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7
8 TITLE

9
10 **Using nano-XRM and high-contrast imaging to inform micro-porosity permeability**
11 **during Stokes-Brinkman single and two-phase flow simulations on micro-CT images.**

12
13 ABSTRACT

14
15 Carbonate rocks have particularly complex and multiscale pore systems which are weakly
16 understood. In this study we use combined experimental, modelling, and pore space generation
17 methods to tackle the impact of micro-porosity on the bulk flow properties of Estailades
18 limestone. First, a nano-core from a microporous grain of Estailades Limestone was scanned
19 using x-ray nano tomography (nano-XRM). The information from the nano-XRM scan was
20 then used as input into an object-based pore network generator, on which permeability fields
21 were simulated for a range of porosities, creating a synthetic Kozeny-Carman porosity-
22 permeability relationship targeted for the specific micro porous system present in
23 Estailades. We found a good match between experimental and simulated Mercury Intrusion
24 Capillary Pressure (MICP) range in the imaged geometry and a good match between the
25 imaged and object generated permeabilities and MICP.

26 A micro-core of Estailades was then scanned using x-ray microtomography (μ CT), the
27 differential pressure was measured during single phase flow, and the rock was flooded with
28 highly doped brine to differentiate connected from unconnected micro-porosity. The
29 differential contrast between the dry and doped images was used to assign a porosity to each
30 voxel of connected micro-porosity. The flow through the pore space was then solved using a
31 Stokes-Brinkman solver while a second segmented image with no micro-porosity was solved
32 a Stokes solver. The differences between the measured permeability and the two computed
33 permeabilities was evaluated. We found that there was good agreement between both the
34 computed permeability of the Stokes and Stokes-Brinkman simulation with the measured
35 permeability. However, there was considerable differences in the velocity fields with the
36 Stokes-Brinkman simulation capturing stagnant regions of the pore space that were not present
37 in the Stokes simulations.

38 Additionally, we investigated the implications of including micro-porosity in
39 estimations of relative permeability. Nitrogen was experimentally co-injected through the core
40 with doped brine at a 50% fractional flow and imaged to the two-phase effective permeability.
41 This experimental measurement was compared with the numerical permeability simulated
42 using both Stokes and Stokes-Brinkman models for several saturation points along a synthetic
43 MICP injection curve. We found that the Stokes simulation was not able to predict relative
44 permeability with this method due to the major flow paths in the macro-porosity being impeded
45 by the injected non-wetting phase. The Stokes-Brinkman simulations, however, allowed flow
46 in the microporous regions around these blocked flow paths and was able to achieve a relative
47 permeability prediction that was a reasonable match to the experimental measurement. This
48 method could be used to predict relative permeability in water wet pore-structures with high
49 micro-porosity.

51

52 INTRODUCTION

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54 Experiments combining X-ray microtomography (μ CT) with *in situ* flow apparatus is now an
55 accepted method of studying pore scale processes in real rocks [1, 2]. Pore-scale imaging
56 experiments coupled with simulation is an increasingly important tool used in industry prediction
57 of geological and petrophysical properties including porosity and connectivity [3],
58 mineralogical heterogeneity [4], and relative permeability [5, 6].

59

60 Typically, these simulations are done on the segmented image and are only concerned with the
61 macro pore space where the fluid solid boundary is fully resolved and able to be segmented
62 into pore and grain on a voxel-by-voxel basis [7]. When the rock grains are solid and the pore
63 throats are large compared to the image resolution, a reasonably accurate segmentation is all
64 that is needed to get a realistic estimation of flow through the rock [8, 9]. However, not all
65 grains are non-porous, and intra-granular micro-porosity is a significant contributor to total
66 carbonate micro-porosity [10]. Most carbonate rocks have grains that are micro-porous [11] –
67 hereafter defined as a grain that has interior porosity that is not fully resolvable at the resolution
68 of the imaging apparatus. Furthermore, over 50% of world oil is stored in carbonate reservoirs
69 [12].

70

71 The Stokes-Brinkman flow simulation technique combines Darcy's law effective media flow
72 with pore-scale Stokes flow has been proposed as a solution to this problem bridging the gap
73 between the fully resolved pore-scale and the partially resolved nano-scale [13, 14],
74 particularly when macro and micro-porosity are effectively separated in spatial length scale. In
75 many variations of Stokes-Brinkman simulations Darcy's law is solved in the semi-solid rock
76 matrix based on an estimated permeability which is derived from the calculated porosity using
77 the relative greyscale between solid grains and pore space.

78

79 Assigning porosity values to partially resolved voxels is well documented [15] and has been
80 used in conjunction with Mercury Intrusion Capillary Pressure (MICP) measurements in many
81 core-scale simulations on x-ray tomography images that do not have sufficient resolution to
82 see the structure and connectivity of the pore space needed to make a Navier-Stokes calculation
83 possible. In this case, reconstructed greyscale values are used as an analogue for porosity and
84 the permeability is assigned to each porosity value based on Kozeny-Carman estimations. This
85 method of assigning a relationship between porosity and permeability, however, is based on an
86 even the assumption that the porous medium is effectively represented by an even packing of
87 equally-sized elliptical beads [16, 17]. Furthermore, this method does not include any influence
88 associated with micro-pore space connectivity [18]. A section of micro-porosity may have high
89 porosity without necessarily being connected to the macro porosity in any significant way.

90

91 To quantify the connected porosity of the pore space Lin, Al-Khulaifi [19] flooded the rock
92 with highly doped brine at varying concentrations. They found that the highest doped brine
93 gave the best contrast and was able to quantify the distribution of connected and unconnected
94 porosity, as well as the porosity distribution of the connected porosity by thresholding the
95 difference between the dry scan and the doped scan. Any differences between the two images
96 must be associated with a change in saturation of the micro-porosity, with the magnitude of the
97 change being associated with the fractional change. This is similar to the method used by Ott,
98 Andrew [20] to quantify pore scale behaviour during salt precipitation.

99

100 The Kozeny-Carman equation related the permeability K to the porosity ϕ by:

101

$$K = \frac{\varphi^3}{c(1 - \varphi)^2 S^2}$$

102 where c is the Kozeny constant and S is the specific surface area based on the solid volume.
103 This relationship can be used to relate local pore structure to macroscopic flow behaviour;
104 however, the Kozeny-Carman method is fundamentally flawed when representing more
105 complex pore structures as it assumes a homogeneous pore structure of evenly packed,
106 uniformly sized spherical grains. Furthermore, the Kozeny-Carman method does not
107 incorporate any geological processes that would change the shape and connectivity of the
108 pore space (i.e. compaction and diagenesis). To properly define this relationship at the pore -
109 scale it is necessary to image the structure of the micro-porosity and numerically calculate the
110 porosity and permeability relationship from a segmented image which well resolves the pore
111 structure at the nano-scale.

112 Nano-scale techniques including FIB-SEM (focused ion beam scanning electron microscopy),
113 helium ion microscopy, and nano x-ray microscopy (nano-XRM) have emerged as
114 technologies capable of resolving this porosity at the resolution of several nm for FIB-SEM
115 and nano-XRM [21] down to tens of angstroms for the helium ion [22, 23]. However, when
116 imaging at this resolution it is only possible to see small volumes of rock on the order around
117 $10 \mu\text{m} \times 10 \mu\text{m} \times 10 \mu\text{m}$ for charged beam instruments and around $65 \mu\text{m} \times 65 \mu\text{m} \times 65 \mu\text{m}$ for
118 nano-XRM. Thus, it is necessary to either image many different parts of the micro pore
119 structure or to find a way of extrapolating these structures synthetically.

120

121 Early digital rock analysis efforts used synthetic pore space generation extensively to examine
122 simple systems at the pore scale, however as imaging technologies have improved, it has
123 largely supplanted synthetic pore network generation for the examination of simple geometries.
124 Nevertheless, synthetic techniques do present specific advantages, especially when examining
125 mechanisms behind various processes while controlling the amount of heterogeneity [24, 25]
126 These synthetic pore spaces can either be constructed physically, usually by glass beads or
127 etchings in glass (e.g. [26-28]) or numerically using a pore space generator, using stochastic or
128 object-based techniques, subject to various constraints (e.g. [29]).

129

130 Recently, Andrew [30] has used a combination of numerical pore space generation and
131 multiscale imaging to investigate the porosity-permeability relationships of shale and
132 sandstones. He found that the (geological) diagenetic processes inherent in the creation of the
133 porosity should dictate how to approach the generation so as to accurately predict the evolution
134 of permeability. Shales have a porosity defined by authigenic growth within a deformable
135 matrix, making the pore structure significantly more spherical than intergranular pore
136 structures, common in sandstones and carbonates. As such, authigenic organic hosted pore
137 networks can be modelled (to a high level of statistical similarity when compared with imaged
138 pore networks) using a network of (overlapping) spherical pores, while sandstones can be
139 modelled similarly accurately by modelling individual grains as convex polyhedra, with the
140 pore network given by the space between the grains.

141

142 The goal of this study is to present a method that combines fluid flow experiments with
143 multiscale imaging of macro and micro-porosity and synthetic pore space generation to
144 increase the accuracy of numerical multiphase pore-scale simulations on microporous rocks
145 using Stokes-Brinkman simulations.

146

147 First, we imaged the micro-porosity of Estailades limestone using nano-XRM at a spatial
148 resolution of 50nm. We then analysed this image to generate a statistical description of the
149 micritic grains which constitute the nano-porous network. This statistical description was then
150 used to generate a range of pore networks using object-based techniques, creating a porosity-
151 permeability map specific to this rock type. A 6-mm diameter, 24-mm long core plug of
152 Estailades Limestone was then imaged using micro-CT at a resolution of 3.9 μm , with and
153 without high contrast brine. This was then segmented into pore, grain, and 12 microporous
154 regions of varying porosity, which was used as the input to a Stokes-Brinkman solver with
155 each of the microporous regions assigned a permeability based on the generated porosity-
156 permeability relationship. The permeability and flow fields of the Stokes-Brinkman simulation
157 were then compared to a Stokes only flow simulation with the same pore space.

158

159 We then ran a steady-state flow experiment on the same core *in situ*. Nitrogen gas and 30 wt.
160 % potassium iodide (KI) brine were injected into the core at a fractional flow of 0.5 and allowed
161 to come to steady-state. The core was then imaged, and the differential pressure was measured,
162 corresponding to a single point on the relative permeability curve. Concurrently, relative
163 permeability was simulated with GeoDict software [31] by using an MICP-like simulated
164 injection method where the non-wetting phase is allowed to occupy regions of the pore network
165 using a maximal inscribed spheres technique. Permeability through only the wetting phase was
166 then simulated using both Stokes and Stokes-Brinkman methods by simulating single phase
167 flow through the wetting phase only. The relative permeability measured *in situ* was then
168 compared to these simulation results.

169

170 MATERIALS AND METHODS

171

172 **Sample Characterisation**

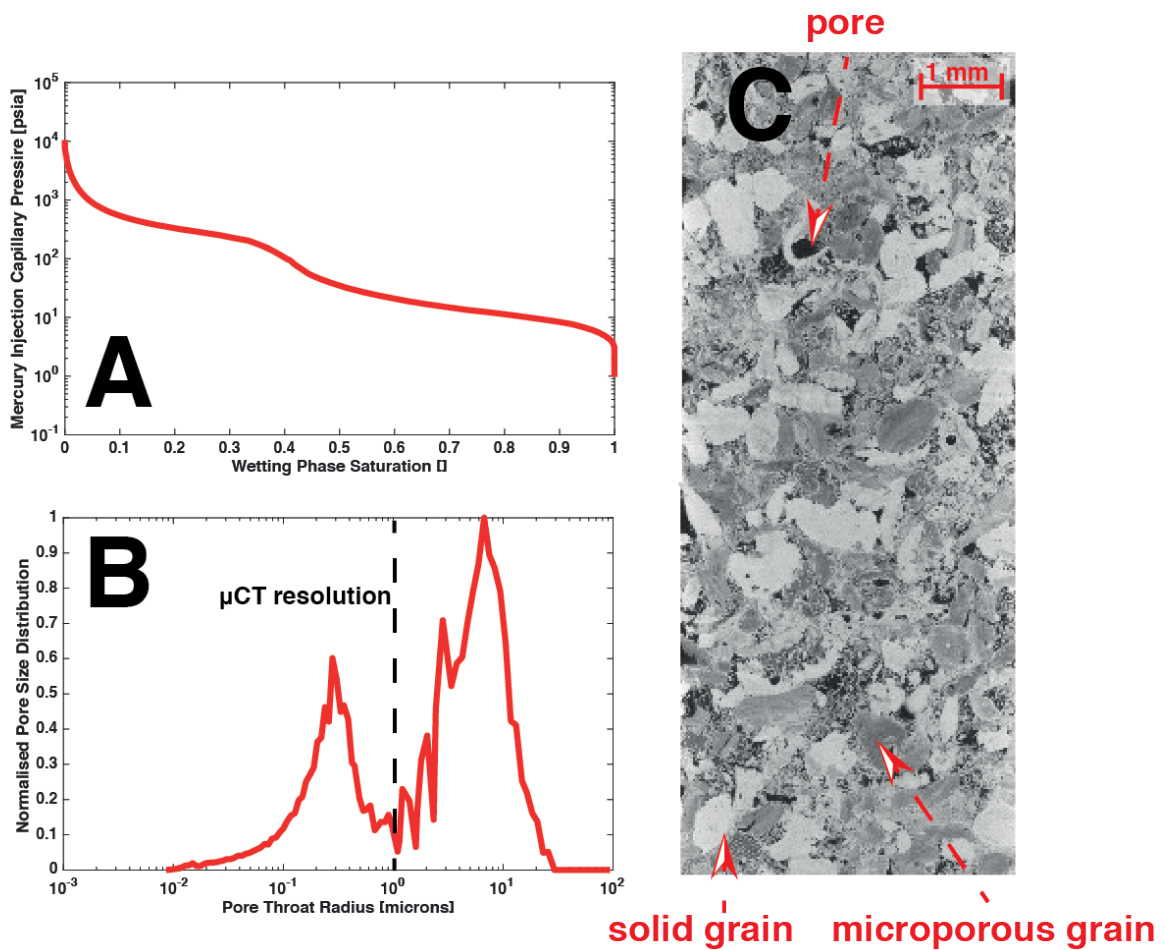
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174 Estailades is a limestone quarried at Oppede, France. It was deposited 22 million years ago
175 and is composed of mostly calcite (>97%) with a minor quartz component. Estailades is a
176 medium to coarse-grained bioclastic grainstone with microporous bioclast grains. The helium
177 porosity is 0.295 and a bulk-scale absolute permeability of $1.490 \times 10^{-12} \text{ m}^2$ (measured at
178 Weatherford Laboratories, East Grinstead, UK).

179

180 Estailades is a well-connected heterogeneous carbonate. The MICP curve and pore-throat
181 distribution show a clear bimodal population of pore throats [Figure 1]. However, only the
182 larger population of throats is accessible to μ -CT imaging, and only contributes around half of
183 the total porosity, with the remainder residing in the microporous bioclasts.

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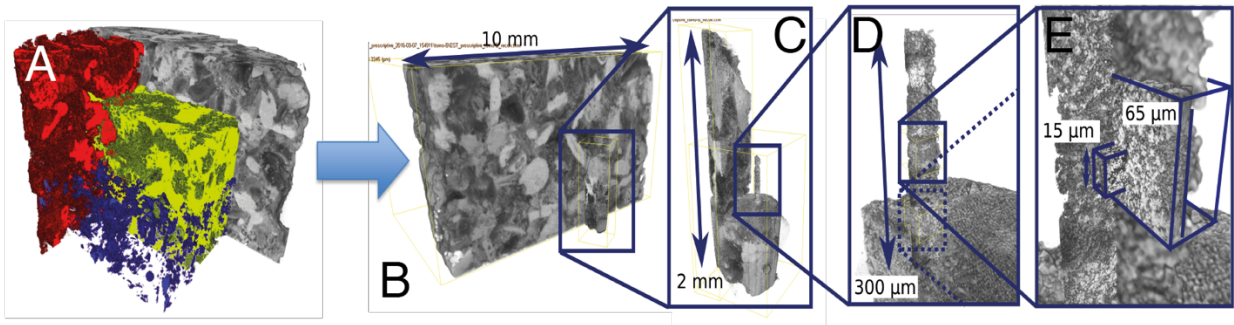


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Figure 1 Estailades limestone MICP curves (A,B) with the μ CT resolution shown as a dashed black line. A μ CT image (C) with labelled pores, solid, and microporous grains.

Nano scale Imaging

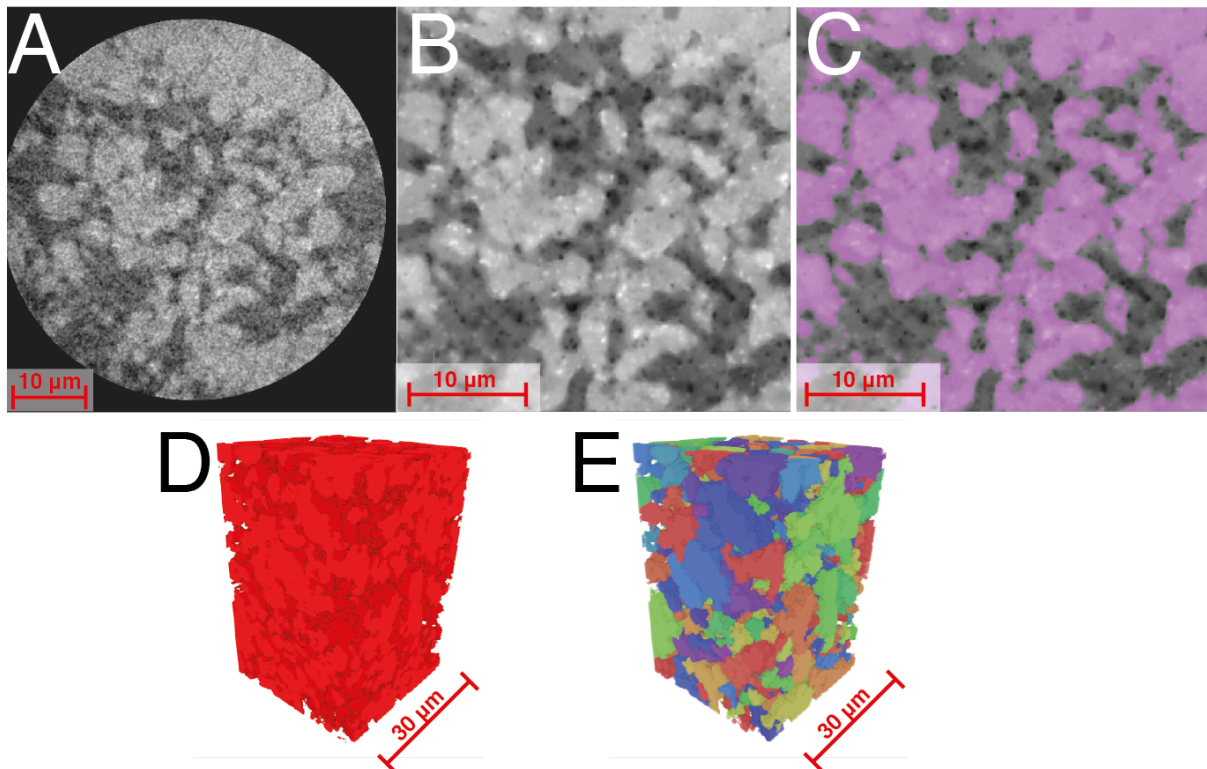
The ZEISS Xradia Ultra 810 nano-XRM was used to image microporous structure down to a resolution of 50 nm [Figure 2]. The extremely high resolution of this system requires relatively stringent sample size restrictions, with samples having a diameter no larger than 100 μ m. Sample preparation of such a small sample is extremely challenging, even in non-heterogeneous samples, and the heterogeneous nature of many geological systems compounds this challenge significantly. To prepare such samples a complex multi-stage sample preparation protocol was performed using an Oxford gimballed laser micro-machining mill model A-532-DW (www.oxfordlasers.com) [32].



200 **Figure 2** A core of Estailades is scanned in the μ CT (A) and the pores (red), solid grains (blue),
 201 and microporous grains (yellow) are identified. An interesting subsection is identified (B) and
 202 milled (C). A section of the milled section (D) is then scanned in the nano-XRM (E).
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 204

205 First a 10mm diameter mechanically drilled sample of (air saturated) Estailades was scanned
 206 low (10 μ m) resolution using a ZEISS XRM-510 μ CT. Fiducial marks made of aluminum tape
 207 were placed on the surface of the sample to enable alignment between the laser micro-
 208 machining stage and the sample. The low-resolution image was then segmented into micro-
 209 porosity, macro-porosity and solid mineral grains using ZEISS Zen Intellesis machine learning
 210 based segmentation [Figure 3]. As Estailades is very simple mineralogically (>97% calcite),
 211 the greyscale of each voxel within the micro-porosity is only associated with the internal
 212 porosity of that voxel, ranging from the value observed within the macro-porosity
 213 (corresponding to a 100% porosity within the voxel) to that observed within the solid grain
 214 (corresponding to a 0% porosity). The greyscale distribution within the microporous phase
 215 therefore corresponds to its internal porosity distribution [Figure 3]. A 30 μ m \times 30 μ m \times 30
 216 μ m region of micro-porosity (corresponding to 3 \times 3 \times 3 voxels within the macroscopic image)
 217 was then identified which corresponded to the modal porosity within the porosity distribution
 218 of the micro-porosity (a porosity of 40%). The offset of this region relative to the sample
 219 fiducial marks was then measured, and the region of interest (ROI) aligned underneath the laser
 220 axis. A coarse pillar of dimensions 800 μ m diameter, 2 mm length was extracted from the
 221 sample using the laser micro-machining in a top-down fashion. This sample was then
 222 transferred to the end of a dowel pin using an automated sample transfer procedure. This coarse
 223 pillar was then imaged within the μ CT with a voxel size of 800 nm along its length. This image
 224 was then registered with the macroscopic (10 μ m voxel size) image of the 10mm diameter
 225 pillar using a normalized mutual information metric.
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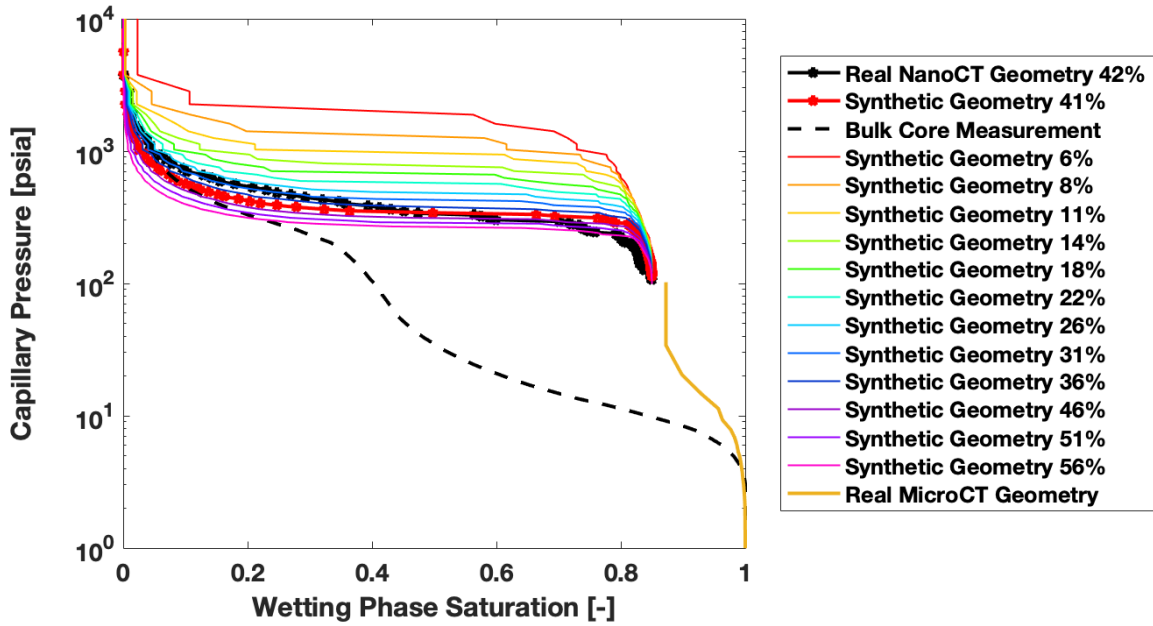
227 The coarse pillar was then transferred to a rotational stage within the laser system with a
 228 rotational axis perpendicular to the laser axis. The sample was then slowly reduced to produce
 229 a fine pillar 65 μ m in diameter operating the laser in lathe-like fashion. This pillar was then
 230 imaged along its length within the ZEISS Ultra NanoCT at low resolution (128nm voxel size).
 231 This dataset was then registered with the lower resolution dataset of the coarse (800 μ m
 232 diameter) pillar (and thereby the macroscopic image of the 10mm diameter core). This multi-
 233 scale representation of the micro-porosity was then inspected to identify the location within the
 234 nano-XRM corresponding to location within the fine pillar of the region of modal (40%)
 235 porosity, initially identified from the macroscopic image. This region was then scanned at the
 236 final, highest resolution (32nm voxel size) non-invasively within the fine pillar. The internal
 237 structure of the imaged micro-porosity consists of subhedral crystals of micrite, consistent with
 238 SEM and transmitted light microscopy analysis of this sample [33].
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 242 **Figure 3** (A) The raw nano-XRM image, (B) cropped and filtered image, (C) grains identified
 243 by machine learning, (D) segmented 3-D image, and (E) separated grains.
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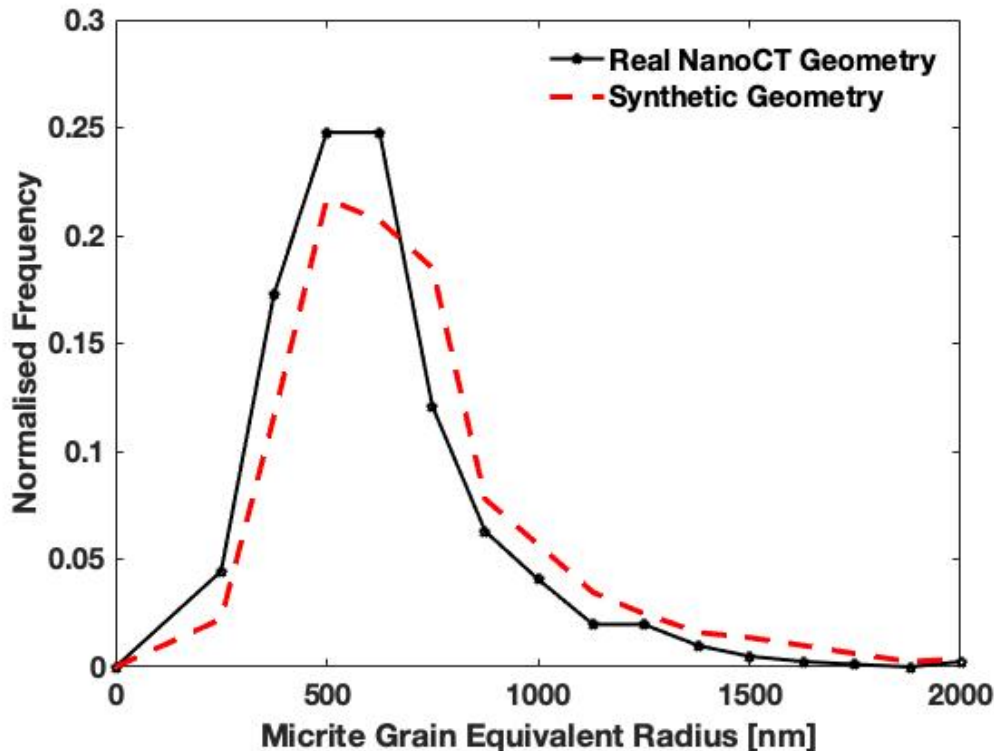
245 The resulting reconstructed image was first denoised using an edge preserving non-local means
 246 filter, then segmented using ZEISS Zen Intellesis machine learning based segmentation. Such
 247 a segmentation technique has been showed in quantitative benchmarks to be significantly more
 248 robust when dealing with such noisy and challenging images [34, 35]. The resulting porosity
 249 observed within the image (41%) matched well with the inferred porosity of the 30μm x 30μm
 250 x 30μm region initially identified from the macroscopic 10μm voxel size image of the 10mm
 251 diameter core. Stokes flow was simulated within this pore geometry using the LIR FlowDict
 252 solver [36] (Math2Market GeoDict), giving a nano-porous permeability of $2.63 \times 10^{-15} \text{ m}^2$.
 253

254 MICP was also simulated on this structure using the SatuDict modules of GeoDict
 255 (Math2Market), showing a good match in peak position between the microporous peak in the
 256 experimental MICP and the simulated MICP through the microporous structure [Figure 4].
 257



258
 259 **Figure 4** The real nano-XRM (solid black), real micro-CT (solid brown), synthetic (red and
 260 rainbow), and bulk core measured (black dashed) MICP curves.
 261

262 To extend this result to cover the porosity range observed within the microporosity a suite of
 263 similar pore networks were constructed using object based techniques [37]. The connected
 264 micritic matrix was separated into a network of discrete, separated micrite grains using a
 265 watershed algorithm [Figure 3E]. The volume and equivalent radius distribution of these grains
 266 was then measured, showing a unimodal distribution with a peak equivalent grain radius of
 267 around 500 nm [Figure 5].
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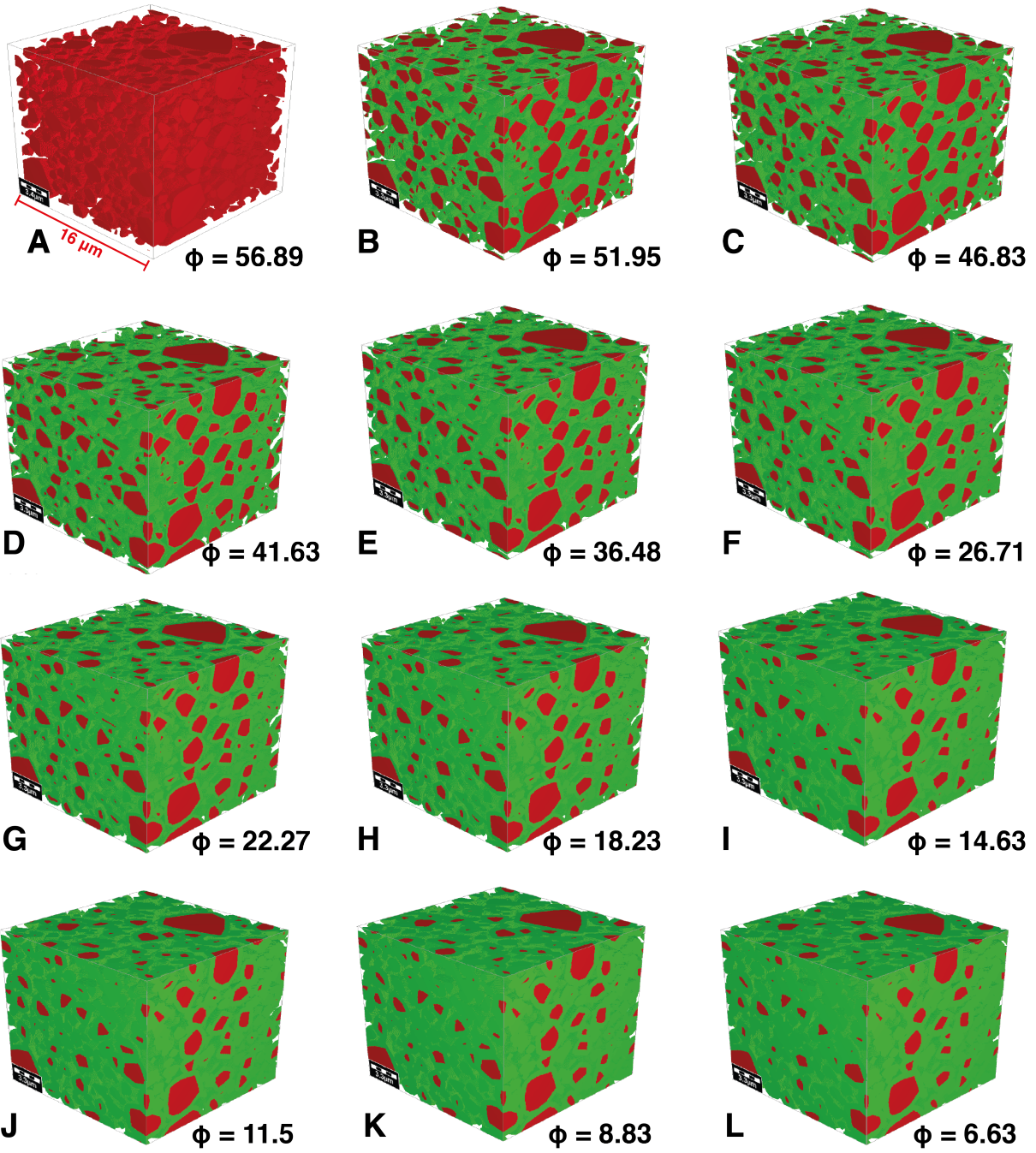
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Figure 5 Micrite grain equivalent radii frequency histograms for the real geometry imaged by nano CT (black) and the 41% porosity synthetic image (red).

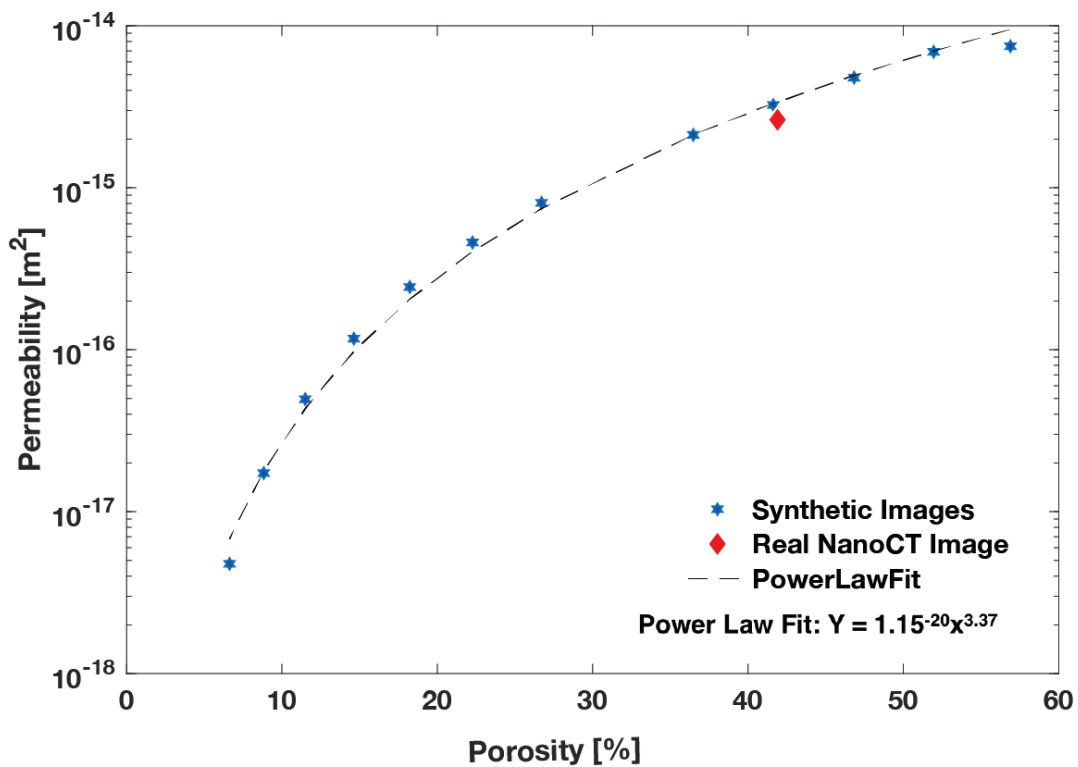
A histogram of the volumes of the separated grains and the porosity and permeability of the nano-XRM image is then used to generate synthetic grains in a pore space [Figure 6A]. All geometry creation was performed in GeoDict software using the GrainGeo module. These grains are then dilated successively to create twelve synthetic pore spaces with porosities ranging from 6.63 to 56.89.

A suite of pore geometries were then created by modelling the micritic grains as convex polyhedra, bounded by spheres with a radius distribution given by the radius distribution of the micritic grains. The polyhedra were placed randomly within a 3D volume of size $16 \times 16 \times 16 \mu\text{m}^3$ without allowing granular overlap until no more polyhedra could be fit within the pore geometry. This structure was then progressively dilated by 1 voxel at a time, with simulations of both MICP and Stokes-flow permeability performed on each successive pore network until no connected pore network remained [Figure 6B-L], creating a porosity-permeability relationship for the intragranular micritic micro-porosity in this sample [Figure 7]. We found that the porosity-permeability relationship corresponded to a power law fit with an exponent of 3.37 which is reasonable when compared to previous published Kozeny-Carmen estimations for porous rocks [38].



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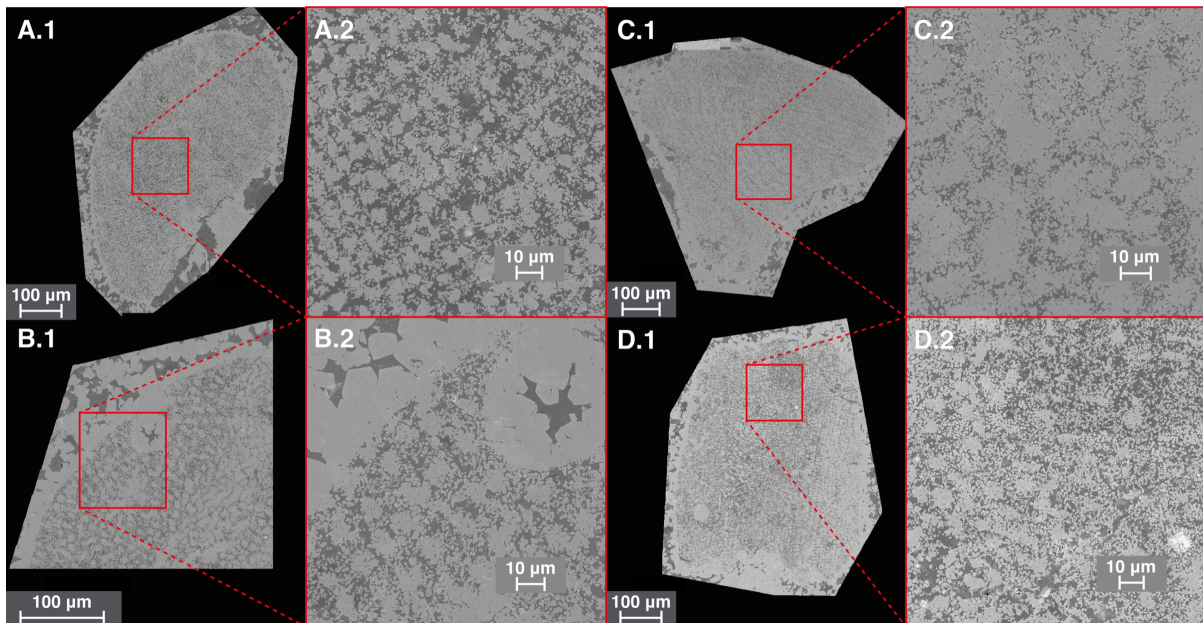
Figure 6 (A) is the synthetic pore space generated from the volumetric grain size distribution from the nano-XRM image. (B-L) Dilated grains (green) with preserved grains (red) and pore space (clear).



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 300 **Figure 7** The synthetic porosity-permeability relationship (blue stars), with the power law fit
 301 of $Y = 10^{-20}x^{3.72}$ (black dashes) and the real image porosity and permeability value (red
 302 diamond).

303
 304 In addition to nano-XRM imaging we also imaged several microporous grains using a Zeiss
 305 Sigma 300 SEM at a pixel resolution of 20 nm to examine the structural heterogeneity inside a
 306 microporous grain [Figure 8]. We found that the micritic structures were reasonably regular
 307 and consistent with our generated synthetic grain packings. However, it is interesting to note
 308 the high-density layers of compacted calcite on the outside of the grains which is likely to be
 309 lower permeability than the interior of the grains.

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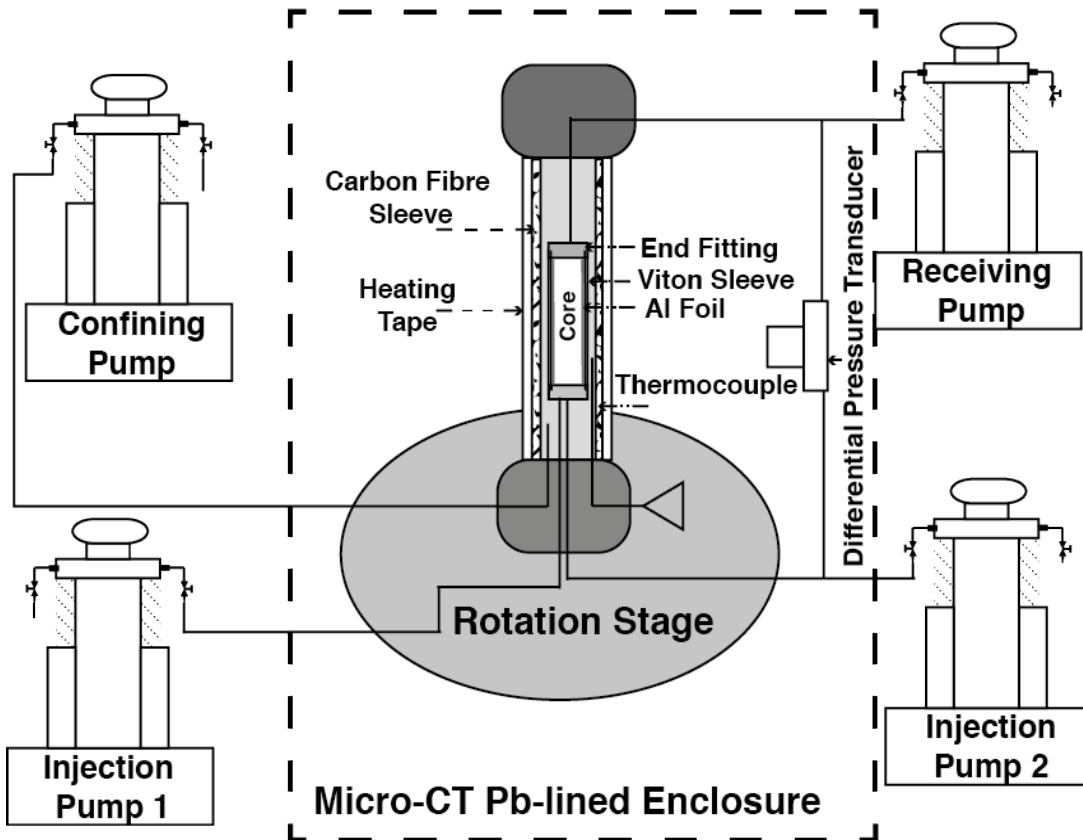


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 312 **Figure 8** Estailades grains (A.1-D.1) and high-resolution sections (A.2-D.2) showing micritic
 313 calcite with some dense calcite around the grain boundaries.
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315 Pore-scale experiments and imaging

316
 317 A new 5mm diameter, 24mm long core of Estailades was then drilled from the same 1 m³
 318 block of limestone as used for the nano scale study. The core was loaded into a carbon fibre
 319 core holder (airborne composites) and then imaged dry [Figure 11A]. The core was confined
 320 using DI water at 10 bar and two high pressure syringe (Teledyne isco) pumps were used to
 321 drive highly doped brine of 30 wt.% KI through the core with a constant back pressure of 2 bar
 322 [Figure 9] for 1000 pore volumes and reimaged with the brine inside. The core was washed
 323 with DI water for 1000 pore volumes and three differential pressure measurements were made
 324 using a Keller PD-33X differential pressure transducer with a total range of 300kbar and an
 325 error of 0.01% across the whole range during flow of 0.5 0.75 and 1.25 mL.min⁻¹ with a 2-bar
 326 back pressure [Figure 12A].
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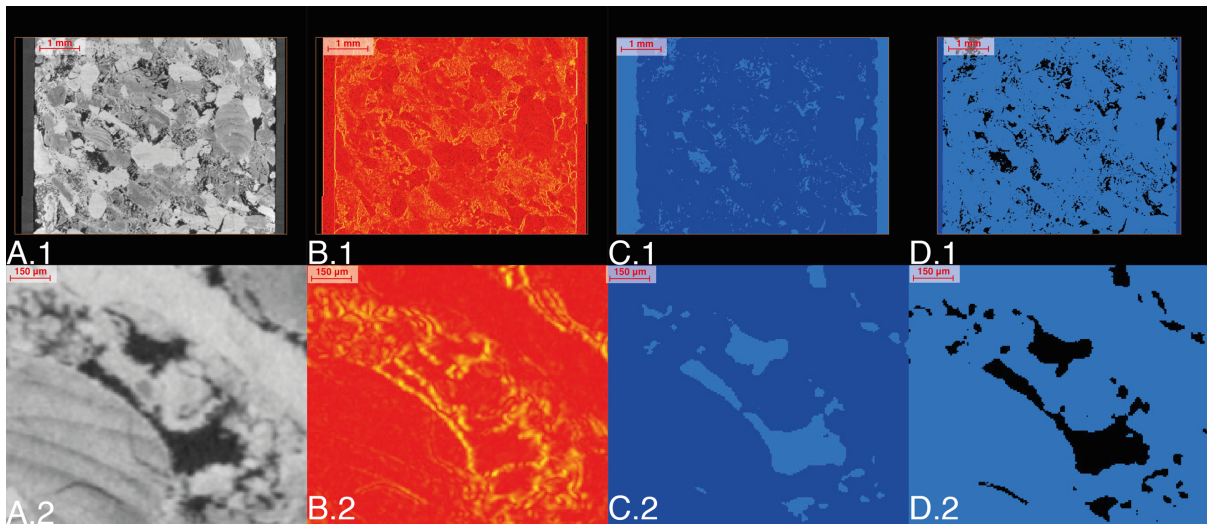
328 The core was then confined at 120 bar, the internal pore pressure raised to 100 bar and the
 329 temperature raised to 50°C. Nitrogen gas (N₂) was co-injected through the core with 30 wt. %
 330 KI brine and allowed to come to steady-state. The differential pressure was measured, and
 331 images of the core were taken *in situ*. Precise details of this experimental apparatus and method
 332 of measuring relative permeability can be found in [39].



333
 334 **Figure 9** The experimental apparatus consists of the injection, receiving, and confining pumps
 335 outside the micro-CT, with a core holder and differential transducer on the rotation stage inside
 336 the micro-CT lead-lined enclosure. The core holder is made of carbon fibre and is equipped
 337 with thermocouples and heating wrap. The core is wrapped in Aluminium foil inside a viton
 338 sleeve which is attached to the end fittings supporting the two injection pumps and receiving
 339 pump.

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 341 **Pore scale image processing**

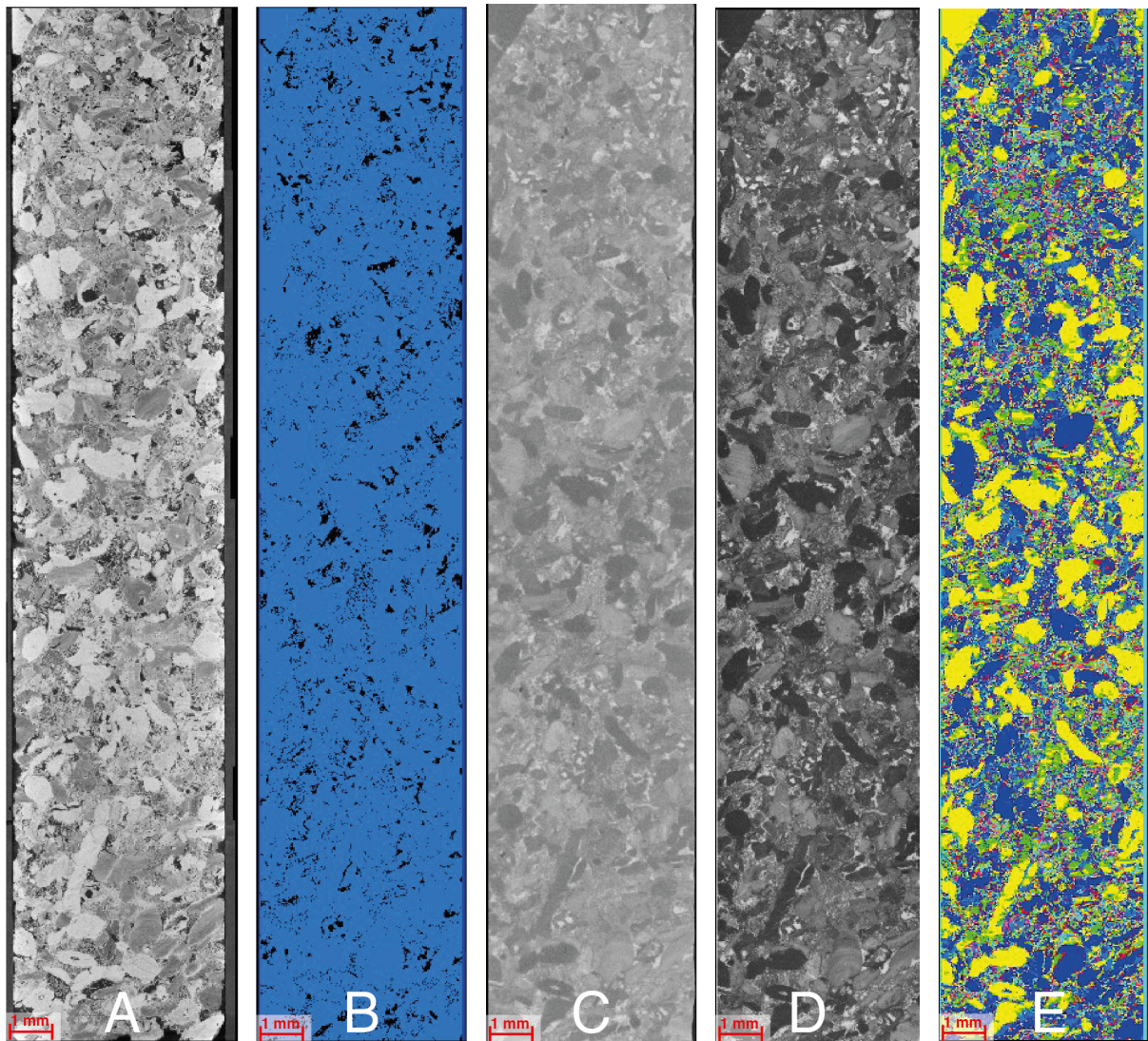
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 343 Initially a watershed segmentation was performed on the dry image Estailades to identify
 344 regions of pore and rock [Figure 10]. While watershed segmentation gave a reasonable
 345 estimation of porosity and visual examination confirmed fidelity of pores, the pore space itself
 346 was unconnected across the length of the domain [Figure 10C]. Without a connected pore
 347 space, Stokes simulations are not possible. The reason for this dysconnectivity is inherent in
 348 the design of the watershed method where the tightest pore throats are most likely to suffer
 349 from partial volume effects and have smaller gradients in the gradient image [Figure 10B].
 350 These smaller gradients are less likely to be identified as pore space and thus the pore throats
 351 may be closed artificially. To combat this problem of closed throats we instead used machine
 352 learning segmentation which uses not only image gradients but texture and other higher order
 353 features to identify phases [Figure 10C]. It is important to note that while segmentation using
 354 machine learning can be more accurate, it takes longer to train the algorithm and is more
 355 computationally expensive compared to watershed [40, 41].
 356



357
 358 **Figure 10** Watershed segmentation vs Weka 3D. The dry scan (A), gradient image (B),
 359 watershed segmentation (C), and Weka 3D machine learning segmentation (D) are shown at
 360 low (1) and high zoom (2).
 361

362 The Weka3D machine learning segmentation algorithm in Fiji was used to segment the macro
 363 pore space for both the Stokes and Stokes-Brinkman simulations [Figure 11B]. The images of
 364 the rock filled with doped brine were then used to identify the solid grains and unconnected
 365 micro-porosity. The pore space, unconnected micro-porosity and solid grains were then
 366 masked and the remaining greyscale values were used to label the connected microporous
 367 grains based on porosity using Avizo 9.3 (www.fei.com) [Figure 11D]. These porosities were
 368 then assigned a permeability based in FlowDict based on the permeability calculated on the
 369 synthetic pore spaces [Table 1]. A similar workflow was followed for the images of imaged *in*
 370 *situ* fluid distributions, with the images registered to the dry scan and then the nitrogen was
 371 segmented inside the pore space using a watershed algorithm on the difference image. The non-
 372 wetting phase saturation can then be calculated based on the number of pore-space voxels filled
 373 with gas. Figure 14 shows the nitrogen in the pore space visualised as small, medium, and large
 374 clusters.

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Figure 11 The image processing workflow. The dry scan (A) is segmented using machine learning (B). The doped scan (C) is subtracted from the dry scan to get the difference image (D). The difference image greyscale is then thresholded to 12 different porosity values and grains and then the pore space of segmented dry scan (B) is masked to create the 14-phase segmentation of solid grains, 12 types of microporous grains, and pores (E).

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Table 1 Porosity and permeability values for micro-porosity calculated from synthetic images.

Porosity Difference [%]	Range in Image	Porosity of Simulation [%]	Simulated Permeability [m^2]	Fraction of Total Core Volume [%]	Segmentation Phase #
100		N/A	N/A (Pore)	9.95	1
54.45 – 99.9		56.89	7.47×10^{-15}	17.16	2
49.4 – 54.4		51.95	6.91×10^{-15}	4.63	3
44.3 - 49.3		46.83	4.79×10^{-15}	4.62	4
39.1 - 44.2		41.63	3.24×10^{-15}	4.86	5
31.6 - 39.0		36.48	2.12×10^{-15}	7.22	6
24.5 – 31.5		26.71	8.06×10^{-16}	6.93	7
20.3 - 24.4		22.27	4.59×10^{-16}	4.09	8
16.5 - 20.2		18.23	2.44×10^{-16}	3.65	9
13.1 - 16.4		14.63	1.17×10^{-16}	3.21	10
10.2-13.0		11.5	4.95×10^{-17}	2.78	11
7.8-10.1		8.83	1.73×10^{-17}	2.36	12
0.1-7.7		6.63	4.76×10^{-18}	7.73	13
0		N/A	N/A (Grain)	20.83	14
N/A		N/A	0 (Viton Sleeve)	N/A	15

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Numerical methods

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All simulations in this paper were completed using modules contained in Math2Market GeoDict. This includes synthetic pore-space generation (GrainGeo), Stokes flow (FlowDict), Stokes-Brinkman flow (FlowDict), and synthetic MICP injection (SatuDict).

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The GrainGeo module in GeoDict [42] can be used to create digital 3D models of ceramics, sintered materials, grain packings or digital rocks. The starting point for modelling are user-defined parameter such as known grain size distribution, pore size distribution and grain shapes. By changing the parameters of the underlying the model, new material structures are designed and their material properties can be studied.

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The LIR solver [36] in the FlowDict module is a very fast and memory efficient iterative finite volume method. The solver computes the permeability, as well as velocity and pressure fields, on large 3D images. The LIR solver can be used for the numerical solution of the Stokes, Stokes-Brinkman, Navier-Stokes, and Navier-Stokes-Brinkman equations. Usually, 3D images are represented as regular voxel grids where the number of grid cells grows cubically. The LIR solver uses an adaptive grid, instead of a regular grid, to reduce significantly the number of grid cells. The basis of the adaptive grid is a data structure called LIR-tree [43] that is used for spatial partitioning of 3D images. The pore space is coarsened in areas with small velocity and pressure variations, while keeping the original resolution near the solid surfaces and in regions where velocity or pressure vary rapidly. Pressure and velocity are discretized on staggered grids and they are arranged in such a way that each cell can satisfy the (Navier-)Stokes(-Brinkman)-equations independently from its neighbour cells.

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The pore morphology method [44] is used in SatuDict and it predicts the distribution of a wetting phase (WP) and a non-wetting phase (NWP) inside a porous medium. The method

415 distributes two fluids by using morphological operations rather than solving partial differential
416 equations. For drainage, it can be envisioned that spheres are pushed into the structure and
417 placed in the pore space where the pore size is greater than a certain radius. The radius is
418 decreased in an iterative process and this corresponds to an increase of the capillary pressure.
419 The superposition of all spheres represents the NWP. The pore morphology method achieves
420 this placement of spheres by dilation and erosion processes of the solid phase with the pore
421 space. Additional connectivity checks [45] with respect to NWP and WP reservoirs can be used
422 to increase the validity of the distributions and they allow to introduce residual phases. The
423 output of the algorithm is a finite sequence of quasi-stationary states. For relative permeability
424 of the WP, for instance, we solve a single-phase flow inside the WP and treat the interface
425 between WP and NWP as immobile no-slip interface.

426

427 RESULTS AND DISCUSSION

428

429 Differential pressure measurements were used with Darcy's Law:

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431

$$k = - \frac{Q\mu L}{A(\Delta P)}$$

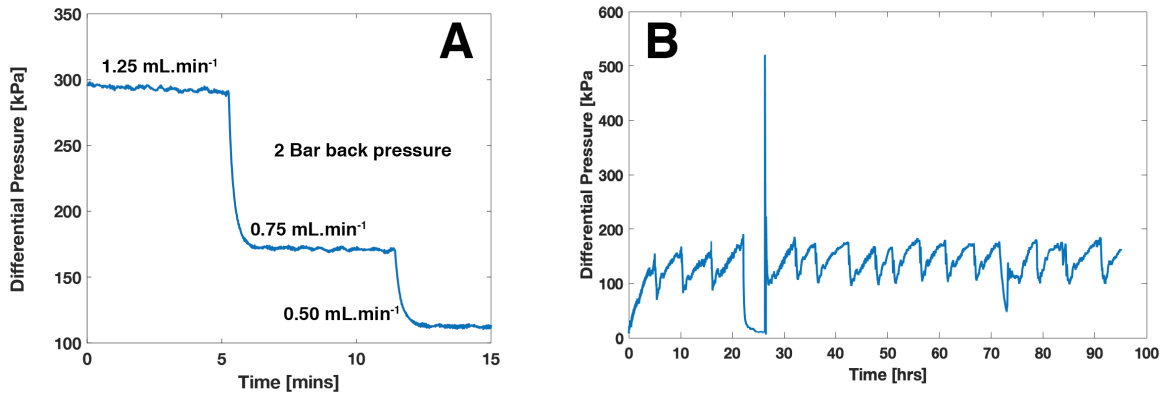
432

433 where k is permeability [m^2], Q is the flowrate [$\text{m}^3 \cdot \text{s}^{-1}$], μ is viscosity [$\text{mPa} \cdot \text{s}^{-1}$], L is the length
434 of the core [m], A is the cross-sectional area of the core [m^2], and ΔP [Pa] is the differential
435 pressure between the inlet and the outlet of the core. The calculated permeability from the
436 differential pressure measurements was $2.43 \times 10^{-14} \text{ m}^2$ [Figure 12A]. Each of the large-scale
437 simulations were run for 162 hours on 24 3.0GHz cores. The Stokes simulation used around
438 80 GB of RAM while the Stokes-Brinkman used around 256 GB of RAM. Unfortunately, due
439 to memory constraints, the least permeable phases (1-5) were set to zero permeability in the
440 Stokes-Brinkman simulations. The estimated permeability of the Stokes simulation was $1.21 \times$
441 10^{-14} m^2 while the Stoke-Brinkman simulation was $3.57 \times 10^{-14} \text{ m}^2$. There values indicate that
442 the Stokes simulation under estimated permeability by 50% while the Stokes-Brinkman
443 simulation over estimated permeability by 46%.

444 There are two likely sources of error in the Stokes simulations – segmentation error and
445 unaccounted-for contributions of micro-porosity to permeability. It is possible that the Weka
446 segmentation needs more training and is still not capturing all of the small pore throats that
447 contribute to flow. However, we believe that it is more likely the lack of microporous regions
448 that closes off flow in places that would otherwise have hydraulic connection as we see in the
449 high-density calcite crystal layer on the SEM images of exterior of the grains in Figure 8.

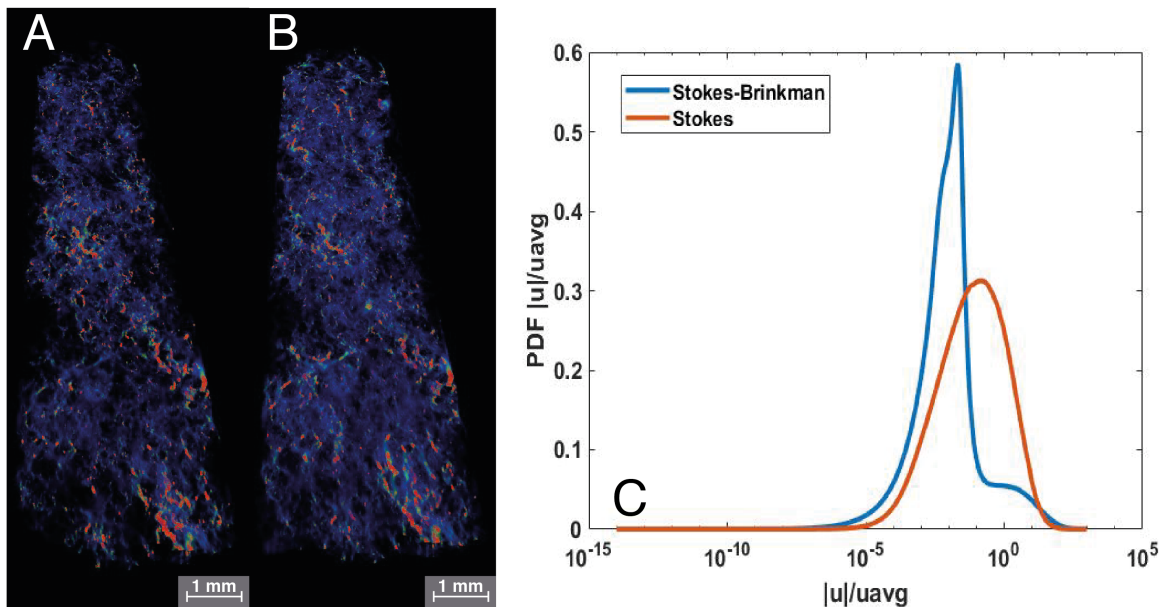
450 In contrast, the Stokes-Brinkman simulation over predicts permeability. We posit could
451 be due to an over prediction of connectivity in the microporous regions which is also consistent
452 with Figure 8. While our method of doped brine flooding should minimise misidentification of
453 completely unconnected areas of micro porosity, if there is a minor hydraulic connection the
454 doped brine would still flood the area very slowly and by the time 100 pore volumes have been
455 flooded through the core the micro-porosity would be completely flooded. A possible solution
456 to this problem would be to do time-resolved imaging during doped brine flooding to have
457 some idea of the local connectivity of each microporous voxel.

458



459
 460 **Figure 12** (A) Differential pressure [kPa] measurements across the core at brine flowrates of
 461 1.25, 0.75, and 0.5 mL.min⁻¹ with a back pressure of 2 Bar. (B) Differential pressure measured
 462 during co-injection of N₂ and KI brine.

463
 464 The velocity fields and probability density functions (PDFs) of velocity are shown in
 465 Figure 13. A visual inspection of the velocity fields does not reveal very much difference.
 466 However, when we compare the PDFs of velocity in Figure 13C we see a distinct difference in
 467 the peak velocities and tail. In the Stokes simulation the velocity PDF is a smooth gaussian
 468 distribution with a peak centred around 1. However, in the Stokes-Brinkman simulation we see
 469 a smaller secondary peak around 1 with the main peak around 10⁻² with a long tail. This
 470 indicates that in the Stokes simulation we are only capturing advective flow while in the Stokes-
 471 Brinkman simulation there is a large amount of slow flow through the micro pore space. This
 472 result has many applications but is particularly important during contaminant transport for
 473 predicting the concentration of contaminants with time. If the slower transport is not
 474 incorporated into the model than the peak and the tail will not be accurately predicted.
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 478 **Figure 13** Velocity fields rendered with high velocities in red and low velocities in blue for
 479 Stokes (A) and Stokes-Brinkman Simulations (B). The PDF's of velocity (C) are shown for
 480 Stokes (red) and Stokes Brinkman (blue) simulations.
 481

482 The segmentation technique employed for the macro pore space may also have a significant
483 control on the simulated velocity PDF. As discussed in the methods section, when a typical
484 watershed segmentation was attempted on this image the macro pore space was unconnected
485 throughout the length of the samples. In previous studies watershed has been used to segment
486 the pore space and the predicted permeability values were far below the ones predicted in this
487 paper. Menke, Bijeljic [46], Menke, Andrew [47], Menke, Bijeljic [48] report values ranging
488 from 1.53×10^{-14} to $1.57 \times 10^{-13} \text{ m}^2$. It is likely that pore space remained connected in these
489 cases because while the samples were imaged at approximately the same resolution, they were
490 significantly shorter (and thus overall contained less heterogeneity). However, the watershed
491 segmentation still did not properly segment the small throats and thus the permeability was
492 predicted to be much lower than would be expected from the bulk measured permeability of
493 $1.490 \times 10^{-12} \text{ m}^2$. For complex pore structures watershed segmentation will be less accurate as
494 the more sophisticated textural and featural segmentation approaches and should be used with
495 caution.

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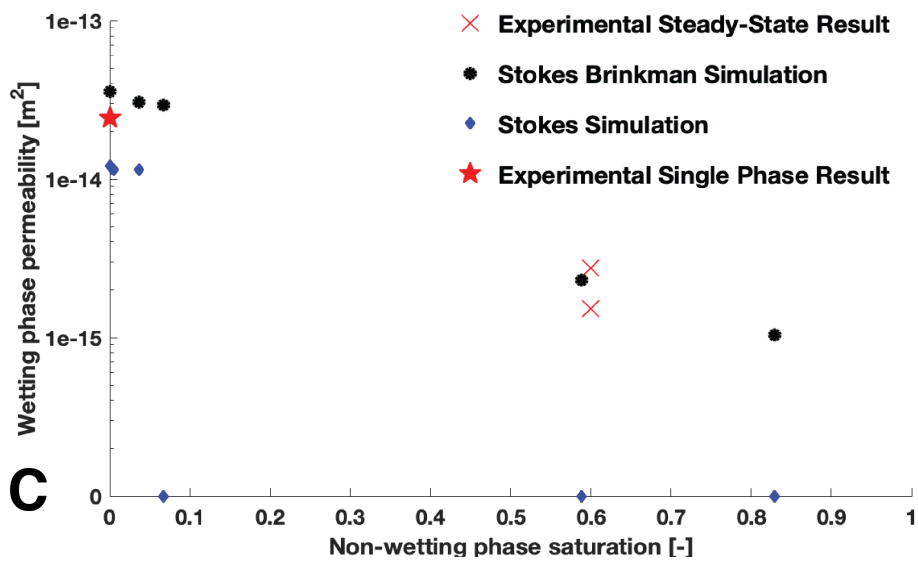
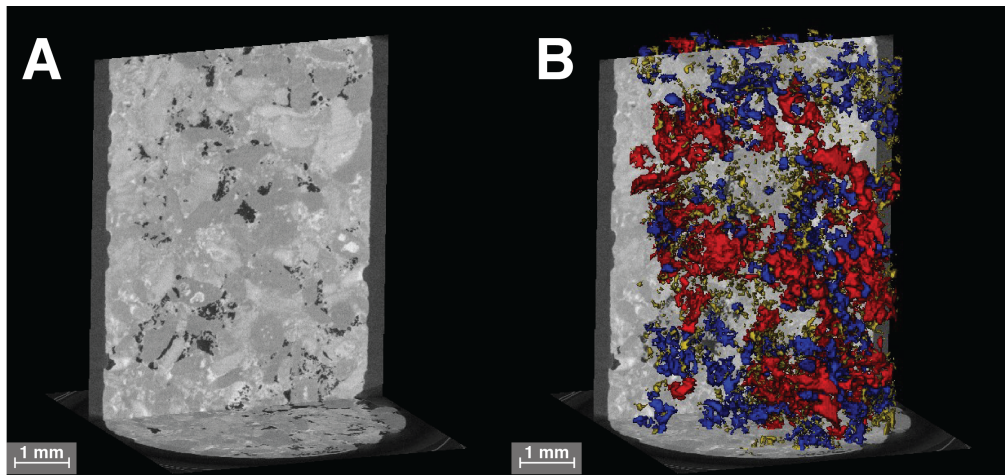
497 During co-injection we measured differential pressure for 95 hours. We observed a cyclic
498 perturbation where pressure builds from $\sim 90 \text{ kPa}$ to $\sim 180 \text{ kPa}$ over the course of ~ 5 hours and
499 then suddenly drops back down. These pressures correspond to wetting phase permeabilities
500 fluctuating between $1.52 \times 10^{-15} \text{ m}^2$ and $2.74 \times 10^{-15} \text{ m}^2$. We imaged the core during flow and
501 observed that the non-wetting phase saturation to be 0.6 in the macro pore space. It is important
502 to note that as each scan took around 5 hours any changes in saturation during this period would
503 be time-averaged. To try and understand why the pressure was building and releasing we
504 modelled the streamlines through the core using FlowDict [Figure 15]. We found that all flow
505 of the non-wetting phase is directed through a single small pore throat about two thirds of the
506 way through the core. We postulate that this small flow impedance was causing capillary
507 pressure to build and then be released as the local capillary pressure built enough to flow
508 through this small pore throat, a theory supported by the approximately periodic nature of the
509 pressure fluctuations [49, 50]. More experiments targeting the investigation of this theory
510 would be an interesting target for future research, however, are out of the scope of this paper.

511

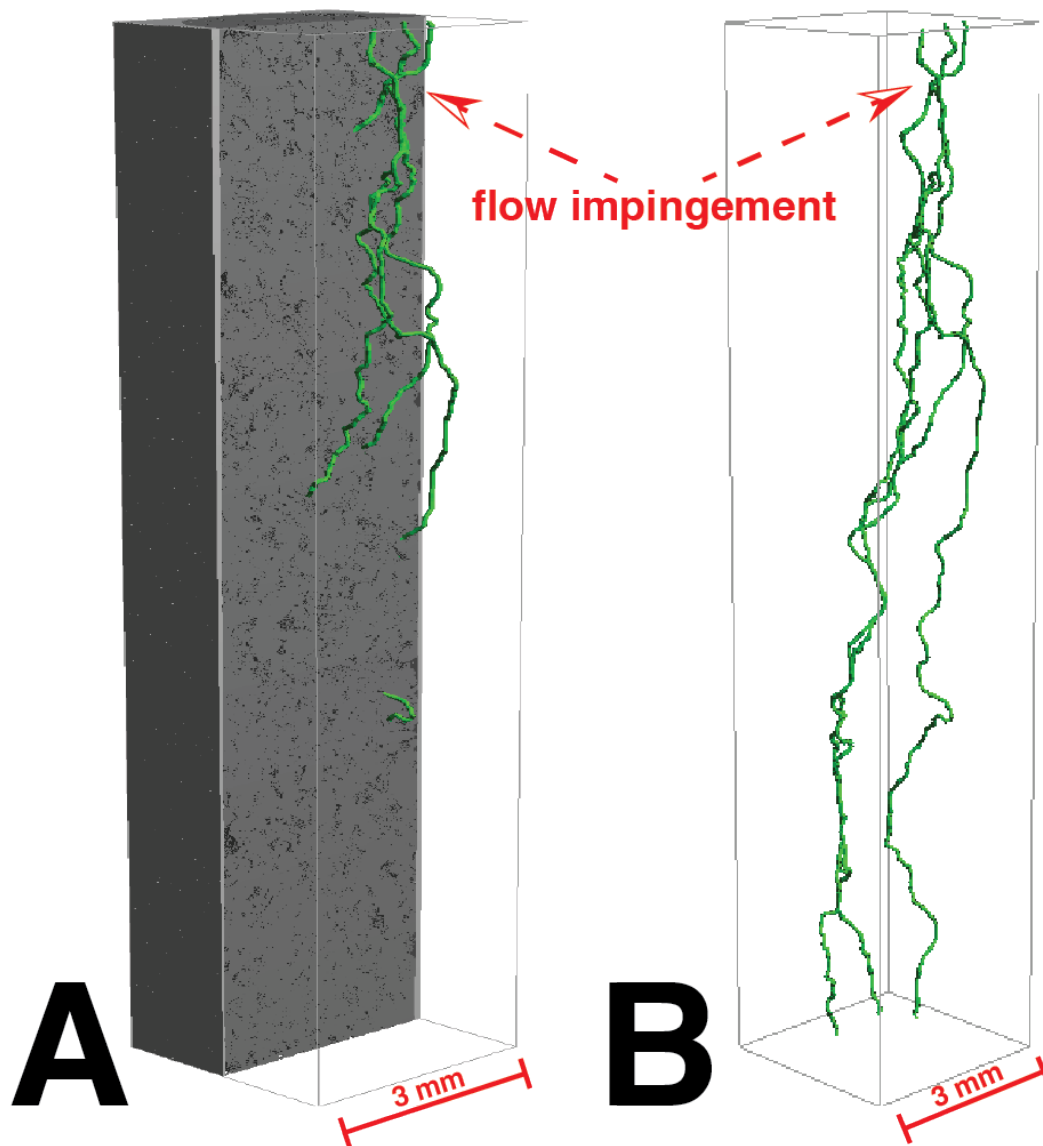
512 Relative permeability was then simulated by simulating fluid distributions using SatuDict,
513 injecting non-wetting phase into the core from both sides using a maxima-inscribed-spheres
514 technique on the connected pore network, slowly increasing the saturation from 0 to 1.
515 Permeability was calculated by simulating flow through the wetting-phase as a single-phase
516 permeability using both Stokes and Stokes-Brinkman methods. We found that initially the
517 permeability estimation ranged between 1.21×10^{-14} and $1.14 \times 10^{-14} \text{ m}^2$ for non-wetting phase
518 saturation of 0 to 0.036, but that after this saturation the non-wetting phase completely blocks
519 all connected pathways and the permeability is predicted as zero. In the Stokes-Brinkman
520 simulations, however, we observe that the initial permeabilities are higher than the Stokes flow
521 with values ranging from 3.57×10^{-14} to $2.92 \times 10^{-14} \text{ m}^2$ for non-wetting phase saturation of 0
522 to 0.067. Furthermore, there is a connected flow path for all saturations, and we find that the
523 predicted permeability of $2.30 \times 10^{-15} \text{ m}^2$ at a saturation of 0.59 is in good agreement with the
524 experimental measurements.

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527
 528 **Figure 14** The wet scan (A) taken during co-injection of N₂ (black) in the pore space of
 529 Estailades. A 3-D rendering of N₂ (B) sieved by size with small (yellow), medium (blue), and
 530 large (red) disconnected clusters. The wetting phase permeability is plotted as a function of
 531 non-wetting phase saturation (C) with the Stokes simulation in blue, the Stokes-Brinkman in
 532 black and the experimental results from single phase shown as a red star and the result from
 533 steady-state co-injection as a red cross.
 534



535
 536 **Figure 15** The three widest percolation paths through the core shown in green with the rock
 537 shown in grey (A) and the rock transparent (B).

538
 539 CONCLUSIONS

540
 541 We have developed a method of using multiscale imaging and experiments to characterize
 542 relative permeability in a microporous carbonate, even at high non-wetting phase saturations.
 543 Intra-granular micro-porosity in this system was characterized using targeted nano X-ray
 544 microscopy, which was then used to generate a suite of synthetic pore geometries
 545 hydrodynamically similar to the imaged network. This was used to generate a customized
 546 Kozeny-Carman porosity-permeability relationship which was used to populate a macroscopic
 547 porosity map, generated from the (macro-scale) X-ray microscopy.

548
 549 By coupling multi-phase flow simulation with a multi-scale description of flow we were
 550 accurately able to predict relative permeability at a fractional flow of 0.5, where a single-scale
 551 simulation failed to capture an effective flow pathway - the wetting phase disconnected in the
 552 macro-pore space, only remaining connected through the micro-porosity. Such a multiscale
 553 approach is particularly powerful when attempting to assess systems with high levels of

554 multiscale structural heterogeneity, such as complex carbonate and shale reservoirs. It also
555 shows that, while these systems can be extremely challenging to characterize, they are tractable
556 by coupling state-of-the-art imaging technologies with stochastic network generation, guided
557 by a geological understanding of the medium in question.

558

559 Future work may include the extension of these analyses across the full experimental relative
560 permeability curve, fast tomography imaging to observe dynamic changes in saturation, further
561 (quantitative) assessment and comparison of micritic structures across several rock types.

562

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564

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567 Technology Park. We thank Professor Martin Blunt for his support in this research.

568

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