Large earthquakes are more predictable than smaller ones

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Abstract
Large earthquakes have been viewed as highly chaotic events regardless of their magnitude, making their prediction intrinsically challenging. Here, we develop a mathematical tool to incorporate multiscale physics, capable of describing both deterministic and chaotic systems, to model earthquake rupture. Our findings suggest that the chaotic behavior of seismic dynamics, that is, its sensitivity to initial and boundary conditions, is inversely related to its magnitude. To validate this hypothesis, we performed numerical simulations with heterogeneous fault conditions. Our results indicate that large earthquakes, usually occurring in regions with higher residual energy and lower b-value (i.e., the exponent of the Gutenberg-Richter law), are less susceptible to be affected by perturbations. This suggests that a higher variability in earthquake magnitudes (larger b-values) may be indicative of structural complexity of the fault network and heterogeneous stress conditions. We compare our theoretical predictions with the statistical properties of seismicity in Southern California; specifically, we show that our model agrees with the observed relationship between the b-value and the fractal dimension of hypocenters. The similarities observed between simulated and natural earthquakes support the hypothesis that large events may be less chaotic than smaller ones; hence, more predictable.

Keywords: Earthquake predictability; Seismic rupture; Chaos theory; Residual energy; b-value; HE-B method.

1. Introduction

Earthquakes are a persistent threat to human society, capable of causing widespread devastation (e.g., Kahandawa et al., 2018). The rapid release of accumulated tectonic stress can result in catastrophic natural disasters with severe human and economic consequences (Knopoff, 1958; Vassiliou and Kanamori, 1982; Gudmundsson, 2014; Aksoy et al., 2024; Silverio-Murillo et al., 2024). To efficiently face seismic risk, a deeper understanding of seismicity is needed. Particularly, a fundamental aspect of earthquake studies is the examination of rupture processes along geological faults (e.g., Christensen and Beck, 1994; Kintner et al., 2018; Otarola et al., 2021; Martínez-Lopez, 2023; Wang et al., 2023), as these can induce notable changes in the soil's physical characteristics, such as variations in ground velocity, acceleration and frequency (Colavitti et al., 2022; Li et al., 2022; Venegas-Aravena, 2024a). Evidence from various studies points to the possibility that seismic rupture processes may exhibit the hallmarks of chaotic systems, suggesting a complex and unpredictable nature of these events. Some perspectives on earthquake generation are rooted in simplified spring-block models which exhibit these chaotic dynamics (e.g., Huang and Turcotte, 1990; Gualandi et al., 2023). This is reflected in computational simulations where small variations in the initial conditions generate completely different rupturing scenarios (e.g., Erickson et al., 2011). That is, causing no correlation between a priori and a posteriori parameter (Venegas-Aravena et al.,2024). Despite this complexity, a consistent finding from these simulations is the emergence of a single dominant parameter: residual energy. The importance of the accumulated/residual strain is also coherent with recent results in the modeling of paleoseismic recordings (Salditch et al., 2020). This parameter, which defines zones where ruptures are prone to occur (Noda et al., 2022), appears to exert a controlling influence on both the spatial extent and temporal evolution of ruptures (Venegas-Aravena, 2023; Venegas-Aravena et al., 2024). Its value is dependent on both the available energy, which is determined by the initial stresses, and the fracture energy, which is associated with the energy required to continue propagating the rupture (Noda et al., 2022). Therefore, when the rupture front approaches a zone with negative (positive) stress release rate, more (less) energy is consumed in generating the rupture, causing the rupture to arrest (continue propagating). Other formalisms associated with friction have also found that rupture arrest can be related to stresses and fracture energy (Barras et al., 2023).

Residual energy has been linked to a parameter called thermodynamic fractal dimension D (Venegas-Aravena and Cordaro, 2023a). This quantity is useful for characterizing fault distribution (Zou and Fialko, 2024) and the spatial distribution of global seismicity (Perinelli et al., 2024). For instance, it has been observed that the fractal dimension decreases prior to a major earthquake, suggesting a transition from a more diffuse, three-dimensional seismicity distribution to a more localized, planar distribution along the fault (Murase, 2004; Wyss et al., 2004; Iaccarino and Picozzi, 2023). Other studies have interpreted this decrease in D as an indicator of an impending larger rupture due to the increase of shear stresses (e.g., Ito and Kaneko, 2023; Venegas-Aravena and Cordaro, 2023a). Furthermore, it has been linked to the b-value, a parameter describing earthquake frequency (Venegas-Aravena and Cordaro, 2023b). Given the proportional relationship between D and the b-value, a decrease in D is also associated with a decrease in the b-value prior to large earthquakes. Given the link between the parameter D and properties associated with chaotic systems, as suggested by lower D values in less chaotic systems (Venegas-Aravena and Cordaro 2024), the b-value is anticipated to provide insights into the chaotic states of faults. To explore this connection, Section 2 delves into the fundamental principles of multiscale thermodynamics applied to faults. Section 3 presents various simulations of heterogeneous ruptures, facilitating the interpretation of parameters such as D and the b-value within the framework of multiscale thermodynamics and chaotic systems. In Section 4, we apply these concepts to a real fault system, specifically in Southern California, to support our theoretical and numerical results. A discussion and conclusion are presented in Sections 5 and 6 respectively.

2. Theoretical background: multiscale thermodynamics

As earthquakes are essentially multi-scale events that may exhibit chaotic behavior, a physical framework is required to fully understand their dynamics. In this regard, Venegas-Aravena and Cordaro (2024), have developed a quantitative relationship linking the sum of the Lyapunov exponents Λ, to the thermodynamic fractal dimension D, expressed as:

$$\Lambda \sim -e^{(D_E - D - 1)/k_V} \qquad (1)$$

The Euclidean dimension is denoted by D_E, while k_V is a constant associated with the system's scale. D is a parameter that characterizes the distribution of systems exhibiting power-law behavior.

While Lyapunov exponents are related to the eigenvalues of the Jacobian matrix, describing the local stability of a system (e.g., Wu and Baleanu, 2015), Equation 1 is inspired by the work of Hoover and Posch (1994), wherein the summation of exponent pairs in non-equilibrium systems is employed to quantify irreversibility and the loss of phase-space dimensionality associated with dissipative processes such as frictional heat generation and the occurrence of earthquakes, thereby providing a complementary perspective to purely local analysis.

It is also paramount to comprehend the physical significance of Equation 1. The parameter Λ, representing the sum of the Lyapunov exponents, describes the global tendency of the system towards contraction or dilation of volume in phase space, reflecting overall dissipation or instability (e.g., Eden et al. (1991)). In contrast, the internal dynamics of a dissipative system, including the thermodynamic forces and fluxes that drive entropy production, are described at the microscopic level by the Onsager coefficients (Onsager, 1931a; 1931b). The introduction of the parameter D within multiscale thermodynamics framework implies that dissipative processes are described by a generalization of the Onsager coefficients, which operate across a range of scales. In this context, D serves as a conceptual bridge, enabling the linkage of dynamics occurring at smaller scales, where the common thermodynamic forces and fluxes manifest, with the global evolution of the system observed in the macroscopic phase space. In this manner, D quantifies the organization of dissipation and fluctuations across these multiple scales, thereby influencing the global stability of the system as characterized by Λ.

This Equation can be observed in Figure 1a for k_V = 1, where low (large) values of Λ are associated with low (large) values of D. A higher value of Λ indicates that the system is more susceptible to the influence of small changes in initial conditions (Ruelle, 1983; Tabor, 1989), whereas a more negative value of Λ suggests that the system is less sensitive to these initial conditions, which could be considered as being more regular. The sign of Λ provides an indication of whether a system is non-reversible/dissipative (negative sum) or conservative (e.g., Hoover and Posch, 1994). Given that the brittle crust is a system characterized by the dissipation of stored energy, the sum of Λ is a more relevant metric than the largest Lyapunov exponent, often used to determine the chaotic nature of a system. Consequently, lower values of D correspond to less chaotic systems, i.e., less sensitive to initial conditions. In the case of earthquakes, D can be related to the magnitude of seismic events through the equation (Venegas-Aravena and Cordaro 2023a):

$$M_W \sim log_{10}\left(e^{-\alpha(D)}\right) \qquad (2)$$

Where $\alpha(D) = pD/k_V$ and $p = 3/(5 - D)$. From Equation 1, D can be written in terms of the sum of the Lyapunov exponent as $D = (D_E - 1) - k_V ln\Lambda$. By substituting this Equation into Equation 2, we can establish a direct relationship between the magnitude and chaotic systems as:

$$M_W = M_W(\Lambda) \qquad (3)$$

This relationship is graphically depicted in the color-coded maps presented in Figures 1a, 1b, and 1c. In these maps, red tones correspond to earthquakes of greater magnitude, while blue tones represent smaller earthquakes. It should be noted that the apparent deviation of Figure 1, particularly Figure 1a where Equation 1 is plotted, from typical exponential functions is attributable to the restricted range employed for D (between 2 and 3) and the specific selection of k_V. A broader range for D and/or alternative values of k_V may result in graphs exhibiting a more visually exponential form.
In Figure 1a, large earthquakes are correlated with lower values of both D and Λ. This finding implies that larger seismic events exhibit a reduced sensitivity to initial conditions. In contrast, smaller earthquakes (represented by blue hues in Figure 1a) are associated with a higher degree of chaos, suggesting that these events originate in a more chaotic environment where even small perturbations can lead to seismic activity of varying scales, from small to intermediate. This phenomenon is coherent with several observations suggesting the strong sensitivity of small seismicity to stress perturbations, e.g., tides and hydrological modulations (Rubinstein et al., 2008; Petrelis et al., 2021), which are not reported for major events (Vidale et al., 1998).

Figure 1a also illustrates that the values of Λ are negative, which might suggest that the system is not chaotic as described. However, it is important to differentiate between individual Lyapunov exponents and their summation. While a negative sum of all Lyapunov exponents indicates a contraction, due to energy dissipation, of the global phase-space volume, the presence of even a single positive Lyapunov exponent is the defining characteristic of chaos. This positive exponent signifies the exponential divergence of initially infinitesimally close trajectories along a specific direction in phase space, leading to the unpredictability and sensitive dependence on initial conditions characteristic of chaotic systems. Therefore, a system can exhibit a net dissipative behavior (negative sum) and still be fundamentally chaotic due to the local instability introduced by at least one positive Lyapunov exponent, which drives the complex and seemingly random evolution of its dynamics.

To delve deeper into this phenomenon, it is imperative to examine the energy conditions within the fault, specifically the concept of residual energy, E^{res} (Noda et al., 2021). This energy parameter serves as a criterion for the initiation of ruptures, indicating that a positive E^{res} value signifies a greater propensity for a fault to generate ruptures, while negative values diminish this likelihood. Mathematically, this energy can be expressed as:

$$E^{res} = \Delta W_0 - G_C \qquad (4)$$

Where ΔW_0 represents the available energy, which can be correlated with the elastic energy stored within the system, and G_C denotes the fracture energy, characterizing the resistance to rupture propagation. It is also important to note that residual energy can be regarded as equivalent to radiated energy, which refers to the energy radiated to the medium which is transported by seismic waves (e.g., Rivera and Kanamori, 2005; Venegas-Aravena, 2024a). Despite this equivalence, the concept of residual energy as defined by Noda et al. (2021) more closely aligns with the processes occurring within the fault and its heterogeneities. Therefore, given the emphasis in this work on the generation of ruptures within faults, rather than the propagation of seismic energy through a medium, the concept of residual energy has been adopted. Equation 4 can be expressed in terms of D as follows (Venegas-Aravena and Cordaro, 2023a):

$$E^{res} \sim e^{-D/2k_V} - d_0^D \qquad (5)$$

Where d_0 is constant.

At this juncture, it is pertinent to elucidate the relationship among the fractal dimension, the Euclidean dimension, and earthquake magnitude through the residual energy as described by Equation 5. Within the framework of this study, D_E represents the dimension of the Euclidean space in which the spatial distribution of earthquake epicenters, and their ruptures, is embedded and subsequently analyzed to derive an empirical fractal dimension. Specifically, D_E defines a volume and is therefore equal to 3. In the context of multiscale thermodynamics, the fractal dimension D serves as a global parameter of the fault, quantifying its geometric irregularities and, consequently, its fracture energy (e.g., Xie, 1994). The connection to magnitude lies in the fact that lower values of D imply a reduced fracture energy, leading to a larger area of positive residual energy and, as a result, a higher probability of the occurrence of earthquakes with greater magnitude M_W (Venegas-Aravena and Cordaro, 2023a; Venegas-Aravena, 2024b). Consequently, the fractal dimension of the spatial distribution of earthquakes, ascertained within a Euclidean space, is indirectly related to magnitude through its association with the global parameter D of the fault.

Figure 1b illustrates the relationship between E^{res}, Λ, and earthquake magnitude (color-coded map). Higher E^{res} values correlate with lower Λ values, suggesting that regions more prone to rupture are also less sensitive to initial conditions. Given that these regions are associated with large earthquakes (red colors), it is proposed that areas with high residual energy have higher chances to host a major event as a response to stress perturbations.

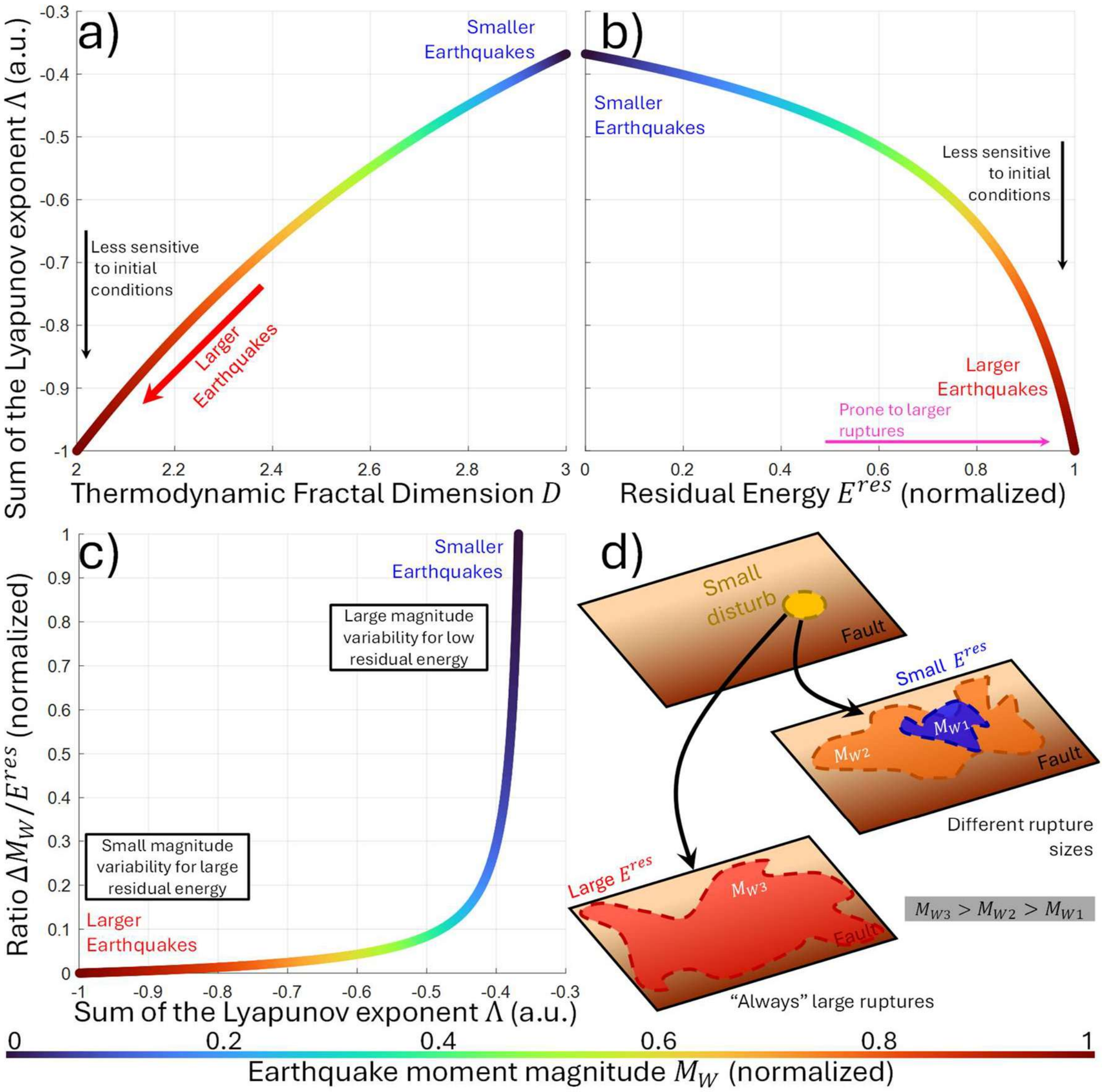

Figure 1: a) Equation 1 reveals a relationship between the sum of Lyapunov exponents Λ and the thermodynamic fractal dimension D. Systems with low sensitivity to initial conditions (highly negative Λ values) correspond to low D values. Colors indicate event magnitudes as calculated by Equation 2. Large events (red hues) are associated with low D and low Λ. b) Equation 5 relates Λ to residual energy (E^{res}). Higher E^{res} values correlate with a higher probability of large earthquakes, which in turn are linked to lower chaos and larger events. c) The plot of magnitude changes for a given E^{res} versus Λ shows that small (large) earthquakes exhibit greater (lesser) magnitude variability for low (high) E^{res}, as indicated by blue (red) hues. d) A schematic illustrates how perturbations can trigger small-to-medium or large earthquakes depending on E^{res}.

To visualize this, Figure 1c shows the variation in magnitude ΔM_W relative to residual energy. This figure illustrates the change in magnitude resulting from the addition of a small quantity of residual energy to a fault, in comparison to the same fault without this increase in energy. The findings indicate that introducing a small amount of residual energy can significantly elevate the expected magnitude (relative to the expected magnitude of the same fault without this additional energy) when the initial residual energy is low. Conversely, if the initial residual energy is already high, the addition of the same small quantity of energy produces a comparatively minor change in the expected magnitude (relative to the fault without this additional energy), suggesting a saturation effect on the magnitude. That is, the variation in M_W is small (large) when E^{res} is high (low). This supports the notion that there is a more restricted range of possible earthquakes when the residual energy in the fault is higher. Figure 1d schematically depicts this concept: a fault with a small E^{res} can generate earthquakes of magnitudes M_{W1} (blue area) and M_{W2} (orange area), whereas in the case of a large E^{res}, only earthquakes of magnitude M_{W3} (red area) can be generated, which is larger than both $M_{W1}$and M_{W2}.

3. Simulations

3.1 Heterogeneous Energy-Based method

The heterogeneous energy-based method (HE-Bm) posits that seismic rupture propagation is governed by the heterogeneous distribution of residual energy (Venegas-Aravena, 2023). This model suggests that rupture velocity and slip magnitude at each point on the fault are directly correlated with the residual energy. Consequently, regions with high residual energy are more prone to experiencing large slip uf and high rupture velocities vr, which can potentially lead to larger magnitude earthquakes. Thus, the relation between slip and residual energy is $u_f \propto E^{res}$.
According to the framework of HE-Bm, E^{res} can be linked to the distribution of interseismic coupling on a fault through the concept of available energy, while fault geometry is related to residual energy via fracture energy (Venegas-Aravena and Cordaro, 2023a). The two-dimensional fractal dimension D of natural fractures have been determined to be 2.3 (Huang et al., 1992). Consequently, due to the proportional relationship between the fracture energy G_C and the geometric variations of the fracture (e.g. Xie,1993), it is expected that G_C will also possess a fractal dimension of 2.3.

3.2 Ruptures in a single distribution of G_C

Figure 2a presents an exemplar G_C distribution exhibiting a fractal dimension of 2.3. The strike and depth are 700 *km* and 150 *km* respectively. The spacing is 349.5 *m* for the strike and 371.6 *m* for the depth. This distribution was constructed through the interpolation of random values, employing the methodology outlined by Chen and Yang (2016). Given the established inverse correlation between elevated G_C values and rupture size (Renou et al., 2022), attributed to the self-arresting nature of ruptures induced by energy depletion, the central region (depicted in blue in Figure 1a) was intentionally constrained to be three orders of magnitude less than the peripheral regions (rendered in red). Note that the arrest of ruptures due to geometric changes (fracture energy) has also been observed in real faults (e.g., Padilla et al., 2024). Consequently, ruptures are invariably localized within the lower G_C value domains (represented by the blue hues in Figure 1a). As coupling seems to be related to stress (Wallace et al., 2012), this implies that for a given level of stress on a fault, it is the fault roughness that primarily determines residual energy. Smoother faults exhibit lower fracture energy, resulting in reduced resistance to rupture initiation and, consequently, larger rupture events. Conversely, rougher faults present higher fracture energy, limiting residual energy and thus constraining rupture size.

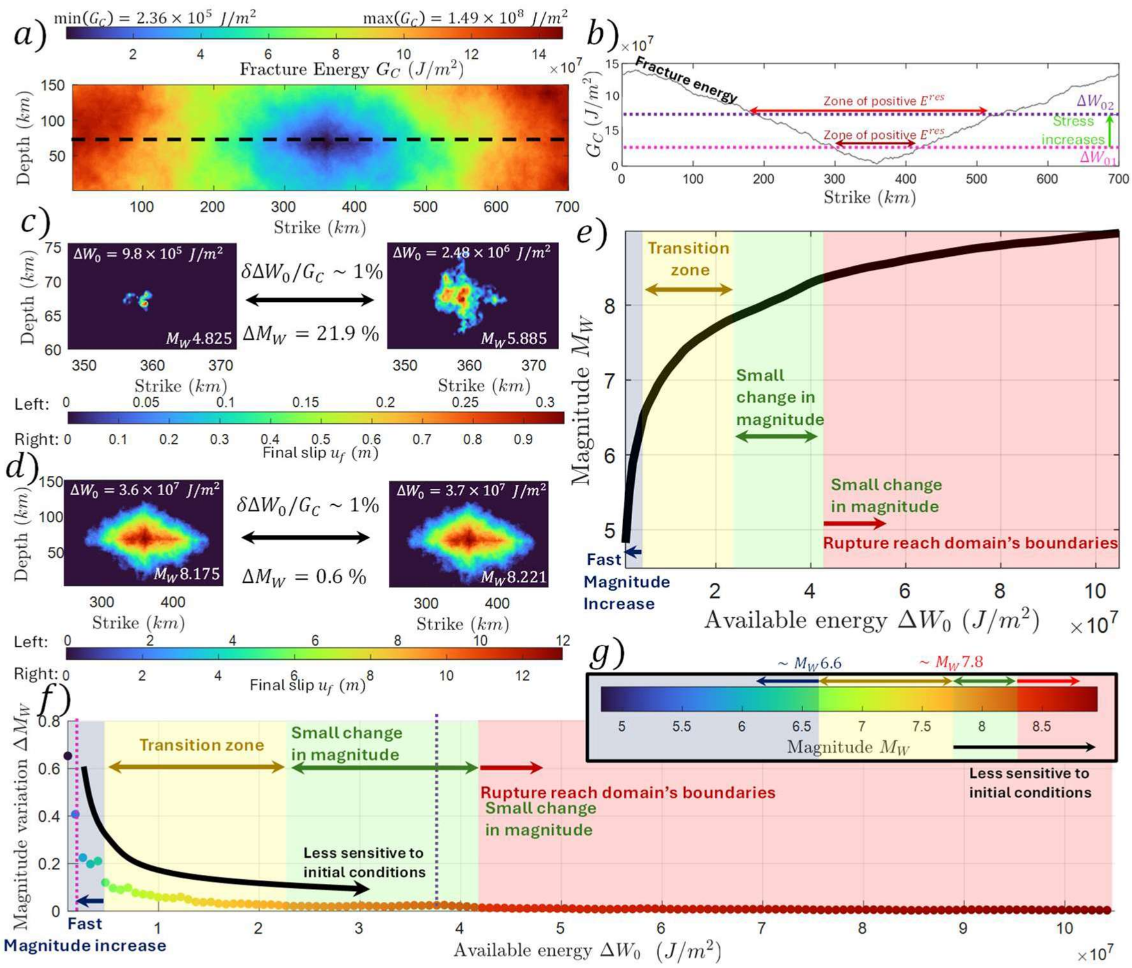

Figure 2: a) Example of fracture energy distribution with D=2.3, where the central region has low values, and the edges have high values. b) Fracture energy profile corresponding to the segmented black line in a). Magenta and purple segmented lines indicate two levels of available energy ΔW_0. Dark red and red double arrows indicate the size of positive E^{res}, potentially corresponding to rupture size. c) and d) shows the final slip distributions for conditions of low and high available energy, respectively. c) reveals larger changes in magnitude than those shown in d). e) The relationship between moment magnitude and ΔW_0 is shown. The gray region highlights a rapid increase in magnitude with increasing ΔW_0, while the yellow and green zones show a decreasing rate of increase. The red zone indicates ruptures that reach the fault edges. f) The variation of magnitude with available energy for different values of the parameter ΔW_0 is shown. Lower values of ΔW_0 result in larger changes in magnitude. g) The color map used in f), which indicates the sensitivity of earthquakes to initial conditions: blue for smaller earthquakes (M_W~6.6), yellow for a transition region (M_W between 6.6 and 7.8), and red for larger earthquakes (M_W > 7.8) that are less sensitive.

Equivalently, for a given fracture energy distribution, the residual one will be determined by the amount of available energy. This example is shown in Figure 2b. The black curve corresponds to a trace indicated by the black segmented line in Figure 2a. The minimum value is $G_C = 2.36 \times 10^5\, J/m^2$, which is found approximately at the midpoint of the fault (strike of 350 km). These values of G_C tend to increase towards the strike equal to zero km and equal to 700 km. The segmented magenta and purple lines represent two uniform distributions of available energy, ΔW_{01} and ΔW_{02}, respectively. In this case, ΔW_{02} is greater than ΔW_{01}, indicating that the first case

has a smaller amount of accumulated stress on the fault than the second case. The dark red double arrow would indicate the zone with positive residual energy given the level of ΔW_{01}, which is equivalent to a potential rupture zone. The red double arrow indicates the zone of positive residual energy given a higher accumulated stress (given by ΔW_{02}). This zone is wider than the region marked by the dark red arrow, highlighting the presence of larger ruptures promoted by high stress values throughout the crustal volume. The increase of available energy also translates into changes in earthquake magnitudes. For instance, Figure 2c illustrates two ruptures initiated with similar available energy ($10^6\ J/m^2$), representing a one percent variation relative to the maximum fracture energy. This excess in available energy defines the positive residual energy area (rupture area A), which can be related to the seismic moment M_0 through the empirical relationship $M_0 = \mu C_2 A^{3/2}$, where μ is the shear modulus with a value of $40\ GPa$ and C_2 is a dimensionless constant equal to 3.8×10^{-5} (Leonard, 2010). The small variation available energy results in a ~22% increase in earthquake magnitude (from M_W 4.8 to M_W 5.9). Conversely, when the available energy is higher ($\sim 3.6 \times 10^7\ J/m^2$), a similar 1% increase produces earthquakes with nearly identical magnitudes (M_W 8.2 representing a variation smaller than 1%, as shown in Figure 2d), suggesting that faults with higher residual energy yield similar magnitude earthquakes.
The dependence of magnitude on available energy is depicted in Figure 2e. This figure shows 140 simulations with different values of ΔW_0. A significant increase in magnitude is observed for low available energy values, indicated by the blue region. In this region, a small increase in ΔW_0 (less than 10 MJ/m^2) can elevate an earthquake from magnitudes less than 5 to approximately M_W6.6. The yellow and green regions show a less pronounced increase in magnitude compared to the blue region. The red range represents events where the ruptures approach the fault boundaries. The magnitude change in this region appears unaffected by fault boundary influences. Figure 2f quantifies these variations, revealing that the blue region experiences ΔM_W values close to 0.7, while the green and red regions show negligible changes. The color map in Figure 2g corroborates these findings, with earthquakes smaller than M_W6.6 predominantly falling within the blue region and larger earthquakes (M_W larger than M_W7.8) exhibiting minimal sensitivity to variations in ΔM_W.

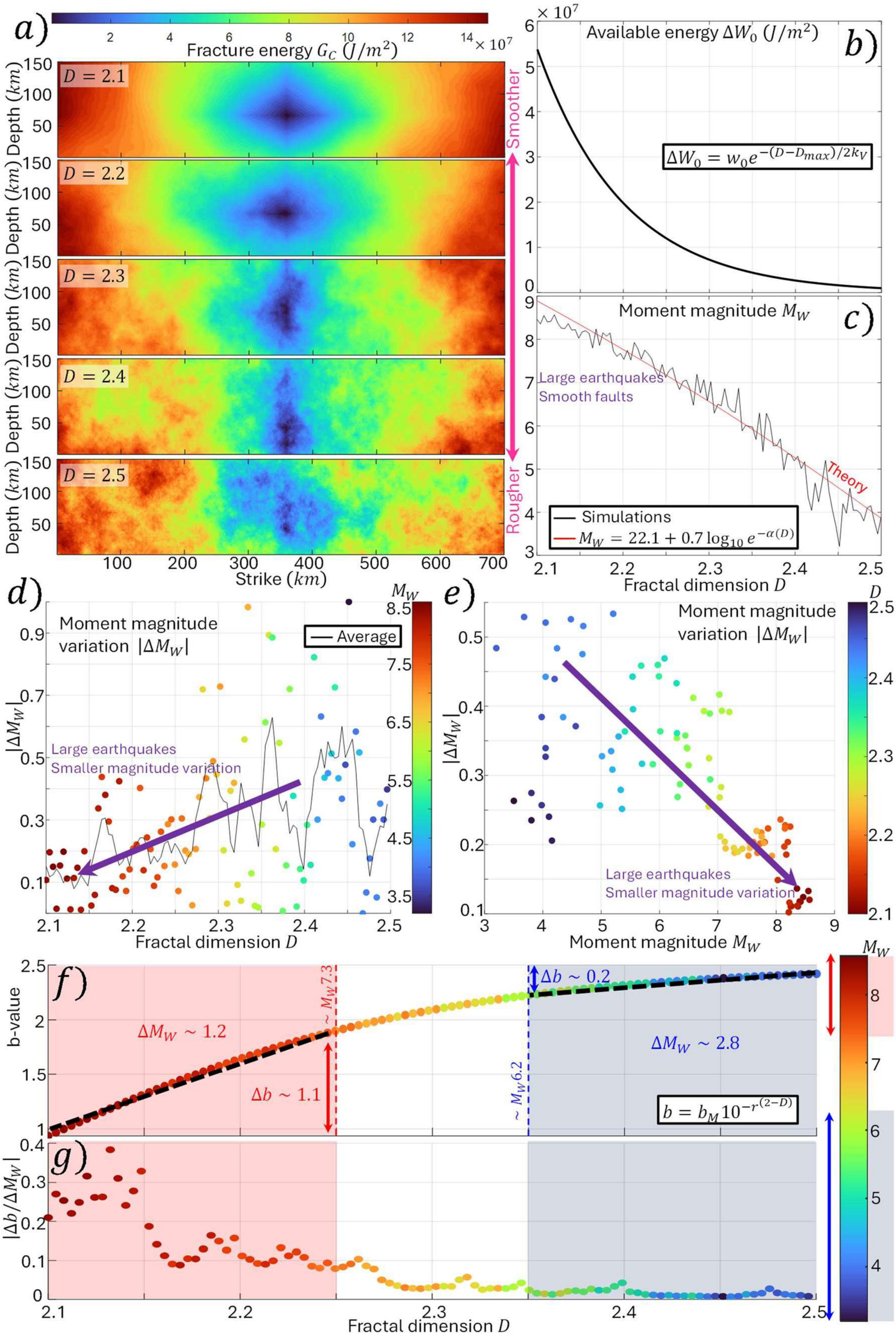

Figure 3: a) Fracture energy of a fault with different fractal dimension (D) values. The distribution becomes rougher as D increases. b) There is an exponential relationship between available energy and D. As D decreases, the available energy increases exponentially. c) The relationship between moment magnitude and D is shown. The simulated data is represented by the black curve, and the theoretical prediction by Venegas-Aravena and Cordaro (2023a) is

shown in red. d) and e) show the relationship between fractal dimension, moment magnitude, and magnitude variation. The purple arrow indicates that in both figures, low values of D are associated with high-magnitude earthquakes and small magnitude variations. That is, a small change in D, when ΔW_0 values are high, almost always generates similar large earthquakes. When ΔW_0 values are low, there is a greater variation in magnitude. f) Relationship between b-value and D. There is a greater decrease in the b-value when there are larger earthquakes. g) Variation of the b-value with changes in M_W. This variation is greater when D is lower.

3.3 Ruptures with different G_C

While natural faults can be characterized by a fractal dimension of approximately 2.2 (e.g. Kagan, 1991), variations in this value are possible. To investigate the impact of fractal dimensions on fracture energy, 100 simulations were conducted with fractal dimensions ranging from 2.1 to 2.5. Figure 3a illustrates examples of fault geometries with fractal dimensions of 2.1, 2.2, 2.3, 2.4, and 2.5, respectively, where maximum and minimum G_C values are consistent with Figure 2a. In these simulations, lower G_C values are maintained at the fault center and higher values at the edges. As shown in Figure 3a, the distribution of G_C becomes smoother as the fractal dimension approaches 2.1.

As suggested by Venegas-Aravena and Cordaro (2023a), ΔW_0 is inversely related to D, with the specific relationship being $\Delta W_0 \sim e^{-D/2k_V}$. Therefore, any change in G_C must be accompanied by a corresponding change in ΔW_0 . Figure 3b visualizes this relationship using parameter values of $w_0 = 9.84 \times 10^5\, J/m^2$, D_{max} = 2.5 and k_V = 0.05. These values yield a range of ΔW_0 consistent with the previous section, ensuring that ΔW_0 is sufficiently large to allow for rupture initiation but not so large as to be influenced by domain boundaries. The figure clearly demonstrates that lower values of D are associated with higher values of ΔW_0, indicating smoother spatial distributions of G_C. The magnitude of these ruptures also varies as a function of D. In Figure 3c, the red curve represents the theoretical relationship proposed by Venegas-Aravena and Cordaro (2023a), given by Equation 2. The observed magnitudes, depicted by the black curve, align well with the theoretical values. However, a higher variability in magnitude $|\Delta M_W|$ is observed for larger values of D (greater than 2.4), while lower values of D (less than 2.3) exhibit lower variability. This variation is visualized in Figure 3d, where the color map indicates magnitude. The black curve in Figure 3d represents a 5-point moving average of $|\Delta M_W|$, with the purple arrow highlighting the trend towards lower magnitude variability for smaller values of D. Figure 3e explicitly shows the relationship between M_W and its average variability (in this case a 10-point moving average), with the color map indicating D values. As the purple arrow suggests, there is an inverse correlation between M_W and its average variability, where earthquakes with magnitudes less than $M_W \sim 5$ can exhibit magnitude differences greater than $0.5 M_W$. In contrast, for earthquakes with magnitudes greater than $M_W \sim 8$, this variability decreases to approximately $0.1 M_W$.

3.4 Chaos and b-value

Both laboratory and field studies have shown a negative correlation between the b-value, which quantifies the frequency of earthquakes of different magnitudes in each region and increasing stress levels. This leads to a decrease in the b-value and may be associated with large magnitude earthquakes (Scholz, 2015; Dong et al., 2022). Studies have established a theoretical link between the b-value and fractal dimension, suggesting that lower b-values correspond to lower D values and vice versa (Aki, 1981; Venegas-Aravena and Cordaro, 2023b). Specifically, this relationship is expressed as b − value= $b_M 10^{-r^{(2-D)}}$, where b_M is 2.5 and r is a constant between 10^3 and 10^4 (in this study, r is set to medium value 5000 for simulations). This law is illustrated in Figure 3f. The color map indicates earthquake magnitudes, with blue transparency representing events from M_W 3.4 to M_W 6.2, corresponding to a D variation of 1.5. The magnitude variation within this zone is ΔM_W 2.8M_W, while the b-value decrease is Δb 0.2. Red transparency indicates the same variation of D, but with earthquakes ranging from M_W 7.3 to M_W 8.5, corresponding to a ΔM_W 1.2 and a Δb 1.1 decrease. The reduction in the rate of change of the b-value with respect to magnitude is clearly displayed in Figure 3g. The blue transparent area emphasizes a region where the absolute value of the b-value remains relatively constant, even as the magnitude of earthquakes fluctuates. It is important to note that a 5-point moving average was applied to the data. This suggests that the b-value is less sensitive to changes when the fault system predominantly generates smaller earthquakes. In contrast, the red transparent area reveals a more pronounced relationship between b-value and magnitude, with the b-value fluctuating more rapidly as the magnitude increases. These results imply that a more abrupt decrease in the b-value is associated with smaller changes in magnitude but, likely, also with changes in fault conditions leading to less chaotic behavior.

4. A reality check: comparison with seismicity in Southern California

We have already compared our theoretical predictions with the output of dynamic simulations of earthquakes; here we make a reality check with the statistical properties of seismic catalogs. Specifically, we validate the compatibility of the relationship b-value = $b_M 10^{-r^{(2-D)}}$, between the b-value of the Gutenberg-Richter law and the fractal dimension of faulting. Since it is not possible to directly investigate the fractal properties of faults, we calculate the fractal dimension of hypocenters (hereafter referred as D), which are expected to be distributed within a subset of the fracture network; hence, D is equal or lower than the value for the fault system. Nevertheless, even with different coefficients (b_M and r), the empirical law of the b-value follows the same trend because seismic events are supposed to occur throughout the whole investigated crustal volumes. Thus, we specify that we are not interested either in assessing the true fractal dimension of the networks of faults hosting seismicity (an accurate estimation is not feasible) nor the true fractal dimension of seismic events in their long-term behavior (which would require much longer catalogs than available nowadays and accurate declustering). Here, our goal is just the observational validation of the mathematical relationship b-value = $b_M 10^{-r^{(2-D)}}$. It requires a high-quality relocated seismic catalog produced by a roughly uniformly distributed network of seismic stations (i.e., uniform completeness magnitude). Both the background and triggered components are considered, otherwise the spatial variations of the b-value and fractal dimension vanish preventing any investigation of their relationship with the available catalogs.

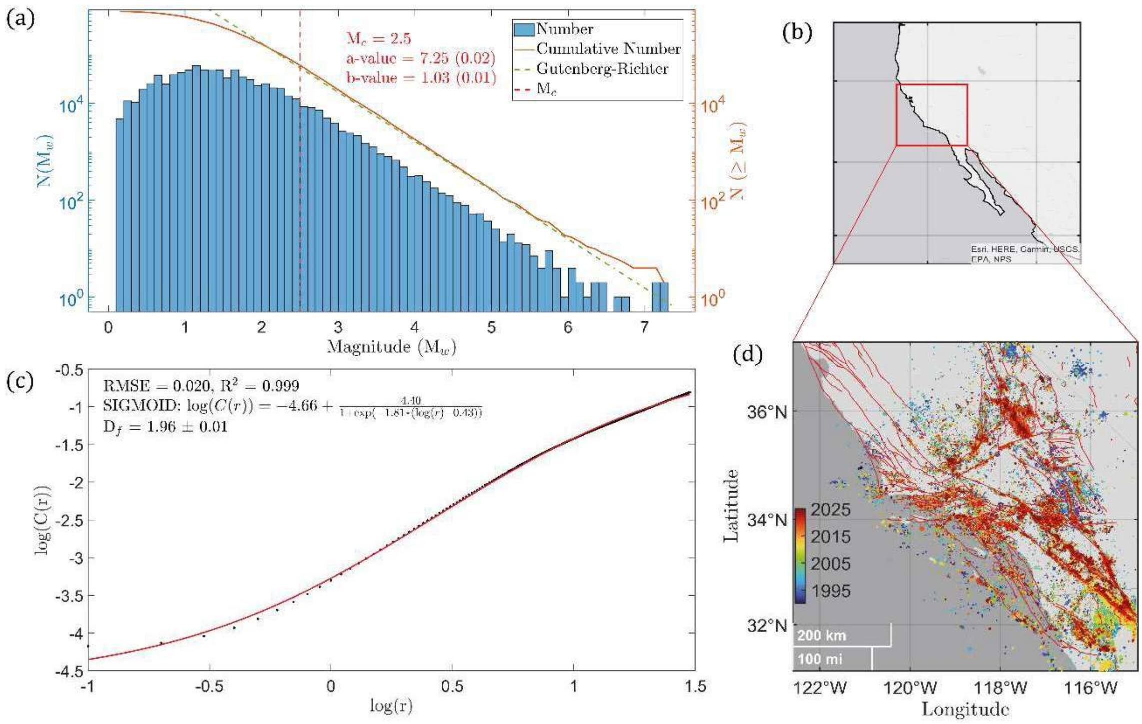

Figure 4: Seismicity in Southern California (SCEC Catalog, 1990-2025, latitude 31°-37° N, longitude 115°-122° W, depth lower than 30 km). (a) Frequency-magnitude distribution of seismicity, the completeness magnitude is highlighted by the red vertical dashed line. In (b) and (d) is the map with the spatial distribution of seismicity. (c) log-log representation of the correlation function vs the threshold radial distance (*km*). The plot shows a range of scales where the curve is well approximated by a line, i.e., hypocenters follow a fractal distribution in space.

We analyze the shallow crustal seismicity (depth lower than 30 *km*) in Southern California between 1/1/1990 and 20/1/2025 listed in the Waveform Relocated Earthquake Catalog for Southern California (Hauksson et al., 2012). A visual representation of seismicity considered in this study is given in Figure 4a, b and d. The catalog is divided into several squared regions. The number and selection procedure used to define the structure of the subsets do not significantly affect the final output provided that the fractal probability and the b-value are investigated only for regions with at least 500 events to get stable and reliable results. Only events above the completeness magnitude are considered, with Mc = 2.5. It is estimated according to the EMR method (Woessner and Wiemer, 2005). Since short-term aftershocks incompleteness (STAI) after the occurrence of major events is still present even if a great part of the catalog contains reliable information, the b-value is calculated using the b-positive algorithm (van der Elst, 2021) with the b-more positive correction (Lippiello and Petrillo, 2024) to avoid bias. The uncertainty of the b-value is found using bootstrapping over 100 simulations with acceptance probability equal to 0.5. The fractal dimension of the hypocenters is measured using the Grassberger and Procaccia algorithm (Grassberger and Procaccia, 1983). Here, we introduce a new method to remove possible sources of bias in its estimation due to the arbitrary selection of the lower and upper cut-offs for the linear region in the log-log plot. The curve of the correlation function $C(r)$ as a function of the threshold radius r is fitted using the sigmoid function $y = y_0 + k/\left(1 + e^{-\beta x}\right)$, where $y = log\left(C(r)\right)$ and $x = log(r)$, while k, β and y_0 are left as free parameters, so that the fractal dimension (i.e., the derivative of the sigmoid in its symmetry saddle point) is given by $D = k\beta/4$. The uncertainty is calculated by propagating the fit errors of k and β. The estimation of the fractal dimension of hypocenters for the whole catalog is in Figure 4c. The analysis performed over a wide range of possible grids (both uniformly spaced and nested according to the number of seismic events within them) shows that the b-value and the fractal dimension of hypocenters are positively correlated. To improve the reliability of the result, we only consider subregions in the grid containing at least 500 events above the completeness magnitude. Moreover, the curve b-value = $b_M 10^{-r^{(2-D)}}$ provides a good fit of the relationship between b and D, in agreement with our model. The output of our investigation is summarized in Figure 5. In this plot, we use a uniformly spaced 50x50 grid.

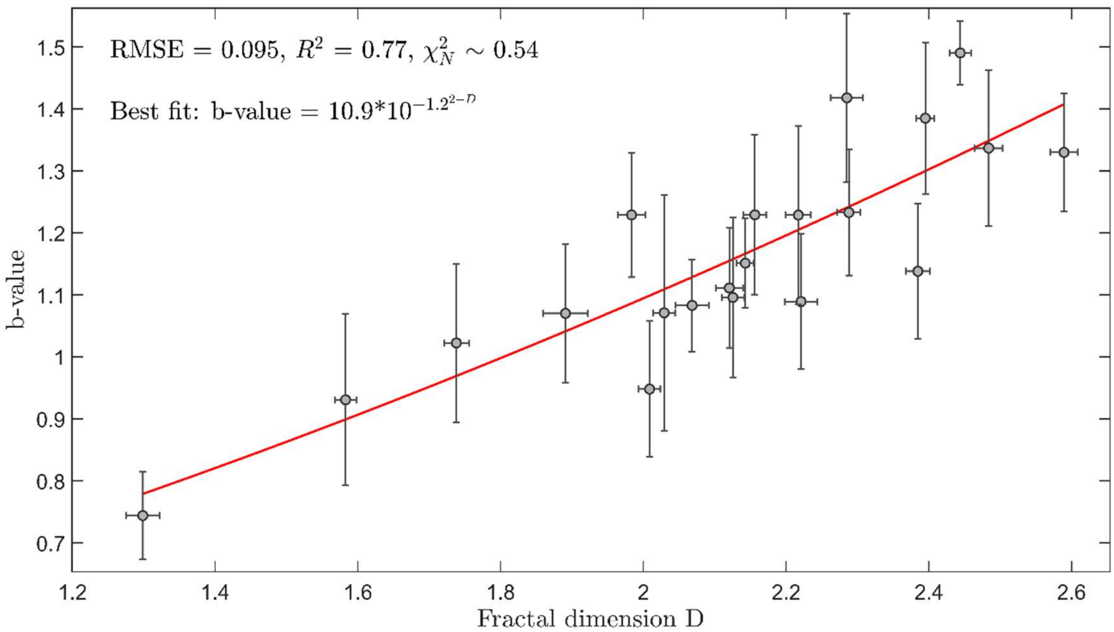

Figure 5: b-value vs fractal dimension of hypocenters (D) in Southern California. The b-value of the Gutenberg-Richter law is found to be positively correlated with the fractal dimension of hypocenters in Southern California (SCEC Catalog, Hauksson et al., 2012) coherently with previous literature on the topic. The plot represents shallow crustal seismicity from 1/1/1990 to 20/1/2025 (depth lower than 30 km) in between latitude 31°-37° N and longitude 115°-122° W and above the completeness magnitude Mc =2.5. Error bars represent 2σ uncertainty. The b-value is estimated using the b-more-positive approach, while the fractal dimension of hypocenters is found by applying the Grassberger & Procaccia algorithm (Grassberger and Procaccia, 1983). The red line is the output of the non-linear fit b-value = $b_M 10^{-r^{2-D}}$ whose trend is predicted in our model and derived in Venegas-Aravena and Cordaro, 2023b.

5. Discussions

5.1 The chaotic nature of earthquakes

The chaotic nature of earthquakes has been a subject of intense debate within the scientific community (e.g., Scholz, 1990; Huang and Turcotte, 1992; Goltz,1997; Vieira, 1999; Yılmaz et al., 2023). Traditionally, earthquakes have been considered highly unpredictable due to the complexity of the processes involved in fault rupture (e.g., Geller et al., 1997; Kagan 1997). However, a growing body of research, spanning conceptual frameworks, crustal stress, thermodynamics, artificial intelligence, and GNSS measurements, suggests that fault stability may be investigated, with a potential influence on precursory activity, may be achieved (e.g., Wyss, 1997; Crampin and Gao, 2010; Posadas et al., 2021; Bhatia et al., 2023; Bletery and Nocquet 2023; Devi et al., 2024), especially in the case of larger magnitude seismic event (Kaveh et al., 2024). In this study, we propose a novel perspective, grounded in multi-scale thermodynamics, suggesting that the chaotic nature of earthquakes may be modulated by the residual energy stored within faults volumes (Venegas-Aravena and Cordaro 2023a). These primarily theoretical developments suggest that as the thermodynamic fractal dimension (D) decreases, the residual energy in the system increases. This correlation is further supported by the framework presented in Section 2, where we demonstrate that a higher residual energy is linked to a lower sum of Lyapunov exponents Λ (red regions in Figure 1b), a hallmark of reduced chaoticity. Consequently, earthquakes with higher residual energy exhibit more deterministic behavior, as larger magnitude earthquakes (lower Λ) show a smaller change in magnitude per unit of residual energy (Figure 1c). This suggests that these earthquakes are less susceptible to the exponential growth of small perturbations, a hallmark of chaotic systems. Figure 1c presents the analysis of $\Delta M_W/E^{res}$, revealing an interesting parallelism with the findings of Kanamori and Rivera (2004). In their study, they defined the ratio E_R/M_0, which is associated with the radiative efficiency of an earthquake, representing the fraction of energy released during rupture that is radiated as seismic waves. Their results indicated that this ratio increases with earthquake size, suggesting proportionally greater radiated energy for larger magnitude events. In this regard, both the work of Kanamori and Rivera (2004) and the present study (Figure 1c) imply a scale-dependent behavior of earthquakes, where larger events exhibit different characteristics concerning the role of energy in the rupture and radiation processes compared to smaller ones. However, this study focuses on the change in sensitivity $\Sigma(\Sigma = \Delta M_W/E^{res})$ of a fault to a perturbation, whereas Kanamori and Rivera (2004) investigated the radiative efficiency ε ($\varepsilon = E_R/M_0$). Abstracting from the evident unit discrepancies, a potential compatibility between these findings can be inferred by positing an inverse relationship between seismic sensitivity and radiative efficiency, such that higher efficiency corresponds to a system less sensitive to initial perturbations. This hypothetical relationship can be expressed as:

$$\Sigma = \frac{\varepsilon_0}{\varepsilon} \qquad (6)$$

where ε_0 represents a quantity with the necessary physical units to establish the equality of the Equation. Despite this, more analyses need to be done in order to establish a deeper understanding between sensibility and efficiency. Section 3 evaluates the proposed hypothesis through numerical simulations of kinematic rupture using the HE-B method. This method has been effective in modeling self-arrested earthquakes that comply with observational constraints, such as the asperity criterion of Somerville et al. (1999), which links the zone of high slip (known as asperity) to the release of a large amount of seismic energy. The results obtained show a clear relationship between residual energy and the variability in earthquake magnitude. For instance, the blue zone in Figure 2e demonstrates that smaller earthquakes (M_W < 6.6) exhibit a high sensitivity to variations in available energy. This means that small increases in available energy can lead to significant increases in the magnitude of the resulting earthquake, as evidenced by the high values of ΔM_W and the regions of high magnitude variability (transparent blue zone in Figure 2f). Specifically, this indicates that a 1% increase in available energy can result in an increase in the magnitude of the resulting earthquake greater than 0.5M_W. In other words, when a fault has a specific low value of available energy (e.g., $0.1 \times 10^7\ J/m^2$) and has the potential to generate a magnitude M_W= 5 earthquake, it can produce a larger earthquake (M_W = 5.5) or a smaller one (M_W = 4.5) if the available energy is slightly increased or decreased. In contrast, larger earthquakes (M_W > 7.8) show a notable lack of sensitivity to changes in available energy (transparent green and red zones in Figure 2f). These zones indicate that a slight increase or decrease in available energy does not produce significant changes in the magnitude of the resulting earthquake. This low sensitivity demonstrates less chaotic behavior in the simulations, where larger earthquakes are more predictable and less influenced by small perturbations. This also suggests that these larger events may also be more predictable. In line with this, analyses conducted in seismic rupture simulations with a rate-and-state friction law on simple faults (Kaveh et al., 2024) are consistent with the results shown in this work, which is based on the distribution of residual energy. An interesting aspect of the work by Kaveh et al. (2024) comes from the threshold above which predictions can be made. They observed that it was possible to make forecasts of earthquakes with magnitudes greater than approximately $M_W 6.9$. In contrast, Figures 2e and 2f indicate that the variation in magnitude due to a change in available energy begins to be less than $0.1 M_W$ when earthquakes begin to have magnitudes greater than $M_W 6.6$ (transparent yellow zone). This suggests a similar threshold for predictability in both studies.

5.2 Insights from numerical simulations

To further assess the link between residual energy and chaos in seismic activity, numerical simulations were performed varying both the fracture energy distribution and the system's available energy, in accordance with Equation 5. The findings strongly corroborate the proposed hypothesis. Figure 3c, for example, demonstrates an excellent agreement between the simulated magnitude-parameter D relationship (black curve) and the theoretically predicted one (Equation 2, red curve), affirming the established theoretical connection between parameter D, residual energy, and simulated earthquake magnitudes. Additionally, Figures 3d and 3e corroborate the trend observed in the earthquakes of Figure 2: larger magnitude earthquakes exhibit a smaller variation in their magnitude, suggesting a lower degree of chaos in these events. To gain further credibility, a parameter more commonly used in seismology was needed. The b-value of the Gutenberg-Richter law has traditionally been used as an indicator of the relative occurrence rate of earthquakes of different magnitudes (Ito and Kaneko, 2023; Lacidogna et al., 2023). In this study, the relationship between the b-value and the degree of chaos in seismicity has been explored. In particular, Figure 3f shows that the b-value decreases more abruptly for earthquakes with magnitudes greater than $M_W 7.3$ (red zone), indicating a decrease in the occurrence rate of smaller earthquakes relative to larger ones. This behavior is associated with b-value changes on the order of 0.4 and suggests a less chaotic regime. On the other hand, for earthquakes with magnitudes less than $M_W 6.3$ (blue zone), the b-value exhibits less pronounced changes, indicating a greater variability in the occurrence rate of earthquakes of different magnitudes, and therefore, a more chaotic regime. This relationship between the b-value and seismic chaos is consistent with the interpretation of parameter D. Low D values (associated with higher-magnitude earthquakes) imply a more homogeneous distribution of residual energy over a larger area within faults, thereby reducing the probability and number of smaller events. Consequently, the ratio of large to small events, known as the b-value, is directly influenced by D. Therefore, it can be argued that a typical decrease in the b-value (e.g., Rivière et al., 2018; Sharon et al., 2022; Chan et al., 2024) is a measure of the chaos of a system, supporting the notion that low b-values are associated with imminent larger magnitude earthquakes, coherently with previous research (e.g., Gulia and Wiemer, 2019).

5.3 Implications of our new relationship between b-value and fractal dimension and comparison with previous observations

We also prove the theoretical relationship between the b-value and fractal dimension b-value $= b_M 10^{-r^{(2-D)}}$, also supporting the idea that large earthquakes tend to occur within networks with low fractal dimensions (i.e., along major faults). See Figure 5 for the output in the case of the SCEC Catalog in Southern California (1990-2025). Observations show good agreement with theory and, even though with relatively large uncertainties, are statistically robust. It is important to note that, according to Venegas-Aravena and Cordaro (2023b), the case where b-value and D are approximately proportional can be obtained, as shown in Figure 5. The direct effect of the fractal dimension of faulting on the maximum magnitude is more difficult to observe since large earthquakes are rare events and the available seismic catalogs only contain a few cases, if any, of events with the largest expected magnitude for each fault system, preventing a reliable analysis. Moreover, the results would be rescaled for the size of the largest seismogenic source in each fault network, which is tricky to estimate. Conversely, the b-value can nowadays be evaluated by robust and unbiased estimators. This is the reason why we choose to validate directly the relationship between b and D.

Our finding that an increase in b corresponds to an increase in D implies that regions with more frequent small earthquakes (higher b-value) also exhibit more spatially diffuse seismicity, whereas areas dominated by larger events (lower b-value) display tighter hypocenter clustering.
This relationship is not an unprecedented result, and it is consistent with previous studies that have linked stress heterogeneity to both earthquake size distribution and spatial patterns. Among them, Hirata (1989) demonstrated that fault network complexity influences seismicity clustering, suggesting that structural heterogeneity affects both the b-value and hypocenter distributions. Wiemer & Wyss (1997) further established that the spatial variations in b-value reflect differences in stress regimes, with lower b-values often found in high-stress zones where earthquakes may nucleate along preferential fault planes, leading to stronger clustering (lower D). Nanjo et al. (1998) provided direct evidence that higher b-values correlate with more uniformly distributed seismicity, supporting our observed positive b-value-Dcorrelation. While Tormann et al. (2014) argued that regions with homogeneous stress conditions (higher b-value) tend to produce less clustered seismicity, reinforcing the idea that stress state modulates both earthquake size and spatial organization. Finally, Zaccagnino and Doglioni (2022) showed that the fractal properties of faulting affect the earthquake rupture processes that, in turn, reveal themselves as different scaling exponents of the Gutenberg-Richter law. The new advance here is that our mathematical derivation allows us to relate fractal dimension and scaling properties of seismicity in the framework of dynamical systems and chaos theory. These findings, together with previous observational ones, underscore the importance of structural heterogeneity in governing both the frequency-magnitude distribution and the spatial complexity of seismicity as well as its chaotic properties, offering a unified framework for interpreting earthquake dynamical patterns.

5.4 Impact on the predictability of larger events

The analyses conducted in this study, which involves theoretical, numerical and observational data, contrast with the traditional view of earthquakes as highly chaotic systems. However, it is important to note that our proposal does not dismiss the role of chaos in seismic rupture dynamics. Rather, it suggests that the chaotic behavior may be modulated by the amount of residual energy stored in the fault. In other words, when residual energy is high, the probability of releasing all that energy in a sudden event increases due to the coalescence of different fault segments ready to nucleate or that can be dynamically activated during the coseismic phase. Conversely, when residual energy is low, the fault has more options for releasing that energy, which can lead to seismic ruptures of varying sizes. Our findings have significant implications for understanding the precursor seismicity, known as foreshocks (e.g., Lippiello et al., 2019; Bolton et al., 2023). If we can accurately quantify the residual energy in a fault, we may be able to estimate the probability of large magnitude earthquakes and assess their destructive potential.

This is particularly significant as recent research indicates that approximately half of large-magnitude seismic events may be preceded by precursor seismic activity, although the magnitude difference between foreshocks and mainshocks does not appear to be substantial (Wetzler et al., 2023). This suggests that states of higher energy preferentially evolve into large earthquakes. These states are associated with a greater amount of available energy, which is directly related to stress. Here, it is important to note that low values of D also indicate an accumulation of stress in localized areas (Venegas-Aravena et al., 2022). Geodetic measurements have confirmed this accumulation of localized stresses between earthquakes of magnitudes greater than $M_W 7$ (Kato and Ben-Zion, 2020). Additionally, foreshocks also appear to be related to the geometric conditions of faults (e.g., McLaskey and Kilgore, 2013; Cattania and Segall, 2021), which can be incorporated into the residual energy through fracture energy. Subsequently, residual energy can be used to estimate the physics of seismic precursors. For instance, it has been estimated that foreshocks may not be reliable when estimating the probability of subsequent mainshocks (Zaccagnino et al., 2024). Here, residual energy in the context of multi-scale thermodynamics can offer two explanations for the lack of clarity regarding foreshocks. Firstly, foreshock-type activity should arise as a stress perturbation, which, when considering a state of residual energy, can trigger events of different magnitudes but within a range of magnitudes close to that of the potential future mainshock. However, the magnitude of these foreshocks can be chaotic, limiting the ability to conduct statistical analyses and thus declaring them as foreshocks in real-time measurements. Secondly, the increase in residual energy implies a lower variability in the magnitude of earthquakes, in agreement with Lippiello et al., 2024. This suggests that when residual energy may be very high in a fault, stress perturbation has higher chances to trigger large earthquakes, limiting the existence of foreshocks. That is, the probability that larger earthquakes are affected or associated with foreshocks could decrease with an increase in the magnitude of the mainshock.
Here, one way to associate foreshocks with large magnitude mainshocks is if the rupture area of a foreshock reaches a zone of the fault with very high residual energy, which could be seen as a perturbation leading to a single large subsequent earthquake. This scenario could occur in the so-called "Mogi Doughnut" of subduction zones (Mogi, 1969), where the shallowest zones of the plate interface accumulate large amounts of energy while most seismicity occurs at deeper locations with lower accumulated energy (Schurr et al., 2020).

The findings and interpretation carried out in this work agree with recent modeling and observational findings in the literature (e.g., Nielsen, 2024) and could also influence future research, which may focus on developing methods to directly measure residual energy in natural faults, creating more sophisticated models that incorporate parameter D and allow for the simulation of the evolution of residual energy over time. Finally, exploring the implications of our results for seismic risk assessment and the design of earthquake-resistant structures.

5.5. Limitations of our model, challenges and future directions

While the dynamical framework presented in this study offers insights into earthquake predictability through chaos theory and thermodynamics, several limitations must be acknowledged. We list them hereafter:

1) Earthquakes emerge from spatially extended, heterogeneous systems where stress interactions, geometric complexities, and multiscale processes challenge deterministic models. Our framework suggests reduced chaoticity for large events essentially promoted by the control of the residual energy on the final size of the mainshock. This result is in conflict with self-organized criticality (SOC) (Bak and Tang, 1989), which argues that scale-

invariant earthquake statistics arise from stochastic processes under critical conditions rather than deterministic chaos. Conversely, our model is consistent with recent results suggesting that seismicity usually operates well below criticality and that large earthquakes show special features different from smaller ones which make them more predictable (Sornette, 2009; Sornette and Ouillon, 2012; Nandan et al., 2021).

2) The link between Lyapunov exponents (Λ), residual energy, and predictability relies on numerical simulations (e.g., HE-B method) with idealized friction laws and boundary conditions. Real faults also exhibit complicated friction laws, off-fault plasticity, and long-range interactions which are challenging to be fully incorporated.
3) The proposed predictability threshold (M 6.6-6.9) agrees with Kaveh et al. (2024), but universal applications remain uncertain. Regional variations in fault maturity, stress accumulation, and tectonic setting may modulate chaotic behavior, limiting generalizations.
4) Empirical validation relies on seismic catalogs with incomplete records of large events (due to their rarity) and potential biases in b-value and fractal dimension estimation. While Southern California catalog supports our $b - D$ positive correlation, global applicability requires testing across diverse tectonic regimes.

6. Conclusions

The results obtained in this study suggest that the chaotic behavior of earthquakes can be modulated by the amount of residual energy stored in the fault. Our findings indicate that larger earthquakes, associated with higher residual energy, exhibit less chaotic behavior. This new perspective challenges traditional conceptions about the nature of earthquakes and opens new avenues of research in seismology. While these results are promising, further research is required to confirm and deepen our findings. Specifically, methods need to be developed to directly measure residual energy in natural faults and to construct more sophisticated models that incorporate the parameter D. Indeed, our approach advances a deterministic perspective on large earthquakes, even though limitations highlight the need for more advanced models combining chaos theory and statistical seismology. Future works should address multiscale fault physics and observational uncertainties to refine our predictive framework. This research will allow us to advance our understanding of the mechanisms governing the generation and propagation of earthquakes and pave the way for a better assessment and mitigation of seismic risk.

Data and Code Availability

Data and codes are available upon reasonable requests to both the authors.

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Competing interests

The authors declare no competing interests.

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