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Tidal flexure reveals effective elasticity in grounding zones on the Ross Ice Shelf

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	independent of the hydrostatic assumption, with implications for basal melt rate estimates and future sea-level rise projections.

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Tidal flexure reveals effective elasticity in grounding zones on the Ross Ice Shelf

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ABSTRACT.

The grounding zones of Antarctic ice shelves are among the continent's most dynamic regions, where floating ice shelves buttress grounded upstream ice and tidal forcing drives cyclic flexure at the ice-ocean-bed interface. We use ICESat-2 altimetry and airborne ice-penetrating radar to constrain the effective Young's modulus E^* of ice in the flexure zone at three sites on the Ross Ice Shelf. By modeling ice as an elastic beam of variable thickness, we infer a single effective elastic parameter, E^* , that encapsulates the combined flexural response of the ice-bed-ocean system. Our results show considerable spatial variability in E^* , with values ranging from 1–9 GPa across sites, and an average of 3.6 ± 2.5 GPa. This variability reflects intersecting basal, oceanographic, and mechanical processes in the grounding zone, including bed stiffness, subglacial hydrology, and viscoelasticity of ice. Because flexure of bed and ice cannot readily be distinguished in observations, we argue for a bulk interpretation of E^* that allows uncertainty to be quantified in terms of a single parameter. These results offer a new method for estimating ice shelf thickness and thickness gradient near the grounding line, independent of the hydrostatic assumption, with implications for basal melt rate estimates and future sea-level rise projections.

27 INTRODUCTION

28 Grounding zones are transition regions where grounded ice sheets become floating ice shelves and ice, ocean,
 29 and the solid Earth all meet. The dynamics of the grounding zone are among the most sensitive indicators of
 30 change in the Antarctic (Pattyn, 2017; Scambos and others, 2017; Gudmundsson and others, 2019; Seroussi
 31 and others, 2020). The satellite era has revealed widespread, rapid grounding line retreat (Joughin and
 32 others, 2014; Rignot and others, 2014), especially in West Antarctica, where ice shelves are increasingly
 33 melted from below by warm, salty Circumpolar Deep Water (Nakayama and others, 2018; Jourdain and
 34 others, 2017). Basal melt rate in the grounding zone is one of the largest sources of uncertainty in future
 35 projections of the rate and amount of future sea-level rise (Pritchard and others, 2012; Adusumilli and
 36 others, 2020; Seroussi and others, 2020; Hill and others, 2021).

37 Basal melt rate m can be calculated by conservation of mass provided that the local ice thickness h is
 38 well-known such that:

$$\frac{\partial h}{\partial t} = (a - m) - \nabla \cdot (h\mathbf{u}) \quad (1)$$

39 where a is the surface mass balance, and u is the ice velocity. Ice shelf thickness is typically calculated by
 40 a freeboard approach, where the ice height above the ocean surface is measured and the total ice thickness
 41 is calculated as the freeboard height times $\rho_{sea}/(\rho_{sea} - \rho_{ice})$, with a correction for firn densification (Smith
 42 and others, 2020; Chartrand and Howat, 2023). This works well near the calving front, where ice is in
 43 hydrostatic equilibrium and floats up and down on ocean tides. However, near the grounding line, the ice
 44 shelf is mechanically coupled to the upstream ice sheet and flexes, rather than floats (figure 1).

45 The flexure of an elastic beam under a load depends on its flexural rigidity, proportional to the product
 46 of its thickness cubed and Young's modulus E . In 1995, David Vaughan modeled the tidal flexure of ice
 47 with an elastic beam of constant thickness and rode a snowmobile back over its own tracks at different
 48 points in the tidal cycle on the Rutford Ice Stream while measuring the vertical position of the ice surface
 49 and found that a value for E of 0.88 ± 0.35 GPa adequately minimized the misfit between modeled and
 50 observed flexure using a constant ice thickness h from contemporaneous radar sounding measurements, as
 51 only one of h or E can be inferred from this inversion given the other (Vaughan, 1995).

52 This result is somewhat surprising, as the value of the Young's modulus of ice is commonly taken to be
 53 9 GPa: an order of magnitude greater (Cuffey and Paterson, 2010). The Young's modulus of a material

54 is an elastic parameter that reflects the ratio of an applied uniaxial normal stress to instantaneous normal
55 strain, and has its physical basis in the electromagnetic forces between molecules in its crystal lattice. It
56 is measured in the laboratory by mechanical stress-strain curve testing, or by wave speed measurement
57 through the lattice, or by interferometric phonon scattering measurements (Schulson and Duval, 2011;
58 Rathmann and others, 2022; Gammon and others, 1983). As ice is a viscoelastic material, there is a
59 dependence of E on the rate of the applied stress due to creep and anelastic effects happening over time,
60 as well as on temperature and presence of water or inclusions (Sinha, 1989, 1978). Sinha (1978) uses the
61 term *effective* Young's modulus to distinguish between the "true" instantaneous elastic modulus at high
62 frequencies and the relationship between stress and strain of in situ ice. Gold (1977) compiles a number of
63 laboratory experiments and finds an approximately linear relationship between Young's modulus and the
64 frequency of applied stress leading to $E \approx 3$ GPa at 10^{-3} Hz. The tidal frequency is about 10^{-5} Hz, which
65 would give $E \approx 1$ GPa if the linear relationship continues, as noted by Vaughan (1995).

66 Numerous approaches have been employed to resolve the apparent discrepancy between the laboratory
67 values of $E = 9$ GPa and the much smaller results from tidal flexure models. Reeh and others (2003)
68 develops a linear viscoelastic model similar to a Burgers model with four elastic parameters and uses
69 flexure data on Nioghalvfjærdsfjorden glacier from tiltmeters and GPS, with airborne ice-penetrating radar
70 thickness, and temperature measurements from a borehole to determine the local ice viscosity. Detailed
71 information on the phase of the viscous adjustment to the tide is also needed, which is possible with
72 tiltmeters but not with satellite laser altimetry. The launch of ICESat in 2005, followed by ICESat-2 in
73 2018, made it possible to difference repeat track measurements of ice surface height at different points
74 in the tidal cycle to observe tidal flexure near the grounding line on many ice shelves around Antarctica
75 (Fricker and Padman, 2006). Sayag and Worster (2013) modeled the flexure as an elastic beam of constant
76 thickness and used ICESat altimetry in two places on the Ronne-Filchner Ice Shelf and found a best fitting
77 value for E of 1.8 GPa in a "stiff-fixed" model, where the grounding line is fixed and ice rests on a stiff
78 frozen bed, but a best fitting E of 9.33 GPa with a "soft-free" model, where the grounding line is allowed
79 to move and ice rests on an elastic bed having an estimated spring constant of 1 MPa/m. Marsh and
80 others (2014) used differential interferometric synthetic aperture radar (InSAR) to observe tidal flexure on
81 Beardmore Glacier and allowed the ice thickness to vary and using a value for E of 1.4 GPa are able to
82 invert for local ice thickness using COMSOL, a commercial multi-physics finite-element software package.

83 A number of studies have incorporated viscoelastic effects and compared these to elastic models. Wild

84 and others (2018) compare a viscoelastic and an elastic model, with an elastic bed of spring constant 5
85 MPa/m, and find the viscoelastic model is very sensitive to the tidal model used to determine the phase of
86 the tide. Rosier and others (2017) compare a simple elastic beam model to full-Stokes viscoelastic models
87 with inclusion of basal crevasses and density dependent ice stiffness and find that the elastic solution model
88 can produce similarly excellent fits to data, as well as that a factor of two of variation, ranging from about
89 2-4 GPa, in estimated effective Young's modulus exists even when local ice thickness is well constrained.

90 Here, we use satellite observations of the surface flexure of ice along with a simple physical model of
91 ice in the flexure zone as an elastic beam of varying thickness, motivated by the need for better estimates
92 of ice thickness and thickness gradient near the grounding line, to constrain the effective Young's modulus
93 of ice in three places on the Ross Ice Shelf. We infer a single effective elastic parameter, the effective
94 Young's modulus E^* , to parameterize the observable flexure of the combined ice-bed-ocean system near
95 the grounding line, and argue that we can not readily distinguish flexure of the ice from flexure of the bed.
96 This allows much of the uncertainty inherent to the problem to be wrapped into one effective parameter
97 whose associated uncertainty can be quantified. We use airborne ice-penetrating radar measurements of
98 ice thickness $h(x)$ on the Ross Ice Shelf (RIS) (Das and others, 2020) to constrain the effective rheology of
99 ice in the flexure zone by minimizing the misfit between modeled flexure and observed ice flexure with laser
100 altimetry data from ICESat-2. We consider this work to be the first major step in a method towards an
101 independent, observationally constrained estimate of ice thickness near the grounding line for basal melt
102 rate, which can ultimately be linked to reducing uncertainty about the rate and amount of future sea level
103 rise.

104 We show that a simple, observationally constrained model of the grounding zone as an elastic beam
105 under tidal forcing reveals substantial variation in effective Young's modulus in three areas on the Ross Ice
106 Shelf. The in situ environment of ice near the grounding line is complex and influenced by physical forces
107 on intersecting length and timescales (Figure 1). Ocean tides cycle around Antarctica every 12-24 hours
108 (Padman and others, 2018). Tides pump seawater in and out of the flexure zone, acting as a forcing to
109 the hydrodynamics of a poroelastic bed with sediment on a spectrum from fully frozen to fully deformable
110 (Warburton and others, 2020). There is tidal mixing of incoming and outgoing water masses: warm, salty
111 Circumpolar Deep Water increasingly encroaches on the Antarctic continental shelf and melts ice from
112 below, releasing cold glacial meltwater which can refreeze onto the ice shelf (Nakayama and others, 2018).
113 There are surface and basal crevasses and channels that direct the flow of water (Alley and others, 2023).

114 The first direct underwater observations of the grounding zone have begun to show a complex system where
 115 these and other processes interact on different length and timescales to produce angular terraced forms
 116 on the undersides of ice shelves, as well as swooping, curved regions where convective and oceanographic
 117 forcings act differently on the flexure zone system (Wåhlin and others, 2024).

118 The observable surface flexure of ice near the grounding line reflects the combined effects of these
 119 competing processes, as well as any flexure of the sediment and bed. For this reason, as well as the viscous
 120 and anelastic processes that necessarily affect the stress-strain relationship in real time, we will refer only
 121 to an effective Young's modulus E^* , to reflect the fact that the relationship between stress and strain that
 122 can be inferred from tidal flexure is a bulk parameter not identical to the Young's modulus of ice as can
 123 be measured in the lab, and may indeed vary considerably in space.

124 METHODS

125 We employ an elastic beam bending model constrained by repeat track flexure data from ICESat-2 to
 126 estimate the flexural rigidity of ice in the flexure zone of the Ross Ice Shelf. These locations are selected to be
 127 far from confining topography, perpendicular to the grounding line, and close to ice thickness measurements,
 128 in order to narrowly apply the linear elastic approximation. Using ice-penetrating radar thickness data
 129 from the ROSETTA-Ice airborne geophysical survey (Das and others, 2020), we then calculate the effective
 130 Young's modulus of ice in the flexure zone.

131 Model

132 We model ice in the flexure zone as an elastic beam of varying thickness, under small deformations due to
 133 tidal forcing, such that the linear elastic approximation holds and the resultant vertical deflection $w(x)$ is
 134 described by the Euler-Bernoulli beam bending equation, after Holdsworth (1969):

$$\frac{d^2}{dx^2} \left[D(x) \frac{d^2 w}{dx^2} \right] = \rho_w g [A_0 - w(x)] \quad (2)$$

135 where A_0 is the far-field sea level, ρ_w is the mass density of seawater, g is gravitational acceleration, and
 136 D is the spatially variable flexural rigidity of the beam:

$$D(x) = \frac{Eh^3}{12(1-\nu^2)} \quad (3)$$

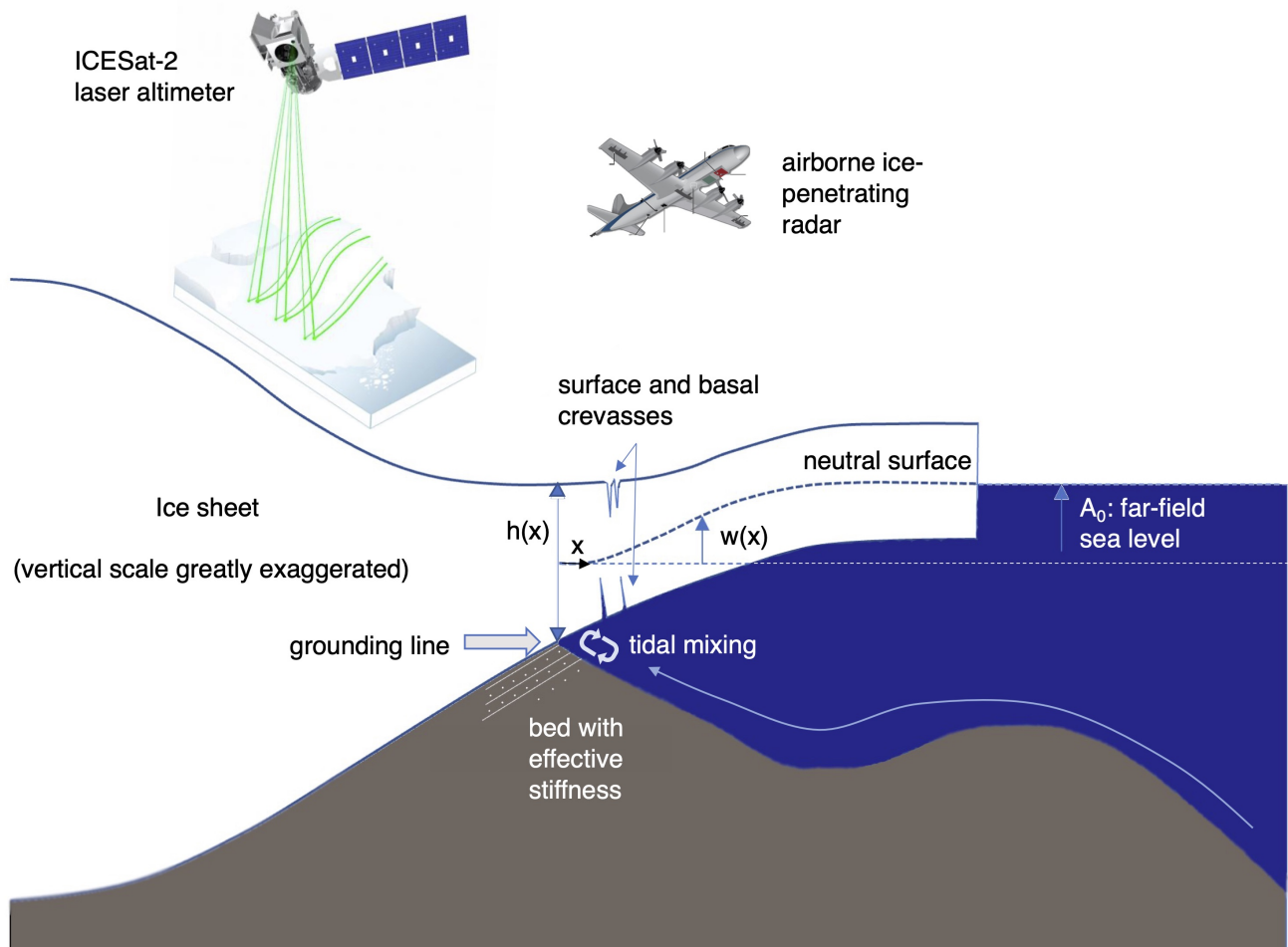


Fig. 1. We model the flexure zone as an elastic beam under tidal forcing with effective Young's modulus E^* . The far-field sea level is $A_0(t)$. At $A_0 = 0$, $w(x) = 0$, and the neutral surface of our model beam rests on the x -axis. As the tide changes, the upward force on the beam is the hydrostatic pressure proportional to the difference between A_0 and $w(x)$. We can observe this flexure by differencing repeat track ice elevation measurements from ICESat-2. We allow the ice thickness in the grounding zone to vary as $h(x)$ and model the resultant combined flexure of such a beam with an effective Young's modulus E^* . Surface and basal crevasses may be present. Tidal mixing takes place at the ice-ocean interface, and the grounding line may move back and forth.

137 where ν is the Poisson's ratio of ice, here taken to be 0.3.

138 To solve, we discretize Eq. 2 by central differences after Jacquot and Dewey (2001) (see supplement
139 S1). Given full information about D , that is, h and E , the flexure of a beam under an applied forcing can
140 be readily calculated. This is called the forward problem. The inverse problem seeks to infer properties
141 of the beam given the flexure w . However, we will only ever be able to invert or optimize for the flexural
142 rigidity D of the beam, that is, only one of the Young's modulus or the thickness, assuming the other is
143 known. This problem is also ill-posed, making it sensitive to noise and resulting in solutions that may
144 oscillate about the correct one (Lucchinetti and Stüssi, 2002).

145 Solving Eq. 2 requires four boundary conditions. The model results are sensitive to these boundary
146 conditions and where they are applied. Here, we select our problem area as the flexure zone: everywhere
147 flexure is observed to occur on tidal timescales. The landward boundary is where vertical tidal deflection
148 vanishes. The seaward boundary is where the vertical tidal deflection reaches the constant value of A_0 ,
149 the far-field sea level, which varies at each repeat track measurement. That is, $\partial w/\partial x = 0$ and $w = 0$ at
150 $x = 0$, and $\partial w/\partial x = 0$ and $w = A_0$ at $x = L$, where L is the length of the flexure zone. This is the same
151 set of boundary conditions used by Vaughan (1995). The length of the flexure zone and A_0 are inferred
152 from ICESat-2 data, as described in the ICESat-2 grounding zone deflection data section.

153 Then, using spatially varying ice thickness from DICE (see below), we use the MATLAB patternsearch
154 solver (The MathWorks Inc., 2022), a global optimization algorithm, to solve Eq. 2 many (e.g., hundreds or
155 thousands) of times while varying E^* to find the value of E^* that minimizes the square of the misfit between
156 the model and data. In order to manage the sensitivity of the problem to the location of the boundaries, we
157 repeat the optimization with the landward boundary at a range of discrete points up to several kilometers
158 upstream and downstream of its initial estimated position. This is akin to a free grounding line position,
159 though we note that the landward extent of tidal flexure may well be upstream of the grounding line.
160 Our method includes a grid search that sweeps across the parameter space of E^* and the position of the
161 landward boundary, seeking a minimum in the misfit space that is sensible among beams and tracks (see
162 below boundary value grid search section). For each beam and track at each site, we infer a value for E^* .

163 Variable ice shelf thickness data

164 Here we use ice thickness data from the ROSETTA-Ice airborne survey from 2015–2017 (Das and others,
165 2020), which uses deep ice radar (DICE) with a 188 MHz center frequency, 60 MHz bandwidth, and 1.4

166 m range resolution to produce a dataset of ice thickness on the Ross Ice Shelf (Figure 2). To get spatially
167 varying ice thickness from DICE at points that are coincident with ICESat-2 ground tracks in our selected
168 regions of interest, we use all the DICE datapoints as a scattered interpolant in order to linearly interpolate
169 between them.

170 Interpreting radargrams in the grounding zone is more difficult than it is near the calving front because
171 seawater intrusion upstream of the grounding line and grounding zone hydrodynamics can complicate
172 delineation of the bottom surface of the ice (MacGregor and others, 2011). However, crossover analysis in
173 Das and others (2020) was robust over the three years of data collection and were consistent to 2 m. These
174 are among the most direct detailed measurements of ice thickness that presently exist. Continent-wide
175 data products of ice shelf thickness typically calculate ice thickness by assuming that ice downstream of a
176 fixed grounding line is free-floating and in hydrostatic equilibrium, then make a correction based on any
177 existing nearby radar ice thickness measurements and interpolate (Morlighem and others, 2020). This is
178 an excellent assumption far from the grounding line that breaks down in the grounding zone, where ice is
179 mechanically coupled to the grounded ice sheet. In places where ice-penetrating radar data is sparse, the
180 hydrostatic assumption systemically biases inferred ice thickness.

181 **Boundary value grid search**

182 The location of the inward limit of tidal flexure can migrate several kilometers on tidal timescales, as is
183 seen in both observations and models (Freer and others, 2023; Robel and others, 2022; Stubblefield and
184 others, 2021). This is sometimes interpreted as the grounding line position migrating on tidal timescales.
185 While this may indeed sometimes be the case, we argue that in this application the precise location of the
186 grounding line cannot be treated identically as the landward limit of tidal flexure, as ice is thick and some
187 of the flexure will necessarily be transferred upstream of the grounding line. Here, we allow the landward
188 boundary to vary for every track by performing a grid search around the initial estimated position where
189 $\partial w / \partial x = 0$, which we first estimate as described below in the ICESat-2 grounding zone deflection data
190 section. This implicitly allows for tidal migration of the grounding line, as well as includes any flexure of
191 the bed that occurs below the observable surface flexure. This method is akin to a “free/stiff” configuration
192 in the parlance of Sayag and Worster (2013), where the ice shelf rests on a stiff bed but the grounding line
193 position is allowed to vary at different points in the tidal cycle. Note that we do not assume nor require the
194 bed to be stiff, but rather that we use the combined flexure signal to infer the effective Young’s modulus

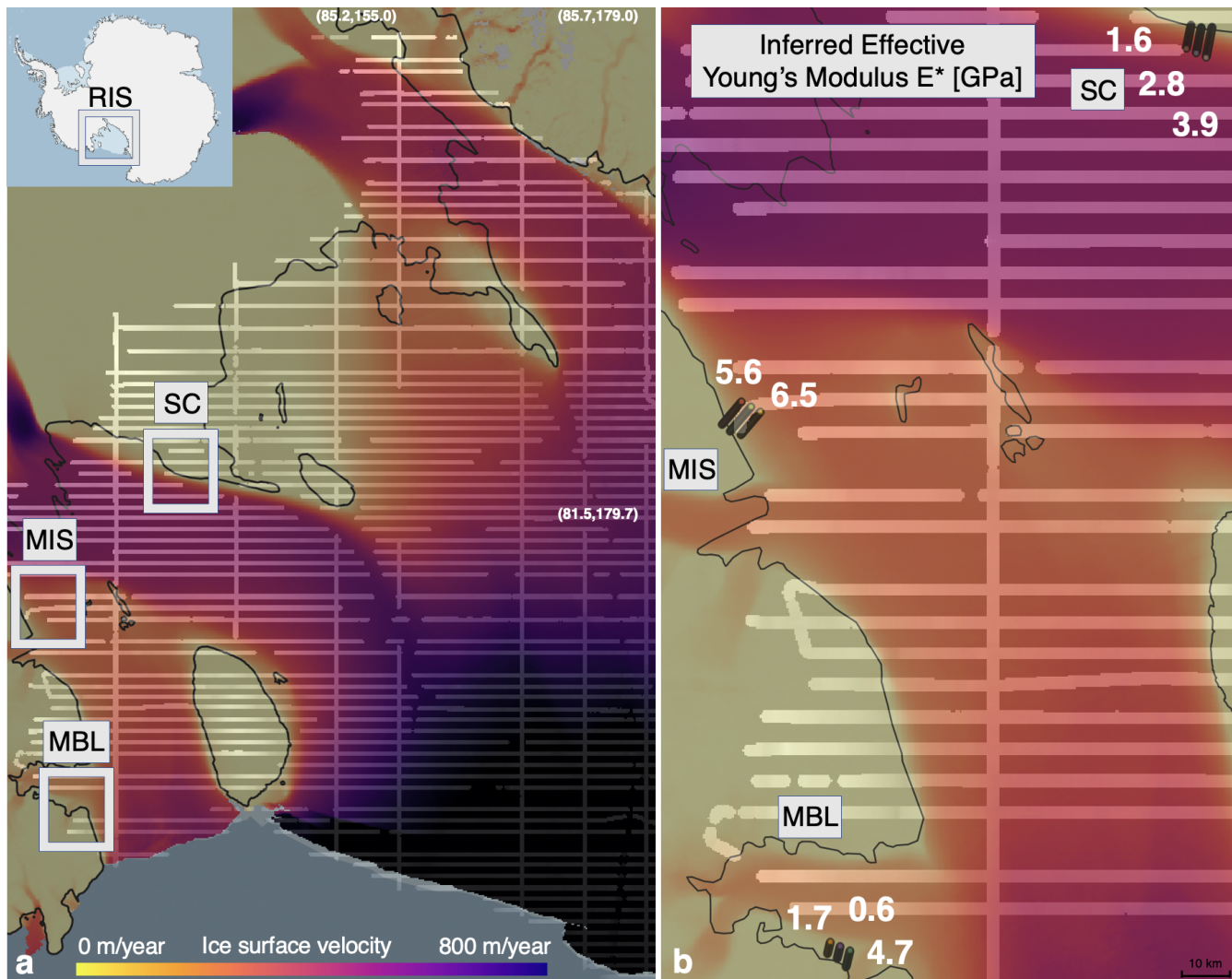


Fig. 2. The Ross Ice Shelf (RIS) and its location on the Antarctic continent (inset). The three regions studied here are highlighted: Siple Coast (SC), MacAyeal Ice Stream (MIS), and Marie Byrd Land (MBL). Study sites were chosen for their proximity to DICE measurements, distance from confining topography, and flexure data consistent with the boundary conditions we apply here. Horizontal and vertical lines show flight paths from the ROSETTA-Ice airborne ice-penetrating radar campaign (Das and others, 2020). Colors in panel (a) show ice surface velocity (Mouginot and others, 2017). Panel (b) shows the average inferred effective Young's modulus along each beam pair ground track at each site. Sections of the ICESat-2 ground tracks used for modeling are shown, dotted. Coordinates in (a) are provided at select points for spatial reference. A 10 km scalebar is shown in (b).

195 E^* .

196 We repeat the optimization described above at discrete points around the initial estimated landward
197 boundary position, first up to 1 km away in both along-track directions. We seek a well-behaved minimum
198 in the the square of the misfit between the modeled and observed deflection. That is, a single distinct
199 minimum within the bounds of the search, and a boundary position that makes physical sense with respect
200 to the neighboring beams. If necessary, we extend the grid search to 2 or 4 km in either direction to meet
201 these conditions. Figures detailing the grid searches for all data used can be found in Supplement S1.

202 Sometimes, there is more than one minimum in the misfit space as we vary the landward boundary
203 position, due to the ill-posedness of the problem. In these cases, great care is taken in selecting the best
204 result that is consistent between neighboring photon ground track pairs. Detailed information on each case
205 is presented in Supplement S1, but generally, we also calculate the derivative of the observed flexure and
206 use it to pick the result containing more information about the flexure.

207 **ICESat-2 flexure zone deflection data**

208 Tidal flexure of ice shelves can be readily observed by making repeated measurements of ice shelf sur-
209 face height at different points in the tidal cycle and differencing those measurements. This has been
210 accomplished by snowmobile (Vaughan, 1995), with ICESat (Fricker and Padman, 2006), or as here, with
211 ICESat-2, a laser altimeter launched in 2018, which has much denser spatial resolution than ICESat, and
212 also with radar interferometry (Freer and others, 2023; Li and others, 2022; Wild and others, 2018). We
213 identify ICESat-2 reference ground tracks (RGTs) that cross the grounding line in three regions of the
214 Ross Ice Shelf: Marie Byrd Land (MBL), MacAyeal Ice Stream (MIS), and the Siple Coast (SC). We used
215 OpenAltimetry (Jodha and others, 2020) to identify ground tracks of interest that were approximately
216 perpendicular to the grounding line in the NASA MEaSUREs Antarctic grounding line dataset (Mouginot
217 and others, 2017) and away from confining topography that might influence the flexure.

218 Repeat track measurements occur at 91-day intervals. After excluding passes from 2018, when satellite
219 pointing was not yet optimized, as well as tracks with obvious discontinuities due to cloudy conditions or
220 other anomalous effects, 3–5 usable passes typically remain for a given photon track pair, which consists
221 of a strong and a weak beam. We use all three photon track pairs aboard ICESat-2, which are spaced 3.3
222 km apart, finding a value for E^* for each track within each pair. We report the average E^* value across all
223 tracks, for each photon track pair separately, in Table 1.

224 After processing and filtering the data as described in Supplement S1, we calculate the mean ice surface
225 height over time using all available ground passes. We then subtract this mean height from each individual
226 pass and apply a final smoothing step to obtain the vertical height anomaly for each track. We interpret
227 this anomaly as the vertical tidal deflection, w , under the assumption that with enough repeat tracks, the
228 mean height approximates the unstressed state with zero tidal amplitude.

229 The approximate locations of the boundaries of the grounding zone are selected visually from the
230 ICESat-2 grounding zone deflection data. As can be seen in Figure 1, the grounding line is the point
231 at which any landward height anomaly is flat and roughly zero. As described above, the grounding line
232 position is allowed to vary in the inversions by up to four kilometers in either direction, which makes its
233 visual selection appropriate as initial estimate. The precise location of the seaward boundary does not
234 sensitively affect the result of the inversion, so long as it is far enough away to not artificially shorten the
235 grounding zone, so it is approximated conservatively then not changed. The value of the far-field sea level,
236 A_0 , is also determined from the data as the deflection at the seaward edge of the grounding zone minus
237 the deflection at the grounding line. When A_0 is small, little information about ice flexure can be gleaned
238 by the inversion and these cases are left out of our analysis.

239 The apparent ice surface height will change as it snows. The best available data products of Antarctic
240 precipitation in this region are available only at 25 km resolution (Wessem and others, 2018), so this could
241 only be modeled as a constant valued offset to the observed surface. While this is visible in some of our
242 data as an approximately constant valued offset in height anomaly landward of the grounding line, since
243 we effectively zero the landward boundary in our model by making A_0 the difference in deflection between
244 the seaward and landward edge of the model, we make no further correction for snow. Snowfall on the
245 Ross Ice Shelf varies by site but is typically on the order of several centimeters to a meter of snow per year
246 (Cohen and Dean, 2013), while the tidal amplitude is approximately one meter about once a day (Padman
247 and others, 2018).

248 RESULTS

249 We identify three grounding zones on the Ross Ice Shelf that are far from confining topography, where DICE
250 thickness measurements exist close to the grounding line, ICESat-2 ground tracks are roughly perpendicular
251 to the grounding line, and where differenced ice elevation profiles show roughly zero displacement upstream
252 of the flexure zone and roughly constant displacement downstream of the flexure zone. These are the Siple

Table 1. Inferred effective Young's modulus (E^*) in GPa at each beam and track used across three study sites on the Ross Ice Shelf. Tracks A_{1-5} correspond to dates listed in Figure 3 and are not the identical at different sites.

Site	Beam	A_1	A_2	A_3	A_4	A_5
SC	1	5.4	–	–	2.3	–
	2	1.4	5.4	2.5	1.6	3.0
	3	1.3	1.8	2.3	1.3	1.2
MIS	2	9.0	7.5	5.4	8.8	1.7
	3	–	4.7	–	6.4	–
MBL	1	4.3	–	–	5.0	–
	2	0.6	–	–	–	–
	3	0.8	–	–	2.6	–

253 Coast (SC), MacAyeal Ice Stream (MIS), and Marie Byrd Land (MBL). They are identified in Figure 2. The
 254 flexure zones are between 6 and 15 km wide between the seaward and landward boundaries as identified
 255 in the methods section. We observe that the landward boundary of the flexure zone often migrates on
 256 kilometer scales on tidal timescales.

257 After smoothing, filtering, and selection of approximate boundaries of the flexure zone, we run the
 258 inversion described in the Model section at each site, using each of the three beam pairs aboard ICESat-
 259 2, which are spaced 3.3 km away from each other on the ground. We find a resultant effective Young's
 260 modulus for each track, which are measured at different points in the tidal cycle. There are between one
 261 and five usable (that is, without discontinuities or other data issues, discussed further in supplement S1)
 262 tracks per beam at the sites. The average effective Young's modulus E^* found across all measurements at
 263 all sites is 3.6 ± 2.5 GPa. Results for all tracks, beams, and sites are presented in Table 1.

264 We do not see any clear differences in effective Young's modulus in observations taken at high versus low
 265 tide. We ensure that solutions presented in Table 1 are self-consistent by verifying that the minima found
 266 in the inversion are accurately selected, that the boundary position grid search has appropriately found a
 267 minimum in misfit space, and that the flexure extent positions measured by different photon ground track
 268 pairs are plausible with respect to each other. Detailed results can be found in supplement S1.

269 Below, we summarize the results at each site and plot the observed and modeled flexure used to invert
 270 for the effective Young's modulus there.

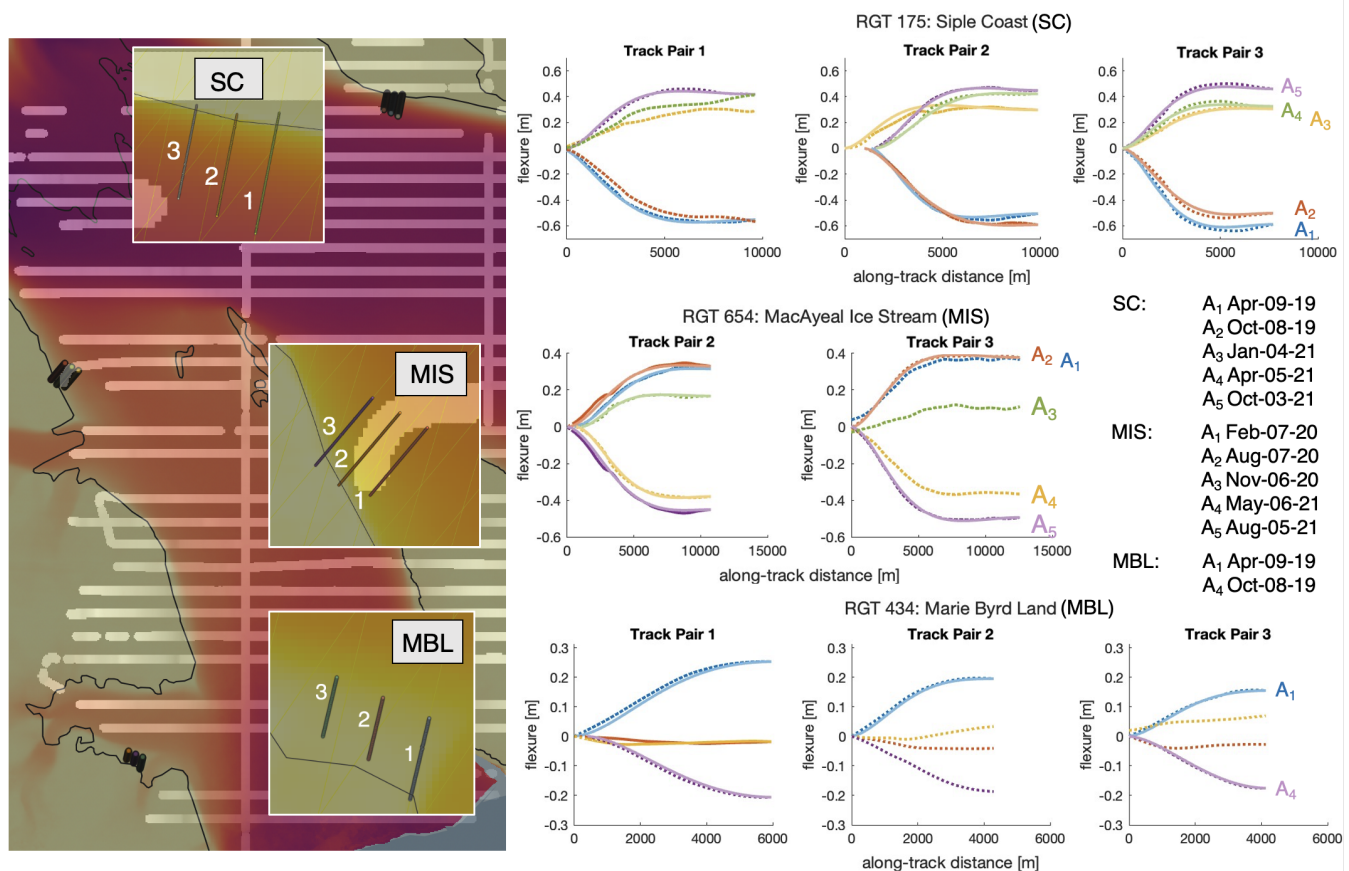


Fig. 3. Observed tidal flexure at three study sites along ICESat-2 ground tracks. (Left) The locations of the three regions: Siple Coast (SC), MacAyeal Ice Stream (MIS), and Marie Byrd Land (MBL), are shown with insets highlighting the three photon ground track pairs (1–3) used at each site. The background color map shows ice velocity (see Fig.2b) with overlying ROSETTA-Ice radar tracks. (Right) Flexure profiles for each beam, grouped by site and Reference Ground Track (RGT): RGT 175 (SC), RGT 654 (MIS), and RGT 434 (MBL). Each curve represents a measurement at a different point in the tidal cycle, labeled A_1 – A_5 , and corresponds to a unique date of observation (listed at right). Vertical displacement $w(x)$ is measured from repeat-track surface elevation anomalies derived from ICESat-2. Only observations with consistent flexural behavior and sufficient spatial coverage were retained for inversion. Dotted lines depict data. Solid lines depict modeled flexure.

271 **Siple Coast**

272 At the Siple Coast (SC) site, we observe qualitative differences in the shape of the flexure profiles across
273 the three photon track pairs, and an average effective Young's modulus of 2.5 ± 1.4 GPa across all three.
274 At photon track pair 1, of the five available passes, tracks 2, 3, and 5 exhibit small features within the
275 flexure data that lead to artificially high estimates of effective Young's modulus, so we exclude them from
276 our analysis. These features are not present at photon pairs 2 or 3, where all tracks are retained. The
277 flexure profiles at pairs 2 and 3 yield similar effective Young's moduli. While flexure extent line positions
278 are similar at pair 3, they migrate several hundred meters across passes at pair 2. Across all pairs and
279 tracks, values of E^* are similar at both high and low tide. We also note a slight decreasing trend in effective
280 Young's modulus moving eastward across the site. The mean E^* at photon track pairs 1, 2, and 3 is 1.6
281 GPa, 2.8 GPa, and 3.9 GPa, respectively.

282 **MacAyeal Ice Stream**

283 At MacAyeal Ice Stream (MIS), two of the three photon track pairs of RGT 654 produce usable results.
284 Values of E^* span both higher and lower than the mean at both high and low tide. Similarly, the inland
285 flexure extent position varies by about a kilometer between tracks, without any clear correlation with tidal
286 phase. Photon track pair 3 presents the greatest difficulty in interpreting the flexure extent grid search:
287 the flexure extent is displaced more than 1 km from the initial estimate, and the misfit space contains
288 multiple minima. As detailed in Supplement S1, we ultimately use tracks 2 and 4 to infer effective Young's
289 moduli of 4.7 GPa and 6.4 GPa, respectively. In both cases, the inland extent of flexure is found to be
290 approximately 1.5 km from the initial estimate, with no systematic difference between high and low tide.
291 Flexure profiles at track pair 1 do not exhibit a clear transition to a flat signal beyond the tidal amplitude,
292 so we assume our model is not applicable there and exclude them.

293 **Marie Byrd Land**

294 At the Marie Byrd Land (MBL) site, we observe qualitatively different flexure profiles at each track, though
295 fewer tracks are usable in our inversion compared to the other two sites. Tracks 2 and 3 are too close to
296 the unstressed (flat) state to be used for modeling. This site is also located closer to potential confining
297 topography than either other site. We observe less than one kilometer of motion in the flexure extent
298 between measurements. We use tracks 1 and 4 from photon track pairs 1 and 3, and track 1 only from

299 pair 2. The average E^* at MBL is 2.7 ± 1.8 GPa. The average inferred E^* for all tracks at photon pairs 1,
300 2, and 3 is 4.7 GPa, 0.6 GPa, and 1.7 GPa, respectively. Potential contributors to the spread in E^* values
301 are discussed below.

302 DISCUSSION

303 We present a method for estimating effective elastic rheology near the grounding zone using tidal flexure
304 observations from ICESat-2 and show that by varying a single effective elastic parameter in the flexure
305 zone we can achieve excellent fits to flexure data with ice-penetrating radar ice thickness that varies in
306 space from the ROSETTA-Ice campaign. Our results suggest that effective rheology near the grounding
307 line may vary significantly in space.

308 Our model is linear elastic. Linear elasticity is an adequate approximation when the forcing timescale is
309 short compared to the viscous relaxation timescale of ice in the grounding zone—that is, when the Deborah
310 number $De = \frac{\tau_{ve}}{\tau_{forcing}}$ is large. Taking the viscous relaxation timescale $\tau_{ve} = 2\mu/E$ (Turcotte and Schubert,
311 2014), where μ is the dynamic viscosity, approximated as 6×10^{14} Pa s from Ranganathan and Minchew
312 (2024), using $E = 3.6$ GPa, the mean value from results found here, and taking the forcing (tidal) timescale
313 on the RIS to be approximately 12 hours (Padman and others, 2018), we find $De \approx O(10)$. This indicates
314 that while the linear elastic approximation may be valid in some grounding zones around Antarctica, care
315 is warranted in selection of those areas, as the effective rheology of ice is increasingly observed to have
316 strong spatial variation (Ranganathan and Minchew, 2024; Millstein and others, 2022) and there may be
317 places where the viscous timescale is comparable to the tidal timescale.

318 There is no indication that the available data support differentiation between the ice and bed elasticity.
319 Using a single effective elastic parameter is a convenient choice that allows the uncertainty associated
320 with doing so to be quantified. In any tidal flexure model, there is a tradeoff in uncertainty in the
321 elastic parameters of the ice or the bed or the tide selected for use in the model, which is evident in the
322 spatial variation we find in effective Young's modulus. In addition to the known temperature and frequency
323 dependence of E , which implicitly accounts for some anelastic effects, and the increasingly evident variation
324 in the viscous properties of ice, there is no consensus yet about how the elastic properties of ice might
325 differ as it exists in situ. There is basal and surface crevassing as well as microcracks, interstitial inclusions,
326 larger grain sizes than laboratory experiments allow, air bubbles, and other effects that might be collectively
327 parameterized as damage. Overall, our mean result of $E^* = 3.6 \pm 2.5$ GPa suggests that until we can

328 better disentangle the intersecting processes in the grounding zone, we must either accept high uncertainty
329 in effective rheology, or that any attempt at parameterization of the grounding zone may require a piecewise
330 approach with different assumptions in different locations. The magnitude of this uncertainty is similar
331 to the result of Rosier and others (2017), who find a factor of two of uncertainty in effective Young's
332 modulus. Laboratory experiments that can more closely emulate in situ ice with inclusions or damage and
333 at forcing frequencies close to the tidal range would be valuable in understanding the natural laboratory
334 of the Antarctic Ice Sheet.

335 We did not find obvious differences in effective Young's modulus at high versus low tide at any site. This
336 is in contrast to Sayag and Worster (2013), who find higher elastic moduli at high tide than at low tide and
337 argue that subglacial lubrication is enhanced at high tides. We cannot rule out due to the small number
338 of sites that tidal effects may exist, and indeed agree that subglacial hydrology may affect observable tidal
339 flexure. However, we argue that the elastic properties of the bed and subglacial till are effectively unknown
340 and cannot be distinguished from the flexure of the ice, and are thus incorporated into our inference of
341 E^* . First principles modeling of the subglacial environment and how it is affected by tides and how this
342 translates to surface flexure will help step towards more accurate representation of the grounding zone in
343 larger ice models.

344 Beam bending models, or plate bending models in 2D or 3D, with different boundary conditions may
345 be able to accommodate a wider subset of ice geometries than presented here. Out of 30 candidate RGTs
346 around the Ross Ice Shelf, only three sites were suitable enough for inclusion in the results. We select and
347 use tracks whose flexure profiles suggest that the linear elastic approximation is a reasonable one: with
348 smooth flexure between a flat upstream region and a flat downstream region that varies with time. We
349 pick tracks that are roughly perpendicular to the grounding line and away from confining topography that
350 might induce flexure in cross-track dimensions. No sites in the Transantarctic Mountains met these criteria:
351 those tracks had different patterns of tidal deflection. Those grounding lines may also be substantially
352 three dimensional as the ice flows off steep topography. The observable flexure on smaller scales sometimes
353 needed to be excluded as well: beam 1 at the MIS site showed a downward flexure feature where the
354 other two tracks remained constant seaward of the flexure zone, indicating there might be influence from
355 other bathymetry features or in cross-track directions. Future work that seeks to parameterize grounding
356 zone processes in larger models may need to take into account the inherent spatial variability in effective
357 grounding zone rheology in order to accurately translate from tidal timescale processes to long-term ice

358 sheet dynamics.

359 The variance in E^* both among and between the sites suggest that a linear elastic parameterization
360 works very well in some sectors and less well in others. The spread in values at the MBL site, for instance,
361 might be noise around a mean value, but also might be related to the ill-posedness of the problem, or reflect
362 that the nearby ice stream (see figure 3) is complicating the flexure pattern, or the subglacial system that
363 is implicit to our parameterization. At the MIS site, values for E^* cluster closely together, while at the SC
364 site, a downward trend in E^* moving eastward is reflected in our results. Finding E^* in more and more
365 continuous regions will help elucidate the true scale of spatial variance in effective Young's modulus.

366 The choice to parameterize the grounding zone in terms of an effective elastic rheology is both motivated
367 by the need for an independent estimation of the ice thickness and thickness gradient there in order to
368 calculate the basal melt rate of ice near the grounding line and also justified by the uncertainty about
369 basal conditions in the grounding zone and of the rheology of ice. The ice flexure reveals that there is
370 evident variability in effective rheology on sub-catchment scales; what causes this variability and how it
371 can be parameterized in large ice models will be more and more elucidated by fundamental physics models
372 constrained by the remote sensing data record that accumulates day by day.

373 CONCLUSION

374 We infer an effective Young's modulus of 3.6 ± 2.5 GPa from ICESat-2 data by modeling the flexure zone as
375 an elastic beam of variable thickness. Surface flexure measurements do not readily allow us to distinguish
376 between deformation of the bed and deformation of the ice. Therefore, the inferred elastic modulus reflects
377 the combined flexural response of both ice and bed, along with any additive tidal or subglacial hydrological
378 effects, as well as the in situ relationship between stress and strain in ice, which is time-dependent due
379 to its viscoelasticity. Understanding how the effective Young's modulus varies in space at kilometer scales
380 offers new insight into the mechanical character of the ice–bed–ocean interface, and here, is the first step
381 in a method for making observationally constrained estimates of ice thickness and thickness gradient in the
382 grounding zone.

383 Making models of Antarctica that accurately reflect its current state and recent past requires careful
384 modeling of the grounding zone, and better parameterizations of the processes that occur there on inter-
385 secting spatial and temporal scales. Our work shows that a linear elastic model of tidal flexure could be
386 one method for doing so, but also that it would likely need to be part of a suite of solutions encompassing

viscous, viscoelastic, and viscoplastic flexure as well. The geometry of the grounding zone is complex and dynamic: different bending geometries that fit the topography on the scales over which it varies could be employed. Extending our elastic beam bending method to a viscoelastic plate bending flexure method is one clear way that more complex geometries could be represented, but the extent to which any of these potential methodologies are differentiable from one another within a small, noisy, observable signal is not yet well constrained.

The results presented here add to the constellation of estimates of the effective rheology of ice from observational and experimental methods. We are able to consider questions of variability of effective Young's modulus on small scales only because this is one of the areas where direct ice-penetrating radar measurements of ice thickness exist. In other places, ice thickness in the grounding zone is estimated by assuming ice shelves are fully free-floating and their thickness can be calculated by measuring the height of the ice above flotation. In the grounding zone, this would bias ice thickness estimates. In order to make an independent estimation of the ice thickness near the grounding line from the surface flexure, we must first make a reasonable estimation of the effective Young's modulus. More direct measurements of ice thickness in grounding zones would be a rich resource for the glaciological community as we attempt to untangle the intersecting physical processes in grounding zones, and deeply pertinent to IPCC goals toward reducing uncertainty about future sea level rise.

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