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Title: Modeling of Magnetotelluric Source Amplitude Effect in a Spherical Coordinate System

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Abstract

.Magnetotelluric (MT) impedances of the three-dimensional (3-D) Earth are typically modeled in a Cartesian coordinate system, neglecting the Earth's curvature. While this approximation is valid for one-dimensional (1-D) structures, its applicability to 3-D Earth remains uncertain. To evaluate the influence of source configuration—particularly polarization and amplitude—on MT responses, we conducted systematic 3-D forward modeling in a spherical coordinate system with external dipole source excitations, using both oceanic and continental models. Our results show that the spherical MT impedance, unlike its Cartesian counterpart, is generally non-unique and depends on the source amplitude even when three independent sources are applied. The associated tipper estimates are even more strongly affected. This amplitude dependence is not limited to forward modeling but may also influence impedance and tipper estimates derived from real MT observations, since natural source amplitudes vary over time and the Earth's curvature is always present. As a result, these source amplitude effects may help account for observed seasonal variations in MT responses. To support accurate modeling under such conditions, this study proposes a practical framework based on a rotated spherical coordinate system to improve numerical stability and ensure consistency with real MT observations. This framework will enable rigorous comparisons with Cartesian models and provide a robust basis for 3-D inversion in spherical geometry in the future. The findings offer an additional perspective for interpreting MT responses in complex Earth structures and may serve as a useful complement to conventional modeling approaches.

Keywords

Magnetotelluric, impedance, tipper, modeling, spherical coordinates, source effect

Main Text

1 Introduction

The magnetotelluric (MT) method (Cagniard, 1953) is one of the most effective natural-source electromagnetic (EM) induction techniques for exploring the Earth's interior. It utilizes the impedance, represented as the complex ratio of horizontal electric to magnetic fields, as a response function that provides information about the Earth's electrical conductivity structure. MT data consist of time series of electric and magnetic fields recorded at observation sites on land and on the seafloor. The MT method enables exploration of depths ranging from the near surface to several hundred kilometers into the upper mantle. It has been extensively applied in various geophysical investigations, including in the petroleum and mineral industries (e.g., Strangway et al., 1973; Livelybrooks et al., 1996; Garcia & Jones, 2000; Jiang et al., 2022), as well as in studies of crustal and deep mantle structures (e.g., Stanley et al., 1977; Rosell et al., 2011; Tada et al., 2016; Zhang et al., 2016; Matsuno et al., 2017).

EM induction studies are generally classified into two approaches based on spatial scale and frequency range. One is the global (or semiglobal) approach, in which the Earth is treated as a spherical conductor and the fundamental equation of EM induction is solved in a spherical coordinate system (e.g., Banks, 1969; Schmucker, 1999a, 1999b; Shimizu et al., 2011; Kuvshinov and Semenov, 2012; Grayver et al., 2017; Guzavina et al., 2019; Zhang et al., 2023). The second, known as local or regional induction approach, focuses on a relatively small area of the Earth's surface and typically assumes a flat Earth. The MT method is a representative example of this category.

Srivastava (1966) and Utada (2018) demonstrated that, when the induction wavenumber dominates the source wavenumber, MT impedances in both spherical and flat Earth models are equivalent, provided the Earth's structure is one-dimensional (1-D). Nonetheless, in practice, most MT studies adopt the flat Earth approximation—even for three-dimensional (3-D) Earth models. This approximation is considered valid for regional and local studies conducted at mid-to-low latitudes and for periods up to a few hours (e.g., Simpson & Bahr, 2005;

Chave & Jones, 2012). Consequently, modeling in a Cartesian coordinate system is commonly preferred in MT studies due to its simplicity and more mature development compared to spherical approaches. Advances in computational power have further driven progress of 3-D MT modeling and inversion techniques, enabling surveys at hundreds of sites and allowing detailed investigations of the Earth's 3-D structure (e.g., Wannamaker et al., 1984; Mackie et al., 1993, 1994; Newman & Alumbaugh, 2000; Siripunvaraporn et al., 2005; Egbert & Kelbert, 2012).

Despite its widespread use, 3-D modeling in a Cartesian coordinate system has its limitations. It assumes the Earth as flat and therefore ignores curvature effects, which may become significant in global or large-scale regional studies. SInce MT observations are conducted on the Earth's surface, it is natural to consider formulating the fundamental theory in a spherical coordinate system. While developing a theoretical framework in Cartesian coordinates is common for computational, historical and practical reasons, ensuring its consistency with the spherical coordinate framework remains important. The primary concern of the present paper is this consistency between MT methods based on spherical and Cartesian frameworks.

Recent studies have identified this issue and attempted 3-D MT modeling of a laterally heterogeneous Earth using spherical models to quantitatively assess the validity of employing Cartesian models in regional studies (Grayver et al., 2019; Luo et al., 2019; Han et al., 2020; Han & Hu, 2023). These investigations evaluated the differences in impedance estimates obtained from Cartesian and spherical models. However, they differ in several aspects, including how the source is treated.

Luo et al. (2019), Han et al. (2020), and Han and Hu (2023) applied a combination of two orthogonal external magnetic dipoles that generate spatially uniform fields, oriented northward and eastward, respectively, at the intersection of the equator and the central meridian. Grayver et al. (2019), on the other hand, introduced a third orthogonal dipole directed radially at the same intersection to ensure that the matrix for calculating impedances remains full rank at any location on the Earth.

These source treatments are notably more constrained than those typically used in Cartesian forward solvers employing the plane-wave approximation, where any pair of linearly independent sources provides unique impedance elements regardless of amplitude and polarization (Berdichevsky and Dmitriev, 1997; Berdichevsky, 1999). Therefore, in a Cartesian coordinate system, the source effect primarily refers to the source wavenumber effect (also known as the source dimension effect), which has been the focus of most related studies in the past (e.g., Schmucker, 1987; Garcia et al., 1997). However, few studies have assessed how source characteristics affect MT impedance estimates in a spherical coordinate system. This gap has motivated the present study.

When the plane-wave source is not assumed, MT source effects can be categorized into three types: harmonic degree (wavenumber), polarization, and amplitude. Although previous studies employing spherical models have used various source combinations, none have systematically examined the source effects on MT impedance. This is a critical issue because MT impedance under the plane-wave approximation is defined as a response function dependent solely on frequency and the Earth's electrical conductivity distribution—not on the source itself.

The objectives of this study are:

(1) to present a mathematical formulation and modeling procedure for 3-D MT simulation in a spherical coordinate system, and

(2) to investigate source effects—specifically, those related to amplitude and polarization.

To simplify the problem, we limit our analysis to two standard source configurations: plane-wave excitation in Cartesian coordinates and external magnetic dipole sources in spherical coordinates. Under these conditions, the source wavenumber (or harmonic degree) is either fixed or implicitly determined by source geometry, making it unnecessary to consider its effect separately. Furthermore, as demonstrated in Section 2.3, polarization effects are inherently included in the amplitude effect under this framework. To explore the amplitude-related source effects under these assumptions—and with a view toward applying spherical-coordinate MT modeling to real data—we conduct systematic numerical experiments based on realistic model setups, which are constructed using survey areas and station layouts from previous MT observations conducted both offshore (e.g., Baba et al., 2010; Tada et al., 2014) and onshore (e.g., Yang et al., 2020).

2 Formulation

2.1 Basic equations and coordinate systems

In this study, we perform MT forward modeling in a spherical coordinate system to account for Earth curvature effects and to enable comparisons with real data. The governing equations for the time-varying electromagnetic (EM) field are Maxwell's equations, which we solve in the frequency domain:

$$\nabla \times \mathbf{E}(\mathbf{r}, \omega) = -i\omega\mu\mathbf{H}(\mathbf{r}, \omega), \tag{1}$$

$$\nabla \times \mathbf{H}(\mathbf{r}, \omega) = \sigma(\mathbf{r})\mathbf{E}(\mathbf{r}, \omega) + \mathbf{j}^{ext}(\mathbf{r}, \omega), \tag{2}$$

where $\mathbf{E}(\mathbf{r}, \omega)$ and $\mathbf{H}(\mathbf{r}, \omega)$ denote the electric and magnetic fields at position $\mathbf{r}$ and angular frequency ω. Here, i denotes the imaginary unit, μ the magnetic permeability, $\sigma(\mathbf{r})$ the electrical conductivity, and $\mathbf{j}^{ext}(\mathbf{r}, \omega)$ the source electric current density. The displacement current is ignored in Eq. (2). We assume $\mu = \mu_0$

everywhere, where μ_0 represents the magnetic permeability of free space. We consider frequencies between 10^{-3} and 10^{-4} Hz for oceanic models and between 10^{-2} and 10^{-4} Hz for continental models.

The position vector $\mathbf{r}$ in the geographic spherical coordinate system is defined as $\mathbf{r} = (r, \theta, \varphi)^t$ relative to the reference coordinate system (ξ, η, ζ), where the center of the Earth is taken as the origin (Figure 1a). Here, r, θ, and φ denote the distance from the origin, the colatitude, and the longitude, respectively. Superscript t indicates the transpose.

In numerical modeling in a spherical coordinate system, the grid spacing in the longitudinal and co-latitudinal directions is defined as $r_e sin\theta d\varphi$ and $r_e d\theta$, respectively, where $r_e (= 6371\ km)$ is the Earth's radius and $d\varphi$ and $d\theta$ are the respective angular grid intervals. Due to the $sin\theta$ factor, the longitudinal grid spacing becomes narrower at higher latitudes. This dependence leads to resolution asymmetry and potential loss of accuracy when the domain center is far from the equator,.

To mitigate this effect, we apply a coordinate rotation from the original reference system (ζ, η, ξ) to a rotated system (ζ', η', ξ') using two Euler angles α and β (see Figure S1). The rotated coordinate system is defined so that the intersection of its equator and central meridian ($\theta' = 90°, \varphi' = 0°$) coincides with the center of the study region (Figure 1b). This re-centering improves numerical stability by evening out grid spacing and enhancing model symmetry.

Importantly, this coordinate rotation serves not only for enhancing accuracy in spherical numerical modeling, but also for enabling direct comparison between Cartesian and spherical formulations with the same structural model. Moreover, the use of rotation is crucial for future applications to real MT data, where numerical consistency and precision are required. For these reasons, incorporating coordinate rotation is essential both for the reliability of this study and for its applicability to future analyses

2.2 MT impedance and deviation

In spherical MT modeling, the impedance $\mathbf{Z}$ is estimated using the solutions of the basic equations, which relates the electric and magnetic fields via the following equation:

$$\mathbf{E}(\mathbf{r}_{ob}, \omega) = \mathbf{Z}(\mathbf{r}_{ob}, \omega)\mathbf{H}(\mathbf{r}_{ob}, \omega) \tag{3}$$

where $\mathbf{r}_{ob}$ denotes the observation site location. The tipper $\mathbf{T}$ which relates the radial and tangential components of the magnetic field is defined as:

$$H_r(\mathbf{r}_{ob}, \omega) = \mathbf{T}(\mathbf{r}_{ob}, \omega)\mathbf{H}(\mathbf{r}_{ob}, \omega). \tag{4}$$

In MT, $\mathbf{E}$ and $\mathbf{H}$ in Equations (3) and (4) consist of two horizontal components; therefore, the impedance is a complex-valued 2×2 (4-element) tensor, while the tipper is a complex-valued 1×2 tensor.

In our modeling, the location of each observation site $\mathbf{r}_{ob}$ defined in the original spherical coordinate system, but the EM fields are computed in the rotated system. Therefore, we convert the impedance elements from the rotated coordinate system back to the geographic system to match the reference frame used in real observations.

The four impedance elements are denoted as $Z_{ij}(\mathbf{r}_{ob}, \omega)$ where $i, j \in \{\theta, \varphi\}$. Hereafter, the position and frequency dependences of electromagnetic fields and impedances, $(\mathbf{r}_{ob}, \omega)$, are omitted for simplicity.

To quantify differences in impedances calculated at a location $\mathbf{r}_{ob}$ and frequency ω under different conditions (e.g., source combinations) 1 and 2, we define the Frobenius norm (F-norm) deviation:

$$dZ^{1-2} = \frac{\|\mathbf{Z}^1 - \mathbf{Z}^2\|_F}{\|\mathbf{Z}^2\|_F}, \quad (5)$$

where $\mathbf{Z}^1$ and $\mathbf{Z}^2$ are the two impedances to be compared. The F-norm of $\mathbf{Z}$ is defined as:

$$\|\mathbf{Z}\|_F = \{\mathrm{tr}(\mathbf{Z}^H\mathbf{Z})\}^{1/2} = \left\{\sum_{i,j} |Z_{ij}|^2\right\}^{1/2}, \quad (6)$$

where superscript H denotes the Hermitian (complex conjugate) transpose.

In certain cases, we calculate the average F-norm deviation over the entire study region, which is defined as:

$$dZ_{avg}^{1-2} = \frac{1}{N}\sum_{n=1}^{N} \frac{\|\mathbf{Z}_n^1 - \mathbf{Z}_n^2\|_F}{\|\mathbf{Z}_n^2\|_F}, \quad (7)$$

where subscript n represents a calculation cell, and N is the total number of calculation cells in the study region. We set the target level of the F-norm deviation of impedance of 0.01, based on typical impedance uncertainty in seafloor measurements (Tada et al., 2012). In subsequent comparisons, two impedances are considered consistent if the F-norm deviation is smaller than this target level.

To visualize and interpret the complex-valued elements of the impedance, we also compute the apparent resistivity and impedance phase:

$$\rho_{a_{ij}} = \frac{\left|Z_{ij}\right|^2}{\omega\mu_0} \tag{8}$$

and

$$\phi_{ij} = \arg\left[Z_{ij}\right]. \tag{9}$$

Similar to the F-norm deviation in Equation (5), the relative differences in ρ_{aij} and ϕ_{ij} are expressed as:

$$d\rho_{a_{ij}}^{1-2} = \frac{\rho_{a_{ij}}^{1}}{\rho_{a_{ij}}^{2}} \tag{10}$$

and

$$d\phi_{ij}^{1-2} = \phi_{ij}^{1} - \phi_{ij}^{2}, \tag{11}$$

respectively.

To compare tippers, we use the L2-norm deviation, defined as:

$$dT^{1-2} = \frac{\|\mathbf{T}^1 - \mathbf{T}^2\|_2}{\|\mathbf{T}^2\|_2}. \tag{12}$$

where the L2-norm of a tipper vector is given by:

$$\|\mathbf{T}\|_2 = \left\{|T_\theta|^2 + \left|T_\varphi\right|^2\right\}^{1/2}. \tag{13}$$

We also defined the averaged L2-norm deviation of tippers over the entire study region as:

$$dT_{avg}^{1-2} = \frac{1}{N} \sum_{n=1,N} \frac{\|\mathbf{T}_n^1 - \mathbf{T}_n^2\|_2}{\|\mathbf{T}_n^2\|_2}, \tag{14}$$

similar to the average F-norm deviation of the MT impedances given by Equation (7).

2.3 External source

To isolate the amplitude-related source effect, we adopt an external magnetic source with spherical harmonic degree one (dipole type) in the rotated spherical coordinate system. This produces a uniform magnetic field—the simplest spatial source structure—allowing us to focus solely on the amplitude effect. We assume that the external magnetic field above the Earth's surface ($r > r_e$) can be obtained from the spatial gradient of a scalar potential, generally expressed by the spherical harmonic expansion as:

$$V\left(r,\theta',\varphi',\omega\right) = r\left(q_1^0(\omega)\cos\theta' + q_1^1(\omega)\cos\varphi' \sin\theta' + s_1^1(\omega)\sin\varphi' \sin\theta'\right), \quad (15)$$

where $q_1^0(\omega)$, $q_1^1(\omega)$, and $s_1^1(\omega)$ are expansion coefficients of the axial and two equatorial dipole terms, respectively.

Using $\mathbf{B} = -\nabla V$ and $\mathbf{H} = \frac{\mathbf{B}}{\mu_0}$, the three components of the external magnetic field are obtained as:

$$H_r\left(r,\theta',\varphi',\omega\right) = -\frac{1}{\mu_0}\left(q_1^0(\omega)\cos\theta' + q_1^1(\omega)\cos\varphi' \sin\theta' + s_1^1(\omega)\sin\varphi' \sin\theta'\right), (16)$$

$$H_\theta\left(r,\theta',\varphi',\omega\right) = -\frac{1}{\mu_0}\left(-q_1^0(\omega)\sin\theta' + q_1^1(\omega)\cos\varphi' \cos\theta' + s_1^1(\omega)\sin\varphi' \cos\theta'\right), (17)$$

and

$$H_\varphi\left(r,\theta',\varphi',\omega\right) = -\frac{1}{\mu_0}\left(-q_1^1(\omega)\sin\varphi' + s_1^1(\omega)\cos\varphi'\right). \quad (18)$$

For convenience, we represent the three basis sources aligned with $\zeta'-$, $\xi'-$, and $\eta'-$directions, denoted as $\hat{\mathbf{S}}_{\zeta'}$, $\hat{\mathbf{S}}_{\xi'}$, and $\hat{\mathbf{S}}_{\eta'}$, corresponding to the harmonic expansion coefficients q_1^0, q_1^1, and s_1^1, respectively (Figure 2a). By setting $\frac{q_1^0(\omega)}{\mu_0} = \frac{q_1^1(\omega)}{\mu_0} = \frac{s_1^1(\omega)}{\mu_0} = -1$, these sources generate uniform magnetic fields of unit amplitude in the positive $\zeta'-$, $\xi'-$, and $\eta'-$directions, respectively.

An arbitrary source in the rotated spherical coordinate system, denoted as $\mathbf{S}(\theta_S', \varphi_S')$, is specified by the pole location (θ_S', φ_S') (Figure 2b). The source $\mathbf{S}$ can be expressed as a linear combination of the three basis sources:

$$\mathbf{S}(\theta_S', \varphi_S') = a_{\zeta'}\hat{\mathbf{S}}_{\zeta'} + a_{\eta'}\hat{\mathbf{S}}_{\eta'} + a_{\xi'}\hat{\mathbf{S}}_{\xi'}, \tag{19}$$

where $a_{\zeta'}$, $a_{\eta'}$, and $a_{\xi'}$ are real-valued arbitrary source scaling factors. This formulation implies that any variation in the source orientation—including changes in polarization—can be captured by appropriate combinations of the basis sources through the coefficients $a_{\zeta'}$, $a_{\eta'}$, and $a_{\xi'}$. In this sense, the so-called source polarization effect is inherently included in the broader source amplitude effect, as it does not require an additional or separate modeling framework.

A source of unit amplitude in an arbitrary direction is given by:

$$\hat{\mathbf{S}}(\theta_S', \varphi_S') = \hat{a}_{\zeta'}\hat{\mathbf{S}}_{\zeta'} + \hat{a}_{\eta'}\hat{\mathbf{S}}_{\eta'} + \hat{a}_{\xi'}\hat{\mathbf{S}}_{\xi'}, \tag{20}$$

where

$$\hat{a}_{\zeta'} = \frac{a_{\zeta'}}{|\mathbf{S}(\theta_S', \varphi_S')|} = \cos\theta_S', \tag{21}$$

$$\hat{a}_{\eta'} = \frac{a_{\eta'}}{|\mathbf{S}(\theta_S', \varphi_S')|} = \sin\theta_S' \sin\varphi_S', \tag{22}$$

and

$$\hat{a}_{\xi'} = \frac{a_{\xi'}}{|\mathbf{s}(\theta'_S, \varphi'_S)|} = \sin\theta'_S \cos\varphi'_S\,. \tag{23}$$

Let the external field generated by an arbitrary source $\mathbf{S}$ be denoted as $\mathbf{H}_{ext}(\mathbf{S})$. Using Eq. (19) and the linearity of the EM field, this can be written as:

$$\mathbf{H}_{ext}(\mathbf{S}) = a_{\zeta'}\mathbf{H}_{ext}\big(\hat{\mathbf{S}}_{\zeta'}\big) + a_{\eta'}\mathbf{H}_{ext}\big(\hat{\mathbf{S}}_{\eta'}\big) + a_{\xi'}\mathbf{H}_{ext}\big(\hat{\mathbf{S}}_{\xi'}\big), \tag{24}$$

where

$$\mathbf{H}_{ext}\big(\hat{\mathbf{S}}_{\zeta'}\big) = -\frac{q_1^0(\omega)}{\mu_0}\begin{pmatrix}\cos\theta' \\ -\sin\theta' \\ 0\end{pmatrix} = \begin{pmatrix}\cos\theta' \\ -\sin\theta' \\ 0\end{pmatrix}, \tag{25}$$

$$\mathbf{H}_{ext}\big(\hat{\mathbf{S}}_{\eta'}\big) = -\frac{q_1^1(\omega)}{\mu_0}\begin{pmatrix}\sin\theta'\cos\varphi' \\ \cos\theta'\cos\varphi' \\ -\sin\varphi'\end{pmatrix} = \begin{pmatrix}\sin\theta'\cos\varphi' \\ \cos\theta'\cos\varphi' \\ -\sin\varphi'\end{pmatrix}, \tag{26}$$

and

$$\mathbf{H}_{ext}\big(\hat{\mathbf{S}}_{\xi'}\big) = -\frac{s_1^1(\omega)}{\mu_0}\begin{pmatrix}\sin\theta'\sin\varphi' \\ \cos\theta'\sin\varphi' \\ \cos\varphi'\end{pmatrix} = \begin{pmatrix}\sin\theta'\sin\varphi' \\ \cos\theta'\sin\varphi' \\ \cos\varphi'\end{pmatrix}. \tag{27}$$

These external fields given in Equations (25)-(27) correspond to $-H^1$, $-H^2$, and $-H^3$ in Grayver et al. (2019).

Using Equations. (25)–(27), the Maxwell's equations are solved by providing a source boundary condition at the outer boundary of the model domain, which is set at sufficiently high altitude where all internally induced components are decayed out (See next section). Let the resulting EM solution for the external dipole source $\hat{\mathbf{S}}_{\zeta'}$, for example, be denoted by $\mathbf{E}(\hat{\mathbf{S}}_{\zeta'})$ and $\mathbf{H}(\hat{\mathbf{S}}_{\zeta'})$. Then, the solutions for an arbitrary dipole source $\mathbf{S}$ can then be obtained as a linear combination of the solutions for the three basis sources:

$$\mathbf{E}(\mathbf{S}) = a_{\zeta'}\mathbf{E}(\hat{\mathbf{S}}_{\zeta'}) + a_{\eta'}\mathbf{E}(\hat{\mathbf{S}}_{\eta'}) + a_{\xi'}\mathbf{E}(\hat{\mathbf{S}}_{\xi'}) \tag{28}$$

and

$$\mathbf{H}(\mathbf{S}) = a_{\zeta'}\mathbf{H}(\hat{\mathbf{S}}_{\zeta'}) + a_{\eta'}\mathbf{H}(\hat{\mathbf{S}}_{\eta'}) + a_{\xi'}\mathbf{H}(\hat{\mathbf{S}}_{\xi'}), \tag{29}$$

respectively.

In this study, the MT impedance is estimated from the EM field solutions for a set of external dipole sources. A set of two or three sources is denoted by $\{\mathbf{S}_1, \mathbf{S}_2\}$ or $\{\mathbf{S}_1, \mathbf{S}_2, \mathbf{S}_3\}$, respectively, where $\mathbf{S}_1$, $\mathbf{S}_2$, and $\mathbf{S}_3$ are external dipole sources with arbitrary directions and amplitudes. In practice, we perform numerical modeling using the three basis sources $\hat{\mathbf{S}}_{\zeta'}$, $\hat{\mathbf{S}}_{\eta'}$ and $\hat{\mathbf{S}}_{\xi'}$ to cover all possible source directions. This approach enables efficient evaluation of source direction and amplitude effects without redundant computations.

3 Model setup

3.1 Modeling methods and surface inhomogeneities

Modeling in a spherical coordinate system was carried out using a global forward code modified from Uyeshima and Schultz (2000). This code employs a staggered-grid finite-difference method to solve Maxwell's equations, with all variables calculated in double precision. The source field, as expressed in Equation (25), (26) or (27), was set at the outer boundary of the model domain ($r = 10r_e$). The altitude was chosen sufficiently far from the Earth's surface to ensure that the internal part of the primary field becomes negligible. The inner boundary was set at the core-mantle boundary ($r = 3479$ km), where the radial magnetic component is assumed to vanish. The numerical solution was iteratively verified for convergence, using a stopping criterion defined by the normalized change in the magnetic field. This was evaluated via the dot product of the magnetic vector field integrated over the entire domain and was required to fall below 10^{-16}.

The model assumes that the Earth's interior consists of two thin, laterally heterogeneous shells on the surface and a 1-D (radially symmetric) structure beneath them. The shallowest layer is a 4-km-thick inhomogeneous shell representing the land-sea electrical conductivity contrast, with variable conductance incorporating bathymetric variations. Land topography is neglected due to its relatively low conductance. The second layer is a 1-km-thick inhomogeneous layer that account for lateral variations in oceanic sediment thickness. The underlying 1-D structure is based on a simplified oceanic mantle model referencing Baba et al. (2010) and Shimizu et al. (2010) (Figure 3).

We assumed electrical conductivities of the sea water, sediment, and crustal rock in the two surface shells to be 3.0, 0.1, and 0.01 S/m, respectively. For the oceanic surface shell, we averaged the bathymetry data from ETOPO1 (Amante & Eakins, 2009) that originally had a resolution of $1' \times 1'$, within each cell. The average conductivity in the 4 km-thick surface shell were then calculated assuming uniform conductivity within each cell. For the sediment shell, we employed the Laske and Masters (1997) model. The sediment thickness, originally provided at a resolution of 1° × 1°, was interpolated to match the model grid and used to compute the average conductivity in a similar manner. In the oceanic model, the MT impedance and tipper were estimated at the seafloor (i.e., the boundary between the top two heterogeneous shells) using the modeled EM solutions. In the continental model, the MT responses were estimated at the land surface.

3.2 Grid configuration

The same grid configuration was used for both the oceanic and continental models to ensure consistent comparison. Because uniform fine gridding over the entire domain is computationally impractical, non-uniform grids were applied both radially and tangentially. The lateral extent of the study region was set to 20° × 20°. The center of the study region was chosen as (25°N, 135°E) for the oceanic model and at (32°N, 90° E) for the continental model. These centers were then relocated to (0°N, 0°E) in the rotated coordinate system.

The radial grid spacing was uniformly set to 500 m in the land-sea contrast layer and to 250 m in the sediment layer. In the underlying 1-D layers, the radial grid spacing increased logarithmically (Fujita et al., 2018) down the CMB. The lateral grid size was set to 0.25° within the core study region, increasing progressively to 1°, 2°, and 5°, with the distance from the center (Figure 4). The (40° × 40°) area of the finest (0.25°) grids was determined via numerical testing (See Figure S2, and Table S1). The radial grid spacing near the seafloor was confirmed to be adequate for the 3-D model (Figure S3). The final grid system consisted of:

- 89 cells in the radial (r) direction, including 17 cells in the air,
- 194 cells in the co-latitudinal (θ') direction,
- 230 cells in the longitudinal (φ') direction.

4 Formulation of Impedance and Tipper estimation from numerical models

In MT studies, subsurface conductivity structures are estimated by fitting numerically calculated impedances and tippers to those derived from observational data. Before investigating the source amplitude dependence of impedance and tipper through numerical calculations which is the central focus of this study, this section formulates the procedure for estimating these EM responses from electric and magnetic field solutions through forward modeling. The formulations are provided in both Cartesian and spherical coordinate systems to highlight their respective characteristics, particularly to aid readers more familiar with Cartesian modeling in understanding the differences between the two frameworks.

4.1 Cartesian Coordinate Case

As is commonly assumed in conventional modeling, we adopt the plane-wave assumption in the Cartesian coordinate system. Two unit-amplitude sources polarized in the north-south (x) and east-west (y) directions, respectively—referred to as basis sources—are applied. These sources induce two horizontal components of the electric field and all three components of the magnetic field at the Earth's surface.

Separating the magnetic field into horizontal and vertical components, we define the 2×2 matrices of electric and magnetic field solutions for respective sources as:

$$\hat{\mathbf{E}}^{(2\times2)} = \begin{pmatrix} \hat{E}_x{}^{(x)} & \hat{E}_x{}^{(y)} \\ \hat{E}_y{}^{(x)} & \hat{E}_y{}^{(y)} \end{pmatrix}, \quad \hat{\mathbf{H}}^{(2\times2)} = \begin{pmatrix} \hat{H}_x{}^{(x)} & \hat{H}_x{}^{(y)} \\ \hat{H}_y{}^{(x)} & \hat{H}_y{}^{(y)} \end{pmatrix}, \quad (30)$$

The impedance is obtained as the exact solution to:

$$\hat{\mathbf{E}}^{(2\times2)} = \hat{\mathbf{Z}}\hat{\mathbf{H}}^{(2\times2)} \quad (31)$$

The vertical magnetic field solution matrix is defined as:

$$\hat{\mathbf{H}}_z^{(1\times2)} = \left(\hat{H}_z^{(x)} \quad \hat{H}_z^{(y)}\right), \quad (32)$$

and the tipper is given by:

$$\hat{\mathbf{H}}_z^{(1\times2)} = \hat{\mathbf{T}}\hat{\mathbf{H}}^{(2\times2)} \quad (33)$$

Here, $\hat{\mathbf{Z}}$ and $\hat{\mathbf{T}}$ represent the impedance and tipper derived from standard forward solver.

Any plane-wave source in the Cartesian system with arbitrary polarization and amplitude can be expressed as a linear combination of the two basis sources as shown in Equation (19). Owing to the linearity of Maxwell's equations, the resulting EM field solutions are also expressed as a linear combination of solutions for the basis sources. Thus, modeling with an arbitrary pair of independent plane-wave sources will provide us impedances and tippers that are identical to $\hat{\mathbf{Z}}$ and $\hat{\mathbf{T}}$, respectively. This property is referred to as the uniqueness of the impedance and tipper. It implies that under the plane-wave approximation, the impedance and tipper are independent of the source polarization and amplitude scaling, and can be treated as response functions depending solely on the electrical conductivity structure and frequency (e.g., Berdichevsky and Dmitriev, 1997).

4.2 Spherical Coordinate Case

In numerical modeling based on spherical coordinates, it is common to apply two or three orthogonal external magnetic dipole sources of unit amplitude. While a separate class of sources with negligible radial magnetic components at the surface has been proposed for tipper estimation (e.g., Kruglyakov and Kuvshinov, 2019), we adopt the same sources for both impedance and tipper estimation. This choice reflects the reasoning that, in practical observations, both are estimated from the same time series data, and using different sources in modeling could introduce inconsistencies.

In the spherical system, we define three orthogonal dipole sources—one aligned with the Earth's rotation axis and two lying in the equatorial plane. These correspond to the $\zeta' -$, $\xi' -$, and $\eta' -$directions in the global reference coordinate system (Figure 1). Forward modeling from the three basis sources at the Earth's surface yields three sets of solutions for horizontal electric and magnetic field components, organized as 2×3 matrices:

$$\hat{\mathbf{E}}^{(2\times3)} = \begin{pmatrix} \hat{E}_\theta{}^{(\zeta')} & \hat{E}_\theta{}^{(\xi')} & \hat{E}_\theta{}^{(\eta')} \\ \hat{E}_\varphi{}^{(\zeta')} & \hat{E}_\varphi{}^{(\xi')} & \hat{E}_\varphi{}^{(\eta')} \end{pmatrix}, \quad \hat{\mathbf{H}}^{(2\times3)} = \begin{pmatrix} \hat{H}_\theta{}^{(\zeta')} & \hat{H}_\theta{}^{(\xi')} & \hat{H}_\theta{}^{(\eta')} \\ \hat{H}_\varphi{}^{(\zeta')} & \hat{H}_\varphi{}^{(\xi')} & \hat{H}_\varphi{}^{(\eta')} \end{pmatrix}. \quad (34)$$

These matrices are related by:

$$\hat{\mathbf{E}}^{(2\times3)} = \hat{\mathbf{Z}}\hat{\mathbf{H}}^{(2\times3)}. \quad (35)$$

Letting a solution matrix of the radial magnetic component be represented as:

$$\hat{\mathbf{H}}_r{}^{(1\times3)} = \begin{pmatrix} \hat{H}_r{}^{(\zeta')} & \hat{H}_r{}^{(\xi')} & \hat{H}_r{}^{(\eta')} \end{pmatrix}, \quad (36)$$

The tipper satisfies the relation:

$$\hat{\mathbf{H}}_r{}^{(1\times3)} = \hat{\mathbf{T}}\hat{\mathbf{H}}^{(2\times3)}. \quad (37)$$

Because $\hat{\mathbf{H}}^{(2\times3)}$ is not a square matrix, Equations (35) and (37) represent overdetermined systems rather than exact solutions. The impedance and tipper are therefore estimated using a least-squares approach:

$$\hat{\mathbf{Z}} = \hat{\mathbf{E}}^{(2\times3)}\hat{\mathbf{H}}^{(2\times3)^{\boldsymbol{H}}}\left(\hat{\mathbf{H}}^{(2\times3)}\hat{\mathbf{H}}^{(2\times3)^{\boldsymbol{H}}}\right)^{-1} \quad (38)$$

$$\hat{\mathbf{T}} = \hat{\mathbf{H}}_r{}^{(1\times3)}\hat{\mathbf{H}}^{(2\times3)^{\boldsymbol{H}}}\left(\hat{\mathbf{H}}^{(2\times3)}\hat{\mathbf{H}}^{(2\times3)^{\boldsymbol{H}}}\right)^{-1}, \quad (39)$$

which correspond to the impedance and tipper estimates commonly used in spherical MT modeling (e.g., Grayver et al., 2019).

Any dipole source of arbitrary polarization and amplitude can be expressed as a linear combination of three basis sources, and a set of EM solutions in Equations (34) and (36) can be expressed as linear combinations of solutions for the three basis sources. However, unlike in Cartesian modeling, impedances and tippers for a set of three external dipole sources with arbitrary polarizations and amplitudes generally satisfy an ill-posed system of Equations (35) and (37). In the next section, we examine through numerical modeling whether the impedance and tipper in the spherical coordinate system can, like in the Cartesian case, be regarded as unique response functions determined solely by the conductivity structure and frequency.

5 Estimating the MT impedance and tipper from a spherical model

Estimating the impedance and tipper in MT modeling requires at least two linearly independent external sources. In Cartesian models under the plane-wave approximation, the resulting impedance and tipper are shown to be well-defined response functions, independent of source amplitude and polarization in the previous section. In contrast, it remains unclear whether the same independence holds in spherical coordinate systems, which necessitates clarification through numerical modeling. This section systematically examines how source configuration influences impedance and tipper estimates in spherical models. In this section, we consider the oceanic model (Figure 5a), which includes the study region of a seafloor MT experiment conducted in the Philippine Sea by Baba et al. (2010).

5.1 Estimating the impedance and tipper using two sources

If we solve the basic equation using two independent sources of arbitrary polarization and amplitude, the conventional impedance satisfies:

$$\mathbf{E}^{(2\times2)} = \mathbf{Z}\mathbf{H}^{(2\times2)}. \quad (40)$$

where $\mathbf{E}^{(2\times2)}$ and $\mathbf{H}^{(2\times2)}$ are 2×2 matrices of two horizontal component solutions due to two independent external dipole sources. Eq. (40) constitutes a well-posed system of equations allowing for an exact solution, and therefore some of recent studies (Luo et al., 2019; Han et al., 2020; Han & Hu, 2023) applied this approach.

The given two sources define a great circle plane. Let two mutually orthogonal external dipole sources of unit amplitude on the plane be denoted as $\hat{\mathbf{S}}^{(1)}$ and $\hat{\mathbf{S}}^{(2)}$, and EM solution matrices for the two sources be denoted as $\hat{\mathbf{E}}^{(2\times2)}$ and $\hat{\mathbf{H}}^{(2\times2)}$. These two solutions satisfy:

$$\hat{\mathbf{E}}^{(2\times2)} = \hat{\mathbf{Z}}_2\hat{\mathbf{H}}^{(2\times2)}, \qquad (41)$$

where $\hat{\mathbf{Z}}_2$ is the impedance determined from these two sources. The EM solutions in Equation (40) can be expressed as linear combinations of the solutions in Equation (41), if and only if both sets of external dipole sources are restricted on to the same great circle plane, in other words, the source degrees of freedom are two. The identity $\mathbf{Z} = \hat{\mathbf{Z}}_2$ does not hold, if either of the two external dipole sources contains a component normal to the great circle plane. Given the complexity of natural geomagnetic disturbances, it is unlikely that the time-varying external fields are constrained to a single great circle plane, i.e., having only two degrees of source freedom. Conversely, MT impedance estimates from two independent sources are generally non-unique and depend on the polarization. The same applies to the tipper estimates.

We conducted a numerical modeling to visually demonstrate the implication above. We calculated the MT impedances in the rotated spherical coordinate system for two frequencies: 10^{-3} and 10^{-4} Hz. We compared two results obtained by using different combinations of external dipole sources of unit amplitude: $\{\hat{\mathbf{S}}_{\zeta'}, \hat{\mathbf{S}}_{\eta'}\}$ and $\{\hat{\mathbf{S}}(45°, 45°), \hat{\mathbf{S}}(135°, 45°)\}$. The first source combination provides the best similarity to MT modeling in a Cartesian coordinate system with two orthogonal plane-wave sources polarized in the N-S and E-W directions. These combinations were selected to span the widest angular separation, thereby maximizing the potential variation in the resulting impedance estimates.

The results at seven selected sites showed significant F-norm deviations (Figure 5b), indicating that the impedance estimated using these two sources is non-uniquely determined and depends on the source polarization.

5.2 Estimating the impedance and tipper using three independent sources

Next, we extend the configuration to include three mutually orthogonal external dipole sources, following the suggestion by Grayver et al. (2019). This configuration yields an overdetermined system that can be solved using the least-squares method. We conduct a numerical modeling to examine the significance of the non-uniqueness suggested in the previous section.

First, we consider a set of three sources, which can be derived from a rigid rotation of the set of three basis sources. The solution matrices are given as:

$$\hat{\mathbf{E}}^{(2\times3)\prime} = \hat{\mathbf{E}}^{(2\times3)}\mathbf{R}^{(3\times3)}, \hat{\mathbf{H}}^{(2\times3)\prime} = \hat{\mathbf{H}}^{(2\times3)}\mathbf{R}^{(3\times3)}, \hat{\mathbf{H}}_r{}^{(1\times3)\prime} = \hat{\mathbf{H}}_r{}^{(1\times3)}\mathbf{R}^{(3\times3)}, \quad (42)$$

where and $\mathbf{R}^{(3\times3)}$ denotes a matrix representing a rigid rotation operator in 3-D space. These solutions satisfy:

$$\hat{\mathbf{E}}^{(2\times3)\prime} = \hat{\mathbf{Z}}'\hat{\mathbf{H}}^{(2\times3)\prime}, \hat{\mathbf{H}}_r{}^{(1\times3)\prime} = \hat{\mathbf{T}}'\hat{\mathbf{H}}^{(2\times3)\prime}. \quad (43)$$

Here, $\hat{\mathbf{Z}}'$ and $\hat{\mathbf{T}}'$ are the impedance and tipper after a rigid rotation of three basis sources, and can be estimated as:

$$\hat{\mathbf{Z}}' = \hat{\mathbf{E}}^{(2\times3)\prime}\hat{\mathbf{H}}^{(2\times3)\prime-1} = \hat{\mathbf{E}}^{(2\times3)}\mathbf{R}^{(3\times3)}\left(\hat{\mathbf{H}}^{(2\times3)}\mathbf{R}^{(3\times3)}\right)^{-1}, \quad (44)$$

$$\hat{\mathbf{T}}' = \hat{\mathbf{H}}_r{}^{(1\times3)\prime}\hat{\mathbf{H}}^{(2\times3)\prime-1} = \hat{\mathbf{H}}_r{}^{(1\times3)\prime}\mathbf{R}^{(3\times3)}\left(\hat{\mathbf{H}}^{(2\times3)}\mathbf{R}^{(3\times3)}\right)^{-1}. \quad (45)$$

Obviously, the identities $\hat{\mathbf{Z}}' = \hat{\mathbf{Z}}$ and $\hat{\mathbf{T}}' = \hat{\mathbf{T}}$ hold for arbitrary rigid rotations.

However, natural source fields (geomagnetic disturbances) are complex; their amplitudes and polarizations change dynamically over time. To examine whether more complex source combinations with different polarizations and amplitudes yield unique impedances as shown above, we conducted further numerical experiments and compared the impedances estimated from various source combinations. We know there are three kinds of MT source effects: dimension, polarization and amplitude effects. However, we opted to omit the experiment of the source dimension effect, because the effect appears regardless of how the response is defined.

As shown in Section 2.3, an arbitrary source can be decomposed into a linear combination of three basis sources. Therefore, the source polarization effect is inherently captured by the source amplitude effect, and only the latter needs to be examined numerically. As a simple approach, we considered cases in which two of the three orthogonal sources had unit amplitudes, while the amplitude of the third source was varied.

The source combinations examined were: $\{a_1\hat{\mathbf{S}}(45°, 45°),\ \hat{\mathbf{S}}(135°, 45°),\ \hat{\mathbf{S}}(90°, 135°)\}$, $\{\hat{\mathbf{S}}(45°, 45°),\ a_2\hat{\mathbf{S}}(135°, 45°),\ \hat{\mathbf{S}}(90°, 135°)\}$ and $\{\hat{\mathbf{S}}(45°, 45°),\ \hat{\mathbf{S}}(135°, 45°),\ a_3\hat{\mathbf{S}}(90°, 135°)\}$, where amplitude factors, a_1, a_2, and a_3 took values $10^{-0.5}$, 1, and $10^{0.5}$, respectively. The impedance is estimated by solving the following overdetermined system:

$$\begin{pmatrix} E_\theta^1 & E_\theta^2 & E_\theta^3 \\ E_\varphi^1 & E_\varphi^2 & E_\varphi^3 \end{pmatrix} = \begin{pmatrix} Z_{\theta\theta} & Z_{\theta\varphi} \\ Z_{\varphi\theta} & Z_{\varphi\varphi} \end{pmatrix} \begin{pmatrix} H_\theta^1 & H_\theta^2 & H_\theta^3 \\ H_\varphi^1 & H_\varphi^2 & H_\varphi^3 \end{pmatrix} + \begin{pmatrix} \delta\epsilon_{E_\theta}^1 & \delta\epsilon_{E_\theta}^2 & \delta\epsilon_{E_\theta}^3 \\ \delta\epsilon_{E_\varphi}^1 & \delta\epsilon_{E_\varphi}^2 & \delta\epsilon_{E_\varphi}^3 \end{pmatrix}, \quad (46)$$

where E_θ^k, E_φ^k, H_θ^k and H_φ^k $(k = 1,2,3)$ are the EM solutions for k'th source, and $\delta\epsilon_{E_\theta}^k$ and $\delta\epsilon_{E_\varphi}^k$ are residuals.

The impedances calculated from these source combinations in the oceanic model were compared with those obtained from the basis source combination $\{\hat{\mathbf{S}}_{\zeta'}$, $\hat{\mathbf{S}}_{\eta'}$, $\hat{\mathbf{S}}_{\xi'}\}$. The F-norm deviations of the impedance obtained at seven selected sites exceeded 0.01 at most sites, when a_1, a_2, or a_3 was $10^{-0.5}$ or $10^{0.5}$ (See Figure S4). Additionally, the largest deviation occurred at T16 or T09, while the smallest deviation was observed at T10, suggesting that the deviation diminishes with increasing distance from the coastlines and the regions with steep bathymetric gradients. Differences in the apparent resistivity and impedance phase also showed significant anomalies not only near coastlines but also in flat basins (see Figure S5). These results suggest that the impedance estimated from the three independent sources depends on the source amplitude.

Next, the average F-norm deviation of the impedance was calculated for the entire study region over a wider range of amplitude scaling factors. The three sets of impedances obtained from different values of a_1, a_2, and a_3 were compared with those from the reference source combination consisting of three basis sources. The amplitude factors a_1 (represented by open blue circles), a_2 (represented by solid blue squares), and a_3 (represented by open blue squares) varied in an extremely wide range between 10^{-5} and 10^{5}. The results are shown in Figure 6 (left). We found that the averaged deviations were approximately 10^{-9} only when a_1, a_2, and a_3 were exactly unity. Outside this narrow range near unity, the average F-norm deviations easily exceeded the typical observation error level (0.01), with the largest deviation at a level of 0.1 (Figure 7). The deviations also exhibited slight asymmetry with respect to the unit amplitudes. These numerical experiments clearly demonstrate that the MT impedance estimates in a spherical coordinate system are not unique, being dependent on the source amplitude.

Tippers were computed using the same sets of source combinations by solving the following system:

$$\begin{pmatrix} {H_r}^{(1)} & {H_r}^{(2)} & {H_r}^{(3)} \end{pmatrix} = \begin{pmatrix} T_\theta & T_\varphi \end{pmatrix} \begin{pmatrix} {H_\theta}^{(1)} & {H_\theta}^{(2)} & {H_\theta}^{(3)} \\ {H_\varphi}^{(1)} & {H_\varphi}^{(2)} & {H_\varphi}^{(3)} \end{pmatrix} + \begin{pmatrix} \delta{\epsilon_{H_r}}^{(1)} & \delta{\epsilon_{H_r}}^{(2)} & \delta{\epsilon_{H_r}}^{(3)} \end{pmatrix}. \quad (47)$$

The average deviation of the tipper was also calculated using the same model setup. As shown in Figure 6 (right), the deviations are approximately 10^{-8} and 10^{-7} only when the amplitude factor is near unity, whereas they increase sharply when the amplitude factor deviates slightly from unity. Outside this narrow range, the deviations remain consistently high, between 10^{-1} and 1. The L2-norm deviations of the tipper obtained at seven selected sites exceeded 0.01 when any of the amplitude factors a_1, a_2, or a_3 was set to $10^{-0.5}$ or $10^{0.5}$ (Figure S6), clearly demonstrating the stronger amplitude dependence of the tippers.

6 A complementary test using a continental model

As a complementary test, we applied the same methodology to a continental model representing the Qinghai-Tibet Plateau region (Figure 8a), aiming to assess whether the source amplitude dependence observed in oceanic settings also arises in continental environments. Two cases were considered: one with a highly conductive anomaly (Yang et al., 2020) and one without. In the model with the anomaly, a polygonal structure with a conductivity of 0.1 S/m was assumed, centered in the study region and to have dimensions of 16° × 6° laterally (represented by the green rectangle in Figure 8a), extending from 4 to 100 km in depth. In the absence of the 3-D anomaly, the model approximates a 1-D structure because the study region is far from the coastlines and the crust and upper mantle are assumed to be laterally homogeneous with a conductivity of 10^{-3} S/m down to the depth of $100\ km$ (Figure 3).

We first analyzed the case without a 3-D anomaly. The impedance elements were estimated at five selected sites, L05, L02, L03, L01, and L04 (Figure 8), where MT measurements were conducted (Yang et al., 2020), for three frequencies, $10^{-2}, 10^{-3}$ and $10^{-4}\ Hz$. Source combinations used were $\{a_1\hat{\mathbf{S}}(45°, 45°), \hat{\mathbf{S}}(135°, 45°), \hat{\mathbf{S}}(90°, 135°)\}$ with a_1 is $10^{-0.5}$, and the basis set $\{\hat{\mathbf{S}}_{\zeta'}, \hat{\mathbf{S}}_{\eta'}, \hat{\mathbf{S}}_{\xi'}\}$. As shown in Figure 8b, the F-norm deviations of all five sites are significantly small (well below the typical observation error level), in the range of 10^{-5} to 10^{-3}.

In contrast, introducing a 3-D conductivity anomaly caused significant changes. As shown in Figure 8c, the F-norm deviations at sites near the edge of the anomaly (L04 and L05) increased by several orders of magnitude, while those at a more distant site (L03) remained small. These substantial deviations surpassed the typical MT observation error threshold of 0.01.

Figure 9 shows maps of the L2-norm of the tippers and the F-norm deviation of the impedances for the model including the anomaly. A strong spatial correlation is observed between the two maps. Figure 10 further illustrates a positive correlation between impedance deviations and tipper norms.

Figure 9 presents maps of the L2-norm of tippers and the F-norm deviation of impedances for the model with the anomaly. Tippers were calculated using a source combination of $\{\hat{\mathbf{S}}_{\zeta'},\ \hat{\mathbf{S}}_{\eta'},\ \hat{\mathbf{S}}_{\xi'}\}$ and deviations are computed between the impedances obtained from $\{a_1\hat{\mathbf{S}}(45°, 45°),\ \hat{\mathbf{S}}(135°, 45°),\ \hat{\mathbf{S}}(90°, 135°)\}$ with $a_1 = 10^{-0.5}$, and those from $\{\hat{\mathbf{S}}_{\zeta'},\ \hat{\mathbf{S}}_{\eta'},\ \hat{\mathbf{S}}_{\xi'}\}$. A strong spatial correlation is observed between the two maps. Figure 10 demonstrates a clear positive correlation between the impedance deviations and tipper norms. The results from these numerical experiments show that the impedance deviation due to the source amplitude effect tends to increase with the tipper norm. This suggest that the impedance deviation is caused by a lateral contrast of the conductivity heterogeneity.

In summary, these findings provide compelling evidence that, in the presence of lateral heterogeneity, the conventional MT impedance and tipper estimated in a spherical coordinate system using three independent sources are not uniquely determinable but depend on the source amplitude.

7 Discussion

This study investigated how MT impedance and tipper estimates depend on source amplitude, using forward modeling in a spherical coordinate system. Although our simulations focused on external magnetic dipole sources, this setup reflects the common approximation that time-varying natural geomagnetic fields are composed of three degree-one (dipolar) sources. Deviations from this ideal situation can be considered small perturbations, stemming from higher spherical harmonic terms or observational noise. In this context, variations in the relative amplitudes of the dipole components alone can produce changes in MT estimates, even without changes in the conductivity structure or source geometry. This amplitude dependence is not limited to synthetic models—it can affect real data analyses as well.

As an application of this result, we examined seasonal variations in MT responses, which have been reported across a broad range of frequencies in various land regions (e.g., Kappler et al., 2010; Brändlein et al., 2012; Araya et al., 2013; Ernst et al., 2020, 2022). Ernst et al. (2020, 2022) reported that seasonal variations in tippers were more significant than those in impedances, attributing this to changes in the external radial (vertical) magnetic field. In Cartesian coordinate modeling, the source can only be characterized in terms of its spatial wavenumber. In contrast, spherical modeling allows for explicit control of the amplitude ratios among dipole components, enabling a more direct investigation of amplitude-related effects. In this study, we adopted a continental model containing a single 3-D anomaly (Figure 8), which yields a relatively simple spatial response pattern, with notably large tippers occurring near the boundaries of lateral conductivity contrasts (Figure 9).

To examine the influence of individual magnetic field components on the tipper estimates, we performed tests using two source combinations: $\{\hat{\mathbf{S}}_{\zeta'},\ \hat{\mathbf{S}}_{\eta'},$

$a_{\xi'}\hat{\mathbf{S}}_{\xi'}\}$ and $\{\hat{\mathbf{S}}_{\zeta'},\ a_{\eta'}\hat{\mathbf{S}}_{\eta'},\ \hat{\mathbf{S}}_{\xi'}\}$, where the amplitude factors $a_{\xi'}$ and $a_{\eta'}$ were set to 1, 2, 5, or 10. This allowed us to isolate the effects of varying the radial and tangential components of the external field, respectively. Site L04, located near the northern edge of the conductivity contrast, was selected for its expected sensitivity to tipper amplitude (Figure 9).

The calculated tipper amplitudes increased with $a_{\xi'}$ (Figure 11a), with changes in T_{φ} being more pronounced than those in T_{θ}. This trend contrasts with the findings of Ernst et al. (2020), who observed greater seasonal variations in T_{θ}. On the other hand, the variations in tipper amplitudes associated with changes in $a_{\eta'}$ (Figure 11b) were considerably smaller, indicating that the radial magnetic component exerts a stronger influence on tipper behavior.

Figure 12a compares the seasonal differences in tippers observed by Ernst et al. (2020), represented by the absolute seasonal differences between tippers estimated from the magnetic field data in summer and winter (solid black circles), with differences in our modeled tippers at site L04 for varying $a_{\xi'}$. Our calculations show that the observed seasonal variations in the real and imaginary parts of T_{θ}

can be approximately explained by a change in the amplitude of the source radial component from 1 (at higher frequencies) to approximately 10 (at lower frequencies).

In contrast, changes in T_{φ} can be explained by a smaller change in $a_{\xi'}$,around unity. While our objective was not to reproduce the observations by Ernst et al. (2020) in details, the comparison suggests that seasonal tipper variations, at least in part, can be interpreted as resulting from changes in the amplitude of the external radial magnetic component using only degree-one (spatially uniform) sources.

We further examined the apparent resistivity differences at site L04 for varying $a_{\xi'}$ values and compared them to those obtained from observed data by Ernst et al. (2022) as shown in Figure 12b. Differences on the order of 0.01 in $\rho_{a_{\varphi\theta}}$ and 0.1 in $\rho_{a_{\theta\varphi}}$ at a frequency of 10^{-4} Hz ($10{,}000$ sec in period) at L04 could be attributed to a case with $a_{\xi'} \approx 10$. A more accurate evaluation of the amplitude-related differences in observed responses would require the use of detailed conductivity model that reflects the true regional structure beneath the observation sites in Europe.

In addition to interpreting observational variations, we turn our attention to the issue of non-uniqueness in impedance estimation that arises when the amplitudes of the three source components are not balanced. In Figures 6 and 7, we saw a marked decrease of the average deviation of the impedance when the amplitude factor is close to unity. Outside this narrow range, the average deviation remains consistently above the typical level of observation error. This behavior can be attributed to the significant residuals that result from solving an overdetermined system of equations when estimating the impedance from three sources with unequal amplitudes. Here, we define the averaged residual across the entire study region for an external source k as:

$$d\boldsymbol{\epsilon}_{\mathbf{E}}^{k} = \frac{1}{N}\sum_{n=1,N}\frac{\left\|\delta\boldsymbol{\epsilon}_{\mathbf{E}}^{k}\right\|_2}{\left\|\mathbf{E}^{k}\right\|_2}, \tag{48}$$

where subscript n represents a calculation cell in the study region, and N is the total number of cells for the oceanic model or the continental model. $\delta\boldsymbol{\epsilon}_{\mathbf{E}}^{k}$ is the residual given in Equation (46) for the external source k.

For a source combination of $\{\hat{\mathbf{S}}_{\zeta'},\ \hat{\mathbf{S}}_{\eta'},\ \hat{\mathbf{S}}_{\xi'}\}$, Table 1 lists the average residuals associated with each source at two frequencies. We observed notably large residuals for $\hat{\mathbf{S}}_{\xi'}$, indicating the significant contribution of t the radial magnetic components in inducing the electric field. This contribution alters the resulting electric fields and, in turn, the estimated impedance elements, thereby reducing the accuracy of the impedance relation.

The amplitude-related source effect described above is specific to spherical impedance estimates, which we refer to as the “source amplitude effect.” Under the dipole source assumption, any source combination can be decomposed into three basis sources with specific amplitude factors. The source amplitude effect in spherical impedance arises when these amplitude factors are unequal. However, questions remain, such as how significant the non-uniqueness of the impedance is in practice, and how the non-uniqueness of the tipper influences the reliability of inversion results. Further investigation is required to address these questions.

In addition to the amplitude-related effect, another type of source effect exist, which we refer to as the “source harmonic degree effect”. This arises from contributions of sources with harmonic degrees higher than one and is conceptually analogous to the source wavenumber (or dimension) effect in Cartesian MT impedances. The large impedance residual for the source $\hat{\mathbf{S}}_{\xi'}$, which increases with decreasing frequency (Table 1), can be partially attributed to this effect. While addressing how to properly account for this effect in a spherical coordinate system is an important problem, it is beyond the scope of the present study.

8 Conclusions

In this study, we performed MT forward modeling in a spherical coordinate system using laterally heterogeneous oceanic and continental models to systematically investigate the effects of source polarization and amplitude on impedance and tipper estimates.

We found that the spherical impedance estimated from two independent sources is generally not unique, except when all source orientations lie within a single great circle plane. In such special cases, the impedance is consistent across different source combinations, but not invariant under arbitrary rigid-body rotations of the source configuration.

When three linearly independent source components are applied, the impedance estimate becomes invariant under arbitrary rigid-body rotation and is uniquely defined with respect to source orientation. However, this uniqueness breaks down when the amplitudes of the three sources are unbalanced, leading to persistent non-uniqueness in both impedance and tipper estimates.

These findings suggest that spherical impedance and tipper responses are inherently non-unique under typical conditions. This source amplitude effect may partly explain observed seasonal variations in tippers and four-element impedances, as it allows for substantial variations even without changes in subsurface conductivity.

While further research is needed to determine how this non-uniqueness influences practical MT inversion, the modeling procedures employed in this study—including coordinate system rotation centered on the study region—offer improved numerical accuracy and stability. This approach is not only essential for rigorous comparisons with Cartesian models but also provides a promising framework for future applications of spherical MT modeling and inversion using real observational data.

List of abbreviations

EM: Electromagnetic

MT: Magnetoteluric

1-D: One-dimensional

3-D: Three-dimensional

Declarations

The authors *must* provide the following sections under the heading "Declarations".

Ethics approval and consent to participate

Not applicable

Consent for publication

Not applicable.

Availability of data and materials

Dataset of modeling results is available at https://doi.org/10.5281/zenodo.10725054

Competing interests

Authors have no competing interest.

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Authors' contributions

XLW mainly conducted this research and prepared the manuscript. HU supervised the research, and provided the basic concepts. HS and PY participated in discussions on the course of the research. All authors read and approved the final manuscript.

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Figure legends

Figure 1. (a) A spherical coordinate system with the center of the Earth as the origin. (b) Coastlines on the spherical Earth. The two great circles shown as red lines represent the equator and central meridian in the rotated spherical coordinate system. Black dots mark the surface intersections of the $\xi'-$, $\eta'-$ and $\zeta'-$ axes. The area enclosed by the red dashed lines indicates the study region for the oceanic model. Red crosses denote locations of the seven selected sites considered in later sections.

Figure 2. (a) Distribution of magnetic field components on a spherical surface for the three basis sources in the rotated coordinate system. The black dots mark the location of the intersection of the equator and central meridian. (b) External dipole sources in the rotated coordinate system. **S** is a source dipole of arbitrary polarization and amplitude with one of its poles located at (θ'_S, φ'_S).

Figure 3. 1-D electrical conductivity structure and the radial gridding (depth from 5 km to 1,000 km) assumed for the numerical models. The structure is designed by

considering the 1-D structures beneath the Pacific obtained by Baba et al. (2010) and Shimizu et al. (2010).

Figure 4. Lateral grid spacing for spherical model in the rotated coordinate system. The red dashed square indicates the study region.

Figure 5. (a) A map showing the bathymetry and site locations of the oceanic model. Red crosses show the seven selected sites (after Baba et al., 2010). (b) F-norm deviations at seven selected sites between calculated impedances with source combinations of $\{\hat{\mathbf{S}}(45°, 45°), \hat{\mathbf{S}}(135°, 45°)\}$ and $\{\hat{\mathbf{S}}_{\zeta'}, \hat{\mathbf{S}}_{\eta'}\}$. Model: the oceanic model in a rotated spherical coordinate system.

Figure 6. Dependence of dZ_{avg}^{1-2} (left) and dT_{avg}^{1-2} (right) on $\log(a_1)$, $\log(a_2)$, or $\log(a_3)$ in a range when a_1, a_2 or a_3 is between 10^{-5} and 10^{5} in the entire study region at a frequency of 10^{-4} sec. Shown are the average deviations between impedances and tippers from source combinations of $\{a_1\hat{\mathbf{S}}(45°, 45°),\ \hat{\mathbf{S}}(135°, 45°),$

$\hat{\mathbf{S}}(90°, 135°)\}$ and $\{\hat{\mathbf{S}}_{\zeta'},\ \hat{\mathbf{S}}_{\eta'},\ \hat{\mathbf{S}}_{\xi'}\}$, those from source combination of $\{\hat{\mathbf{S}}(45°, 45°),\ a_2\hat{\mathbf{S}}(135°, 45°),\ \hat{\mathbf{S}}(90°, 135°)\}$ and from $\{\hat{\mathbf{S}}_{\zeta'},\ \hat{\mathbf{S}}_{\eta'},\ \hat{\mathbf{S}}_{\xi'}\}$, and those from $\{\hat{\mathbf{S}}(45°, 45°),\ \hat{\mathbf{S}}(135°, 45°),\ a_3\hat{\mathbf{S}}(90°, 135°)\}$, and $\{\hat{\mathbf{S}}_{\zeta'},\ \hat{\mathbf{S}}_{\eta'},\ \hat{\mathbf{S}}_{\xi'}\}$. Hatched area corresponds to horizontal range of a_i considered in Section 6. Model: the oceanic model in a rotated spherical coordinate system.

Figure 7. Dependence of dZ_{avg}^{1-2} on log(1–a_i) or log(a_i–1) (i=1, 2 or 3) in a range between –7 and –0.5 (hatched area in Figure 6) in the whole study region at a frequency of 10^{-4} Hz. Symbols are the same with Figure 6. Model: the oceanic model in a rotated spherical coordinate system.

Figure 8. (a) Map of the study region showing locations of the five selected sites (green crosses) and the 3-D anomaly (green rectangle) in the continental model (Yang et al., 2020). (b) The F-norm deviations of spherical impedances using source combinations of $\{a_1\hat{\mathbf{S}}(45°, 45°),\ \hat{\mathbf{S}}(135°, 45°),\ \hat{\mathbf{S}}(90°, 135°)\}$ where a_1 is $10^{-0.5}$ and $\{\hat{\mathbf{S}}_{\zeta'},\ \hat{\mathbf{S}}_{\eta'},\ \hat{\mathbf{S}}_{\xi'}\}$ at

the five selected sites in the continental model for the case without a 3-D anomaly. (c) Same as (b), but for the case with a 3-D anomaly.

Figure 9. Top: Logarithmic distribution of tipper L2 norms calculated using $\{\hat{\mathbf{S}}_{\zeta'},\ \hat{\mathbf{S}}_{\eta'},\ \hat{\mathbf{S}}_{\xi'}\}$. Bottom: Logarithmic distribution of F-norm deviations of impedances computed from $\{a_1\hat{\mathbf{S}}(45°, 45°),\ \hat{\mathbf{S}}(135°, 45°),\ \hat{\mathbf{S}}(90°, 135°)\}$ when a_1 is $10^{-0.5}$ and $\{\hat{\mathbf{S}}_{\zeta'},\ \hat{\mathbf{S}}_{\eta'},\ \hat{\mathbf{S}}_{\xi'}\}$ using the continental model. A strong spatial correlation is evident between the two maps.

Figure 10. Dependence of F-norm impedance deviation on tipper norm. (a) At five selected sites. (b) At all surface grid points in the study region. Tippers were calculated from $\{\hat{\mathbf{S}}_{\zeta'},\ \hat{\mathbf{S}}_{\eta'},\ \hat{\mathbf{S}}_{\xi'}\}$; F-norm impedance deviations were calculated between results from $\{a_1\hat{\mathbf{S}}(45°, 45°),\ \hat{\mathbf{S}}(135°, 45°),\ \hat{\mathbf{S}}(90°, 135°)\}$ where a_1 is $10^{-0.5}$ and those from $\{\hat{\mathbf{S}}_{\zeta'},\ \hat{\mathbf{S}}_{\eta'},\ \hat{\mathbf{S}}_{\xi'}\}$, using the continental model.

Figure 11. Period dependence of tipper at site L04 when amplitudes (a) $a_{\xi'}$ in $\{\hat{\mathbf{S}}_{\zeta'}, \hat{\mathbf{S}}_{\eta'}, a_{\xi'}\hat{\mathbf{S}}_{\xi'}\}$ or (b) $a_{\eta'}$ in $\{\hat{\mathbf{S}}_{\zeta'}, a_{\eta'}\hat{\mathbf{S}}_{\eta'}, \hat{\mathbf{S}}_{\xi'}\}$ has different values. Blue solid triangles, green open squares, red solid circles, and orange open squares indicate results when the source amplitude $a_{\xi'}$ or $a_{\eta'}$ is 1, 2, 5 and 10, respectively. Model: the continental model with a 3-D anomaly in a rotated spherical coordinate system.

Figure 12. Period dependence of (a) absolute differences in tipper norm and (b) apparent resistivity at L04 for different values of the source amplitude ($a_{\xi'}$=1, 2, 5 or 10). Model: the continental model with a 3-D anomaly in a rotated spherical coordinate system.

Tables

Table 1 Least squared residuals.

Shown are averaged residuals $\mathrm{d}\boldsymbol{\epsilon}_{\mathbf{E}}^{\mathrm{k}}$ when calculating impedances from the source combination of $\{\widehat{\boldsymbol{S}}_{\zeta'}, \widehat{\boldsymbol{S}}_{\eta'}, \widehat{\boldsymbol{S}}_{\xi'}\}$. Model: oceanic model in the rotated spherical coordinate system.

Source	$10^{-3}\ Hz$	$10^{-4}\ Hz$
$\hat{\mathbf{S}}_{\zeta'}$	0.0112	0.0080
$\hat{\mathbf{S}}_{\eta'}$	0.0121	0.0090
$\hat{\mathbf{S}}_{\xi'}$	0.5837	0.6174