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Evaluating Turbulence Parameterizations at Gray Zone Resolutions for the Ocean Surface Boundary Layer

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Key Points:

- Ocean simulations with partially resolved boundary layer turbulence exhibit resolution-dependent sensitivity to choice of turbulence closure
- In the gray zone resolutions, k - ϵ can accurately represent mean-state properties but excessively damp small-scale turbulence
- Using Smagorinsky or the implicit method in the gray zone allows more active turbulence despite sacrificing mean state fidelity

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14 Abstract

15 Turbulent mixing in ocean boundary layers is often fully parameterized as a subgrid-
 16 scale process in realistic ocean simulations. However, recent submesoscale modeling stud-
 17 ies have advanced to a horizontal grid spacing of $\mathcal{O}(10\text{ m})$ that is comparable to, or even
 18 smaller than, the typical depth of the turbulent surface boundary layer. Meanwhile, ef-
 19 forts toward realistic large-eddy simulations (LES) nested within regional models require
 20 subdomains with similar grid spacings, where turbulent eddies are partially resolved in
 21 the mixed layer. The range of intermediate grid resolution, often known as the “gray zone”,
 22 presents challenges for model configuration and analysis, including uncertainties regard-
 23 ing the behavior of common turbulence closures outside of their ideal use cases. In this
 24 study, we evaluate three common configurations for subgrid turbulence— k - ϵ , Smagorin-
 25 sky, and an implicit no-closure method—in the gray zone resolutions for the ocean sur-
 26 face mixed layer. Results indicate the k - ϵ closure shows less sensitivity to grid spacing,
 27 producing accurate mean mixed-layer profiles even with partially resolved turbulence.
 28 However, it overly damps turbulent motions, significantly reducing small-scale variabil-
 29 ity that could otherwise be captured. The Smagorinsky closure and the implicit method,
 30 in contrast, exhibit higher sensitivity to grid spacing, initially performing poorly but con-
 31 verging toward baseline solutions at finer grids. Our findings provide guidance for sub-
 32 mesoscale and turbulent-scale modeling, recommending Smagorinsky or implicit meth-
 33 ods for nesting scenarios at turbulence-permitting resolutions. The k - ϵ closure is more
 34 suitable for high-resolution models primarily focused on accurate mean-state represen-
 35 tations rather than explicitly resolving detailed three-dimensional turbulence.

36 Plain Language Summary

37 Turbulence is an important small-scale process that mixes heat, momentum, and
 38 nutrients near the ocean surface and bottom. Typically, ocean models cannot fully re-
 39 solve turbulent motions due to limited grid resolution and instead rely on parameter-
 40 izations to approximate the effects of turbulent mixing. However, as models improve and
 41 begin to partially resolve turbulence at finer resolutions, choosing appropriate param-
 42 eterizations becomes increasingly important. Here, we evaluate three common model con-
 43 figurations—the k - ϵ closure, the Smagorinsky closure, and an implicit no-closure method—across
 44 a range of intermediate grid spacings known as the “gray zone”. The k - ϵ closure accu-
 45 rately represents boundary mixing regardless of resolution but suppresses smaller-scale
 46 turbulence. In contrast, the Smagorinsky closure and the implicit methods are sensitive
 47 to resolution, performing poorly at coarse grids but substantially improving at higher
 48 resolutions. Our findings suggest using Smagorinsky or implicit methods for simulations
 49 aimed at explicitly resolving turbulent motions (e.g., nested large eddy simulations), while
 50 the k - ϵ closure is better suited for scenarios prioritizing accurate mean conditions over
 51 detailed turbulence features.

52 1 Introduction

53 Realistic regional ocean simulations now regularly resolve submesoscale eddies at
 54 a grid spacing of $\mathcal{O}(1\text{ km})$ (Gula et al., 2021; Taylor & Thompson, 2023). Through mul-
 55 tiple nesting steps, many studies have also successfully further refined the model grid spac-
 56 ing to $\mathcal{O}(10\text{ m})$ to investigate mixed-layer processes, such as Langmuir circulation (Hypolite
 57 et al., 2022), symmetric instability (Dong et al., 2022), and submesoscale shelf current
 58 (Dauhajre et al., 2019). Relative to the typical value of mixed layer depth at $\mathcal{O}(100\text{ m})$,
 59 realistic simulations at the horizontal resolution of $\mathcal{O}(10\text{ m})$ can represent, albeit coarsely,
 60 large turbulent eddies in the mixed layer. However, despite being turbulence-permitting,
 61 these simulations generally employ vertical mixing parameterizations from the Reynolds-
 62 averaged Navier-Stokes equations (RANS) framework, for example, K-profile paramete-
 63 rization (KPP, Large et al., 1994) and k - ϵ (Jones & Launder, 1972), which are designed

64 to model boundary turbulence at all scales. This leads to concerns about double-counting
 65 of turbulence from directly resolved eddies and subgrid processes, both in terms of en-
 66 ergetics and effects on dynamics. Likewise, the premature use of LES closures (e.g., Smagorin-
 67 sky) at these intermediate scales is also problematic without representing the energy cas-
 68 cascade in the inertial subrange. This dilemma posed by the lack of scale separation is com-
 69 monly known as the “gray zone” problem, first introduced by the atmospheric sciences
 70 literature (Wyngaard, 2004). In this study, we aim to better understand the sensitiv-
 71 ity of common turbulence closures in the gray zone resolutions for the ocean surface bound-
 72 ary layer, with the goal to inform strategies on closure selection that better fit model-
 73 ing objectives.

74 Representing turbulent flows at the gray zone resolution has been a subject of ex-
 75 tensive research for atmospheric modeling (F. Chow et al., 2019; Honnert et al., 2020).
 76 For example, Zhou et al. (2014) performed simulations for the atmospheric convective
 77 boundary layer and found that the gray zone resolution resulted in overly large convec-
 78 tion cells and a delayed onset of turbulence. Mirocha et al. (2013) and Goodfriend et al.
 79 (2015) focused on nested simulations and found that the choice of subgrid closure affected
 80 the turbulence flow transition at the coarse-fine domain interface. Treatments at the lat-
 81 eral boundaries, such as increasing forcing frequency (Brisson et al., 2016), adding syn-
 82 thetic perturbations (Mazzaro et al., 2017), and optimizing grid aspect ratios (Daniels
 83 et al., 2016), can help improve turbulence statistics in the inner domain. The gray zone
 84 problem has also led to the active development of many scale-aware turbulence closures
 85 for atmospheric modeling (F. K. Chow et al., 2005; Bhattacharya & Stevens, 2016; Kurowski
 86 & Teixeira, 2018), which enable the transition from bulk closures to three-dimensional
 87 turbulence for the planetary boundary layer. While scale-aware parameterizations (built
 88 specifically to span the gray zone) are desirable, these have not yet been studied in depth
 89 for the ocean and are not available in most common ocean models (e.g., ROMS, CROCO,
 90 and MITgcm).

91 For ocean modeling, the gray zone problem has already been a practical concern
 92 and will become more prevalent in the future. Our study thus aims to evaluate the per-
 93 formance of commonly used turbulence closures (k - ϵ , Smagorinsky, and the implicit method)
 94 in the gray zone resolutions that allow the partial representation of mixed layer eddies.
 95 Here, we orient our analysis to two common modeling scenarios that may face the gray
 96 zone problem. First, for stand-alone submesoscale-resolving simulations—where the ob-
 97 jective is to highly resolve processes at $\mathcal{O}(100\text{ m})$ —we focus on the representation of av-
 98 eraged mixed layer profiles and the evolution of mixed layer depth. Second, for nested
 99 LES—where the objective is the best possible representation of turbulence statistics as
 100 the gray zone is traversed with telescoping resolution grids—we evaluate the represen-
 101 tation of turbulent coherent structures, fluxes, and kinetic energy. In the following sec-
 102 tions, we first define the problem setup and numerical simulations in Section 2. Section
 103 3 describes the results and makes recommendations based on the modeling objectives.
 104 Finally, results are summarized in Section 4, along with a discussion of the limitations
 105 and future research directions.

106 2 Methods

107 2.1 Model Description

108 The Coastal and Regional Ocean Community model, CROCO Version 1.3.0 (Auclair
 109 et al., 2022), is used to simulate turbulence in the upper ocean boundary layer. In this
 110 study, we broadly define the gray zone as a range of horizontal grid spacings extending
 111 from the scale of mixed layer depth to well-resolved LES, for which turbulent eddies are
 112 allowed to appear (coarsely resolved) in the mixed layer (further details on grid spac-
 113 ing can be found in Section 2.2). The same non-hydrostatic, non-Boussinesq solver is ap-
 114 plied to all simulations with the fifth-order weighted essentially non-oscillatory (WENO5)

115 scheme for both momentum and tracer advection. This setup follows CROCO’s docu-
 116 mentation on LES applications by Jullien et al. (2022) as well as the best practices ac-
 117 cording to Pressel et al. (2017), although we note below some results that indicate that
 118 a higher-order solver may be preferable for some LES applications. Recently, CROCO’s
 119 non-hydrostatic solver and the WENO5 scheme have been compared to the US National
 120 Center for Atmospheric Research’s LES model (NCAR-LES) and demonstrated good ac-
 121 curacy for surface boundary layer turbulence problems (Fan et al., 2023).

122 Two turbulence closures in CROCO, the k - ϵ model (Jones & Launder, 1972) and
 123 the Smagorinsky-Lilly model (Lilly, 1962), are evaluated for their performance in the gray
 124 zone. While KPP is another common closure for ocean modeling, it underperformed in
 125 our preliminary tests relative to two-equations models like k - ϵ for the gray zone resolu-
 126 tions with large turbulent eddies in the mixed layer. Therefore, we limit our scope to k -
 127 ϵ as the representative RANS closure in this study. As an additional point of compar-
 128 ison, we run simulations without an explicit turbulence closure: an alternative approach
 129 relying instead on the dissipative nature of the monotonic advection scheme (WENO5)
 130 to provide an implicit model of subgrid turbulence. In the LES framework, this is known
 131 as implicit LES (ILES) or monotonically integrated LES (MILES) (Grinstein et al., 2007),
 132 and it has shown good accuracy for turbulent flows in a wide range of atmospheric and
 133 oceanic contexts (Smolarkiewicz et al., 2007; Pressel et al., 2017; Silvestri et al., 2024).
 134 Readers should note that implementations of the closures used here may vary across dif-
 135 ferent models. It is possible that the closure performance in the gray zone could be im-
 136 proved by targeted re-tuning. However, such ad hoc efforts are likely to be challenging
 137 and may not generalize across cases. Hence, we do not include such efforts here (see Sec-
 138 tion 4).

139 The k - ϵ model is a common RANS closure used for ocean modeling, for example,
 140 in the basin-scale submesoscale-permitting simulations of the Atlantic Ocean (GIGATL)
 141 (Gula et al., 2021) and the idealized simulations of ice dynamics in the surface mixed
 142 layer (Herman et al., 2020). In CROCO, k - ϵ is implemented as part of the Generic Length
 143 Scale mixing parameterization (Umlauf & Burchard, 2003), which solves the transport
 144 equations for turbulent kinetic energy (TKE, k) and its dissipation rate (ϵ) to compute
 145 eddy viscosity, K_{gls} . As a vertical mixing parameterization in CROCO, the shear pro-
 146 duction term for TKE, $P = K_{gls} [(\partial_z u)^2 + (\partial_z v)^2]$ accounts only for the vertical shear
 147 of the horizontal velocity rather than the full deformation rate. Meanwhile, the advec-
 148 tion terms in the total derivatives of k and ϵ include both horizontal and vertical com-
 149 ponents. The eddy viscosity and diffusivity are computed with the stability function from
 150 Canuto et al. (2001). The minimum TKE parameter is set to $10^{-10} \text{ m}^2\text{s}^{-2}$ while the back-
 151 ground viscosity and diffusivity are prescribed as a constant $10^{-6} \text{ m}^2\text{s}^{-1}$.

152 Similar to the wide applicability of k - ϵ , the Smagorinsky model is a common clo-
 153 sure in the LES framework where comparatively large 3D turbulent eddies are explic-
 154 itly resolved while smaller-scale motions are modeled. Typically, LES requires the grid
 155 configuration to be fine enough to resolve at least 80% of the energy in turbulent mo-
 156 tions (Pope, 2000). For optimal performance, the filter width should be placed within
 157 the inertial subrange to ensure a proper representation of the energy cascade. In CROCO,
 158 the Smagorinsky model follows the formulation by Lilly (1962) to calculate eddy viscos-
 159 ity but separates the horizontal and vertical directions, namely

$$K_{smag} = \begin{cases} C_s^2 \Delta_x \Delta_y D & \text{(horizontal),} \\ C_s^2 \Delta_z^2 D & \text{(vertical),} \end{cases} \quad (1)$$

160 where $D = \sqrt{2S_{ij}S_{ij}}$ and $S_{ij} = 0.5(\partial_{x_j}u_i + \partial_{x_i}u_j)$ is the strain rate. The Smagorin-
 161 sky coefficient C_s is fixed at the canonical value of 0.16, and the filter width is taken as
 162 the horizontal grid spacings (Δ_x and Δ_y) and the vertical grid spacing (Δ_z). Note that
 163 this anisotropic treatment of the eddy viscosity differs from traditional implementations
 164 of the Smagorinsky model, which assume isotropic eddy viscosity (Chamecki et al., 2019).

165 Additionally, CROCO applies a buoyancy adjustment such that when a form of the crit-
 166 ical Richardson number (N^2/D^2) is larger than 0.25 (strongly stratified regions), the eddy
 167 viscosity is set to a constant background value of $10^{-6} \text{ m}^2\text{s}^{-1}$. The turbulent diffusiv-
 168 ity is set to K_{smag} with a constant turbulence Prandtl number (Pr) equal to 1. Since
 169 LES closures are designed for grid resolutions that permit the resolution of isotropic ed-
 170 dies, their ideal usage excludes the gray zone when the grid has a high aspect ratio (i.e.,
 171 large horizontal grid spacing). Nevertheless, we believe it is necessary to evaluate the per-
 172 formance of the Smagorinsky scheme in the gray zone as part of a nesting-down strat-
 173 egy to achieve realistic LES, despite this application falling outside its intended and val-
 174 idated use.

175 2.2 Simulation Configuration

176 This study focuses on canonical turbulence regimes relevant to the ocean surface
 177 boundary layer, including idealized simulations forced by constant, spatially homogeneous
 178 surface buoyancy flux, wind drag, and surface gravity waves in a doubly periodic domain.
 179 We note that spatially inhomogeneous mean flows, such as submesoscale fronts, can mod-
 180 ify the parameter space of turbulence production and dissipation (Dong et al., 2024; Zheng
 181 et al., 2025); however, these cases are left for future work.

182 A total of five cases are designed to represent different types of turbulent flow (Ta-
 183 ble 1). To differentiate the relative contribution to turbulence kinetic energy, we map
 184 the surface forcings on $La_t - \Lambda$ parameter space (Figure 1). The turbulent Langmuir
 185 number (La_t) quantifies the competition between the wind-driven shear turbulence and
 186 the vortex forcing associated with the Stokes-drift velocity (McWilliams et al., 1997) and
 187 is defined as

$$La_t = (u_* / u_s)^{1/2}, \quad (2)$$

188 where $u_* = (\tau / \rho_0)^{1/2}$ is the friction velocity and $u_s = \omega^3 a^2 g^{-1}$ is the surface Stokes
 189 drift for deep water waves. The surface forcings for wind drag (τ), wave frequency (ω),
 190 and wave amplitude (a) are applied only in the x-direction. The reference density (ρ_0)
 191 is chosen to be 1010 kg m^{-3} (neglecting salinity) along with a standard value of grav-
 192 ity ($g = 9.81 \text{ m s}^{-2}$).

193 The stability parameter (Λ) characterizes the relative impacts of surface wind stress
 194 and buoyancy fluxes and is defined as

$$\Lambda = \kappa w_*^3 / u_*^3, \quad (3)$$

195 where $\kappa = 0.41$ is the von Kármán constant and $w_* = (B_s |h|)^{1/3}$ is turbulent convec-
 196 tive velocity. The surface buoyancy flux, B_s , is related to the heat flux F by

$$B_s = \frac{\alpha g}{\rho_0 c_p} F, \quad (4)$$

197 where $\alpha = 2 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$ is the thermal expansion coefficient used in the linear equa-
 198 tion of state and $c_p = 3985 \text{ J (kg }^\circ\text{C)}^{-1}$ is the specific heat capacity. A boundary layer
 199 depth, h , of 30 m (set from the initial condition) is used to calculate w_* , even though
 200 the evolution of h varies depending on the surface forcings.

201 To represent the surface wave effect, the wave-current interaction was activated in
 202 CROCO (MRL_WCI) for the wave-averaged equations. We prescribe monochromatic wave
 203 forcings ($\omega = 1.01 \text{ rad s}^{-1}$ and $a = 0.8 \text{ m}$) to generate a uniform Stokes drift profile
 204 aligned with the wind forcing ($\tau_x = 0.037 \text{ N m}^{-2}$), resulting in a wind-wave equilib-
 205 rium at $La_t = 0.3$ for both the wind-wave and convection-wind-wave cases. The same
 206 set of forcings was used in the LES study on Langmuir turbulence by McWilliams et al.
 207 (1997) and was revisited with non-hydrostatic CROCO by Herman et al. (2020). The

208 other cases have no wave forcing such that $La_t \rightarrow \infty$. For the stability parameter, we
 209 choose $\Lambda = 0.61$ for the convection-wind case to reflect the shear-dominated turbulence
 210 generation with a weak convection. In the realistic convection-wind-wave case, $\Lambda = 1.76$
 211 falls between the 60% and 90% contours of the joint probability density function reported
 212 by Li et al. (2019), indicating surface forcing conditions representative of the real ocean.
 213 The convection-only case has no wind forcing for $\Lambda \rightarrow \infty$ and the shear-only case has
 214 no surface buoyancy flux giving $\Lambda = 0$.

Forcing Case	F (W m^{-2})	B_s ($\text{m}^2 \text{s}^{-3}$)	w_* (m s^{-1})	τ_x (N m^{-2})	u_* (m s^{-1})	u_s (m s^{-1})	La_t	Λ
Convection-only	-100	4.87×10^{-8}	1.14×10^{-2}	0.00	0.00	0.00	∞	∞
Shear-only	0.00	0.00	0.00	0.10	9.95×10^{-3}	0.00	∞	0.00
Convection-wind	-100	4.87×10^{-8}	1.14×10^{-2}	0.10	9.95×10^{-3}	0.00	∞	0.61
Wind-wave	0.00	0.00	0.00	0.037	6.05×10^{-3}	6.79×10^{-2}	0.30	0.00
Convection-wind-wave	-65	4.87×10^{-8}	9.83×10^{-3}	0.037	6.05×10^{-3}	6.79×10^{-2}	0.30	1.76

Table 1. Surface forcing conditions for the five Scenarios evaluated in the gray zone resolution using the CROCO model.

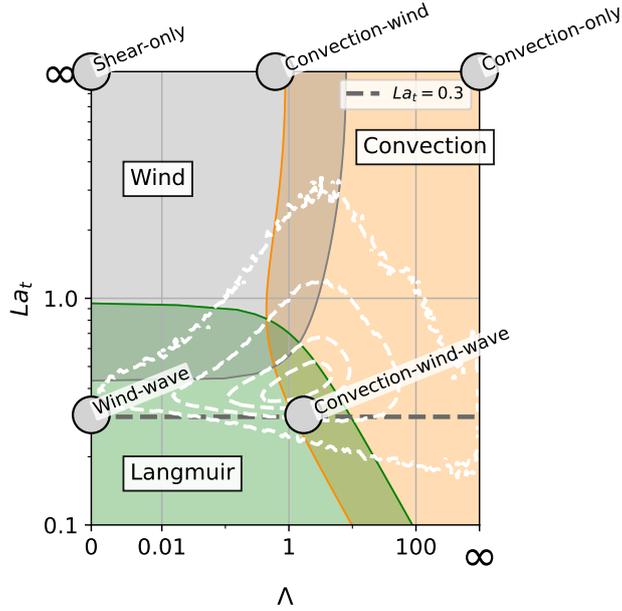


Figure 1. Five forcing cases, shown as gray circles in the $La_t - \Lambda$ space (see also Chor et al., 2021). The solid blue, gray, and red lines denote regions where more than 25% of the turbulence kinetic energy is produced by Langmuir forcing, wind stress, or buoyancy fluxes, respectively. The dashed white lines represent the joint probability distribution function of the realistic ocean from Li et al. (2019).

215 Given that the precise definition of the gray zone range remains an area of ongoing
 216 research (Beare, 2014; F. Chow et al., 2019; Honnert et al., 2020), we do not attempt
 217 to provide a rigorous framing here. Relative to an initial mixed layer depth of 30 m, we
 218 choose horizontal grid spacings to be 4, 12, 24, and 48 m ($\Delta = \Delta_x = \Delta_y$, identical in

219 both the x and y directions). Our post hoc analysis suggests that these grid spacings ef-
 220 fectively span the gray zone in our simulations, as indicated by the emergence of turbu-
 221 lent eddies. To evaluate the gray zone results, each forcing case includes a 1.25 m run
 222 with the Smagorinsky closure, serving as a well-resolved baseline for comparison. The
 223 computational domain is horizontally periodic, with 256×256 grid points for the 1.25
 224 m and 4 m runs and 128×128 for the rest. All runs share the same 100-point vertical
 225 grid configuration (using CROCO’s grid parameters: $\theta_s = 11.97$, $\theta_b = 0$, $hc = 401.69$,
 226 and $h = 131.46$), which maintains an approximately constant 1 m spacing above 60 m
 227 depth and gradually stretches to 4 m near the bottom. A sponge layer is applied to the
 228 bottom 20 grid points, where the velocity field is nudged to zero and the temperature
 229 is relaxed toward the initial stratification. The combination of the stretched vertical grid
 230 and sponge layer damps internal waves and minimizes their reflection at the bottom, help-
 231 ing to isolate surface mixing dynamics.

232 The initial condition for all simulations consists of a resting, stratified ocean. A 30
 233 m mixed layer with a uniform temperature of 20°C is positioned on top of a stratified
 234 interior with $N^2 = 1.96 \times 10^{-5}\text{ s}^{-2}$ (equivalent to $0.01\text{ }^\circ\text{C m}^{-1}$). To accelerate the tran-
 235 sition to turbulence in the mixed layer, small Gaussian noise with a zero mean and a stan-
 236 dard deviation of $5 \times 10^{-7}\text{ }^\circ\text{C}$ is added to the temperature field. All runs are integrated
 237 for four inertial periods with a constant Coriolis frequency of 10^{-4} s^{-1} .

238 The non-hydrostatic solver of CROCO uses a time-splitting method with two user-
 239 defined time steps for the fast and slow modes. For the baseline run at $\Delta=1.25\text{ m}$, we
 240 use a slow mode time step of 0.5 s and a fast mode time step of 0.017 s, ensuring the Courant
 241 number remains below 0.68 for the Courant-Friedrichs-Lewy (CFL) condition. For gray
 242 zone runs at coarser grid spacings, longer time steps are used (1 s for slow mode; 0.05
 243 s for fast mode). A wide range of time steps was tested, and these values were selected
 244 to balance numerical stability and computation efficiency. To further relax the sound-
 245 related CFL constraint, we set the speed of sound $C_s = 3\text{ m s}^{-1}$ and the second viscos-
 246 ity $\lambda = 10\text{ kg s}^{-1}\text{ m}^{-1}$ in the fast mode. Despite these non-physical values, Fan et al.
 247 (2023) shows that they have minimal impact on turbulence statistics.

248 3 Results

249 To evaluate closure performance in the gray zone, we first describe the instantana-
 250 neous coherent turbulence structures under different forcing scenarios (Section 3.1). Next,
 251 spatio-temporally averaged mixed-layer profiles (i.e., temperature, velocity, and fluxes
 252 of temperature and momentum) are analyzed to quantify the impact of grid spacing on
 253 turbulent mixing, followed by a discussion on the evolution of mixed layer depth. Finally,
 254 we compare kinetic energy spectra to assess the closure effect on effective resolution. In
 255 this section, we define the gray zone as the range of horizontal grid spacing (Δ) that al-
 256 lows turbulent eddies to emerge. The vertical grid is fixed in all simulations.

257 3.1 Coherent Turbulence Structure

258 Analyzing flow structures provides a valuable qualitative metric for evaluating marginally
 259 resolved eddies in the gray zone. Accurately representing turbulent structures is crucial
 260 for nested simulations, as grid-dependent flow artifacts can propagate from the parent
 261 domain and contaminate the final solutions (F. Chow et al., 2019). Here, we present co-
 262 herent turbulence structures as instantaneous flow fields in a horizontal plane near the
 263 surface ($z=10\text{ m}$), four inertial periods after model initialization.

264 Convection-driven turbulence exhibits characteristic cell-like patterns, which are
 265 visible in the high-resolution simulations shown in Figure 2. For example, at $\Delta = 4\text{ m}$
 266 (MLD $\sim 14\Delta$), Smagorinsky, k - ϵ , and implicit runs all display the classic signature of
 267 convection cells with strong downwelling boundaries (blue) and broad centers of weak

268 upwelling (red). Driven by surface cooling, the cellular patterns are similar to those found
 269 in other simulations of the surface ocean mixed layer (Chor et al., 2018; Souza et al., 2020)
 270 and the atmospheric boundary layer heated from below (Honnert et al., 2011; Zhou et
 271 al., 2014). At larger grid spacings, the cellular pattern persists at $\Delta = 12$ m but becomes
 272 barely visible at $\Delta = 24$ m and $\Delta = 48$ m. Among the closures, the k - ϵ runs produce
 273 the weakest vertical velocities, suggesting damping of the resolved turbulence. Meanwhile,
 274 in the Smagorinsky runs, the size of the convection cells are resolution-dependent, in-
 275 creasing with the horizontal grid spacing at a rate faster than seen in the implicit case.
 276 This behavior may indicate a limitation of the Smagorinsky model in the gray zone, where
 277 it struggles to represent subgrid dissipation and impacts kinetic energy across different
 278 scales (see Section 3.2.1 for further discussion on subgrid fluxes).

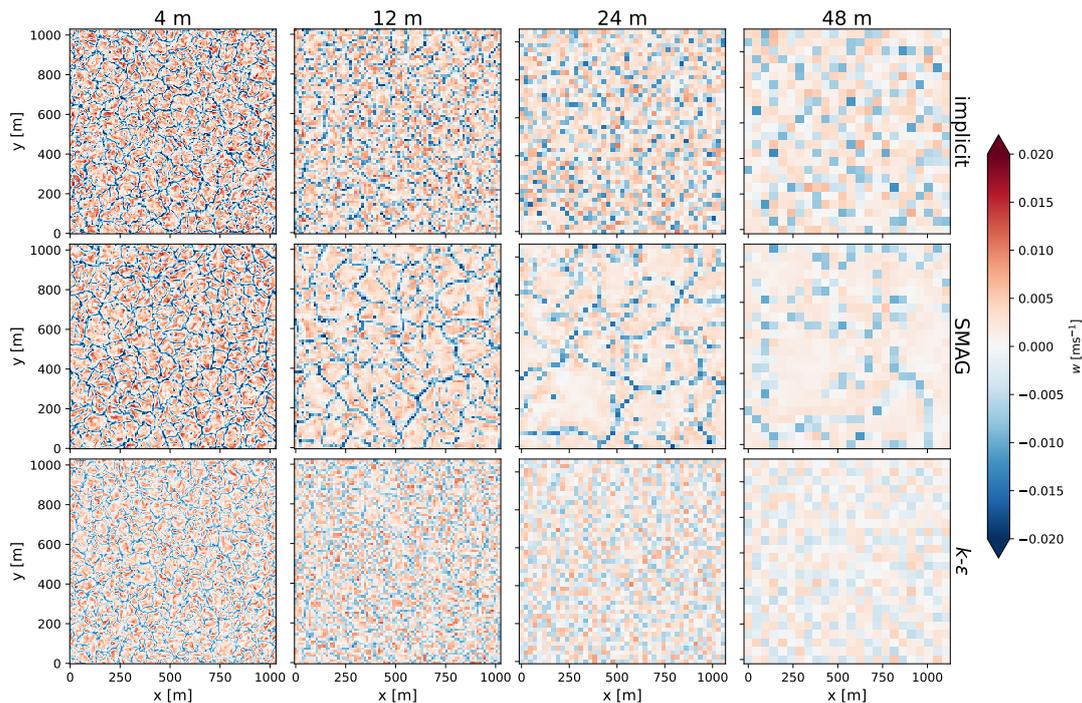


Figure 2. Snapshots of the vertical velocity (w) from the convection-only case at 10 m depth after four inertial periods. Each column is organized to show the results of the same closure at different grid spacings.

279 The shear-driven turbulence case exhibits diverse fine-scale features including streaks
 280 and rolls (Figure 3). Unlike convection, the turbulence patterns are highly sensitive to
 281 both closure and grid spacing. In particular, k - ϵ consistently produces a horizontally ho-
 282 mogeneous flow field with near-zero vertical velocity regardless of grid spacing. Despite
 283 the lack of resolved turbulence, k - ϵ still drives vertical mixing through subgrid processes,
 284 as shown later in the flux profile (Section 3.2.2). For Smagorinsky runs, the turbulence
 285 structure changes at different grid spacings. The flow fields display eddies at $\Delta = 4$ m
 286 and elongated streaks at $\Delta = 12$ m and 24 m. At $\Delta = 48$ m (comparable to the MLD
 287 at 40 m), no turbulence appears. The implicit runs exhibit turbulence patterns across
 288 the gray zone, but the scale of eddies depends on grid spacing. As shown later in Sec-
 289 tion 3.2.2, although both the Smagorinsky and implicit runs allow turbulence to form,
 290 they struggle to generate sufficient vertical mixing to deepen the mixed layer at coarse
 291 grid spacings ($\Delta = 24$ m and 48 m).

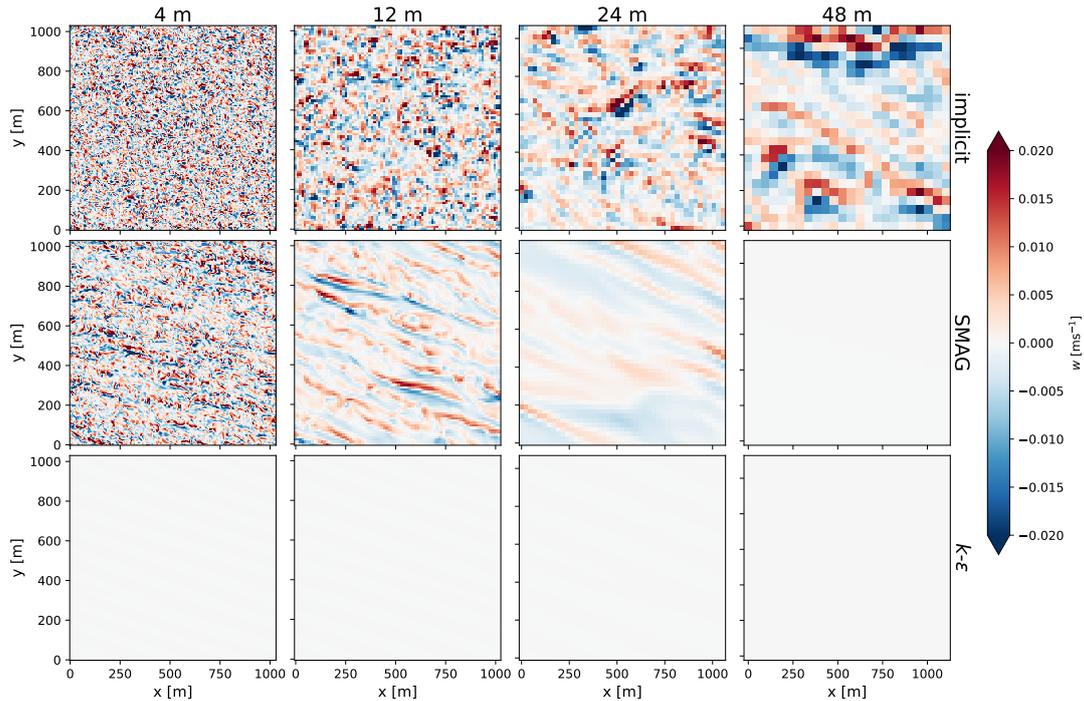


Figure 3. Snapshots of the vertical velocity (w) from the shear-only case at 10 m depth after four inertial periods. Each column is organized to show the results of the same closure at different grid spacings. Note that no turbulence appears in the k - ϵ runs and the 48 m Smagorinsky run.

292 The wind-wave case is designed to generate Langmuir turbulence under strong wind
 293 and wave conditions typical of the surface ocean (Belcher et al., 2012). With $La_t = 0.3$,
 294 the roll-cell pattern of Langmuir circulations (concentrated horizontal convergence zones
 295 and strong downwelling lines) exhibits strong sensitivities to grid spacing and subgrid
 296 closures (Figure 4). Unlike the homogeneous flow in the shear-only case, k - ϵ generates
 297 the roll-cell pattern with the additional forcing of Stokes drift. However, compared to
 298 Smagorinsky and the implicit method, k - ϵ produces relatively smooth velocity gradients,
 299 again indicating excessive damping of resolved turbulence (as discussed in Section 3.4
 300 when analyzing flow spectra). The relative width of roll cells scales with grid spacing for
 301 both Smagorinsky and the implicit method, increasing from about 100 m at $\Delta = 4$ m
 302 to over 400 m at $\Delta = 24$ m. In contrast, k - ϵ shows less sensitivity to grid spacing, con-
 303 sistenty producing roll cells with a characteristic width of about 100 m across all grid
 304 spacings. At $\Delta = 48$ m, the Smagorinsky solution does not show turbulent eddies, whereas
 305 k - ϵ and the implicit method still generate weak but coherent downwelling lines. This aligns
 306 with previous modeling studies by Hypolite et al. (2021, 2022), which reported a simi-
 307 lar sensitivity of roll cell width to model resolution in the realistic simulations of the Cal-
 308 ifornia Current system.

309 The other forcing cases exhibit similar sensitivities in their coherent structure to
 310 closure and grid spacing, with additional figures provided in the supporting information.
 311 While this section only offers a qualitative discussion of flow patterns, the representa-
 312 tion of coherent turbulence structures has important implications for model nesting in
 313 the gray zone. We find that k - ϵ tends to damp turbulent eddies more than Smagorin-
 314 sky and the implicit method, with the shear-only case being the most pronounced ex-
 315 ample. When an LES is nested within a regional simulation, excessively laminar flow at

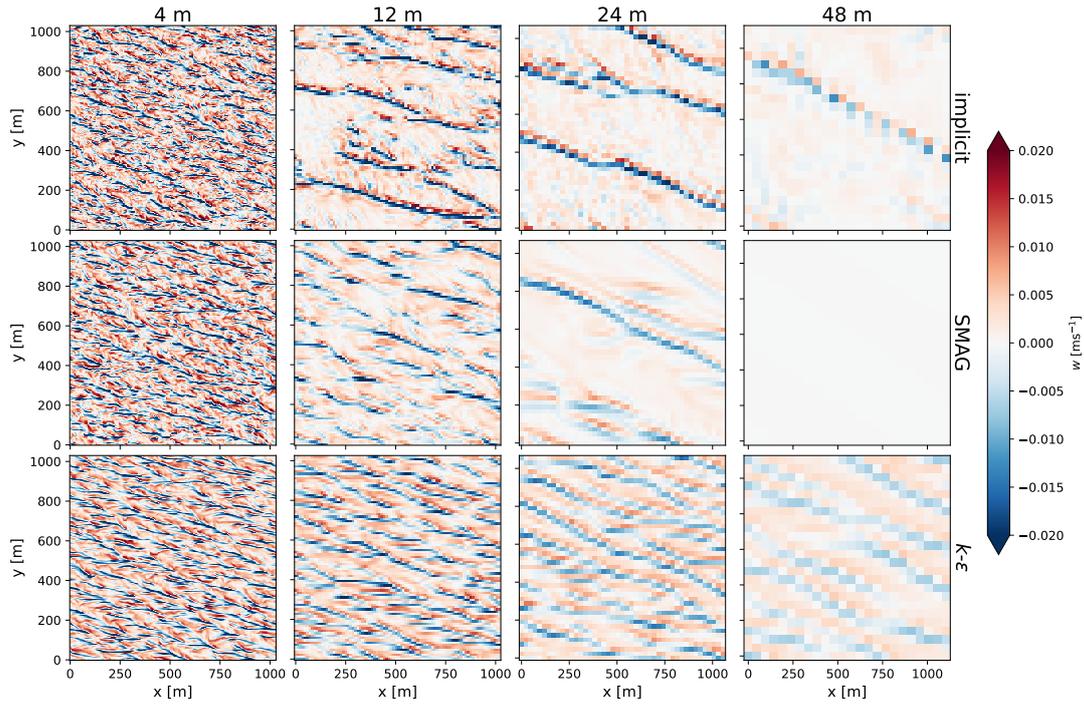


Figure 4. Snapshots of the vertical velocity (w) from the wind-wave case at 10 m depth after four inertial periods. Each column is organized to show the results of the same closure at different grid spacings. Note that no turbulence appears in the 48 m Smagorinsky run.

316 the boundary may suppress turbulence development in the inner domain. Compared to
 317 k - ϵ in the gray zone, using Smagorinsky and the implicit method produces different tur-
 318 bulance patterns, which can propagate into the inner domains and influence the final so-
 319 lutions. In the next section on mixed-layer profiles, we move toward the quantitative eval-
 320 uation of boundary mixing in the gray zone.

3.2 Mixed Layer Profiles

322 This section examines the horizontally-averaged properties of the mixed layer—including
 323 temperature, velocity, and stratification profiles—for the convection-only (3.2.1), shear-
 324 only (3.2.2), and convection-wind-wave cases (3.2.3). Flux profiles are separated into re-
 325 solved and subgrid components. All profiles are horizontally averaged at each depth and
 326 temporally over the last inertial period.

3.2.1 Convection-only

328 Figure 5 presents the temperature and stratification profiles (N^2) from the convection-
 329 only case. Each panel displays the gray zone profiles of different grid spacings, compared
 330 to the baseline solution ($\Delta = 1.25$ m, Smagorinsky) in red. Qualitatively, the averaged
 331 profiles under convection-driven turbulence show little sensitivity to changes in grid spac-
 332 ing and closure, relative to other forcing cases below. For the implicit and Smagorinsky
 333 solutions, the most notable deviations from the baseline solution occur near the surface
 334 above a depth of 5 m. In contrast, the k - ϵ solutions exhibit greater differences at the bot-
 335 tom of the mixed layer, between 45 m and 55 m. A clear convergence of results is ob-
 336 served as the grid spacing is refined.

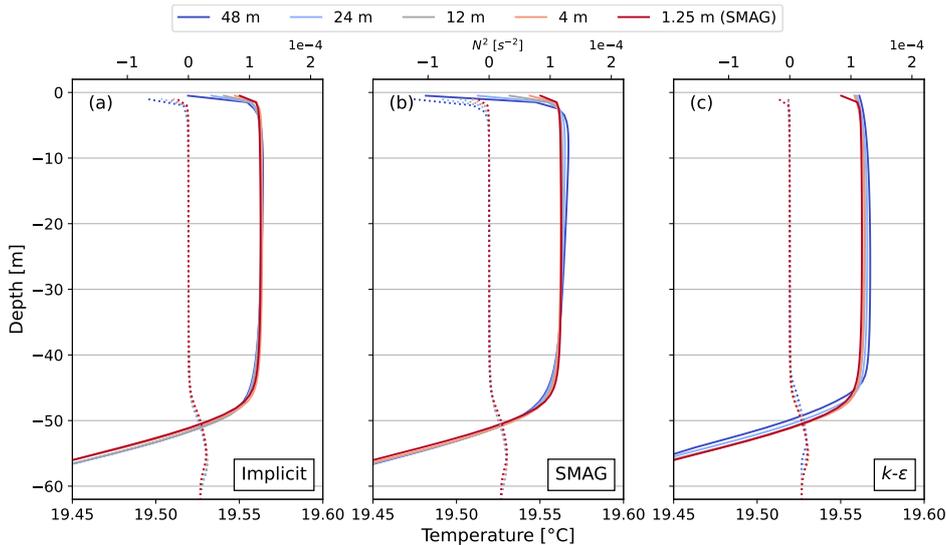


Figure 5. Mean-state temperature (solid lines) and stratification (dotted lines) profiles from the convection case. Along with the temperature profiles (solid lines) in the top panels, the accompanying N^2 profiles are shown as dotted lines. The 1.25 m Smagorinsky run is used as the baseline of comparison (red lines).

337 Figure 6 presents the vertical velocity variance and temperature flux profiles in the
 338 convection case. Regardless of closure, the velocity variances increase with smaller grid
 339 spacings, corresponding to more active turbulent eddies (Section 3.1). The damping ef-
 340 fect of $k-\epsilon$ on resolved turbulence, previously observed in the flow visualization, is also
 341 evident here, with the smallest variances relative to those in the Smagorinsky and im-
 342 plicit runs. For temperature fluxes, all runs show similar total flux magnitudes, regard-
 343 less of closures or grid spacing. This suggests that larger turbulent temperature anomalies
 344 at lower resolutions compensate for reduced vertical velocities.

345 For the temperature profiles of $k-\epsilon$ in Figure 6.f, the total fluxes remain consistent
 346 despite varying degrees of resolved turbulence at different grid spacings. The relative frac-
 347 tion of subgrid component ($-\overline{K_s \partial_z T}$, where K_s is eddy diffusivity) compensates for changes
 348 in the resolved component ($w'T'$). Small differences emerge in the entrainment layer (40–60
 349 m), where the higher-resolution runs produce slightly enhanced negative temperature
 350 flux. Previously, Umlauf and Burchard (2005) have also evaluated the performance of
 351 $k-\epsilon$ for free convection and reported good approximations of buoyancy flux and entrain-
 352 ment depth.

353 In Figure 6.d and e, the temperature flux profiles from the Smagorinsky and the
 354 implicit runs show striking similarities. The subgrid fluxes by Smagorinsky are negligi-
 355 ble, suggesting that numerical dissipation from the advection scheme alone provides most
 356 of the diffusion. While this behavior is acceptable from a model fidelity perspective, it
 357 is not entirely desirable, as the model effectively functions as an implicit LES despite the
 358 prescription of an explicit closure. A higher-order advection scheme (not currently avail-
 359 able in CROCO) could potentially mitigate this issue. However, since this behavior is
 360 observed only in free convection, it may not be a concern in more realistic configurations
 361 (Section 3.2.3).

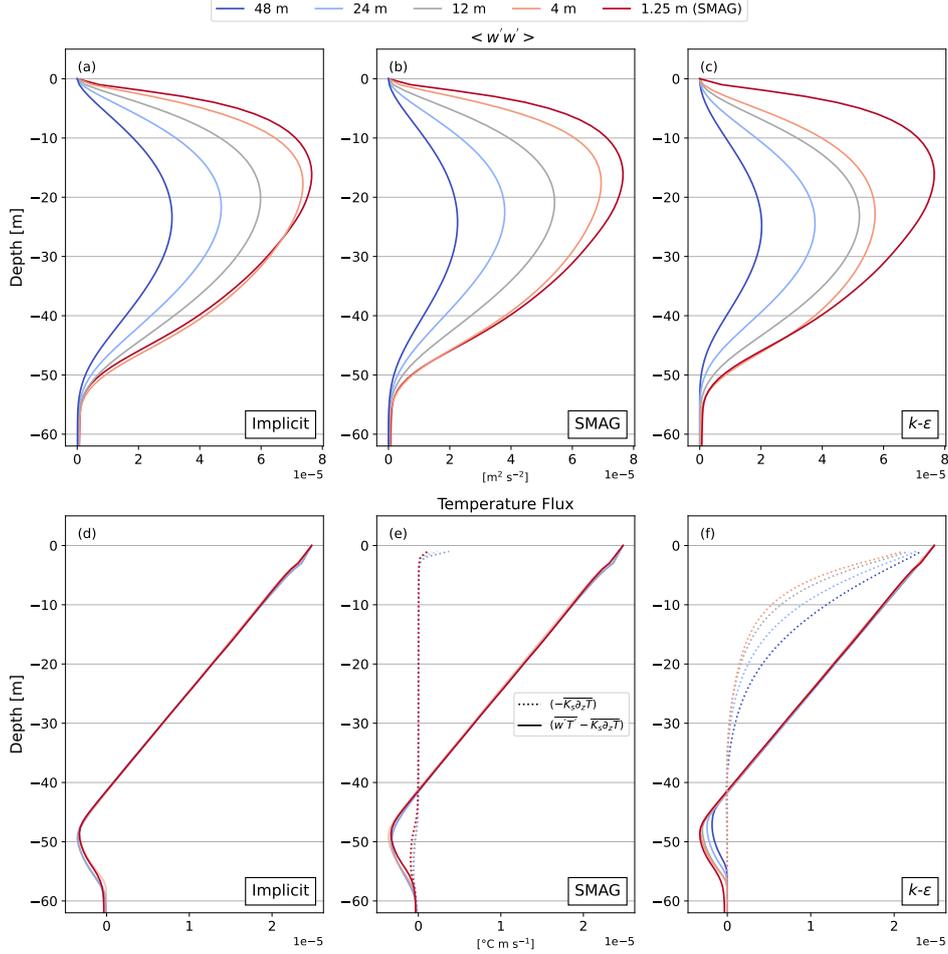


Figure 6. Averaged vertical velocity variance and temperature fluxes in the convection case, where the total flux is denoted as the solid lines and the subgrid component as dotted lines. The 1.25 m Smagorinsky run is used as the baseline of comparison in red lines.

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3.2.2 Shear-only

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Compared to free convection, the shear-only case is far more sensitive to closure choice. In the $k-\epsilon$ runs, turbulent mixing is entirely represented by subgrid processes. Figure 7.c shows that $k-\epsilon$ produces identical, well-mixed temperature and velocity profiles at different grid spacings, with zero velocity variance (Figure 8.c) and only subgrid fluxes $(-\overline{K_m \partial_z u})$ where K_m is eddy viscosity; See Figure 8.f). The lack of resolution sensitivity reflects the expected behavior of $k-\epsilon$ as a RANS turbulence closure, which is well validated for free-shear flows. Compared to Smagorinsky and the implicit solutions (Figure 7.a, b), $k-\epsilon$ generates a deeper mixed layer, with strong stratification in the entrainment layer at 45 m. The maximum N^2 is almost twice as large as the baseline solution (Figure 7.c).

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The sensitivity to grid spacing is particularly pronounced for Smagorinsky and the implicit method. At $\Delta = 48$ m, the Smagorinsky solution shows no turbulent eddies (Figure 3). The temperature profile remains at 30 m from the initial condition, while the velocity profile shows strong surface shear above 10 m (Figure 7.e); At $\Delta = 48$ m, the momentum flux is entirely subgrid above 10 m in response to wind drag, while the tem-

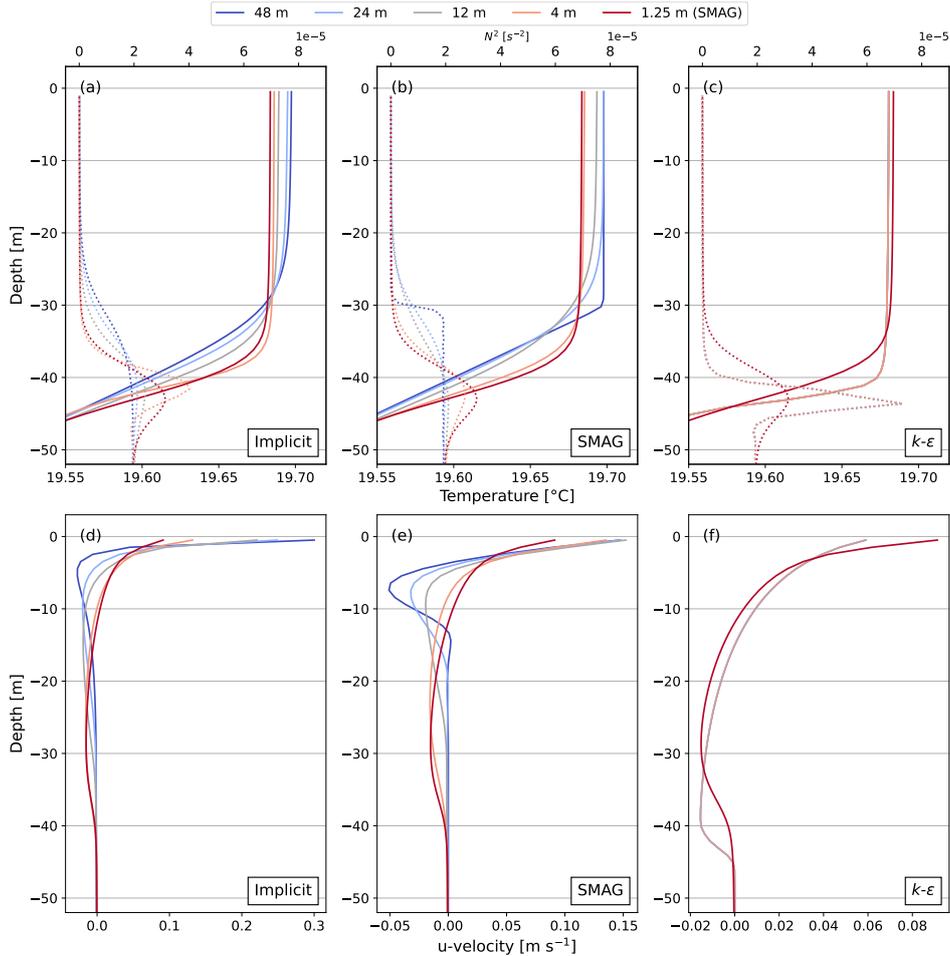


Figure 7. Averaged temperature and u-velocity profiles for the shear-only case. Along with the temperature profiles (solid lines) in the top panels, the accompanying N^2 profiles are shown as dotted lines. The 1.25 m Smagorinsky run is used as the baseline of comparison in red lines. Note the differences in the x-axis scale in the bottom row.

378 perature flux is negligible at all depths (Figure 8.e, h). These profiles suggest that the
 379 mixed layer is not being deepened effectively, as further reflected in the MLD time series
 380 (Figure 12.b). Similarly, the implicit method at $\Delta = 48$ m also struggles to drive
 381 mixed-layer deepening, despite some turbulence appearing in the velocity field in Fig-
 382 ure 3 (so is the non-zero variance in Figure 8.a).

383 Despite difficulties in representing mixed-layer deepening at coarse grid spacings,
 384 both Smagorinsky and the implicit method demonstrate better performance at finer grids.
 385 From $\Delta = 48$ m to 4 m, the temperature and velocity profiles (Figure 7.a, b, d, and e),
 386 as well as the temperature and momentum fluxes (Figure 8.d, e, g, and h), coverage to
 387 the baseline solution. The momentum fluxes increase within the mixed-layer interior (10–30
 388 m), indicating stronger downward mixing of momentum input from surface wind stress.
 389 Near the surface (above 10 m), the Smagorinsky subgrid flux for momentum contributes
 390 less to the total momentum flux with smaller grid spacing, suggesting that vertical mix-
 391 ing is increasingly dominated by resolved turbulence rather than the subgrid closure, as
 392 expected for LES. However, the opposite trend is observed in temperature flux, where

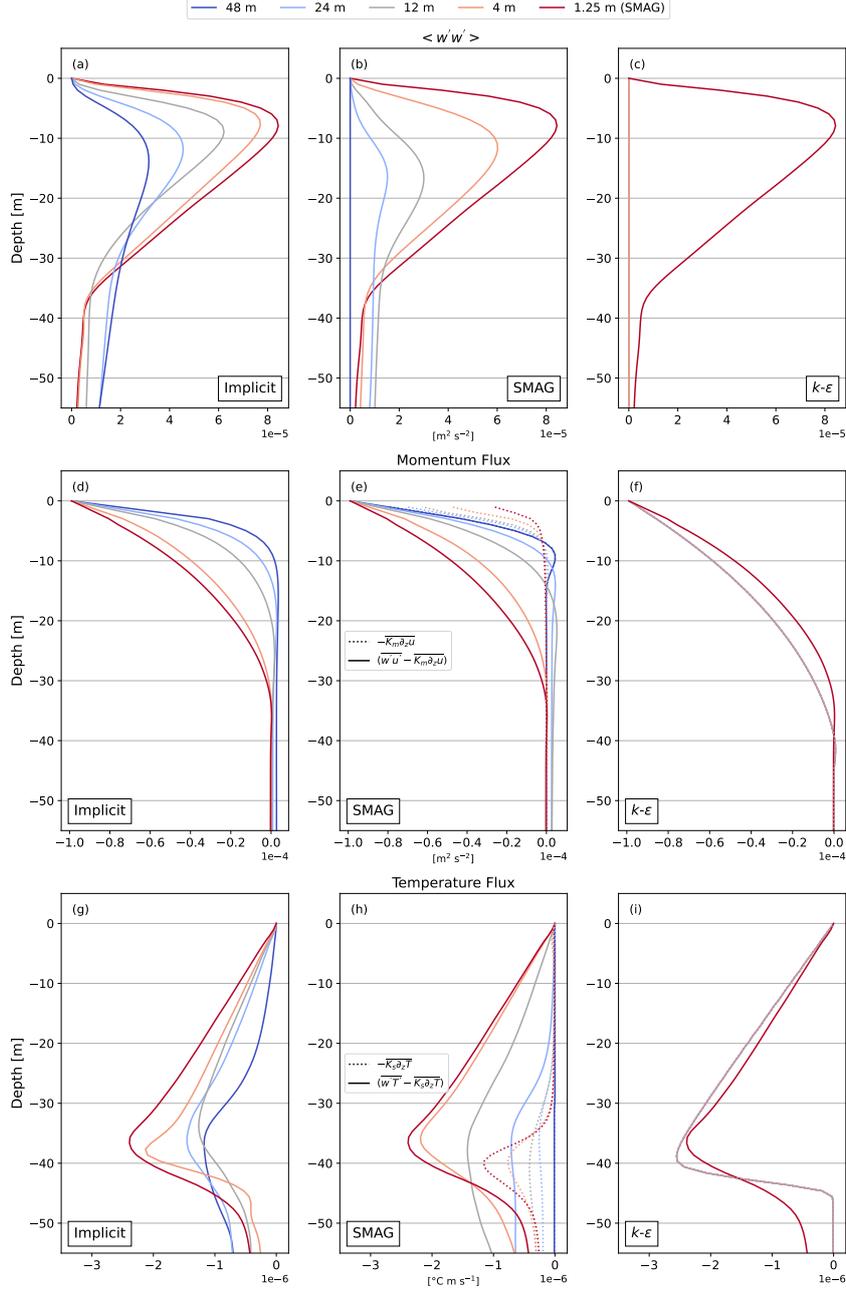


Figure 8. Averaged vertical velocity variance, together with the momentum and temperature fluxes from the shear case. In the flux profiles, the solid lines show the total flux and the dotted lines show the subgrid component. The 1.25 m Smagorinsky run is used as the baseline of comparison in red lines. Note that all $k-\epsilon$ runs produce the same profile consisting only of the subgrid component.

393 the subgrid contribution in the Smagorinsky runs increases with resolution near the en-
 394 trainment layer at about 40 m.

395 When comparing the performance of different closures, the shear-only case high-
 396 lights $k-\epsilon$'s ability to represent mixed-layer deepening with minimal resolution sensitiv-

ity in the gray zone. However, for nested simulations, k - ϵ may suppress the emergence of turbulent eddies, making it problematic for intermediate nesting steps. Instead, transitioning from bulk closures to Smagorinsky or the implicit method may improve turbulence representation. Nevertheless, this approach introduces a trade-off with mean-state accuracy, which must be considered despite the benefits at higher resolutions. While shear-only forcing is uncommon at the ocean surface (Belcher et al., 2012), these findings may have implications for bottom boundary layer simulations (Umlauf et al., 2015; Wenegrat & Thomas, 2020).

3.2.3 Convection-wind-wave

Beyond the single forcing cases, we extend our analysis to a more oceanographically relevant case driven by combined convection-wind-wave forcing (Figure 1). Similar figures for the convection-shear and wind-wave cases are available in the supporting information.

When comparing the mixed layer profiles of different closures for this case in Figure 9, the k - ϵ solutions show the smallest deviation from the baseline, highlighting their accuracy in representing mean mixed-layer profiles even with partially resolved turbulence (see the instantaneous flow field in Figure S2 from the supporting information). Further, k - ϵ exhibits minimal sensitivity to grid spacing and maintains good accuracy throughout the gray zone. In comparison, the Smagorinsky and implicit solutions show strong sensitivity to grid spacing. For example, at $\Delta = 48$ m, instead of producing a well-mixed boundary layer, both the Smagorinsky and the implicit solutions exhibit strong temperature inversions and velocity shear near the surface (Figure 9.a, b, d, and e), indicating poor performance when turbulent eddies are barely resolved. When Δ is refined to 4 m, both Smagorinsky and the implicit method generate a well-mixed boundary layer, with nearly identical profiles converging to the baseline solution.

The variance and flux profiles in Figure 10 highlight the role of subgrid closure in driving mixing relative to resolved motions. Similar to the convection-only case, the vertical velocity variance, which corresponds to the intensity of turbulent eddies, increases with resolution for all three closures (Figure 10.a, b, and c). k - ϵ again stands out for its consistency in representing total fluxes across the gray zone range. k - ϵ also maintains a reasonable partition between resolved and subgrid fluxes, with a smaller relative contribution from the subgrid closure at higher resolution, suggesting its adaptability to grid refinement (Figure 10.f and i). For Smagorinsky and the implicit method, despite poor performance at coarse resolutions, the total flux converges with smaller grid spacings (Figure 10.d, e, g, and h). Notably, the subgrid fluxes by Smagorinsky are considerably smaller than those by k - ϵ at the same grid. At $\Delta = 4$ m, the Smagorinsky solutions and the implicit solutions achieve a similar accuracy relative to the baseline. Here, the small subgrid contribution in the Smagorinsky runs suggests that vertical mixing is well represented by resolved eddies rather than the subgrid closure.

Overall, among different closures for the realistic forcings, k - ϵ best represents mixed-layer profiles but damps turbulent eddies. This may be advantageous for submesoscale-resolving simulations that require an accurate mixed-layer representation without explicitly resolving turbulent eddies. Despite poor performance at coarse grids, the Smagorinsky and the implicit solutions quickly converge, with a greater fraction of total flux carried by resolved turbulence. These properties may be useful for nesting applications transitioning through gray zone resolutions toward well-resolved LES, where parent solutions can influence turbulence in the nested domain (F. Chow et al., 2019).

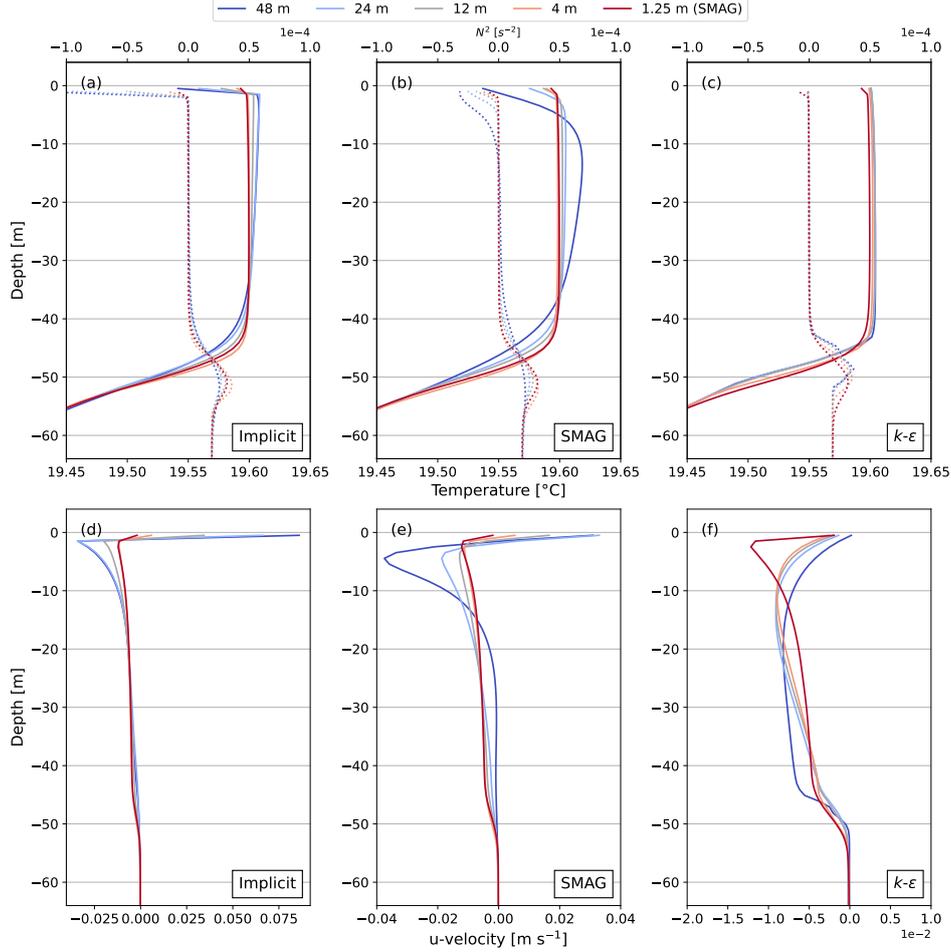


Figure 9. Averaged temperature and u-velocity profiles for the convection-wind-wave case. Along with the temperature profiles (solid lines) in the top panels, the accompanying N^2 profiles are shown as dotted lines. The 1.25 m Smagorinsky run is used as the baseline of comparison in red lines. Note the differences in the x-axis scale in the bottom row.

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3.3 Time dependence

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While the previous analysis focuses on instantaneous and temporally-averaged properties, in this section we compare the MLD time series to highlight the joint impacts of grid spacing and closure on the rate of mixed-layer deepening. There are many different methods to diagnose MLD, and the criteria based on a density difference from the surface is commonly used in regional models (Courtois et al., 2017). In this study, the surface value is not used, given that some gray zone simulations are not well mixed near the surface (see Figure 9 for the temperature inversion in the 48 m Smagorinsky run from the convection-wind-wave case). Instead, we define MLD as the depth where the density exceeds the mixed layer average (taken between 15 m and 25 m) by 0.005 kg m^{-3} . A range of density thresholds ($0.01\text{-}0.001 \text{ kg m}^{-3}$) was tested, and the results were found to be qualitatively robust to any reasonable choice of thresholds. We have tested another common MLD criterion based on a density gradient (i.e., depth of N^2 maximum) following Fan et al. (2023). However, we find the coarse resolution runs tend to generate less pronounced N^2 maxima near the mixed layer bottom (see the stratification profiles in Figure 7.a and b), leading to large fluctuations in the MLD time series at the initial

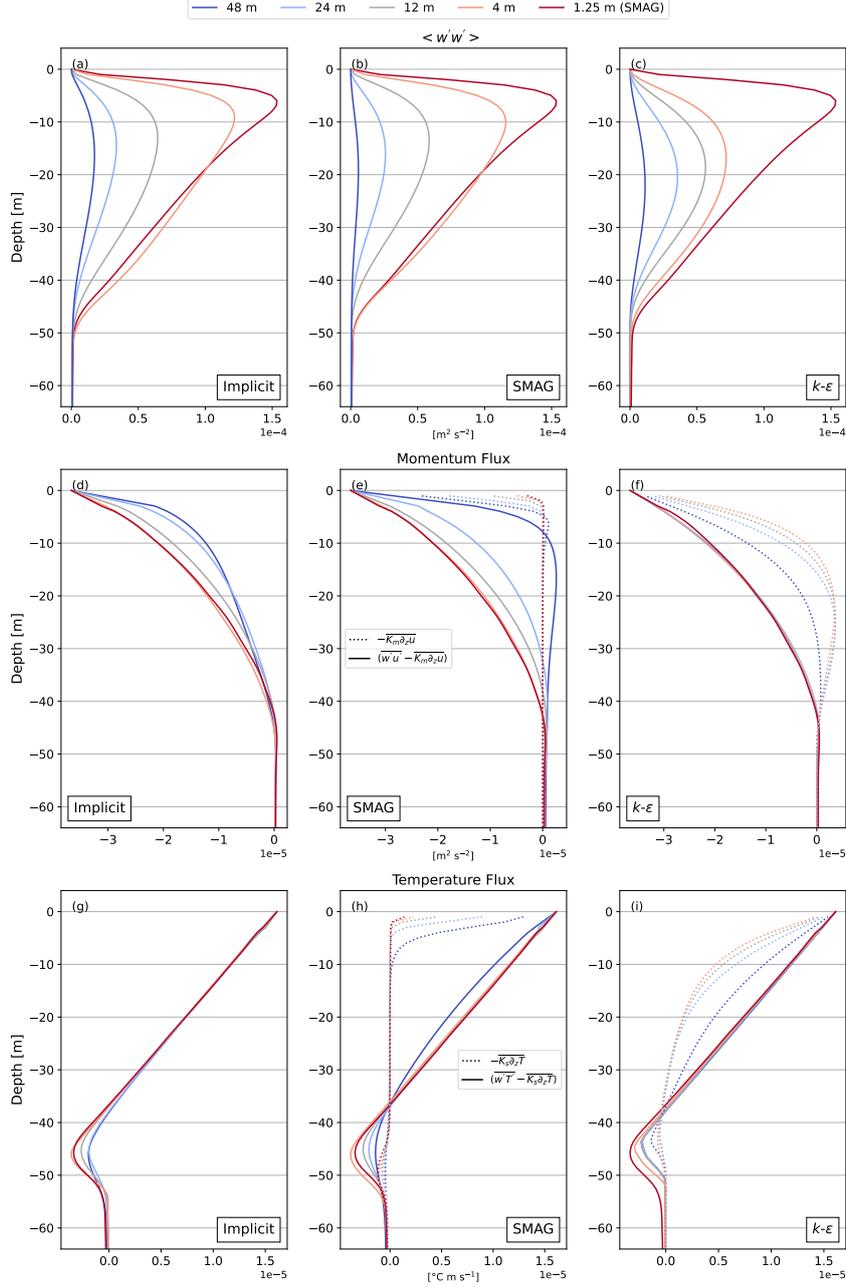


Figure 10. Averaged vertical velocity variance, together with the momentum and temperature fluxes from the convection-wind-wave case. In the flux profiles, the solid lines show the total flux and the dotted lines show the subgrid component. The 1.25 m Smagorinsky run is used as the baseline of comparison in red lines.

460 time steps. Therefore, to allow a better comparison of MLD evolution across different
 461 gray zone simulations, we choose the criteria of density threshold.

462 Figure 11 shows the MLD time series for the convection-only case comparing dif-
 463 ferent closures (panel a-c) and summarizing the mean errors over the last inertial period
 464 (panel d). Similarly to the mixed-layer profiles discussed in Section 3.2.1, the mixed-layer

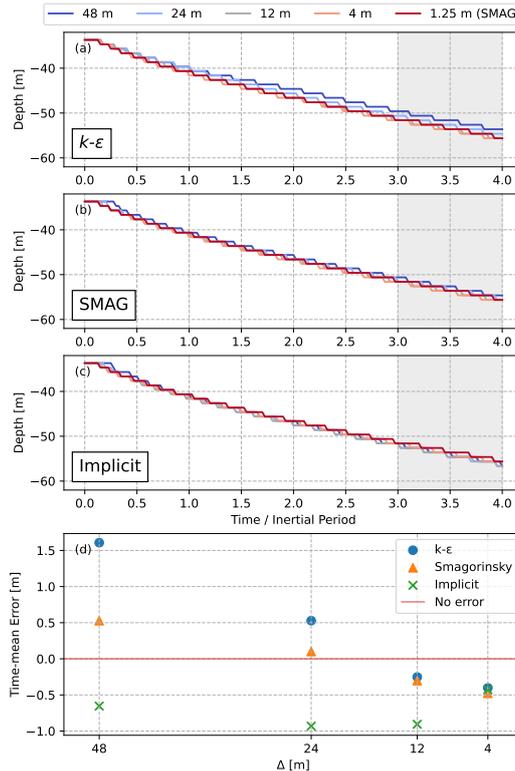


Figure 11. Time series of MLD in the convection-only case by different closures and grid resolutions in panel a-c. The mean MLD errors relative to the baseline run in the last inertial period are shown in panel d.

465 deepening rate shows minimal sensitivity to grid spacing for all closures. Shortly after
 466 model initialization, MLD deepens in all gray zone simulations due to surface buoyancy
 467 loss, reaching approximately 55 m after four inertial periods. While all closure solutions
 468 converge at smaller grid spacing, the $k-\epsilon$ solution at $\Delta = 48$ m exhibits the largest er-
 469 ror from the baseline, though still relatively small (1-2 m). This error is consistent with
 470 earlier observations in the temperature profiles (figure 6.c), where $k-\epsilon$ struggles in the
 471 entrainment layer between 45 m and 55 m.

472 In contrast to convection, the mixed-layer evolution under wind forcing (Figure 12)
 473 is more sensitive to closure and grid spacing. As discussed earlier, the $k-\epsilon$ runs in the shear-
 474 only case do not produce turbulent eddies, modeling turbulence mixing entirely as a sub-
 475 grid process. This results in an identical solution regardless of grid spacing. In Figure
 476 12, the $k-\epsilon$ solutions exhibit a deeper mixed layer after four inertial periods, consistent
 477 with the stratification seen in the temperature profiles (Figure 7.c). For Smagorinsky and
 478 the implicit solutions, their inefficiency in driving vertical mixing at coarse grid spacings
 479 (e.g., $\Delta = 48, 24, 12$ m) is evident for the lack of mixed-layer deepening in the MLD time
 480 series (Figure 12.b, c), although we note these simulations also had a thick transition layer
 481 between the mixed-layer and interior with reduced stratification (Figure 7.a, b). Over-
 482 all, MLD evolution in the shear-only case highlights the accumulation of mean-state er-
 483 ror in gray zone simulations, which could become particularly problematic for long-time
 484 integrations or applications where the fidelity of large-scale conditions is critical for LES.

485 Figure 13 shows mixed-layer evolution under more realistic convection-wind-wave
 486 forcing conditions. In addition to mean-state drift, the MLD time series reveals notable

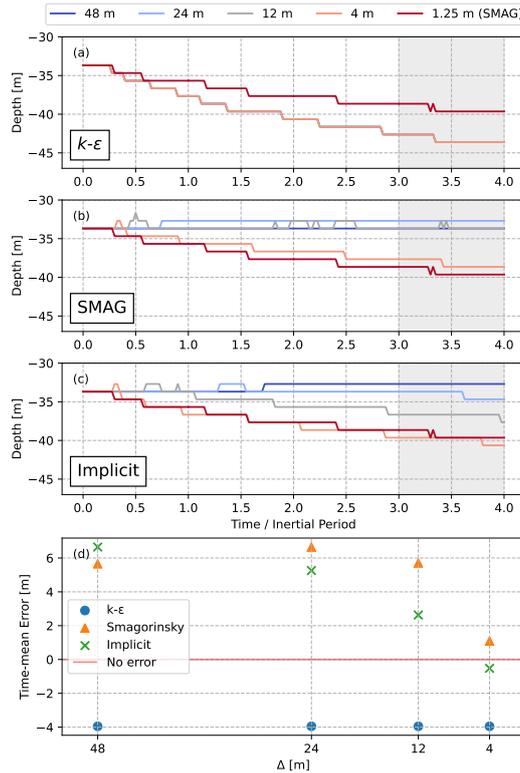


Figure 12. Time series of MLD in the shear-only case by different closures and grid resolutions in panel a-c. The mean MLD errors relative to the baseline run in the last inertial period are shown in panel d.

487 differences in the onset of mixed-layer deepening. For $k-\epsilon$, the mixed layer begins to deepen
 488 immediately, following the baseline solution. However, for Smagorinsky and the implicit
 489 method, the deepening of MLD is delayed at coarse resolutions. For example, in the Smagorin-
 490 sky solution at $\Delta = 48$ m, MLD remains at its initial value for 1.5 inertial periods, lead-
 491 ing to a shallower MLD for about 5 m ($\sim 10\%$ MLD) after four inertial periods. This
 492 suggests that the transition to turbulence also depends on grid spacing and closure (see
 493 Figure S9 for the time series of vertical velocity variance), which can be an important
 494 consideration for strongly time-dependent turbulence problems such as the diurnal cycle
 495 (Wenegrat & McPhaden, 2015; Sutherland et al., 2016). At coarse grid spacings that
 496 barely resolve turbulent eddies, $k-\epsilon$ may offer a better mean-state representation of the
 497 mixed layer. When using Smagorinsky or the implicit method in the intermediate nest-
 498 ing steps toward LES, introducing additional perturbations to initial and boundary condi-
 499 tions may help spin up turbulent eddies and improve inner-domain solutions.

500 3.4 Effective Resolution

501 A model’s effective resolution is commonly defined as the horizontal length scale
 502 where dissipation—due to both the numerical discretization and turbulence parameterizations—
 503 begin to significantly influence the solution, such that the underlying dynamics are no
 504 longer properly represented. It is often diagnosed by where the kinetic energy spectrum
 505 departs from the expected slope at small scales, for example, $k^{-5/3}$ for atmospheric mesoscale
 506 dynamics (Skamarock, 2004) and k^{-2} for oceanic submesoscale (Soufflet et al., 2016).
 507 Prior work indicates the effective model resolution is often reasonably approximated as

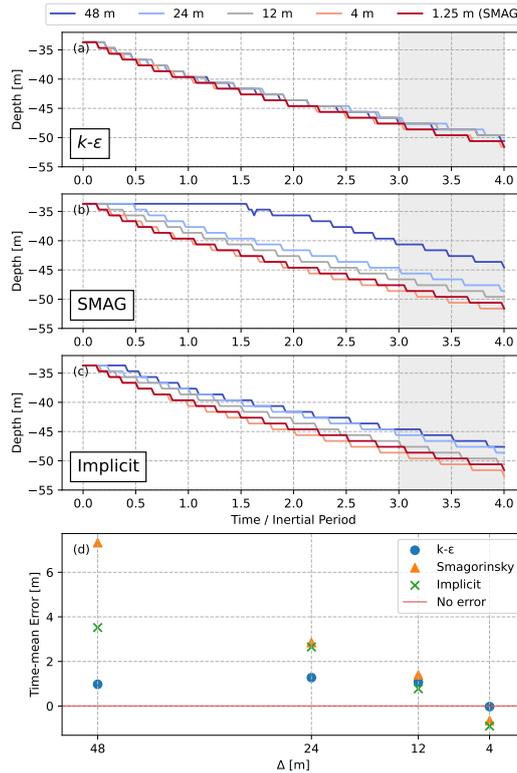


Figure 13. Time series of MLD in the convection-wind-wave case by different closures and grid resolutions (panel a-c). The mean errors relative to the baseline run is calculated for the last inertial period (panel d).

508 multiples of grid spacing ($\approx 7-10\Delta$), although the precise value can be a complicated
 509 factor of many choices of numerical setup, including advection schemes for momentum
 510 and tracers, as well as time-stepping routine (Soufflet et al., 2016). Here, we highlight
 511 how gray zone simulations can incorrectly represent the dynamics near the turbulence
 512 injection scale (e.g., section 3.1), such that even scales much larger than those expected
 513 to be affected directly by model dissipation may be contaminated. In this section, we
 514 highlight this behavior focusing on the convection-wind-wave case. The results of other
 515 forcing cases display similar characteristics and can be found in Figure S10-14 from the
 516 supporting information.

517 The effective resolution of the baseline run—due to explicit dissipation from the
 518 eddy viscosity and the implicit numerical dissipation from the WENO5 scheme—can be
 519 identified by the high-wavenumber roll-off of the spectral kinetic energy (Figure 14). The
 520 baseline KE spectra, labeled as 1.25 m (SMAG) in all panels, follow the expected slope
 521 of the inertial subrange ($k^{-5/3}$) starting from 50 m until wavelengths of approximately
 522 10 m at which point there is a faster roll-off. The clear separation between the inertial
 523 subrange and the numerical range indicates an effective numerical resolution of about
 524 7Δ , similar to the range found previously for much larger-scale simulations (Soufflet et
 525 al., 2016). However, only the baseline run has an unambiguous effective resolution scale
 526 that can be identified from the spectral slopes. For the gray-zone runs of larger grid spac-
 527 ings (e.g., the implicit runs $\Delta > 4$ m), the inertial subrange is not well resolved while
 528 still impacted by the numerical dissipation, making it impossible to identify the effec-
 529 tive resolution based on the expected spectral slope. This highlights the presence of under-
 530 resolved large eddies and indicates the gray zone nature of these simulations. The mis-

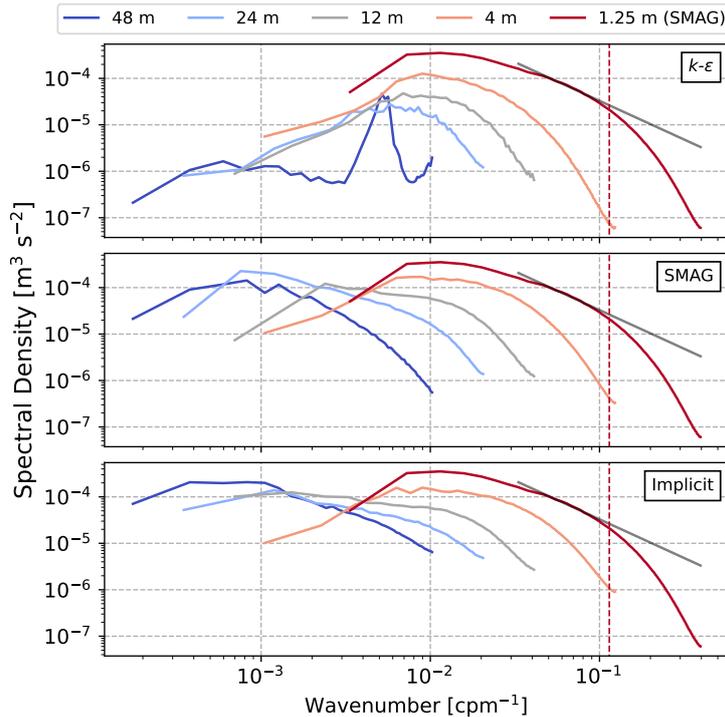


Figure 14. Turbulent kinetic energy spectra from the convection-wind-wave case at 20 m depth, derived from the 3D velocity fields and averaged over the last inertial period. The dashed vertical lines mark the effective numerical resolution of the 1.25 m Smagorinsky run at 7Δ , and the characteristic $-5/3$ slope of the inertial subrange is shown in gray.

531 representation of large eddies and model dissipation is likely to contribute to errors in
 532 the mean-state profiles (see Section 3.2.3).

533 In Figure 14, the $k-\epsilon$ spectra display lower levels of energy than the baseline and
 534 the implicit and Smagorinsky spectra at the same grid spacings, suggesting that the use
 535 of $k-\epsilon$ in the gray zone substantially damps turbulent eddies. This is consistent with the
 536 smoothed velocity gradients found in the flow structure (Section 3.1). At $\Delta = 48$ m,
 537 the $k-\epsilon$ spectrum becomes overly flat with few turbulent eddies explicitly resolved. The
 538 damping issue of $k-\epsilon$ is the most pronounced in the shear-only case where the closure com-
 539 pletely suppresses the emergence of turbulent eddies regardless of grid spacings (Figure
 540 S11). For the $k-\epsilon$ simulations, the effective resolution cannot be meaningfully defined here,
 541 as the dynamics of all scales up to the domain size are not properly represented. For nested
 542 simulations across the gray zone, the use of $k-\epsilon$ may be problematic for supplying overly
 543 laminar flows for boundary conditions, affecting the turbulence statistics in inner solu-
 544 tions. The importance of this will vary depending on the role of turbulence advection
 545 between parent and child domains.

546 Compared to $k-\epsilon$, the Smagorinsky closure and the implicit method generate simi-
 547 lar spectra of higher energy levels for the resolved turbulent motions (Figure 14). This
 548 is valuable during intermediate nesting steps in the gray zone to better represent tur-
 549 bulance in the inner domain despite the trade-off in the mean-state accuracy (Section
 550 3.2.3). However, instead of aligning with the baseline solution, the spectral peaks in the
 551 Smagorinsky and the implicit runs shift towards longer wavelengths with larger grid spac-
 552 ings, representing the model’s attempt at representing turbulence even on the coarse grids.

553 The spectrum shift highlights how the spectral definition of effective resolution (based
 554 on roll-off of the slope) may at times be misleading, as there can be shifts in wavenum-
 555 ber space or energy levels that are independent of the spectral shape (see also Figure S11).
 556 For instance, the 24 m grid solution using Smagorinsky has a spectral shape broadly sim-
 557 ilar to the 1.25 m baseline run, but being shifted such that the spectral peak is at ap-
 558 proximately 1200 m wavelength rather than 150 m. It would be incorrect to conclude
 559 that the effective resolution of this simulation is near 200 m (where there is a roll-off in
 560 the spectrum), as comparison with the baseline simulation indicates inaccurate repre-
 561 sentation of turbulent motions at scales well exceeding 10Δ (in the sense of both the vari-
 562 ance contained at a given spatial scale and in terms of the coherent structures as dis-
 563 cussed in section 3.1). This should be considered for nesting applications, where inac-
 564 curate representations of boundary layer turbulence from the outer domain can be ad-
 565 vected into the inner domain through the boundary conditions, contaminating the final
 566 solution, an issue identified previously for nested LES of the atmospheric boundary layer
 567 (Mirocha et al., 2013; Mazzaro et al., 2017).

568 4 Summary and Discussion

569 This study evaluates the gray zone performance of two common turbulence closures,
 570 k - ϵ and Smagorinsky, as well as the implicit method using the WENO5 advection scheme,
 571 for ocean surface mixed layer under different forcing conditions. With marginally resolved
 572 turbulent eddies, we find that coherent turbulent structures, mean mixed-layer profiles,
 573 and kinetic energy spectra exhibit closure-dependent resolution sensitivities. In general,
 574 k - ϵ solutions provide the highest accuracy in representing bulk properties such as mixed
 575 layer depth, maintaining well-mixed temperature and velocity profiles across the gray
 576 zone range. The total fluxes (subgrid plus resolved) by k - ϵ also remain mostly consis-
 577 tent, with reasonable adjustment to resolution in the partitioning between the subgrid
 578 and resolved fluxes. However, the instantaneous flow fields and the kinetic energy spec-
 579 tra suggest k - ϵ damps the turbulence motions, representing mixing primarily as a sub-
 580 grid process, even when the grid is fine enough to begin resolving turbulence. The most
 581 striking example is the shear-only case, where k - ϵ suppresses turbulence entirely, pro-
 582 ducing an identical laminar flow field at all grid spacings tested. In comparison, the Smagorin-
 583 sky and the implicit solutions show greater sensitivity to grid spacing. With coarse grids,
 584 they fail to drive sufficient vertical mixing to deepen the mixed layer, resulting in strong
 585 velocity shear and temperature inversions near the surface. However, with smaller grid
 586 spacing, both Smagorinsky and the implicit method demonstrate improvement as their
 587 solutions converge toward the baseline LES. Unlike the k - ϵ solutions, which are domi-
 588 nated by subgrid fluxes, the Smagorinsky and the implicit solutions primarily represent
 589 turbulent mixing through the resolved flow motions, exhibiting enhanced kinetic energy
 590 at the smaller scales.

591 The gray zone simulations are designed to inform modeling strategies on turbulence
 592 closures during the intermediate scale between the typical RANS and LES paradigms.
 593 Depending on modeling objectives, the consistency of k - ϵ is valuable for simulations aim-
 594 ing to resolve processes larger than boundary layer turbulence (e.g., Langmuir cells and
 595 submesoscale fronts). Even at gray-zone resolutions with partially resolved turbulent ed-
 596 dies, k - ϵ shows good performance in representing bulk mixing in the boundary layer. How-
 597 ever, the k - ϵ solutions are associated with damped turbulence in the resolved velocity
 598 fields. While the absence of resolved turbulence may be acceptable for some submesoscale-
 599 resolving simulations, it can be problematic for nested simulations that need to traverse
 600 the gray zone to achieve realistic LES. For example, in a nested setup, an overly lam-
 601 inar parent-domain flow can inhibit small-scale turbulence development in the child do-
 602 main (Zhou et al., 2014). In this case, using k - ϵ in the gray zone may effectively act as
 603 a much lower-resolution simulation than the grid spacing suggests. A potential remedy
 604 is to switch from k - ϵ to Smagorinsky or use no closure (implicit method) during inter-

605 mediate nesting steps, allowing for more active turbulence at the boundaries. However,
606 this comes with the trade-off of less accurately resolved mean-state evolution, potentially
607 leading to solution drift in larger-scale properties. Given the fast convergence at smaller
608 grid spacing, this trade-off may be acceptable, especially considering that high-resolution
609 nests are typically run for short durations, limiting mean-state error accumulation. Strongly
610 time-dependent problems introduce an additional challenge: turbulence onset exhibits
611 sensitivity to both grid spacing and closure, making it difficult to predict a priori. This
612 sensitivity should be carefully considered when designing nesting strategies.

613 Here we have focused on the performance of common RANS and LES closures; how-
614 ever, several open questions remain regarding best practices for modeling in the ocean
615 gray zone. First, further case studies should investigate submesoscale frontal configura-
616 tions and the associated instabilities (e.g., symmetric instability), as their length scales
617 likely fall within the gray zone and significantly impact turbulence properties in ways
618 not well captured by RANS closures (Bachman et al., 2017; Dong et al., 2022; Chor et
619 al., 2022). Second, despite our effort to design surface forcing representative of the re-
620 alistic ocean, similar sensitivity analyses at the gray zone resolution should be applied
621 to realistic regional simulations formally validated by observational data. Third, the pre-
622 scription of open boundary conditions—including issues such as the forcing frequency
623 and nudging strength—likely influences the fidelity of gray zone simulations. While this
624 issue has been recognized in atmospheric simulations (F. Chow et al., 2019), its impact
625 in ocean simulations remains largely unexplored (Scotti, 2010). Finally, while we have
626 tested existing RANS and LES closures in the gray zone outside their intended appli-
627 cations, the existence of the gray zone problem ultimately necessitates the development
628 of scale-aware turbulence parameterizations and hybrid RANS/LES methods, such as
629 detached-eddy simulation (Spalart, 2009). High-resolution ocean simulation will bene-
630 fit from a uniform turbulence model that can be seamlessly applied across the gray zone
631 without strong trade-offs between mean-state and turbulence representation, while also
632 minimizing the need for extensive sensitivity testing and ad hoc tuning. Efforts in this
633 direction can be guided by existing research on scale-aware turbulence closures for at-
634 mospheric modeling (F. Chow et al., 2019; Honnert et al., 2020).

635 **Open Research Section**

636 The original source code for Coastal and Regional Ocean COmmunity model (CROCO)
637 v1.3.0 used to generate the gray zone simulations is preserved at Zenodo (Auclair et al.,
638 2022). The archiving of model data is underway, and will become publicly available at
639 Zenodo (<https://zenodo.org/records/15116237>).

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Supporting Information for “Evaluating Turbulence Parameterizations at Gray Zone Resolutions for the Ocean Surface Boundary Layer”

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1. Text S1 - S2
2. Figures S1 - S13

Introduction

The supporting information includes supplementary texts and figures from our ocean surface mixed layer simulations, which explore different atmospheric and oceanic forcing

scenarios. These additional figures, presented here for completeness, were generated using the same analytical methods and visual formats as the figures in the main text.

Text S1: Convection-shear Case

In the convection-shear case, the vertical velocity in Figure S1 displays patterns of turbulent eddies for all grid spacings and closures tested. Similar to other forcing cases, $k-\epsilon$ damps the small-scale eddy features. For the mean state profiles in Figure S3 and the flux profiles in Figure S4, the gray-zone behavior of $k-\epsilon$ is similar to the convection-only case where the mean states converge to the baseline solution with smaller grid spacings. The flux profiles also indicate a reasonable partition between the resolved and subgrid components. For Smagorinsky and the implicit method, the solutions suggest insufficient mixing, with significant temperature inversion and velocity shear near the surface. The mean-state solutions are inferior to $k-\epsilon$ especially at coarse grid spacing, although they converge to the baseline with grid refinement.

Text S2: Wind-wave Case

The wind-wave case was designed to investigate the gray zone behavior of a classical set up of Langmuir turbulence at $La_t \sim 0.3$ - a process that is often fully parametrized or resolved by ocean modeling studies. Among the temperature and velocity profiles in Figure S6, the most interesting feature is that the sensitivity of $k-\epsilon$ solutions to grid spacing, unlike the insensitivity found in the shear-only case. In addition, $k-\epsilon$ drives vertical mixing more effectively than Smagorinsky and the implicit method, especially for the coarse grid spacings where the boundary layer eddies could barely be resolved (i.e., $\Delta > 12$ m, relative to a mixed layer depth of 40 m). In Figure S7, the flux profiles by different closures display the similar characteristic as the convection-wind-wave case. The

total fluxes generally agree with the baseline solution, and the subgrid component from the closure reduces slightly in response to larger resolved turbulence at higher resolution.

1. Turbulence Flow Visualization

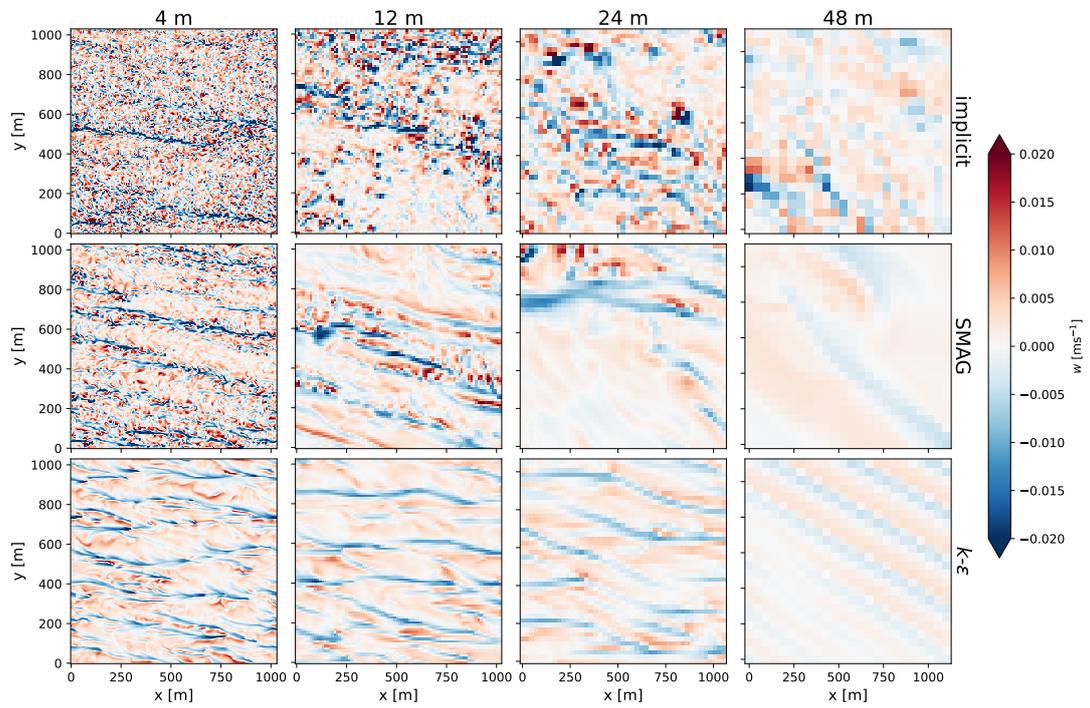


Figure S1. Snapshots of the vertical velocity (w) at 10 m depth for the convection-shear case.

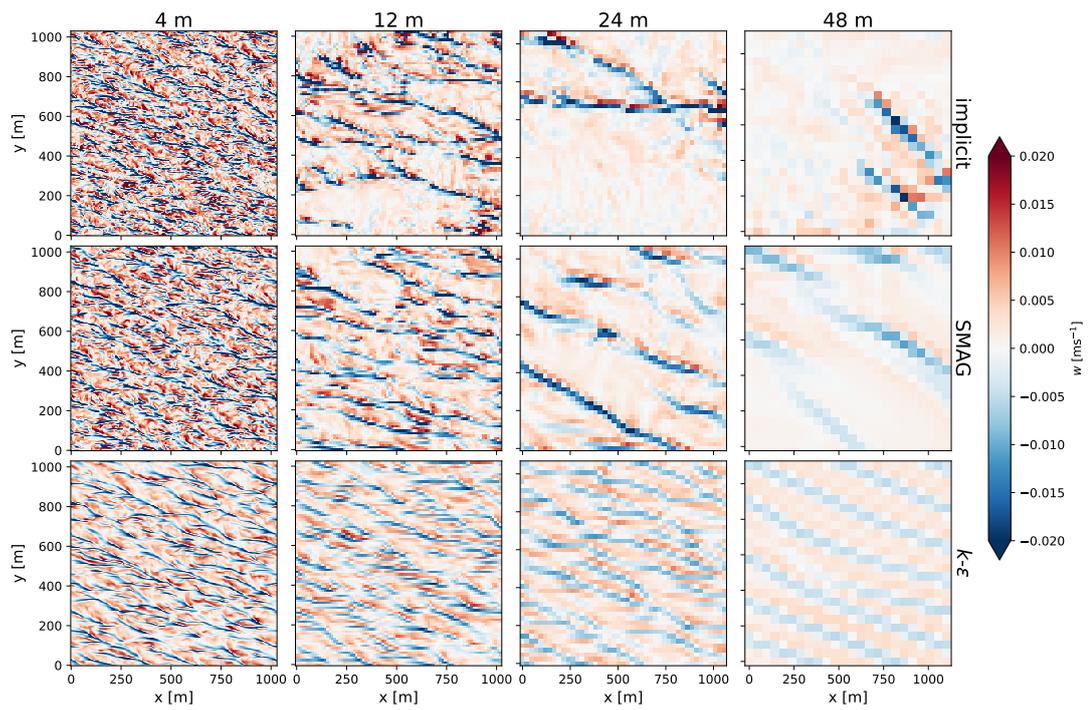


Figure S2. Snapshots of the vertical velocity (w) at 10 m depth from the convection-wind-wave case.

2. Mixed Layer Profiles

2.1. Convection-shear

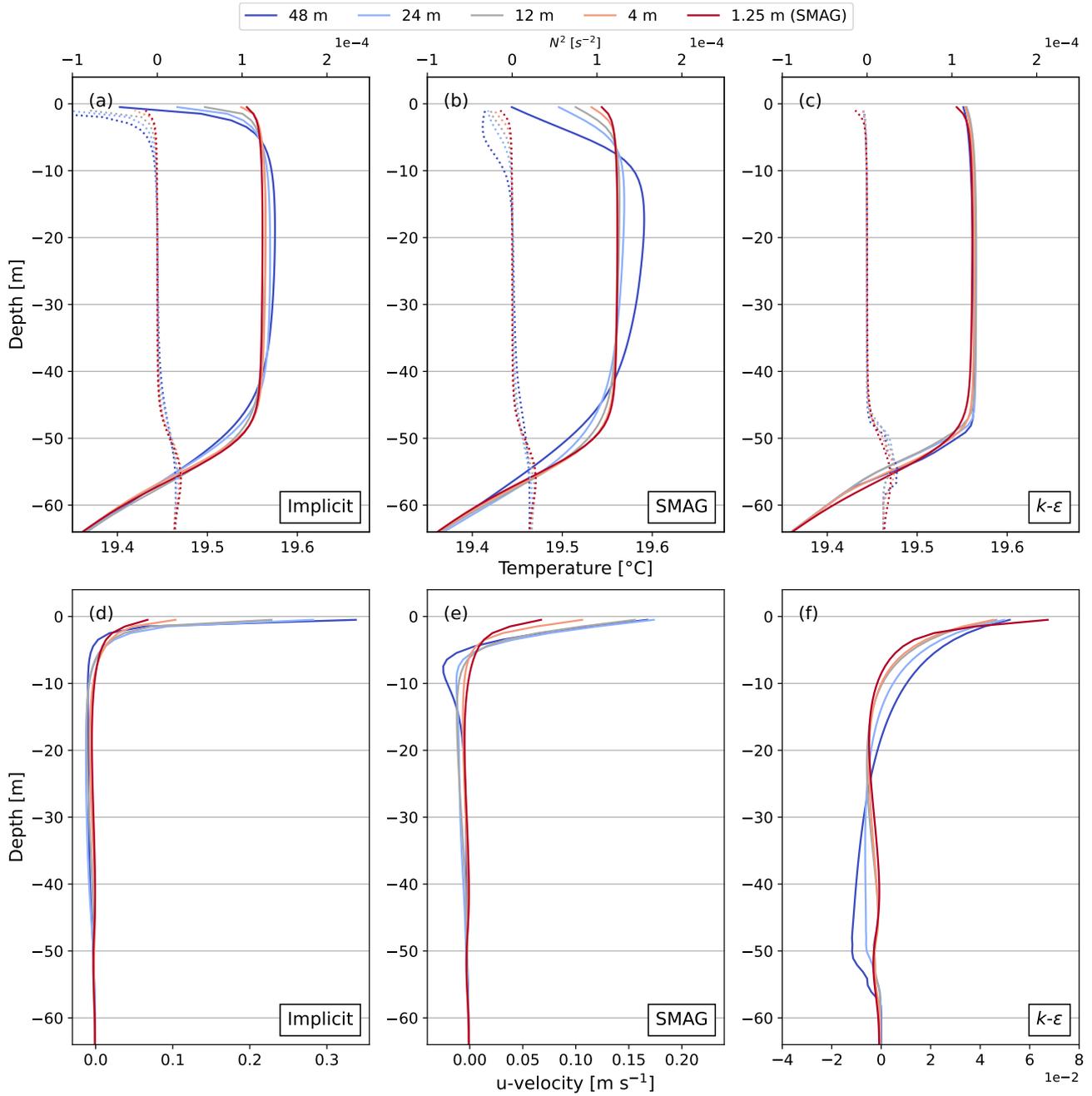


Figure S3. Averaged temperature and u-velocity profiles for the convection-shear case. Note the differences in x-axis scale in the bottom row.

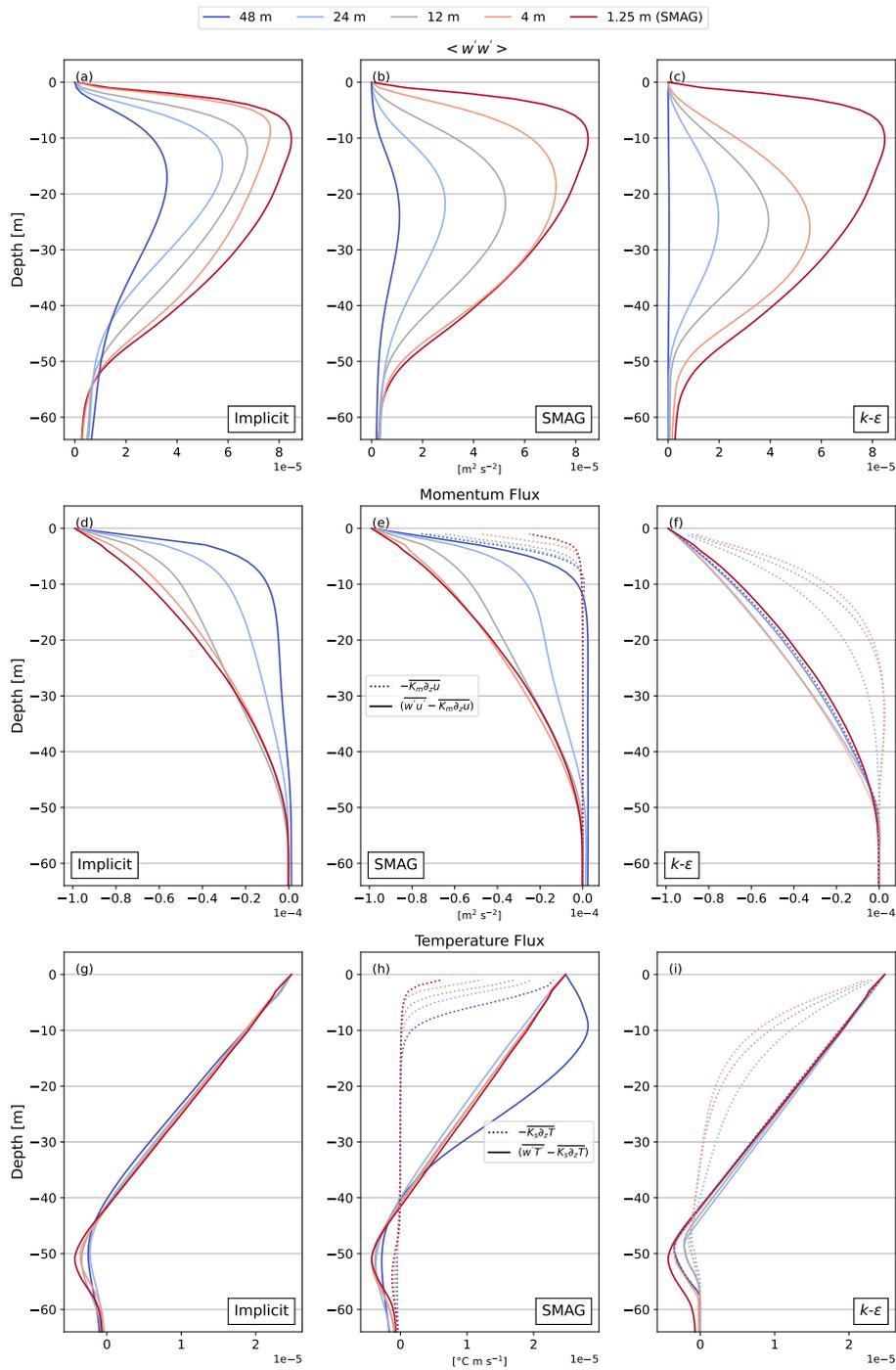


Figure S4. Averaged vertical velocity variance, together with momentum and temperature fluxes from the convection-shear case. In the flux profiles, the solid lines show the total flux and the dotted lines show the subgrid component.

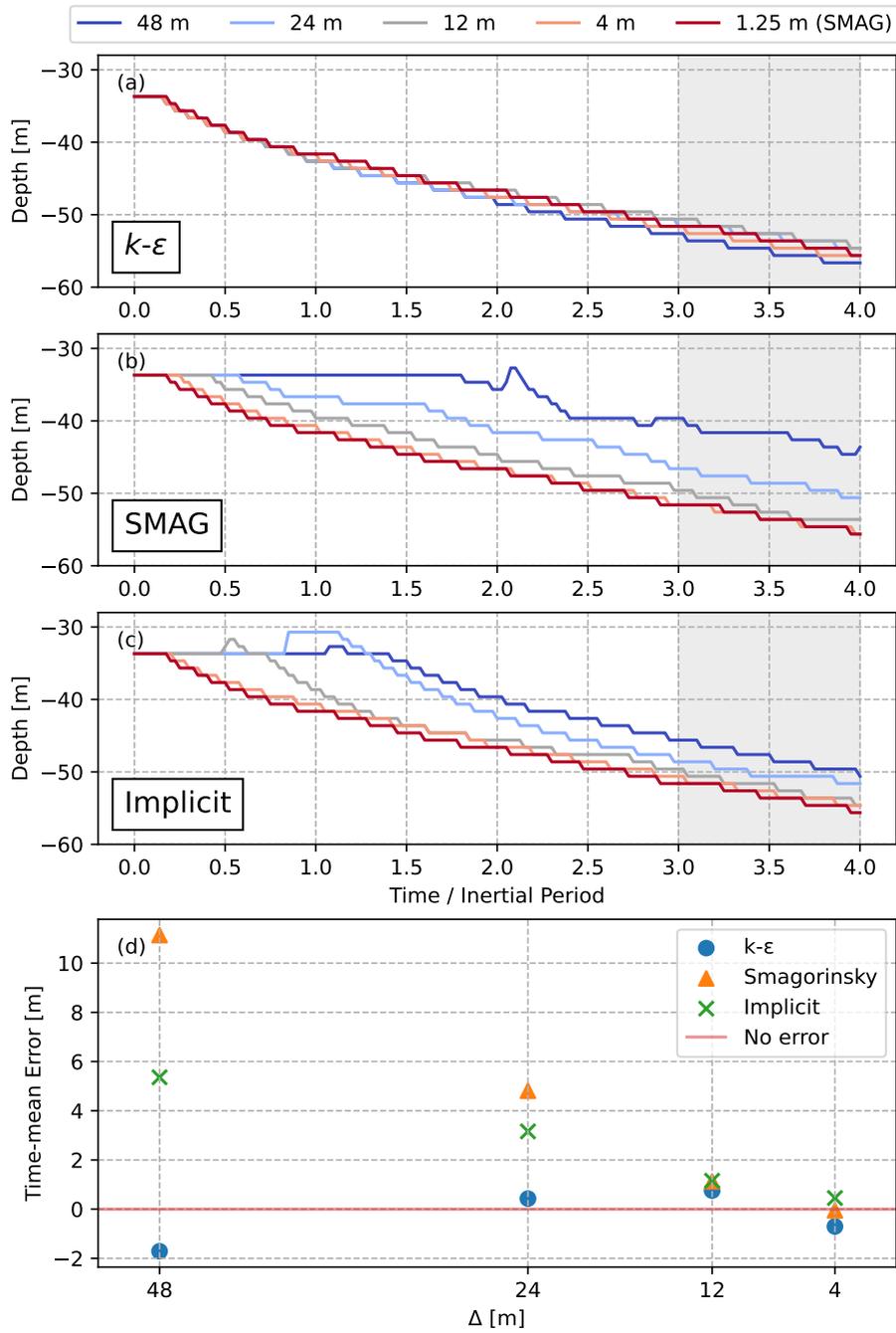


Figure S5. Time series MLD in the convection-shear case by different closures at the gray zone resolutions (Panel a-c). The mean error of the MLD time series relative to the baseline run in the last inertial period are shown in Panel d.

2.2. Wind-wave

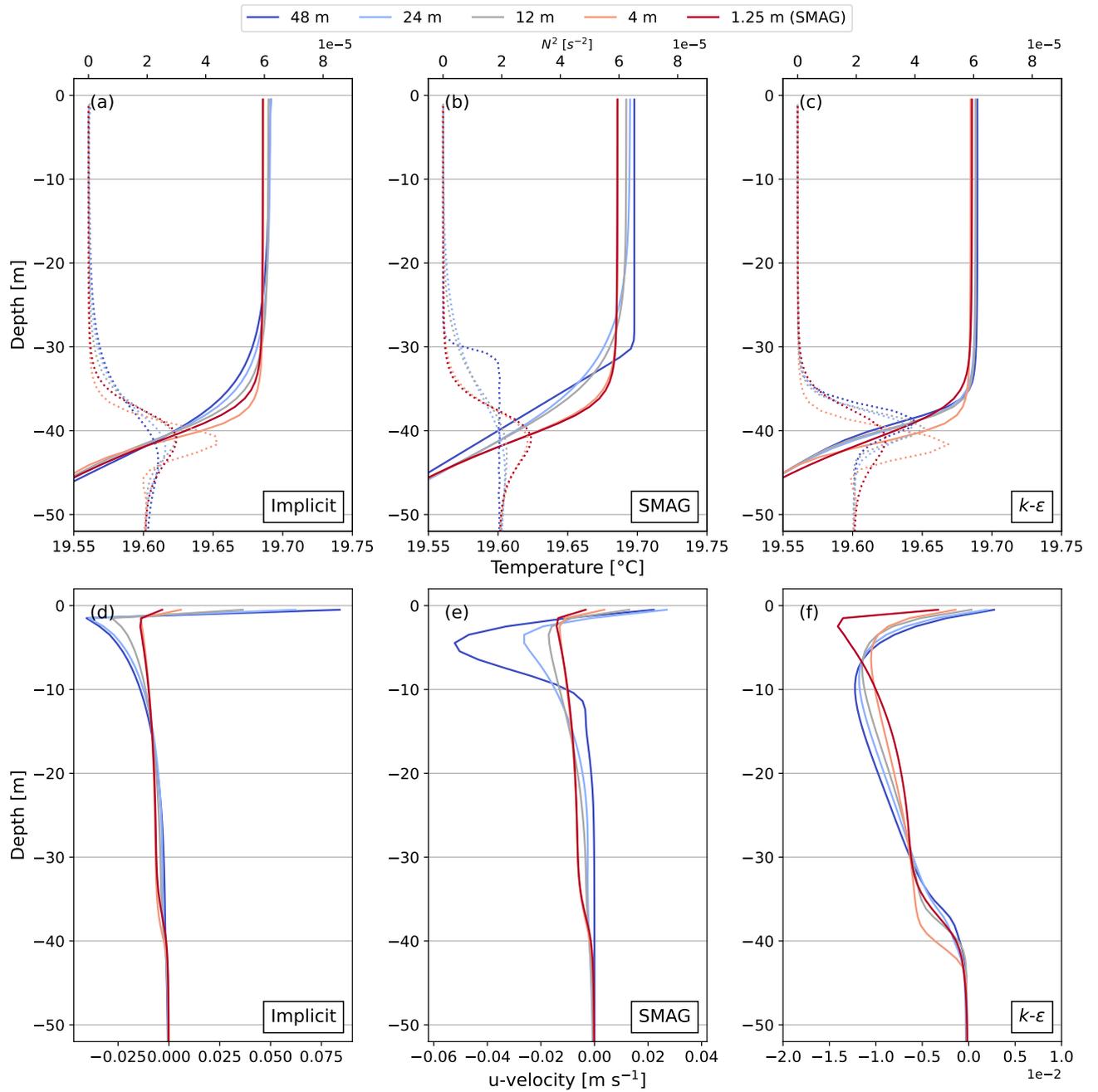


Figure S6. Averaged temperature and u-velocity profiles for the wind-wave case of Langmuir turbulence. Note the differences in x-axis scale in the bottom row.

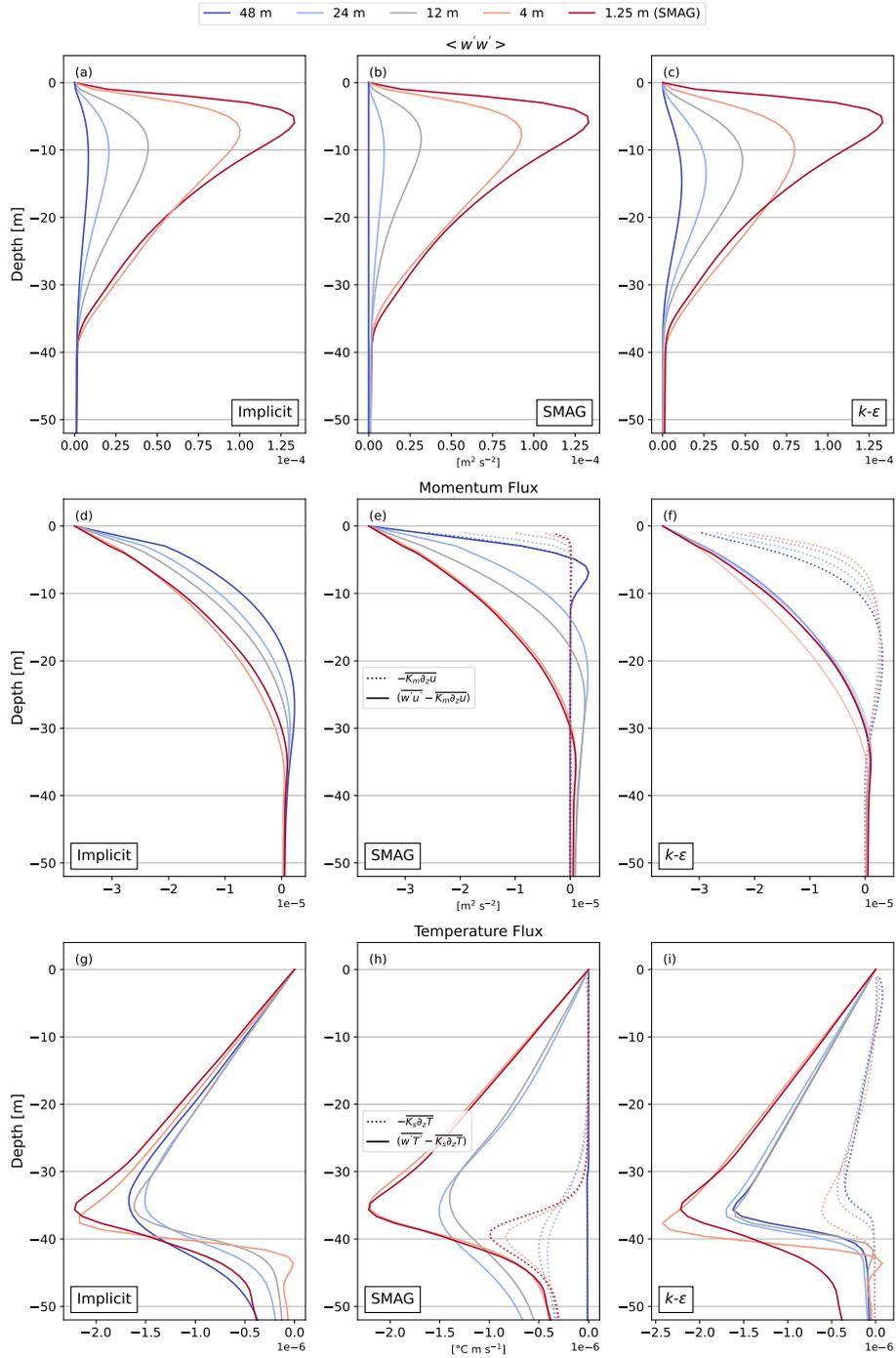


Figure S7. Averaged vertical velocity variance, together with momentum and temperature fluxes from the wind-wave case of Langmuir turbulence. In the flux profiles, the solid lines show the total flux and the dotted lines show the subgrid component.

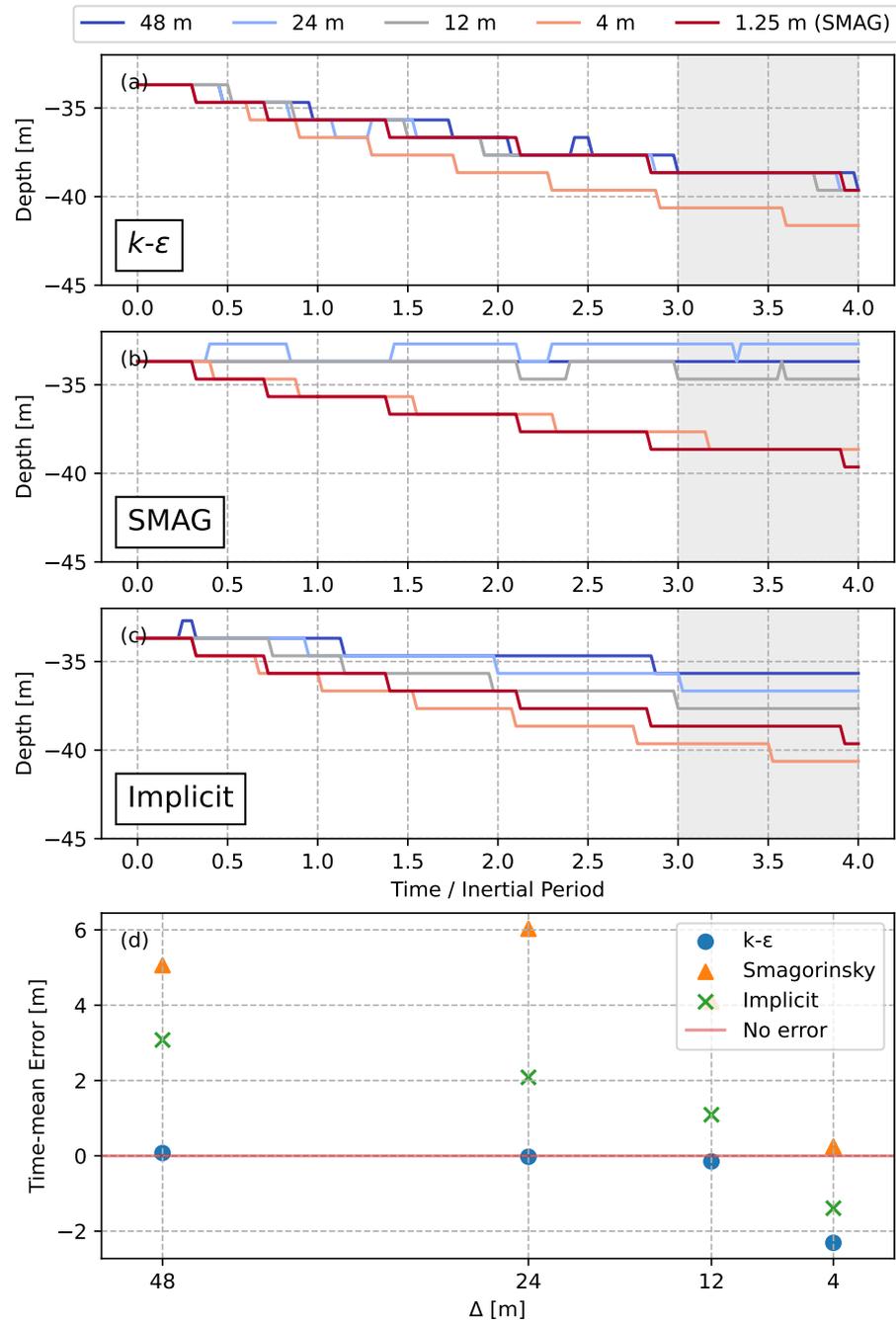


Figure S8. Time series MLD in the convection-shear case by different closures at the gray zone resolutions (Panel a-c). The mean error of the MLD time series relative to the baseline run in the last inertial period are shown in Panel d.

3. Turbulence onset

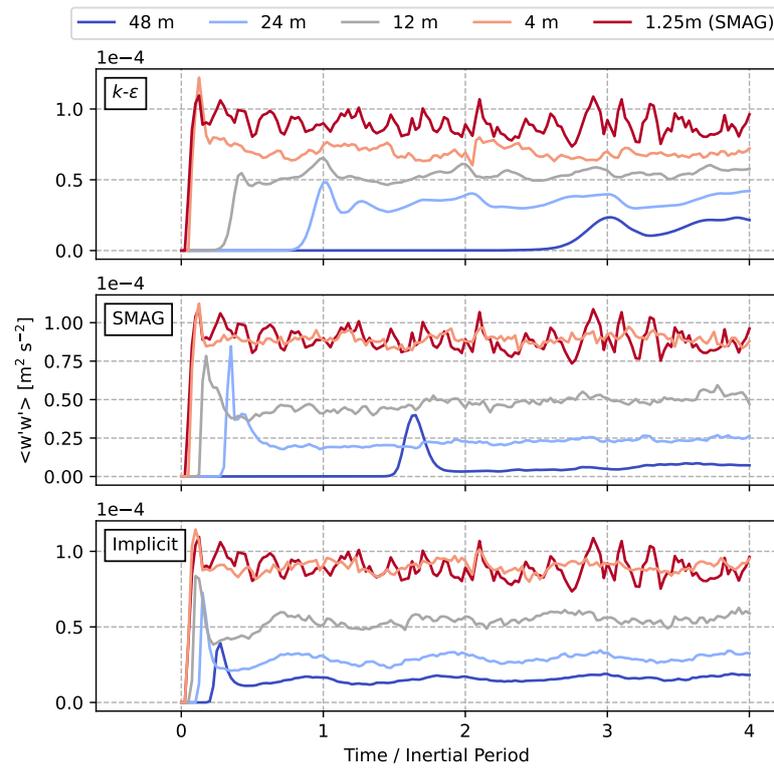


Figure S9. Time series of vertical velocity variance in the convection-wind-wave case. The variance is calculated as the horizontal average at the middle of the mixed layer depth, which evolves throughout the simulations.

4. Kinetic Energy Spectra

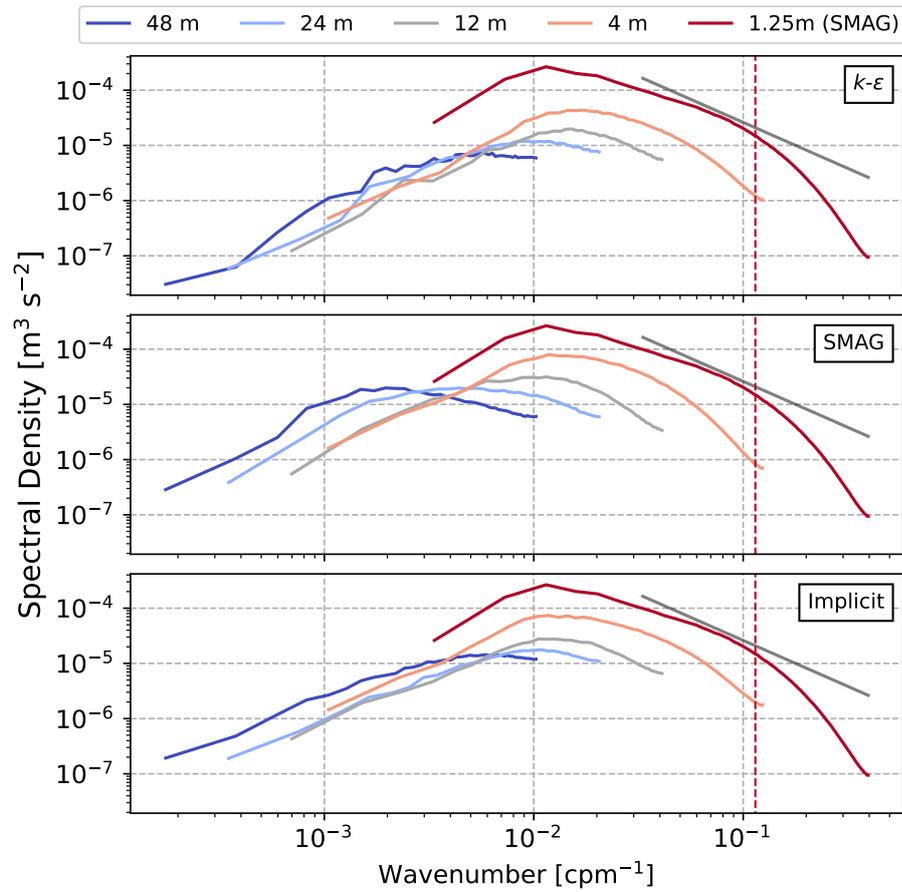


Figure S10. Turbulent kinetic energy spectra from the convection-only case at 20 m depth, derived from the 3D velocity fields and averaged over one inertial period. The red vertical line marks the range of numerical dissipation at 7Δ , and the characteristic $-5/3$ slope of the inertial subrange is shown in gray.

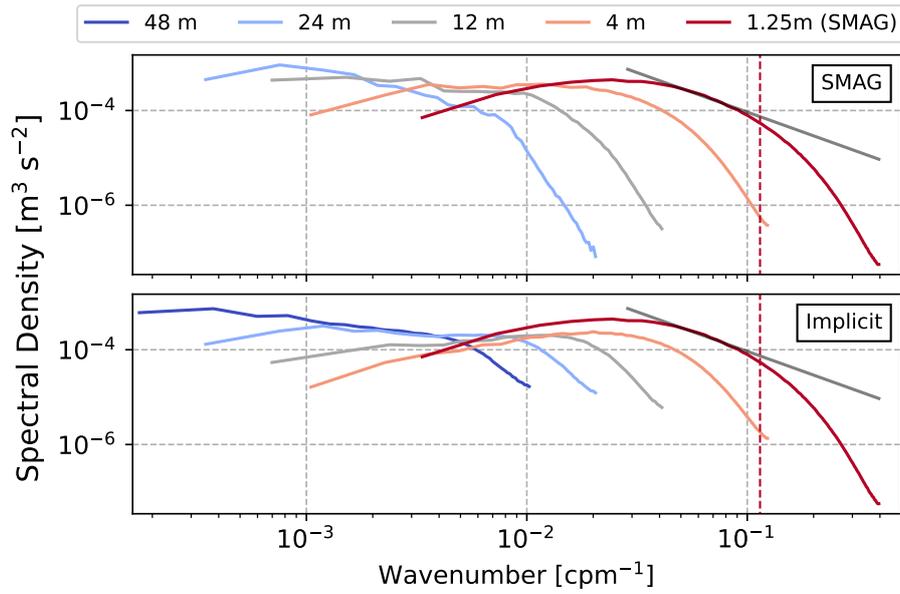


Figure S11. Turbulent kinetic energy spectra from the shear-only case at 10 m depth, derived from the 3D velocity fields and averaged over one inertial period. The red vertical line marks the range of numerical dissipation at 7Δ , and the characteristic $-5/3$ slope of the inertial subrange is shown in gray.

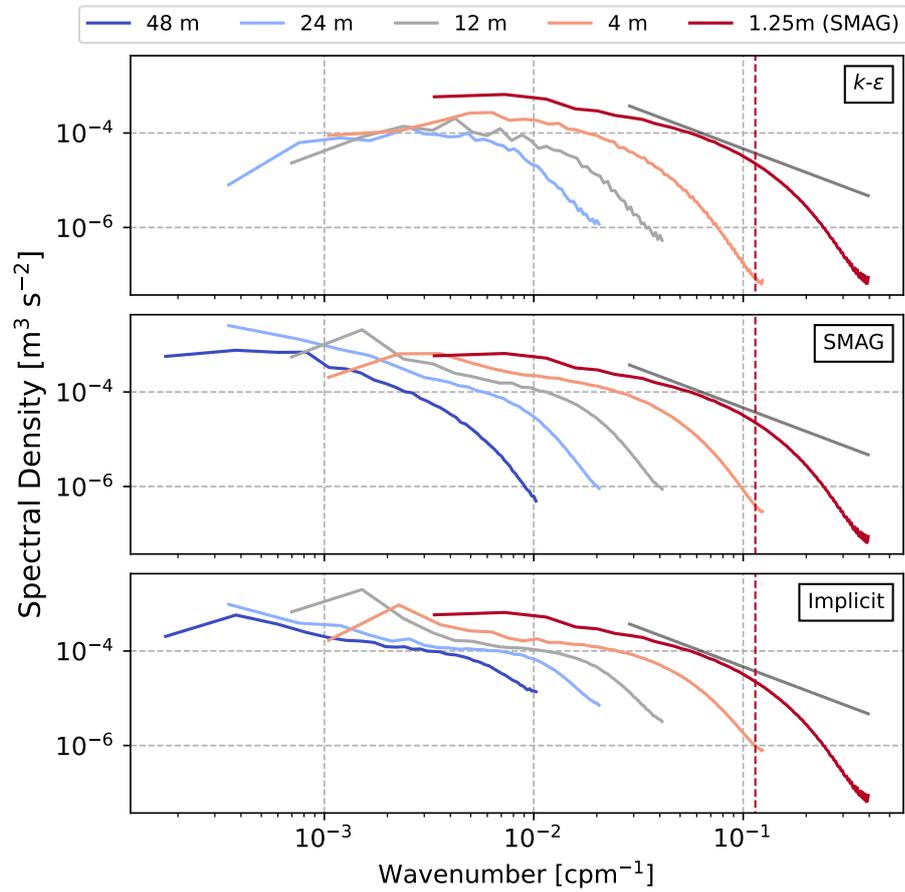


Figure S12. Turbulent kinetic energy spectra from the convection-shear case at 20 m depth, derived from the 3D velocity fields and averaged over one inertial period. The red vertical line marks the range of numerical dissipation at 7Δ , and the characteristic $-5/3$ slope of the inertial subrange is shown in gray.

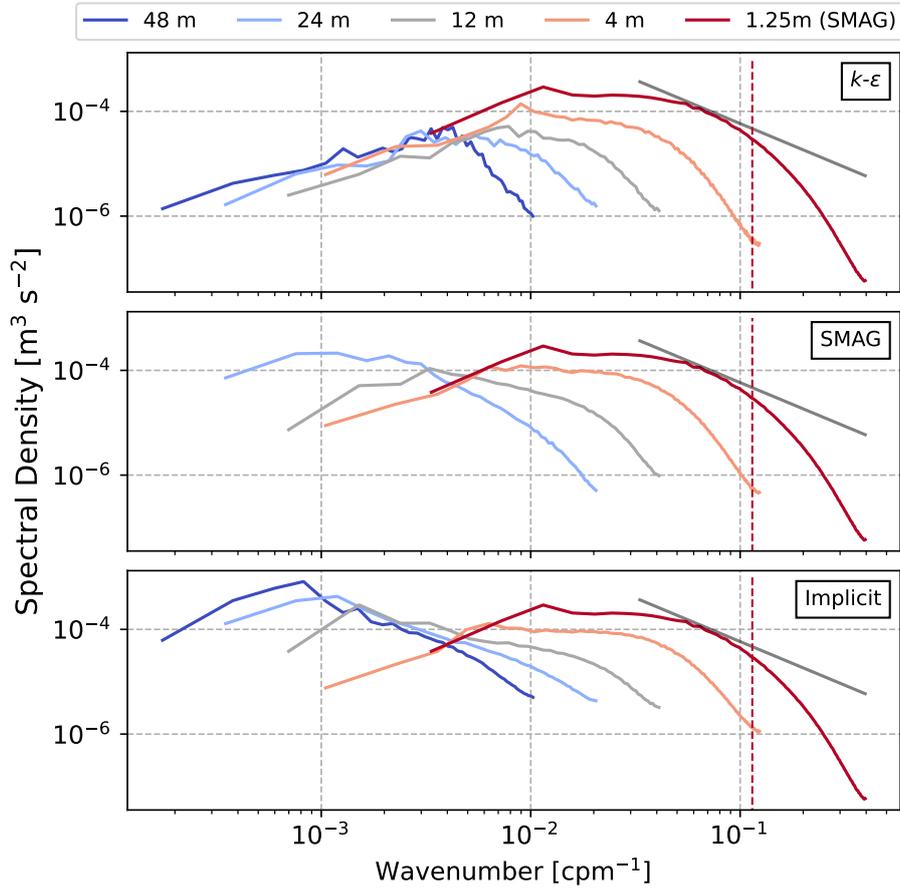


Figure S13. Turbulent kinetic energy spectra from the wind-wave case at 10 m depth, derived from the 3D velocity fields and averaged over one inertial period. The red vertical line marks the range of numerical dissipation at 7Δ , and the characteristic $-5/3$ slope of the inertial subrange is shown in gray.