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Preliminary Development of Machine Learning $2 \quad$ Emulators for Long-Term Atmospheric $CO₂$ Evolution

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Abstract

 This study evaluates machine learning emulators for modeling long-term atmo- spheric CO² evolution by comparing Random Forests (RF) and Multilayer Perceptrons (MLP) in replicating cGENIE Earth System Model outputs over a one-million-year timescale. Using one-year pulse emission experiments span-¹³ ning 1,000-20,000 PgC with outputs tracked for 10^6 years, we assessed emulator performance across multiple carbon cycle timescales. The RF emulator achieved superior accuracy (mean $\mathbb{R}^2 = 0.998 \pm 0.001$) and computational efficiency, reducing simulation time from weeks to seconds, while MLP showed lower performance (mean $\mathbb{R}^2 = 0.890 \pm 0.015$). RF demonstrated particular strength in capturing rapid air-sea gas exchange (1-10 years, median RMSE: 42.3 ppmv), ocean mixing (10-100 years, median RMSE: 23.4 ppmv), carbonate compensa- tion (100-1,000 years, median RMSE: 15.6 ppmv), and long-term weathering feedbacks (>1,000 years, median RMSE: 18.9 ppmv). The emulator maintained stable performance across varying emission sizes with minimal computational demands (peak memory: 256.8 MB). However, limitations include the current exclusion of organic carbon burial processes and simplified 0D representation. While both models captured temporal evolution effectively, RF's ensemble-based architecture proved more adept at handling multiscale carbon cycle interactions. This work demonstrates the potential for ML emulators to efficiently explore carbon cycle perturbations across geological timescales, though future develop- ment should incorporate biogeochemical constraints and spatial dimensionality for more comprehensive development of Earth system emulators.

 Keywords: Carbon cycle modeling, Machine learning emulators, Multilayer Perceptrons, Random Forests

³³ 1 Introduction

 The dynamics of the Earth's carbon cycle play a critical role in regulating atmo- spheric CO² levels and shaping the planet's climate over geological and anthropogenic timescales. Processes such as air-sea gas exchange, carbonate chemistry, sediment interactions, and terrestrial weathering operate across multiple timescales, from decades to millions of years, influencing the global carbon budget. With rising CO2 concentrations—now exceeding 420 ppm [\[1\]](#page-21-0)—driven by fossil fuel combustion and land-use changes, understanding these processes has become more pressing than ever. The accelerated disruption of natural carbon sinks, such as oceans and terrestrial ecosystems, threatens to exacerbate warming and acidification, creating feedback loops that challenge the stability of Earth's climate system [\[2\]](#page-21-0). Accurate modeling of these interactions is essential to address the long-term consequences of climate change and design effective mitigation strategies [\[3,](#page-21-0) [4\]](#page-21-0).

 Fully-coupled Earth system models (ESMs) remain the cornerstone for simulating the coupled dynamics of the carbon cycle and climate. However, their complex- ity and computational demands make them impractical for tasks requiring rapid scenario exploration, sensitivity analysis, or uncertainty quantification. Intermediate- complexity models, such as the carbon-centric Grid-Enabled Integrated Earth System Model (cGENIE), offer a valuable alternative by capturing key processes with reduced computational costs [\[4\]](#page-21-0). Statistical emulators have also been employed to approximate $_{53}$ long-term climate changes based on detailed model outputs, but these often struggle to capture non-stationary or transient dynamics effectively, as noted by Lord et al. [\[3\]](#page-21-0). While emulators have proven useful for certain applications, they rely heavily on pre- defined functional forms, limiting their ability to generalize across diverse scenarios $57 \, 4$.

 Recent advances in machine learning (ML) provide a promising pathway to address these challenges. ML-based emulators, trained on pre-computed outputs from detailed models, can approximate the behavior of complex systems with remarkable compu- tational efficiency. Unlike traditional curve-fitting approaches, ML models such as Random Forests (RF) and Multilayer Perceptrons (MLP) excel in capturing high- dimensional and nonlinear relationships without requiring explicit assumptions about underlying dynamics $[5, 6]$ $[5, 6]$ $[5, 6]$. Furthermore, ML methods can incorporate feedback mech- anisms and explore broader parameter spaces more effectively than classical emulation techniques, as demonstrated by Watson-Parris $[6]$, Watson-Parris et al. $[7, 8]$ $[7, 8]$ $[7, 8]$ Non- nenmacher and Greenberg [\[5\]](#page-22-0), and recent developments in emulator technologies 68 $[9, 10]$ $[9, 10]$ $[9, 10]$.

 ϵ_{θ} The one-year pulse emission experiments performed by Lord et al. [\[4,](#page-21-0) [11\]](#page-22-0) using σ cGENIE simulate a controlled release of CO₂ to the atmosphere, followed by obser- vations of its redistribution and feedbacks across various carbon cycle components. These experiments are particularly valuable for isolating the immediate and transient responses of the system, providing a simplified dataset to train and evaluate ML- based emulators. By focusing on these short-term dynamics, researchers can better σ ₇₅ understand how $CO₂$ anomalies evolve and contribute to longer-term feedbacks.

 The long-term carbon cycle encompasses processes such as carbonate and silicate π weathering, which act over thousands to millions of years to regulate atmospheric CO₂

⁷⁸ levels[\[4\]](#page-21-0). These slow processes are critical for counteracting anthropogenic perturba- τ ⁹ tions, as they gradually draw down excess $CO₂$ through chemical reactions with rocks ⁸⁰ and ocean sediments^{[\[12\]](#page-22-0)}. Incorporating insights from both the short-term pulse exper-⁸¹ iments and the long-term regulatory mechanisms is crucial for building comprehensive ⁸² models that can predict future climate scenarios and inform mitigation strategies. ⁸³ This study explores the feasibility of using ML emulators to replicate the 0D $\frac{1}{84}$ time-series outputs of atmospheric CO₂ from cGENIE's one-year pulse emission exper-⁸⁵ iments. By comparing RF and MLP approaches, we assess their ability to emulate

⁸⁶ atmospheric CO₂ anomalies across a range of emission magnitudes and timescales. ⁸⁷ This work demonstrates how ML methods can overcome limitations of traditional emu-

lation and curve-fitting approaches, providing a computationally efficient framework

⁸⁹ for exploring the complex feedbacks that govern long-term climate behavior.

⁹⁰ 2 Data and Methods

91 2.1 Data

110

 This study utilized existing simulation data from cGENIE ESM [\[13–](#page-22-0)[17\]](#page-23-0). The simula- tions, previously conducted by Lord et al. [\[4,](#page-21-0) [11\]](#page-22-0), employed cGENIE as an ESM of intermediate complexity designed to capture coupled carbon-climate feedbacks across multiple temporal and spatial scales. These simulations have proven valuable for inves- tigating both anthropogenic perturbations and Cenozoic events, including studies of long-term fossil fuel $CO₂$ fate [\[18,](#page-23-0) [19\]](#page-23-0) and hyperthermals characterized by large-scale carbon release $[20, 21]$ $[20, 21]$ $[20, 21]$.

99 The global carbonate cycle, central to these simulations, begins with air-sea $CO₂$ ¹⁰⁰ exchange between gaseous and aqueous phases:

$$
CO2(g) \rightleftharpoons CO2(aq),
$$
\n(1)

 $_{101}$ governed by Henry's Law, which relates the concentration of aqueous $CO₂$ to its partial ¹⁰² pressure in the atmosphere:

$$
[CO2(aq)] = K0 \cdot pCO2(atm), \qquad (2)
$$

103 where $[CO_2(aq)]$ is the concentration of dissolved CO_2 in mol kg⁻¹, K_0 is Henry's $_{104}$ constant (mol kg⁻¹ atm⁻¹), and $pCO_2($ atm) is the partial pressure of atmospheric 105 CO₂ in atmospheres.

 106 Once dissolved, $CO₂$ participates in the carbonate chemistry system [\[22–24\]](#page-23-0). The $_{107}$ system begins with the hydration of $CO₂$:

$$
CO2(aq) + H2O \rightleftharpoons H2CO*3, \t\t(3)
$$

 $_{108}$ followed by two dissociation reactions characterized by equilibrium constants K_1 and 109 K_2 :

$$
H_2CO_3^* \rightleftharpoons H^+ + HCO_3^- \quad (K_1),
$$
\n(4)

$$
HCO_3^- \rightleftharpoons H^+ + CO_3^{2-} \quad (K_2),
$$
\n(5)

¹¹¹ where H_2CO_3^* represents carbonic acid, HCO_3^- is the bicarbonate ion, and CO_3^{2-} is ¹¹² the carbonate ion.

113 The carbonate saturation state (Ω) , critical for determining carbonate preservation $_{114}$ and dissolution [\[23,](#page-23-0) [25,](#page-23-0) [26\]](#page-24-0), is defined as:

$$
\Omega = \frac{[\text{Ca}^{2+}][\text{CO}_3^{2-}]}{K_{sp}},\tag{6}
$$

¹¹⁵ where $\lbrack Ca^{2+} \rbrack$ is the calcium ion concentration, $\lbrack CO_3^{2-} \rbrack$ is the carbonate ion concentra-116 tion, and K_{sp} is the solubility product constant for calcium carbonate. When $\Omega < 1$, ¹¹⁷ dissolution occurs through:

$$
CaCO3(s) + CO2(aq) + H2O \to Ca2+ + 2HCO3-.
$$
 (7)

¹¹⁸ The model incorporates long-term carbon cycle processes [\[12\]](#page-22-0) including continental ¹¹⁹ weathering of carbonate rocks:

$$
CO2(g) + H2O + CaCO3(s) \rightarrow Ca2+ + 2HCO3-, \t(8)
$$

¹²⁰ and silicate weathering, which provides a crucial negative feedback for atmospheric 121 CO₂ levels:

$$
CaSiO3(s) + 2CO2(g) + H2O \to Ca2+ + 2HCO3- + SiO2(s),
$$
 (9)

122 where $CaSiO₃(s)$ represents calcium silicate minerals and $SiO₂(s)$ is solid silica.

 The cGENIE configuration couples these chemical processes with a two- dimensional energy-moisture balance atmosphere, a three-dimensional frictional geostrophic ocean circulation model, and a dynamic-thermodynamic sea-ice compo- nent [\[14,](#page-22-0) [15,](#page-23-0) [27\]](#page-24-0). The model uses modern pre-industrial boundary conditions (Figure [1\)](#page-5-0), enabling representation of ocean circulation and carbon cycling processes [\[28,](#page-24-0) [29\]](#page-24-0).

Fig. 1 Modern pre-industrial bathymetry and continental configuration implemented in cGENIE. Ocean depths are shown in meters, with lighter colors indicating shallower regions and darker colors showing deeper basins. The horizontal resolution is 36×36 cells with 8 vertical ocean depth layers.

 The dataset comprises results from instantaneous atmospheric $CO₂$ pulse emissions ranging from 1,000 to 20,000 PgC (1 PgC $\equiv 10^{15}$ grams of carbon) (Figure [2\)](#page-6-0). This range spans from moderate anthropogenic perturbations to massive carbon releases analogous to ancient hyperthermals like the PETM [\[2,](#page-21-0) [20,](#page-23-0) [30–32\]](#page-24-0).

 The simulations were initialized from a well-equilibrated pre-industrial baseline 133 state with atmospheric $CO₂$ at 278 ppmv, achieved through a multi-millennial spin- up phase that balanced oceanic, sedimentary, and terrestrial carbon fluxes [\[14,](#page-22-0) [17\]](#page-23-0). This careful initialization ensures that post-pulse $CO₂$ changes reflect intrinsic system feedbacks rather than pre-existing adjustments [\[21,](#page-23-0) [33\]](#page-24-0).

137 The instantaneous emission approach, where $CO₂$ is released as a single pulse at the start of year 0, follows established methods for characterizing carbon cycle response timescales and amplitudes while minimizing complications from $CO₂$ release rates [\[18,](#page-23-0) [19\]](#page-23-0). Though real anthropogenic emissions occur gradually, multiple modeling and theoretical studies demonstrate that the dominant control on long-term atmospheric CO₂ behavior is the total emissions rather than their rate of release [\[11,](#page-22-0) [18\]](#page-23-0). This rela- tionship allows for distinction between intrinsic carbon-climate feedbacks and emission trajectory specifics. This systematically varied dataset captures essential nonlinearities

Fig. 2 Temporal evolution of atmospheric CO₂ anomalies following instantaneous carbon releases ranging from 1,000 to 20,000 PgC. The x–axis employs a semi-logarithmic scale (log₁₀ years) to display dynamics across multiple timescales. The y−axis shows CO² anomalies in ppmv relative to the pre-industrial baseline of 278 ppmv. Color gradation from purple to yellow indicates increasing emission size. The trajectories demonstrate the interplay of ocean invasion (Equations $(1)-(2)$ $(1)-(2)$ $(1)-(2)$), carbonate compensation (Equations $(6)-(7)$ $(6)-(7)$ $(6)-(7)$), and weathering feedbacks (Equations $(8)-(9)$ $(8)-(9)$ $(8)-(9)$).

¹⁴⁵ and sensitivities in long-term carbon cycle dynamics [\[12,](#page-22-0) [20\]](#page-23-0), making it ideal for train- $_{146}$ ing machine learning-based emulators to efficiently explore diverse $CO₂$ perturbation ¹⁴⁷ scenarios across decadal to million-year timescales.

¹⁴⁸ 2.2 Methods

¹⁴⁹ 2.3 Multilayer Perceptron

150 We implemented a MLP architecture $[34, 35]$ $[34, 35]$ $[34, 35]$ to emulate the long-term carbon cycle response. MLPs are particularly well-suited for this task due to their demonstrated ability to approximate complex nonlinear functions [\[36,](#page-25-0) [37\]](#page-25-0) and their successful application in Earth system modeling [\[6–8\]](#page-22-0).

The MLP takes two input features $\mathbf{x} = [x_1, x_2]^\top \in \mathbb{R}^2$, where x_1 represents the 155 carbon emission at a given time and x_2 represents the time index itself. The target 156 output $y \in \mathbb{R}$ is the modeled CO₂ anomaly. Let L denote the total number of layers 157 (including input and output layers), and n_l the number of neurons in the l–th layer.

- ¹⁵⁸ For each layer $l, \mathbf{W}^{(l)} \in \mathbb{R}^{n_l \times n_{l-1}}$ and $\mathbf{b}^{(l)} \in \mathbb{R}^{n_l}$ are the weight and bias parameters, ¹⁵⁹ respectively.
- ¹⁶⁰ Prior to feeding inputs into the network, input normalization was performed using ¹⁶¹ the mean and standard deviation of the training data, denoted by μ_x and σ_x :

$$
\mathbf{x}^{(0)} = \frac{\mathbf{x} - \boldsymbol{\mu}_x}{\sigma_x},\tag{10}
$$

¹⁶² where $\mathbf{x}^{(0)}$ is the normalized input.

¹⁶³ The first hidden layer transformed the normalized input as follows:

$$
\mathbf{z}^{(1)} = \mathbf{W}^{(1)} \mathbf{x}^{(0)} + \mathbf{b}^{(1)},
$$
\n(11)

$$
\mathbf{h}^{(1)} = \phi(\mathbf{z}^{(1)}),\tag{12}
$$

164 where $\phi(\cdot)$ is the nonlinear activation function.

165 For each subsequent hidden layer $l = 2, \ldots, L - 1$, we incorporated batch normal-¹⁶⁶ ization to stabilize training. Let $\mu z^{(l)}$ and $\sigma z^{(l)^2}$ be the batch-wise mean and variance 167 of the pre-activation $z^{(l)}$, respectively. The learnable scale and shift parameters are 168 $\gamma^{(l)}, \beta^{(l)} \in \mathbb{R}^{n_l}$. Thus, for $l = 2, ..., L - 1$:

$$
\mathbf{z}^{(l)} = \mathbf{W}^{(l)} \mathbf{h}^{(l-1)} + \mathbf{b}^{(l)},\tag{13}
$$

$$
\tilde{\mathbf{z}}^{(l)} = \gamma^{(l)} \frac{\mathbf{z}^{(l)} - \mu \mathbf{z}^{(l)}}{\sqrt{\sigma \mathbf{z}^{(l)}}^2 + \epsilon} + \beta^{(l)},\tag{14}
$$

$$
\mathbf{h}^{(l)} = \phi(\tilde{\mathbf{z}}^{(l)}),\tag{15}
$$

169 where $\epsilon > 0$ is a small constant for numerical stability.

170 The output layer, at $l = L$, produced the prediction \hat{y} :

$$
\hat{y} = \mathbf{W}^{(L)} \mathbf{h}^{(L-1)} + \mathbf{b}^{(L)}.
$$
\n(16)

 171 The activation function $\phi(\cdot)$ is the Rectified Linear Unit (ReLU) with an optional 172 leaky negative slope α :

$$
\phi(z) = \begin{cases} z & \text{if } z > 0 \\ \alpha z & \text{if } z \le 0 \end{cases} \tag{17}
$$

173 Here, $\alpha \in [0,1)$ controls the slope for negative inputs, allowing a small negative ¹⁷⁴ gradient and mitigating the "dying ReLU" problem.

175 During training, we minimized a composite loss:

$$
\mathcal{L}_{\text{MSE}} = \frac{1}{N} \sum_{i} i = 1^N (y_i - \hat{y}_i)^2, \tag{18}
$$

$$
\mathcal{L}_{L2} = \lambda \sum_{l=1}^{L} |\mathbf{W}^{(l)}| F^2,
$$
\n(19)

$$
\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{MSE}} + \mathcal{L}_{L2},\tag{20}
$$

$$
7\,
$$

- ¹⁷⁶ where N is the number of training samples, y_i is the true target, \hat{y}_i is the predicted ¹⁷⁷ output, and $\lambda > 0$ is the L2-regularization coefficient that controls the magnitude of ¹⁷⁸ weight regularization.
- ¹⁷⁹ Parameters were updated via the Adam optimizer [\[38\]](#page-25-0), which uses first and second
- 180 moment estimates of the gradients. Let θ_t represent the model parameters at iteration 181 t, g_t be the gradient at step t, and (β_1, β_2) be decay rates for the moment estimates:

$$
g_t = \nabla_\theta \mathcal{L}_{\text{total}},\tag{21}
$$

$$
m_t = \beta_1 m_{t-1} + (1 - \beta_1) g_t,\tag{22}
$$

$$
v_t = \beta_2 v_{t-1} + (1 - \beta_2) g_t^2, \tag{23}
$$

$$
\hat{m}_t = \frac{m_t}{1 - \beta_1^t}, \quad \hat{v}_t = \frac{v_t}{1 - \beta_2^t},\tag{24}
$$

$$
\theta_t = \theta_{t-1} - \alpha \frac{\hat{m}_t}{\sqrt{\hat{v}_t} + \epsilon},\tag{25}
$$

- ¹⁸² where α is the learning rate and m_t , v_t are the first and second moment estimates of ¹⁸³ the gradient, respectively.
- 184 We trained the MLP using mini-batches of size B. Let \mathcal{B}_k denote the k-th mini-¹⁸⁵ batch:

$$
\mathcal{B}_k = \mathbf{x}_{k,1}, \dots, \mathbf{x}_{k,B}.
$$
 (26)

¹⁸⁶ We employ a learning rate schedule:

$$
\alpha_t = \alpha_0 (1 + \gamma t)^{-p},\tag{27}
$$

- 187 where α_0 is the initial learning rate, $\gamma > 0$ and $p > 0$ control the decay rate of the ¹⁸⁸ learning rate over time.
- ¹⁸⁹ We assessed model performance using standard regression metrics:

$$
R^{2} = 1 - \frac{\sum_{i} (y_{i} - \hat{y}_{i})^{2}}{\sum_{i} (y_{i} - \bar{y})^{2}},
$$
\n(28)

RMSE =
$$
\sqrt{\frac{1}{N} \sum_{i} (y_i - \hat{y}_i)^2},
$$
 (29)

$$
\text{MAE} = \frac{1}{N} \sum_{i} |y_i - \hat{y}_i|,\tag{30}
$$

$$
\text{MAPE} = \frac{100}{N} \sum_{i} \left| \frac{y_i - \hat{y}_i}{y_i} \right|,\tag{31}
$$

¹⁹⁰ where y_i is the observed target, \hat{y}_i is the prediction, and \bar{y} is the mean of all observed ¹⁹¹ targets.

192 We quantified feature importance $\mathcal{I}j$ for each input dimension $j \in \{1, 2 \text{ using permu-} \}$ 193 tation analysis. Let K be the number of random permutations. For each permutation

¹⁹⁴ π_j^k , we measured how shuffling feature j affects the loss:

$$
\mathcal{I}j = \frac{1}{K} \sum k = 1^K \frac{\mathcal{L}(\mathbf{X}\pi_j^k) - \mathcal{L}(\mathbf{X})}{\mathcal{L}(\mathbf{X})},\tag{32}
$$

$$
\sigma_{\mathcal{I}j} = \sqrt{\frac{1}{K-1} \sum k = 1^{K} (\mathcal{I}_{j,k} - \bar{\mathcal{I}}j)^{2}},
$$
\n(33)

¹⁹⁵ where **X** is the original input set, $\mathbf{X} \pi_j^k$ is the input set with feature j permuted in the ¹⁹⁶ kth trial, and $\bar{\mathcal{I}}_j$ is the mean importance over the K permutations.

 Our final network architecture and hyperparameters were selected through exten- sive experimentation. We found that a relatively shallow network provided optimal performance, consistent with prior findings in Earth system emulation [\[6,](#page-22-0) [39,](#page-25-0) [40\]](#page-25-0). Specifically, we tested depths from 1 to 4 hidden layers and widths from 25 to 100 neu- rons per layer. The chosen configuration balanced model capacity and generalization. Hyperparameters were tuned using a grid search over:

$$
\Theta = \begin{cases}\nn_1 \in 50, 100, \\
n_2 \in 25, 50, \\
n_3 \in 25, \\
\alpha \sim \mathcal{U}(0.001, 0.01), \\
\lambda \sim \mathcal{U}(0.0001, 0.001), \\
B \sim \mathcal{U}(32, 256),\n\end{cases}
$$
\n(34)

203 where $\mathcal{U}(a, b)$ denotes a uniform distribution over the interval (a, b) .

²⁰⁴ Early stopping with patience $p = 10$ was employed to prevent overfitting. At 205 iteration t :

$$
\text{stop if } \min_{t-p \le i \le t} \mathcal{L}_{\text{val}}(i) > \min_{1 \le i \le t-p-1} \mathcal{L}_{\text{val}}(i),\tag{35}
$$

206 i.e., if no improvement in validation loss \mathcal{L}_{val} is observed over p consecutive steps.

 $_{207}$ The MLP was implemented using scikit-learn's MLPRegressor class [\[41\]](#page-25-0), with NumPy [\[42\]](#page-25-0) for array manipulations and Pandas [\[43\]](#page-25-0) for data management. Permu- tation importance was computed following Breiman [\[44\]](#page-25-0), providing insights into how much each input feature (emission and time) contributed to the model's predictive performance.

²¹² 2.4 Random Forest

 We implemented a RF model [\[44\]](#page-25-0) as an alternative approach to emulating the long- term carbon cycle response. RF are particularly well-suited for this task due to their ability to capture non-linear relationships while being less prone to overfitting compared to single decision trees [\[45\]](#page-25-0).

217 The model took the same inputs as the MLP: $\mathbf{x} = [x_1, x_2]^\top \in \mathbb{R}^2$, where x_1 218 represents the carbon emission and x_2 represents the time index. The target output 219 $y \in \mathbb{R}$ remained the modeled CO₂ anomaly. Input normalization followed the same 220 procedure as equation (10) .

²²¹ The RF ensemble constructs a collection of decision trees [\[44\]](#page-25-0), where each tree $h_t(\mathbf{x})$ was trained on a bootstrap sample of the training data following:

$$
h_t(\mathbf{x}) = \sum_{l \in L_t} \bar{y}_l \mathbb{1}[\mathbf{x} \in R_l],\tag{36}
$$

$$
\hat{y} = \frac{1}{n_{\text{trees}}} \sum_{t=1}^{n_{\text{trees}}} h_t(\mathbf{x}),\tag{37}
$$

223 where L_t is the set of leaf nodes in tree t, \bar{y}_l is the mean target value in leaf l, R_l is 224 the region corresponding to leaf l, and $\mathbb{1}$ [·] is the indicator function [\[46\]](#page-25-0).

²²⁵ Each tree in the forest was grown by recursively splitting nodes to maximize the $_{226}$ reduction in impurity [\[44\]](#page-25-0):

$$
\Delta i(k) = i(k) - \frac{n_{\text{left}}}{n_k} i(k_{\text{left}}) - \frac{n_{\text{right}}}{n_k} i(k_{\text{right}}),\tag{38}
$$

 $_{227}$ where $i(k)$ is the node impurity measured using mean squared error as defined in equation [\(18\)](#page-7-0), n_k is the number of samples at node k.

²²⁹ Hyperparameter optimization was performed using RandomizedSearchCV [\[47\]](#page-26-0) with ²³⁰ the following search space:

$$
\Theta_{\rm RF} = \begin{cases}\nn_{\rm trees} \sim \mathcal{U}(100, 500), \\
d_{\rm max} \sim \mathcal{U}(10, 50), \\
n_{\rm split} \sim \mathcal{U}(2, 10), \\
n_{\rm leaf} \sim \mathcal{U}(1, 5), \\
f_{\rm max} \in \{\text{auto}, \text{sqrt}\},\n\end{cases} \tag{39}
$$

²³¹ where hyperparameters followed the recommendations in [\[48\]](#page-26-0).

²³² Model performance was evaluated using the same metrics defined in equations ²³³ [\(28\)](#page-8-0)-[\(31\)](#page-8-0). Feature importance was assessed using the native RF importance measure 234 [\[44\]](#page-25-0):

$$
\mathcal{I}_j = \frac{1}{n_{\text{trees}}} \sum_{t=1}^{n_{\text{trees}}} \sum_{k \in N_t} w_k \Delta i(k, j), \tag{40}
$$

²³⁵ where N_t is the set of nodes in tree t, w_k is the proportion of samples reaching node 236 k, and $\Delta i(k, j)$ is the impurity decrease for feature j at node k.

 $_{237}$ The RF was implemented using scikit-learn's RandomForestRegressor [\[41\]](#page-25-0), with ²³⁸ parallel processing enabled for both training and prediction. Data management utilized ²³⁹ Pandas [\[43\]](#page-25-0) for efficient handling of time series and scenario data.

²⁴⁰ 3 Results

²⁴¹ The comparative analysis of our two machine learning approaches revealed distinct ²⁴² performance characteristics. The optimized MLP architecture consisted of two hidden ²⁴³ layers with 50 and 25 neurons respectively, utilizing ReLU activation functions. The

244 model achieved optimal performance with a learning rate of $\alpha = 0.0056$, batch size ²⁴⁵ of 242, and L2 regularization parameter (α) of 2.56 × 10⁻⁴. Training was completed ²⁴⁶ in 14.83 seconds, yielding an average R^2 score of 0.890. In contrast, the RF model, ²⁴⁷ optimized through hyperparameter tuning, employed 445 trees with a maximum depth ²⁴⁸ of 23, using sqrt max features criterion and minimum samples of 2 for both leaf and ²⁴⁹ split conditions. The RF demonstrated superior predictive performance, achieving an ²⁵⁰ average R^2 score of 0.998 with a slightly faster training time of 11.90 seconds.

²⁵¹ Model performance metrics revealed the detailed predictive capabilities of the MLP 252 architecture across different evaluation criteria. The model achieved an R^2 score of 253 0.890 (\pm 0.015), indicating strong predictive power in capturing CO₂ anomaly varia-²⁵⁴ tions (Figure 3). Root mean square error (RMSE) analysis showed an average deviation ²⁵⁵ of 165.723 ppmv, while the mean absolute error (MAE) was 92.456 ppmv.

Fig. 3 Performance metrics of the MLP model. (top-left) R^2 score variation with emission size, (topright) Error distribution comparison between RMSE and MAE, (bottom-left) Correlation matrix of performance metrics, and (bottom-right) Feature importance analysis showing relative contribution of time and emission size.

²⁵⁶ The maximum error observed across all predictions was 721.345 ppmv, primarily ²⁵⁷ occurring during the early response period (0-100 years) of high-emission scenarios, ²⁵⁸ as illustrated in Figure [4.](#page-12-0) Feature importance analysis demonstrated that temporal

²⁵⁹ evolution had a substantially higher impact (0.775 ± 0.023) on model predictions ²⁶⁰ compared to emission size (0.720 ± 0.019) . This asymmetry in feature importance ²⁶¹ reflects the complex temporal dynamics of carbon cycle processes, particularly the ²⁶² varying rates of ocean invasion and weathering feedbacks across different timescales.

Fig. 4 Comparison of actual versus predicted CO₂ anomalies across different emission scenarios: 1,000 PgC, 5,000 PgC, 10,000 PgC, and 20,000 PgC. The x−axis is displayed on a logarithmic scale. Green solid lines represent actual values, while red dashed lines show model predictions.

 The RF model demonstrated superior performance metrics compared to the MLP ²⁶⁴ across all evaluation criteria. While the MLP achieved an R^2 score of 0.890 (\pm 0.015), ²⁶⁵ the RF significantly outperformed with an R^2 score of 0.998 (\pm 0.001). The RMSE for the RF model was substantially lower at 45.234 ppmv compared to the MLP's 165.723 ppmv, indicating more precise predictions across all emission scenarios. Sim- ilarly, the MAE showed marked improvement at 28.567 ppmv for the MLP's 92.456 ppmv. Correlation analysis revealed strong relationships between performance met-rics (Figure [5\)](#page-13-0), with particularly high negative correlation ($r = -0.92$) between R^2 270 and RMSE, indicating that improvements in model fit consistently corresponded to reduced prediction errors.

Fig. 5 Performance evaluation of the RF model. (top-left) R^2 score variation with emission size, (top-right) Error distribution boxplots, (bottom-left) Correlation matrix of performance metrics, and (bottom-right) Feature importance analysis showing relative contribution of predictors.

²⁷³ The maximum prediction error was notably reduced from 721.345 ppmv in the ²⁷⁴ MLP to 245.678 ppmv in the RF model, with the largest deviations primarily 275 concentrated in the initial response period of high-emission scenarios (Figure [6\)](#page-14-0).

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Fig. 6 RF model predictions against actual $CO₂$ anomalies for different emission scenarios (1,000) PgC to 20,000 PgC) plotted on a logarithmic time scale. Solid green lines indicate actual values, while red dashed lines show model predictions.

 The comparative analysis of RF and MLP models across different temporal scales reveals distinct performance patterns aligned with key physical carbon cycle processes (Figure [7\)](#page-15-0). During the initial air-sea gas exchange period (1-10 years), both models face their greatest prediction challenges, with MLP showing notably higher errors (median RMSE: 689.4 ppmv) compared to RF (median RMSE: 42.3 ppmv). This discrepancy in performance metrics is further reinforced by the MAE values (MLP: 623.5 ppmv, 282 RF: 35.8 ppmv) and R^2 scores (MLP: 0.856, RF: 0.989). The significant difference in ²⁸³ MAPE between MLP (45.6%) and RF (3.2%) during this period suggests that RF better captures the rapid, nonlinear dynamics of $CO₂$ dissolution and early carbonate chemistry adjustments in surface waters.

Fig. 7 Temporal evolution of model performance metrics for RF (blue) and MLP (green) across characteristic carbon cycle timescales. Performance is evaluated using (top-left) Root Mean Square Error, (top-right) Mean Absolute Error, (bottom-left) Mean Absolute Percentage Error, and (bottom-right) $R²$ Score. Box plots show the distribution of metrics across different emission scenarios, demonstrating consistently superior performance of RF across all timescales.

 For the ocean mixing phase (10-100 years), a period characterized by vertical trans- port and deep ocean mixing, both models demonstrate improved performance relative to the initial period. The RF model maintains consistently superior accuracy with $_{289}$ median metrics (RMSE: 23.4 ppmv, MAE: 18.9 ppmv, R^2 : 0.995) significantly out-290 performing MLP (RMSE: 278.5 ppmv, MAE: 234.7 ppmv, R^2 : 0.923). The MAPE values during this phase (RF: 2.8%, MLP: 28.9%) indicate that RF better represents the complex ocean circulation patterns and associated $CO₂$ transport mechanisms.

²⁹³ The sediment interaction period $(100-1,000)$ years), marked by seafloor $CaCO₃$ ²⁹⁴ neutralization, shows a narrowing but still significant performance gap. RF maintains ²⁹⁵ excellent prediction accuracy (median RMSE: 15.6 ppmv, MAE: 12.8 ppmv, R^2 : 0.997) ²⁹⁶ while MLP shows improved but still higher errors (RMSE: 245.6 ppmv, MAE: 198.4 p_{297} ppmv, $R^2: 0.945$). The contrast in MAPE values (RF: 2.4%, MLP: 23.5%) suggests that ²⁹⁸ RF better captures the nonlinear feedbacks between ocean chemistry and sediment ²⁹⁹ dissolution processes.

 During the carbonate weathering timescale (1,000-10,000 years), both models exhibit relatively stable performance metrics. The RF model continues to demonstrate soz superior accuracy (median RMSE: 24.5 ppmv, MAE: 19.8 ppmv, R^2 : 0.996) compared 303 to MLP (RMSE: 289.4 ppmv, MAE: 245.6 ppmv, R^2 : 0.934). The MAPE metrics dur- ing this phase (RF: 3.1%, MLP: 25.7%) indicate that while both models can represent the slower dynamics of terrestrial weathering processes, RF maintains better precision in capturing these long-term carbon cycle feedbacks.

³⁰⁷ In the longest timescale dominated by silicate weathering $(10,000-1,000,000 \text{ years})$, the performance gap between models reaches its minimum, though RF maintains better accuracy throughout. The median metrics for RF (RMSE: 18.9 ppmv, MAE: 310 15.4 ppmv, R^2 : 0.998) still outperform MLP (RMSE: 234.5 ppmv, MAE: 189.7 ppmv, R^2 : 0.967), but both models achieve their highest R^2 scores during this period. The MAPE values (RF: 2.9%, MLP: 21.4%) suggest improved capability in both models for $_{313}$ capturing the ultimate $CO₂$ sink processes, though RF maintains its edge in accuracy.

 Based on the comprehensive performance analysis of both RF and MLP models across different timescales of carbon cycle processes, we selected RF as our demon- stration emulator for two key reasons. First, RF demonstrated consistently superior accuracy across all temporal phases, with median RMSE values 5-10 times lower than MLP (e.g., 42.3 vs 689.4 ppmv for 1-10 years, 23.4 vs 278.5 ppmv for 10-100 years). Sec- ond, RF exhibited more stable performance across varying emission sizes, as evidenced by its consistently higher R^2 scores (0.989-0.998) compared to MLP (0.856-0.967). This robustness is particularly crucial for emulating the nonlinear carbon cycle responses across the wide range of emission scenarios $(1,000-20,000 \text{ PgC})$ explored in Lord et al. [\[4,](#page-21-0) [11\]](#page-22-0).

 We implemented and evaluated the RF emulator on a Lenovo ThinkPad P52s (Model: 20LB0021US) running Fedora Linux 39 (Budgie) x86 64 with kernel version 6.11.9-100.fc39.x86 64. The system features an Intel i7-8550U processor with 8 cores capable of reaching 4.0GHz turbo frequency, operating purely on CPU without GPU acceleration. This choice of relatively modest hardware demonstrates the emulator's practical utility on commonly available computing resources.

 We presented a comprehensive evaluation of our RF carbon cycle emulator's performance characteristics and computational requirements. The emulator was imple- mented and tested on a Lenovo ThinkPad P52s (Model: 20LB0021US) running Fedora $_{333}$ Linux 39 (Budgie) x86.64 with kernel version 6.11.9-100.fc39.x86.64. The system fea- tures an Intel i7-8550U processor with eight cores capable of reaching 4.0GHz turbo frequency, operating purely on CPU without GPU acceleration.

 The RF emulator demonstrated remarkable computational efficiency in reproduc-³³⁷ ing the complex carbon cycle dynamics originally simulated by cGENIE. The total $_{338}$ runtime for emulating the complete set of 20 emission scenarios $(1,000-20,000 \text{ PgC})$ was 345.67 seconds, with an average per-scenario runtime of 16.46 seconds. Memory utilization remained modest throughout the emulation process, with a peak memory footprint of 256.8 MB for the entire simulation ensemble. Individual scenario mem- ory requirements ranged from 178.4 MB for the 1,000 PgC case to 256.8 MB for the 20,000 PgC scenario, indicating efficient scaling with problem size.

 Detailed timing analysis reveals consistent performance across emission scenarios. For example, the 1,000 PgC scenario completed in 15.23 seconds, while the 20,000 PgC scenario required 17.89 seconds, demonstrating only minimal overhead for larger emission sizes. The prediction output shapes remained constant at (1000, 2) across $_{348}$ all scenarios, representing time points and corresponding $CO₂$ concentrations. The emulator achieved this performance while maintaining high numerical precision, with output predictions stored in 64-bit floating-point format.

 The computational advantage becomes particularly evident when compared to the original cGENIE simulations, which typically require weeks of computing time per scenario on comparable hardware. Our RF emulator thus achieves approximately four orders of magnitude speedup while maintaining high accuracy across all tem- poral scales and emission scenarios (Figure 8). This efficiency makes it practical to explore large parameter spaces and conduct uncertainty analyses that would be computationally prohibitive with the full ESMs.

Fig. 8 RF emulator predictions of atmospheric $CO₂$ decay curves for emission scenarios from 1,000 to 20,000 PgC, generated on a standard desktop PC. The semi-logarithmic x –axis spans from 10⁰ to ⁶ years, revealing multi-scale carbon cycle dynamics. Each trajectory represents a different emission scenario (color-coded from purple to yellow for increasing emissions), showing $CO₂$ anomalies in ppmv relative to the pre-industrial baseline (278 ppmv). These results were generated with an average runtime of 16.46 seconds per scenario, compared to several weeks for the original cGENIE simulations.

 The pickle-serialized RF model, including all trained parameters and preprocessing transformations, requires only 24.5 MB of storage space. This compact size facilitates

 easy distribution and deployment across different computing environments. Load- ing the serialized model takes approximately 0.34 seconds on our test system, with negligible memory overhead during the loading process.

³⁶³ 4 Discussion

 This study demonstrates the potential of ML techniques, specifically RF and MLP, $\frac{365}{365}$ to emulate the long-term atmospheric CO₂ evolution as simulated by cGENIE. By focusing on the simple one-year pulse response following carbon release outputs, as outlined by Lord et al. [\[4,](#page-21-0) [11\]](#page-22-0), this work provides an efficient and scalable alternative for understanding the fundamental processes governing atmospheric $CO₂$ anomalies over decadal to millennial timescales.

 The one-year pulse approach isolates the system's intrinsic feedback mechanisms, $_{371}$ including rapid air-sea $CO₂$ exchange, carbonate compensation in ocean sediments, and long-term silicate weathering on land. These mechanisms are key to regulating $\frac{373}{373}$ atmospheric CO₂ and are central to understanding carbon cycle feedbacks [\[19,](#page-23-0) [21\]](#page-23-0).

 A fundamental limitation of this study is its focus on emulating only 0D time-series outputs from cGENIE. While this approach successfully captures temporal dynam- ics of atmospheric $CO₂$ anomalies, it does not account for spatial heterogeneity in carbon cycle processes or regional variations in climate feedbacks. The 0D nature means we lose important information about spatial patterns of carbon uptake, regional differences in weathering rates, and geographical variations in ocean acidification. This simplification, while computationally advantageous, limits the emulator's ability to represent spatially dependent processes such as regional ocean circulation pat- terns, localized weathering responses, and spatial variations in marine carbon burial. Future work should extend these emulation techniques to include spatial dimensions, though this would significantly increase both the complexity of the machine learning architecture and the computational resources required for training.

 Furthermore, it is important to note that this ML emulation approach is fundamen- tally limited by relying solely on the one-year pulse experiments from Lord et al. [\[4,](#page-21-0) [11\]](#page-22-0) that only account for inorganic carbon burial through carbonate compensation. While this captures a key long-term carbon cycle feedback, it neglects the critical role of organic carbon burial in marine sediments. Recent work has demonstrated that organic carbon burial and associated nutrient cycling can create unexpected instabilities in Earth's climate regulation [\[49\]](#page-26-0). The coupling of organic matter degradation, phospho- rus cycling, and redox conditions can lead to non-monotonic responses in atmospheric CO₂ following perturbations, including potential cooling overshoots - dynamics that cannot be captured by models focused only on inorganic carbon burial.

 The development of the Organic Matter ENabled SEDiment (OMEN-SED) model and its coupling to cGENIE [\[50\]](#page-26-0) now enables explicit simulation of organic carbon burial and associated biogeochemical dynamics in marine sediments. This includes representation of organic matter degradation pathways, nutrient recycling, and redox- dependent processes that Vervoort et al. [\[51\]](#page-26-0) showed are essential for accurately simulating negative carbon isotope excursions and associated environmental changes

 throughout Earth's history. Future ML emulation efforts would benefit from incorpo- rating these organic carbon burial processes to provide a more complete representation of long-term carbon cycle feedbacks, though this would require expanding the training dataset beyond simple pulse experiments to capture the complex interactions between organic and inorganic carbon burial.

 The RF model effectively captures these nonlinear processes due to its ensemble- based architecture, which handles heterogeneous data and multiscale interactions. However, RF's reliance on decision trees and their aggregates results in abstrac- tion, which limits mechanistic interpretability [\[52,](#page-26-0) [53\]](#page-26-0). While RF provides feature ⁴¹¹ importance insights, such as the dominance of temporal dynamics over emission size, it lacks the ability to represent geochemical pathways explicitly. This abstraction poses challenges when exploring novel scenarios outside the training dataset, such as combinations of biological and chemical feedbacks under extreme anthropogenic forcing.

 MLP, on the other hand, theoretically offers greater flexibility in approximat- ing nonlinear relationships but struggled in this application due to both scientific and computational limitations. Scientifically, the carbon cycle involves hierarchical and multiscale feedbacks that are challenging for neural networks to capture without tailored architectures or explicit temporal encoding. For example, during the rapid- response phase (1–10 years), dominated by highly dynamic air-sea gas exchange and carbonate chemistry, MLP displayed higher errors than RF. This discrepancy arises from MLP's lack of mechanisms to prioritize temporal dynamics or represent hier- archical relationships [\[54\]](#page-26-0), which are critical for modeling the progression of carbon cycle feedbacks [\[15,](#page-23-0) [23\]](#page-23-0). Computationally, MLP relies on backpropagation for train- ing, which is prone to issues such as vanishing or exploding gradients, especially in deeper networks [\[55\]](#page-26-0). Furthermore, MLP often requires extensive datasets to general- ize effectively, a limitation in scenarios where simulations like those from cGENIE are computationally expensive to produce.

 Over longer timescales (100–10,000 years), processes such as carbonate dissolu- tion in sediments and silicate weathering on land dominate the carbon cycle. These processes act as negative feedbacks, gradually restoring equilibrium to the Earth sys- tem after a perturbation [\[11,](#page-22-0) [33\]](#page-24-0). RF successfully captures these trends, reflecting its ability to approximate the cumulative effects of long-term feedbacks. However, the simplifications inherent in the one-year pulse response dataset exclude interactions between feedback mechanisms, such as the coupling between terrestrial weathering and ocean alkalinity, which are critical for predicting carbon cycle dynamics under prolonged or multiple perturbations [\[12,](#page-22-0) [21\]](#page-23-0). These limitations highlight the need for expanded training datasets that incorporate a broader range of emission scenarios and interactions between feedback mechanisms.

 Both RF and MLP emulators demonstrate the power of ML in providing com- putationally efficient alternatives to full Earth system simulations. For example, RF achieves a four-orders-of-magnitude speedup compared to cGENIE while maintaining high predictive accuracy. However, the abstraction inherent in these models limits their generalizability and interpretability. RF's static architecture is particularly limited in handling temporal dependencies, as it does not explicitly encode the sequential nature

 of feedbacks. In contrast, MLP's black-box nature hinders scientific understanding, as it lacks interpretable mechanisms to explain model predictions [\[56,](#page-26-0) [57\]](#page-26-0).

 Emerging methods such as physics-informed neural networks (PINNs) provide a promising solution to these challenges [\[58,](#page-26-0) [59\]](#page-27-0). PINNs embed physical constraints into the learning process, ensuring that predictions remain consistent with known geochem- ical and climatic dynamics. This hybrid approach bridges the gap between empirical accuracy and mechanistic insight, offering significant advantages for applications requiring both computational efficiency and scientific validity. Additionally, advanced architectures like recurrent neural networks (RNNs) and attention-based transform- ers could enhance the handling of temporal dependencies in carbon cycle feedbacks, particularly for scenarios involving overlapping or cascading processes [\[54,](#page-26-0) [60\]](#page-27-0).

 Finally, the limitations identified in both RF and MLP highlight the importance of developing hybrid approaches that integrate domain-specific knowledge with ML techniques. By combining data-driven modeling with process-based insights, future emulators can achieve greater interpretability, generalizability, and robustness, making them invaluable tools for understanding and managing the complexities of the Earth system.

5 Conclusion

 This study demonstrates the successful application of ML approaches, particularly RF, in emulating atmospheric $CO₂$ evolution across multiple timescales through a 0D time series framework. Our RF emulator achieves remarkable computational effi- ciency—reducing simulation time from weeks to seconds—while maintaining high 469 predictive accuracy $(R^2 = 0.998 \pm 0.001)$ across emission scenarios ranging from 1,000 $470 \text{ to } 20,000 \text{ PgC}$. While focusing on temporal evolution rather than spatial heterogeneity, the model shows superior performance in capturing rapid air-sea gas exchange (1-10 years), ocean mixing (10-100 years), carbonate compensation (100-1,000 years), and long-term weathering feedbacks (>1,000 years), representing a significant advancement in Earth system modeling capabilities. However, limitations in mechanistic inter- pretability, the current exclusion of organic carbon burial processes, and the simplified 0D representation highlight opportunities for future development, particularly through PINNs and hybrid approaches that combine data-driven efficiency with process-based understanding.

 The implications of this work extend beyond computational gains, offering a path- way for rapid exploration of carbon cycle perturbations across geological timescales. This capability is particularly crucial for understanding anthropogenic climate change and past hyperthermal events, enabling efficient parameter space exploration and uncertainty quantification that would be computationally prohibitive with traditional ESMs. As we advance toward more sophisticated emulation approaches, the integration of biogeochemical constraints, organic carbon dynamics, and spatial dimensionality will further enhance our ability to predict and understand Earth system responses to carbon cycle perturbations across multiple temporal and spatial scales.

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490 Code and Data Availability. All relevant code, data (used and generated in this work), pickle files for both the trained RF and MLP emulators, and figures are stored in the following OSF repository: [https://doi.org/10.17605/OSF.IO/3G74U.](https://doi.org/10.17605/OSF.IO/3G74U) Additionally, the complete outputs of the cGENIE experiments are available upon request.

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₄₉₇ Declarations

Conflict of interest. The author declares there is no conflict.

⁴⁹⁹ Competing interests. Author does not have any competing financial interest to declare.

References

 [1] Friedlingstein, P., Jones, M.W., O'Sullivan, M., Andrew, R.M., Bakker, D.C.E., Hauck, J., Le Qu´er´e, C., Peters, G.P., Peters, W., Pongratz, J., Sitch, S., Canadell, J.G., Ciais, P., Jackson, R.B., Alin, S.R., Anthoni, P., Bates, N.R., Becker, M., Bellouin, N., Bopp, L., Chau, T.T.T., Chevallier, F., Chini, L.P., Cronin, M., Currie, K.I., Decharme, B., Djeutchouang, L.M., Dou, X., Evans, W., Feely, R.A., Feng, L., Gasser, T., Gilfillan, D., Gkritzalis, T., Grassi, G., Gregor, L., Gru- ber, N., G¨urses, O., Harris, I., Houghton, R.A., Hurtt, G.C., Iida, Y., Ilyina, T., Luijkx, I.T., Jain, A., Jones, S.D., Kato, E., Kennedy, D., Klein Goldewijk, K., Knauer, J., Korsbakken, J.I., K¨ortzinger, A., Landsch¨utzer, P., Lauvset, S.K., L ₅₁₁ Lefèvre, N., Lienert, S., Liu, J., Marland, G., McGuire, P.C., Melton, J.R., Munro, D.R., Nabel, J.E.M.S., Nakaoka, S.-I., Niwa, Y., Ono, T., Pierrot, D., Poul- ter, B., Rehder, G., Resplandy, L., Robertson, E., R¨odenbeck, C., Rosan, T.M., 514 Schwinger, J., Schwingshackl, C., Séférian, R., Sutton, A.J., Sweeney, C., Tan- hua, T., Tans, P.P., Tian, H., Tilbrook, B., Tubiello, F., Werf, G.R., Vuichard, N., Wada, C., Wanninkhof, R., Watson, A.J., Willis, D., Wiltshire, A.J., Yuan, W., Yue, C., Yue, X., Zaehle, S., Zeng, J.: Global Carbon Budget 2021. Earth Syst. Sci. Data 14(4), 1917–2005 (2022) <https://doi.org/10.5194/essd-14-1917-2022>

 [2] IPCC: Climate Change 2021: The Physical Science Basis. Cambridge University Press, Cambridge, UK (2021). <https://doi.org/10.1017/9781009157896>

 [3] Lord, N.S., Crucifix, M., Lunt, D.J., Thorne, M.C., Bounceur, N., Dowsett, H., O'Brien, C.L., Ridgwell, A.: Emulation of long-term changes in global climate: application to the late Pliocene and future. Clim. Past 13(11), 1539–1571 (2017) <https://doi.org/10.5194/cp-13-1539-2017>

 [4] Lord, N.S., Ridgwell, A., Thorne, M.C., Lunt, D.J.: The long tail of anthropogenic $SO₂$ decline in the atmosphere and its consequences for post-closure performance

- assessments for disposal of radioactive wastes. Mineral. Mag. 79(6), 1613–1623 (2015) <https://doi.org/10.1180/minmag.2015.079.6.37>
- [5] Nonnenmacher, M., Greenberg, D.S.: Deep Emulators for Differentiation, Fore- casting, and Parametrization in Earth Science Simulators. J. Adv. Model. Earth Syst. 13(7), 2021–002554 (2021) <https://doi.org/10.1029/2021MS002554>
- [6] Watson-Parris, D.: Machine learning for weather and climate are worlds apart. Philos. Trans. R. Soc. A 379(2194), 20200098 (2021) [https://doi.org/10.1098/](https://doi.org/10.1098/rsta.2020.0098) [rsta.2020.0098](https://doi.org/10.1098/rsta.2020.0098)
- [7] Watson-Parris, D., Williams, A., Deaconu, L., Stier, P.: Model calibration using ESEm v1.1.0 – an open, scalable Earth system emulator. Geosci. Model Dev. 14(12), 7659–7672 (2021) <https://doi.org/10.5194/gmd-14-7659-2021>
- 538 [8] Watson-Parris, D., Rao, Y., Olivié, D., Seland, , Nowack, P., Camps-Valls, G., Stier, P., Bouabid, S., Dewey, M., Fons, E., Gonzalez, J., Harder, P., Jeg- gle, K., Lenhardt, J., Manshausen, P., Novitasari, M., Ricard, L., Roesch, C.: ClimateBench v1.0: A Benchmark for Data-Driven Climate Projections. J. Adv. Model. Earth Syst. 14(10), 2021–002954 (2022) [https://doi.org/10.1029/](https://doi.org/10.1029/2021MS002954) [2021MS002954](https://doi.org/10.1029/2021MS002954)
- [9] Ukkonen, P.: Exploring Pathways to More Accurate Machine Learning Emulation of Atmospheric Radiative Transfer. J. Adv. Model. Earth Syst. 14(4), 2021– 002875 (2022) <https://doi.org/10.1029/2021MS002875>

 [10] Barthel Sorensen, B., Charalampopoulos, A., Zhang, S., Harrop, B.E., Leung, L.R., Sapsis, T.P.: A Non-Intrusive Machine Learning Framework for Debiasing Long-Time Coarse Resolution Climate Simulations and Quantifying Rare Events Statistics. J. Adv. Model. Earth Syst. 16(3), 2023–004122 (2024) [https://doi.org/](https://doi.org/10.1029/2023MS004122) [10.1029/2023MS004122](https://doi.org/10.1029/2023MS004122)

- [11] Lord, N.S., Ridgwell, A., Thorne, M.C., Lunt, D.J.: An impulse response function $\frac{553}{553}$ for the "long tail" of excess atmospheric CO_2 in an Earth system model. Glob. Biogeochem. Cycles 30(1), 2–17 (2016) <https://doi.org/10.1002/2014GB005074>
- [12] Ridgwell, A., Zeebe, R.E.: The role of the global carbonate cycle in the regulation and evolution of the Earth system. Earth Planet. Sci. Lett. 234(3-4), 299–315 (2005) <https://doi.org/10.1016/j.epsl.2005.03.006>
- [13] Lenton, T.M., Williamson, M.S., Edwards, N.R., Marsh, R., Price, A.R., Ridgwell, A.J., Shepherd, J.G., Cox, S.J., team, G.: Millennial timescale carbon cycle and climate change in an efficient Earth system model. Clim. Dyn. 26, 687–711 (2006) <https://doi.org/10.1007/s00382-006-0109-9>
- $_{562}$ [14] Ridgwell, A., Hargreaves, J.C.: Regulation of atmospheric CO₂ by deep-sea sed-iments in an Earth system model. Glob. Biogeochem. Cycles 21(2), 2008 (2007)

<https://doi.org/10.1029/2006GB002764>

- [15] Ridgwell, A., Hargreaves, J.C., Edwards, N.R., Annan, J.D., Lenton, T.M., Marsh, R., Yool, A., Watson, A.: Marine geochemical data assimilation in an effi- cient Earth System Model of global biogeochemical cycling. Biogeosciences 4(1), 87–104 (2007) <https://doi.org/10.5194/bg-4-87-2007>
- [16] Colbourn, G., Ridgwell, A., Lenton, T.M.: The Rock Geochemical Model (RokGeM) v0.9. Geosci. Model Dev. 6(5), 1543–1573 (2013) [https://doi.org/10.](https://doi.org/10.5194/gmd-6-1543-2013) $5194/\text{gmd-6-1543-2013}$
- [17] Colbourn, G., Ridgwell, A., Lenton, T.M.: The time scale of the silicate weath- $\frac{573}{29}$ ering negative feedback on atmospheric CO₂. Glob. Biogeochem. Cycles 29(5), 583–596 (2015) <https://doi.org/10.1002/2014GB005054>
- [18] Eby, M., Zickfeld, K., Montenegro, A., Archer, D., Meissner, K.J., Weaver, A.J.: Lifetime of Anthropogenic Climate Change: Millennial Time Scales of Potential CO₂ and Surface Temperature Perturbations. J. Clim. $22(10)$, 2501–2511 (2009) <https://doi.org/10.1175/2008JCLI2554.1>
- [19] Archer, D., Kheshgi, H., Maier-Reimer, E.: Atmospheric lifetime of fossil fuel carbon dioxide. Annu. Rev. Earth Planet. Sci. 37, 117–134 (2009) [https://doi.](https://doi.org/10.1146/annurev.earth.031208.100206) [org/10.1146/annurev.earth.031208.100206](https://doi.org/10.1146/annurev.earth.031208.100206)
- [20] Zeebe, R.E., Zachos, J.C.: Reversed deep-sea carbonate ion basin-gradient dur- ing Paleocene-Eocene Thermal Maximum. Paleoceanography 22(3), 3201 (2007) <https://doi.org/10.1029/2006PA001395>
- [21] Zeebe, R.E., Zachos, J.C., Dickens, G.R.: CO₂ forcing alone insufficient to explain Palaeocene–Eocene Thermal Maximum warming. Nat. Geosci. 1(8), 576–579 (2008) <https://doi.org/10.1038/ngeo195>
- [22] Zeebe, R.E., Wolf-Gladrow, D.A.: $CO₂$ in Seawater: Equilibrium, Kinetics, Isotopes. Elsevier, Amsterdam, Netherlands (2001)
- [23] Feely, R.A., Sabine, C.L., Lee, K., Berelson, W., Kleypas, J., Fabry, V.J., Millero, $F.J.:$ Impact of Anthropogenic $CO₂$ on the CaCO₃ System in the Oceans. Science 305(5682), 362–366 (2004) <https://doi.org/10.1126/science.1097329>
- [24] Jiang, L.-Q., Feely, R.A., Carter, B.R., Lauvset, S.K., Olsen, A.: Surface ocean pH and buffer capacity: Past, present and future. Sci. Rep. 9, 18624 (2019) [https:](https://doi.org/10.1038/s41598-019-55039-4) [//doi.org/10.1038/s41598-019-55039-4](https://doi.org/10.1038/s41598-019-55039-4)
- [25] H¨onisch, B., Ridgwell, A., Schmidt, D.N., Thomas, E., Gibbs, S.J., Sluijs, A., Zeebe, R., Kump, L., Martindale, R.C., Greene, S.E., Kiessling, W., Ries, J.B., Zachos, J.C., Royer, D.L., Barker, S., Marchitto, T.M.J., Moyer, R., Pelejero, C., Ziveri, P., Foster, G.L., Williams, B.: The geological record of ocean acidification.
	-
- Science 335(6072), 1058–1063 (2012) <https://doi.org/10.1126/science.1208277> [26] Kwiatkowski, L., Torres, O., Bopp, L., Aumont, O., Chamberlain, M., Christian, J.R., Dunne, J.P., Gehlen, M., Ilyina, T., John, J.G., Lenton, A., Li, H., Loven-⁶⁰³ duski, N.S., Orr, J.C., Palmieri, J., Santana-Falcón, Y., Schwinger, J., Séférian, R., Stock, C.A., Tagliabue, A., Takano, Y., Tjiputra, J., Toyama, K., Tsujino, H., Watanabe, M., Yamamoto, A., Yool, A., Ziehn, T.: Twenty-first century ocean warming, acidification, deoxygenation, and upper-ocean nutrient and primary production decline from CMIP6 model projections. Biogeosciences 17, 3439–3470 (2020) <https://doi.org/10.5194/bg-17-3439-2020>
- [27] Marsh, R., M¨uller, S.A., Yool, A., Edwards, N.R.: Incorporation of the C- GOLDSTEIN efficient climate model into the GENIE framework: "eb go gs" ϵ_{611} configurations of GENIE. Geosci. Model Dev. $4(4)$, 957–992 (2011) [https://doi.](https://doi.org/10.5194/gmd-4-957-2011) $_{612}$ [org/10.5194/gmd-4-957-2011](https://doi.org/10.5194/gmd-4-957-2011)
- [28] Gattuso, J.-P., Hansson, L. (eds.): Ocean Acidification. Oxford University Press, Oxford, UK (2011). <https://doi.org/10.1093/oso/9780199591091.001.0001>
- ⁶¹⁵ [29] Gattuso, J.-P., Magnan, A., Billé, R., Cheung, W.W.L., Howes, E.L., Joos, F., Allemand, D., Bopp, L., Cooley, S.R., Eakin, H., Hoegh-Guldberg, O., Kelly, R.P., P¨ortner, H.-O., Rogers, A.D., Baxter, J.M., Laffoley, D., Osborn, D., Rankovic, A., Rochette, J., Sumaila, U.R., Treyer, S., Turley, C.: Contrasting futures for $\frac{619}{100}$ ocean and society from different anthropogenic CO_2 emissions scenarios. Science 349(6243), 4722 (2015) <https://doi.org/10.1126/science.aac4722>
- [30] Kirtland Turner, S., Ridgwell, A.: Recovering the true size of an Eocene hyper- $\frac{622}{28(4)}$ thermal from the marine sedimentary record. Paleoceanography $28(4)$, 700–712 (2013) <https://doi.org/10.1002/2013PA002541>
- [31] Doney, S.C., Fabry, V.J., Feely, R.A., Kleypas, J.A.: Ocean acidification: The ⁶²⁵ other $CO₂$ problem. Annu. Rev. Mar. Sci. 1, 169–192 (2009) [https://doi.org/10.](https://doi.org/10.1146/annurev.marine.010908.163834) [1146/annurev.marine.010908.163834](https://doi.org/10.1146/annurev.marine.010908.163834)
- [32] Doney, S.C., Busch, D.S., Cooley, S.R., Kroeker, K.J.: The impacts of ocean acidification on marine ecosystems and reliant human communities. Annu. Rev. Environ. Resour. 45, 83–112 (2020) [https://doi.org/10.1146/](https://doi.org/10.1146/annurev-environ-012320-083019) [annurev-environ-012320-083019](https://doi.org/10.1146/annurev-environ-012320-083019)
- [33] Archer, D.: An atlas of the distribution of calcium carbonate in sediments of the 632 deep sea. Glob. Biogeochem. Cycles $10(1)$, $159-174$ (1996) [https://doi.org/10.](https://doi.org/10.1029/95GB03016) [1029/95GB03016](https://doi.org/10.1029/95GB03016)
- [34] Rosenblatt, F.: Principles of Neurodynamics: Perceptrons and the Theory of Brain Mechanisms. Spartan Books, Washington, DC (1962)

- [35] Rumelhart, D.E., Hinton, G.E., Williams, R.J.: Learning representations by back- propagating errors. Nature 323(6088), 533–536 (1986) [https://doi.org/10.1038/](https://doi.org/10.1038/323533a0) [323533a0](https://doi.org/10.1038/323533a0)
- [36] Cybenko, G.: Approximation by superpositions of a sigmoidal function. Math. Control Signals Syst. 2(4), 303–314 (1989) <https://doi.org/10.1007/BF02551274>
- [37] Hornik, K., Stinchcombe, M., White, H.: Multilayer feedforward networks are $\frac{642}{100}$ universal approximators. Neural Netw. $2(5)$, 359–366 (1989) [https://doi.org/10.](https://doi.org/10.1016/0893-6080(89)90020-8) [1016/0893-6080\(89\)90020-8](https://doi.org/10.1016/0893-6080(89)90020-8)
- [38] Kingma, D.P., Ba, J.: Adam: A Method for Stochastic Optimization. arXiv:1412.6980 (2014). <https://doi.org/10.48550/arXiv.1412.6980>
- [39] Castruccio, S., McInerney, D.J., Stein, M.L., Liu, Z., Moyer, E.J.: Statistical Emulation of Climate Model Projections Based on Precomputed GCM Runs. J. Clim. 27(5), 1829–1844 (2014) <https://doi.org/10.1175/JCLI-D-13-00099.1>
- [40] Tebaldi, C., Arblaster, J.M.: Pattern scaling: its strengths and limitations, and ϵ_{650} an update on the latest model simulations. Clim. Change 122(3), 459–471 (2014) <https://doi.org/10.1007/s10584-013-1032-9>
- [41] Pedregosa, F., Varoquaux, G., Gramfort, A., Michel, V., Thirion, B., Grisel, O., Blondel, M., Prettenhofer, P., Weiss, R., Dubourg, V., Vanderplas, J., Passos, A., Cournapeau, D., Brucher, M., Perrot, M., Duchesnay, E.: Scikit-learn: Machine Learning in Python. J. Mach. Learn. Res. 12, 2825–2830 (2011)
- [42] Harris, C.R., Millman, K.J., Walt, S.J., Gommers, R., Virtanen, P., Cournapeau, D., Wieser, E., Taylor, J., Berg, S., Smith, N.J., Kern, R., Picus, M., Hoyer, S., ⁶⁵⁸ Kerkwijk, M.H., Brett, M., Haldar, J., Río, J.F., Wiebe, M., Peterson, P., Gérard- Marchant, P., Sheppard, K., Reddy, T., Weckesser, W., Abbasi, H., Gohlke, C., Oliphant, T.E.: Array programming with NumPy. Nature 585, 357–362 (2020) <https://doi.org/10.1038/s41586-020-2649-2>
- [43] McKinney, W.: Data Structures for Statistical Computing in Python. In: Walt, S., Millman, J. (eds.) Proceedings of the 9th Python in Science Conference, pp. 56–61 (2010). <https://doi.org/10.25080/Majora-92bf1922-00a>
- ₆₆₅ [\[](https://doi.org/10.1023/A:1010933404324)44] Breiman, L.: Random Forests. Mach. Learn. $45(1)$, 5-32 (2001) [https://doi.org/](https://doi.org/10.1023/A:1010933404324) [10.1023/A:1010933404324](https://doi.org/10.1023/A:1010933404324)
- [45] Hastie, T., Tibshirani, R., Friedman, J.: The Elements of Statistical Learning: Data Mining, Inference, and Prediction, 2nd edn. Springer, New York, NY (2009). <https://doi.org/10.1007/978-0-387-84858-7>
- [46] Friedman, J., Hastie, T., Tibshirani, R.: Additive Logistic Regression: A Statisti- cal View of Boosting. Ann. Stat. $28(2)$, 337–407 (2000) [https://doi.org/10.1214/](https://doi.org/10.1214/aos/1016218223)
	-

[aos/1016218223](https://doi.org/10.1214/aos/1016218223)

- [47] Bergstra, J., Bengio, Y.: Random Search for Hyper-Parameter Optimization. J. μ_{674} Mach. Learn. Res. 13(10), 281–305 (2012)
- [48] Probst, P., Wright, M.N., Boulesteix, A.-L.: Hyperparameters and Tuning Strate- gies for Random Forest. Wiley Interdiscip. Rev. Data Min. Knowl. Discov. 9(3), 1301 (2019) <https://doi.org/10.1002/widm.1301>
- [49] Hülse, D., Ridgwell, A.: Instability in the geological regulation of Earth's climate. EarthArXiv (2023) <https://doi.org/10.31223/X5F13X>
- [50] H¨ulse, D., Arndt, S., Daines, S., Regnier, P., Ridgwell, A.: OMEN-SED 1.0: a novel, numerically efficient organic matter sediment diagenesis module for coupling to Earth system models. Geosci. Model Dev. 11, 2649–2689 (2018) <https://doi.org/10.5194/gmd-11-2649-2018>
- [51] Vervoort, P., Adloff, M., Greene, S.E., Kirtland Turner, S.: Negative carbon iso- tope excursions: An interpretive framework. Environ. Res. Lett. 14(8), 085014 (2019) <https://doi.org/10.1088/1748-9326/ab3318>
- [52] Cutler, D.R., Edwards, T.C.J., Beard, K.H., Cutler, A., Hess, K.T., Gibson, J., Lawler, J.J.: RANDOM FORESTS FOR CLASSIFICATION IN ECOLOGY. Ecology 88(11), 2783–2792 (2007) <https://doi.org/10.1890/07-0539.1>
- [53] Wright, M.N., Ziegler, A.: ranger: A fast implementation of random forests for μ_{691} high-dimensional data in R and C++. J. Stat. Softw. 77(1), 1–17 (2019) [https:](https://doi.org/10.18637/jss.v077.i01) [//doi.org/10.18637/jss.v077.i01](https://doi.org/10.18637/jss.v077.i01)
- [54] Hochreiter, S., Schmidhuber, J.: Long Short-Term Memory. Neural Comput. 9(8), 1735–1780 (1997) <https://doi.org/10.1162/neco.1997.9.8.1735>
- [55] Goodfellow, I., Bengio, Y., Courville, A.: Deep Learning. MIT Press, Cambridge, MA (2016). <https://www.deeplearningbook.org/>
- [56] Rudin, C.: Stop explaining black box machine learning models for high stakes decisions and use interpretable models instead. Nat. Mach. Intell. 1(5), 206–215 $\frac{(2019)}{\text{https://doi.org/10.1038/s42256-019-0048-x}}$ $\frac{(2019)}{\text{https://doi.org/10.1038/s42256-019-0048-x}}$ $\frac{(2019)}{\text{https://doi.org/10.1038/s42256-019-0048-x}}$
- [57] Lundberg, S.M., Lee, S.-I.: A Unified Approach to Interpreting Model Pre- dictions. In: Guyon, I., Luxburg, U.V., Bengio, S., Wallach, H., Fergus, R., Vishwanathan, S., Garnett, R. (eds.) Advances in Neural Information Processing Systems, vol. 30 (2017). [https://proceedings.neurips.cc/paper](https://proceedings.neurips.cc/paper_files/paper/2017/file/8a20a8621978632d76c43dfd28b67767-Paper.pdf) files/paper/2017/ [file/8a20a8621978632d76c43dfd28b67767-Paper.pdf](https://proceedings.neurips.cc/paper_files/paper/2017/file/8a20a8621978632d76c43dfd28b67767-Paper.pdf)
- [58] Raissi, M., Perdikaris, P., Karniadakis, G.E.: Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving

- partial differential equations. J. Comput. Phys. 378, 686–707 (2019) [https://doi.](https://doi.org/10.1016/j.jcp.2018.10.045) [org/10.1016/j.jcp.2018.10.045](https://doi.org/10.1016/j.jcp.2018.10.045)
- [59] Karniadakis, G.E., Kevrekidis, I.G., Lu, L., Perdikaris, P., Wang, S., Yang, L.: Physics-informed machine learning. Nat. Rev. Phys. 3(6), 422–440 (2021) [https:](https://doi.org/10.1038/s42254-021-00314-5) [//doi.org/10.1038/s42254-021-00314-5](https://doi.org/10.1038/s42254-021-00314-5)
- [60] Vaswani, A., Shazeer, N., Parmar, N., Uszkoreit, J., Jones, L., Gomez, A.N., Kaiser, L., Polosukhin, I.: Attention is All you Need. In: Guyon, I., Luxburg, U.V., Bengio, S., Wallach, H., Fergus, R., Vishwanathan, S., Garnett, R. (eds.) Advances in Neural Information Processing Sys- tems, vol. 30 (2017). [https://proceedings.neurips.cc/paper](https://proceedings.neurips.cc/paper_files/paper/2017/file/3f5ee243547dee91fbd053c1c4a845aa-Paper.pdf) files/paper/2017/file/ [3f5ee243547dee91fbd053c1c4a845aa-Paper.pdf](https://proceedings.neurips.cc/paper_files/paper/2017/file/3f5ee243547dee91fbd053c1c4a845aa-Paper.pdf)