 3.2$ km). The trench column x_T , maximum-moment column x_M , first isostatic column x_I , and forebulge are marked. (a) Topography w , with $(dV/dx)/(\Delta\rho g)$ overlain. (b) The normal-stress difference $\sigma_{xx} - \sigma_{zz}$, with yield contours and principal-stress crosses. (c) Equivalent density $\hat{\rho} = g^{-1}\tau_{zx,x}$, with its signed centre of mass (black) and the mid-plate depth $h/2$ (dashed). The track is blanked near x_I , where $\int \hat{\rho} dz = 0$ and its centre of mass is undefined. (d) The resultants V , with dM/dx overlain, M , and $\Delta GPE^*(x)$, with $\Delta N_D(x)$ overlain. Plotting conventions are given in Section 3.3.

uated only for complete vertical columns. Its column integrals are smoothed along the plate with a 6-km Gaussian, and the track is drawn where the net integral exceeds one quarter of the absolute integral; it is blanked where the ratio is ill-conditioned. Throughout these figures, $\sigma_{xx} - \sigma_{zz}$ runs from brown (compression) to teal (tension), τ_{zx} from purple (negative, the sign of the applied trench load) to green (positive), and $\hat{\rho}$ from blue (negative) to red (positive).

Use of AI tools

The central analytical framework of this study was developed without AI assistance and appeared in preprint version 1 (D. Sandiford, 2025). During preparation of this revised manuscript, Claude Code (Anthropic) and OpenAI Codex were used to assist in developing and reviewing the finite-element code, analysis and figure-generation scripts, and reproducibility package. The same tools assisted with language editing and with checking the manuscript's notation and numerical values against the released code and model output. The author designed the study, evaluated all scientific and modelling decisions, reviewed all AI-assisted material, and takes full responsibility for the manuscript and released software.

4 Results

Section 2 shows that the trench-pull force—or, more generally, ΔGPE^* between non-isostatic columns—depends on the depth distribution of $\tau_{zx,x}$. For uniform bending of an elastic plate, equilibrium gives the familiar parabolic $\tau_{zx,x}$ profile centred at mid-plate. Supporting Information Text S4 extends this construction to symmetric yield-stress envelopes: yielding concentrates the distribution toward the elastic core without shifting its centroid from mid-plate. These analytical results give a reference prediction of an effective arm near $h/2$ and a trench-pull force that scales with deflection. The results below examine the sensitivity of that prediction to load, strength model, background N_D , and plate thickness.

4.1 Stress structure and trench pull in the reference model

Figure 3 presents the reference model as a detailed example before the four model suites described in Section 3.2 are compared. The reference model is a 60-km-thick Tresca plate loaded by $V = 4 \text{ TN m}^{-1}$. It develops a 3.18-km-deep trench. Because neither the upper nor lower plate surface carries horizontal shear traction, $\bar{\sigma}_{xx}$ is constant and the cumulative differences measured from the trench remain locked along the plate: $\Delta N_D(x) = \Delta\text{GPE}^*(x)$ (Figure 3d). At the first isostatic column, their common value gives the trench-pull force, $\Delta\text{GPE}^* = -\Delta\bar{\sigma}_{zz}$, of 2.54 TN m^{-1} ; the equality has a residual of 0.019%.

Figure 3 also connects the two-dimensional stress fields within the plate to the integrated force balance. Panel (a) shows $dV/dx = \Delta\rho gw$, the signed counterpart of the spatially varying upward restoring traction $-\Delta\rho gw$, while panel (d) shows $dM/dx = V$. These are the familiar thin-plate expressions of vertical-force and moment balance. Panels (b) and (c) show the information not retained in that description: the normal-stress difference and the depth distribution of the horizontal gradient of vertical shear stress, expressed as the equivalent density $\hat{\rho} = g^{-1}\tau_{zx,x}$. The solid black line in panel (c) traces the signed local centre of mass of this distribution, which remains near mid-plate through the trench region. The thin-plate relationships fix its integral through dV/dx , but not this depth distribution, which determines its first moment and hence the trench-pull force.

Supporting Information Figure S5 shows the corresponding corrected-density (ρ^*) profiles at the trench, maximum-moment, and first isostatic columns. Beneath the trench, the equivalent-density contribution retains approximately the broad parabolic form anticipated by the schematic construction in Figure 2. At the maximum-moment column, deep yielding instead concentrates it within the remaining elastic core. The simple profile is therefore a useful approximation beneath the trench, but does not describe every section through the bending region.

4.2 Trench pull is controlled primarily by trench depth

The magnitude of the applied vertical load at the trench, V , does not uniquely determine the trench-pull force (Figure 4a). Suite 2 (open blue symbols) varies V , across which the trench-pull force increases from 0.51 to 3.20 TN m⁻¹; the curved relationship reflects the increasing compliance of the plate as yielding develops. Suites 1 and 3 provide the complementary fixed-load experiments. At $V = 4$ TN m⁻¹, changing the strength model (green symbols) or the imposed background N_D (red symbols) produces different trench geometries and hence different trench-pull forces.

Most of this variation disappears when the pull is plotted against trench deflection (Figure 4b). Models with different loads, strength distributions, and background stresses fall close to a common trend. Thus, for a given plate thickness, the interplay between loading and strength influences the trench-pull force mainly by controlling the trench depth; the depth then fixes the amplitude of the pressure deficit. At small loads and deflections, Suite 2 closely follows the uniform-plate $h/2$ prediction shown by the dashed line. The departure grows as yielding redistributes the equivalent load toward the plate base in the bending region, moving its centre below mid-plate and increasing the effective arm. For the reference model, the direct ratio $\Delta GPE^*/(\Delta \rho g w_T)$ gives an effective arm of approximately 35 km, or $0.59h$, compared with $h/2 = 30$ km. The asymmetric strength model (DD-VM asym) departs furthest from the common pull-deflection relation; the most compressive background model ($N_D = -3$ TN m⁻¹) is the clearest departure within Suite 3. Even these models remain within approximately 10% of the common relation.

Suite 3 shows that background N_D affects the trench-pull force mainly through its influence on trench deflection. Tension-like values of +1 to +3 TN m⁻¹ change the force by about 1%, whereas $N_D = -3$ TN m⁻¹ increases both deflection and trench pull. Once expressed against deflection, however, these models remain close to the common relation, indicating little change in the effective arm over the range tested.

4.3 The dipole reconstruction recovers the trench-pull force

Equation (14) shows that the trench-pull force obtained directly from $-\int \sigma_{zz} dz$ can also be recovered from the first moment of the corrected-density difference. Figure 5 tests this equivalence continuously through the bending region in each of the four Suite 1 models. The thick grey curves show the vertical-normal-stress calculation; the solid black curves reconstruct the same force from the first moment of the finite-element shear-stress-gradient distribution about the deformed plate top. This is its unnormalised first moment, rather than a force-based arm inferred from ΔGPE^* .

The reconstruction agrees with the direct vertical-normal-stress calculation to within 1.7–2.3% across the four models, including those with strongly contrasting yield-strength profiles. The dashed curves replace the calculated first moment with the simpler mid-plate approximation. This fixed arm captures the scale of the force but differs from the vertical-normal-stress calculation by 9–20%. Figure 5 therefore confirms the first-moment reconstruction throughout the bending region and quantifies the error introduced by the $h/2$ approximation.

4.4 The pull is more sensitive to mechanical thickness than to strength distribution

Suite 1 compares one uniform elastic model with three elasto-plastic models at the same plate thickness and applied vertical load. The elasto-plastic models have the same nominal plastic moment based on their prescribed yield-strength profiles. Under the plane-strain von Mises criterion, however, the DD-VM limit is $2\sigma_Y/\sqrt{3}$, so those models carry approximately 10–15% more plastic moment than the Tresca model. The profiles remain

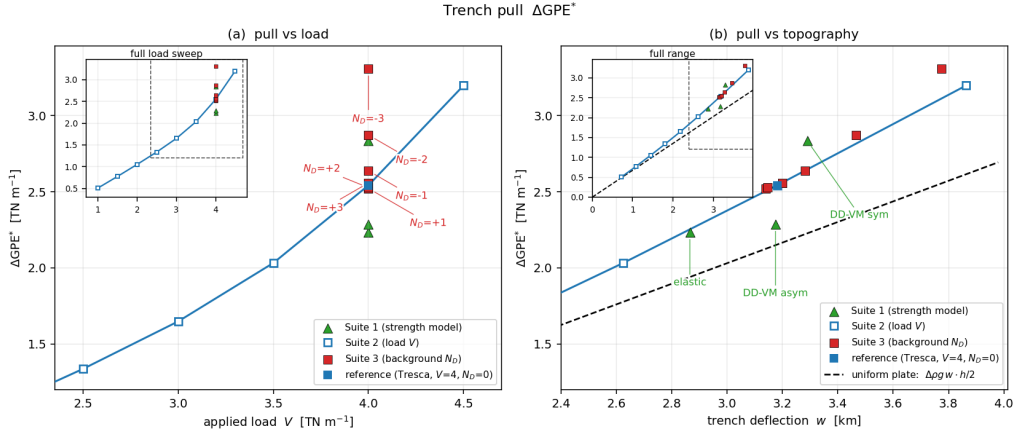


Figure 4. Trench-pull force ΔGPE^* for Suites 1–3, all with a plate thickness of 60 km, plotted against (a) the applied vertical load V and (b) the trench deflection w . Symbols distinguish Suite 1 (strength model at fixed $V = 4\ TN\ m^{-1}$), Suite 2 (load sweep from $V = 1$ to $4.5\ TN\ m^{-1}$, producing deflections from 0.73 to 3.95 km), and Suite 3 (background N_D); the square is the reference model. Variation associated with load, strength model, and background stress largely collapses when expressed against deflection. The insets show the full load and deflection ranges, with the plotted windows boxed. The dashed line is the uniform-plate estimate $(\Delta \rho g w)(h/2)$. The models lie above it and imply the deflection-normalised arm $\Delta GPE^*/(\Delta \rho g w) \approx 35\ km$, or approximately $0.59h$.

sharply contrasting: the Tresca model has a uniform yield strength of 150 MPa; the asymmetric depth-dependent von Mises model strengthens downward from approximately 30 MPa at the upper surface to approximately 180 MPa at mid-plate; and the symmetric depth-dependent model rises from approximately 37 MPa at both surfaces to approximately 380 MPa at mid-plate. Despite these marked differences, the three elasto-plastic models form similar trenches (3.18–3.29 km), while the elastic model deflects 2.87 km. All four remain within about $\pm 10\%$ of the common pull-deflection trend, with pulls of 2.23–2.84 $TN\ m^{-1}$. Thus, even large changes in the magnitude, depth dependence, and symmetry of the yield strength have a comparatively small effect on the trench-pull force.

Figure 3b helps explain this clustering. Yielding is most extensive near the maximum-moment location and diminishes toward both the trench and the first isostatic column. The trench column lies outside the yielded region in each elasto-plastic model. The third column of Figure 6 shows the consequence: the equivalent-density profiles and their signed centres of mass differ markedly among the models at the maximum-moment location (red), but are more similar beneath the trench (bold black). The strength envelopes therefore reorganize the stress field most strongly within the actively yielding part of the bending region, while having less effect at the bounding columns used to calculate the trench-pull force. Full-field views of the individual models are provided in Supporting Information Text S5.

Plate thickness has a much larger effect. In Suite 4, 30–60-km-thick Tresca plates are loaded to the same 3.2-km trench depth. The trench-pull force increases from 1.40 to 2.54 $TN\ m^{-1}$, nearly in proportion to thickness (Figure 7). Because the trench deflection is matched, the trench pressure deficit, $\Delta \rho g w_T$, is the same in every model. The increase in trench-pull force shown in Figure 7 therefore records a longer effective moment arm, a_{eff} (Equation 15). Normalized by plate thickness, this arm decreases slightly from $0.66h$ for the 30-km plate to $0.59h$ for the 60-km plate. Across the models tested here,

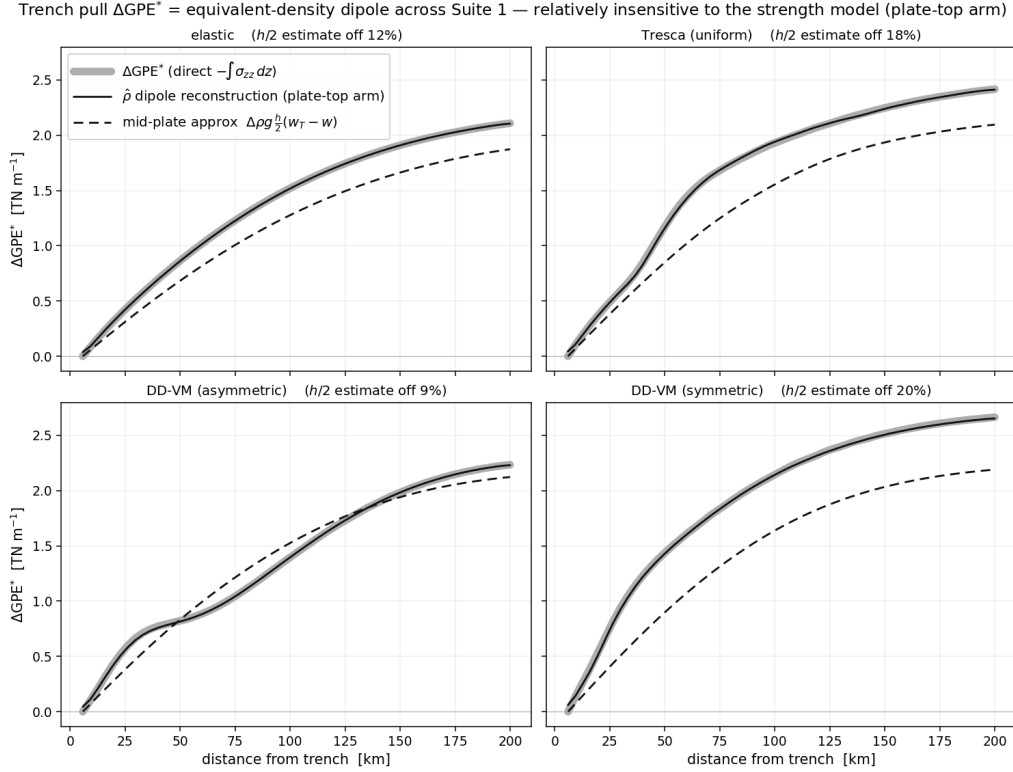


Figure 5. The change in GPE^* along the plate, calculated from the vertical normal stress (thick grey; $-\int \sigma_{zz} dz$) and reconstructed from the first moment of the finite-element shear-stress gradient about the deformed plate top (solid black), for the four Suite 1 strength models at $V = 4 \text{ TN m}^{-1}$. Both curves are referenced to a complete column 5 km inboard of x_T ; the reconstruction is lightly Gaussian-smoothed along x . The two calculations agree to within 1.7–2.3%. Dashed curves show the mid-plate approximation $\Delta\rho g(h/2)\Delta w(x)$ using the same reference column. Each panel title gives the maximum absolute departure of this approximation from the vertical-normal-stress calculation, normalized by the maximum of that calculation over the plotted interval.

trench depth fixes the amplitude of the pressure anomaly, while the thickness of the stress-supporting plate sets the principal length scale over which it acts.

5 Discussion

5.1 Trench pull as a topographic pressure-gradient force

The vertically integrated force balance developed in Section 2 identifies differences in vertically integrated vertical normal stress as the generally applicable force contribution: $\Delta GPE^* = -\Delta\bar{\sigma}_{zz}$. For two isostatic columns treated as lithostatic, this quantity equals their true, density-based GPE difference; conventional GPE is therefore a special case. Beneath a non-isostatic trench, σ_{zz} must instead be the full vertical normal stress that satisfies equilibrium. Force contributions based on this full stress are referred to here as GPE-like forces: for non-isostatic columns, $-\Delta\bar{\sigma}_{zz}$ no longer equals the GPE difference sensu stricto. GPE-like forces need not involve topography. Topographic pressure-gradient forces are the subset for which relief creates one lobe of the density dipole by replacing rock with air or water, or conversely (Figure 2a,c). Ridge push and trench pull

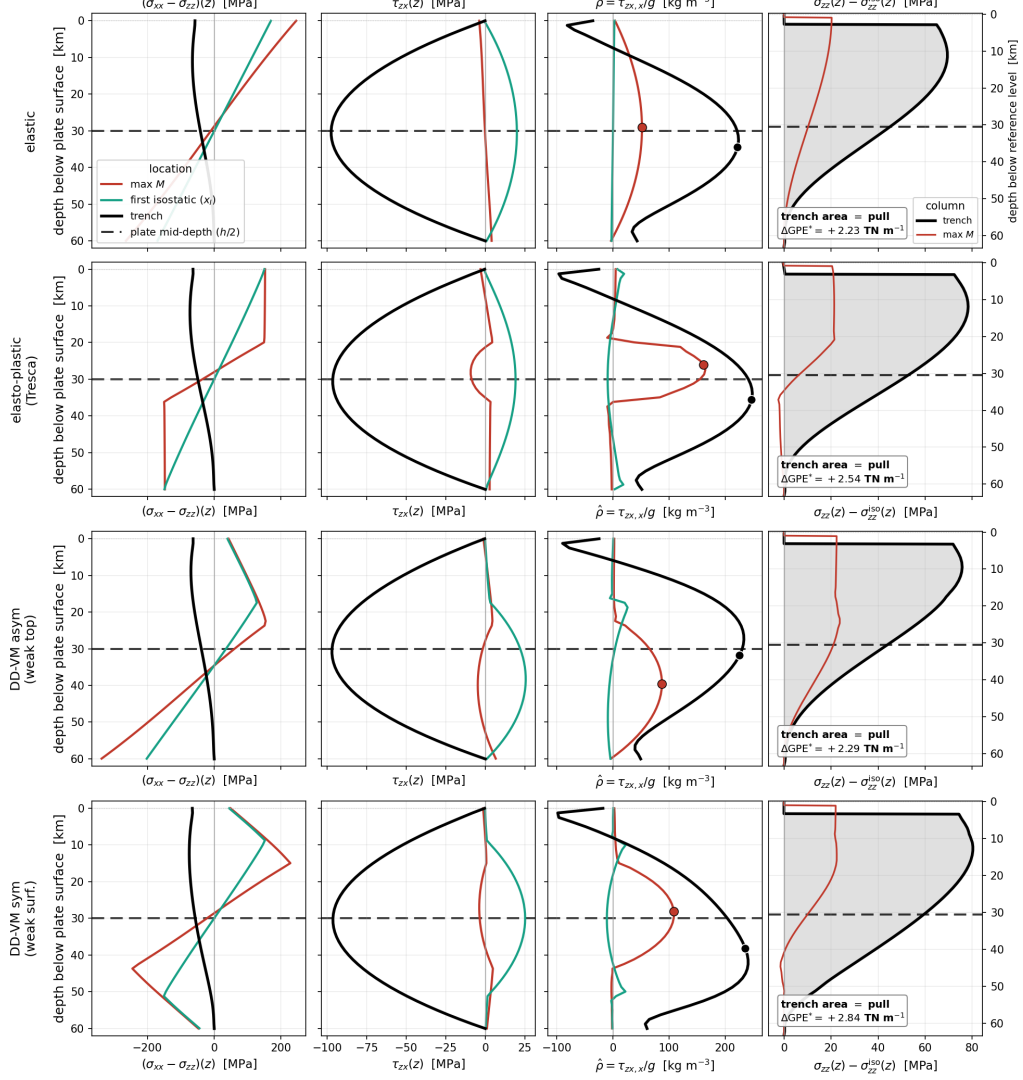
Depth profiles at the flexure locations (trench · max M · first isostatic x_I); (a)-(c) below plate surface, (d) below reference level ● = centroid of $\tau_{zx,x}$


Figure 6. Vertical profiles for the four Suite 1 constitutive models (rows), evaluated at three positions: the trench column x_T (bold black), maximum-moment column x_M (red), and first isostatic column x_I (green). These locations are marked for the reference Tresca model in Figure 3. From left to right, the columns show (1) the normal-stress difference $(\sigma_{xx} - \sigma_{zz})(z)$; (2) the vertical shear stress $\tau_{zx}(z)$; (3) the equivalent density $\hat{\rho}(z) = \tau_{zx,x}/g$, with dots marking its signed centre of mass; and (4) the vertical-normal-stress difference $\sigma_{zz}(x, z) - \sigma_{zz}(x_I, z)$. The shaded depth integral of the trench profile in the fourth column gives the trench-pull force. The trench profiles and their centres of mass vary less among the constitutive models than those at the maximum-moment column.

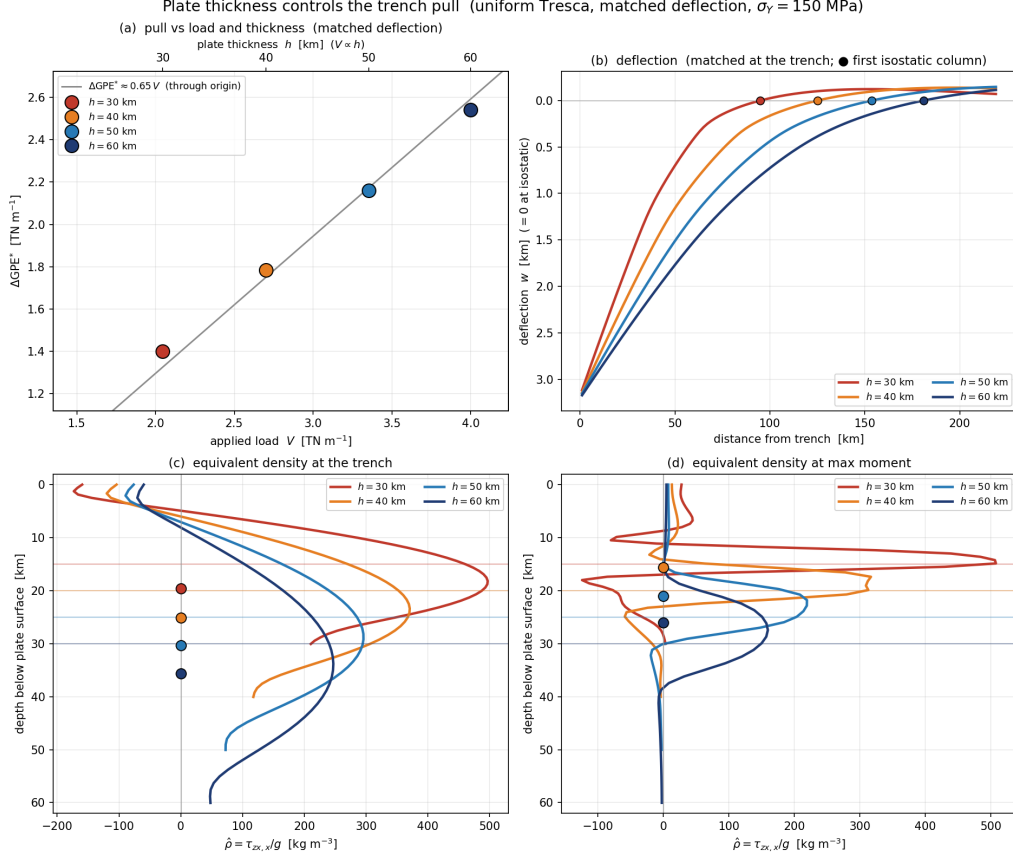


Figure 7. Plate-thickness suite for uniform Tresca plates with a yield strength of 150 MPa. For each thickness h , the applied load V is adjusted to produce the same trench deflection of approximately 3.2 km. (a) Trench-pull force ΔGPE^* plotted against the adjusted load (lower axis) and plate thickness (upper axis). The through-origin relation is $\Delta GPE^* \approx 0.65V$, and ΔGPE^* also increases nearly in proportion to h . (b) The matched deflection profiles; dots mark the first isostatic column and show the increase in flexural wavelength with plate thickness. (c,d) Vertical profiles of the equivalent density $\hat{\rho}(z) = \tau_{zx,x}(z)/g$ at the trench and maximum-moment columns. Horizontal lines mark the mid-plate depth $h/2$, and dots mark the signed centre of mass of $\hat{\rho}$. For the 30-km plate in panel (d), positive and negative contributions across the fully plastic hinge nearly cancel, so the signed centre of mass is undefined and no dot is shown (Supporting Information Figure S7). At fixed trench deflection, the trench-pull force therefore increases nearly in proportion to plate thickness as the shear-stress-gradient distribution extends deeper through the plate.

both belong to this class, but differ in how the pressure anomaly is equilibrated. For ridge push, compensation is provided by the thermal density structure of isostatic lithosphere; beneath a flexural trench, equilibration depends principally on the shear-stress-gradient distribution within the deflected plate. Figure 2b shows why lithostatic pressure cannot be substituted for the full σ_{zz} : the trench pressure deficit then persists with depth and the inferred force continues to grow with the integration depth. The full vertical normal stress includes the shear-stress-gradient contribution and gives the finite trench-pull force shown by the bounded, shaded area. For trench pull, the surface deficit sets the amplitude of the pressure anomaly, whereas its equilibration with depth sets the integrated force (Figure 2b).

Richter and McKenzie (1977) recognized that replacing rock with water creates a pressure deficit that might drive the trailing plate, but identified the unknown stress distribution beneath the trench as the obstacle to calculating the force. Bercovici et al. (2015) identify horizontal pressure gradients as the primary driver of lateral flow within convective thermal boundary layers. Where the upper boundary is deformable, part of this pressure gradient is expressed through surface topography, as observed in convecting lava lakes (Valade et al., 2018). The analysis here identifies and quantifies its non-isostatic, trench-side contribution. Together, ridge push and trench pull provide a plate-wide topographic pressure-gradient force that, as discussed in the following section, is of sufficient magnitude to balance plausible estimates of basal drag.

5.2 Magnitude and extrapolation to oceanic lithosphere

In the reference model, which has a 60-km-thick plate and a trench deflection of 3.2 km, the trench-pull force is approximately 2.5 TN m^{-1} , or 0.8 TN m^{-1} per kilometre of deflection. At fixed plate thickness, models with different loading conditions, background N_D , and strength distributions remain within about $\pm 10\%$ of a common pull-deflection relation (Figure 4). By contrast, the force-based effective arm, a_{eff} , increases nearly in proportion to plate thickness (Figure 7). Thus, trench depth sets the amplitude of the pressure anomaly, whereas the thickness of the stress-supporting plate sets the principal length scale over which it acts (close to, but not exactly, $h/2$).

Oceanic lithosphere has no sharply defined lower boundary, but the depth range capable of supporting significant stress is commonly described by its mechanical thickness, h_m . This is the part of the lithosphere that contributes materially to its integrated strength and bending moment, making h_m the natural analogue of the finite plate thickness h used in the models. For old oceanic lithosphere, estimates of h_m from strength envelopes agree broadly with the depth distribution of intraplate seismicity, suggesting values of approximately 50–70 km (Goetze & Evans, 1979; Chapple & Forsyth, 1979; Craig et al., 2014; Kohlstedt et al., 1995; D. Sandiford & Craig, 2023). Thermal plate models place the thickness of old oceanic lithosphere at approximately 90–120 km (Parsons & Sclater, 1977; Stein & Stein, 1992). The hotter basal part of this thermal boundary layer contributes little to the supported stress and bending moment, so the thermal lithosphere is substantially thicker than the mechanical lithosphere. The extrapolation therefore rests on a transfer hypothesis: h_m plays the same role as the finite thickness h in the models. Across the constitutive models tested here, the force arm depends much more strongly on the total thickness of the stress-supporting plate than on the distribution of yield strength within it, supporting the representative value $h_m = 60 \text{ km}$ adopted here.

Grellet and Dubois (1982) report trench deflections of approximately 2.5–5.5 km, with the larger values characteristic of old subducting lithosphere. More recent flexural analyses likewise show substantial variability within old Pacific lithosphere: trench deflections are approximately 2.5 km at the Japan and Philippine trenches but exceed 4 km at parts of the Kuril and Mariana trenches (Zhang et al., 2014, 2018). The 3.18-

km deflection produced by the representative $h_m = 60$ km model therefore lies toward the conservative end of observed values. For deflections of 3–3.5 km, the mid-plate estimate gives 2.1–2.4 TN m^{-1} . The model-calibrated effective arms span $a_{\text{eff}}/h_m = 0.52$ –0.66, giving a representative force of approximately 2.5 TN m^{-1} . At a specified mechanical thickness, the observed deflection directly sets the amplitude of the trench-pull force.

The results here give a representative trench-pull estimate of approximately 2.5 TN m^{-1} , within the range of conventional ridge-push estimates of 2–4 TN m^{-1} (Lister, 1975; Dahlen, 1981; Turcotte & Schubert, 2002). Together, the two topographic pressure-gradient forces provide approximately 4.5–6.5 TN m^{-1} . The estimated topographic pressure-gradient contributions from the upwelling ridge and downwelling trench are therefore similar in magnitude, giving the plate-wide forcing an approximate symmetry despite the fundamentally different character of the topography at either end. For comparison, a mean basal shear stress of 1 MPa—near the upper end inferred for large plates by Bird et al. (2008)—integrated over 5000 km corresponds to 5 TN m^{-1} ; several other estimates are lower (Melosh, 1977; Wiens & Stein, 1985; Chen et al., 2021). The combined topographic pressure-gradient force is therefore sufficient to balance plausible estimates of basal drag.

5.3 Trench pull, slab pull, and the trailing-plate force balance

In the conventional slab-pull model, net slab pull is a tension-like stress resultant transmitted through the subduction hinge to the trailing plate (Orowan, 1965; Elsasser, 1969; Davies, 2022). This resultant is measured by $N_D(x_T)$: a positive value is driving, whereas a negative value is resisting. Trench pull is measured by the positive increment across the trench–first-isostatic-column interval. Because basal traction is negligible over this short distance,

$$N_D(x_I) - N_D(x_T) \approx \Delta \text{GPE}^* > 0.$$

Suite 3 shows that background N_D affects this increment mainly through its effect on trench deflection; at a specified deflection, the effective arm changes little. The trench-pull contribution can therefore be estimated from trench topography and mechanical thickness without knowing $N_D(x_T)$ precisely. The limiting case $N_D(x_T) \approx 0$ makes the consequence clear: trench pull alone produces a positive N_D seaward of x_I , indistinguishable there from the tension-like resultant conventionally attributed to net slab pull. This gives a concrete interpretation to the observation of Bercovici et al. (2015) that low pressure above a downwelling makes the slab appear to pull the plate.

Slab buoyancy is likely the ultimate source of the boundary resultants N_D , V , and M that load and deflect the trailing plate. In the conventional slab-pull model, a tension-like N_D is transmitted through the hinge. In the trench-pull mechanism, the slab stress guide need only load the trailing plate vertically. Flexural deflection then creates a pressure deficit whose equilibration through the mechanical plate couples the vertical loading to the horizontal force balance. The depth of this equilibration scales with mechanical thickness and, in the equivalent-density description, is summarized by the force-based effective arm, a_{eff} . Trench pull can therefore be regarded as a variant of the stress-guide model in which slab buoyancy is coupled to a horizontal resultant through the trench pressure deficit rather than through direct transmission of N_D around the hinge.

Beyond the first isostatic column, the two contributions cannot be separated from N_D alone. Their distinction lies in the change in N_D between x_T and x_I , but this relatively small depth-integrated change is superposed on the much larger local stresses associated with flexural bending. Seismic observations nevertheless provide some constraint. For old lithosphere, an apparent neutral-plane depth of about 30 km is difficult to reconcile with a bending-region-averaged N_D greater than about 2 TN m^{-1} (D. Sandiford & Craig, 2023). This argues against conventional slab-pull resultants of order 5 TN m^{-1} or more, as implied by the estimates of Conrad and Lithgow-Bertelloni (2002). Seismicity is unlikely, however, to distinguish conventional slab pull from trench pull when their

contributions are each only a few TN m^{-1} , because both produce the same positive N_D seaward of the first isostatic column.

Numerical subduction models can evaluate this partition directly by integrating the stresses through successive columns. Models in which the slab and trailing plate form a continuous body are particularly useful because their loading emerges from slab buoyancy. Thermo-mechanical models also allow the stress-supporting lithosphere to emerge from temperature-dependent constitutive behaviour rather than an imposed plate thickness. In the model of D. Sandiford and Craig (2023), for example, $N_D(x_T)$ is weakly positive, about 0.6 TN m^{-1} , and increases by about 2 TN m^{-1} across the flexural region. By this diagnostic, the trench-pull contribution exceeds the conventionally transmitted component. A single value reported seaward of the trench would combine the two and obscure their origins.

5.4 Model limitations and further tests

The strength models span substantial differences in yield strength and its depth distribution, but are completely rate-independent. The uniform Tresca model is perfectly plastic, while the depth-dependent von Mises models include only weak linear hardening. The same general elasto-plastic treatment is used above and below the neutral plane, whereas deformation at greater lithospheric depths is more appropriately described by semi-brittle flow and temperature- and strain-rate-dependent creep.

The models resolve a sequence of static equilibrium states rather than the advection of material through the bending region. They therefore omit the loading and unloading history acquired as material passes the maximum-moment region and its curvature subsequently decreases toward the trench, including any residual plastic strain. The loaded boundary is also kept elastic through a local increase in yield strength, further idealising the stress state nearest the trench.

Loading is applied solely through a vertical shear-stress resultant at the trench; no independent bending-moment resultant is imposed there. This isolates the key mechanism considered here: the coupling between a purely vertical load on the trailing plate, as might be expected from slab buoyancy, and the horizontal force balance. Moment-dominated and mixed loading remain untested. Finally, the numerical plate has a prescribed finite thickness, unlike the gradational strength structure of oceanic lithosphere. Treating the model thickness h as analogous to mechanical thickness h_m remains a transfer hypothesis. Advecting thermo-mechanical subduction models would test these assumptions together by incorporating strain-rate-dependent deformation, evolving thermal structure, and a continuous slab-plate system.

6 Conclusions

Richter and McKenzie (1977) recognized that the pressure reduction in the column beneath a trench could contribute to driving the plate, and identified the unknown distribution of the supporting stresses as the central obstacle to quantifying that force. The trench column is non-isostatic: the rock-water mass deficit is balanced by stresses within the deflected plate rather than by subsurface density compensation. A GPE calculation built on the lithostatic pressure therefore fails at the trench, because the lithostatic pressure difference between the trench column and the trailing plate does not equilibrate with depth. Within the vertically integrated horizontal force balance (Section 2.2), the resolution is to retain the full vertical normal stress that satisfies equilibrium. The vertical-normal-stress difference between the trench and the first isostatic column does equilibrate, and Equation 8 identifies the contribution responsible: horizontal gradients in the vertical shear stress. The depth integral of this difference is therefore finite and defines the trench-pull force ($\Delta\text{GPE}^* = -\Delta\bar{\sigma}_{zz}$). The trench pressure deficit fixes the near-

surface amplitude of the vertical-normal-stress difference; quantifying the force requires knowing how that difference equilibrates with depth.

Expressing the shear-stress-gradient distribution as an equivalent density converts this depth dependence into an effective dipole arm. The trench-pull force is therefore set by the pressure deficit and the arm. What theory leaves open is how strongly the stress distribution — and hence the arm — depends on the strength model and loading. In the models, over a five-fold range of load, background in-plane stress from -3 to $+3$ TN m^{-1} , and four yield strength models, the relationship between the trench pull and the topography sits effectively on a straight line; the identity $\Delta N_D = -\Delta \bar{\sigma}_{zz}$ holds to better than 0.1% across all models; the force-based arm remains near $h_m/2$, giving 2.5 TN m^{-1} for the reference 3.2-km trench, comparable to ridge push; for markedly different yield-strength profiles with similar nominal plastic moments, the trench-pull force changes by only about 10%; tension-like backgrounds change it by about 1%; and at matched deflection, the pull grows nearly in proportion to the plate thickness, as the arm scaling requires.

Like ridge push, trench pull is a topographic pressure-gradient force. Trench topography sets the near-surface pressure anomaly, while its equilibration with depth determines the resulting force. Thin-plate theory retains only the resultant of the vertical shear stress, so a flexural model can reproduce the trench topography without providing any estimate of the force. Written as an equivalent density, the flexural support restores the isostatic mass balance to the bending plate — every column carries the same corrected mass, and the force is the dipole of the difference; the classical GPE framework is recovered as the isostatic special case. The force can therefore be evaluated either as the area between the vertical-normal-stress curves of the two columns or as the first moment of their corrected-density difference.

In the conventional slab-pull model, the slab drives the trailing plate through a tension-like force resultant passed through the hinge—a positive $N_D(x_T)$, carried irrespective of the bending stresses superposed on it. Trench pull provides a distinct route by which slab buoyancy can drive the trailing plate. In this mode, the slab loads the plate vertically; the resulting deflection creates a pressure deficit, which horizontal equilibrium balances through a positive change in N_D , $\Delta N_D > 0$, between x_T and x_I . The two modes can coexist, but trench pull does not require a tension-like resultant to be transmitted through the hinge. It is the topographic pressure-gradient force associated with the non-isostatic trench region.

Together, ridge push and trench pull provide a topographic driving force of order 5 TN m^{-1} , sufficient to balance basal drag across a 5000-km-long trailing plate for plausible basal shear stresses (Melosh, 1977; Wiens & Stein, 1985; Chen et al., 2021). Topographic pressure gradients may therefore provide the primary component of the force driving the lateral motion of oceanic thermal boundary layers. Given an estimate of mechanical thickness, observed trench depth provides a direct proxy for the trench-pull force. Existing numerical models can distinguish the two modes by evaluating $N_D(x_T)$ for conventional slab pull and $-\Delta \bar{\sigma}_{zz}$ between x_T and x_I for trench pull.

Open Research

The model code, all data produced by the Julia models, and the scripts used to reproduce the analysis, tables and figures are available in the GitHub repository https://github.com/dansand/trench_pull_ferrite. The repository will also be archived in a citable persistent repository. The finite-element models were solved with Ferrite.jl v1.4.1 (Carlsson et al., 2024).

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Table 2. Principal symbols and reference values.

Symbol	Meaning
w_T	trench deflection (~ 3.2 km reference)
h	model plate thickness (60 km reference)
h_c	oceanic crustal thickness (7 km reference)
h_m	mechanical thickness of the lithosphere (~ 60 km)
h_t	thermal thickness (~ 100 km)
α	flexural parameter, $(4D/\Delta\rho g)^{1/4}$ (124 km at $h = 60$ km)
α_T	volumetric thermal expansivity
x_T, x_M, x_I	trench, maximum-moment and first isostatic column positions
V	vertical shear resultant, $\int \tau_{zx} dz$; model labels quote the positive magnitude of the applied downward t
$Q(z)$	shear function, $\int_0^z \tau_{zx,x} d\xi$
$\hat{\rho}$	equivalent density, $\tau_{zx,x}/g$
ρ^*	corrected density, $\rho + \hat{\rho}$
$\Delta\rho$	rock–water density contrast, $\rho_m - \rho_w$
ΔP	pressure magnitude of either dipole lobe ($\Delta\rho g w_T$ for trench pull)
a_{eff}	force-based effective moment arm, $\Delta\text{GPE}^*/\Delta P$
N_D	normal-stress-difference resultant $\int (\sigma_{xx} - \sigma_{zz}) dz$
GPE^*	corrected GPE, $-\bar{\sigma}_{zz}$
$\Delta(\cdot)$	column difference, $(\cdot)(x_I) - (\cdot)(x_T)$

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Supporting Information for “The trench pull force: constraints from elasto-plastic bending models”

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Table S1. Additional symbols used in the shear-stress derivation of Text S4.

Symbol	Meaning
ζ	plate-centred vertical coordinate, $ \zeta \leq h/2$
c	elastic-core half-thickness in a partially yielded plate
φ_a	unit-integral parabolic shear-stress profile over $ \zeta \leq a$
V'	horizontal gradient dV/dx of the vertical shear resultant
c'	horizontal gradient dc/dx of the elastic-core half-thickness

S1 Integral force balance, isostatic columns and dipole construction

This text derives the vertically integrated horizontal force balance from its boundary tractions, distinguishes isostasy from lithostatic stress, and gives the first-moment and dipole forms of the vertical-normal-stress contribution.

S1.1 The vertically integrated horizontal force balance

Static equilibrium of a two-dimensional region Ω requires its boundary tractions and body force to sum to zero:

$$\oint_{\partial\Omega} \boldsymbol{\sigma} \mathbf{n} ds + \int_{\Omega} \rho \mathbf{g} dA = \mathbf{0}. \quad (\text{S1})$$

Consider the non-isostatic rectangle $x_T \leq x \leq x_I$, $z_0 \leq z \leq z_c$ shown in Figure 1 of the main text, with z positive downwards. Its outward normals on the left, right, upper and lower boundaries are respectively $-\mathbf{e}_x$, $+\mathbf{e}_x$, $-\mathbf{e}_z$ and $+\mathbf{e}_z$. Gravity has no horizontal component, and the upper boundary carries no horizontal shear traction. Taking the horizontal component of Equation (S1) therefore gives

$$0 = - \int_{z_0}^{z_c} \sigma_{xx}(x_T, z) dz + \int_{z_0}^{z_c} \sigma_{xx}(x_I, z) dz + \int_{x_T}^{x_I} \tau_{zx}(x, z_c) dx. \quad (\text{S2})$$

Using the main-text definitions $\Delta(\cdot) \equiv (\cdot)(x_I) - (\cdot)(x_T)$, $\bar{\sigma}_{xx} \equiv \int_{z_0}^{z_c} \sigma_{xx} dz$, and $F_B \equiv \int_{x_T}^{x_I} \tau_{zx}(x, z_c) dx$, Equation (S2) reduces to

$$\Delta \bar{\sigma}_{xx} + F_B = 0, \quad (\text{S3})$$

which is Equation (2) of the main text.

The connection with pressure follows by writing $\sigma_{ij} = -p\delta_{ij} + \tau_{ij}$. Thus,

$$\sigma_{xx} = -p + \tau_{xx} = (\sigma_{zz} - \tau_{zz}) + \tau_{xx} = \sigma_{zz} + (\tau_{xx} - \tau_{zz}). \quad (\text{S4})$$

For two columns, a difference in the full vertical normal stress therefore contributes to the difference in horizontal stress, while the normal-stress difference contains the remaining contribution. This is not a unique separation of pressure from deviatoric stress: changing σ_{xx} while holding σ_{zz} fixed changes both. The utility of the decomposition is instead that σ_{zz} can be constrained from vertical equilibrium before the vertically integrated horizontal balance is evaluated. After integration through depth,

$$\bar{\sigma}_{xx} = \bar{\sigma}_{zz} + \overline{\sigma_{xx} - \sigma_{zz}}. \quad (\text{S5})$$

The two terms on the right are represented by

$$\text{GPE}^* \equiv -\bar{\sigma}_{zz}, \quad N_D \equiv \overline{\sigma_{xx} - \sigma_{zz}} = \int_0^{z_c} (\sigma_{xx} - \sigma_{zz}) dz. \quad (\text{S6})$$

Substitution into Equation (S3) gives

$$-\Delta \text{GPE}^* + \Delta N_D + F_B = 0, \quad \text{or} \quad \Delta N_D = \Delta \text{GPE}^* - F_B, \quad (\text{S7})$$

identical to Equation (7) of the main text.

The terms in Equation (S7) are signed horizontal force contributions on the chosen region; positive values act to the right. A positive ΔN_D is therefore a rightward contribution: the integrated normal-stress difference is more positive in the right-hand column than in the left-hand column. A positive $\Delta \bar{\sigma}_{zz}$ is likewise rightward. The definition $\text{GPE}^* = -\bar{\sigma}_{zz}$ retains the conventional GPE sign based on positive lithostatic pressure. Accordingly, $\Delta \text{GPE}^* > 0$ means $\Delta \bar{\sigma}_{zz} < 0$ and represents a leftward force. In the ridge-side domain of Figure 1, for example, GPE increases from x_I towards the elevated ridge at x_R , and ridge push acts to the left. The same convention applies between x_T and x_I : positive x is seaward, so a positive trench-pull force is directed leftwards, towards the trench.

S1.2 Isostatic and lithostatic columns

The shear function $Q(z)$ is defined in Equation (8) of the main text. A column is isostatic at z_c when

$$Q(z_c) = \int_0^{z_c} \tau_{zx,x} dz = 0. \quad (\text{S8})$$

This is an integral condition: finite positive and negative shear-stress gradients may occur at intermediate depths and cancel by z_c . A lithostatic column satisfies the stronger pointwise condition $Q(z) = 0$, or equivalently $-\sigma_{zz}(z) = P_L(z)$, at every depth. Isostasy is often conflated with this lithostatic vertical-normal-stress condition (e.g., Dahlen, 1981). In the body-force-free numerical models, the common lithostatic stress is omitted and σ_{zz} denotes only the perturbation produced by flexure. A lithostatic column therefore appears as $\sigma_{zz} = 0$ at every depth. The relevant rock–water body-force difference is not discarded: it is represented by the Winkler restoring pressure $\Delta \rho g w$.

To obtain the simple analytical construction in Section 2.4 of the main text, the first isostatic column is approximated as lithostatic throughout, so that $\hat{\rho}(x_I, z) = 0$.

This gives the two-lobe representation in Figure 2(a); panel (b) shows the corresponding vertical-normal-stress difference, and Equation (17) gives its mathematical form. The approximation is used only in this analytical development; it is not imposed when the numerical models are analysed. Instead, the trench-pull force is calculated from the full vertical normal stress in both columns, including any internally cancelling shear-stress gradients at x_I . Such gradients are visible in Figure 3(c) of the main text and Figure S5(a), even though their depth integral at x_I is nearly zero. Approximating this column as lithostatic means setting $\tau_{zx,x}(x_I, z) = 0$ at every depth. Its effect on the trench-pull force can be tested by replacing $\bar{\sigma}_{zz}(x_I)$ with zero in post-processing. This changes the inferred force by only 0.13% in the reference model. The change reaches 2.0% across Suite 1 and 3.8% across all numerical models. Thus, although the first isostatic column is not lithostatic pointwise, $-\sigma_{zz}(x_I, z) \neq P_L(x_I, z)$, its depth-integrated contribution is small and $\text{GPE}^*(x_I) \approx \text{GPE}(x_I)$ for the trench-pull calculation.

S1.3 Direct and equivalent-density forms

The vertical component of local equilibrium is

$$\frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} + \rho g = 0. \quad (\text{S9})$$

Integrating downward gives Equation (8) of the main text. For two columns x_0 and x_1 , let $\Delta(\cdot) \equiv (\cdot)(x_1) - (\cdot)(x_0)$, as in the main text. If their vertical normal stresses coincide at z_c , vertical equilibrium requires

$$\int_0^{z_c} (g \Delta \rho + \Delta \tau_{zx,x}) dz = 0. \quad (\text{S10})$$

At a depth $z < z_c$, their vertical-normal-stress difference is

$$-\Delta \sigma_{zz}(z) = \int_0^z [g \Delta \rho(\xi) + \Delta \tau_{zx,x}(\xi)] d\xi. \quad (\text{S11})$$

Let $F(z) \equiv -\Delta \sigma_{zz}(z)$. Equation (S11) gives $F'(z) = g \Delta \rho(z) + \Delta \tau_{zx,x}(z)$, while Equation (S10) gives $F(z_c) = 0$: the zeroth moment of the density and shear-stress-gradient differences vanishes. Integration by parts then gives their contribution to the vertically integrated horizontal force balance (Equation (7) of the main text):

$$\begin{aligned} \Delta \text{GPE}^* &= \int_0^{z_c} F(z) dz \\ &= [zF(z)]_0^{z_c} - \int_0^{z_c} z F'(z) dz \\ &= -g \int_0^{z_c} z \Delta \rho dz - \int_0^{z_c} z \Delta \tau_{zx,x} dz. \end{aligned} \quad (\text{S12})$$

The boundary term vanishes because the upper equipotential is the coordinate origin and $F(z_c) = 0$. Equation (S10) also makes the moment independent of the chosen depth origin. Once their depth integrals balance, the force depends only on where the density and shear-stress-gradient differences occur within the columns. The shear-stress-gradient difference enters both the closure condition and the first-moment force in the same mathematical position as the density difference.

Defining $\hat{\rho} = \tau_{zx,x}/g$ and $\rho^* = \rho + \hat{\rho}$ combines these terms, giving

$$\int_0^{z_c} \Delta \rho^* dz = 0, \quad \Delta \text{GPE}^* = -g \int_0^{z_c} z \Delta \rho^* dz. \quad (\text{S13})$$

Equivalent density is therefore a representation of shear support in density units, not true mass and not an additional force. Equality of the full vertical normal stress at z_c

requires any difference in integrated true density between the columns to be balanced by an equal and opposite difference in equivalent density. Their sum has zero integral but can retain a finite first moment because the two contributions may occur at different depths. Where each has a well-defined centre of mass, that first moment is their common integrated magnitude multiplied by the separation between their centres. Either column may contain finite $\hat{\rho}$; only the complete difference $\Delta\rho^*$ is required. The clean two-lobe form used for the idealised trench follows after the additional assumptions stated below.

S1.4 Topographic pressure-gradient forces and the dipole picture

Both conventional GPE forces, $\Delta\bar{P}_L$, and GPE-like forces, $-\Delta\bar{\sigma}_{zz}$, can be finite without any surface relief. A zero-integral difference between two columns can still have a finite first moment. For example, two equal-thickness, two-layer columns remain isostatic and have the same surface elevation if their density layers are exchanged, but their density moments differ. Not all GPE or GPE-like forces are therefore topographic.

Ridge push and trench pull form a more specific class. A *topographic pressure-gradient force* is defined here as a GPE or GPE-like force for which one physically identifiable part of the density-difference distribution is the rock–water or rock–air exchange created by surface relief. Topography sets the mass of this lobe and the near-surface pressure anomaly. Closure requires the remainder of the corrected-density difference to have an equal and opposite integral (Equation (13) of the main text). At a ridge, that compensation is supplied by the thermal density structure of isostatic lithosphere; at a trench, it is supplied by vertical shear-stress gradients within the deflected plate.

Once the topographic lobe is specified, the remaining corrected-density difference is its compensation. If their common integrated magnitude is M and their centres of mass are z_{top} and z_{comp} , Equation (S13) becomes

$$|\Delta\text{GPE}^*| = gM|z_{\text{comp}} - z_{\text{top}}|. \quad (\text{S14})$$

The first moment is unique, although its decomposition into arbitrary dipoles is not. The topographic decomposition is physically distinguished because the surface lobe is independently fixed by relief and the rock–water or rock–air density contrast.

The two lobes need not be vertically separated. True- and equivalent-density differences may overlap, either column may contain shear support, and either part may contain several positive and negative intervals. The moment relation remains valid provided that the complete corrected-density difference is used and Equation (S10) is satisfied.

The main-text analytical estimate makes three additional simplifications: below the level of the trench axis, $z = w_T$, the two columns (x_T and x_I) have the same true-density structure; the reference column has $\hat{\rho} = 0$; and the trench shear support has one centre of mass near the middle of a mechanically connected load-bearing plate. The numerical calculation does not impose these three simplifying assumptions. It obtains the trench-pull force directly from $-\Delta\bar{\sigma}_{zz}$, where $\sigma_{zz}(x, z)$ is the Cauchy vertical normal stress recovered from the finite-element solution and integrated through the deformed columns (Section S2.4). Equivalent density is calculated separately from the modelled shear-stress gradient, and its first moment provides a second calculation of the same force.

S2 Numerical formulation, extraction and verification

This section provides details of the density simplification, boundary loading, constitutive models and post-processing used for the numerical experiments. Numerical benchmarks and convergence tests follow below.

S2.1 Body-force-free formulation and restoring traction

The plate occupies $0 \leq x \leq L$, $0 \leq z \leq h$ in its reference configuration and is treated in plane strain. No body force is applied within the plate; the computed stresses arise from the imposed trench-edge loading and the resulting flexure. The rock–water density contrast produced by deflection is represented only through a vertical Winkler restoring traction, $\Delta\rho g w(x)$, applied at the upper surface and directed opposite to the deflection, with stiffness $k_f = \Delta\rho g$ (Turcotte & Schubert, 2002; Watts, 2001). The base is traction-free, the far end is fixed, and the prescribed trench-edge traction is described in Text S2.2.

Finite rotation of the plate geometry is retained, and the reported stress resultants are calculated from Cauchy stresses on vertical sections through the deformed plate (Text S2.4). Because these sections begin at the deflected plate top, they exclude the water wedge between that surface and sea level, $\frac{1}{2}\Delta\rho g w_T^2$ (4.5% of the reference trench-pull force). Density differences associated with displacement of the crustal and thermal structure are likewise omitted; both contributions are assessed separately in Text S3.

S2.2 Trench-edge loading

The numerical model represents slab loading of the trailing plate as a vertical shear traction on the trench-edge column. The applied traction follows the parabolic shear-stress profile of a uniform elastic plate (Text S4.1). Using the plate-centred coordinate $\zeta = z - h/2$ and the positive load magnitude V , the imposed traction is

$$\tau_{zx}(0, \zeta) = -\frac{3V}{2h} \left(1 - \frac{4\zeta^2}{h^2} \right), \quad |\zeta| \leq \frac{h}{2}. \quad (\text{S15})$$

Thus the signed resultant at the loaded edge is $\int_{-h/2}^{h/2} \tau_{zx} d\zeta = -V$, and the traction vanishes at the upper and lower surfaces. Horizontal traction on the trench face is zero in the reference model; Suite 3 applies a uniform horizontal traction to impose the stated background N_D . This specifies τ_{zx} only at the loaded boundary; its horizontal gradient within the plate is determined by the mechanical solution. This construction allows the vertical force transmitted by the slab to be varied independently. A model that continues through the slab would replace the boundary load with a continuous stress field, but would not permit the same isolation of the applied resultant. At the opposite end, the displacement is fixed at L ; for the reference model, $L \approx 13\alpha$, where $\alpha = (4D/\Delta\rho g)^{1/4}$ is the flexural parameter. $D = Eh^3/[12(1 - \nu^2)]$ is the elastic flexural rigidity.

To keep the loaded edge elastic, the plastic models add 100 MPa to the yield strength at the trench. This increment is uniform through the plate thickness and decays exponentially away from the trench, with a characteristic decay length of 10 km; it is negligible by the maximum-bending-moment region.

S2.3 Constitutive models and solution procedure

All models use the same elastic constants and finite-element solution path. The elastic behaviour is represented by a St. Venant–Kirchhoff material with modulus E and Poisson ratio ν , which reduces to linear elasticity at small strain. The elastic model is obtained by setting the yield stress effectively to infinity.

The uniform elasto-plastic model uses the in-plane Tresca yield surface

$$(\sigma_{xx} - \sigma_{zz})^2 + 4\tau_{zx}^2 = \sigma_Y^2, \quad (\text{S16})$$

where $\sigma_Y = 150$ MPa is the in-plane principal-stress difference at yield; the corresponding maximum shear stress is $\sigma_Y/2 = 75$ MPa. The criterion excludes the out-of-plane stress σ_{yy} , which remains elastic. This model is perfectly plastic ($H = 0$). The two depth-dependent models instead use a von Mises yield criterion under plane-strain conditions

and replace the uniform strength by

$$\sigma_Y(z) = C + \mu \rho_m g f(z). \quad (\text{S17})$$

The asymmetric model uses $f(z) = z$, $\mu = 0.15$ and $C = 31.5$ MPa, producing a weak upper surface and a strong base. The symmetric model uses $f(z) = h/2 - |z - h/2|$, $\mu = 0.35$ and $C = 36.7$ MPa, producing weak surfaces and maximum strength at mid-depth. The depth-dependent envelopes are calibrated to the uniform-Tresca plastic moment,

$$M_p = \frac{\sigma_Y h^2}{4} = 1.35 \times 10^{17} \text{ N}. \quad (\text{S18})$$

The two depth-dependent models use linear isotropic hardening, $\sigma_Y(z) + H\kappa_p$, where κ_p is the accumulated plastic multiplier and $H = 7.0 \times 10^8$ Pa. This modest hardening is retained to prevent the weak surface yield bands from localising at the mesh scale. It increases the local yield strength by at most 5.3 MPa over the strains reached in these models. None of the models includes time-dependent ductile deformation or creep.

The equations are solved using the Ferrite finite-element library (Carlsson et al., 2024). The mesh contains 800×48 biquadratic quadrilaterals, giving $\Delta x = 2$ km and $\Delta z = 1.25$ km. The trench load is applied incrementally, with equilibrium solved at each step and the plastic state carried forward between steps. The increment-convergence test and the counts used for each model class are reported in Section S2.6.

S2.4 Extraction of force resultants

Because the upper and lower surfaces carry no horizontal traction, the horizontal equilibrium equation gives

$$\frac{d\bar{\sigma}_{xx}}{dx} = 0. \quad (\text{S19})$$

The vertical equilibrium equation and the Winkler restoring traction give

$$\frac{dV}{dx} = \Delta \rho g w. \quad (\text{S20})$$

Thus Equation (18) of the main text is exact for the numerical model. These integrated relations constrain the resultants but do not specify the depth distribution of the shear-stress gradient.

The solver exports the second Piola–Kirchhoff stress $\mathbf{S}$ and nodal displacement on the reference mesh; the resultants are formed in post-processing. At each node, the Cauchy stress is obtained by the push-forward $\boldsymbol{\sigma} = J^{-1} \mathbf{F} \mathbf{S} \mathbf{F}^T$, where $\mathbf{F}$ is the deformation gradient and $J = \det \mathbf{F}$. The Cauchy stresses are interpolated onto vertical lines through the deformed plate and integrated over deformed depth to obtain $\bar{\sigma}_{zz}$ and $\bar{\sigma}_{xx} - \bar{\sigma}_{zz}$. These resultants give GPE^* and N_D , respectively, through Equation (S6). The trench-pull force is ΔGPE^* between the trench and first isostatic columns. The trench column, x_T , is the leftmost vertical line that passes continuously through the full plate thickness, so its position accounts for rotation of the loaded face. The first isostatic column, x_I , is the first position outboard of the trench where the deflection and foundation load return to zero; no assumption about its internal stress structure enters the extraction.

Independently, the signed centre of mass of the equivalent-density distribution is calculated from the finite-element shear-stress gradient as

$$z_{\hat{\rho}} = \frac{\int z \tau_{zx,x} dz}{\int \tau_{zx,x} dz}, \quad (\text{S21})$$

wherever the denominator is non-zero. This diagnostic is distinct from the force-based arm reported in the main text and is undefined at columns where the net gradient vanishes.

For the reference model, the directly extracted resultants satisfy $\Delta N_D = \Delta \text{GPE}^*$ with a residual of 0.019%. The residual is defined as

$$\frac{|\Delta N_D - \Delta \text{GPE}^*|}{|\Delta \text{GPE}^*|}. \quad (\text{S22})$$

Equation (16) of the main text defines the trench pressure deficit as $\Delta \rho g w_T$, and Equation (19) writes the trench-pull force as this pressure deficit multiplied by a dipole arm. The force-based effective arm is therefore $\Delta \text{GPE}^*/(\Delta \rho g w_T)$ for each model. The local centre-of-mass curve in Figure 3(c) is instead calculated directly from the equivalent-density profile, $\hat{\rho}(z) = g^{-1}\tau_{zx,x}(z)$, at each horizontal position. The plotted centre of mass is signed and is compared with the dashed mid-plate reference $h/2$. Its denominator, $g^{-1}dV/dx$, vanishes at isostatic columns, so the curve is not plotted there.

S2.5 Benchmarks against classical solutions

The elastic model is benchmarked against the closed-form thin-beam solution for an end-loaded beam on an elastic foundation (Hetényi, 1946) (Figure S1; Table S2). The shear resultant differs from the analytical solution by at most 0.8%, and the vertical shear stress follows the predicted parabolic depth profile (Figure S1(c)), on which the development of Text S4 is based. The finite-element deflection is 1.6% greater than the thin-beam prediction, consistent with the contribution of transverse-shear deformation in a finite-thickness model, which thin-beam theory omits (S. P. Timoshenko, 1921).

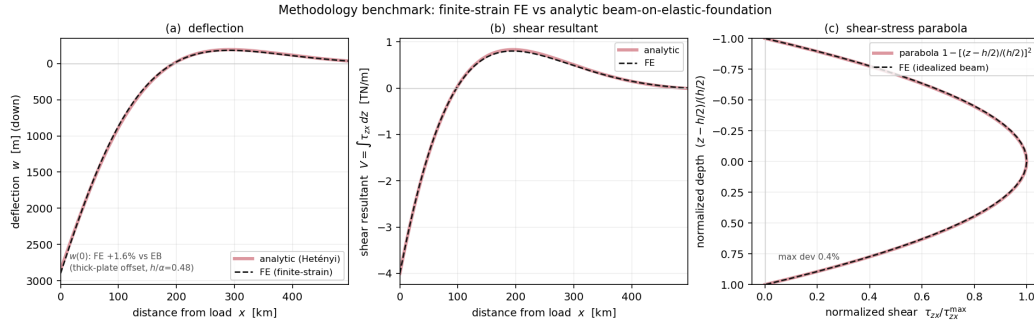


Figure S1. The elastic model of Suite 1 against the classical flexure solutions ($V = 4 \text{ TN m}^{-1}$, $\alpha = 124 \text{ km}$). (a) Deflection against the closed-form thin-beam solution: the finite-element deflection exceeds the thin-beam prediction by 1.6% — the finite-thickness offset discussed in the text. (b) The shear resultant V against the analytic form: maximum error 0.8%, and the end value, 4.000 TN m^{-1} , recovers the applied-load magnitude. (c) The vertical shear stress against the elastic parabola (maximum deviation 0.39%; body-force-free, constant-shear idealized beam, $\sigma_{zz} \approx 0$, normalized). Both idealized-beam benchmarks (Figures S1 and S2) use the exported material-frame stress on reference sections by design; the production-model resultants instead use Cauchy stress on vertical lines through the deformed plate (Text S2.4).

The plasticity is benchmarked against the elastic-perfectly-plastic pure-bending moment-curvature relation for a rectangular section (e.g., Govindjee, 2012): the moment grows in proportion to curvature until first yield, then flattens as yielding spreads inward from the surfaces, approaching the fully plastic moment M_p as a limit; for a rectangular section $M_p = \sigma_Y h^2/4$ and first yield sits at $M_y = \frac{2}{3}M_p$. The test uses a separate model configuration, run with the same Tresca constitutive model and finite-element plasticity implementation as the reference flexure model, but without an isostatic restoring force. One end is fixed; the other is displaced vertically while remaining free to move

Table S2. Solver verification and benchmarks (elastic model of Suite 1, $V = 4 \text{ TN m}^{-1}$, unless stated).

Check	FE result	Reference
end-shear deflection $w(0)$	2899 m	2853 m thin-beam (+1.6%; see t
deflection profile $w(x)$, 0–4 α	max error 1.6%	Hetényi (the same offset)
shear resultant $V(x)$	max error 0.8%	analytic; end magnitude 4.000 T
shear-stress parabola $\tau_{zx}(z)$	max deviation 0.39%	elastic parabola (idealized beam)
plastic return map	returned stress lies on the prescribed yield surface	relative tolerance 10^{-9}
consistent tangent	agrees with centred finite differences	relative tolerance 10^{-5} (Tresca a

horizontally. The nondimensional beam has $L/h = 10$, $E = 60$, $\nu = 0.25$ and $\sigma_Y = 1$, and is discretized by 200×40 biquadratic elements. The end displacement is increased to $2.2h$ in 44 increments. The Tresca yield strength is uniform through the beam thickness and along most of its length, but increases exponentially towards each end with a characteristic length of $0.05L$, reaching nine times the interior value at the boundaries to suppress corner yielding. Each section is normalized by its local yield strength, and the model has no hardening. Sections at different positions are bent by different amounts, so a single run samples the whole moment–curvature curve. Curvature is obtained from the slope of σ_{xx} through the elastic core, divided by the plane-strain modulus $E' = E/(1-\nu^2)$. The beam also carries a constant shear resultant and is therefore not in exact pure bending; this accounts for the small residual departure from the analytical relation. The resulting section values track the analytical curve (Figure S2), with a mean deviation of 0.09% of M_p and a maximum of 0.85%.

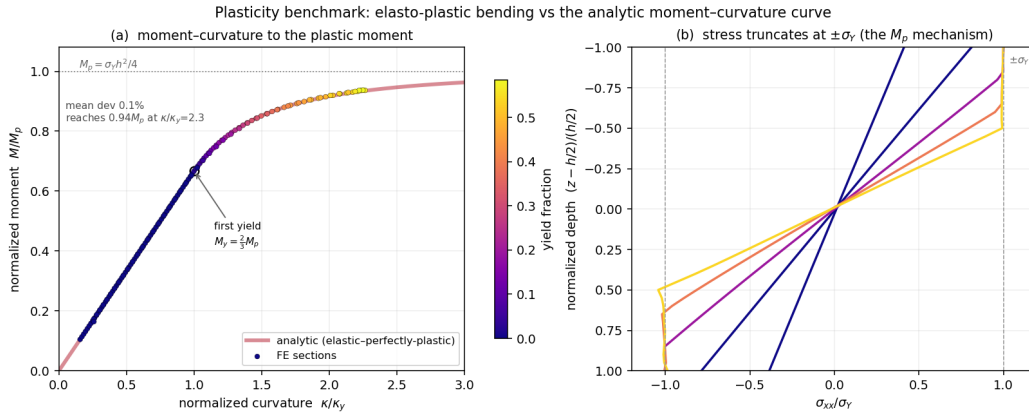


Figure S2. The elasto-plastic benchmark: moment–curvature to the plastic moment (a Tresca beam without an isostatic restoring force or hardening, bent by an imposed end displacement). (a) M/M_p against κ/κ_y — each section normalized by quantities formed from its own local yield stress σ_Y : the plastic moment $M_p = \sigma_Y h^2/4$ for the moment, the yield curvature $\kappa_y = 2\sigma_Y/(E'h)$ for the curvature, with $E' = E/(1-\nu^2)$ — for sections spanning one run (mean deviation from the analytic curve 0.09% of M_p): the elastic line $M/M_p = \frac{2}{3} \kappa/\kappa_y$, the onset of yield at $M_y = \frac{2}{3} M_p$, and the flattening toward M_p along $M/M_p = 1 - \frac{1}{3}(\kappa_y/\kappa)^2$. (b) $\sigma_{xx}(z)$ truncating at $\pm\sigma_Y$ about a shrinking elastic core — the mechanism behind M_p .

S2.6 Mesh convergence

To check that the results are not limited by the numerical resolution, the reference Tresca model is repeated at three mesh resolutions, spanning a factor of three, and with the number of load increments varied (Table S3). The trench deflection is the same to the metre at all three resolutions, and the pull at the 800×48 mesh is within 0.08% of its value at the highest resolution. Reducing the number of load increments in the reference Tresca model from 24 to 12 changes the pull by only 0.005%. The elastic and Tresca models use 24 increments, whereas the depth-dependent models use 22. The third column of the table is the residual of the equilibrium identity $\Delta N_D = \Delta \text{GPE}^*$ — the statement of the vertically integrated horizontal force balance between two columns (Eq. 18 of the main text). This residual shrinks as the mesh is refined. It is 0.019% for the reference calculation and remains below 0.1% across all 20 production models.

Table S3. Mesh convergence and equilibrium consistency (reference Tresca model, $V = 4 \text{ TN m}^{-1}$, 24 load increments).

mesh $n_x \times n_z$	w_T [m]	ΔGPE^* [TN m^{-1}]	identity residual [%]
400×24	3233	2.550	0.097
800×48 (all model suites)	3233	2.542	0.019
1200×72	3233	2.540	0.010

S3 Scales, assumptions and subsidiary contributions

This text expands the idealizations summarized in Section 2.1 of the main text. The natural length scale of the problem is the separation between the trench and the first isostatic column. It is set by the flexural parameter and is generally $L_{TI} = O(10^2 \text{ km})$ (Caldwell et al., 1976; Hunter & Watts, 2016; Watts, 2001); in the reference numerical model, x_I is located at approximately 182 km in the model coordinate. The trench-pull balance is evaluated over the interval from x_T to x_I .

Cartesian (flat-Earth) geometry. Because $L_{TI}/R_E \ll 1$, Earth curvature is neglected over the trench-isostatic-column interval. The same approximation allows treating the equipotentials as horizontal planes across the domain.

Two-dimensional plane strain. For a uniform plate, the flexural support of a deflection of wavenumber k scales as $Dk^4 w$. Its characteristic wavelength is therefore set by the flexural parameter $\alpha = (4D/\Delta\rho g)^{1/4}$, a small multiple of the plate thickness (Caldwell et al., 1976; Turcotte & Schubert, 2002; Watts, 2001). The trench-outer-rise profile varies across strike on this scale, whereas trenches generally vary along strike over distances many times greater (Hunter & Watts, 2016). Flexural support consequently becomes negligible in the along-strike curvature contribution, providing the basis for a trench-normal plane-strain section. The approximation is least appropriate at arc junctions and subducting ridges, where along-strike variations approach the flexural wavelength.

A hydrostatic asthenosphere. Classical trench-flexure models assume that the material beneath the plate is hydrostatic: pressure is constant on equipotentials and basal shear is absent (Caldwell et al., 1976; Turcotte et al., 1978; Chapple & Forsyth, 1979; Turcotte & Schubert, 2002). Trench relief is therefore treated as flexurally supported, not as dynamic topography sustained by lateral variations in vertical normal stress beneath the plate. Pressure gradients do occur elsewhere in subduction systems, including within the mantle wedge and in plate-scale asthenospheric return flow (Tovish et al., 1978; Schubert et al., 1978; Holt, 2022), but these are not the support assumed for the short-wavelength trench-outer-rise topography. Here the trench pressure deficit is in-

stead maintained by stresses within the flexing plate — in the words of Richter and McKenzie (1977), “maintained by elastic forces.”

Density simplification and the Winkler load. The analytical estimate retains only the rock–water density difference created by deflection. The numerical problem therefore includes no gravitational body force within the plate and represents this difference by the Winkler restoring pressure $\Delta\rho g w$. Any common lithostatic contribution would be identical in the material columns and is therefore omitted. In real oceanic lithosphere, deflection also displaces the crust and thermal structure, producing additional density contrasts between the columns. The estimates in Section S3.1 show that these make contributions of a few per cent.

A plate of finite thickness. This idealization enters both the uniform-plate estimate in Section 2.5 and the numerical models in Section 3 of the main text. The numerical plate has a thickness h and a discrete base; the lithosphere, whose strength degrades continuously with temperature, has no such boundary. Its counterpart in the analytical scaling is the mechanical thickness h_m , referring to the part of the thermal lithosphere that can support significant differential stress. It is sometimes defined by a threshold in the strength envelope at a chosen differential-stress limit; the depth of the neutral plane can also be used as a proxy for half the mechanical thickness when the strength structure is approximately symmetric. The consequences of this idealization, and the graded-strength models that test it, are discussed in Section 5.4 of the main text.

A statics problem. Section 2 of the main text is built entirely on static equilibrium, without inertia or flow. The GPE-like force associated with non-isostatic columns, ΔGPE^* , is quantified by integrating that equilibrium over a finite region; the resulting identity is independent of constitutive behaviour. Flow models of subduction do develop trench-pull forces of a few TN m^{-1} (Sandiford & Craig, 2023; Sandiford, 2026), but in them the load, the plate thickness, and the trench geometry co-evolve, so their effects cannot be isolated in the same way. The static models used here allow the load, plate thickness and constitutive behaviour to be varied separately.

S3.1 Subsidiary contributions to the trench pull

The uniform-plate approximation neglects density contrasts produced when the crustal and thermal structures are displaced, and the plate-column calculation omits the sea-level wedge included in the theoretical integration convention. The following estimates use the same dipole construction and show that each contribution is an order of magnitude smaller than the $\approx 2.5 \text{ TN m}^{-1}$ trench-pull force. Reference values are $w_T = 3.2 \text{ km}$ and $h_m = 60 \text{ km}$. Here $\delta(\Delta\text{GPE}^*)$ denotes the correction to the uniform-plate estimate.

Displaced crust.

For two columns of the same age, the trench column’s density profile is, to first order, the isostatic column’s profile displaced downward by w_T , so $\rho_I(z) - \rho_I(z - w_T)$ gives their density difference: every internal interface contributes a block of thickness w_T . With a crust of thickness h_c and density ρ_c , the anomaly consists of $(\rho_c - \rho_w) w_T$ at the seafloor and $(\rho_m - \rho_c) w_T$ at the Moho (depth h_c). The net anomaly remains $(\rho_m - \rho_w) w_T$, so the integrated magnitude of the compensating equivalent-density lobe is unchanged. Only the deficit centroid moves, by $(\rho_m - \rho_c) h_c / (\rho_m - \rho_w)$. The resulting change in the trench-pull force is

$$\delta(\Delta\text{GPE}^*) = -g (\rho_m - \rho_c) h_c w_T \approx -0.10 \text{ TN m}^{-1} \quad (\rho_m - \rho_c = 400 \text{ kg m}^{-3}, h_c = 7 \text{ km}), \quad (\text{S23})$$

a reduction of approximately 4%. Part of the mass deficit occurs at the Moho rather than the seafloor, shortening the moment arm.

Displaced thermal density profile.

Let $\delta\rho_{\text{th}}(z)$ be the thermal density excess of the cold plate over the asthenospheric adiabat, decreasing from $\delta\rho_{\text{th}}(0) = \rho_m \alpha_T \Delta T$ at the surface over the thermal thickness. Displacing this profile downward by w_T produces the distributed anomaly $w_T d\delta\rho_{\text{th}}/dz$, whose integral, $-w_T \delta\rho_{\text{th}}(0)$, is a mass excess in the trench column. The compensating equivalent-density lobe is approximated as acting at $z_s \approx w_T + h_m/2$, following the uniform-plate scaling of Equation 19 in the main text. Evaluating the dipole gives

$$\delta(\Delta\text{GPE}^*) = g w_T \delta\rho_{\text{th}}(0) (z_{\text{th}} - z_s), \quad z_{\text{th}} \equiv \frac{\int_0^{z_c} \delta\rho_{\text{th}} dz}{\delta\rho_{\text{th}}(0)}. \quad (\text{S24})$$

For half-space cooling at 100 Myr ($\alpha_T = 3 \times 10^{-5} \text{ K}^{-1}$, $\Delta T \approx 1350 \text{ K}$ and $z_{\text{th}} = 2\sqrt{\kappa_{\text{th}} t}/\sqrt{\pi} \approx 60 \text{ km}$, where κ_{th} is thermal diffusivity), $\delta\rho_{\text{th}}(0) \approx 130 \text{ kg m}^{-3}$ and $\delta(\Delta\text{GPE}^*) \approx +0.1 \text{ TN m}^{-1}$, approximately 5% of the trench-pull force. This estimate is sensitive to the depth of the compensating support for mass perturbations below the mechanical plate and is therefore quoted as an order of magnitude.

Sea-level wedge and plate-column convention.

The theoretical construction in Section 2 of the main text begins at the sea-level equipotential. It therefore includes the wedge between that level and the deflected plate top, whose first-moment contribution is

$$F_w = \frac{1}{2} \Delta\rho g w_T^2 = 0.11 \text{ TN m}^{-1}, \quad (\text{S25})$$

for the reference $w_T = 3.18 \text{ km}$. By contrast, the massless finite-element model begins each column at the deflected plate top and omits this wedge. Equation (S25) is 4.5% of the reference trench-pull force. The reported model forces are plate-column values, and their comparison with the $h/2$ scaling uses the same $w_T = w(x_T)$ in both numerator and denominator, so that comparison is internally consistent.

The displaced-crust and thermal-profile corrections are each approximately 5% of the trench-pull force and have opposite signs, supporting the uniform-plate approximation at approximately the 10% level. The sea-level wedge is instead a defined difference between integration conventions; Equation (S25) allows the theoretical and plate-column values to be compared directly.

S4 The shear-stress distribution and its horizontal gradient

This text develops the depth distribution of the vertical shear stress τ_{zx} and its horizontal gradient $\tau_{zx,x}$. Together with density, the depth integral of this gradient sets σ_{zz} . Its depth distribution therefore determines how the vertical-normal-stress difference between the trench and reference columns *equilibrates*, and hence sets the trench-pull force (Section 2.3 and Figure 2b of the main text). For a uniform elastic plate of finite thickness, the vertical shear stress and its gradient are both parabolic in depth, as developed in Section S4.1 (Tanimoto, 1957). This derivation uses the small-slope, linear-geometry approximation of thin-plate theory: the stress distribution is evaluated in the undeformed plate geometry, omitting the geometrical effects of section rotation and deflection. The subsequent development asks how yielding modifies this distribution. It is restricted to symmetric geometry, loading and yield patterns, for which the narrowing of the elastic core and the variation of the shear resultant along the plate can be treated analytically.

S4.1 The elastic plate

Throughout this text we use the plate-centred coordinate ζ , measured from the plate mid-surface and positive downward, with the plate of thickness h occupying $|\zeta| \leq h/2$.

Under the small-slope approximation stated above, ζ is treated as vertical. It differs from the absolute depth z of Section 2 of the main text only by a shift, so $\partial/\partial\zeta = \partial/\partial z$.

Thin-plate flexure theory retains the vertical shear stress only through its resultant $V = \int_{-h/2}^{h/2} \tau_{zx} d\zeta$ and does not specify its depth distribution (Turcotte & Schubert, 2002). That distribution can be recovered from a full two-dimensional elasticity solution using Airy stress functions (S. Timoshenko & Goodier, 1970). Here beam theory gives the bending part of the normal-stress difference directly: $\sigma_{xx} - \sigma_{zz} = M(x)\zeta/I$, where $I = \int_{-h/2}^{h/2} \zeta^2 d\zeta$ is the second moment of area per unit length. The difference varies linearly with depth and vanishes at the neutral plane $\zeta = 0$. The common lithostatic stress has no horizontal gradient and, in the beam-theory approximation, the transverse normal-stress perturbation is neglected, giving $\sigma_{xx} \simeq M\zeta/I$. Substituting into horizontal equilibrium (the subscripts on τ_{zx} label the stress component; vertical derivatives in ζ and z coincide, as above),

$$\frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\tau_{zx}}{\partial\zeta} = 0, \quad (\text{S26})$$

gives $\partial\tau_{zx}/\partial\zeta = -(dM/dx)\zeta/I = -V\zeta/I$, using $V = dM/dx$. Integrating downward from the traction-free upper surface ($\tau_{zx}(-h/2) = 0$),

$$\tau_{zx}(x, \zeta) = \frac{V(x)}{I} \left(\frac{h^2}{8} - \frac{\zeta^2}{2} \right), \quad (\text{S27})$$

which also vanishes at the traction-free lower surface, $\tau_{zx}(+h/2) = 0$, and carries the resultant $\int \tau_{zx} d\zeta = V$: the vertical shear stress in an elastic plate is parabolic. To compare this full-thickness profile with the profile in a yielded plate, define the unit-integral parabola

$$\varphi_a(\zeta) = \frac{3}{4a^3} (a^2 - \zeta^2), \quad \int_{-a}^a \varphi_a(\zeta) d\zeta = 1, \quad (\text{S28})$$

where a is the half-thickness of the region carrying vertical shear. For the elastic plate $a = h/2$, and the distribution is

$$\tau_{zx}(x, \zeta) = V(x) \varphi_{h/2}(\zeta), \quad (\text{S29})$$

the shape function carries no amplitude, and the resultant is V by construction. The same family, with a shrunk below $h/2$, will describe the yielded plate in Section S4.2. Since $\varphi_{h/2}$ is independent of x , the horizontal gradient is parabolic as well (Tanimoto, 1957),

$$\tau_{zx,x}(\zeta) = \varphi_{h/2}(\zeta) \frac{dV}{dx}. \quad (\text{S30})$$

In the thin-plate equations dV/dx balances the Winkler foundation load associated with the non-isostatic deflection; Eq. (S30) shows how that balance is distributed over depth. For the Winkler foundation defined in Section 3.1 of the main text, $dV/dx = \Delta\rho g w$, so $dV/dx > 0$ where the plate is deflected downward and $dV/dx < 0$ beneath the forebulge, and $dV/dx = 0$ exactly at the isostatic columns ($w = 0$). For a uniform elastic plate, then, the vertical shear stress is always parabolic and symmetric about the plate centre, and its horizontal gradient $\tau_{zx,x}$ shares this shape, reaching its greatest magnitude at (and being symmetric about) the midplane.

S4.2 The elastic–perfectly-plastic plate: Horne’s extension

In the trench region the trailing plate yields extensively and approaches moment saturation, as predicted by yield-stress envelopes (Chapple & Forsyth, 1979; McNutt & Menard, 1982) and seen in numerical models with lithospheric strength profiles (Bessat et al., 2020; Sandiford & Craig, 2023). The development below follows the relevant part of the elastic–perfectly-plastic beam solution of Horne (1951). Horne’s reduced beam solution neglects transverse normal stress, so $\sigma_{zz} = 0$, and includes neither a transverse

body force nor a spatially varying restoring pressure. The flexure models instead represent the rock–water density contrast through the Winkler restoring pressure described above. Yield occurs when the maximum shear stress reaches a fixed limit (a Tresca condition), the same criterion and uniform strength used in the numerical Tresca model. Horne’s solution retains undeformed, small-slope geometry, and its geometry and yield pattern are symmetric about the plate midplane.

As the load increases, the outer fibres yield first, leaving an elastic core of half-thickness $c(x) \leq h/2$ that narrows toward the section of greatest moment.

In Horne’s solution, the yielded-zone longitudinal stress is fixed at $\sigma_{xx} = \pm\sigma_Y$ and is independent of x . Horizontal equilibrium and the traction-free surface condition therefore give $\tau_{zx} = 0$ throughout those zones (Horne, 1951).

With the shear vanishing in the plastic zones, the whole shear resultant is carried by the elastic core, and equilibrium delivers a parabola confined to the core. Within the core the longitudinal stress is linear, $\sigma_{xx} = \sigma_Y \zeta/c$; substituting into Eq. (S26) and integrating with $\tau_{zx}(\pm c) = 0$ (continuity with the shear-free yielded fibres) gives the *same* unit-integral shape function (S28), with the elastic-core half-thickness $h/2$ replaced by c :

$$\tau_{zx}(x, \zeta) = V(x) \varphi_c(\zeta), \quad |\zeta| \leq c \quad \left(\text{equivalently } \frac{V}{I_c} \frac{c^2 - \zeta^2}{2}, \quad I_c = \int_{-c}^c \zeta^2 d\zeta = \frac{2}{3}c^3 \right), \quad (\text{S31})$$

with $\tau_{zx} = 0$ for $c < |\zeta| \leq h/2$. It reduces to the elastic distribution (S29) when $c = h/2$ and carries the full resultant $\int \tau_{zx} d\zeta = V$ for every c . Yielding therefore confines the vertical shear stress to the narrowing elastic region, substantially changing its depth distribution while preserving its symmetry and resultant V . Horne’s truncated parabola is the limiting form obtained when $\partial_x \sigma_{zz} = 0$. The horizontal gradient of this distribution, including the effect of the changing elastic–plastic boundary, is considered next.

S4.3 The gradient across the moving elastic–plastic front

The trench-pull force depends on the horizontal gradient $\tau_{zx,x}$, not on τ_{zx} itself (Section 2 of the main text). Even in the symmetric plate considered here, this gradient need not retain the simple shape of the truncated parabola because both the resultant $V(x)$ and the elastic-core half-thickness $c(x)$ vary along the plate. Because the distribution remains even in ζ , its signed centroid lies at the midplane wherever its resultant is non-zero. The calculation below shows the more complex depth profiles introduced by yielding and a changing elastic–plastic boundary while retaining that symmetry.

The shear stress is continuous at the elastic–plastic boundary $\zeta = \pm c$, although its depth gradient is not, and the boundary changes position with x . Because the core parabola reaches zero at the boundary, its changing position produces no singular term in $\tau_{zx,x}$; it appears instead as the finite step derived below. The gradient is the ordinary derivative of $V(x) \varphi_{c(x)}(\zeta)$, and because τ_{zx} depends on x only through the resultant $V(x)$ and the elastic-core half-thickness $c(x)$, the chain rule (with $\partial \varphi_c / \partial c = 3(3\zeta^2 - c^2)/4c^4$) separates it into two terms:

$$\tau_{zx,x} = \varphi_c \frac{dV}{dx} + V \frac{\partial \varphi_c}{\partial c} \frac{dc}{dx} = \underbrace{\frac{3V'}{4c^3} (c^2 - \zeta^2)}_{\text{flexural support } (\propto V')} + \underbrace{\frac{3Vc'}{4c^4} (3\zeta^2 - c^2)}_{\text{core-narrowing } (\propto Vc')}, \quad (\text{S32})$$

where the prime denotes d/dx .

The two terms describe distinct physical processes. Evaluating them at the moving boundary shows that *varying flexural support* (V') rescales the parabola while it stays pinned to zero at the elastic–plastic boundary ($c^2 - \zeta^2 = 0$ at $\zeta = \pm c$), carrying the full resultant and no discontinuity. *Motion of the plastic front* (c') redistributes the field

because the elastic region itself changes size: its term has zero resultant, and at the front it takes the finite value $3Vc'/2c^2$ ($3\zeta^2 - c^2 = 2c^2$ there) against the zero exterior, so it alone is discontinuous:

$$\tau_{zx,x}(\zeta \rightarrow c^-) - \tau_{zx,x}(\zeta \rightarrow c^+) = + \frac{3V}{2c^2} \frac{dc}{dx} \quad (\text{S33})$$

— a step, finite and integrable, not a spike. Within this construction, front motion is the only mechanism that can produce a discontinuity in $\tau_{zx,x}$, and the shear vanishes exactly where the front moves.

Both terms are even in ζ . The flexural-support term integrates to V' , whereas the core-narrowing term integrates to zero, so $\int \tau_{zx,x} d\zeta = V'$ irrespective of the elastic-core half-thickness. Their first moment about the plate midplane also vanishes:

$$\int \zeta \tau_{zx,x}(\zeta) d\zeta = 0. \quad (\text{S34})$$

Thus, where $V' \neq 0$, yielding can substantially change the depth profile of the shear-stress gradient without changing its resultant or shifting its signed centroid from the midplane. The fixed midplane centroid follows directly from symmetry, rather than from the particular parabolic form. Where $V' = 0$, as at an isostatic column, the signed centroid is undefined, although Eq. (S34) still holds.

Four limiting depth profiles follow according to whether V' and c' are active and to the relative signs of the two coefficients V' and Vc' ; Figure S3 shows each case. Within the yielded part of Horne's section, $M = s\sigma_Y(h^2/4 - c^2/3)$, where s is the sign of M . It follows that $V = dM/dx = -(2/3)s\sigma_Y c c'$ and hence $Vc' = -(2/3)s\sigma_Y c(c')^2$: the sign of the core-narrowing term is controlled by $-M$, not by c' alone. The term therefore retains the same sign on both sides of the maximum-moment section while M retains its sign, even though c' and V both reverse. At the maximum moment $c' = 0$, leaving the V' term alone. At the first isostatic column — the reference column of the trench-pull definition (Eq. 1 of the main text) — $w = 0$ and therefore $V' = 0$, leaving a zero-resultant redistribution. The opposing combination occurs when V' reverses beneath the forebulge.

The calculation illustrates the range of shear-stress-gradient structures that can arise through yielding even in this simple symmetric plate. The midplane signed centroid follows from the retained symmetry and does not extend automatically to an asymmetric strength profile; the numerical results for such profiles are assessed in Section 4.4 of the main text. The construction also describes the stress distribution during monotonic bending and does not retain the strain history acquired as material passes the maximum-moment region and its curvature decreases toward the trench. This limitation is discussed in Section 5.4 of the main text.

S4.4 Comparison with the finite-element models

The main analytical prediction carried into the numerical models is that, for a symmetric plate, the signed centroid of the shear-stress-gradient distribution lies at the plate midplane wherever its resultant is non-zero. Its first moment about the plate top is therefore $h/2$ times its resultant. In the equivalent-density representation, this places the compensating lobe at mid-plate and gives a force-based effective arm of approximately $h/2$ when $w_T \ll h$ (Eq. 19 of the main text).

The numerical models differ from Horne's solution because the Winkler restoring pressure, and hence σ_{zz} , varies along the plate. Within yielded fibres the Tresca criterion keeps $\sigma_{xx} - \sigma_{zz}$ close to $\pm\sigma_Y$. Horizontal equilibrium then gives

$$\frac{\partial \tau_{zx}}{\partial \zeta} = -\frac{\partial \sigma_{xx}}{\partial x} \approx -\frac{\partial \sigma_{zz}}{\partial x}, \quad (\text{S35})$$

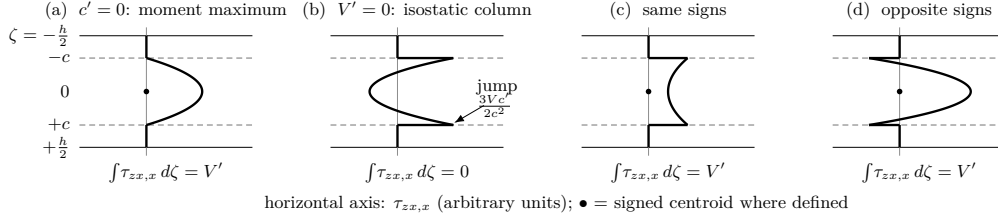


Figure S3. Depth profiles of the shear-stress gradient $\tau_{zx,x}(\zeta)$ in a partially yielded plate for four combinations of the coefficients V' and Vc' (Equation S32); the elastic core occupies $|\zeta| \leq c$, and only the relative signs and profile shapes are represented. The flexural-support term integrates to V' , whereas the core-narrowing term has zero signed integral. Yielding can therefore redistribute the gradient with depth without changing its resultant. (a) At the maximum-moment section, $c' = 0$ and the V' term gives a parabola confined to the elastic core. (b) At the first isostatic column, $w = 0 \Rightarrow V' = 0$, and the core-narrowing term alone gives a zero-resultant redistribution terminating in the finite jump $3Vc'/2c^2$. (c) When V' and Vc' have the same sign, the profile is weighted toward the core edges; this combination occurs on both sides of the maximum moment in the downbent region. (d) When the coefficients have opposite signs, the profile is weighted toward the centre and develops lobes of opposite sign at the core edges; this combination occurs after V' reverses beneath the forebulge. In every case the profile is even in ζ , so its first moment about the midplane is zero. The signed centroid (•) lies at the midplane where the resultant is non-zero; no marker is shown in panel (b), where the centroid is undefined (Eq. S34).

which allows τ_{zx} to remain non-zero in those fibres; it remains small in the numerical solutions. This departure from Horne's truncated parabola is visible in Figure 6 of the main text.

Figure S4 compares the computed $\tau_{zx,x}$ profiles with the two-term analytical form at five locations along the reference elasto-plastic model. For this comparison, the neutral plane is located from the zero crossing of $\sigma_{xx} - \sigma_{zz}$, and $c(x)$ is inferred from its local slope. The analytical profile is centred on that plane. The functions $c(x)$ and $V(x)$ are smoothed along the plate with a 10-km Gaussian before c' and V' are evaluated. The edge-weighted profiles on both sides of the maximum moment illustrate why the physical regime depends on Vc' , rather than on c' alone. Figure 6 of the main text shows that their signed centroids remain near the midplane and that the vertical normal stress anomaly equilibrates within the strong plate.

S5 Additional figures

Figure S5 shows the corrected-density reconstruction for the reference model. Figure S6 gives the complete five-panel view of that model, and Figures S7–S9 use the same layout for the 30-km plate of Suite 4 and the two depth-dependent strength models of Suite 1. The common plotting conventions are defined in Section 3.3 of the main text. These figures show how the yield pattern and the depth distributions of vertical shear stress and equivalent density vary among the models; their final panels show that $\Delta N_D(x)$ and $\Delta GPE^*(x)$ remain closely matched along each plate.

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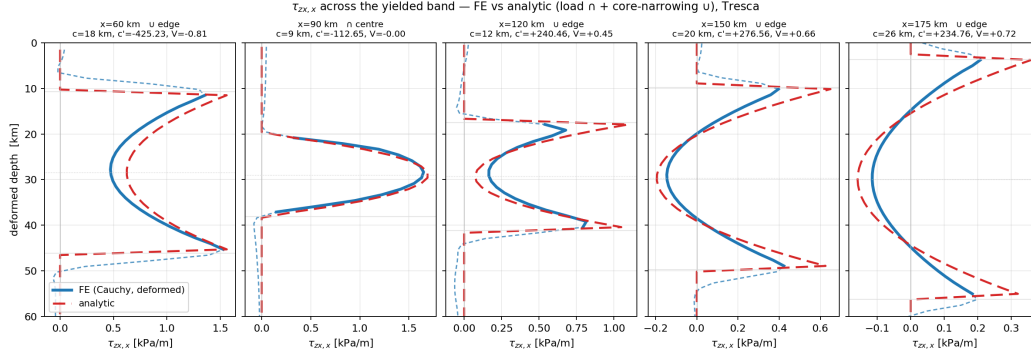


Figure S4. Depth profiles of $\tau_{zx,x}$ at five locations along the reference elasto-plastic model (Figure S6). The finite-element gradient of Cauchy shear stress is solid within the inferred elastic core and finely dashed through the yielded wings. The two-term analytical profile from Equation (S32) is shown by the heavier dashed line, centred on the located neutral plane and evaluated using the local V and c and their smoothed horizontal gradients V' and c' . Panel headers give c in kilometres, c' in metres per kilometre, and V in TN m^{-1} ; the $\cap$ and $\cup$ symbols denote centre- and edge-weighted analytical profiles. The edge-weighted form occurs on both sides of the maximum moment, while the $c' = 0$ limit is centre-peaked. The finite-element profiles retain the same broad structure, with their signed centroids near mid-plate.

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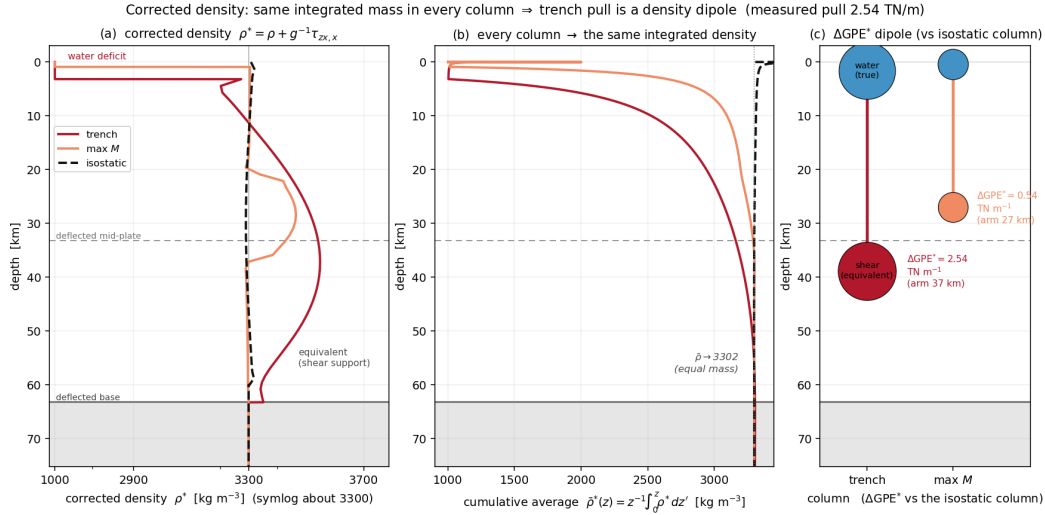


Figure S5. Corrected-density reconstruction of the trench pull at three columns of the reference model. The density $\rho^* = \rho + g^{-1}\tau_{zx,x}$ combines a prescribed rock–water density structure with the equivalent density obtained from the finite-element shear-stress gradient; the true density is introduced in post-processing and is not a body force in the numerical model (plane-strain Tresca, $\sigma_Y = 150$ MPa, $h = 60$ km, $V = 4$ TN m $^{-1}$): the trench, the moment maximum (max M), and the first isostatic column ($w = 0$), which serves as the reference. Horizontal lines mark the trench-column mid-plate (≈ 33 km) and base (≈ 63 km); the grey band continues the prescribed uniform rock density below that depth. (a) Corrected-density profiles, plotted on a symmetric-log scale about 3300 kg m $^{-3}$ so that both the large water deficit over the deflected top and the milder equivalent density (shear-support) excess within the plate are visible. (b) The cumulative depth average $\langle \rho^* \rangle_z = z^{-1}\int_0^z \rho^* dz'$ converges closely among the three columns once the profiles extend below the plate, illustrating the corrected-density closure required by vertical equilibrium. The small residual separation is greatest at the loaded edge. (c) The difference from the isostatic column is separated into an upper true lobe (the water deficit, centred at $w/2$, known from the deflected depth; blue) and a lower equivalent-density lobe (the shear support, from the finite-element stress field; column colour); disk area is proportional to the integrated magnitude of each lobe and bar length is their signed-centroid separation. At the trench, the two reconstructed lobe magnitudes differ slightly because the shear-stress gradient is evaluated at the loaded edge; the plotted separation is therefore not identical to the force-based effective arm. Both lobe magnitude and separation grow toward the trench: $\Delta GPE^* = 2.54$ TN m $^{-1}$ at the trench and 0.54 TN m $^{-1}$ at the moment maximum, against zero at the reference. The quoted ΔGPE^* values are the direct model σ_{zz} integrals relative to the isostatic column; the force-based arm a_{eff} is defined independently by Equation 15 of the main text. The accuracy with which the finite-element gradient reconstructs the direct force is assessed away from the loaded edge in Figure 5 of the main text.

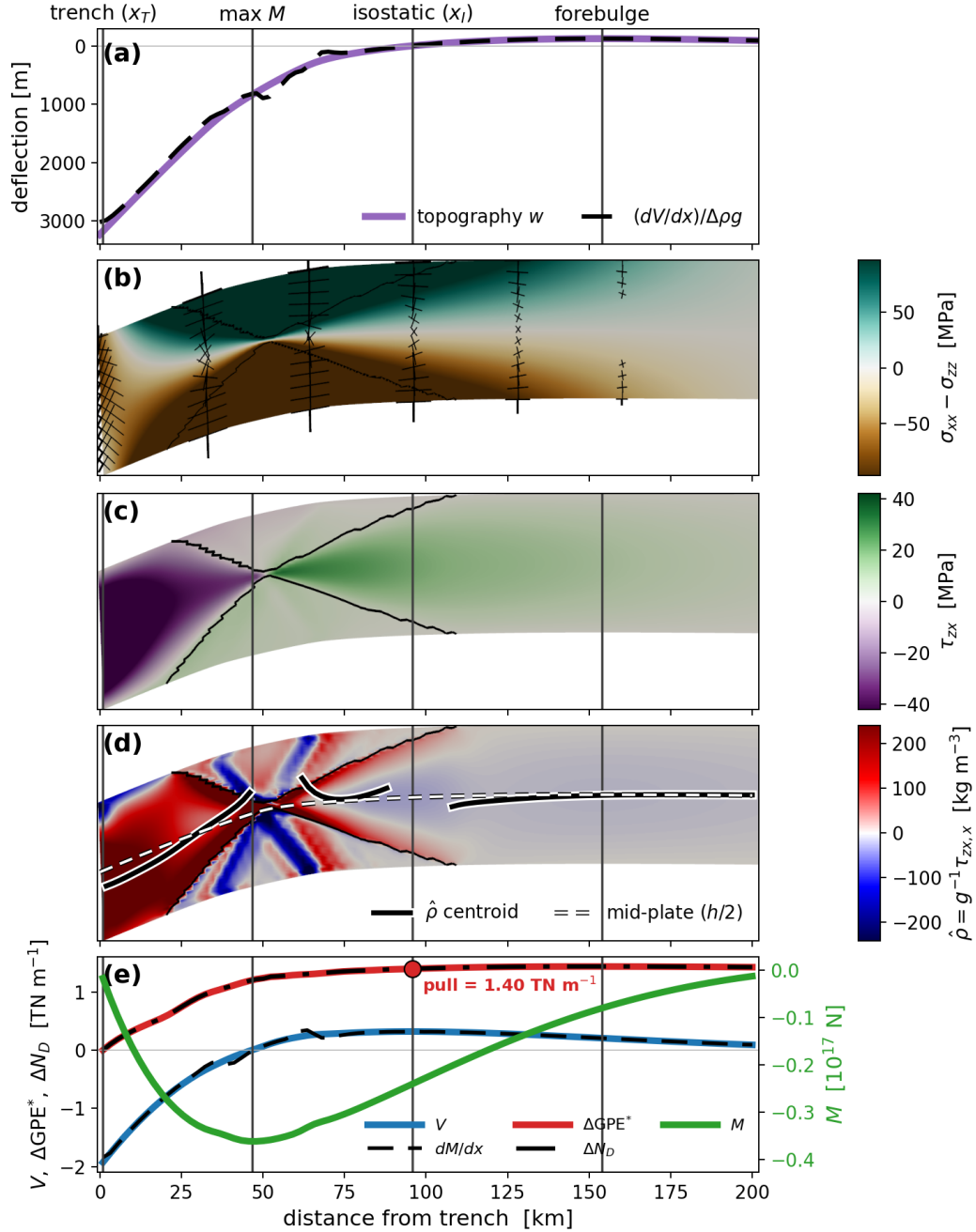


Figure S7. Flexure of the 30-km model in the thickness suite (Suite 4 of the main text; uniform Tresca, $\sigma_Y = 150$ MPa), with $\Delta GPE^* = 1.40$ TN m⁻¹. The layout and plotting conventions follow Figure S6. In panel (d), the signed-centroid track is blanked across the yielded hinge beginning near the maximum-moment column, where alternating lobes of $\hat{\rho}$ nearly cancel within each column, and over a second interval about x_I , where the net equivalent-density integral vanishes. The signed-centroid ratio is ill-conditioned in both intervals.

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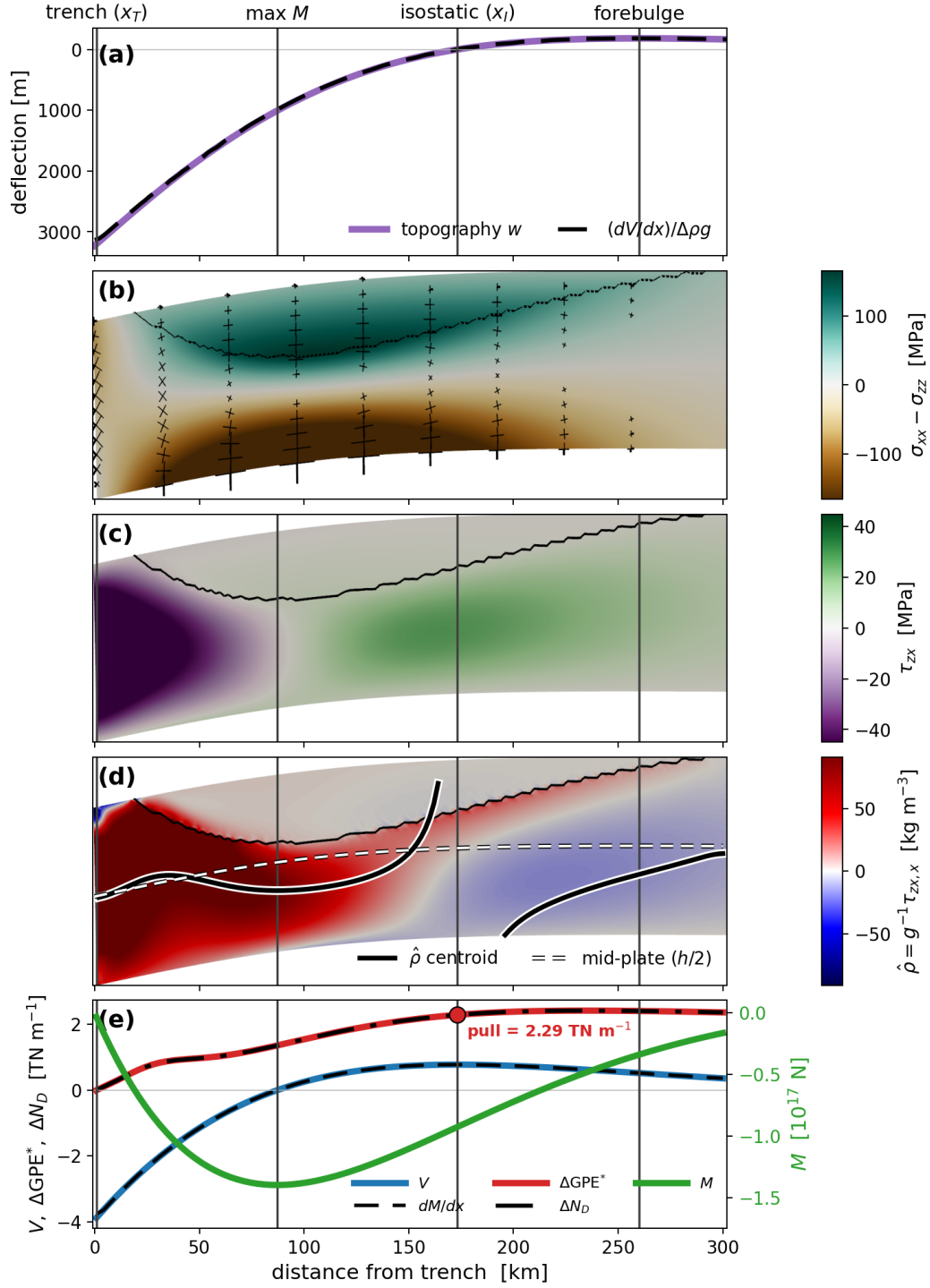


Figure S8. Flexure of the depth-dependent (DD-VM, asymmetric) model in Suite 1, in the five-panel layout and plotting conventions of Figure S6; the trench-pull force between x_T and x_I is 2.29 TN m⁻¹. In panel (d), the signed-centroid track bends toward the surface as it approaches the isostatic gap and re-emerges near the plate base beyond it. The two lobes of $\hat{\rho}$ nearly balance there, so this excursion reflects an ill-conditioned centroid ratio.

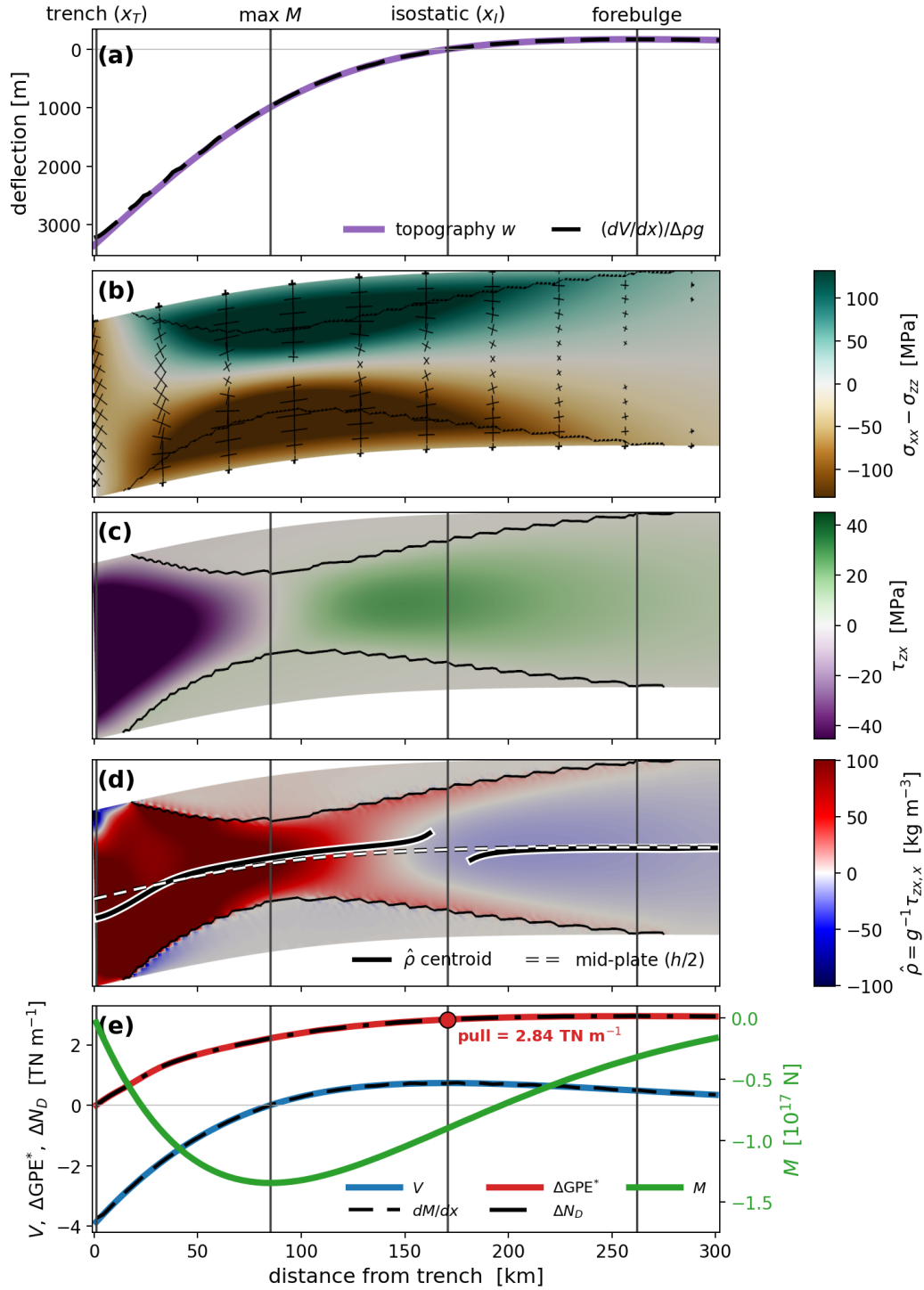


Figure S9. Flexure of the depth-dependent (DD-VM, symmetric) model in Suite 1, in the five-panel layout and plotting conventions of Figure S6; the trench-pull force between x_T and x_I is 2.84 TN m^{-1} .

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