A Hysteresis-Based Framework for Evaluating and Selecting Streamflow Monitoring Approaches Under Unsteady Gradually Varied Flow Conditions

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Abstract

Streamflow estimates derived from stage–discharge rating curves (HQRC) inadequately represent the spatiotemporal complexity of unsteady gradually varied flows (UGVF) and are commonly reported without uncertainty, encouraging their treatment as deterministic. These limitations are increasingly problematic as data users demand defensible confidence in streamflow estimates for cross-agency comparisons and for scientific applications. One of the dominant yet poorly quantified sources of HQRC error is hysteresis arising from ubiquitous unsteady and nonuniform flow conditions characteristic of UGVFs. Hydrometric agencies attempt to mitigate these effects through using alternative approaches such as HQRC corrections or use of stand-alone multivariable methods that augment stage with velocity or free-surface slope measurements. However, evaluating the effectiveness of these alternatives remains challenging, as HQRC corrections rely on simplified flood-wave representations, while multivariable methods retain the semi-empirical underpinnings of traditional HQRC formulations. To address this evaluation gap, we introduce a hysteresis-centric performance metric that quantifies departures between bivariate relationships obtained using HQRC correction and multivariable methods and those derived from conventional HQRC ratings. The metric integrates three complementary indicators: (i) flow deviation magnitude, (ii) hydrograph peak phasing, and (iii) cumulative volumetric differentiation, collectively capturing the hysteresis severity, variable peak timing, and integrated impacts of hysteresis on discharge estimates. The framework is applied to three U.S. Geological Survey gaging stations spanning diverse hydro-geomorphic settings. Results provide practical guidance for evaluating and selecting alternative monitoring methods in relation to dominant flood-wave behavior (kinematic versus non-kinematic) and are synthesized into a structured decision-making workflow for monitoring-system selection under UGVF conditions.

Keywords

streamflow monitoring, stage-discharge rating, index-velocity method, slope-area method; unsteady gradually varied flows, hysteresis

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1 INTRODUCTION

Streamflow records produced by hydrometric agencies constitute one of the most heavily used environmental datasets worldwide. These records underpin applications ranging from water-resources assessment and flood forecasting to ecosystem management, hydraulic engineering, and hydrologic model development. Despite their central role in science and practice, streamflow data are commonly disseminated without explicit uncertainty estimates, encouraging their treatment as deterministic quantities. Such premises become increasingly problematic as stakeholders demand transparent and defensible estimates suitable for cross-agency comparisons, scientific benchmarking, and risk-informed decision making.

More than 95% of the world's streamflow gaging stations continue to rely on stage-*discharge rating curves (HQRCs*), a monitoring approach developed more than a century ago and fundamentally based on the assumptions of steady and uniform flow. The semi-empirical method establishes a unique, site-specific relationship between stage and discharge using direct flow measurements guided by simple hydraulic equations and refined through various statistical and graphical techniques (Kennedy, 1984; Herschy, 2009). Although HQRCs remain attractive because of their simplicity, low operational cost, and long historical record, their underlying assumptions are often violated under natural river conditions. Consequently, streamflow records generated with HQRCs may contain substantial errors when flow behavior departs from the kinematic-wave regime implicitly embedded in the rating relation.

Departures from idealized steady-uniform conditions arise from spatial and temporal variations in river hydraulics, leading to complex flow patterns and dynamic hydraulic responses (Holmes, 2016). Variations in channel geometry at both the stream-reach and cross-sectional scales, together with downstream hydraulic controls (backwater effects) and propagating runoff waves, create flow conditions that are nonuniform and, in many cases, unsteady. Throughout this paper, these complex hydraulic conditions are collectively referred to as *unsteady gradually varied flows (UGVFs*), distinguishing them from the conventional definition of gradually varied flow (GVF), which is limited to steady, nonuniform flow conditions. UGVFs encompass unsteady shallow flows, such as flood-waves that are inherently non-uniform. Under UGVFs, hydraulic variables become path-dependent, producing *hysteretic relationships* whereby identical stages correspond to different discharges on the rising or receding of the flood wave (Prowse, 1984). Such hysteresis is not an exceptional phenomenon but a ubiquitous feature of natural rivers, particularly in low-gradient systems where backwater, storage effects, and wave diffusion exert strong hydraulic controls (Muste et al., 2025).

Recognition of these limitations has motivated the development of numerous alternatives to conventional HQRC monitoring. One class of approaches seeks to correct existing stage-discharge ratings through analytical formulations that account for unsteady or nonuniform flow effects. Examples include the methods of Jones (1915), Boyer (1937), Rátky (2000), Fenton (2018), and Fread (1975), each representing different levels of simplification of the Saint-Venant equations. However, despite these costly and labor-intensive efforts, large-sample studies from Canada, the United Kingdom, Norway, and Australia (e.g., Coxon et al., 2015; McMahon & Peel, 2019; Gharari et al., 2024) indicate that streamflow records reported at HQRC-based stations frequently fail to achieve the 5–8% uncertainty targets recommended by the WMO (2010) and ISO (2020) guidelines, sometimes by wide margins. The methods described above are referred to herein as *HQRC correction methods*; importantly, they are not typically applied in real time.

A second class of approaches augments stage measurements with additional hydraulic observations, leading to *multivariable monitoring protocols* such as the Index Velocity Rating Curve (Levesque & Oberg, 2012) - denoted IVRC; and Continuous Slope-Area (Smith et al., 2010) - denoted CSA. These methods have demonstrated promise for improving streamflow estimation under UGVF conditions while benefiting from advances in acoustic sensing, pressure transducers, and automated data acquisition systems. We refer to these emerging approaches as multivariable monitoring methods to distinguish them from the conventional HQRC, which relies solely on stage measurements. The evaluation of these real-time protocols remains ongoing, as they retain the quasi-steady flow assumption and continue to rely on complementary statistical analyses like those underpinning traditional HQRC formulations.

Despite decades of development, a fundamental gap remains. Existing studies have largely focused on evaluating individual monitoring approaches, whereas few efforts have established a common framework for assessing how effectively alternative methods reproduce the hydraulic signatures generated by UGVFs. In particular*, hysteresis is typically treated as a source of rating-curve error rather than as a measurable manifestation* of flood-wave dynamics capable of revealing the strengths and weaknesses of competing monitoring strategies. As a result, practitioners currently lack objective criteria for determining when conventional HQRC monitoring is adequate and when more advanced approaches are justified.

This study addresses that gap by introducing a *hysteresis-centered framework* for evaluating streamflow monitoring methods operating under UGVF conditions. Rather than comparing monitoring approaches solely through discharge magnitudes, the framework evaluates their ability to reproduce fundamental signatures of flood-wave dynamics. Three complementary indicators are proposed: (1) flow-deviation magnitude, quantified through rising- and falling-limb departures from conventional HQRC estimates; (2) *hydrograph peak phasing*, expressed through the temporal sequencing of free-surface slope, velocity, and stage maxima; and (3) *cumulative volumetric differentiation*, representing integrated hydrograph-scale consequences of hysteresis. Together, these indicators provide a unified basis for characterizing monitoring performance in environments where nonuniform and unsteady flow processes dominate.

The study contributes four advances. First, it introduces a hysteresis-centered methodology for evaluating streamflow monitoring approaches under UGVF conditions. Second, it establishes flow deviation, hydrograph phasing, and volumetric differentiation as practical indicators for quantifying monitoring performance. Third, it provides a unified comparison of correction-based and multivariable monitoring strategies within a common physical framework grounded in the Saint-Venant equations. Finally, it synthesizes the findings into a decision-support workflow that assists hydrometric practitioners in selecting fit-for-purpose monitoring strategies according to flood-wave behavior and site-specific hydraulic complexity.

2 METHODS

2.1 Grounding Streamflow Monitoring in Governing Equations

Streamflow monitoring protocols have evolved over centuries through incremental advancements that balance available measurement technologies with theoretical understanding of river hydraulics (USGS, 1994). The current monitoring methods are based on the simplifying assumption implying that flows in the streams and rivers can be treated as piece-wise, quasi-steady at any time, which is not a valid assumption for monitoring UGVFs. Anchoring streamflow monitoring in a robust physics-based framework valid for the ubiquitous UGVFs can be attained by relying on the well-tested *Saint-Venant equations* for shallow open-channel flows that distinguish between various types of wave types that occur during runoff events (Muste et al., 2017):

$$\underbrace{\frac{1}{g}\frac{\partial V}{\partial t}+\underbrace{\frac{V}{g}\frac{\partial V}{\partial x}+\underbrace{\frac{\partial H}{\partial x}=\underbrace{S_0-S_f}_{kinematic}}_{diffusive}}_{dynamic\ quasi\text{-}steady}}_{dynamic} \quad (1)$$

where S_0 is the slope of the channel bed, S_f is the energy (friction slope), H is the water surface elevation (a.k.a., stage) referenced to a local datum, V is the mean cross-sectional velocity, and g is the gravitational acceleration (see also Figure 1a).

The relative contributions of terms in the momentum equation determine the type of fluvial wave observed at a given site. The right-hand side terms representing gravity (S_0) and friction (S_f) forces constitute the *kinematic wave* component. When $S_0 = S_f$, the wave is purely kinematic - characteristic of steady, uniform flows as those in steep-gradient rivers where the flood propagation is dominated by this wave type (Henderson, 1966). The left-side also accounts for the pressure gradient determined from the free-surface slope ($FSS = \partial H / \partial x$). If these three terms are of similar magnitudes, they generate *diffusive waves* that account for downstream wave dispersion. If the convective and local accelerations (first two terms on the left side) are non-negligible in the total momentum budget, the waves are termed as quasi-steady or *fully dynamic*. The dynamic wave propagates both upstream and downstream relative to the kinematic wave core (House et al., 2025b). Widely used in hydrologic routing is the local inertial formulation of Equation (1) whereby the convective acceleration term is neglected while still capturing backwater and inertia effects (Ferrick, 1985).

Because the derivatives of the primitive variables can be both positive and negative during a flood-wave propagation, rising and falling hydrograph limbs follow distinct hysteretic stage-discharge relationships rather than the unique curve assumed by a simple HQRC (Ferrick, 1985). Consequently, the conventional HQRC approach, which relies primarily on kinematic-wave assumptions, can substantially oversimplify flow conditions when other momentum terms become significant.

2.2 Conventional methods for monitoring nonuniform unsteady UGVFs

This section reviews approaches for monitoring unsteady flow, focusing on two widely used classes of methods in the United States: (1) HQRC correction techniques and (2) multivariable monitoring protocols. Implementing these alternatives to the simple HQRC generally requires additional hydraulic measurements and/or analytical or numerical models to represent unsteady gradually varied flow. Their performance is best evaluated against direct discharge measurements collected throughout storm events; however, such validation is uncommon because continuous flood-period discharge monitoring is costly, logistically challenging, and often unsafe.

Figure 1 summarizes key terminology and instrumentation configurations for autonomous streamflow monitoring. Typical layouts for measuring stage, index velocity, and free-surface slope are shown in Figure 1b, although modern instruments such as Acoustic Doppler Current Profilers (ADCPs) can simultaneously measure stage and velocity with a single unit. Table 1 provides a concise overview of the mathematical formulations underlying the monitoring methods discussed herein, along with the wave types they represent. Throughout this study, Q denotes the actual discharge, while Q_o denotes the discharge estimated using the simple HQRC method.

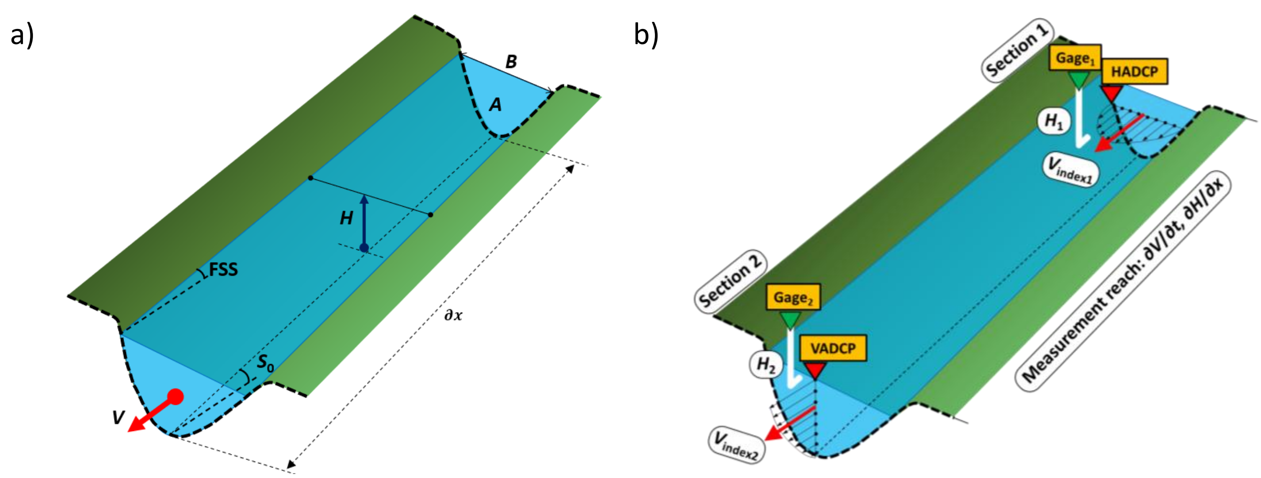


Figure 1. Streamflow monitoring terminology and instrumentation: (a) hydraulic variable notation; (b) instrument layouts for acquiring variables used in conventional monitoring methods. *Legend: B* – channel width; *A* – cross section area; *R* - hydraulic radius; *FSS* – free-surface slope; V_{index} – index velocity; ∂x – spacing between end cross sections; *Gage* – instrument for free-surface elevation measurement (*H*); S_0 - bed slope; Horizontal ADCP (*HADCP*), Vertical ADCP (*VADCP*) – instruments for *measuring* V_{index} *along horizontal or vertical paths in a water body.*

The stage-discharge rating (HQRC). The stage-discharge rating curve (HQRC) remains the most widely used streamflow monitoring method, requiring only stage measurements at a single location. By relating stage observations to direct discharge measurements, it produces a semi-empirical rating that is simple, reliable, and cost-effective to implement. Rating development combines hydraulic principles for steady, uniform flow (Eq. 2 in Table 1) with statistical analysis of field discharge measurements (Rantz et al., 1982; Kennedy, 1984). The resulting rating, Q_o, reflects site-specific hydraulic controls and channel geometry and is typically represented by two or three manually fitted segments informed by measurements and hydrologic judgment (Rozos et al., 2022). While effective for routine discharge reporting, HQRCs are less suitable for applications requiring sub-daily resolution on flood-wave propagation when flow conditions vary rapidly (Holmes, 2016). Nevertheless, despite their underlying quasi-steady flow assumption, HQRCs remain the foundation of streamflow monitoring worldwide.

HQRC corrections. Conventional HQRC ratings assume kinematic-wave conditions and therefore become unreliable under UGVF, where pressure and acceleration terms contribute significantly to momentum transfer. To address these limitations, numerous correction methods have been developed that modify the baseline rating (Q_o) using auxiliary measurements, analytical formulations, or numerical models. Approaches by Jones (1915), Boyer (1937), Henderson (1966), Fread (1975), Rátky (2000), Fenton (2001), Petersen-Øverleir (2006), and Schmidt & Yen (2008) differ primarily in their treatment of wave dynamics, ranging from kinematic and diffusive to quasi-steady and fully dynamic representations. Despite their long history, evaluating HQRC corrections remains difficult because continuous direct discharge measurements during flood-wave passage, the only reliable benchmark, are rarely available. Consequently, most operational gauging stations continue to rely on uncorrected HQRC ratings, leaving hysteresis-induced errors largely unquantified. The most common corrections in USGS practice treat flood waves as diffusive rather than kinematic by incorporating stage-fall relationships, stage rate-of-change, or wave celerity estimates. While operationally useful, these methods require additional measurements and introduce subjectivity that can limit reproducibility. In this study, we evaluate the kinematic correction of Jones (1915) and the diffusive correction of Boyer (1937) (Table 1).

More advanced methods by Rátky (2000) and Fenton (2001) represent flood waves as quasi-steady dynamic flows, retaining most momentum terms while requiring additional hydraulic information. Fenton's formulation further demonstrates the value of free-surface slope measurements, enabling stage-conveyance relationships and advection-diffusion representations of flood-wave propagation. For reconstructing actual flows, we adopt the dynamic-wave approach of Fread (1975), which explicitly accounts for hysteresis through variable energy slopes associated with pressure and inertia effects (Table 1). Implemented through numerical iteration, the method has been refined to incorporate detailed cross-sectional geometry (Lee & Muste, 2017) and successfully applied at several USGS stream gages (Domanski et al., 2025).

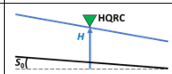
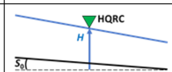
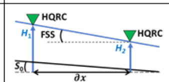
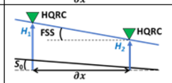
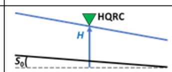
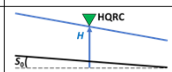
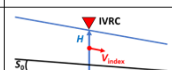
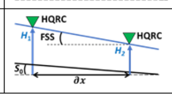
The index-velocity method (IVRC). The index-velocity method (IVRC) has become a widely adopted approach for monitoring rivers affected by backwater and unsteady flow while maintaining strong performance under steady conditions (Muste et al., 2019, Muste et al., 2020). Its widespread use followed the introduction of acoustic instruments in the 1980s, which enabled continuous, non-intrusive velocity measurements and significantly improved streamflow monitoring. By incorporating velocity in addition to stage (a geometrical flow descriptor), IVRC generally provides more reliable tracking of UGVFs than stage-only methods.

The IVRC implementation requires a stage-area rating (HARC) derived from channel surveys and paired measurements of stage, discharge, and index velocity. Discharge is then computed as the product of cross-sectional area from the HARC and mean velocity estimated from the index-velocity rating. Unlike HQRCs, IVRC ratings are typically developed using statistical regression rather than subjective graphical fitting, making them more reproducible. However, they are commonly constructed without distinguishing between rising and falling hydrograph limbs, which may limit accuracy at strongly hysteretic sites (Muste et al., 2022).

Continuous slope-area method (CSA). The availability of low-cost pressure transducers has renewed interest in the continuous slope-area (CSA) method for discharge monitoring (Smith et al., 2010; Stewart et al., 2012). CSA extends the traditional slope-area approach (Dalrymple & Benson, 1967) by replacing bed slope, S_0, with FSS, estimated from stage measurements at multiple locations, typically at least three sensors along a short reach. The method has been successfully applied in both small streams and large rivers under steady and unsteady flow conditions, although it is generally limited to short reaches where FSS can be reliably estimated (Muste et al., 2016; Lee & Muste, 2017; Muste et al., 2019; Muste et al., 2025). The method is only applicable to short river reaches (i.e., less than 5 channel widths). Using excessively long reaches might include natural flow changes (e.g. bed slope breaks, bends, tributary inflows) that can lead to significant errors in the free-surface slope estimation (Schmidt & Garcia, 2003; House et al., 2025a).

As summarized in Table 1, the Jones, Boyer, Rátky, Fenton, and Fread methods are HQRC-based corrections that require an existing HQRC (Q_o). The Jones and Boyer methods rely on an additional rating or FSS estimates, which may be derived either from paired stage measurements or from temporal stage derivatives. In contrast, IVRC and CSA are multivariable, semi-empirical methods that do not rely on an HQRC. IVRC uniquely requires continuous index-velocity measurements, whereas Boyer, Rátky, and CSA require stage measurements at multiple locations to determine FSS.

Table 1. Essential specifications for the HQRC alternative methods analyzed in this study

Method (assumed wave type)	Code	Input measurements & site specifications			Monitored output, *Q(t)*, based on:
		Arrangement	Variable(s) & parameters	Cross-sections	
Stage-discharge – (Kennedy, 1984) (*kinematic*)	Q_0		H	1	HQRC (Q_0) semi-empirical rating guided by: $Q_0 = a(H - H_0)^b$ (2) *a*, *b* coefficients accounting for station control
Corrected stage-discharge – (Jones, 1915) (*quasi-steady kinematic*)	Q_{JON}		H & Q_0, $A(H)$, S_0	1	HQRC (Q_0) semi-empirical rating along with $Q = Q_0\left[1 + \frac{1}{S_0 c_0}\frac{\partial H}{\partial t}\right]^{\frac{1}{2}}$ (3) $c_0 \approx \partial Q_0/\partial A$ (4)
Corrected stage-discharge – (Boyer, 1937) (*diffusive*)	Q_{BOY}		H_1, H_2 & Q_0, $A(H)$, S_0, ∂x	2	HQRC (Q_0) semi-empirical rating & $Q = Q_0\left[1 + \frac{1}{(\mathrm{FSS})c_0}\frac{\partial H}{\partial t}\right]^{\frac{1}{2}}$ (5) & Equation (4)
Corrected stage-discharge – (Rátky, 2000) (*quasi-steady dynamic*)	Q_{RAT}		H_1, H_2 & Q_0, $A(H)$, $B(H)$, S_0, ∂x	2	$Q = Q_0\left[1 + \frac{1}{c_0\sqrt{S_s}}\frac{\partial H}{\partial t} - \frac{Q_0}{2Bc_0 S_s}\frac{\partial^2 H}{\partial x^2}\right]^{\frac{1}{2}}$ (6) & Equation (3); $S_s = S_0\,\partial H/\partial x$ (7)
Corrected stage-discharge – (Fenton, 2001) (*quasi-steady dynamic*)	Q_{FEN}		H & Q_0, $A(H)$, $B(H)$, S_0	1	$Q = Q_0\left[1 + \frac{1}{2}\frac{1}{c_0 S_0}\frac{\partial H}{\partial t} - \frac{1}{2}\frac{D}{c_0^3 S_0}\frac{\partial^2 H}{\partial t^2}\right]$ (8) & Equation (4); $D = Q_0/2BS_0$ (9)
Corrected stage-discharge – (Fread, 1975) (*dynamic*)	Q_{FRE}		H & Q_0, $A(H)$, $B(H)$, S_0, n_0	1	$Q - \frac{AR^{\frac{2}{3}}}{n}\left[S_0 + \frac{A}{KQ}\frac{\partial H}{\partial t} + \frac{Q}{gA^2}\left(\frac{\partial A}{\partial t} - \frac{B}{K}\frac{\partial H}{\partial t}\right) + \left(\frac{\frac{Q'}{A'} - \frac{Q}{A}}{g\Delta t}\right)\right] = 0$ (10)
Index-velocity – (Levesque & Oberg, 2012) (*empirical diffusive*)	Q_{IVRC}		H, IVRC & $A(H)$	1	Q (t) time series
Continuous slope-area – (Smith et al., 2010) (*diffusive*)	Q_{CSA}		H_1, H_2 & $A(H)$, n, ∂x	2	$Q = \frac{1}{n}AR^{2/3}(\mathrm{FSS})^{1/2}$ [SI units] (11) n – Manning's roughness coefficient

2.3 Hysteresis-centered evaluation framework

To assess the ability of HQRC corrections and multivariable monitoring methods to capture UGVFs, we adopt a *hysteresis-centered framework* focused on the non-unique stage-discharge relationships characteristic of unsteady flow. Unlike the conventional HQRC, which assumes a unique one-to-one stage-discharge relation, UGVFs generate hysteretic loops that reflect different hydraulic controls on the rising and falling limbs of a hydrograph. Figure 2 illustrates these concepts (Muste et al., 2025). The conventional HQRC is constructed from measured stage-discharge pairs and represented by a piecewise rating curve (Figure 1a). Typically, the upper segment of the rating contains less data points, especially for extreme flows, hence the rating in this area is obtained by extrapolating lower flow data pairs. Departures of alternative methods from this baseline are quantified as *discharge differences on the rising (DDR)* and *falling (DDF)* limbs (Figure 2d). These deviations arise from momentum terms in Eqn. (1) neglected by the HQRC and manifest as hysteretic loops surrounding the rating curve. Similar hysteresis is expected in relationships between stage and other hydraulic variables, including velocity and free-surface slope (Figures 2b, 2c), and is inherently linked to temporal offsets among hydrograph peaks (Figure 2e). The latter hysteretic feature is rarely (if at all) reported in published literature.

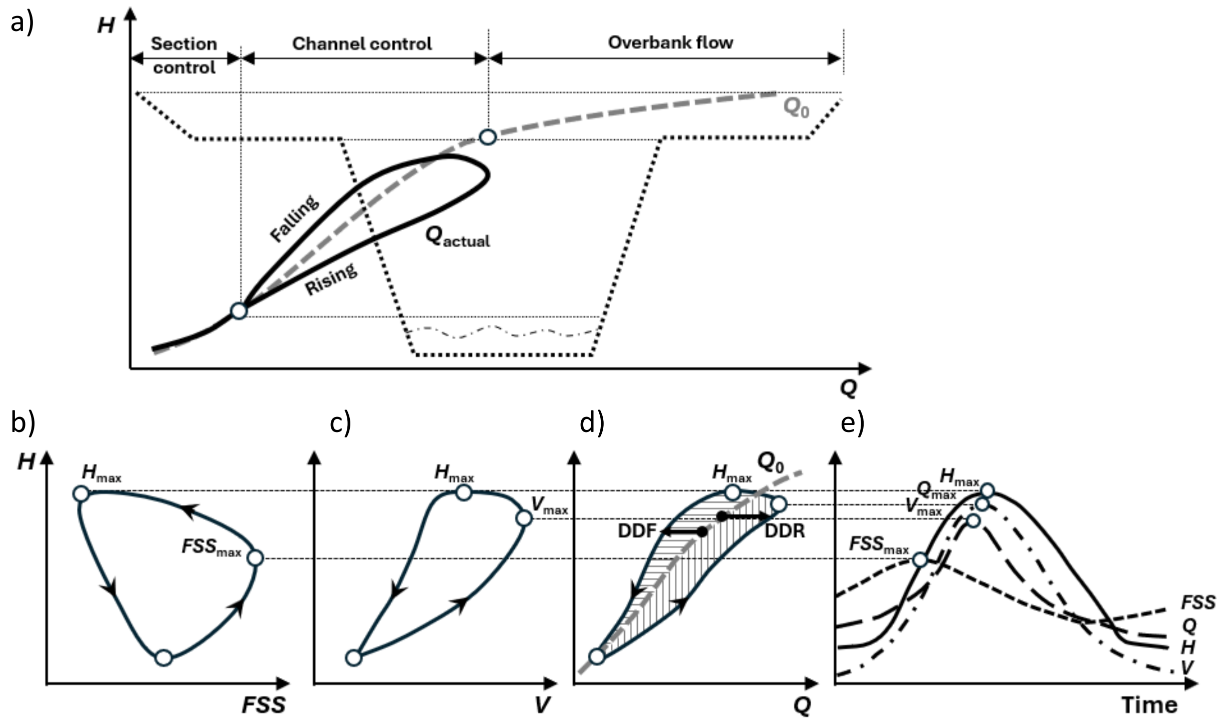


Figure 2. Hysteretic features during unsteady UGVFs: a) one-to-one (Q_0) relationship vs. hysteretic bivariate stage-discharge relationship (Q_{actual}); b), c), and d): looped relationships between stage and free-surface slope (*FSS*), mean velocity (*V)* and discharge (*Q*), respectively. *DDR* and *DDF* display departure of Q_{actual} from the HQRC discharges on the rising and falling limbs; and e) phasing of the hydraulic variable hydrographs.

Monitoring method performance is evaluated using three complementary indicators:

1. *Flow deviation magnitude*: the extent to which estimated discharges depart from the HQRC, quantified by DDR and DDF and visualized as hysteretic loops.
2. *Hydrograph peak phasing consistency*: the degree to which observed peak timing follows the expected sequence of free-surface slope, velocity, and stage.
3. *Cumulative volumetric differentiation*: integrated differences in discharge volume over the rising, peak, and falling phases of a flood wave.

Together, these indicators provide a unified framework for comparing monitoring methods and their ability to resolve unsteady, nonuniform flow behavior. Their proof-of-concept application is presented in Section 4, while more rigorous statistical characterization of paired hysteretic relationships is left for future work.

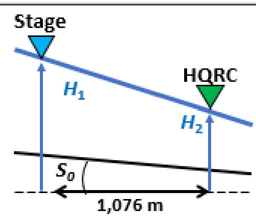
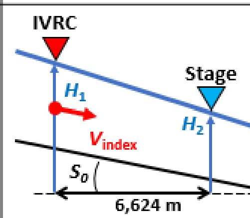
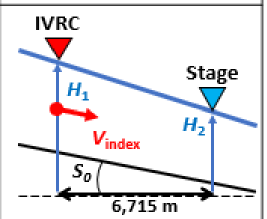
3 RESULTS

3.1 Study sites and instrumentation layouts

We evaluated selected HQRC correction algorithms and multivariable monitoring methods (Table 1) at three USGS gaging stations representing different river sizes: the Chattahoochee River at Atlanta, GA (USGS 02336000; smallest), the Kanawha River at Charleston, WV (USGS 03198000; intermediate), and the Ohio River at Ironton, OH (USGS 03216070; largest). Atlanta station is located in a humid subtropical climate, whereas the

Charleston and Ironton stations are in humid continental climates. Key hydraulic characteristics are summarized in Table 2. The stations use different monitoring approaches. Atlanta operates as a slope-gaging station (Kennedy, 1984), while Charleston and Ironton are stand-alone IVRC stations operated according to USGS guidelines (Levesque & Oberg, 2012). To enable consistent comparison among sites, free-surface slope (*FSS*) at the IVRC stations was estimated using stage data from nearby gaging stations.

Table 2. Site characteristics and instrumentation for the study sites

Gage site (River)	Atlanta (Chattahoochee River, GA)	Charleston (Kanawha River, WV)	Ironton (Ohio River, OH)
Hydraulic specifications			
S_0*	**0.0017**	**0.00005**	**0.00001**
***B* (m)**	**50**	**190**	**440**
B / h*	**10/2 = 5**	**190/5 = 38**	**440/10 = 44**
$Q_{min\text{-}max}$ (m^3s^{-1})	**≈ 35 – 1158**	**≈ 350 – 4,190**	**≈ 350 – 13,200**
Catchment area (km^2)	**3,755**	**27,099**	**157,536**
Catchment length (km)	**120**	**300**	**800**
Manning's n	**0.030 – 0.045**	**0.028 – 0.040**	**0.025 – 0.035**
Measurement equipment	**Stage:** stilling well & float-tape sensor (https://www.usgs.gov/media/images/float-tape-gages) **HQRC stage:** stilling well & float-tape sensor	**IVRC Velocity:** SonTek SL 500 (https://www.xylem.com/en-us/products) **IVRC stage:** SonTek SL 500 **Stage:** FTS SDI-RADAR 26GHz (https://www.environmental-expert.com/products/fts-model-sdi-radar-stage-sensor-737919)	**IVRC Velocity:** SonTek SL 500 **IVRC stage:** SonTek SL 500 **Stage:** YSI WaterLOG H-3611 (https://www.clean.com.br/Menu_produtos/Hidrologia/Medidores_Nivel/H-3611-i_H-3613-i.pdf)

*Determined from measurements at base flow
** Use of trade, firm, or product names is for descriptive purposes only and does not imply endorsement by the authors.

Although IVRC can be implemented with various instruments and deployment configurations, the Charleston and Ironton sites use HADCPs (Figure 1b). These sites were selected because nearby stations with active stage measurements allowed opportunistic estimation of *FSS*. Mean velocity was derived from index-velocity ratings developed by pairing HADCP index velocities with discharge measurements obtained from moving-boat ADCP surveys. Data from all stations are collected at 15-minute intervals and are publicly available through the USGS National Water Information System (waterdata.usgs.gov/nwis/rt). For improved visualization, a 5-point moving average was applied to the published data.

3.2 Experimental Evidence

To maintain consistency with the assumptions underlying the equations in Table 1, analyzed events were generally limited to flows below bankfull stage. The largest storm on the Chattahoochee River was included as an exception to demonstrate the effects of floodplain inundation on flow-variable relationships. Results are presented in both dimensional and nondimensional forms to support cross-site comparisons under varying river size, channel slope, and wave intensity, and to highlight hysteresis across events.

Water-surface elevations (WSE) are referenced to the NAVD 88 datum (www.ngs.noaa.gov/datums). Because Charleston and Ironton lack established HQRC ratings, the Fenton (2018) method was uniformly used to estimate the baseline discharge term of the simple HQRC, denoted Q_{0-FEN}, by fitting polynomials or piecewise splines to the data used in developing the IVRC ratings. Although this approach is not considered best practice (House et al., 2025a), it was necessary because free-surface slope (FSS) was estimated over relatively long reaches due to the absence of more closely spaced stage stations. For all sites, the reference discharge (Q_{actual}) was represented by the Fread (1975) correction applied to Q_0, denoted Q_{FRE}, because it accounts for the full dynamic behavior of the propagating flood wave.

3.2.1. USGS #02336000, Chattahoochee River at Atlanta, Georgia

Three storm events (2009-2020), lasting 3-5 days, were analyzed at Atlanta station (Figure 3a). Maximum stages for the small and medium events remained below bankfull, whereas the largest event exceeded it (Figure 3b). The HQRC developed using the Fenton (2018) method is shown in Figure 3c, where overbank flows produce a noticeable discontinuity in the Fread-corrected discharge (Q_{FRE}).

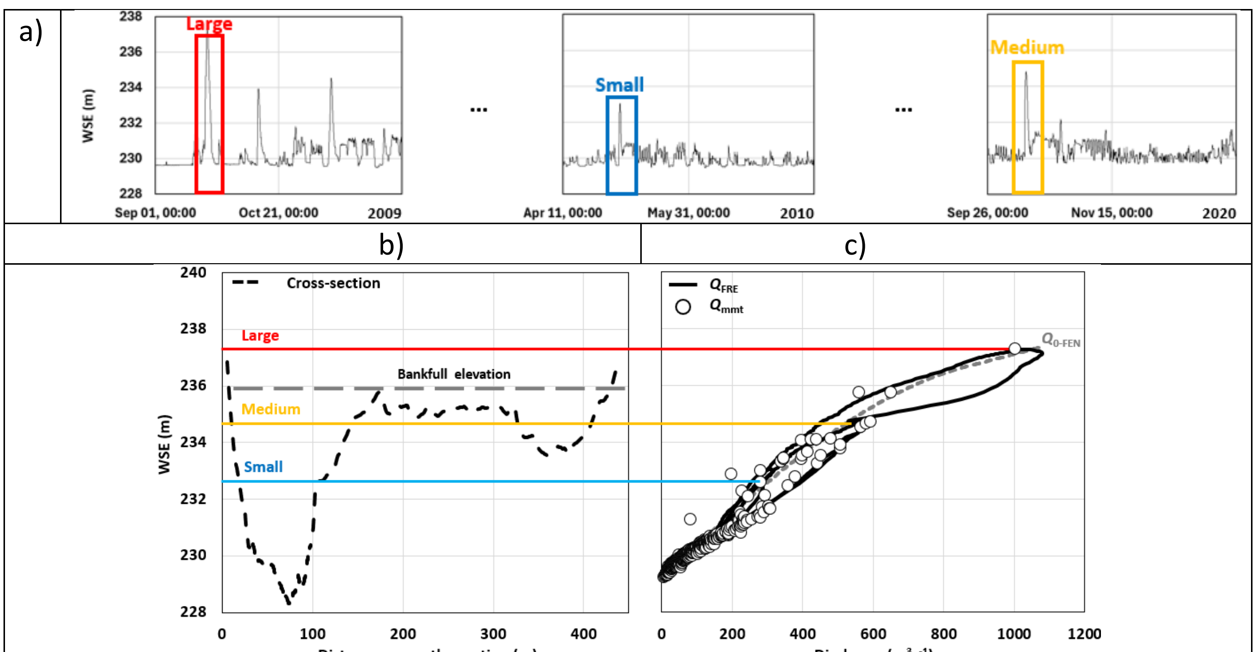


Figure 3. Key analysis elements for the Atlanta station: (a) selected storm events; (b) maximum event stages superposed on the channel cross-section; and (c) reference HQRC ($Q_{0\text{-}FEN}$) and Fread-corrected discharge (Q_{FRE}).

Figure 4 shows time-dependent and hysteretic relationships among hydraulic variables. Stage and FSS peaks exhibit clear phase lags that increase with event magnitude, resulting in larger hysteresis loops. The WSE-FSS relationship is particularly sensitive to UGVF effects and displays loop orientations opposite to those of stage-discharge relationships.

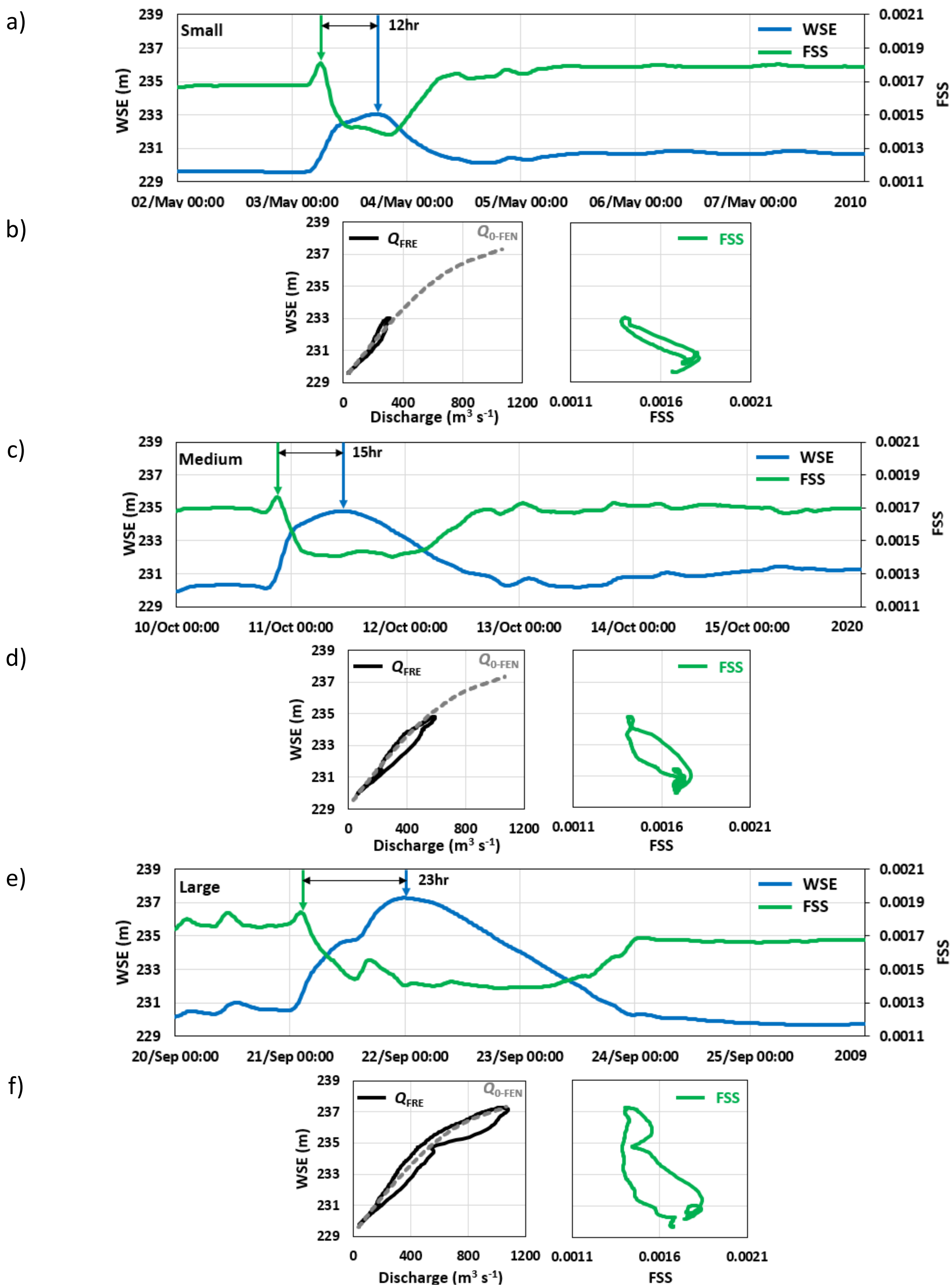


Figure 4. Dimensional representations of: (a, c, e) time series of the principal hydraulic variables for the three selected storm events shown in Figure 3a; and (b, d, f) corresponding stage-based hysteresis loops.

Figure 5 demonstrates that all five HQRC correction methods produce similar dynamic flow patterns, especially along the falling limb, with only minor differences during the rising limb. The Jones (1915) method results agree with those obtained through a prior study by Petersen-Øverleir (2006). Discharges from the stand-alone slope method (Q_{CSA}) closely match Q_{FRE}, and hysteresis remains evident even under overbank conditions.

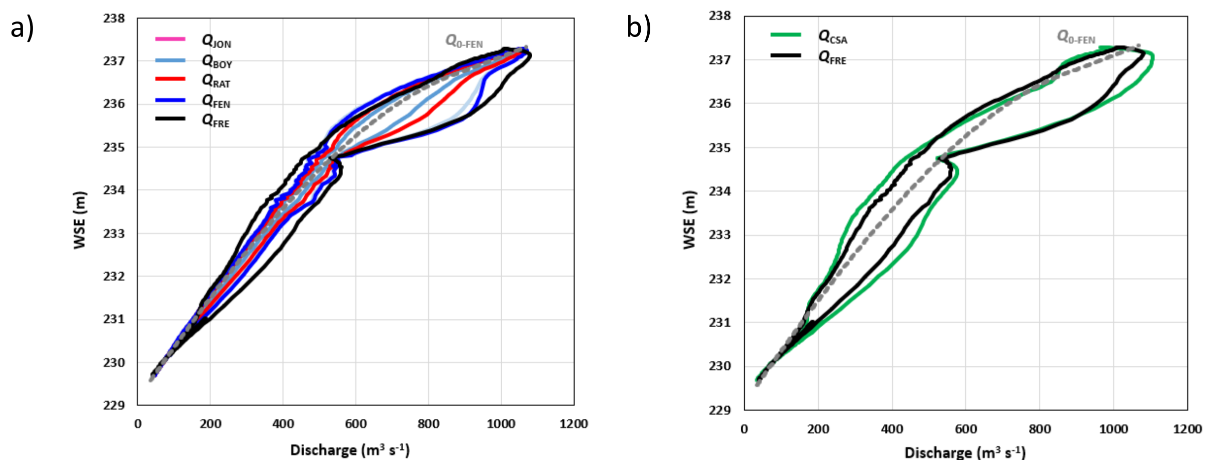


Figure 5. Performance of alternative monitoring methods at the Atlanta station: (a) HQRC correction methods (Table 1) and (b) the stand-alone CSA method.

3.2.2. USGS #03198000, Kanawha River at Charleston, West Virginia

Three storm events from 2020-2024, each lasting 6-8 days, were analyzed using IVRC and CSA data produced by USGS. Figures 6-8 follow the same presentation format as for Atlanta. Compared with Atlanta, Charleston exhibits smaller phase lags and hysteresis loops, indicating weaker hysteretic effects even during larger events. Nevertheless, the results confirm that hysteresis is inherent to UGVFs, with its magnitude controlled by local factors such as channel slope and storm flashiness. The phase-lag sequence between FSS and velocity peaks is similar to that observed at Atlanta but less pronounced. Differences in some FSS patterns and WSE-FSS loop orientations are attributed to stage-sampling limitations discussed in Section 4. For the largest event, all HQRC-correction methods successfully reproduce flow dynamics, although IVRC and CSA estimates differ noticeably from the theoretical Fread-based discharge (Q_{FRE}).

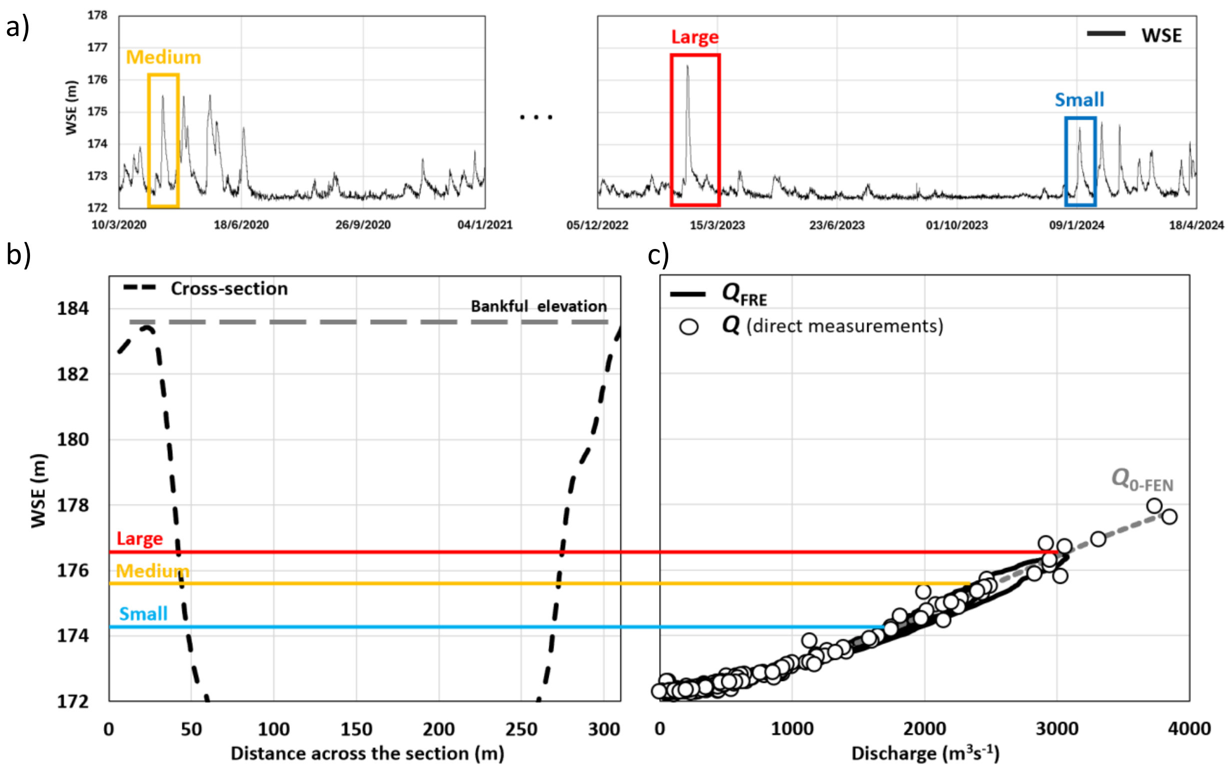
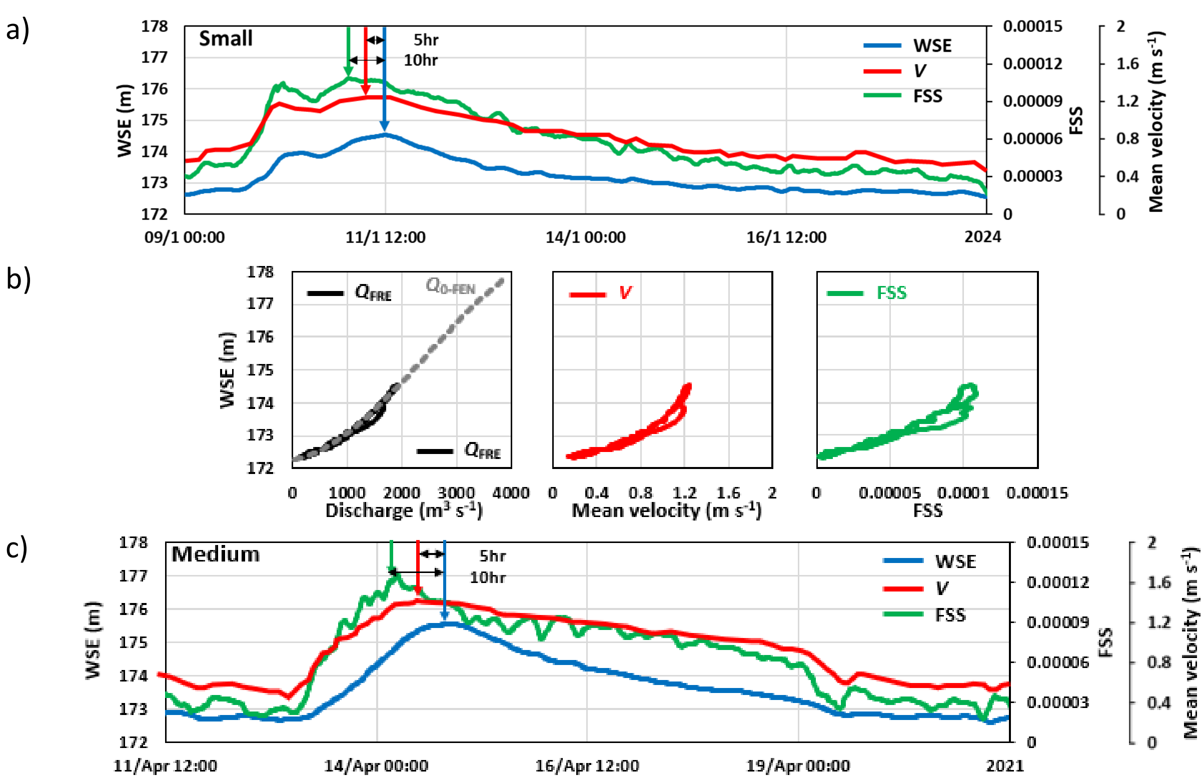


Figure 6. Key analysis elements for the Charleston station: (a) selected storm events; (b) maximum event stages superposed on the channel cross-section; and (c) reference HQRC ($Q_{0\text{-}FEN}$) and Fread-corrected discharge (Q_{FRE}).



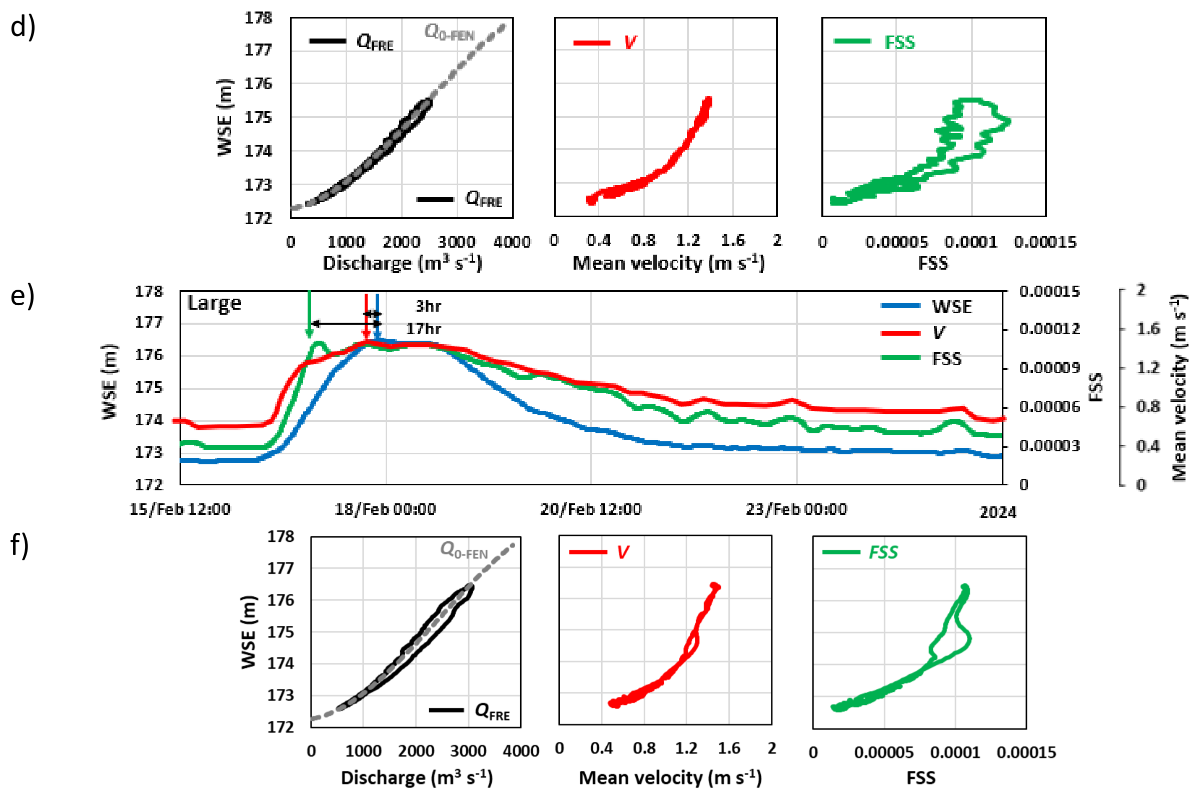


Figure 7. Dimensional representations of: (a, c, e) time series of the principal hydraulic variables for the three storm events shown in Figure 6a; and (b, d, f) corresponding stage-based hysteresis loops.

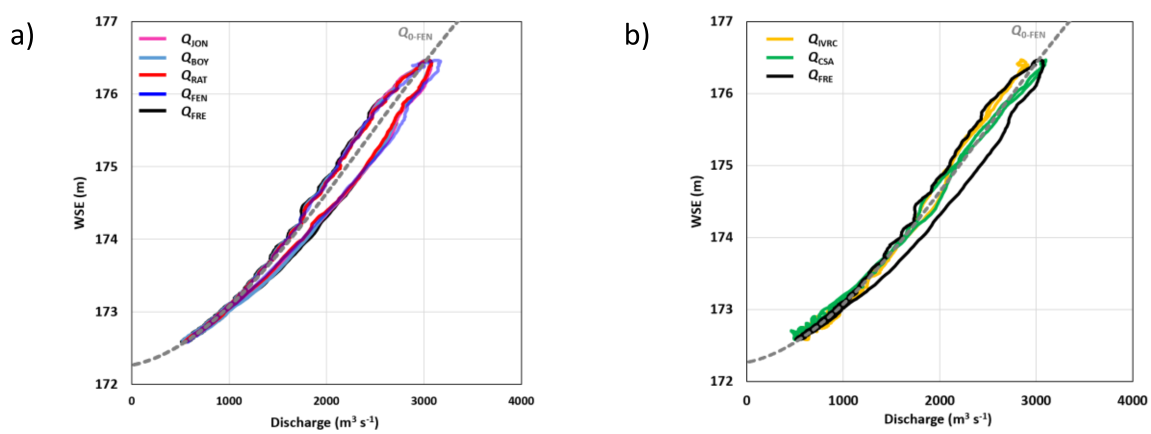


Figure 8. Performance of alternative monitoring methods at the Charleston station: (a) HQRC correction methods (Table 1) and (b) the stand-alone IVRC and CSA methods.

3.2.3. USGS #03216070, Ohio River at Ironton, Ohio

Three storm events from 2016-2023, lasting 5-6 days, were analyzed at Ironton (Figure 9a). Figure 9b shows progressively larger stage-discharge hysteresis loops as event magnitude increases.

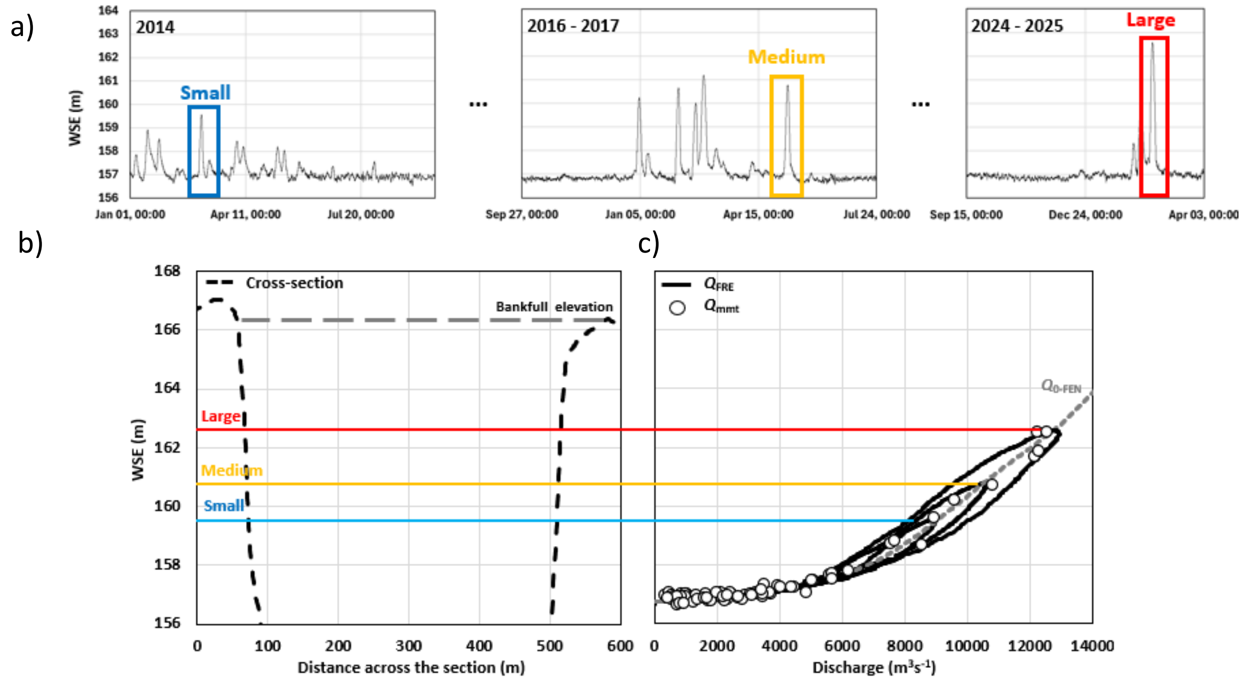
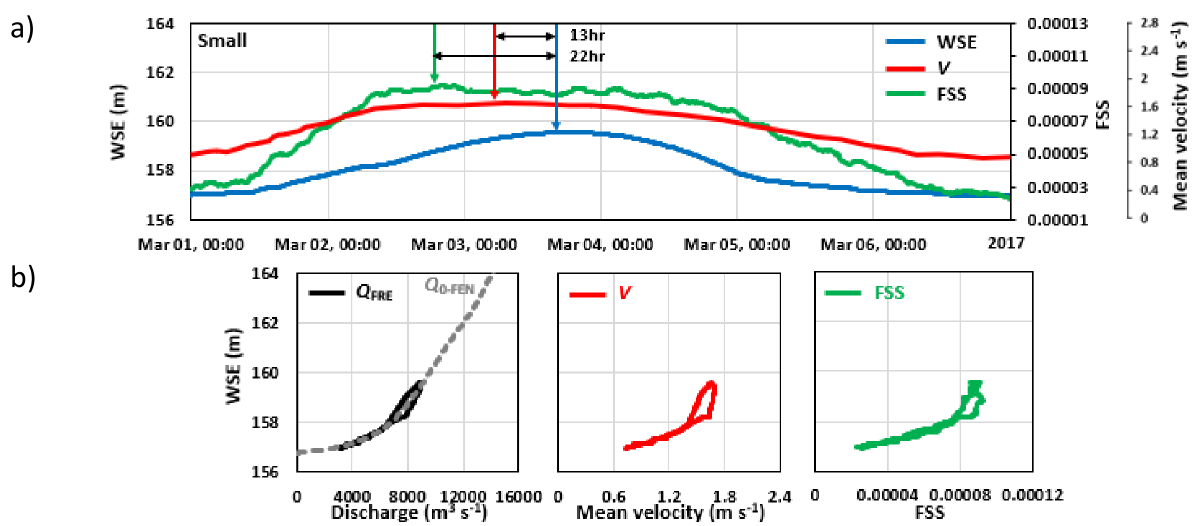


Figure 9. Key analysis elements for the Ironton station: (a) selected storm events; (b) maximum event stages superposed on the channel cross-section; and (c) reference HQRC ($Q_{0\text{-}FEN}$) and Fread-corrected discharge (Q_{FRE}).

Because Ironton has the lowest bed slope of the three sites, it exhibits the strongest hysteretic behavior. Increasing storm magnitude is associated with larger delays between FSS and velocity peaks relative to stage peaks and with larger hysteresis loops among hydraulic variables (Figure 10). These results emphasize the strong relationship between phase lag and loop size and their systematic growth with increasing event intensity.



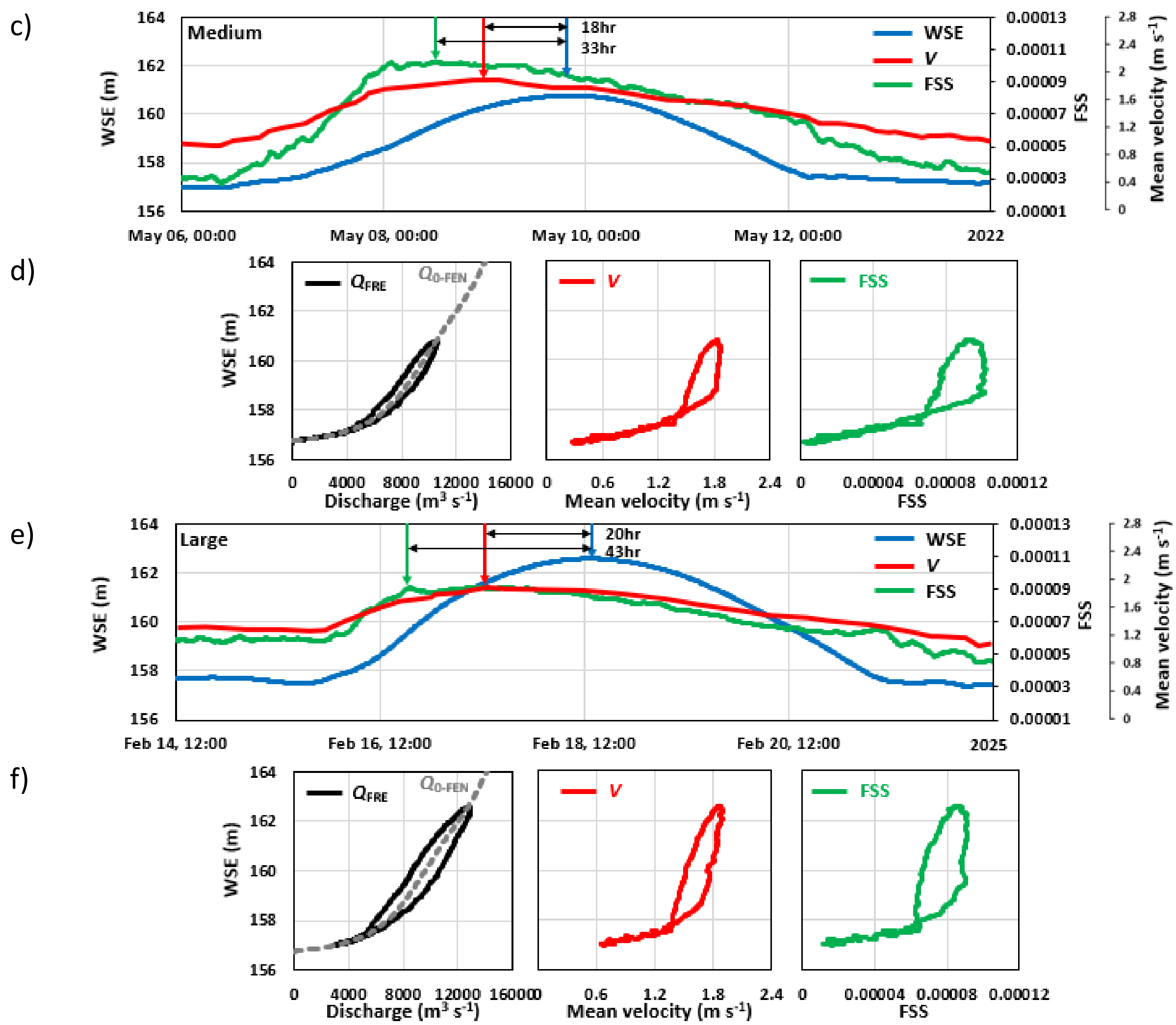


Figure 10. Dimensional representations of: (a, c, e) time series of the principal hydraulic variables for the three storm events shown in Figure 9a; and (b, d, f) corresponding stage-based hysteresis loops.

Figure 11a shows that all five HQRC correction methods reproduce the reference Fread-corrected discharge (Q_{FRE}) satisfactorily. Comparisons with the independent multivariable methods (Q_{CSA} and Q_{IVRC}) in Figure 11b indicates slight underestimation of rising-limb flows but good agreement during recession periods. Unlike Q_{FRE}, which is derived from the baseline HQRC, both Q_{CSA} and Q_{IVRC} are independent estimates based on additional velocity and FSS measurements.

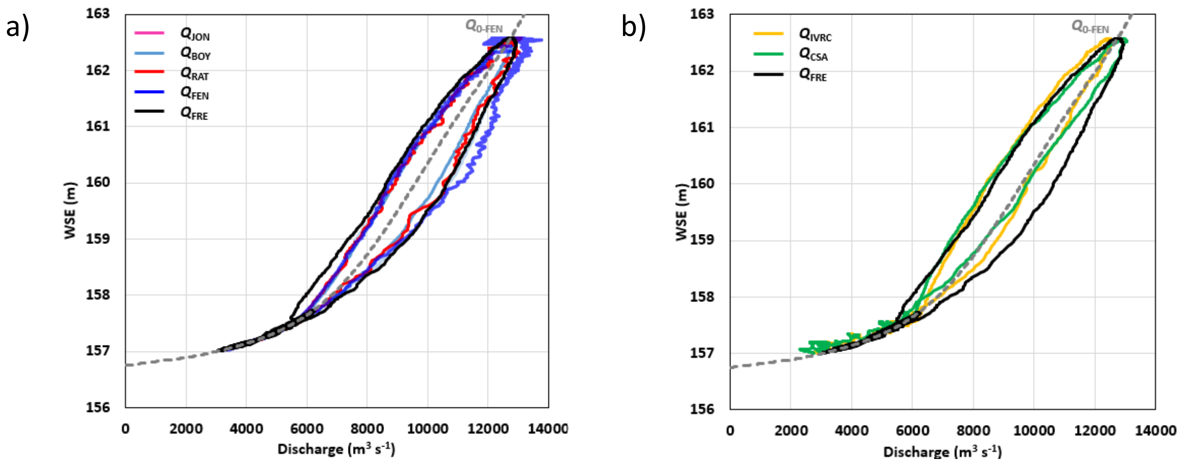


Figure 11. Performance of alternative monitoring methods at the Ironton station: (a) HQRC correction methods (Table 1) and (b) the stand-alone IVRC and CSA methods.

4 DISCUSSION

4.1 Hysteresis as a Diagnostic Signature of UGVF Dynamics

The analyses presented in Section 3 demonstrate that hysteresis is a persistent manifestation of UGVF dynamics and provides a physically meaningful basis for evaluating streamflow monitoring approaches. The hysteretic patterns summarized in Figure 12 reveal systematic departures among hydraulic variables during flood-wave propagation that are not represented in conventional HQRCs. Presenting the hydraulic variables in nondimensional coordinates highlights both the universality and the site dependence of these behaviors.

The observed hysteretic signatures are consistent with previous analytical and field studies conducted across diverse river systems and hydrologic areas, while extending those findings through a comparative assessment of monitoring approaches (Holmes, 2016; Muste et al., 2017; Lee & Muste, 2017; Muste et al., 2020; Muste et al., 2025). Collectively, the results suggest that:

- HQRC-derived streamflow estimates during storm-driven UGVFs frequently exceed the commonly accepted ±5% uncertainty threshold.
- HQRC use introduces systematic biases into discharge estimates and hydrograph-derived quantities, including peak flows, flood timing, event volumes, and constituent loads.
- Monitoring approaches incorporating additional measured variables generally outperform stage-only methods, although broader validation across sites, events, and technologies remain necessary.

These findings establish hysteresis not only as a hydraulic phenomenon but also as a practical physics-based diagnostic tool for assessing the suitability of streamflow monitoring strategies.

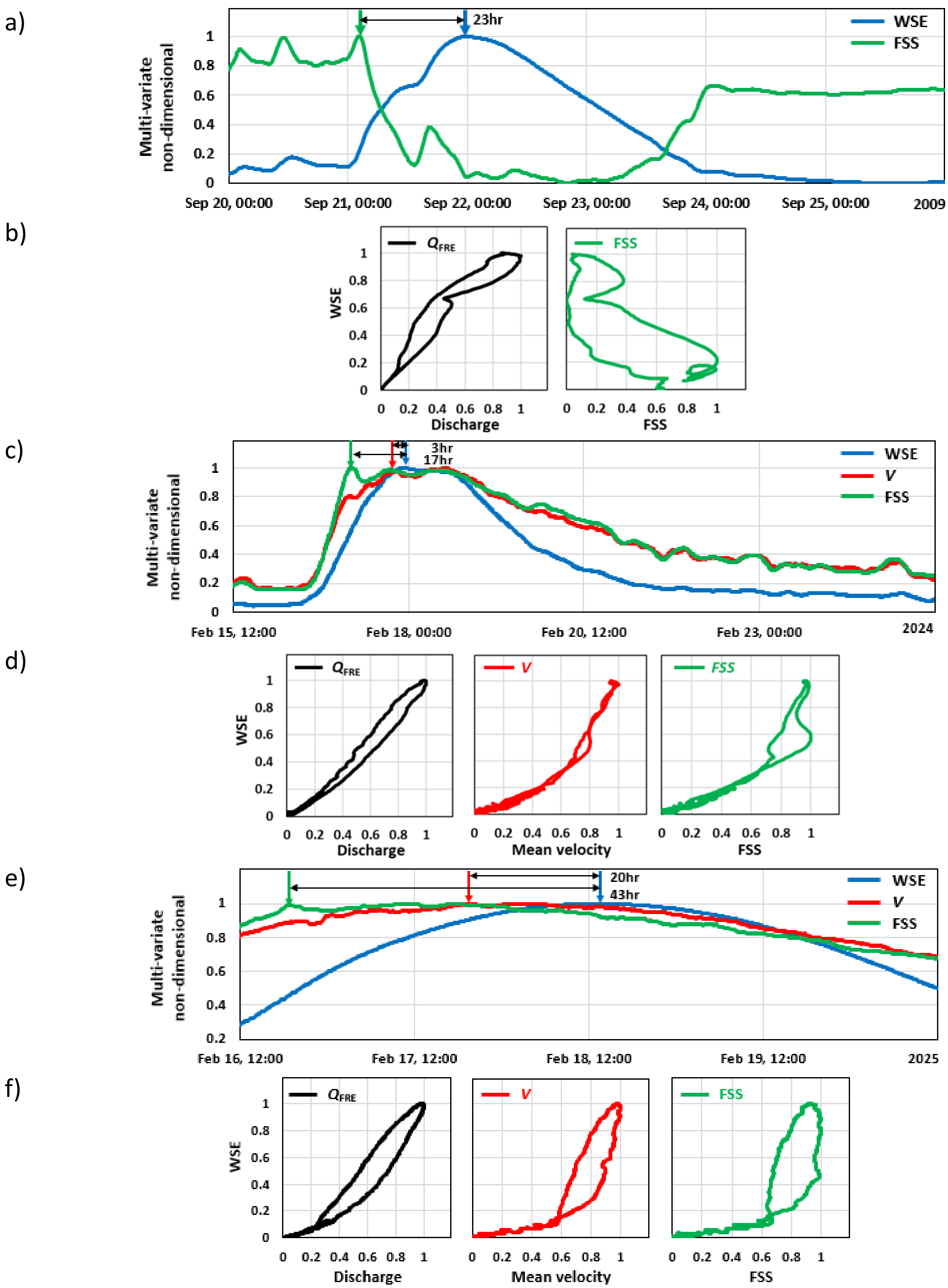


Figure 12. Non-dimensional plots of the hydraulic variable phasing and hysteresis loops for the largest event at each study site: (a, b) Atlanta; (c, d) Charleston; and (e, f) Ironton.

4.2 A Hysteresis-centered Framework for Evaluating Monitoring Performance

Flow deviation magnitude. The flow-deviation indicators DDR and DDF (Figure 2d) provide a quantitative characterization of hysteresis magnitude and, by extension, of the limitations associated with HQRC-based monitoring. Across the analyzed sites, deviations from HQRC estimates ranged from 11 to 46% for the most severely hysteresis-affected site (Atlanta) and between 3 and 12% for the mildest hysteresis-affected site (Charleston). The average DDF values range between 5 and 23% for Atlanta and from 6 to 7% for Charleston. Notable, similar analysis conducted at Henry gaging station on Illinois River in Illinois (USGS #05558300) displayed absolute differences of 65% for DDR and 18% for DDF (Muste et al., 2022). Figure 13 shows the maximum absolute DDR and DDF values and the mean deviation for all discussed monitoring alternatives with reference to the Fenton (2018) discharge estimate ($Q_{0\text{-}FEN}$) for the largest event at each study site. The cylinders in the figure denote the average deviations on the rising and falling limbs, while the spigots represent the corresponding maximum deviations.

All alternative methods consistently indicate higher discharges than the HQRC during the rising limb and lower discharges during the falling limb confirming that HQRC biases are not isolated to peak conditions but persist throughout the event. Although the largest discrepancies often persist only for a limited period during flood-wave passage, their impacts may be disproportionate because they coincide with the periods of greatest hydraulic and geomorphic activity. If overlooked, such departures at an HQRC-based gaging station may be sufficient to trigger abrupt local geomorphic responses (e.g., bank failures or vegetation washout) and to adversely affect aquatic habitat by inducing hydraulic conditions that stress or displace aquatic organisms.

The combined magnitude of DDR and DDF over the rising and falling limbs of an event, determined using the best-performing alternative monitoring method, can be interpreted as a first-order estimate of the uncertainty associated with applying the HQRC under UGVF conditions for the specific event. This approach provides a practical entry point for uncertainty assessment, that does not, however, replace a rigorous uncertainty analysis as errors associated with HQRC-based estimates also arise from measurement inaccuracies, rating-curve representation, extrapolation, and temporal variability. Under UGVF conditions, an additional source of uncertainty emerges from the structural inability of a single-valued stage-discharge relation to represent hysteretic behavior. This source of uncertainty is fundamentally epistemic because it originates from limitations in the monitoring framework rather than from natural variability in the flow process. Quantifying this component remains a significant challenge because uncertainty-analysis methodologies lack consistency across studies, frequently resulting in substantially different estimates (Kiang et al., 2018).

Classification of hysteresis severity. The hysteresis metrics provide a practical basis for classifying site behavior and evaluating the adequacy of existing monitoring approaches. Charleston exhibited predominantly weak hysteresis, with average flow deviations close to 5%, suggesting that HQRC application remains acceptable for many operational purposes. In contrast, Atlanta and Ironton displayed larger DDR and DDF values and stronger peak-phasing effects, indicating that HQRC assumptions become increasingly inadequate. This distinction suggests a framework in which hysteresis severity serves as a screening criterion for determining whether a site can continue to rely on HQRC ratings or requires alternative monitoring approaches.

Performance of alternative monitoring methods. Comparison of alternative methods against the Fread approach illustrated in Figure 14 indicates that approximately 80% of the evaluated DDR and DDF differences fall below 5%. While the performance ranking varies among sites, all tested methods improve upon the simple HQRC approach. The CSA method exhibits inconsistent trends between Atlanta and the Charleston/Ironton sites, likely resulting from issues in FSS determination, as discussed below. The results further show that methods incorporating a more complete representation of UGVF physics consistently outperform approaches based on stronger simplifying assumptions. This finding supports the use of hysteresis metrics as an objective framework for comparing monitoring strategies and evaluating tradeoffs between complexity, cost, and performance.

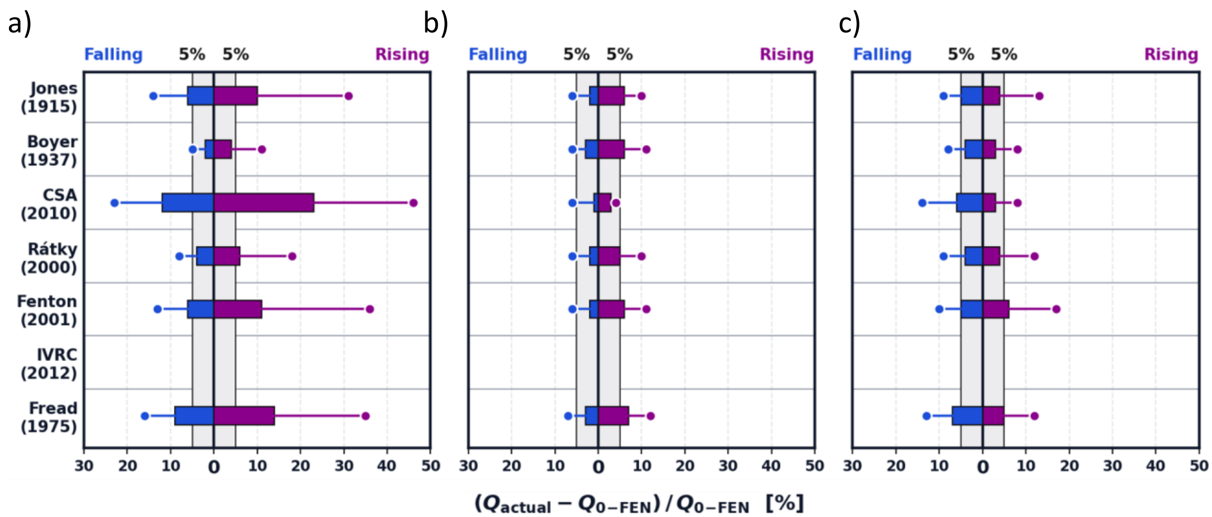


Figure 13. Flow deviation (%) of alternative monitoring methods vs. $Q_{o\text{-}FEN}$ for the largest storm events at (a) Atlanta, (b) Charleston, and (c) Ironton.

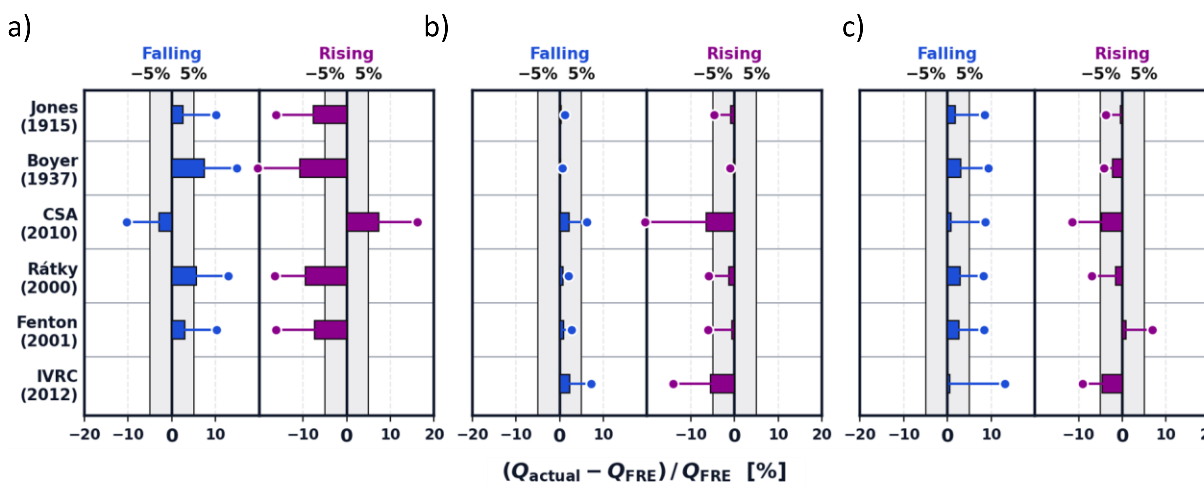


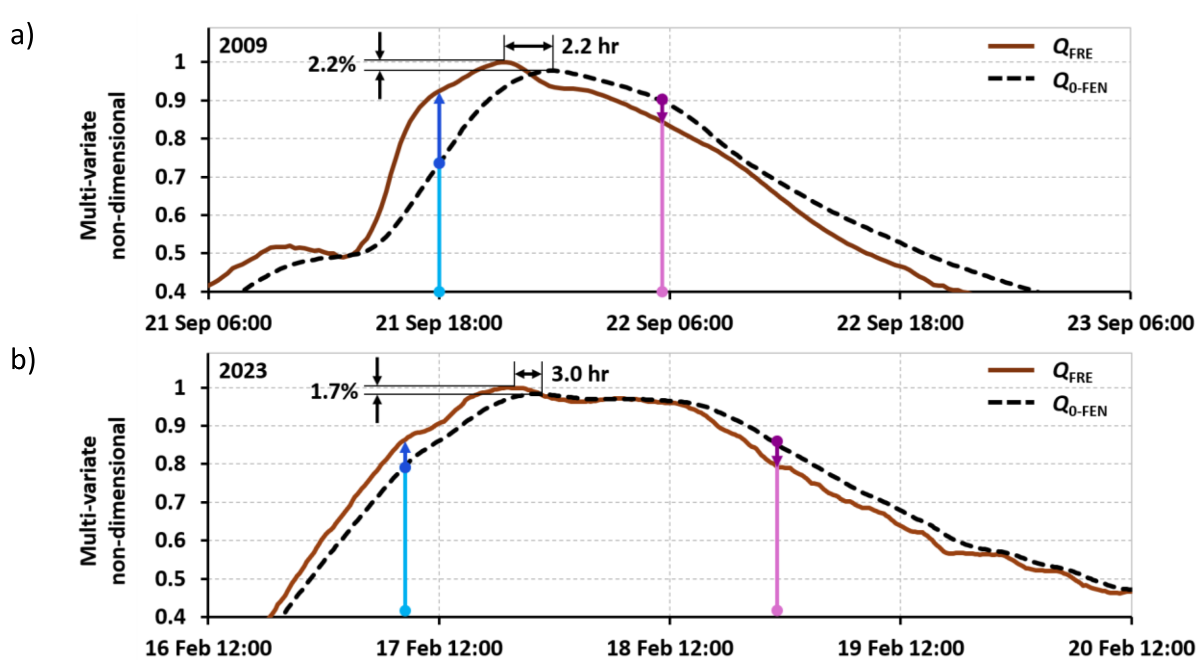
Figure 14. Flow deviation (%) of alternative monitoring methods vs. Q_{FRE} for the largest events at (a) Atlanta, (b) Charleston, and (c) Ironton.

4.3 Implications of Hysteresis-induced Bias

Hydrograph peak phasing. Hydrograph peak phasing, another form of hysteresis manifestation, is apparent in Figures 12 a, c, and e for the largest storms at the gaging sites. Across all analyzed events, free-surface slope (*FSS*), velocity (*V*), and stage (*WSE*) peaks occurred sequentially rather than simultaneously. The largest delay observed in this study reached 43 hours between FSS and WSE peaks.

Consistent with these phasing relationships, UGVF-based discharge estimates produced earlier and larger flow peaks than HQRC-derived estimates. Although the differences were moderate at the study sites, substantially larger discrepancies have been documented under severe hysteretic conditions (e.g., 89 hours between *V* and *FSS* at Henry gaging station). These results demonstrate that HQRC ratings can distort both the magnitude and timing of flood peaks, with implications for forecasting, flood-risk assessment, and operational decision-making.

Event volume biases. Systematic discrepancies between observed and HQRC-estimated flows also affect cumulative hydrograph volume estimates. Figure 15 shows that HQRC rating curves tend to underestimate discharge during the rising limb and overestimate discharge during the falling limb. Since these biases are not symmetric, conservation of event-scale flow volume is not preserved, leading to a systematic underestimation of the total flood volume conveyed during storm events. The resulting volume deviations are summarized in Table 3 as Volume Difference on the Rising Limb (*VDR*), Volume Difference on the Falling Limb (*VDF*), and Total Event Volume Difference (*VDE*), presented in Columns 2, 3, and 4, respectively.



c)

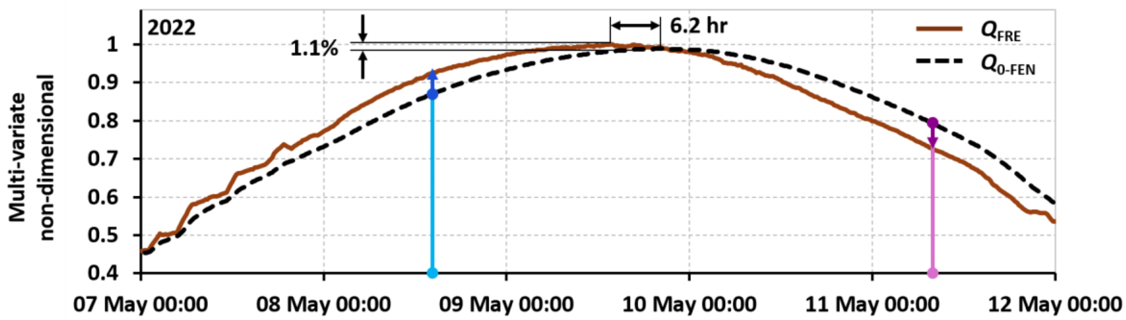


Figure 15. Streamflow time series from Q_{FRE} and HQRC (via $Q_{0\text{-}FEN}$) for the largest storm event at (a) Atlanta, (b) Charleston, and (c) Ironton.

Although the volumetric biases observed at the study sites were relatively modest, their magnitude increases substantially with hysteresis severity. For example, at the more severe hysteretic site at Henry on Illinois River (USGS #05558300) we observed volumetric differences of −26% during the rising limb, +17% during the falling limb, and an overall underestimation of −6% for the total event volume (House et al., 2025b). Consequently, errors in hydrograph-derived products may become significant in applications involving flood accounting, reservoir operations, watershed water balances, and environmental flow assessments.

Table 3. Water Volume Differences (*VD*) among the discharge hydrographs shown in Figure 15.

Site	VDR (%)	VDF (%)	VDE (%)
Atlanta	-14	8	-0.1
Charleston	-6	2	-0.2
Ironton	-4	4	-0.4

Transport of sediment and constituent loads. The implications of these biases extend beyond streamflow estimation. The transport of sediment and other particulate pollutants and of dissolved water-quality constituents (chemical, organic, biologic) are commonly derived from discharge records that rely on HQRC ratings (e.g., Gao & Josefson, 2012; Wilson et al., 2012). Deviations of the actual flow hydrographs from those produced by HQRC method propagates directly into load calculations. As a result, the hydraulic hysteresis affecting streamflow may also contribute to systematic differences in sediment and constituent transport, warranting re-evaluation where more physically consistent UGVF alternative monitoring approaches are available.

4.4 Monitoring Design Insights from Hysteresis Analysis

***Additional measurements for process representation*.** The results demonstrate that stage-only monitoring is insufficient for accurately characterizing UGVFs. Incorporating additional hydraulic variables partially recovers the flow dynamics lost when nonuniform flows are represented through a single-valued stage-discharge relation. More broadly, monitoring protocols should seek to represent a greater portion of the governing UGVF processes.

The superior performance of methods such as Fread, Rátky, and Fenton relative to simpler corrections supports this principle. Among the additional variables evaluated, FSS appears particularly attractive because it captures a substantial portion of flood-wave dynamics while requiring less instrumentation and maintenance than velocity-based approaches (Fenton, 2001; Aricò et al., 2009; Dottori et al., 2009).

Importance of sampling strategy. The effectiveness of UGVF monitoring depends not only on the variables measured but also on the sampling strategy used to capture them. This is evident in Figure 12a, c, and e. Although all FSS traces begin and end at nearly identical pre- and post-storm values, and their maximum occurs before the peaks of the other hydraulic variables, consistent with Figure 2e, the FSS response at Atlanta differs markedly from those at Charleston and Ironton. Specifically, the Chattahoochee record (Figure 12a) exhibits a pronounced drop followed by rapid recovery during storm-wave propagation. This behavior is absent at the other sites (Figures 12c and 12e), where the FSS hydrographs more closely resemble the temporal patterns of the other measured hydraulic variables.

These differences highlight the sensitivity of FSS estimates to sensor spacing. Excessive distances between stage sensors can progressively distort both the magnitude and timing of estimated water-surface slopes, reducing the apparent extent of hysteresis and altering peak-phasing relationships (House et al., 2025a). Therefore, sensor spacing should be based on the spatial and temporal scales of the flow wave, not merely on obtaining a measurable stage difference, particularly for calibration, forecasting, and management applications.

4.5 Selecting Fit-for-Purpose Monitoring Strategy

The principal contribution of this study is the integration of hysteresis analysis into a decision framework for streamflow monitoring strategy selection (Figure 16). The framework combines hydraulic diagnostics with economic considerations to determine whether an existing HQRC rating is sufficient or whether enhanced monitoring approaches are warranted.

The workflow begins with a hysteresis-based evaluation of historical observations to assess the prevalence of kinematic, diffusive, and dynamic wave behavior at a site. This diagnostic uses routinely available stage and streamflow records to identify the dominant fluvial-wave regime and builds on previous studies demonstrating the value of hysteretic signatures for distinguishing among wave types (Ferrick, 1985; Ponce, 1991; Moussa & Bocquillon, 1996; Moramarco et al., 2008; House et al., 2025b). Lee (2013) reviewed approximately a dozen related diagnostic approaches developed for differentiating kinematic, diffusive, and dynamic wave behavior.

Sites exhibiting weak hysteresis and predominantly kinematic behavior may continue to rely on conventional HQRC ratings with acceptable accuracy. In contrast, sites characterized by persistent diffusive or dynamic-wave behavior should be considered for monitoring approaches capable of capturing UGVF processes. The final monitoring strategy is then selected through a cost-benefit assessment that balances anticipated improvements in accuracy against the costs of instrumentation, data processing, and operation. Because the framework is based on observable hydraulic behavior rather than specific technologies, it is applicable to both existing gaging stations and the design of new monitoring networks.

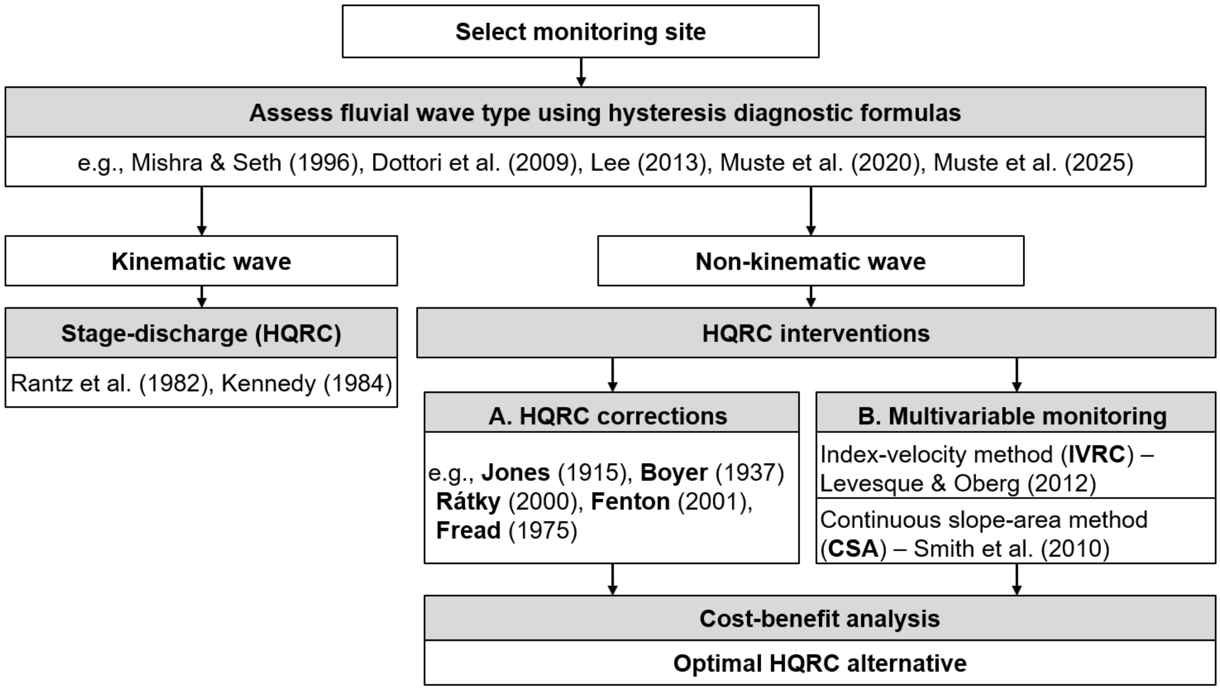


Figure 16. Decision tree for assessing the suitability of an existing HQRC gaging station and identifying when an enhanced streamflow monitoring configuration is warranted

5 CONCLUSIONS

The results presented in this study, interpreted through a hysteresis-centered framework, provide substantial evidence of the challenges associated with accurately estimating streamflow during storm-wave propagation using conventional HQRC methods. At the same time, they demonstrate that hysteresis is more than a byproduct of UGVF dynamics; it also serves as a valuable diagnostic and evaluation tool. By quantifying hysteretic behavior through flow-deviation, peak-phasing, and volumetric metrics, the proposed methodology enables objective assessment of streamflow monitoring performance and provides a physically based foundation for selecting fit-for-purpose monitoring strategies under non-uniform flow conditions.

The observed limitations of HQRC-based estimates under UGVF conditions underscore the need for scalable uncertainty-quantification frameworks that account for flow-regime dependence across diverse hydrologic settings. Such frameworks are essential for improving the reliability of streamflow information used in water-resources management, flood forecasting, infrastructure design, and hydrologic research. Moreover, the performance metrics introduced here provide practical insight into monitoring-system design, highlighting the importance of appropriate spatial and temporal sampling, the benefits of augmenting stage measurements with additional hydraulic observations, and the need to match monitoring approaches to site-specific geomorphic and hydraulic conditions.

An important outcome of this work is the integration of hysteresis analysis into a decision framework for monitoring-strategy selection. By linking observable hysteretic signatures to anticipated monitoring performance, the framework offers a systematic approach for determining when conventional HQRC methods are adequate and when enhanced monitoring configurations capable of representing UGVF processes are justified.

The conclusions presented herein are based on a limited number of sites, storm events, and monitoring approaches. Additional testing across a broader range of hydroclimatic conditions, channel morphologies, and hysteresis severities is needed to establish generalized performance thresholds and improve transferability. Furthermore, the use of Fread-based estimates as a reference highlights the need for benchmark datasets containing continuous direct discharge observations collected throughout complete flood-wave events.

Future research should expand the hysteresis-based methodology to additional monitoring technologies, develop standardized uncertainty metrics linked to hysteresis severity, and establish transferable classification schemes that support systematic selection of fit-for-purpose streamflow monitoring strategies across diverse river systems. Collectively, these efforts would advance the transition from technology-driven monitoring toward hydraulically informed monitoring design, thereby improving the accuracy, efficiency, and defensibility of streamflow observations in dynamic river environments.

Acknowledgments

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Open Research

All hydraulic data analyzed in this study are publicly available, without access restrictions, from the U.S. Geological Survey National Water Information System (USGS, 2025). Stage, discharge, index-velocity, and auxiliary stage records at 15-minute resolution were retrieved for the Chattahoochee River at Atlanta, Georgia (station 02336000), the Kanawha River at Charleston, West Virginia (station 03198000), and the Ohio River at Ironton, Ohio (station 03216070), together with the neighboring stations used to estimate the free-surface slope. As works of the U.S. Government, these data are in the public domain.

The analytical formulations underlying all monitoring methods evaluated in this study—the Jones (1915), Boyer (1937), Rátky (2000), Fenton (2001), and Fread (1975) HQRC corrections and the IVRC and CSA multivariable methods—are reported in full in Table 1, together with the auxiliary variables, the wave type represented, and the number of cross sections each method requires. The reference rating $Q_{0\text{-FEN}}$ was obtained by fitting polynomials or piecewise splines to the stage–discharge pairs following Fenton (2018); the Fread correction was solved by numerical iteration using surveyed cross-sectional geometry, as described by Lee and Muste (2017); and the hysteresis metrics *DDR*, *DDF*, *VDR*, *VDF*, and *VDE* were computed from the definitions given in Sections 2.3 and 4.2–4.3. All computations were performed on the 15-minute USGS records described above. The research code implementing these formulations was developed in Python by the authors and is not publicly distributed; it is available from the corresponding author on reasonable request.

Conflict of Interest Disclosure

The authors declare no conflicts of interest relevant to this study.

References

Aricò, C., Nasello, C., & Tucciarelli, T. (2009), Using unsteady-state water level data to estimate channel roughness and discharge hydrograph. *Advances in Water Resources*, 32(8), 1223–1240. https://doi.org/10.1016/j.advwatres.2009.05.001

Boyer, M. C. (1937), Analysis of methods of adjusting stage and discharge for measurements during changing stage. Unpublished manuscript, Water-Resources Branch, U.S. Geological Survey, pp. 192–200.

Coxon, G., Freer, J., Westerberg, I. K., Wagener, T., Woods, R., & Smith, P. J. (2015), A novel framework for discharge uncertainty quantification applied to 500 UK gauging stations. *Water Resources Research*, 51, 5531–5546. https://doi.org/10.1002/2014WR016532

Dalrymple, T., & Benson, M. A. (1967), Measurement of peak discharge by the slope-area method. *U.S. Geological Survey Techniques of Water-Resources Investigations*, book 3, chap. A2, 12 pp. https://doi.org/10.3133/twri03A2

Domanski, M. M., Holmes, R. R., Jr., Heal, E. N., & Knight, T. M. (2025), Dynamic rating method for computing discharge and stage from time-series data. *U.S. Geological Survey Scientific Investigations Report* 2024–5129, 92 pp. https://doi.org/10.3133/sir20245129

Dottori, F., Martina, L. V., & Todini, E. (2009), A dynamic rating curve approach to indirect discharge measurements. *Hydrology and Earth System Sciences*, 13, 847–863. https://doi.org/10.5194/hess-13-847-2009

Fenton, J. D. (2001), Rating Curves: Part 1 - Correction for Surface Slope in *Proceedings of the Conference on Hydraulics in Civil Engineering*, 28–30 November, Institution of Engineers, Australia, Hobart, pp. 309–317.

Fenton, J. D. (2018), On the generation of stream rating curves. *Journal of Hydrology*, 564, 748–757. https://doi.org/10.1016/j.jhydrol.2018.07.025

Ferrick, M. G. (1985), Analysis of river wave types. *Water Resources Research*, 21(2), 209–220. https://doi.org/10.1029/WR021i002p00209

Fread, D. L. (1975), Computation of stage-discharge relationship affected by unsteady flow. *Water Resources Bulletin, 11(2), 213–228.* https://doi.org/10.1111/j.1752-1688.1975.tb00674.x

Gao, P., & Josefson, M. (2012), Event-based suspended sediment dynamics in a central New York watershed. *Geomorphology*, 139–140, 425–437. https://doi.org/10.1016/j.geomorph.2011.11.007

Gharari, S., Whitfield, P. H., Pietroniro, A., Freer, J., Liu, H., & Clark, M. P. (2024), Exploring the provenance of information across Canadian hydrometric stations: Implications for discharge estimation and uncertainty quantification. *Hydrology and Earth System Sciences*, 28, 4383–4405. https://doi.org/10.5194/hess-28-4383-2024

Henderson, F. M. (1966), *Open channel flow*. Macmillan, New York, NY, 522 pp.

Herschy, R. W. (2009), *Streamflow measurement* (3rd ed.). CRC Press. https://doi.org/10.1201/9781482265880

Holmes, R. R. (2016), River rating complexity, in *Proceedings of River Flow Conference*. Taylor & Francis Group, St. Louis, MO.

House, E., Kim, K., Muste, M., Meselhe, E., & Demir, I. (2025a), Back to basics: On the proper determination of free-surface slope in gradually varied open channel flows. *EarthArXiv*. https://doi.org/10.31223/X50X5M

House, E., Meselhe, E., Muste, M., & Demir, I. (2025b), Streamflow hysteresis analysis through a deep dive budget of the St. Venant momentum terms. *EarthArXiv*. https://doi.org/10.31223/X5W44C

ISO (2020), *Hydrometry—Measurement of liquid flow in open channels—Determination of the stage–discharge relationship*. International Organization for Standardization, ISO 18320(en), Geneva, Switzerland.

Jones, B. E. (1915), A method of correcting river discharge for a changing stage. *U.S. Geological Survey Water-Supply Paper* 375-E, 117–130. https://doi.org/10.3133/wsp375E

Kennedy, E. (1984), Discharge ratings at gaging stations. *U.S. Geological Survey Techniques of Water-Resources Investigations*, book 3, chap. A10, 59 pp. https://doi.org/10.3133/twri03A10

Kiang, J. E., Gazoorian, C., McMillan, H., Coxon, G., Le Coz, J., Westerberg, I. K., et al. (2018), A comparison of methods for streamflow uncertainty estimation. *Water Resources Research*, 54. https://doi.org/10.1029/2018WR022708

Lee, K. (2013), *Evaluation of methodologies for continuous discharge monitoring in unsteady open-channel flows*. Doctoral dissertation, The University of Iowa. https://ir.uiowa.edu/etd/5012

Lee, K., & Muste, M. (2017), Refinement of Fread's method for improved tracking of stream discharges during unsteady flows. *Journal of Hydraulic Engineering*, 143(9). https://doi.org/10.1061/(ASCE)HY.1943-7900.0001280

Levesque, V. A., & Oberg, K. A. (2012), *Computing discharge using the index velocity method*. U.S. Geological Survey, U.S. Department of the Interior. https://doi.org/10.3133/tm3A23

McMahon, T. A., & Peel, M. C. (2019), Uncertainty in stage–discharge rating curves: Application to Australian Hydrologic Reference Stations data. *Hydrological Sciences Journal*, 64(3), 255–275. https://doi.org/10.1080/02626667.2019.1577555

Mishra, S. K., & Seth, S. M. (1996), Use of hysteresis for defining the nature of flood wave propagation in natural channels. *Hydrological Sciences Journal*, 41(2), 153–170. https://doi.org/10.1080/02626669609491489

Moramarco, T., Pandolfo, C., & Singh, V. P. (2008), Accuracy of kinematic wave approximation for flood routing. II: Unsteady analysis. *Journal of Hydrologic Engineering*, 13(11), 1089–1096. https://doi.org/10.1061/(ASCE)1084-0699(2008)13:11(1089)

Moussa, R., & Bocquillon, C. (1996), Criteria for the choice of flood-routing methods in natural channels. *Journal of Hydrology*, 186(1–4), 1–30. https://doi.org/10.1016/S0022-1694(96)03045-4

Muste, M., Bacotiu, C., & Thomas, D. (2019), Evaluation of the slope-area method for continuous streamflow monitoring, in *Proceedings of the 38th World Congress*, Panama City, Panama.

Muste, M., Cheng, Z., Firoozfar, A. R., Tsai, H.-W., Loeser, T., & Xu, H. (2016), Impacts of unsteady flows on monitoring streamflows, in *Proceedings of River Flow Conference*, St. Louis, MO.

Muste, M., Kim, D., & Kim, K. (2022), Insights into flood wave propagation in natural streams as captured with acoustic profilers at an index-velocity gaging station. *Water*, 14, 1380. https://doi.org/10.3390/w14091380

Muste, M., Kim, K., Kim, D., & Fleit, G. (2025), Decoding the hysteretic behavior of hydraulic variables in lowland rivers with multivariate monitoring approaches. *Hydrological Processes*. https://doi.org/10.1002/hyp.70008

Muste, M., Lee, K., Kim, D., Bacotiu, C., Rojas Oliveros, M., Cheng, Z., & Quintero, F. (2020), Revisiting hysteresis of flow variables in monitoring unsteady streamflows. *Journal of Hydraulic Research*, 58(6), 867–887. https://doi.org/10.1080/00221686.2020.1786742

Muste, M., Lyn, D. A., Admiraal, D., Ettema, R., Nikora, V., & Garcia, M. H. (2017), *Experimental hydraulics: Methods, instrumentation, data processing and management, Volume I*. CRC Press. https://doi.org/10.4324/9781315158839

Petersen-Øverleir, A. (2006), Modelling stage–discharge relationships affected by hysteresis using the Jones formula and nonlinear regression. *Hydrological Sciences Journal*, 51(3), 365–388. https://doi.org/10.1623/hysj.51.3.365

Ponce, V. M. (1991), The kinematic wave controversy. *Journal of Hydraulic Engineering*, 117(4), 511–525. https://doi.org/10.1061/(ASCE)0733-9429(1991)117:4(511)

Prowse, C. W. (1984), Some thoughts on lag and hysteresis. *Area*, 16(1), 17–23.

Rantz, S. E., et al. (1982), *Measurement and computation of streamflow* (Vols. 1–2). U.S. Geological Survey Water-Supply Paper 2175. https://doi.org/10.3133/wsp2175

Rátky, I. (2000), Árvízi hurokgörbék közelítő számítása. *Vízügyi Közlemények*, 82(2), 232–261 (in Hungarian).

Rozos, E., Leandro, J., & Koutsoyiannis, D. (2022), Development of rating curves: Machine learning vs. statistical methods. *Hydrology*, 9. https://doi.org/10.3390/hydrology910016

Schmidt, A. R., & Garcia, M. H. (2003), Theoretical examination of historical shifts and adjustments to stage–discharge rating curves, in *Proceedings of the ASCE World Water & Environmental Resources Congress*, Philadelphia, PA.

Schmidt, A. R., & Yen, B. C. (2008), Theoretical development of stage-discharge ratings for subcritical open-channel flows. *Journal of Hydraulic Engineering*, 134(9), 1245–1256. https://doi.org/10.1061/(ASCE)0733-9429(2008)134:9(1245)

Smith, C. F., Cordova, J. T., & Wiele, S. M. (2010), *The continuous slope-area method for computing event hydrographs*. U.S. Geological Survey Scientific Investigations Report 2010–5241, 37 pp. https://doi.org/10.3133/sir20105241

Stewart, A. M., Callegary, J. B., Smith, C. F., Gupta, H. V., Leenhouts, J. M., & Fritzinger, R. A. (2012), Use of the continuous slope-area method to estimate runoff in a network of ephemeral channels, southeast Arizona, USA. *Journal of Hydrology*, 472–473, 148–158. https://doi.org/10.1016/j.jhydrol.2012.09.022

USGS (1994), *A history of the Water Resources Branch, U.S. Geological Survey*, Vol. I: From predecessor surveys to 30 June 1919, Follansbee, R. (Ed.). U.S. Geological Survey, Denver, CO.

USGS (2025), USGS water data for the Nation: U.S. Geological Survey National Water Information System database [Dataset]. U.S. Geological Survey. https://doi.org/10.5066/F7P55KJN

Wilson, C., Papanicolaou, A., & Denn, K. (2012), Quantifying and partitioning fine sediment loads in an intensively agricultural headwater system. *Journal of Soils and Sediments*, 12(6), 966–981. https://doi.org/10.1007/s11368-012-0504-2

WMO (2010), *Manual on stream gauging*, Vol. I: Fieldwork. World Meteorological Organization, WMO-No. 1064, Geneva, Switzerland.