

Precursory slow slip and foreshocks on rough faults

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Key Points:

- Rough fault simulations exhibit simultaneous foreshocks and creep caused by heterogeneity in normal stress induced by roughness
- Stress transfer between foreshocks and creep produces a positive feedback and $1/t$ acceleration prior to the mainshock
- The precursory phase is characterized by migratory seismicity and creep over an extended region

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Abstract

11 **Abstract**
12 Foreshocks are not uncommon prior to large earthquakes, but their physical mechanism
13 controversial. Two interpretations have been put forward: 1. foreshocks are driven by
14 aseismic nucleation; 2. foreshocks are cascades, with each event triggered by earlier ones.
15 Here we study seismic cycles on faults with fractal roughness at wavelengths exceeding
16 the nucleation length. We perform 2-D quasi-dynamic simulations of frictionally uniform
17 rate-state faults. Roughness leads to a range of slip behavior between system-size rup-
18 tures, including widespread creep, localized slow slip, and microseismicity. These processes
19 are explained by spatial variations in normal stress (σ) caused by roughness: regions with
20 low σ tend to creep, while high σ regions remain locked until they break seismically. Fore-
21 shocks and mainshocks both initiate from the rupture of locked asperities, but mainshocks
22 preferentially start on stronger asperities. The preseismic phase is characterized by a feed-
23 back between creep and foreshocks: episodic seismic bursts break groups of nearby as-
24 perities, causing creep to accelerate, which in turns loads other asperities leading to fur-
25 ther foreshocks. A simple analytical treatment of this mutual stress transfer, confirmed
26 by simulations, predicts slip velocities and seismicity rates increase as $1/t$, where t is the
27 time to the mainshock. The model reproduces the observed migration of foreshocks to-
28 wards the mainshock hypocenter, foreshock locations consistent with static stress changes,
29 and the $1/t$ acceleration in stacked catalogs. Instead of interpreting foreshocks as either
30 driven by coseismic stress changes or by creep, we propose that earthquake nucleation
31 on rough faults is driven by the feedback between the two.

Plain Language Summary

The occurrence of premonitory seismicity leading up to large earthquakes has been a central problem in seismology for several decades. In spite of constantly improving observational networks and data analysis tools, we are still grappling with the fundamental question: what causes foreshocks? Do they represent a chain of isolated events, or are they driven by slow slip over a large fault area, gradually accelerating before the main-shock? In this study, we tackle this question with numerical simulations of slip on a fault with a realistic (fractal) geometry. This geometrical complexity causes spatial variations in stress: compression or extension occur as irregularities on opposite sides of the fault are pressed closer together or further apart. This spatial heterogeneity modulates slip stability across the fault, causing simultaneous occurrence of slow slip and foreshocks. The two processes are linked by a positive feedback, since each increases stresses at the location of the other; under certain conditions, this can culminate in a large earthquake. Our model reproduces a number of observed foreshock characteristics, and offers new insights on the physical mechanism driving them.

1 Introduction

Foreshocks have been observed before many moderate and large earthquakes (Abercrombie & Mori, 1996; Jones & Molnar, 1976; Trugman & Ross, 2019; Ende & Ampuero, 2020), and even though modern seismic networks and analysis techniques have imaged foreshocks sequences in unprecedented detail (Ellsworth & Bulut, 2018; Tape et al., 2018), the physical mechanisms driving them remains debated (Gomberg, 2018; Mignan, 2014). One interpretation is that foreshocks represent failures of seismic sources (asperities) driven by an otherwise aseismic nucleation process (Tape et al., 2018; Bouchon et al., 2013, 2011; Schurr et al., 2014; N. Kato, 2014; Suga et al., 2014; McGuire et al., 2005; Abercrombie & Mori, 1996). Aseismic acceleration prior to instability is predicted by theory (Ruina, 1983; Dieterich & Linker, 1992; Rubin & Ampuero, 2005; Ampuero & Rubin, 2008) and has been observed in laboratory experiments (Dieterich & Kilgore, 1996; McLaskey & Lockner, 2014; McLaskey, 2019) and numerical simulations (e.g. Dieterich & Linker, 1992; Lapusta et al., 2000; Lapusta, 2003). On the other hand, foreshocks have been interpreted as a cascade of events triggered by one another, not mediated by an aseismic process (Helmstetter & Sornette, 2003; Hardebeck et al., 2008). Recent studies have shown that the relative locations of foreshocks are in fact consistent with static stress triggering (Ellsworth &

64 Bulut, 2018; Yoon et al., 2019), and the lack of detectable aseismic slip preceding most
65 moderate to large earthquakes supports the view of a triggering cascade.

66 The occurrence of foreshocks implies fault heterogeneity: if they are driven aseis-
67 mically, heterogeneity leads to simultaneous occurrence of seismic and slow slip; in cas-
68 cade model, it is required to explain why foreshocks remain small, while the mainshock
69 evolves into a large rupture. Previous modeling studies of foreshocks have considered var-
70 ious sources of heterogeneity: velocity weakening asperities in a velocity strengthening
71 fault (Dublanche, 2018; Yabe & Ide, 2018); spatial variations in nucleation length on
72 a velocity weakening fault caused by heterogeneous state evolution distance (Noda et al.,
73 2013) or effective normal stress (Schaal & Lapusta, 2019). In these studies, aseismic slip
74 can take place around the asperity due to either velocity strengthening behavior or fric-
75 tional properties that lead to large nucleation dimensions; however, the presence of as-
76 perities with a small nucleation dimension can nevertheless lead to a cascade sequence (Noda
77 et al., 2013).

78 Perhaps the most ubiquitous and best characterized source of heterogeneity is ge-
79 ometrical roughness: faults are fractal surfaces (Power et al., 1987, 1988; Power & Tullis,
80 1991; Sagy et al., 2007; Candela et al., 2009, 2012; Brodsky et al., 2016). Numerical and
81 theoretical studies have shown that fault roughness has a first order effect on rupture
82 nucleation (Tal et al., 2018), propagation and arrest (Fang & Dunham, 2013; Dunham
83 et al., 2011; Heimisson, 2020).

84 Here we focus on the effect of long wavelength roughness (exceeding the nucleat-
85 ing length) on the nucleation phase and precursory seismicity leading up to a mainshock.
86 We perform quasi-dynamic simulations of rough but otherwise uniform velocity-weakening
87 faults embedded in a linear elastic medium. Numerical simulations show that a rich slip
88 behavior ranging from slow slip to seismic ruptures arises as a consequence of normal
89 stress heterogeneity induced by fault roughness, which causes spatial variations in strength
90 and fault stability. Early in the cycle, low normal stress regions start to creep stably while
91 high normal stress regions (from now on referred to as “asperities”) remain locked. The
92 nucleation phase is characterized by an interplay between accelerating creep and episodic
93 foreshocks: creep loads asperities, until they fail seismically; foreshocks increase stress
94 on nearby asperities and creeping areas, causing the latter to accelerate in turn trigger-
95 ing subsequent foreshocks; asperities don’t fully relock after failure, gradually unpinning

106 the fault and increasing the creeping area and velocities. We introduce a simple analytical
 107 model based on these interactions, which predicts acceleration in seismicity rate and
 108 creep as $1/t$, where t is the time to the mainshock. Simulated sequences reproduce a num-
 109 ber of observations, such as the relative location of foreshocks, their migration towards
 100 the mainshock hypocenter and the power-law acceleration of foreshocks in a stacked cat-
 101 alog.

102 2 Numerical model

103 We run 2-D plane strain simulations with the quasi-dynamic boundary element code
 104 *FDRA* (?). The following equation of motion governs fault slip:

$$\tau_{el}(\mathbf{x}) - \tau_f(\mathbf{x}) = \frac{\mu}{2c_s}v(\mathbf{x}), \quad (1)$$

105 where μ is the shear modulus, τ_f the frictional resistance, and τ_{el} the shear stress due
 106 to remote loading and stress interactions between elements. The stress from each ele-
 107 ment is computed from dislocation solutions (e.g., Segall, 2010), accounting for variable
 108 element orientation. The right hand side is the radiation damping term, which repre-
 109 sents stress change due to radiation of plane S-waves (Rice, 1993), with c_s the shear wave
 110 speed. Earthquakes are defined as times when the slip velocity anywhere on the fault
 111 exceeds the threshold velocity $V_{dyn} = 2a\sigma c_s/\mu$ (Rubin & Ampuero, 2005), here ~ 4
 112 cm/s.

113 Frictional resistance evolves according to rate-state friction (Dieterich, 1978):

$$\tau_f(v, \theta) = \sigma \left[f_0 + a \log \frac{v}{v^*} + b \log \frac{\theta v^*}{d_c} \right], \quad (2)$$

114 where, a , b and are constitutive parameters; d_c is the state evolution distance; σ is the
 115 effective normal stress; v_0 a reference slip velocity; f_0 the steady-state friction coefficient
 116 at $v = v^*$, and θ is a state-variable. Model parameters are listed in table 1. We em-
 117 ploy the ageing law (Ruina, 1983) for state evolution:

$$\frac{d\theta}{dt} = 1 - \frac{\theta v}{d_c}, \quad (3)$$

118 such that steady-state friction at sliding velocity v is

$$f_{ss}(v) = f_0 + (a - b) \log \frac{v}{v_0}. \quad (4)$$

119 We apply remote loading such that that the stress rate tensor is pure shear:

$$\dot{\sigma}_1 - \dot{\sigma}_3 \equiv \dot{\sigma}_D \quad (5)$$

$$\dot{\sigma}_1 + \dot{\sigma}_3 = 0, \quad (6)$$

120 where $\sigma_{1,3}$ are the principal stresses and σ_D the differential stress. Resolving these on
 121 to the fault yields shear and normal stressing rates:

$$\dot{\tau} = \frac{\dot{\sigma}_D}{2} \sin(2\Psi + 2\theta) \quad (7)$$

$$\dot{\sigma} = \frac{\dot{\sigma}_D}{2} \cos(2\Psi + 2\theta), \quad (8)$$

122 where Ψ is the average fault angle with respect to σ_1 and $\theta(x)$ the local slope. In gen-
 123 eral, both shear and normal stress vary in time; here we take $\Psi = 45^\circ$, so that the spa-
 124 tially average normal stress is constant and equal to a uniform value $\sigma_0 = 10\text{MPa}$. In
 125 addition to the remote loading, slip on a rough fault causes normal stress changes, and
 126 normal stresses can locally become negative and induce opening if a purely elastic re-
 127 sponse is assumed. In contrast, tensile stresses are reduced or entirely inhibited in an
 128 elasto-viscoplastic medium with Drucker-Prager rheology. We approximate this behav-
 129 ior by setting a minimum value σ_{min} for normal stress, $\sigma_{min} = 1\text{ kPa} \ll \sigma_0$.

130 The fault profile is fractal, characterized by power spectral density

$$P_h = C_h |k|^{-\beta} \quad (9)$$

132 with $\beta = 2H + 1$, where H is the Hurst exponent. For natural faults this is typically
 133 between 0.4–0.8 (Renard & Candela, 2017); here we set $H = 0.7$. For computational
 134 reasons, we only include wavelengths greater than 100m, close to the nominal nucleation
 135 length defined below, unless otherwise specified.

136 2.1 Model resolution

137 To correctly describe rupture behavior, both the nucleation length and the cohe-
 138 sive zone Λ_0 need to be well resolved (e.g. Lapusta et al., 2000; Perfettini & Ampuero,
 139 2008). Erickson et al. (2020) found that a suite of planar fault models, including *FDRA*,
 140 produced well resolved simulations with $\Lambda_0/\Delta x \geq 3$, with $\Lambda_0 = \mu' d_c / b\sigma$ (Rubin, 2008),
 141 in agreement with previous studies (Day et al., 2005). A resolution of $\Lambda_0/\Delta x \approx 1.7$ pro-
 142 duced similar temporal patterns, but slight differences in the frequency-magnitude dis-
 143 tribution of simulated events. On a rough fault, normal stresses change with time and
 144 can locally be higher than the average, requiring a higher resolution. Moreover, we found
 145 that rough fault simulations are less forgiving than may expected from the results above.

Table 1. Model parameters

| Parameter | Value |
|----------------|---------------------------|
| a | 0.015 |
| b | 0.02 |
| d_c | 10^{-4}m |
| σ_0 | 10 MPa |
| $\dot{\tau}_0$ | 0.004 Pa s^{-1} |
| μ | 30GPa |
| ν | 0.25 |
| L_{min} | 100 m |
| L | 5.2 km |
| C_h | 0.013 |
| H | 0.7 |

146 For instance, a simulation resolving the nominal cohesive zone size with 4 grid points and
147 a small fraction (10 – 15%) of the fault with $\Lambda_0/\Delta x \approx 1 - 2$ produced abundant mi-
148 croseismicity and no full ruptures, while doubling the number of grid points generated
149 full ruptures. Since earthquakes tend to arrest where σ is high and the cohesive zone is
150 small, a few under-resolved regions can determine the event size statistics. Here we use
151 a nominal $\Lambda_0/\Delta x \approx 8$, and for the foreshock sequence discussed through most of the
152 paper $\Lambda_0/\Delta x > 2$ everywhere. We tested a few individual foreshocks and verified that
153 their rupture length does not change when doubling the resolution.

154 **3 Summary of simulation results**

155 The first order effect of fault roughness during the interseismic phase is a decrease
156 in fault locking: as seen in Fig. 1(a), and previously noted by Tal et al. (2018), the max-
157 imum slip velocity on the fault is several orders of magnitude larger for a rough fault than
158 for its planar counterpart. Fig. 1(b) shows that this is due to patches of higher veloc-
159 ity between locked patches. For simplicity, in the remainder of the paper we refer to these
160 slowly slipping regions as “creeping”, even though their slip velocity (estimated in sec-
161 tion 4) can be several orders of magnitude lower than typically measurable fault creep.

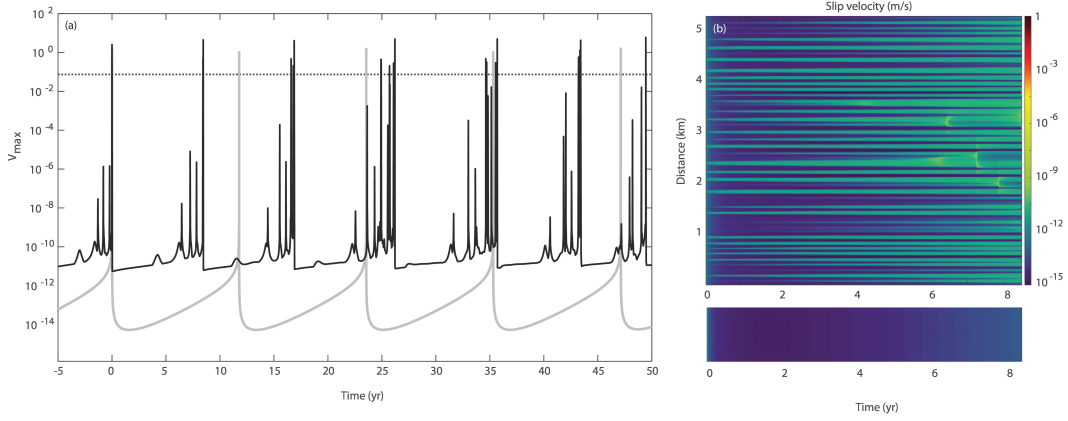


Figure 1. (a) Maximum slip velocity across multiple cycles on a rough (black) and comparable planar (grey) fault. The dotted line is the threshold velocity used to define earthquakes. (b) Slip velocity across the entire fault during one cycle showing alternating creeping and locked patches. The lower panel shows the slip velocity on a planar fault during the same time period (only a small region is shown, since velocity is effectively uniform).

162 During most of the interseismic phase the average slip velocity slowly increases, as
 163 creeping patches widen; this process is entirely aseismic, even though brief slow slip episodes
 164 with velocities up to about $10^{-6} - 10^{-5}$ m/s occur as creep fronts coalesce and break
 165 asperities (Fig. 1, 6–8 years into the cycle). Only in the final part of the cycle do as-
 166 perities rupture in seismic events while creep rates increase (Fig. 2). During the accel-
 167 eration leading up to the mainshock slip velocity on the fault does not increase gradu-
 168 ally but in abrupt steps, associated with bursts of microseismicity. This pattern repeats
 169 at increasingly short temporal scales as the background slip velocity increases.

170 Foreshocks only occur once sufficient slip has accrued on the fault, and the first few
 171 sequences consist of single system-size ruptures. This is due to an increase in the am-
 172 plitude of normal stress perturbations with total slip, quantified in Appendix A: micro-
 173 seismicity starts when the root-mean-square normal stress perturbation $\Delta\sigma_{rms}$ is of the
 174 order of the background normal stress σ_0 . In the rest of the paper we will focus on one
 175 of the first sequences with foreshocks ($\Delta\sigma_{rms}/\sigma_0 = 1.1$), since later sequences, with more
 176 net slip, may not be well resolved (as discussed in 2). Other sequences are qualitatively
 177 similar (Supplementary Figure 1).

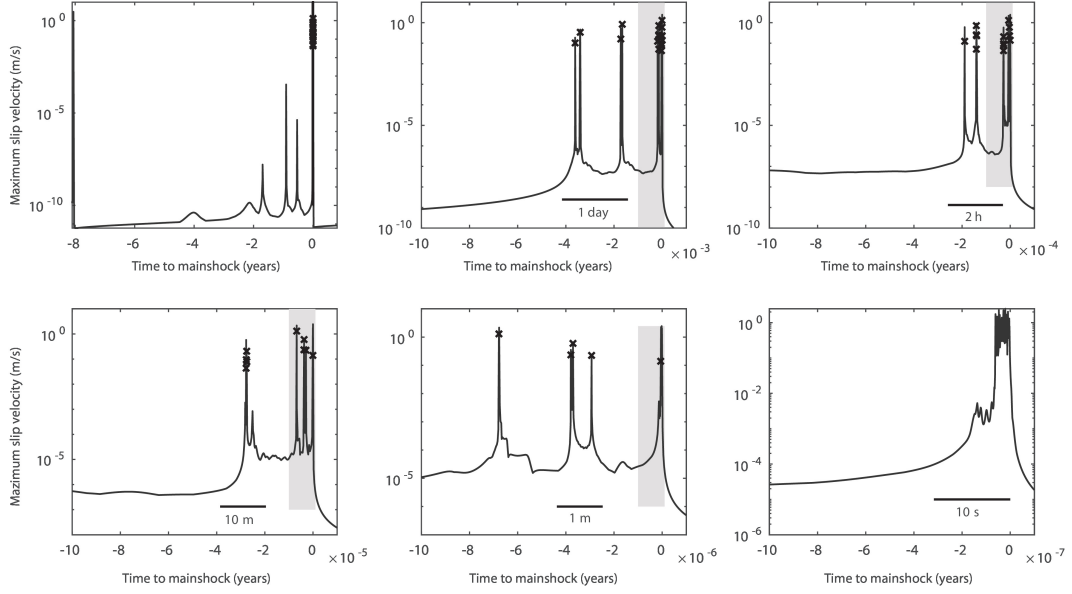


Figure 2. Average slip velocity on the fault leading up to the mainshock, showing a similar pattern across multiple temporal scales. Earthquakes are marked with crosses, and each grey box indicates the extent of the next panel.

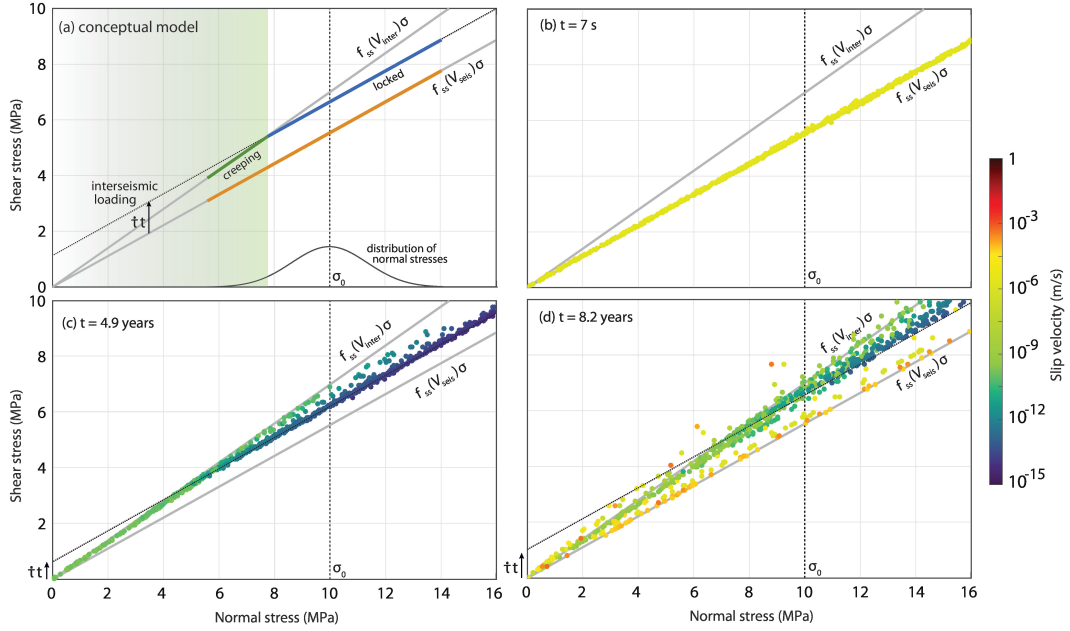


Figure 3. Conceptual model and simulation results for the evolution of stress on the fault. (a) Expected state of stress after the entire fault has ruptured (orange) and later in the cycle: points at low σ reach the end of their cycle first and start creeping (green), while asperities are still locked (blue). σ_0 is the unperturbed normal stress, and the grey lines indicate the static and dynamic strength. (a-c) Shear and normal stresses from the simulation, right after an earthquake (b); during the aseismic phase of the cycle (c); towards the end of the nucleation phase (d).

4 Relationship between fault roughness and interseismic locking

Previous studies have shown that slip on a rough surface leads to perturbations in normal stress (Chester & Chester, 2000; Sagy & Lyakhovsky, 2019; Dunham et al., 2011). In Appendix A we summarize these findings and derive a simple expression for normal stress perturbations as a function of cumulative slip and fault topography. Normal stress perturbations on a fractal fault with uniform slip S have a Gaussian distribution; for a fractal fault with Hurst exponent H , its standard deviation is given by

$$\Delta\sigma_{rms} = \frac{\mu'\alpha S}{2} \sqrt{\frac{H}{2-H}} (2\pi)^H k_{max}^{2-H}, \quad (10)$$

where $\mu' = \mu/(1-\nu)$ and ν is Poisson's ratio and α the roughness (section A1). These variations in normal stress are responsible for the occurrence of alternating creeping and locked regions, as shown in Fig. 4: creep takes place where roughness decreases the normal stress, while regions with increased σ remain locked.

A simple model illustrating the heterogeneous response of a rough fault loaded at uniform rate is shown in Fig. 3. After a system-wide rupture, friction at all points on the fault is at steady-state $f_{co} = f_{ss}(V_{co})$, given by eq. 4 (this applies if fault healing occurs on a much longer timescale than the earthquake itself, as in the case of the ageing rate-state friction). As the fault is loaded at a uniform rate, points with low σ reach static strength sooner than those at high σ (Fig. 3(a)). A creeping patch may then become unstable if it exceeds a critical elasto-frictional length, or creep at constant stress otherwise. The steady-state velocity is $V_{cr} = \dot{\tau}/\kappa$, where κ is the stiffness, which for a region of size L is of the order of L/μ' so that $V_{cr} \approx L\dot{\tau}/\mu'$. The critical length for instability (nucleation length) was first estimated from a spring-slider linear stability analysis (Ruina, 1983); later, Rubin and Ampuero (2005) used energy balance arguments to derive expressions for ageing rate-state faults. In general, this critical length has the form

$$L_c = f(a, b) \frac{\mu' d_c}{\sigma} \quad (11)$$

where $f(a, b)$ is a function of rate-state parameters a, b ; for rate-state friction with the ageing law and $a/b = 0.75$ (as in our case), $f(a, b) = b/[\pi(b-a)^2]$ and the nucleation length is denoted by L_∞ (Rubin & Ampuero, 2005). Expressions for nucleation length derived for a homogeneous fault cannot directly be applied to an heterogeneous one. However, linear fracture mechanics can be used to derive alternative expressions for these cases, as done by Tal et al. (2018) for rough faults with small scale (sub- L_c) roughness, and Dublanquet (2018) for heterogeneous friction. With these caveats in mind, here we appeal to the con-

cept of an heterogeneous nucleation length as an intuitive way to relate spatial variations in normal stress to slip behavior.

Due to the inverse proportionality between L_c and σ , the first patches to reach static strength are the most stable ones (large L_c), thus favouring stable creep. During this phase we expect the average slip velocity on the fault to increase for several reasons: 1) the area of creeping patches increases as more points reach static strength, since the time to failure is given by $T_f \simeq \Delta\tau/\dot{\tau}$, where $\Delta\tau = [f_{ss}(V_{dyn}) - f_{ss}(V_{cr})]\sigma$ is the difference between the dynamic and “static” strength (Fig. 3); 2) Creep on low σ patches redistribute stresses onto locked patches, contributing to the acceleration by causing points to be closer to failure than predicted from tectonic loading in Fig. 3(c); 3) The steady state slip velocity on each patch increases as it widens, since $V_{cr} \sim L_{cr}$. This leads to the interseismic acceleration seen in Fig. 1. As creep occurring in low σ regions penetrate into asperities, it can cause them to fail in localized slow slip or earthquakes (velocity peaks in Fig. 1). Microseismicity occurs late in the cycle since the most locked patches, where the nucleation length is small enough to allow seismic rupture, are the last to reach failure.

5 Seismicity on strong patches

Foreshocks occur in subclusters at multiple temporal scales: Figs. 2 and 4 show 3 events occurring a few days before the mainshock, followed by quiescence and a second cluster about a day later; more clusters occur a few hours and a few minutes before the mainshock. Each burst represents the rupture of a group of nearby asperities (Fig 4 and Supplementary Figure), and the relative location of each event is consistent with static stress transfer from previous ones. This gives rise to migration (e.g. events 1-8, 9-14), which can also reverse due to repeated rupture of the same asperity (e.g. events no. 1,13,14 and 2,12,14 among others). A seismic cluster is bounded by stronger or wider asperities, which typically fail in later bursts: the increase in shear stress imparted by earthquakes on surrounding low σ patches leads to a sudden acceleration, which in turn loads nearby asperities until they fail (see for example accelerated creep at the edge of previous foreshocks leading up to events 6, 11 and 14 in Fig. 4). Similarly, the mainshock initiates at the edge of the previous events and creep. The asperity on which it nucleates has a higher normal stress than nearby asperities and previous foreshocks.

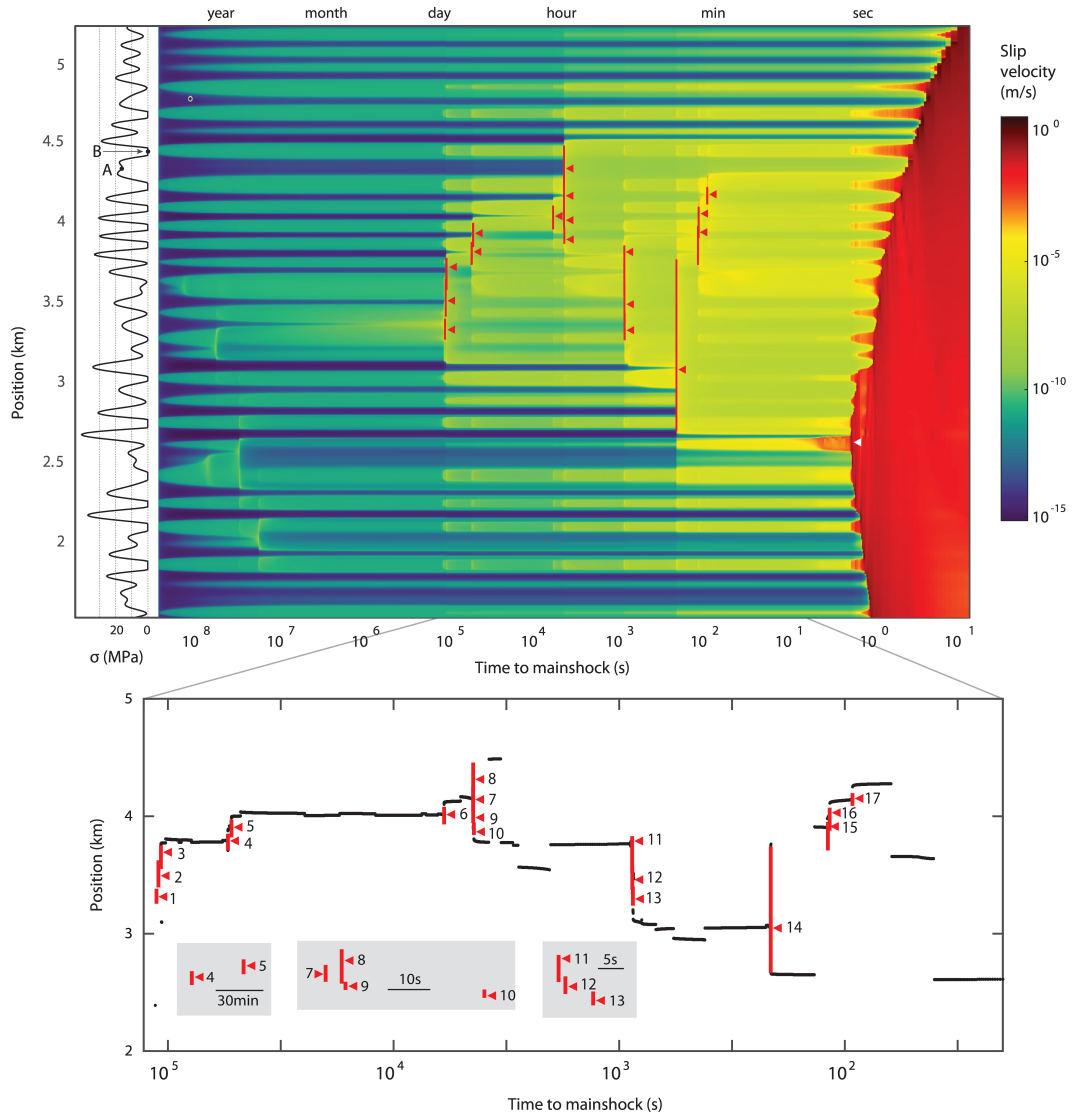


Figure 4. Creep acceleration and seismicity leading up to the mainshock. Top: slip velocity on the fault vs. time to the end of the mainshock, with red bars marking the rupture length and triangles marking the nucleation point (mid-point of the region where $v > V_{dyn}$ during the first earthquake time step). Note the sudden acceleration in nearby creeping patches and the widening of the fast slipping region with each successive seismic burst. Bottom: subset of the top panel, with events numbered by occurrence time. Small black dots indicate the location of maximum slip velocity at each time step, showing accelerated creep at the edges of each burst, where the subsequent ones initiate. Grey panels show close ups of a few clustered foreshocks.

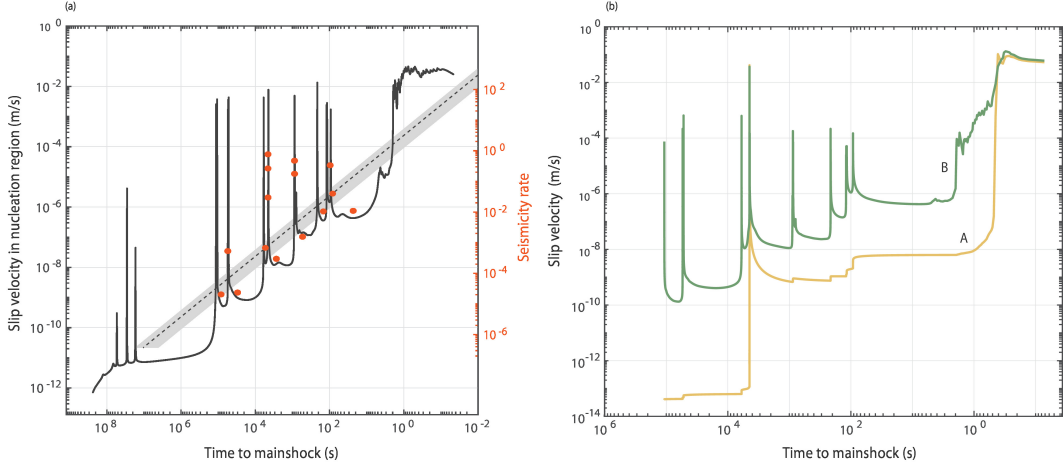


Figure 5. Slip velocity and seismicity rates during the foreshock sequence shown in Fig. 4. (Left) Black solid line: average slip velocity in the nucleation region vs. time to the end of the mainshock. Red circles: seismicity rates estimated by the inverse of interval times, plotted at the midpoint between each pair of events. The y-axes are scaled with respect to one another according to eq. C1. The theoretical evolution of slip velocity (eq. 14) is indicated by the dotted line (for the median value of foreshock stress drop) and grey line (for the entire range of stress drops). (Right) Slip velocity vs. time for the asperity (A) and a nearby creeping patch (B).

240 In spite of the elevated normal stress on asperities, foreshocks don't have partic-
 241 ularly high stress drops (0.1-2MPa): in agreement with Schaal and Lapusta (2019), who
 242 observed a similar behavior in 3-D simulations, we find that foreshocks are not confined
 243 to asperities, but propagate into the surrounding low σ regions, thus lowering the av-
 244 erage stress drop. The presence of such low stress-drop regions is also responsible for the
 245 partial overlap between consecutive events, even though in some cases asperities them-
 246 selves rerupture (Fig. 4).

247 **5.1 Feedback between creep and foreshocks**

248 The average slip velocity during the foreshock sequence increases in sudden steps
 249 when asperities fail (Fig. 4, 5). The acceleration occurs even at large distances from the
 250 foreshocks compared to their rupture dimension, so that foreshocks contribute to widen-
 251 ing the fast creeping area. Average slip velocities on the fault increase approximately with
 252 the inverse of time to mainshock (Fig. 5), similar to studies of velocity weakening asper-
 253 ities embedded in a velocity strengthening (creeping) fault (Dublanche, 2018; Yabe &

254 Ide, 2018). However, neither asperities nor creeping patches follow this trend individ-
 255 ually (Fig. 5).

256 To understand the effect of a seismic rupture on weak patches, consider the change
 257 in velocity caused by an instantaneous shear stress perturbation $\Delta\tau$ through the direct
 258 effect:

$$V = V_0 e^{\Delta\tau/a\sigma}, \quad (12)$$

259 where V_0 is the starting velocity. For a given stress change, areas at low normal stress
 260 are particularly susceptible to stress increases due to foreshocks, even if they are several
 261 rupture lengths away. As an example, Fig. 5(b) shows slip velocities on the asperity which
 262 ruptured in a foreshock (event no.8 in Fig. 4) and a nearby creeping patch, marked in
 263 Fig. 4. After the earthquake, the asperity does not fully relock, but continues slipping
 264 about 4 orders of magnitude faster than it did before. This behavior can be explained
 265 by the faster loading rate from the nearby creeping patches, which prevents the asper-
 266 ity from fully relocking. We can gain some intuition into this by treating the asperity
 267 as a spring-slider driven at a constant stressing rate, which in turn depends on the creep
 268 rate around it. The solution for velocity evolution derived in Appendix B predicts that
 269 the minimum slip speed right after an earthquake grows with stressing rate $\dot{\tau}$:

$$V_{lock} = V_{dyn} e^{b/a} \left(\frac{d_c \dot{\tau}}{b\sigma V_{dyn}} \right)^{b/a}. \quad (13)$$

270 After a mainshock, $\dot{\tau} \approx \dot{\tau}_0$ (the background loading rate); during the nucleation
 271 phase, creep velocities adjacent to the asperities increase (in this case, $V_{cr} \sim 1 \times 10^{-8}$ m/s;
 272 see Fig.5), giving a stressing rate on the asperity of the order of $\tau_{cr} \approx \mu' V_{cr} / L_{asp} \approx$
 273 $\mu' V_{cr} / L_{min} = 4$ Pa/s, here about 10^3 times larger than the background loading rate $\dot{\tau}_0$.
 274 Plugging these numbers in the expression above, we expect V_{lock} after the foreshock to
 275 be about $\sim 10^4$ times larger than its minimum value early in the cycle, consistent with
 276 the simulation (Fig. 5). The creeping patches and asperities subsequently decelerate,
 277 but the asperity slip velocity remains several orders of magnitude larger than before rup-
 278 ture (Fig. 4, 5).

279 The positive feedback between creep rates and seismicity rates leads to an over-
 280 all acceleration and expansion of the creeping region. In Appendix C we derive a sim-
 281 ple analytical model based on the observations described above. It relies on the follow-
 282 ing assumptions: 1. seismicity rate is proportional to average creep rate; 2. creep rates
 283 increase by a constant factor after each foreshock (derived from eq. 12), and don't change

284 otherwise. This simple model predicts that the average slip velocity evolves as

$$\langle V \rangle = \frac{2L_{min}^2 \Delta\tau}{L\mu' \log(\beta)} \frac{1}{t_0 - t} \quad (14)$$

285 where L is the dimension of the nucleation region, $\Delta\tau$ the foreshock stress drop and β
 286 a factor quantifying the increase in creep velocity after each foreshock; t is time since the
 287 first foreshocks and t_0 the time to instability, given by

$$t_0 = \frac{2L_{min}^2 \Delta\tau}{L\mu' \log(\beta) \langle V_0 \rangle}. \quad (15)$$

288 We estimated β by applying eq. 12 to the creep patches in the nucleation region, and
 289 treating foreshocks as uniform stress drop cracks of fixed size, and we obtained values
 290 between 1.1–1.3 (the range is given by variability in foreshock stress drops). Overall,
 291 the average slip velocity in the nucleation region increases approximately as predicted
 292 by this expression (Fig. 5).

293 5.2 Stacked foreshock and aftershock catalogs

294 The prediction of $1/t$ acceleration in creep rates and seismicity rates does not ac-
 295 count for temporal clustering due to elastic interactions between asperities, visible in Fig. 5.
 296 Therefore, the $1/t$ acceleration in seismicity rates may not be readily visible in individ-
 297 ual catalogs. To better capture temporal patterns, we stack the catalogs from all cycles.
 298 All foreshocks-aftershock sequences are shifted so the mainshock occurs at $t = 0$, and
 299 then combined in a single catalog. As shown in Fig. 6(a), the rate of foreshocks increases
 300 with the inverse time to the mainshock, as observed for stacked catalogs of natural se-
 301 quences (Jones & Molnar, 1979; Ogata et al., 1995).

302 5.3 Onset of foreshocks and mainshock

303 The occurrence of foreshocks in the vicinity of the mainshock hypocenter raises the
 304 following question: why do some ruptures arrest, while others in the same region grow
 305 into large events? Fig. 4 shows that the mainshock, like most foreshocks, nucleates at
 306 the edge of a fast creeping region, on an asperity which arrested the previous event. The
 307 mainshock nucleation asperity has the highest normal stress on the entire fault. To ver-
 308 ify whether other mainshocks also nucleate on high σ asperities, we compare normal stresses
 309 in the nucleation region of mainshocks and nearby foreshocks. Fig. 7 shows that main-
 310 shocks tend to nucleate on stronger asperities than most of their foreshocks. This may

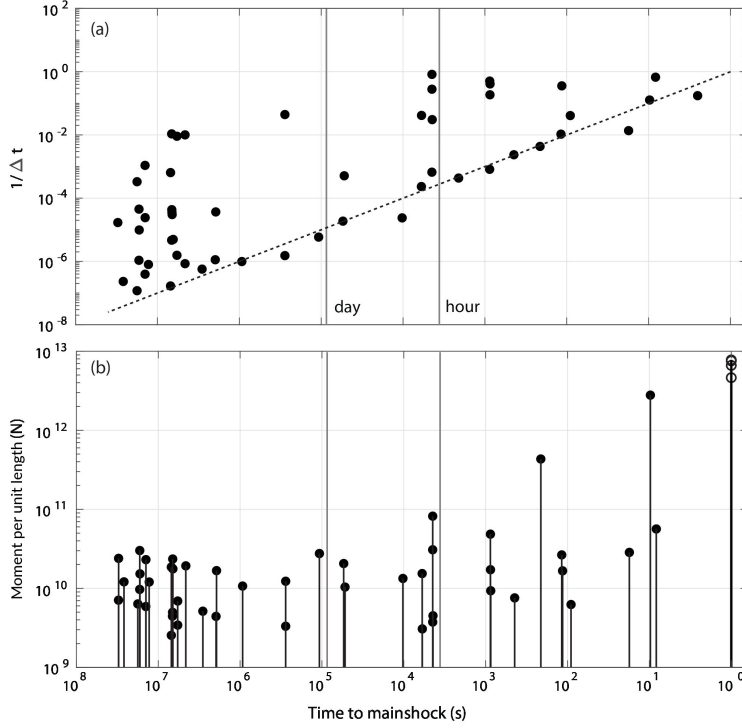


Figure 6. Moment per unit length and interevent times in the stacked catalog. (a) Seismicity rates estimated as the inverse of interevent time showing power-law acceleration. The dotted line is proportional to $1/t$. (b) moment per unit length as a function of time to mainshock. Open circles indicate mainshocks.

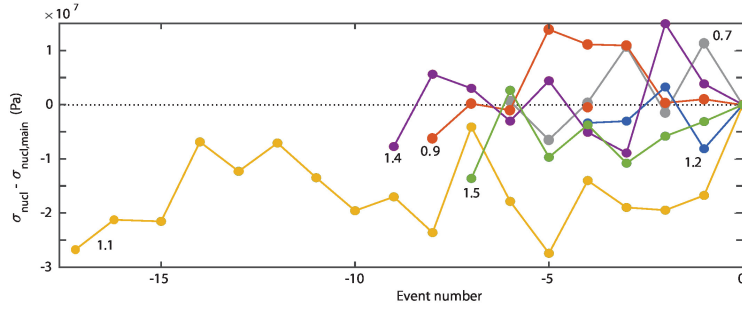


Figure 7. Difference between average normal stress in the nucleation region of foreshocks and their respective mainshocks. Nucleation is defined as the region between points exceeding a velocity threshold at the beginning of an earthquake (see section 2). We consider mainshocks all events with a rupture length exceeding 2km, and only select foreshocks within the mainshock rupture area. Numbers indicate $\Delta\sigma_{rms}/\sigma_0$ for each sequence.

311 not surprising in light of the simple model shown in Fig. 3, since patches with higher nor-
 312 mal stress take longer to reach static strength. Once a strong asperity breaks, its stress
 313 drop is high and leads to a more pronounced stress concentration at its edge, allowing
 314 it to grow further than earlier events. This also explains why larger foreshocks tend to
 315 occur later in the cycle (Fig. 6(b)).

316 Rupture arrest is also determined by the strength of asperities ahead of the rup-
 317 ture tip, which act as barriers. We consider all asperities which are either within or ad-
 318 jacent to a rupture, and as expected we find that stronger asperities are more likely to
 319 arrest ruptures. We also find that a rupture nucleating at normal stress σ_{nuc} has a 62%
 320 probability of breaking an asperity with normal stress exceeding σ_{nuc} , and a 77% chance
 321 of breaking an asperity with normal stress lower than σ_{nuc} . A selection bias could origi-
 322 nate when grouping asperities according to this criterion: on average, asperities with
 323 $\sigma_{asp} > \sigma_{nuc}$ for a given earthquake are stronger than those with $\sigma_{asp} < \sigma_{nuc}$. How-
 324 ever, we find that a difference remains when comparing asperities with approximately
 325 the same normal stress, indicating that σ_{nuc} also affects rupture arrest.

326 6 Discussion

327 The results presented above show that the preseismic phase on a velocity-weakening
 328 fault with fractal roughness is characterized by a complex interplay between slow slip
 329 and foreshocks. Most of the period between mainshocks is devoid of seismicity, and char-
 330 acterized by localized patches of slow slip; late in the cycle, strong asperities start fail-
 331 ing in short bursts, each of them in turn accelerating creep in its neighbourhood. This
 332 process leads to acceleration over an extended region (here about 20 times larger than
 333 the nominal nucleation dimension), with migration of seismicity towards the mainshock
 334 hypocenter.

335 6.1 Model limitations

336 The central result of this study is the coexistence and interaction of slow slip and
 337 foreshocks during nucleation on a rough fault. The primary control on this mixed be-
 338 havior are normal stress perturbations due to roughness, and their effect on fault sta-
 339 bility and slip patterns (section 4). These findings are not specific to rate-state (ageing
 340 law) friction, and likely apply for other frictional laws and weakening mechanism. On
 341 the other hand, certain simplifications in our study may be more consequential and de-

342 serve further investigation. The quasi-dynamic approximation can affect rupture arrest
 343 and foreshock rupture lengths, even though based on previous planar fault studies (Lapusta
 344 et al., 2000; Thomas et al., 2014) we don't expect the qualitative pattern to change dra-
 345 matically with ageing-law rate-state friction. Considering the 3-dimensional nature of
 346 fault surfaces can modify certain aspects of fault dynamics, such the ability of an asper-
 347 ity to arrest rupture or the migration patterns caused by stress redistribution. Another
 348 significant assumption in our study is the purely elastic response: inelastic processes would
 349 limit the amplitude of stress perturbations, in particular at the smallest length scales (e.g.
 350 Dunham et al., 2011).

351 6.2 Conditions for foreshock occurrence

352 The dimension of asperities relative to characteristic elasto-frictional length scales
 353 is expected to affect foreshock behavior. Previous numerical studies of foreshocks on het-
 354 erogeneous faults found that foreshocks only occur in a particular regime (Schaal & La-
 355 pusta, 2019; Dublanchet, 2018): asperities must be larger than the local nucleation di-
 356 mension for seismic slip to occur, but smaller than a critical dimension (such as the nu-
 357 cleation dimension outside the asperity) to arrest without generating system-size rup-
 358 tures. Here, the amplitude of spatial variations in σ controls the range of local nucleation
 359 lengths L_c . As more slip accrues and normal stress perturbations grow, the nucleation
 360 length shrinks on the asperities and grows around them: therefore microseismicity only
 361 appears for sufficiently large normal stress perturbations (here $\Delta\sigma_{rms} \approx \sigma_0$).

362 A similar transition from few large ruptures to many smaller ones was found by Heimisson
 363 (2020) when increasing k_{max} ; since the amplitude of normal stress perturbations grows
 364 with k_{max} (eq. 10), this is consistent with our findings. Similarly, we expect that increas-
 365 ing fault roughness would have the same effect, since $\Delta\sigma_{rms}$ increase with the product
 366 of roughness and accrued slip. In our simulations, we chose $k_{max} \sim 2\pi/L_\infty$, for com-
 367 putational efficiency. To verify the effect of smaller wavelengths, we run also simulations
 368 for a smaller domain and k_{max} up to 4 times higher (Supplementary Figure 2). We find
 369 that the presence of sub- L_∞ asperities leads to more frequent aseismic ruptures (sim-
 370 ilar to those in Fig. 1). Both seismic and aseismic failures contribute to a gradual un-
 371 pinning of the fault, as described above. The temporal evolution of slip velocities, with
 372 an abrupt increase during bursts and an overall $1/t$ trend, is similar to the previous
 373 case.

6.3 Preslip vs. nucleation on rough faults

In the “preslip” model, aseismic slip is generally understood to occur at the location of the mainshock hypocenter, reflecting the notion that seismic instabilities develop over a region of finite size, as predicted by spring-slider stability analysis (Ruina, 1983) or fracture mechanics arguments for a finite fault (e.g. Rubín & Ampuero, 2005). It is conceivable that heterogeneity *within* the nucleation region could lead to foreshocks driven by accelerating slip; however, our results favor a different interpretation. Here the large scale precursory accelerating slip is not mainshock nucleation in the classical sense: since slow slip occurs in stable low σ patches which do not accelerate when subject to slow loading, it does not directly evolve into a seismic rupture. Instead, slow slip triggers smaller scale nucleation on locked asperities, which can remain small or grow into a mainshock.

A similar relationship between preslip and mainshock initiation in presence of heterogeneity has been inferred in laboratory experiments. McLaskey and Lockner (2014) observed acoustic emissions (analogous to foreshocks) and slow slip leading up to failure in a centimeter-scale laboratory sample, and noted that system-size ruptures begin as acoustic emissions, with local strength variations perhaps controlling whether they evolve into larger ruptures. Similarly, meter-scale experiments by McLaskey (2019) show evidence of abrupt earthquake initiation caused by creep penetration from weak regions into a locked patches, “igniting” large ruptures.

The migratory behavior of microseismicity, and the earthquake hypocenter on the edge of the creeping region, also indicate of a different mechanism than self-nucleation. Recent observations of precursory slip leading up to glacial earthquakes by Barcheck et al. (n.d.) are similar to our results: slow slip and microseismicity migrate towards the mainshock hypocenter. Similar seismicity migration has also been observed prior to several events (Tohoku, 2011, A. Kato et al. (2012); Iquique, N. Kato (2014); Brodsky and van der Elst (2014); l’Aquila, Sugan et al. (2014)), and it is sometimes interpreted as evidence for aseismic slip.

On the other hand, migratory behavior can also be interpreted as evidence for direct triggering between foreshocks: seismicity prior to the 1999 Izmit (Ellsworth & Bulut, 2018) and 1999 Hector Mine (Yoon et al., 2019) exhibit a cascade behavior similar to what we observed here (Fig. 4): successive failure of neighbouring asperities, with each event nucleating at the edge of the previous ones, and in one case a rerupture of the same

406 asperity (as in Fig. 4). Here we find that the migration is in some cases caused by di-
 407 rect stress triggering (leading to rapid failure of nearby asperities in a short burst), but
 408 it can also be mediated by accelerated creep between asperities.

409 An intriguing observation is the occurrence of earthquakes in the vicinity of a fu-
 410 ture mainshock hypocenter. The 2004 M_w 6 Parkfield and the M_w 9 Tohoku earthquakes
 411 were both preceded by moderate events within few years of the mainshock, a much shorter
 412 timescale than the respective earthquake cycles. Based on our results, which should be
 413 further verified with fully dynamic simulations, we suggest that local strength variations
 414 between potential nucleation patches within a small region may determine which earth-
 415 quakes evolve into destructive events.

416 **7 Conclusions**

417 We find that fault roughness can lead to simultaneous occurrence of aseismic slip
 418 and foreshocks in the precursory phase of mainshocks, modulated by normal stress vari-
 419 ations caused by fault geometry. The precursory phase can be described as a gradual un-
 420 pinning of the fault by episodic asperity failure, mediated by aseismic slip. The creep-
 421 ing area widens and accelerates through each seismic burst, leading to migration of seis-
 422 micity towards the eventual mainshock hypocenter. A simple model for the positive feed-
 423 back between creep and seismicity predicts that slip accelerates as $1/t$, as confirmed by
 424 the simulations.

425 This process results in precursory slip on a larger scale than, and spatially distinct
 426 from, classical rate state nucleation on flat faults. Our results provide a physical inter-
 427 pretation for laboratory and field evidence of migratory preslip and foreshocks in the vicin-
 428 ity of a future mainshock hypocenter.

429 **Appendix A Normal stress variations**

430 Here we derive the spatial distribution of normal stresses due to slip on a rough
 431 fault with small perturbations in elevation. Fang and Dunham (2013) derived the fol-
 432 lowing expression for normal stress perturbations due to uniform unit slip:

$$433 \quad \Delta\sigma(x) = \frac{\mu'}{2\pi} \int_{-\infty}^{\infty} \frac{y''(\xi)}{x - \xi} d\xi \quad (\text{A1})$$

434 where $\mu' = \mu/(1 - \nu)$ and compressive stresses are positive. The elevation profile can
 435 be written as

$$436 \quad y(\xi) = \int_{k_{min}}^{k_{max}} \hat{y}(k) e^{ik\xi} dk \quad (A2)$$

437 Taking the second derivative and inserting into eq. A1 gives

$$438 \quad \begin{aligned} \Delta\sigma(x) &= \frac{\mu'}{2\pi} \int_{-\infty}^{\infty} \frac{1}{\xi - x} \int_{k_{min}}^{k_{max}} k^2 \hat{y}(k) e^{ik\xi} dk d\xi \\ 439 &= \frac{\mu'}{2\pi} \int_{k_{min}}^{k_{max}} k^2 \hat{y}(k) e^{ikx} \int_{-\infty}^{\infty} \frac{1}{u} e^{iku} du dk, \end{aligned}$$

440 where $u = \xi - x$. We use the following results:

$$441 \quad \int_{-\infty}^{\infty} \frac{\sin(kx)}{x} dx = \pi$$

$$442 \quad \int_{-\infty}^{\infty} \frac{\cos(kx)}{x} dx = 0.$$

443

444 Thus, the inner integral takes the value of $i\pi$ and

$$445 \quad \Delta\sigma(x) = \frac{\mu' S}{2} \int_{k_{min}}^{k_{max}} k^2 \hat{y}(k) e^{i(kx+\pi/2)} dk, \quad (A3)$$

446 where we have reinserted the total slip S . The integral has a form similar to the second
 447 derivative of the topography, but a phase shift of $\pi/2$ in each Fourier component. This
 448 result is consistent with the findings of (Romanet et al., 2019), who demonstrated that
 449 normal stress perturbations on a curved fault are proportional to the local curvature (which
 450 to first order is equal to the second derivative of the slope). The phase shift can be in-
 451 tuitively understood by considering a sinusoidal profile: a phase shift of $\pi/2$ places max-
 452 imum compressive and tensile stresses at the inflection point of restraining and releas-
 453 ing bends (see in fig. A1). Since stress perturbations depend on the second derivative
 454 of the elevation profile, they are dominated by the shortest wavelengths.

455 **A1 Self-similar roughness**

456 Consider a fault with a profile y characterized by power spectral density

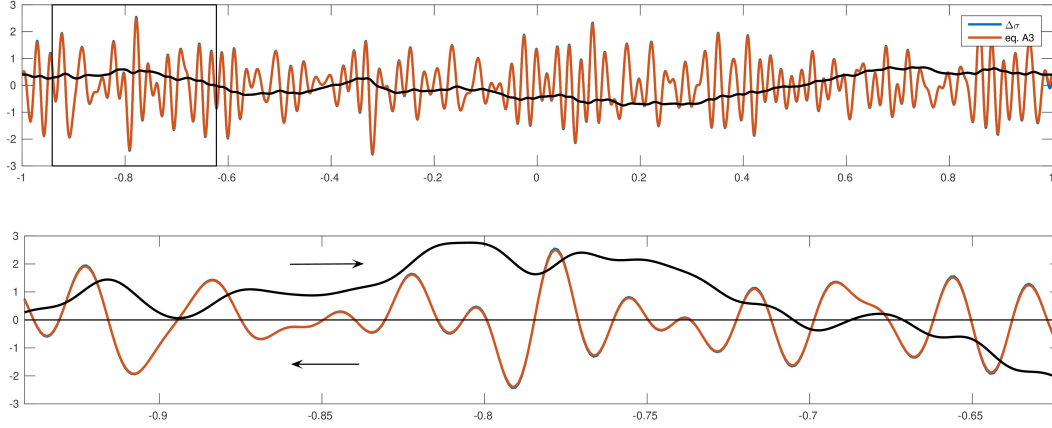


Figure A1. Top: Normal stresses from BEM calculations (blue) and eq. A3 (red), with unit slip and divided by $\mu'/2$. Black: fault profile rescaled by a factor of 500. Bottom: zoomed in (inset in top figure), with fault profile shifted and rescaled by 4000, showing normal stress perturbations corresponding to releasing and restraining bends.

$$457 \quad P_h = C_h |k|^{-\beta} \quad (A4)$$

458 between $k_{min} = 2\pi/L$ and k_{max} , with $\beta = 2H + 1$ and H the Hurst exponent. Using
 459 Parseval's theorem it can be shown that the root mean square elevation in the limit $k_{max} \gg$
 460 k_{min} is

$$461 \quad y_{rms} = \sqrt{\frac{C_h}{\pi(\beta - 1)}} \left(\frac{L}{2\pi}\right)^H = \alpha L^H \quad (A5)$$

462 where α is the surface roughness. Similarly, by applying Parseval's theorem to the sec-
 463 ond derivative of y we obtain the the root mean square value:

$$464 \quad y''_{rms} = \alpha \sqrt{\frac{H}{2-H}} (2\pi)^H k_{max}^{2-H} \quad (A6)$$

465 Here we used fractal surfaces with random phases, resulting in a Gaussian distri-
 466 bution in $y(x)$ and $y''(x)$ is a Gaussian with standard deviation y''_{rms} (e.g. Persson et al.,
 467 2005). Combining this result with eq. A3, we find that normal stress perturbations are
 468 Gaussian distributed with zero mean and standard deviation $\mu' S y''_{rms}/2$, where S is the
 469 accrued slip.

Appendix B Spring slider

To obtain the interseismic evolution of slip velocity, we consider a spring-slider with stiffness κ driven at constant rate $\dot{\tau}_L$:

$$\frac{\tau_0 + t\dot{\tau}_L - \kappa\delta}{\sigma} = [\mu_0 + a \ln(V/V^*) + b \ln(\theta V^*/d_c)] , \quad (\text{B1})$$

where δ is the slip and τ_0 is the stress at time $t = 0$ (see also Rubin and Ampuero (2005), eq.A12). Since we are interested in the velocity during the interseismic phase, the inertial term is not included. Time is measured since the last earthquake, and τ_0 is the residual stress after rupture. More specifically, we define $t = 0$ as the moment when the system last crossed steady-state, and

$$\frac{\tau_0}{\sigma} = f + (a - b) \log(V_{dyn}/V^*) \quad (\text{B2})$$

where $V_{dyn} = (2a\sigma/\mu)v_s$ is the velocity above which inertial effects play a role, in the no-healing regime (Rubin & Ampuero, 2005). Inserting eq. B2 into eq. B1 and solving for V gives

$$V(t) = V_{dyn} \exp\left(\frac{t\dot{\tau}_L - k\delta}{a\sigma}\right) \left(\frac{d_c}{\theta V_{dyn}}\right)^{b/a} \quad (\text{B3})$$

further assuming that the fault is locked ($k\delta/a\sigma \ll 1$) and far below steady-state ($\theta \sim t$), velocity evolves as

$$V(t) = V_{dyn} \exp\left(\frac{t\dot{\tau}_L}{a\sigma}\right) \left(\frac{d_c}{tV_{dyn}}\right)^{b/a} . \quad (\text{B4})$$

The minimum velocity occurs at $t = b\sigma/\dot{\tau}_L$ and is given by

$$V_{lock} = V_{dyn} e^{b/a} \left(\frac{d_c \dot{\tau}_L}{b\sigma V_{dyn}}\right)^{b/a} . \quad (\text{B5})$$

Appendix C Preseismic acceleration

As discussed in section 5.1, the acceleration leading up to the mainshock is controlled by a feedback between creep in low normal stress patches and foreshocks on as-

491 perities. Here we develop a simple model of these interactions and the temporal evolu-
 492 tion of acceleration.

493 Seismicity rate is controlled by the surrounding creep rate, which for simplicity we
 494 take as uniform. The interevent time on a single asperity is of the order of $\Delta\tau/\dot{\tau}$, where
 495 $\Delta\tau$ is the stress drop. Note that this expression does not apply if some interseismic slip
 496 takes place within the rupture area; however, Cattania and Segall (2019) obtained a sim-
 497 ilar expression, within a factor of order one, allowing for creep to penetrate the asper-
 498 ity. The overall seismicity rate on the fault is therefore $N\dot{\tau}/\Delta\tau$, where $N \approx L/L_{min}$
 499 is the number of asperities in the nucleation region. During nucleation we can neglect
 500 tectonic loading, so $\dot{\tau} \approx \dot{\tau}_{cr} = \kappa V(t)$, with $\kappa \sim \mu'/2L_{min}$ so that the seismicity rate
 501 is

$$\frac{dn}{dt} = \frac{L \mu'}{2L_{min}^2 \Delta\tau} \langle V \rangle. \quad (C1)$$

502 where n is the cumulative number of foreshocks, and $\langle V \rangle$ denotes average slip velocity.
 503 We further assume that each earthquake increases the average creep rate by a constant
 504 factor β , derived below, and we neglect self-acceleration of creeping patches. Slip veloc-
 505 ities are then given by

$$\langle V(n) \rangle = \langle V_0 \rangle \beta^n \quad (C2)$$

506 where V_0 is the average slip velocity before the first foreshock. Differentiating eq. C2 and
 507 combining with eq. C1 results in

$$\frac{d\langle V \rangle}{dt} = \frac{L\mu' \log(\beta)}{2L_{min}^2 \Delta\tau} \langle V \rangle^2 \quad (C3)$$

508 which has solution

$$\langle V \rangle = \frac{2L_{min}^2 \Delta\tau}{L\mu' \log(\beta)} \frac{1}{t_0 - t} \quad (C4)$$

509 where t is time since the first foreshocks and t_0 the time to instability, given by

$$t_0 = \frac{2L_{min}^2 \Delta\tau}{L\mu' \log(\beta) \langle V_0 \rangle}. \quad (C5)$$

510 Note that we assumed that the creep velocity remains high after each foreshock. For a
 511 creep patch of fixed dimension (stiffness) subject to a sudden stress increase, we would
 512 instead expect velocity to decay to the steady-state value determined by the background
 513 loading rate; however, simulations show that creep velocities remain high after each step
 514 (Fig. 4, 5), possibly due to the reduction in stiffness after each foreshock described in
 515 section 5.1.

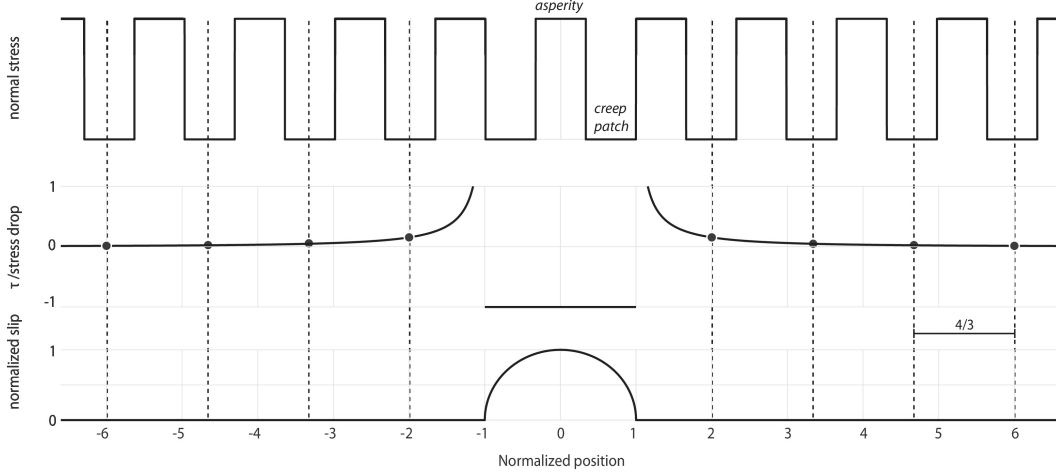


Figure C1. Simple model used to estimate changes in creep rate after a foreshock. Top: schematic spatial distribution of normal stress. Middle: shear stress change caused by a constant stress drop crack normalized by stress drop. Bottom: foreshock slip distribution. Dotted lines and circles indicate the center of creeping patches and locations at which stress changes are calculated.

516 The functional form of eq. C1 and C2 is not expected to change in 3D (even though
 517 α and the prefactor in eq. C1 will differ). Therefore we expect the main result of this
 518 analysis, which is the growth of velocity as the inverse of time to instability, to remain
 519 valid.

520 **C1 Estimating β**

521 To obtain a rough estimate of β , the fractional change in creep rate due to a fore-
 522 shock, we consider a simple model of periodic locked asperities alternating creeping patches,
 523 each region with length $2l$ (Fig. C1). We assume that asperities break in events with uni-
 524 form stress drop, confined to a single asperity and the creeping patch on each side, with
 525 the next asperity acting as barrier. Since the response to stress changes is dominated by
 526 regions with low σ , we consider the change in velocity in creeping patches only.

527 The stress field outside a constant stress drop crack of length $2l$ and stress drop
 528 $\Delta\tau$ is (Bonafede et al., 1985):

$$\Delta\tau_{out} = \Delta\tau \frac{|x| - \sqrt{x^2 - l^2}}{\sqrt{x^2 - l^2}} \tag{C6}$$

529 where x is the distance from the crack center. Since the system is symmetric around $x =$
 530 0, in what follows we consider $x > 0$. We approximate the stress change within each
 531 creeping patch by the value at its center; as shown in Fig. C1, creeping patches are cen-
 532 tered at positions $x = 2l, (2+4/3)l, (2+8/3)l, \dots$. The stress change at position $x = nl$
 533 is given by

$$\Delta\tau_{out} = \Delta\tau \frac{n - \sqrt{n^2 - 1}}{\sqrt{n^2 - 1}}. \quad (C7)$$

534 The local velocity after a stress step given by the direct effect is

$$V = V_0 \exp(\Delta\tau_{out}/a\sigma), \quad (C8)$$

535 where V_0 is the velocity before the stress step and σ the normal stress in creeping patches.
 536 Assuming the same initial velocity V_0 in all creeping patches, the new average velocity
 537 is the sum of the velocity change in each patch divided by the total number of creeping
 538 patches N_p

$$\langle V \rangle = \frac{V_0}{N_p} \sum_{i=0}^{N_p-1} \exp \left[\frac{\Delta\tau}{a\sigma} \left(\frac{n_i - \sqrt{n_i^2 - 1}}{\sqrt{n_i^2 - 1}} \right) \right], \quad (C9)$$

539 where $n_i = 2 + 4i/3$. The fractional change in slip velocity is simply $\beta = \langle V \rangle / V_0$. At
 540 the onset of the foreshock sequence considered in the main text, slip velocities in creep-
 541 ing patches are of the order of 10^{-11} m/s (as expected from $V_{cr} \sim \dot{\gamma} / \mu' L_{cr}$), and their
 542 average normal stress is about 5 MPa. Foreshocks have stress drops between 0.1–2 MPa,
 543 with a median value of 0.5 MPa. Considering the nucleation region between 1.7–4.7 km
 544 (Fig. 4), the number of creeping patches is $\approx 3 \text{ km} / L_{min} = 30$; and since the analysis
 545 above only considers one side of the fault, $N_p = 15$. Plugging these values into eq. C9
 546 gives β between 1.1 and 1.3, depending on the stress drop.

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