

# Complex earthquake behavior on simple faults

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## Key Points:

- Numerical simulations and fracture mechanics predict the occurrence of partial ruptures on sufficiently large fault loaded by creep
- Earthquake statistics is controlled by the ratio of fault dimension to a critical length related to the nucleation dimension
- Large faults exhibit afterslip driven temporal clustering and power-law distribution of seismic moments with a theoretical b-value of 3/4

## Abstract

While power-law distributions in seismic moment and interevent times are ubiquitous in regional catalogs, the statistics of individual faults remains controversial. Continuum fault models typically produce characteristic earthquakes or a narrow range of sizes, leading to the view that the regional statistics originates from interaction of multiple faults. I present theoretical arguments and numerical simulations demonstrating that seismicity on homogeneous planar faults can span several orders of magnitude in rupture dimensions and interevent times, if the fault dimension  $W$  is sufficiently large compared to a characteristic length  $L_{crit}$ , related to the nucleation dimension. Large faults are increasingly less characteristic, with the fraction of system-size ruptures proportional to  $(L_{crit}/W)^{1/2}$ . Earthquake statistics for large  $W/L_{crit}$  is remarkably close to nature, exhibiting Omori decay and power-law distributed rupture lengths. Simple crack models are consistent with a Gutenberg-Richter distribution with  $b = 3/4$ , and provide a physical basis for these distributions on individual faults.

## 1 Introduction

Seismic hazard models are one of the most societally relevant products of earthquake research, but they often rely on poorly tested assumptions due to the scarcity of data on the recurrence interval of large earthquakes. Whether individual faults are more characteristic than predicted by Gutenberg-Richter distribution remains a subject of debate (Parsons & Velasco, 2009; Schwartz, 2010; Page, 2010; Parsons et al., 2012; Kagan et al., 2012; Page & Felzer, 2015; Mulargia et al., 2017; Parsons et al., 2018; Stirling & Gerstenberger, 2018).

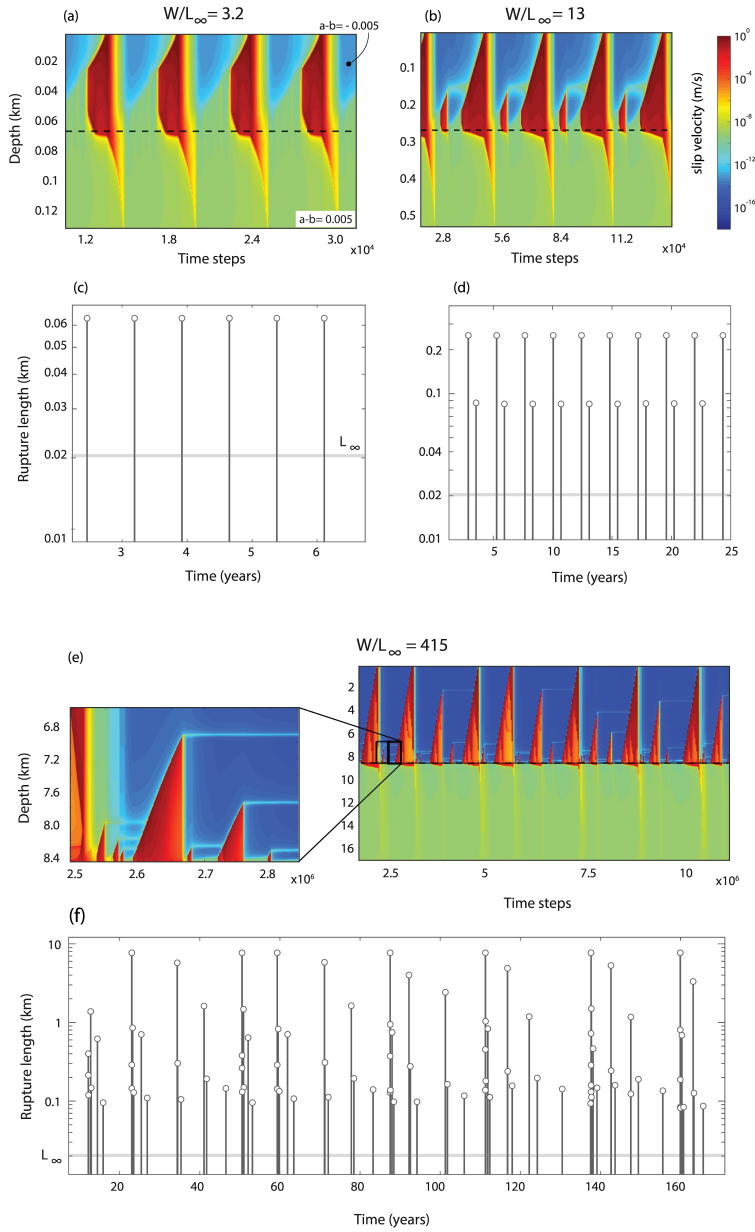
The debate on the frequency-size distribution of earthquakes is echoed in the earthquake physics community. Early earthquake cycle simulations of planar faults in an elastic medium produced sequences of periodic, system-size events (Tse & Rice, 1986; Rice, 1993). Subsequent studies (Lapusta et al., 2000; Lapusta, 2003) found that sub-system-size events (partial ruptures) occur for a small enough slip weakening distance. While simple limit cycles with few or no partial ruptures are produced by correctly discretized models, under resolved simulations exhibited richer slip complexity, including a power-law distribution of rupture dimensions (Rice, 1993; Lapusta et al., 2000; Ben-Zion & Rice, 1995). Even though these models do not correctly solve the continuum equations, one interpretation is that oversized cells represent distinct fault segments, and can be con-

41 considered a proxy for geometrical heterogeneity (Ben-Zion & Rice, 1995). Similarly, dis-  
 42 crete models of faults (such as cellular automata (Bak & Tang, 1989; Olami et al., 1992)  
 43 or discrete elastic models (Burridge & Knopoff, 1967)) produce a power-law distribution  
 44 of earthquake size, analogous to what is observed in nature.

45 The discrepancy between the characteristic and periodic behavior of continuum mod-  
 46 els and the rich complexity of discrete models led to the view that the statistics of seis-  
 47 micity on a regional scale is controlled by the discrete nature of faults (Ben-Zion & Rice,  
 48 1995; Ben-Zion, 2008) or by frictional/geometrical fault heterogeneity (Hillers2007,  
 49 Aochi2009, Kaneko2010, Dublanche2013; the latter is also understood to be responsi-  
 50 ble for partial ruptures on megathrust faults (Li et al., 2018; Qiu et al., 2016; Dal Zilio  
 51 et al., 2019). An important question is then: if fault roughness, segmentation and the  
 52 interaction between separate fault segments are responsible for earthquake statistics on  
 53 a regional scale, are relatively smooth, isolated faults more likely to exhibit character-  
 54 istic quasi-periodic behavior?

55 Here I address this question from a fracture mechanics perspective, and challenge  
 56 the view that a simple fault geometry in a linear elastic medium implies strict period-  
 57 icity or a predominance of characteristic ruptures. From simple energy balance arguments,  
 58 I demonstrate that an homogeneous velocity-weakening (stick-slip) fault in an elastic medium,  
 59 adjacent to or overlying a velocity strengthening (creeping) region can rupture in earth-  
 60 quakes of variable magnitude, and exhibit temporal clustering, if it exceeds a critical di-  
 61 mension relative to the nucleation size.

62 For 2-D earthquake cycle simulations on vertical, antiplane rate-state faults loaded  
 63 by downdip creep (Supplementary Information 1), the transition is illustrated in Fig. 1  
 64 and was first observed by Werner and Rubin (2013). The smallest fault ( $W = 3.2L_\infty$ ,  
 65 with  $L_\infty$  the nucleation half-length defined below) exhibits simple cycles of system-size  
 66 events, while two earthquakes per cycle occur at  $W = 13L_\infty$  and about 12 earthquakes  
 67 per cycle at  $W = 415L_\infty$  (where a “cycle” is the time between two full ruptures). Small  
 68 faults are characteristic and periodic (Fig. 1a,c), and 2 rupture cycles have a bimodal  
 69 rupture distribution of rupture length and interevent times (Fig. 1b,d). In contrast, earth-  
 70 quakes on a large fault span two orders of magnitudes in rupture length and exhibit tem-  
 71 poral clustering (Fig. 1e,f). Below I present a simple theoretical model for these results,



**Figure 1.** Examples of simulated cycles on antiplane vertical faults with variable  $W/L_\infty$ , with color indicating slip speed on a log scale. The dotted line marks the velocity-weakening to velocity-strengthening transition. The x-axis shows computational time steps. (a,c) For  $W/L_\infty = 3.2$  periodic full ruptures occur; (b,d)  $W/L_\infty = 13$  results in two ruptures per cycle; (e,f)  $W/L_\infty = 415$  has wider range of rupture dimensions and temporal clustering. The parameters for this simulation are  $b = 0.02$ ,  $a - b = \pm 0.005$  in the VS/VW regions,  $\sigma = 50$  MPa,  $d_c = 10^{-4}$  m.

72 compare them with empirical power-law distributions observed in nature, and discuss  
 73 the implications for earthquake cycle models and seismic hazard.

## 74 **2 Theory**

75 Consider a velocity-weakening (VW) fault segment loaded by creep from an adja-  
 76 cent velocity-strengthening region: for example, an isolated small asperity embedded in  
 77 a creeping fault, or a long anti-plane fault overlying a velocity-strengthening (VS) layer.  
 78 The stress state in the VW region is determined by the slip that has taken place in the  
 79 creeping region during the interseismic period since the last full rupture,  $S(t)$ . I argue  
 80 that the seismic behavior is controlled by the ratio of interseismic slip required to nu-  
 81 cleate an event ( $S_n$ ) to the slip required for a system-size event ( $S_{full}$ ). Intuitively, we  
 82 may expect  $S_n$  to increase with the nucleation dimension  $L_\infty$ , and the slip deficit for a  
 83 full rupture  $S_{full}$  to increase with the size of the VW region. Since  $S_n$  is constant and  
 84  $S_{full}$  increases with  $W$ , we expect two regimes: for  $S_n \geq S_{full}$ , all events are full rup-  
 85 tures, with simple characteristic cycles. For  $S_n < S_{full}$ , partial ruptures occur. The ra-  
 86 tio of partial ruptures to full ruptures increases with  $S_{full}/S_n$ . From dimensional argu-  
 87 ments (and confirmed more rigorously below), we may expect  $S_{full}/S_n$  to be an increas-  
 88 ing function of  $W/L_\infty$ : this ratio defines the different regimes and degree to which the  
 89 fault is characteristic.

90 To make this argument quantitative, consider a 1D crack in an infinite medium loaded  
 91 by end-point displacement  $S$  on a fault of total extent  $W$ . Quasi-static crack propaga-  
 92 tion is controlled by an energy balance criterion (Griffith, 1921), equivalent to requir-  
 93 ing the stress intensity factor (SIF)  $K$  at the crack tip to be equal to the fracture tough-  
 94 ness  $K_c$  (Irwin, 1957). The stress intensity factor can be written as  $K = K_l - K_{\Delta\tau}$ ,  
 95 where  $K_l$  is the SIF due to loading a stress free crack, and  $K_{\Delta\tau}$  accounts for uniform stress  
 96 changes within the crack. The quasi-static equation of motion for the crack tip at dis-  
 97 tance  $l$  from the load point is given by:

$$98 \quad K_l(l) - K_{\Delta\tau}(l) = K_c \quad (1)$$

99 After each earthquake, the stress near the VS-VW transition is low and the fault  
 100 is locked; as slip accumulates in the VS region, creep penetrates within the VW region  
 101 (Fig. 1a,b). When it reaches a critical distance  $L_n$ , nucleation occurs. During creep prop-  
 102 agation, the fracture energy term is typically negligible (Cattania & Segall, 2018), so eq. 1

103 can be written as  $K_l(l) \approx K_{\Delta\tau}(l)$ . The stress intensity factors are  $K_l = \mu' S / \sqrt{2\pi l}$  and  
 104  $K_{\Delta\tau} = \Delta\tau \sqrt{\pi l / 2}$  (Tada et al., 2000), where  $\mu' = \mu$  for antiplane and  $\mu' = \mu / (1 - \nu)$   
 105 for plane strain deformation ( $\mu$  is the shear modulus,  $\nu$  the Poisson ratio);  $\Delta\tau$  is the stress  
 106 increase behind the creep front, which is equal and opposite to the stress drop in the pre-  
 107 vious event (Supplementary Information, section 2). Therefore the displacement required  
 108 for the creep front to penetrate a distance  $L_n$  is:

$$109 \quad S_n = \frac{\pi \Delta\tau}{\mu'} L_n . \quad (2)$$

110 As proposed by Werner and Rubin (2013), eq. 1 can also be used to estimate the  
 111 minimum time between full ruptures. The condition for full rupture can be simplified  
 112 noticing that  $K_{\Delta\tau} = 0$  over an entire cycle (Cattania & Segall, 2018; Werner & Ru-  
 113 bin, 2013), so that  $K_l = K_c$ . With the expression for  $K_l$  given above, the SIF is min-  
 114 imum at the top of the fault ( $l = W$ ). Therefore the displacement required for a full  
 115 rupture is:

$$116 \quad S_{full} = \frac{\sqrt{2\pi W} K_c}{\mu'} . \quad (3)$$

117 These critical displacements can be used to estimate the relative number of earthquake  
 118 nucleations and full ruptures. Suppose that slip in the creeping region accumulates at  
 119 a rate  $\dot{S} = V_{pl}$  (averaged across a cycle). The average seismicity rate and the rate of  
 120 full ruptures are simply  $r = \dot{S} / S_n$  and  $r_{full} = \dot{S} / S_{full}$ , and the total number of earth-  
 121 quakes per cycle is

$$122 \quad \alpha = \frac{r}{r_{full}} = \frac{S_{full}}{S_n} \sim \frac{K_c \sqrt{W}}{\Delta\tau L_n} \quad (4)$$

123 Partial ruptures occur when  $\alpha > 1$ : the critical slip required for nucleation is smaller  
 124 than the slip for a full rupture. This condition can be expressed in terms of the ratio  $W / L_{crit}$   
 125 with  $L_{crit} \sim (L_n \Delta\tau / K_c^2)$ . Note that the quantities  $L_n$ ,  $K_c$  and  $\Delta\tau$  are determined by  
 126 the elastic and frictional properties of the fault, and do not depend on any actual, mea-  
 127 surable length scale. For certain frictional laws, this expression can be simplified by con-  
 128 sidering how  $L_n$  depends on  $K_c$  and  $\Delta\tau$ . Here I assume this length to be proportional  
 129 to the critical nucleation length  $L_\infty$  derived by Rubin and Ampuero (2005) for ageing  
 130 law simulations by considering the stability of a constant stress drop crack overcoming  
 131 a toughness  $K_c$ , analogous to eq. 1 with  $K_l = 0$ . Taking  $K_{\Delta\tau} \sim \Delta\tau \sqrt{L_\infty} = K_c$ , gives

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$$L_\infty \sim \left( \frac{K_c}{\Delta\tau} \right)^2, \quad (5)$$

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and with  $L_n \sim L_\infty$  the number of ruptures per cycle is

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$$\alpha \sim \sqrt{\frac{W}{L_\infty}}. \quad (6)$$

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I therefore propose that the ratio  $W/L_\infty$  determines the seismic regime of the fault: characteristic and periodic if  $\alpha(W/L_\infty) \leq 1$ , and with a vanishingly small fraction of system-size ruptures as  $\alpha(W/L_\infty) \gg 1$ . While this result was derived for cracks in a 2-D medium, I argue that the scaling may remain valid for other geometries. The 2-D result for the nucleation criterion (eq. 2) is also the limit for a circular asperity with  $R \gg L_n$ , or indeed any geometry with a local curvature radius  $\gg L_n$ . The scaling of the critical slip required for a full rupture (eq. 3) is also common to other geometries, and varies only by a geometrical factor of order 1, as demonstrated by Cattania and Segall (2018) for circular asperities and in the Supplementary Information, section 2 for a vertical fault reaching the free surface.

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The dependence on  $W/L_\infty$  found here is due to the scaling of the nucleation length with stress drop and fracture energy for the ageing law and  $a/b > 0.378$  (Rubin & Ampuero, 2005). More generally, assuming that the number of events per cycles is  $S_{full}/S_n$ , with  $S_{full} \sim \sqrt{W}K_c$  and  $S_n \sim \Delta\tau L_n$  where  $L_n$  is the penetration length, the number of events scales with  $\sqrt{W/L_{crit}}$  where  $L_{crit} = (L_n \Delta\tau / K_c)^2$ .

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I test these predictions against numerical simulations of vertical antiplane faults (Fig. 1), described in the Supplementary Information (section 1). For this simple geometry, the only modification to the theory above consists of including the effect of the free surface, which modifies eq. 3 by a factor  $\phi = 0.71$  (Supplementary Information, section 2). Using expressions for fracture energy from rate-state friction the condition  $\alpha = 1$  is satisfied by

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$$\alpha \approx 0.45 \sqrt{\frac{W}{L_\infty}}. \quad (7)$$

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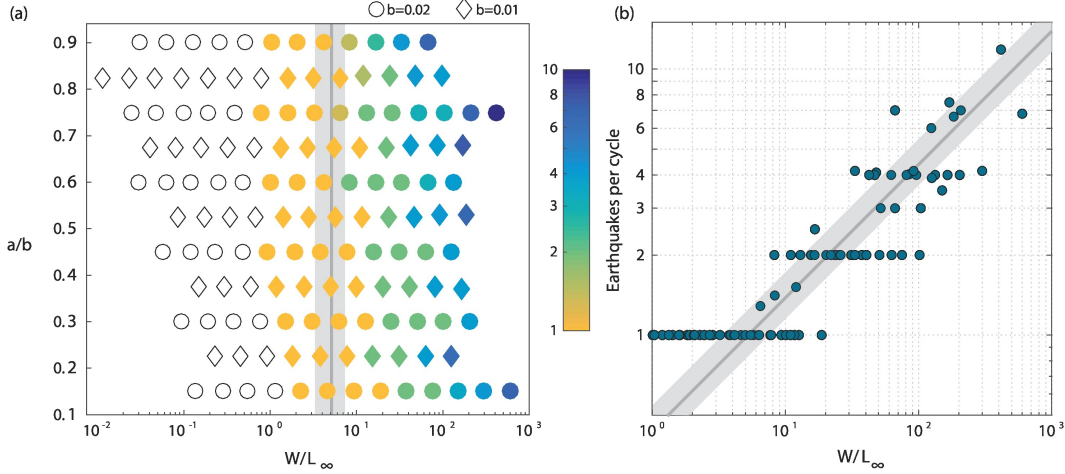
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Therefore partial ruptures are possible when  $\alpha > 1$  or  $W > (5 \pm 2)L_\infty$  (the range corresponds to the standard deviation of  $L_n/L_\infty$ , as described in the Supplementary Information, section 2). Fig. 2a shows a set of simulations with variable frictional (rate-state) parameters and fault dimension:  $W/L_\infty = 5 \pm 2$  is a reasonable approximation of the transition between single to double rupture cycles. The number of earthquakes per cycle is also well fit, to first order, by eq. 7 (Fig. 2b). Note that eq. 7 is derived assuming



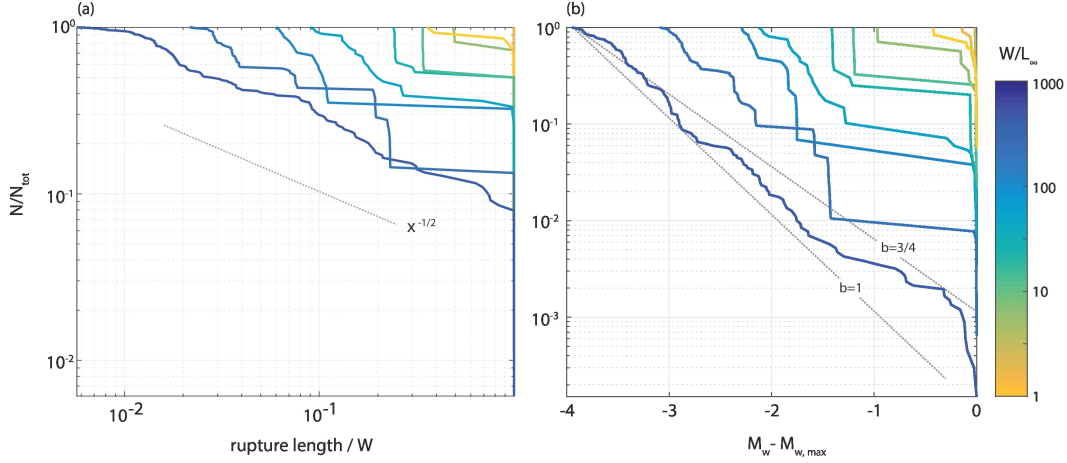
**Figure 2.** (a) number of earthquakes per cycle with variable  $W/L_\infty$  and  $a/b$ . The transition occurs near the value of  $W/L_\infty$  predicted by the crack model (grey line, indicating the estimated value for  $L_n = (2.9 \pm 0.6)L_\infty$ ). Symbols refer to two values of rate-state parameters  $b$  (0.01, diamonds; 0.02, circles), and white symbols on the left represent aseismic simulations. (b) number of ruptures per cycle vs.  $W/L_\infty$ , compared with eq. 7 (dotted line). Fractional values in the simulations are caused by the alternation of cycles with a different number of events.

164 that each displacement increment  $S_n$  corresponds to a single rupture; as discussed in the  
 165 next session, this may not be true in 3-D, and the actual number of events per cycle can  
 166 be higher than this (and may be estimated by geometrical arguments, see Supplemen-  
 167 tary Information 3). Moreover, in addition to the interseismic displacement accrued in  
 168 the creeping region, the stress field is modified by the occurrence of partial ruptures: for  
 169 example, the area of lower stress that can stop propagation of a further rupture (Lapusta2003, I find that this can lead to the occurrence of partial ruptures even  
 170 when the energy criterion above is satisfied ( $S(t) > S_{full}$ ), and increase the number of  
 171 partial ruptures relative to eq. 7.

### 173 3 Distribution of rupture lengths and magnitudes

174 The distribution of rupture lengths is characteristic for small asperities and bimodal  
 175 for asperities with a partial rupture per cycle (Fig. 3). On asperities exceeding hundreds  
 176 of nucleation lengths, and multiple ( $> 10$ ) ruptures per cycle, the distribution appears  
 177 close to a power-law truncated at the characteristic length  $W$ , and spans two orders of  
 178 magnitude. I do not attempt to derive this distribution from first principles, but instead





**Figure 3.** (a) Distribution of rupture lengths showing characteristic distribution at small  $W/L_{\infty}$ ; a bimodal distribution for  $W/L_{\infty} \sim 10$ ; a truncated power-law distributions for large  $W/L_{\infty}$  (up to 415).  $N/N_{tot}$  is the normalized survival function (fraction of events exceeding a certain rupture length). The dotted line shows the the power-law exponent consistent with the scaling in eq. 6. (b) Frequency-magnitude distribution from ruptures in a 3-D medium estimated as outlined in section 3 of the Supplementary Information. The dotted lines show a GR b-value of 0.75 and 1.0.

179 seek the power-law exponent consistent with previous results. Consider a survival func-  
 180 tion of the form  $N(l) = Al^{-\gamma}$  between  $L_{min}$  and  $L_{max}$ , where  $A$  is a constant and  $N$   
 181 is the number of events with rupture length  $\geq l$ . The total number of events is simply  
 182  $AL_{min}^{-\gamma}$ , and the number of events with characteristic length  $L_{max}$  is  $AL_{max}^{-\gamma}$  (there are  
 183  $AL_{max}^{-\gamma}$  greater than or equal to  $L_{max}$ , and zero events greater than or equal to  $(L_{max})+$   
 184  $\varepsilon$ , with  $\varepsilon$  an arbitrary small positive number, since the distribution is truncated). There-  
 185 fore the number of events per cycle is  $(L_{min}/L_{max})^{-\gamma}$ . Setting  $L_{min} \sim L_{\infty}$  and  $L_{max} =$   
 186  $W$ , and comparing this result with eq. 6, implies  $\gamma = 1/2$ , which is indeed the value  
 187 in the simulations (Fig. 3(a)).

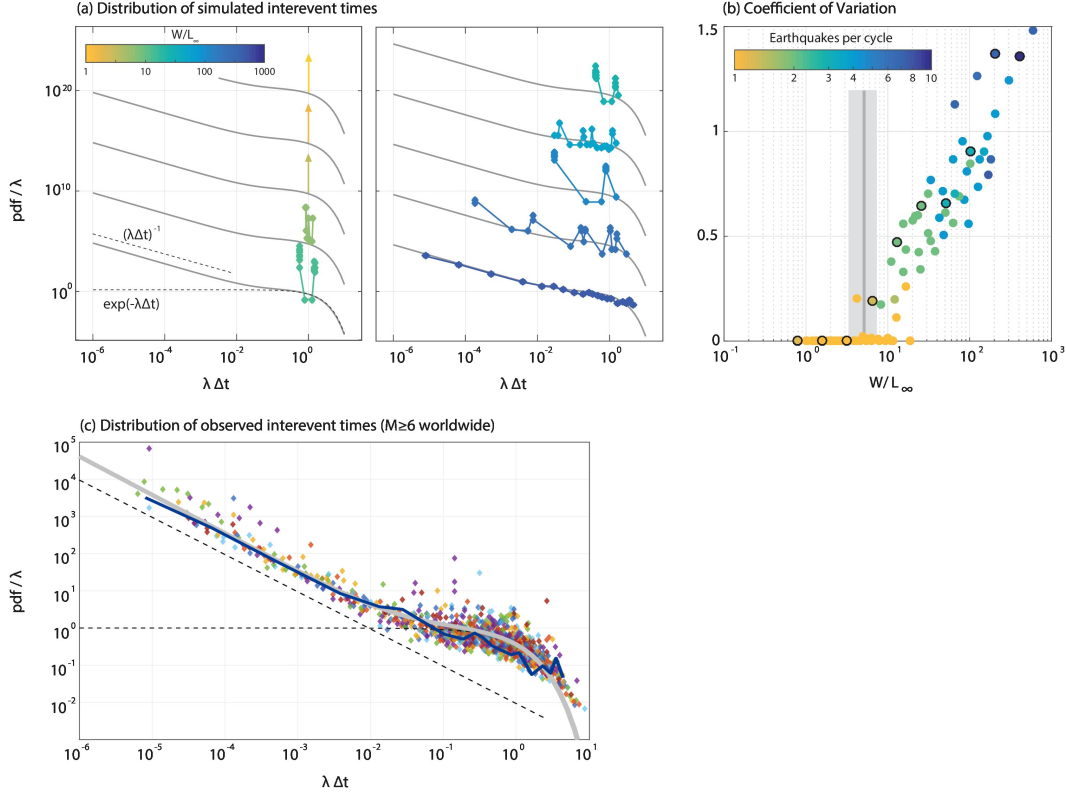
188 Estimating the frequency-magnitude distribution for 2-D fault embedded in a 3-  
 189 D medium requires some assumptions on the rupture length along strike, and the num-  
 190 ber of rupture corresponding to each event simulated in 2-D. Assuming that all ruptures  
 191 have the same aspect ratio and stress drop, the survival function in terms of seismic mo-  
 192 ments has the form:  $N(M_0) \sim M_0^{-1/2}$ , corresponding to a Gutenberg-Richter  $b$ -value  
 193 of 3/4 (Supplementary Information 3). Fig. 3(b) shows the frequency-magnitude dis-

194 tribution obtained after weighting each simulated event by the number of equivalent rup-  
 195 tures in 3-D, assuming constant aspect ratio. The  $b$ -value is close to the theoretical value  
 196 of  $3/4$ , but slightly larger; this seems to be related to a slightly sublinear scaling of slip  
 197 with rupture length. This range is remarkably close to the typical  $b$ -value of 1. However,  
 198 the assumptions made when converting ruptures simulated in 2-D into equivalent 3-D  
 199 ones should be verified by running simulations in 3-D, and considering rupture propa-  
 200 gation along strike after reaching the free surface.

#### 201 4 Inter-event time distributions

202 The distribution of interevent times also undergoes a similar transition from peaked  
 203 at low  $W/L_\infty$ , to power-law at high  $W/L_\infty$  (Fig. 4(a-b)). The coefficient of variation in  
 204 interevent time is  $\sim 10^{-4} - 10^{-3}$  for  $W/L_\infty < 4$ , indicating almost perfectly periodic  
 205 behavior; it increases to 1.5 for the largest  $W/L_\infty$  (415), indicating clustering. I com-  
 206 pare the interevent time distributions with those typically found in seismic catalogs: Fig. 4(c)  
 207 shows the distribution of interevent times, normalized by the average rate, for  $M_w \geq$   
 208 6.0 events in the ANSS Comprehensive Earthquake Catalog. Corral (2004) first noted  
 209 that the interevent time distribution rescaled by the total rate  $\lambda$  follows a universal form  
 210 independent of location and selection criteria; later, Hainzl et al. (2006); Saichev and Sor-  
 211 nette (2007) demonstrated that this behavior is well described by a function derived from  
 212 short term Omori clustering and background Poissonian seismicity. This is given by eq. 15  
 213 in the Supplementary Information (section 4) and shown by the grey lines in Fig. 4(a,c).  
 214 For  $W/L_\infty \gg 1$ , the simulated interevent time distribution approaches this expression,  
 215 with a clear power-law decay at short  $\lambda\Delta t$ , consistent with  $1/t$  Omori-decay, and a slower  
 216 decay at  $\lambda\Delta t \gtrsim 0.1$ . As shown in Fig. 4(c), the distribution at  $W/L_\infty = 415$  is con-  
 217 sistent with the theoretical expression and observed seismicity.

218 Temporal clustering is a direct consequence of afterslip: at a constant creep rate,  
 219 it would always take the same time to accumulate the displacement required for nucle-  
 220 ation  $S_n$  and interevent times would be constant even for multi-rupture cycles. The in-  
 221 crease in clustering on larger faults is caused by faster afterslip, caused by a wider af-  
 222 terslip region for larger ruptures. In the Supplementary Information (section 5) I use a  
 223 simple spring-slider model introduced by Perfettini (2004) to estimate the postseismic  
 224 creep rate at the loading point  $S(t)$ . This yields the following expression for the time to



**Figure 4.** (a) Probability density function of normalized interevent times calculated between any two consecutive events (full or partial ruptures) for the simulations with  $a/b = 0.75$ . They grey lines indicates eq. 15 in the Supplementary Information, compared with a  $1/t$  decay and the exponential distribution typical of Omori decay and a Poisson process respectively (black dotted lines). Simulations with a coefficient of variation smaller than  $10^{-3}$  are indicated by an arrow (delta function). Each curve is offset by  $10^5$  for clarity. (b) coefficient of variation (standard deviation in  $\Delta t$  divided by the mean) as a function of  $W/L_\infty$ , color coded by number of events per cycle. Circled simulations are those shown in the left panel. (c) Distribution of normalized interevent times for  $M_w \geq 6$  earthquakes in the ANSS Comprehensive Earthquake Catalog (Com-Cat), 1980-2018. The catalog was subdivided into 60 regions of 2000km x 2000km, shown by the different colors. Grey and dashed lines as in panel (a); the black line is the distribution for  $W/L_\infty = 415$ , also shown in dark blue in panel (a).

225 the first nucleation after a full rupture:

$$226 \quad T_n = \frac{\sigma(a-b)L_p}{\mu'V_{pl}} \log \left[ \left( \frac{V_{co}}{V_{pl}} \right)^{\pi \frac{L_n}{L_p} - 1} + 1 \right], \quad (8)$$

227 where  $t_0 = \sigma(a-b)_{vs}/k(L_p)V_{pl}$ .  $k(L_p)$  is the spring stiffness, given by  $k = \mu'/L_p$  where

228  $L_p$  is the penetration distance of a rupture into the VS region, and proportional to  $W$ .

229 The duration of a cycle can be estimated from the amount of slip required for a full rup-

230 ture: since creep rate averaged across a cycle is simply  $V_{pl}$ , one can write

$$231 \quad T_{full} = \phi S_{full} / V_{pl} \quad (9)$$

232 with  $S_{full}$  given by eq. 3 and  $\phi = 0.71$  a factor accounting for the free surface (Sup-

233plementary Information, section 2). As shown in Supplementary Figure S3, this expres-

234sion is a lower bound to the occurrence of full ruptures in the simulations, and the time

235 to the first nucleation in the simulations is well approximated by eq. 8. The strong tem-

236poral clustering for  $W/L_\infty$  is due to the fact that the nucleation timescale (eq. 8) and

237 the full rupture timescale (eq. 9) differ by more than 6 orders of magnitude for large  $W/L_\infty$ .

## 238 5 Implications for seismic hazard

239 In this study I explore the statistical properties of seismic sequences on a fault ad-

240jacent to creep. I show that: 1. larger faults are intrinsically less characteristic, with the

241fraction of system size ruptures decreasing as  $\sqrt{L_\infty/W}$ ; 2. for sufficiently large  $W/L_\infty$ ,

242the frequency-size distribution approaches a truncated power-law, with a theoretical Gutenberg-

243Richter  $b$  value of 3/4; 3. the interevent times display Omori type clustering driven by

244afterslip.

245 On a fundamental level, the occurrence of partial ruptures is due to gradients in

246the stress field: in this case, the  $1/x$  decay of stress from a dislocation representing the

247slip accumulated in the VS region (which results in the stress intensity factor  $K_I \sim 1/\sqrt{x}$ ).

248 The observation that the stress concentration ahead of a propagating rupture is large

249enough to make the rupture unstoppable (Rice, 1993; Ben-Zion, 2008) is valid if the fault

250is relatively small, so that the difference between the stress at the nucleation point and

251the minimum stress along the rupture path is less significant. At the bottom of the seis-

252mogenic zone,  $a-b$  can be close to 0 and the nucleation length may be rather large (of

253the order of 0.1 – 1 km for the ageing law with  $d_c \sim 0.1$  m  $\sigma \sim 10$  MPa). This would

254result in  $W/L_\infty \lesssim 100$  and few ruptures per cycle, as found by Lapusta (2003) for age-

255ing law simulations with  $d_c = 0.14$  mm. On the other hand, the nucleation length in

256 nature may be smaller than typically assumed in numerical studies due to smaller val-  
 257 ues of  $d_c$  (close to typical laboratory values of 0.001–0.01 mm, Dieterich (1979); Marone  
 258 (1998)). This would promote larger  $W/L_\infty$ , leading to more events per cycle and the power  
 259 law distributions emerging at  $W/L_\infty \gtrsim 10^2$ – $10^3$ . For other frictional weakening mech-  
 260 anisms, the ratio  $W/L_{crit}$  may be different. For example, the slip law tends to result in  
 261 smaller nucleation lengths (Ampuero & Rubin, 2008), favoring partial ruptures; but the  
 262 smaller fracture energy would instead promote full ruptures. Dynamic weakening also mod-  
 263 ifies the fracture energy. Viesca and Garagash (2015) derived expressions for the fracture  
 264 energy due to thermal pressurization, and I verified that the recurrence interval of full  
 265 ruptures and its scaling with  $W$  can be estimated from the argument above (Cattania  
 266 & Segall, 2016), with their expression for fracture energy.

267 The results presented here are in agreement with, and generalize, previous numer-  
 268 ical studies. Lapusta (2003) first showed that decreasing the value of  $d_c$  partial ruptures  
 269 appear, and their number decreases slowly with  $d_c$ ; this was later confirmed by Werner  
 270 and Rubin (2013). Recent studies found an increase in partial ruptures and complex-  
 271 ity with fault dimension in subduction zones Herrendorfer et al. (2015) and 2-D antiplane  
 272 faults loaded from both sides Wu and Chen (2014); Erickson et al. (2011), confirming  
 273 that the results of the present study can be generalized to other fault geometries and fric-  
 274 tional laws.

275 To summarize, I show that simple, isolated faults do not necessarily produce limit  
 276 cycles of characteristic and periodic ruptures. Power-law distributions commonly observed  
 277 in nature (Gutenberg-Richter distribution and Omori decay) can occur on a planar, ho-  
 278 mogeneous fault as long as the ratio of its size to the nucleation length is large ( $10^2$ –  
 279  $10^3$ ). Natural faults additionally present geometrical and frictional heterogeneity, which  
 280 can give rise to even more variability in rupture lengths and more complex temporal pat-  
 281 terns; however, such heterogeneity is not *required* to arrest a rupture and produce com-  
 282 plex seismic sequences. The fraction of characteristic ruptures is a decreasing function  
 283 of  $W/L_\infty$ , with faults much larger than the nucleation length increasingly less charac-  
 284 teristic. Simple energy arguments, which can be refined by considering the particular ge-  
 285 ometry of interest, can provide insight into the statistics of earthquakes on a fault and  
 286 its seismic hazard.

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