Geoscientific Input Feature Selection for CNN-driven Mineral Prospectivity Mapping

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Abstract In recent years, machine learning techniques such as convolutional neural networks have been used for mineral prospectivity mapping. Since a diverse range of geoscientific data are often available for training, it is computationally challenging to select a subset of features that optimizes model performance. Our study aims to demonstrate the effect of optimal input feature selection on convolutional neural network model performance in mineral prospectivity mapping applications. We demonstrate results from both exhaustive and algorithmic feature selection methods in the context of copper porphyry prospectivity modeling. Using the QUEST dataset from central interior British Columbia, such feature selection technique improves model performance by 7% over models that use all available features, yet consumes around 2.2% of the computational resources needed to exhaustively search for the optimal feature subset.

Keywords Convolutional Neural Networks, Mineral Prospectivity Mapping, Multi-armed Bandits, Feature Selection, Porphyry Copper

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1 Introduction

Mineral Prospectivity Mapping (MPM) is a systematic approach used to assess the potential occurrence of mineral deposits in a geographic area. It involves the integration of geoscientific information such as geological, geochemical, geophysical, and remote sensing data to create predictive models that identify favorable areas for mineral exploration. The primary objective of MPM is to guide efficient and cost-effective exploration strategies, focusing activities on areas with the highest probability of success while minimizing costs and reducing environmental impact. Classic efforts in MPM consist of empirical data integration through expert knowledge [1].

Supervised techniques such as decision trees [2] random forest [3] and Support Vector Machines (SVM) [4] are widely used for MPM, but such methods do not take spatial relationships into account directly. However, mineral deposits and their respective geoscientific signatures are highly spatially correlated, and accounting for such relationships is relevant for MPM. One supervised machine learning model that accounts for spatial correlations is a Convolutional Neural Network (CNN). In recent years, a diverse range of CNN-based techniques have been used for MPM problems [5-7]. However, one of the crucial steps in all forms of MPM is the appropriate and optimal selection of data in relation to the mineral deposit of interest. Various sets of geoscientific data categories are deemed optimal for different exploration targets and corresponding mineral systems [3, 5]. We seek to understand how this choice of input data affects mineral prospectivity models.

A simple and motivating example of the importance of input feature selection can be given through the binary classification of dog and cat images. A successful and widely implemented

CNN architecture is AlexNet [8], which accepts images in RGB format, as a collection of three separate colour input layers. We train an AlexNet model on dog images, together with four altered and unaltered versions of cat images [9]: cat images with a noisy green layer, cat images in which the green layer is replaced by that of a random dog, and cat images with the green layer fully removed. All network hyperparameters as well as the number of training epochs remain fixed. Fig. 1 demonstrates examples of a cat image under each alteration category as well as the corresponding model validation accuracy.

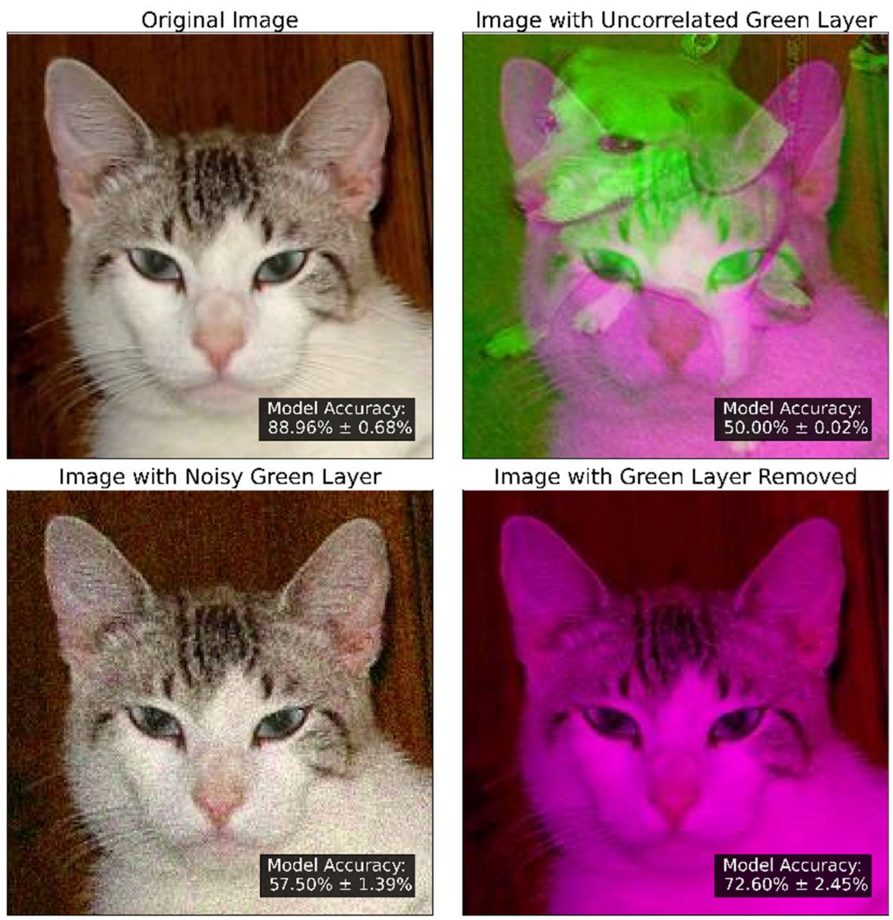

Fig. 1 Validation performance of AlexNet cat/dog classifier trained on original and altered set of cat images. The green component of the cat images is either altered by the Gaussian noise or a full replacement with that of a random dog image. Alternatively, the green layer of the cat images can be fully removed prior to model training, leading to improved model accuracy.

The performance of the AlexNet CNN architecture in these four scenarios shows that even as little as 8.5% noise can reduce model validation accuracy by more than 30%, while adding a completely uncorrelated colour layer to the RGB images reduces the model to a random classifier. On the other hand, removing the altered layer produces better validation accuracy, though subpar compared to models trained on the unaltered data. This case represents a very simple evaluation and selection process, in which only three or less layers of data are used. In most MPM cases, tens of geoscientific data layers are often used as input features, where one or more input features could significantly degrade validation performance. This can be caused by inherently high levels of uncertainty in certain types of geoscientific data (e.g. geological boundary delineations), or lack of strong correlation of some data types to the mineral system of interest (e.g. mismatch in the apparent depth scales of surveys and that of the mineral system). This example motivates a corresponding study for input feature selection in CNN-driven MPM.

The selection of the most appropriate subset of features is effectively an optimization problem. In its most basic form, the preferred set of data can be discretely broken down further, with each subset being exhaustively evaluated [10]. Other more statistically sophisticated methods include using pairwise correlations of input features to reduce the cardinality of the feature space [11], embedding a Lasso and L^2-norm penalty in the neural network's loss function to ensure small weight (i.e., weights related to features with weak predictive potential) are zeroed after training [12] or more novel methods such as Global Sensitivity Analysis (GSA) [13]. In this study, an exhaustive search for the best input data as well as a simple yet effective selection technique called Multi-armed Bandit (MAB) is employed to further illustrate the effect of input feature selection on CNN-based MPM.

2 Methodology

2.1 Data and CNN model

In this study, we use data from the QUEST project for copper porphyry prospectivity mapping. The QUEST project is an extensive data collection campaign, which includes geological, geochemical and geophysical surveys designed to attract the mineral exploration industry to an under-explored region of British Columbia between Williams Lake and Mackenzie [14]; data from the acquisition program have been used for MPM of central British Columbia in recent years [5, 15-16]. The QUEST project is focused on the Quesnel Terrane, which has a number of known copper and gold porphyry occurrences.

The QUEST data can be broken down into geological, geochemical and geophysical categories. The geological category consists of the distance to the closest fault, binary indicators for 5 geological bedrock classes (e.g. intrusive, metamorphic, sedimentary, ultramafic and volcanic rocks), 55 geological bedrock subclasses (e.g. alkaline volcanic rocks, limestone, meta-sediments), and the minimum/maximum geological ages. The geochemical category consists of trace quantity data of 42 elements (e.g. Au, Ni, Pb). Lastly, the geophysical category consists of 5 gravity products, 5 magnetics products and 7 channels of Versatile Time Domain Electromagnetics (VTEM) data (a total of 17 geophysical input features).

In the case of our MPM problem, the input is a so-called data cube, which is a collection of 2D geoscientific data mentioned above (i.e. [spatial information, geoscientific data]). The entire geographic area of interest is cropped into 572212 patches (i.e. sub-cubes of the entire data cube); with an extent of [0.114 degrees North × 0.114 degrees East] (Note that these patches are allowed to geographically overlap). In addition, training, validation and inference stages use 1950 randomly sampled data sub-cubes each. The magnetics data, together with a sample crop are provided as an example for an individual layer of the data cube in Fig. 2.

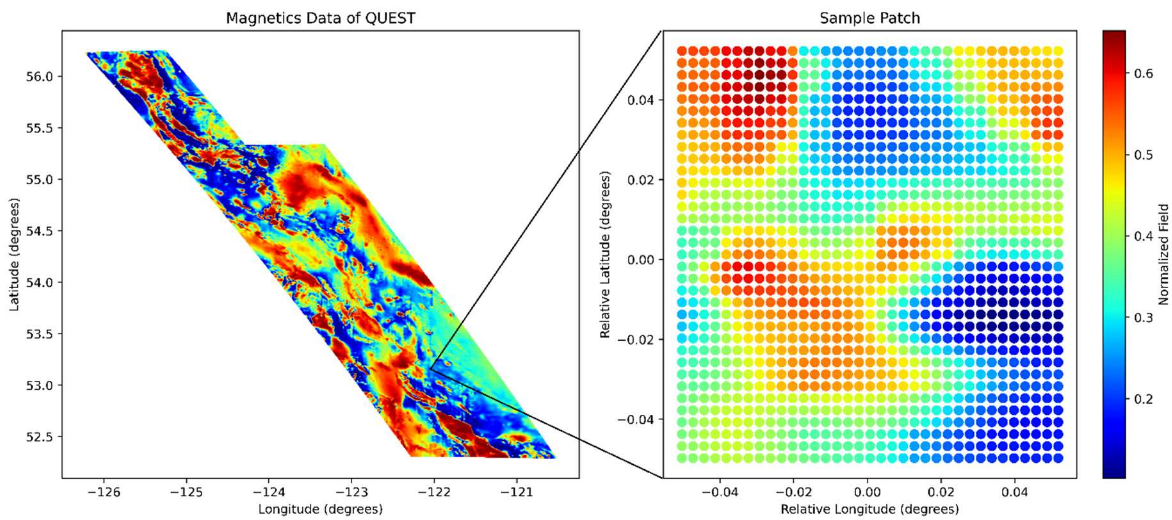

Fig. 2 (Left) QUEST normalized magnetic field strength data of central interior British Columbia. (Right) Example of a data crop used as part of model training and validation (Note that the axes are relative latitudes and longitudes.

The labels of the patches in the training and validation set are assigned by their containment of mineral occurrences, as well as the position of the deposit relative to the frame of the patch. We defined a labeling criterion, where a geographic patch is assigned a positive label if a mineral occurrence falls within a fourth crop size distance of the patch center. Those patches that have mineral occurrences within a 10km distance outside of that region are labeled as interim patches. Note that these interim batches represent the uncertainty in label assignment and are not used for training or validation of CNN models. The rest of possible patches are assigned a negative label. The CNN models are trained on the labels south of the 53.2 degrees and validated on the labels north of 54.7 degrees. Model predictions are made in the central region between these latitudes. (See Fig. 3 for the architecture of the CNN).

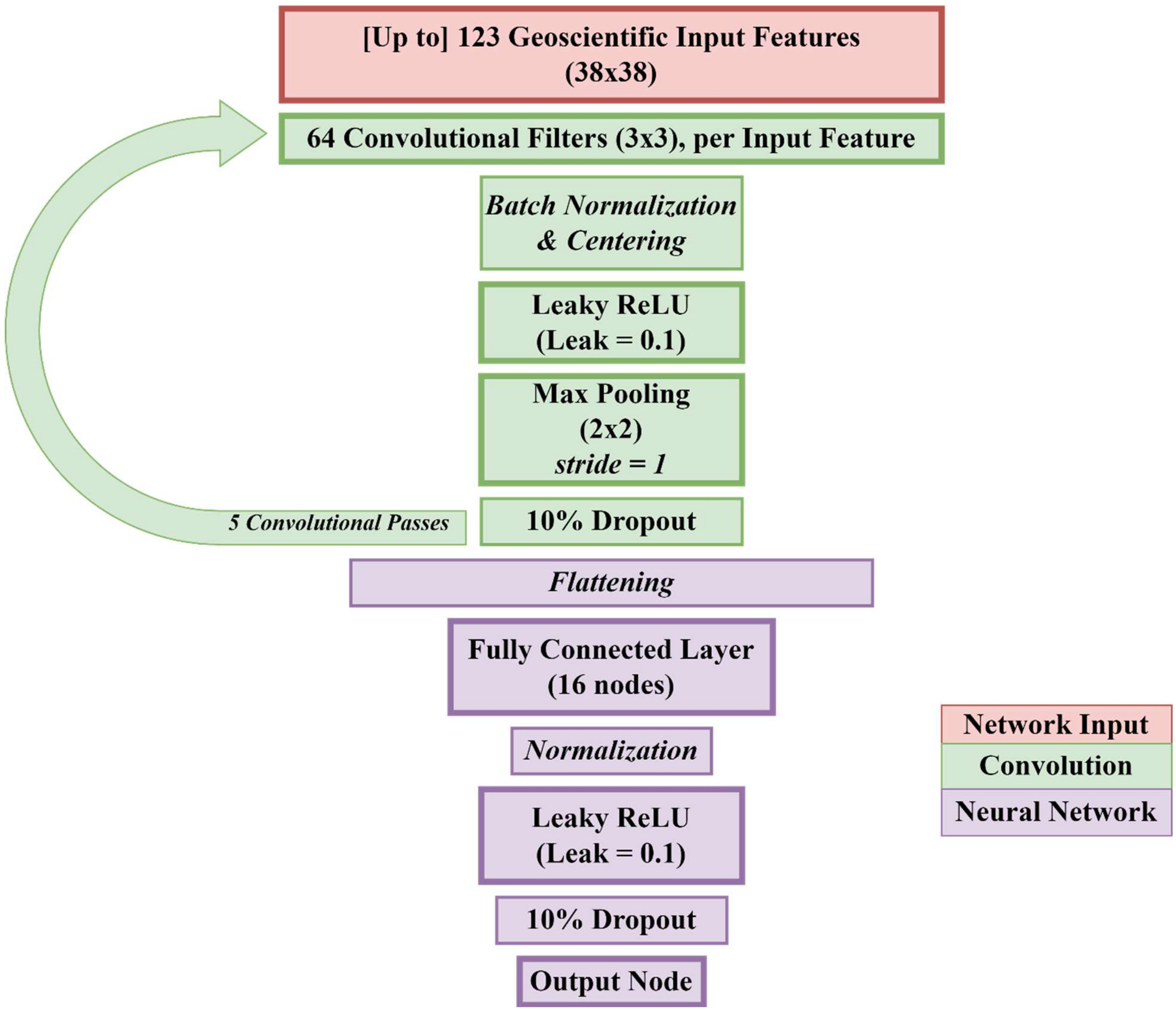

Fig. 3 CNN architecture of the model used for copper porphyry prospectivity modeling in central interior British Columbia. Input features are pre-processed, uniformly cropped and labeled according to the positions of known mineral occurrences in the region. Crops are 38x38 pixels, with a physical dimension of 19.1km. This CNN consists of three components: input feature (red), five convolutional passes (green) and a fully connected artificial neural network (purple). After five convolutional passes and poolings have been performed, the obtained channels are flattened from a 2D format into a single 1D array. This array is used as the input of a fully-connected artificial neural network, which returns an output layer. Note that the model uses a square loss function, and outputs a regression score as its predictions.

2.2 Optimal input feature selection

2.2.1 Optimization Parameter

In this study, the optimization parameter is designed such that it reflects the overall goodness of a prospectivity model. A good prospectivity model is one that makes the most true positive predictions, while making the least false positive predictions. The consequences of false positive predictions can have detrimental financial implications for any exploration effort, leading to misallocation of resources. An effective tool to numerically frame this objective is through the Receiver Operating Characteristic (ROC), which is a curve describing the relationship between the rate of true positive predictions (TPR) and the rate of false positive predictions (FPR) as a function of a classification threshold p_{thr}.

A regression model is chosen for the CNN (i.e. returning scores that range from 0 to 1), and can be validated if a threshold value p_{thr} is defined, with regression scored above p_{thr} validated as a positive prediction and negative otherwise (In this work, p_{thr}= 0.5). We define a parameter to unify the notion of maximizing the rate of true positive predictions and minimizing the rate of false positive predictions of the prospectivity model.

$$R = 1 - \sqrt{\frac{(1-TPR)^2+(FPR)^2}{2}}. \quad (1)$$

where R represents an adjusted Euclidean distance from (FPR,TPR) = (0,1) to the (FPR,TPR) corresponding to , p_{thr}= 0.5 (R = 1 indicates a perfect predictor). This parameter is used to represent model performance throughout the rest of this study. Note that this optimization parameter is different from the loss of the CNN model. The calculated loss of the CNN model incorporates false and true predictions for both positive and negative validation labels, whereas the optimization parameter R only incorporates false positive and true positive predictions (e.g., a model can have a very high rate of false negative predictions and still obtain a high R score, given its false positive prediction rate is minimal).

2.2.2 Exhaustive Search for Optimal Input

An exhaustive search of all possible training feature subsets is performed over geological class, geological subclass, minimum/maximum geological age, distance to nearest fault, geochemical data, gravity, magnetics and VTEM data, to create a benchmark for optimization efficiency. According to the breakdown in Table 1, there are 255 possible combinations to investigate. The categories in Table 1 can be further broken down to allow for more input feature combinations, as many categories encompass multiple sub-features, particularly geochemistry. However, this presents little benefit compared to the associated growth of the search space.

Since stochasticity is central to the machinery of CNNs (mainly due to stochastic gradient descent, dropout and stochastic network weight initializations) [17], the model performance for a particular training feature subset cannot be evaluated from a single CNN run (i.e. complete training and validation operations) of the CNN algorithm, and the result of multiple CNN runs are needed for calculating statistically significant performance figures. Thus, the number of independent trainings and validations (i.e. N_{run}) need to be high (180 CNN runs for each input feature configuration).

2.2.3 Multi-armed Bandits

The Multi-armed Bandit (MAB) gets its name from the idea of a gambler who wishes to maximize their total winnings over time given a row of slot machines (often called one-armed-bandits). In MAB problems, a decision making agent is faced with a set of actions (i.e. the "arms" of the MAB), and must decide which action to select at each step with the goal of maximizing its total cumulative reward (See eqn. 1) [18]. MAB algorithms efficiently allocate limited resources (in this case, computing power and time) by balancing exploration (i.e. taking a random action) and exploitation (i.e. taking the perceived optimal action at the time), and are used to optimize decision-making in dynamic and uncertain environments. Examples range from infill drilling in mining [19] to recommendation systems [20] and portfolio management in finance [21].

The action is defined as a tuple, containing the chosen data within each geoscientific data category (See Table 1 & Fig. 5). The MAB has a "lever" for each possible data subset derived by Table 1, and each can be pulled independently. In this case, pulling a lever is equivalent to training the CNN model on the chosen input feature subset, and observing the validation performance of the model.

An action-value method is applied to numerically relate the concepts of action to reward. The premise of action-value functions is the determination of an action's value in respect to prior experiences. A simple action-value function is defined as the average rewards of previous instances.

$$Q_n(a) = \frac{\left[\sum_{i=1}^{n} R_i(a)\right]}{n} \qquad (2)$$

where $Q_n(a)$ is the average value of action a after taking it n times, and R_i is the reward received after the i^{th} choice of action a (See eqn. 1). The action associated with the maximum Q value (i.e. Q_{max}) is considered the most optimal action until that point, which would be exploited by the agent to maximize future rewards. Fig. 4 demonstrates how the MAB algorithm is integrated with the CNN training and validation process.

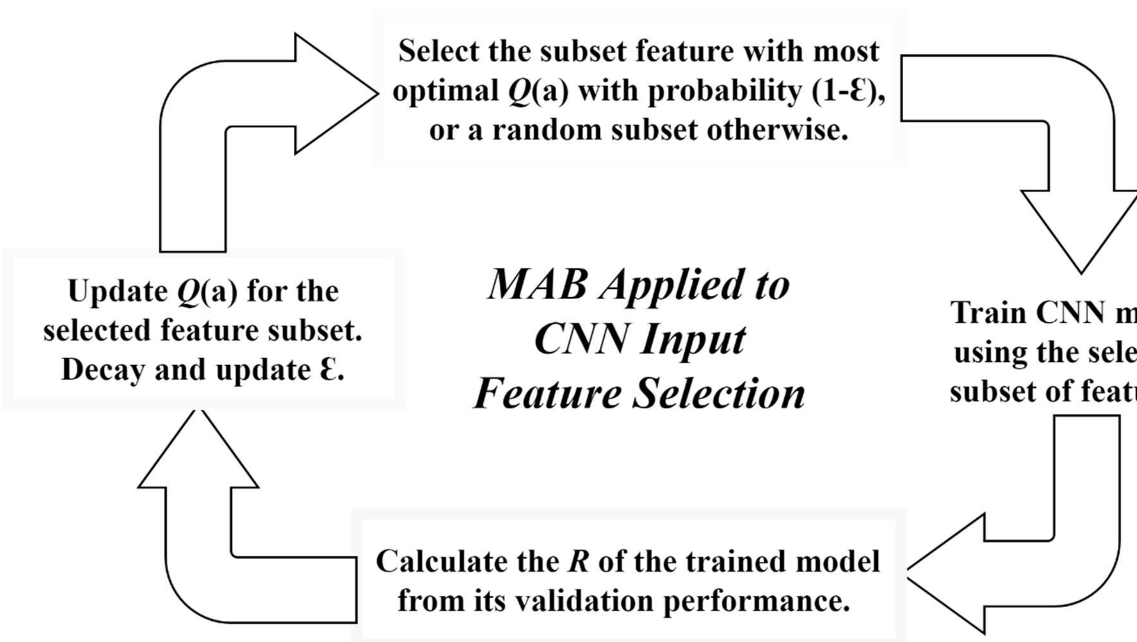

Fig. 4 MAB algorithm applied to input feature selection for a CNN model training and validation process. This diagram represents one step of the MAB algorithm.

MAB agents cannot simply exploit an action they currently believe to be optimal - they must also explore other potentially optimal actions. The trade-off between exploration and exploitation can be managed through a simple and widely used algorithm called Ɛ-greedy. In this method, the agent exploits its experience by taking the action associated with Q_{max} with a 1- Ɛ probability (randomly breaking ties if multiple actions are believed to be equally optimal), and explores by taking a random action with a probability Ɛ. The conventional Ɛ -greedy strategy is modified to let the agent explore a wide range of actions at early MAB steps, and gradually become exploitative as it gains experience. This can be done by decaying the value of Ɛ over MAB steps ($\varepsilon_n = 10^{\frac{-n}{N}}$). The decay rate is set such that epsilon reaches 0.1 at the 1000th step, at which point $argmax[Q(a)]$ is the result of the optimization process.

3 Results

3.1 Exhaustive Search for Optimal Input

The exhaustive search concludes that the best training input feature subset is geological class, geochemical and VTEM data (See Table-1 and Fig. 5). The completion of such number of CNN model trainings and validations necessitates the use of high performance computing (HPCs with NVIDIA A100SXM4 GPUs), taking a total of 3364 GPU hours. In addition, any further breakdown of data can drastically increase the number of needed trainings and validations (See Fig. 5 and Table 1). These challenges motivate the use and incorporation of alternative data selection techniques, such as an MAB.

0	No Geological Data
1	Geological Class
2	Geological Subclass
3	Geological Age
4	Distance to the Nearest Fault
5	Geological Class and Subclass
6	Geological Class and Age
7	Geological Class and Distance to the Nearest Fault
8	Geological Subclass and Age
9	Geological Subclass and Distance to the Nearest Fault
10	Geological Age and Distance to the Nearest Fault
11	Geological Class, Subclass and Age
12	Geological Class, Subclass and Distance to the Nearest Fault
13	Geological Class, Age and Distance to the Nearest Fault
14	Geological Subclass, Age and Distance to the Nearest Fault
15	Geological Class, Subclass, Age and Distance to the Nearest Fault

A0	No Geophysical and Geochemical Data
A1	Geochemical Data
B0	Gravity
B1	Gravity and Geochemistry
C0	Magnetics
C1	Magnetics and Geochemistry
D0	VTEM
D1	VTEM and Geochemistry
E0	Gravity and Magnetics
E1	Gravity, Magnetics and Geochemistry
F0	Gravity and VTEM
F1	Gravity, VTEM and Geochemistry
G0	Magnetics and VTEM
G1	Magnetics, VTEM and Geochemistry
H0	Gravity, Magnetics and VTEM
H1	Gravity, Magnetics, VTEM and Geochemistry

Table 1. Indicators for the QUEST Data Subsets in Fig. 5.

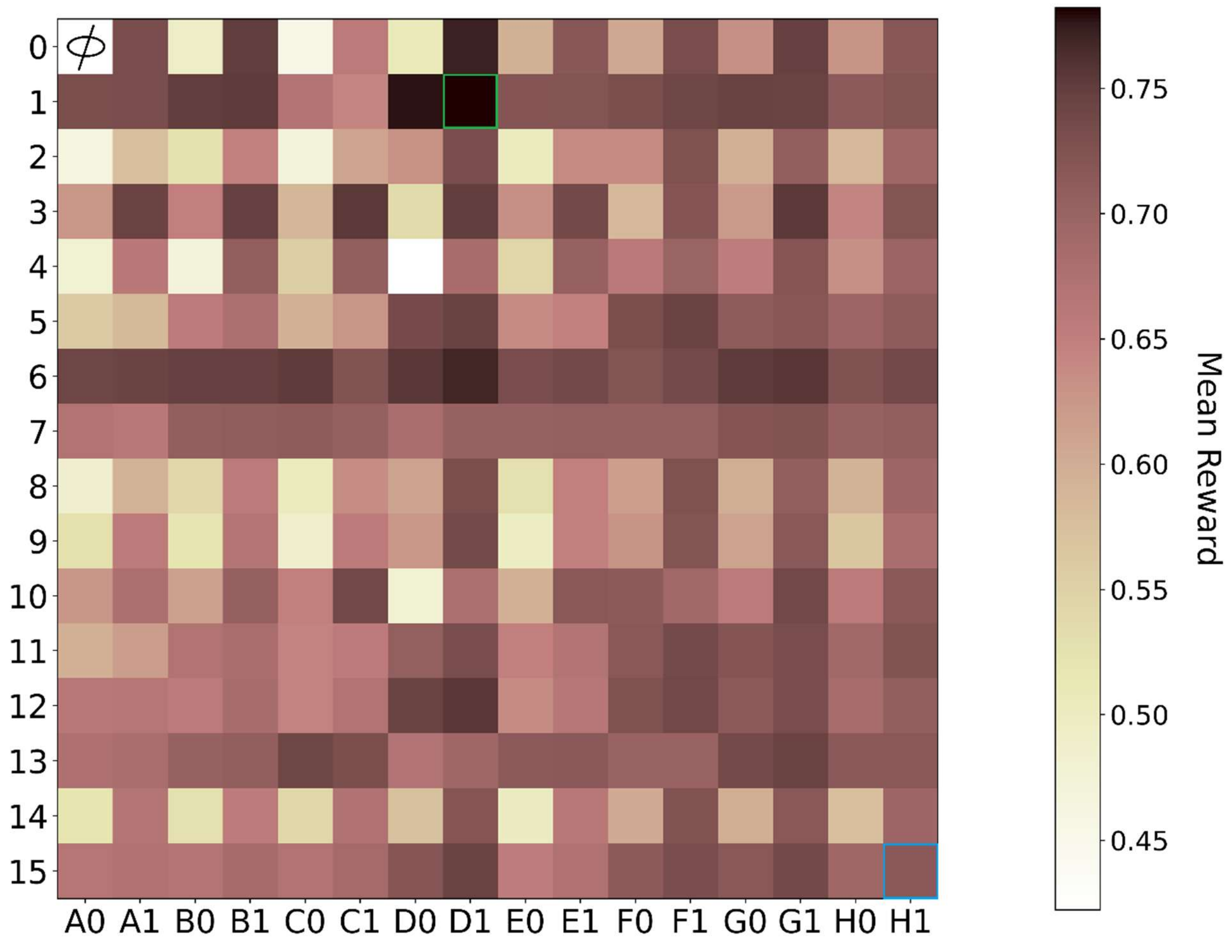

Fig. 5 Mean reward matrix of all data subsets after an exhaustive search (i.e. 180 trainings per training feature subset). The null combination is marked by the phi symbol. The most optimal data combination is boxed in green, and the configuration that includes all data types is boxed in blue. A glossary of data subsets is provided in Table I.

3.2 MAB Search for Optimal Input

The MAB agent concluded that using geological class, VTEM together with geochemistry is the most optimal set of training feature for the CNN-based MPM, which agrees with the exhaustive search (Note that MAB results were consistent over 5 separate iterations of the MAB algorithm). In addition, this result is fully consistent for several runs of the MAB algorithm. Using this choice of geoscientific training features, we obtain an average model reward of more than 78.2%, which is a 7% improvement over training on all features (see Fig. 5 & 6.).

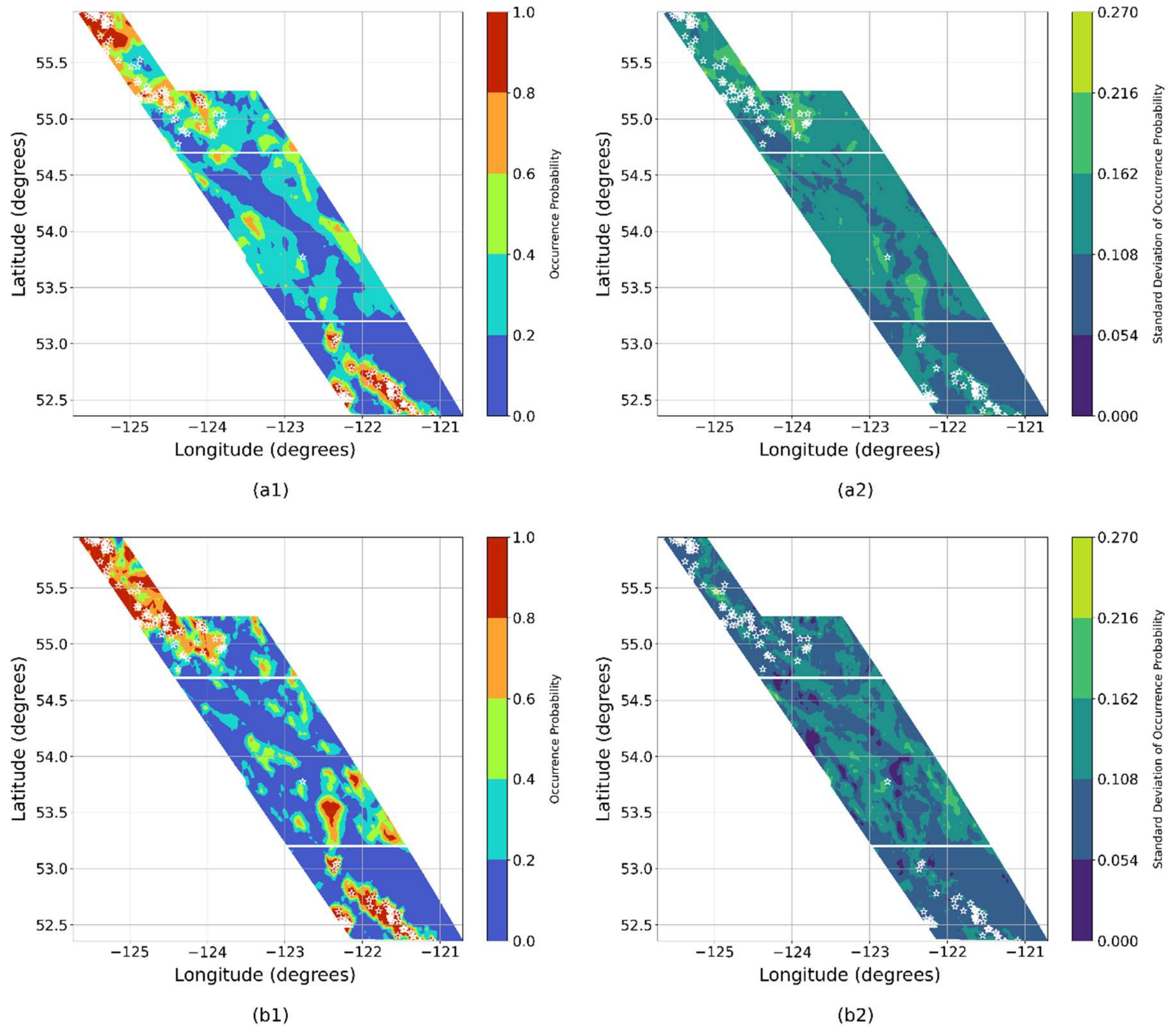

Fig. 6 Copper porphyry prospectivity of central interior British Columbia. (a1) and (a2) show the average and standard deviation prospectivity maps of CNN models trained on all feature types respectively, while (b1) and (b2) show the average and standard deviation prospectivity maps of CNN models trained on the optimal subset of feature types selected by the MAB. The CNN was trained 180 times independently on each training features. Models are trained on the labels south of the 53.2 degrees latitude and validated on the labels north of 54.7 degrees from their respective features (marked by horizontal white lines).

In addition, the optimization improved model stability (i.e. standard deviation of the model reward) from 3.9% in the case of using all training features, to 2.0%. The MAB algorithm is able to pick the most optimal training feature subset after only 1000 MAB steps (average of 4 CNN model trainings per input feature subset, compared to 180 for the exhaustive search), by choosing sub-optimal actions much less than others. Therefore, the MAB algorithm utilized around 74 hours of GPU time, which is only 2.2% of the computational resources that was required for the exhaustive input feature optimization. In other words, the MAB algorithm was able to identify and abandon sub-optimal input feature configurations early, and focus on more promising inputs instead.

3.3 Copper Porphyry Prospectivity Maps

Interesting similarities and contrasts between the two optimized maps are observed (refer back to Fig. 6. for subsequent references). The most prominent prediction is that of a highly prospective area at [-122.4 degrees East, 53.5 degrees North] after training on the optimal data features. This feature is totally absent in the unoptimized prospectivity map. However, the unoptimized prospectivity map contains an area of high variance in the same region, contrary to the optimized model, which produced significantly lower prediction variance in the region. Other features that exclusively appear in the optimized map include two small high prospectivity areas to the south east and north east of the feature at [-122.4 degrees East, 53.5 degrees North] (Note that these features are coupled with relatively high standard deviations).

4 Discussion

The results of the input feature selection for our MPM problem has direct analogues to that of the binary classifier example outlined in section 1. In the binary classifier problem, the input is an RGB image consisting of three colour layers, with the green layer being altered. Therefore, in this context, the decision is to choose between the inclusion of the altered green layer together with the other two layers, or its full exclusion from the input feature set. The example showcases two types of scenarios: 1) random noise and 2) lack of meaningful correlation to labels and other input features. The former can represent MPM cases where some input features have logical correlation with labels and other features, yet contain noise. Uncertainty in determining geological age and ambiguity in the delineation of fault lines are examples of such a case. The optimal input feature set for our MPM excludes three out of four available geological features and their respective combinations. On the other hand, the inclusion of uncorrelated features into the input feature set (case 2) can be destructive to the training process. The lack of correlation can be purely logical and based on the particular mineral system of interest.

The optimal input feature set for our MPM problem includes VTEM and geochemical data, which have much higher spatial resolution and direct correlation to labels compared to features such as gravity and broad geological classifications, reflecting the scattered distribution of copper porphyry deposits and the nature of their underlying mineral system.

5 Conclusion

Modern computational tools such as machine learning capture sophisticated patterns in geoscientific data, generating robust prospectivity maps. This study successfully demonstrated the constructive effect of optimal input feature selection for CNN applications in copper porphyry prospectivity modeling of central interior British Columbia. The data selection optimization via the MAB results in noticeable improvements in model performance (7% better model performance and a 1.9 % reduction in global model variance compared to using all available data), yet only requires 2.2% of computational resources needed to exhaustively search for the optimal data subset. Future work can benefit from recently developed recurrent attention models (RAMs) using deep reinforcement learning, which focus the attention of a convolutional neural network towards certain portions of the input features during the training process.

Declaration

Conflicts of interest/Competing interests

Not applicable.

Availability of data and materials

All data used in this research is public and cited in this manuscript.

Code Availability

The code will be made available upon request for research or academic purposes.

References

[1] Knox-Robinson, C.M., Wyborn, L.A.I., 1997. Towards a holistic exploration strategy: Using Geographic Information Systems as a tool to enhance exploration. *Australian Journal of Earth Sciences* 44(4), 453–463. https://doi.org/10.1080/08120099708728326.

[2] Reddy, R.K.T., Bonham-Carter, G.F., 1991. A Decision-Tree Approach to Mineral Potential Mapping in Snow Lake Area, Manitoba. *Canadian Journal of Remote Sensing* 17(2), 191–200. https://doi.org/10.1080/07038992.1991.10855292.

[3] Lawley, C.J.M., Tschirhart, V., Smith, J.W., Pehrsson, S.J., Schetselaar, E.M., Schaeffer, A.J., Houlé, M.G., Eglington, B.M., 2021. Prospectivity modelling of Canadian magmatic Ni (±Cu ± Co ± PGE) sulphide mineral systems. *Ore Geology Reviews* 132, 103985. https://doi.org/10.1016/j.oregeorev.2021.103985.

[4] Cracknell, M. J., & Reading, A. M. (2014). Geological mapping using remote sensing data: A comparison of five machine learning algorithms, their response to variations in the spatial distribution of training data and the use of explicit spatial information. *Computers and Geosciences*, 63, 22–33. https://doi.org/10.1016/j.cageo.2013.10.008.

[5] McMillan, M., Haber, E., Peters, B., Fohring, J., 2021. Mineral prospectivity mapping using a VNet convolutional neural network. *The Leading Edge* 40(2), 99–105. https://doi.org/10.1190/tle40020099.1.

[6] Yang, N., Zhang, Z., Yang, J., Hong, Z., 2022. Applications of data augmentation in mineral prospectivity prediction based on Convolutional Neural Networks. *Computers & Geosciences* 161, 105075. https://doi.org/10.1016/j.cageo.2022.105075.

[7] Li, Q., Chen, G., Luo, L., 2023. Mineral prospectivity mapping using attention-based convolutional neural network. *Ore Geology Reviews* 156, 105381. https://doi.org/10.1016/j.oregeorev.2023.105381.

[8] Krizhevsky, A., Sutskever, I., Hinton, G.E., 2012. ImageNet classification with deep convolutional Neural Networks. *Advances in Neural Information Processing Systems*.

[9] Cukierski, W., 2013. Dogs vs. Cats. Kaggle. https://kaggle.com/competitions/dogs-vs-cats.

[10] Sahu, R.K., Müller, J., Park, J., Varadharajan, C., Arora, B., Faybishenko, B., Agarwal, D., 2020. Impact of input feature selection on groundwater level prediction from a multi-layer perceptron neural network. *Frontiers in Water* 2. https://doi.org/10.3389/frwa.2020.573034.

[11] Hall, M.A., 1999. Correlation-based feature selection for Machine Learning. PhD thesis, The University of Waikato, Hamilton, New Zealand.

[12] Egwu, N., Mrziglod, T., Schuppert, A., 2022. Neural network input feature selection using structured L2L_2L2-Norm Penalization. *Applied Intelligence*. https://doi.org/10.1007/s10489-022-03539-8.

[13] Zhang, P., 2019. A novel feature selection method based on global sensitivity analysis with application in machine learning-based prediction model. *Applied Soft Computing* 85, 105859. https://doi.org/10.1016/j.asoc.2019.105859.

[14] Geoscience BC, 2020. Quest Project. https://www.geosciencebc.com/major-projects/quest/.

[15] Mitchinson, D.E., Fournier, D., Hart, C.J.R., Astic, T., Cowan, D.C., Lee, R.G., 2022. Identification of New Porphyry Potential Under Cover in British Columbia. *Geoscience BC Report* 2022-07, MDRU Publication 457.

[16] Montsion, R.M., Saumur, B.M., Acosta-Gongora, P., Gadd, M.G., Tschirhart, P., Tschirhart, V., 2019. Knowledge-driven mineral prospectivity modeling in areas with glacial overburden: Porphyry Cu exploration in Quesnellia, British Columbia, Canada. *Applied Earth Science* 128(4), 181–196. https://doi.org/10.1080/25726838.2019.1675403.

[17] Shalev-Shwartz, S., Ben-David, S., 2014. Stochastic Gradient Descent. In *Understanding Machine Learning: From Theory to Algorithms* (pp. 150-166). Cambridge: Cambridge University Press. https://doi.org/10.1017/CBO9781107298019.015.

[18] Sutton, R.S., Barto, A., 2020. Multi-armed Bandits. In *Reinforcement learning: An introduction* (pp. 25–47). The MIT Press.

[19] Dirkx, R., Dimitrakopoulos, R., 2018. Optimizing Infill Drilling Decisions Using Multi-Armed Bandits: Application in a Long-Term, Multi-Element Stockpile. *Mathematical Geosciences* 50, 35–52. https://doi.org/10.1007/s11004-017-9695-9.

[20] Silva, N., Werneck, H., Silva, T., Pereira, A. C., & Rocha, L. (2022). Multi-armed bandits in recommendation systems: A survey of the state-of-the-art and future directions. *Expert Systems with Applications*, 197, 116669.

[21] Moeini, M. (2019). Orthogonal bandit learning for portfolio selection under cardinality constraint. In *Lecture Notes in Computer Science* (pp. 232–248).