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# Observations of Rayleigh and Love wave Azimuthal Anisotropy Across Alaska

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#### **Key Points:** 17

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- Rayleigh wave and Love wave  $2\psi$  and  $4\psi$  azimuthal anisotropy are observed from 10 to 50 s period based on ambient noise data.
- Love wave  $2\psi$  and Rayleigh wave  $4\psi$  with complementary trends in Rayleigh and Love 20 wave amplitudes are caused by Rayleigh-Love coupling. 21
  - The fast axes of various components of anisotropy are related, consistent with expectations for a tilted transversely isotropic medium.

#### 24 **Abstract**

- Using ambient noise data from 10 s to 50 s period across Alaska, we confirm previous estimates 25
- of Rayleigh wave  $2\psi$  azimuthal anisotropy and present the first estimates of Rayleigh wave  $4\psi$ 26
- 27 and Love wave  $2\psi$  and  $4\psi$  azimuthal anisotropy, where  $\psi$  is the angle of propagation. As in
- earlier studies, the fast axis orientations of Rayleigh wave  $2\psi$  are mainly parallel to major faults 28
- 29 in Alaska at all periods. We also find that on average the fast axis of Love wave  $4\psi$  is rotated  $45^{\circ}$
- 30 relative to Rayleigh wave  $2\psi$ , the fast axis of Rayleigh wave  $4\psi$  aligns with Love wave  $2\psi$ , and
- the fast axis differences of Rayleigh and Love wave  $2\psi$  range between 0 and 90 degrees with 31
- many between 40 to 60 degrees. These observations are consistent with non-elliptical anisotropy 32
- with the ellipticity parameters  $\eta_K$  and  $\eta_X$  considerably smaller than 1. Observations of Love 33
- wave  $2\psi$  and Rayleigh wave  $4\psi$  reflect strong Rayleigh-Love coupling, which causes the 34
- observed complementary trends with period of the amplitudes of Rayleigh and Love wave  $2\psi$ 35
- and Rayleigh and Love wave  $4\psi$ . Recent theories of Rayleigh-Love coupling based on a quasi-36
- degenerate theory allow these observations to be understood and to be used in the future to 37
- 38 improve models of the elastic tensor in the crust and mantle.

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### 1 Introduction

Because surface waves propagate horizontally, surface wave phase speed in anisotropic media depends on the azimuth of propagation  $\psi$ . Based on non-degenerate perturbation theory (or Rayleigh's Principle) in which Rayleigh and Love waves interact only weakly, Smith and Dahlen (1973) demonstrated that in a weakly anisotropic medium the azimuthal variation of Rayleigh and Love wave phase and group speeds at angular frequency  $\omega$  is of the form

$$c(\psi) = c_0 \left[ 1 + \frac{A_2}{2} \cos(2(\psi - \psi_2)) + \frac{A_4}{2} \cos 4(\psi - \psi_4) \right]$$
 (1)

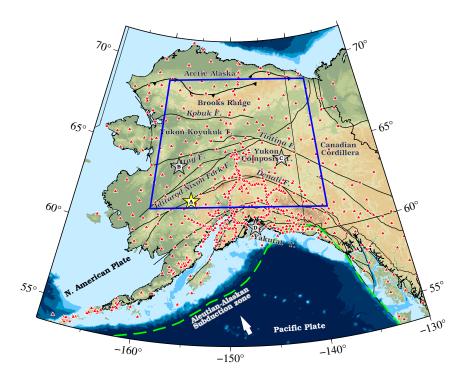
where  $c_0$  is the isotropic speed,  $\psi$  is measured clockwise from north,  $\psi_2$  and  $A_2$  are the fast axis and peak-to-peak amplitude for  $2\psi$ , and  $\psi_4$  and  $A_4$  are the fast axis and peak-to-peak amplitude for  $4\psi$ , respectively. They argued that the azimuthal dependence of Rayleigh wave speed will be dominated by the  $2\psi$  term in equation (1) whereas the Love wave speeds will be dominated by the  $4\psi$  term. Montagner and Nataf (1986) presented straightforward integral expressions so that observations of the frequency dependence of the coefficients in equation (1) can be used to invert for depth-dependent components of the elastic tensor. They also argued that fast axes for the  $2\psi$  terms for Rayleigh and Love waves should be out of phase by 90 degrees. The observation of odd-symmetry components (e.g.,  $1\psi$ ) has been explained by scattering or body-wave interference (Lin & Ritzwoller, 2011; Mauerberger et al., 2020; Zeng et al., 2024).

Based on these studies, focus has been placed on observing and interpreting the modes of anisotropy expected if Rayleigh and Love waves couple only weakly: the  $2\psi$  component of Rayleigh wave anisotropy and the harder to observe  $4\psi$  component of Love wave anisotropy. Many studies have presented and interpreted the  $2\psi$  component of Rayleigh wave anisotropy observed with earthquake data, dating back to the mid-1970s (e.g., Forsyth, 1975; Tanimoto & Anderson, 1985; Montagner and Jobert, 1988; Leveque et al., 1988; Nishimura and Forsyth, 1988, and many others). More recently, these observations have been expanded to include ambient noise observations at higher spatial resolution (e.g., Yao et al., 2010; Lin et al., 2011, and others) and full waveform inversion (e.g., Yuan & Romanowicz,, 2010; Zhu & Tromp 2013, and others). Inversions based on it have been performed to estimate apparent azimuthal anisotropy (Lin et al., 2011; Zhu et al., 2020) as well as inherent anisotropy represented by the elastic tensor (e.g., Xie et al., 2015; Xie et al., 2017; C. Liu & Ritzwoller 2024). Observations of the  $4\psi$  component of Love wave anisotropy are much more rare (e.g., Montagner & Tanimoto, 1990; Trampert and Woodhouse, 2003; Visser et al., 2008; Ekstrom, 2011; Yuan & Beghein 2014; Russell et al., 2019).

Less effort has been devoted to observing the  $2\psi$  variation of Love wave phase speeds and the  $4\psi$  component of the Rayleigh waves. Nevertheless, several studies have found that fundamental mode surface waves appear to possess both  $2\psi$  and  $4\psi$  variations with azimuth (e.g., Montagner & Tainimoto, 1990; Trampert & Woodhouse, 2003; Visser et al., 2008; Polat et al., 2012; Russell et al., 2019). Most of these studies have been performed on a global scale, whereas the local area study of Russell et al. (2019) was performed in a narrow period range (5 - 7.5 s). Recently, X. Liu and Ritzwoller (2025) show theoretically based on a quasi-degenerate theory that Love wave  $2\psi$  and Rayleigh wave  $4\psi$  anisotropy, which are unexpected based on non-degnerate perturbation theory, are expected when Rayleigh-Love coupling is strong and discuss the nature of the anisotropy that will produce such coupling.

In this study, we use data from the USArray Transportable Array (TA) and regional networks across Alaska (**Fig. 1**) to investigate surface wave anisotropy. Other studies have investigated surface wave anisotropy across Alaska before this study, but have focused on Rayleigh wave  $2\psi$  (e.g., Wang & Tape, 2014; Feng et al., 2020; Liu et al., 2022; C. Liu et al., 2024; Liu et al., 2025). We focus on investigating the existence and nature of anisotropy previously not considered, which includes Love wave  $4\psi$  and anisotropy previously considered to be unexpected: Love wave  $2\psi$  and Rayleigh wave  $4\psi$  anisotropy.

We ask four principal questions for Alaska. (1) Are the expected components of azimuthal anisotropy (Rayleigh wave  $2\psi$  and Love wave  $4\psi$ ) observed? (2) Are the unexpected components of azimuthal anisotropy (Love wave  $2\psi$  and Rayleigh wave  $4\psi$ ) observed? (3) Are the fast directions of some of these observables related; in particular are the fast axes for the  $2\psi$  components of Rayleigh and Love waves rotated by 90 degrees relative to each other? (4) Do the amplitudes of Rayleigh and Love wave azimuthal anisotropy vary as a function of period consistent with the existence of Rayleigh-Love coupling? We focus on a discussion of the observations. A discussion of their meaning for earth structure will be the subject of a later contribution.

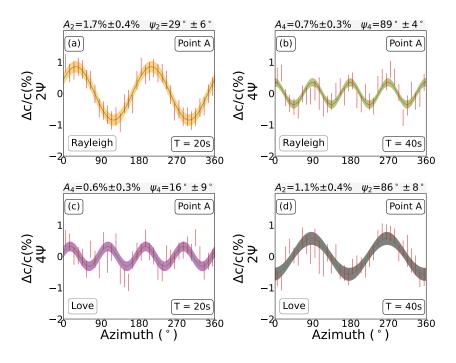


**Figure 1.** Seismic stations used in the study are shown with red triangles, the yellow star is Point A referred to in **Figure 2**, the gray stars are Point B, C, and D referred to in **Figure 6**, and the blue rectangle is the region used to compute the average amplitude of anisotropy in **Figure 4**. Black lines are major faults.

### 2 Data and Methods

We use the ambient noise database constructed by Liu et al. (2022), including Rayleigh and Love waves from 10 to 50 s, as input for the tomography and subsequent analysis. This database is based on both traditional two-station ambient noise interferometry (Bensen et al., 2007) and the more recently developed three-station interferometry method (Zhang et al., 2020, 2021). The tomography and observational methods are discussed in section S1 of Supplementary Materials and methods to estimate uncertainty are discussed in section S4. These methods are similar to those applied by Liu et al. (2022), but here we use 36 azimuthal bins rather than 18.

Examples of measurements of the azimuthal variation of Rayleigh and Love wave phase speed at periods of 20 s and 40 s for a location in western Alaska near the Denali fault (Point A of Figure 1) are shown in Figure 2. At 20 s period, anisotropy is similar to what is expected in the absence of Rayleigh-Love coupling: the Rayleigh wave mainly displays  $2\psi$  anisotropy (Figure 2a) and the Love wave mainly  $4\psi$  anisotropy (Figure 2c). However, at longer periods (e.g., 40 s), which are more sensitive to the mantle, the Rayleigh wave mainly shows  $4\psi$  anisotropy (Figure 2b) and the Love wave mainly  $2\psi$  anisotropy (Figure 2d). This result is an example of the effect of Rayleigh-Love coupling through anisotropy as discussed by X. Liu and Ritzwoller (2025).



**Figure 2.** Examples of measurements of Rayleigh and Love anisotropy in western Alaska near the Denali fault, Location A in **Figure 1**. The left column is at 20 s period, the right column is at 40 s, the top row is for Rayleigh waves, and the bottom row is for Love waves. The red bars are the  $2\psi$  or  $4\psi$  components of observations, whichever is dominant, displaying one standard-deviation of the mean in each of 36 azimuthal bins. The estimated amplitude and fast axis of the dominant component of anisotropy are shown at the top of each panel. Each shaded corridor

- represents the one standard deviation uncertainty in the estimated phase speed. More complete
- observations at Point A are shown in Figures S1-S5.

#### 3 Results and Discussion

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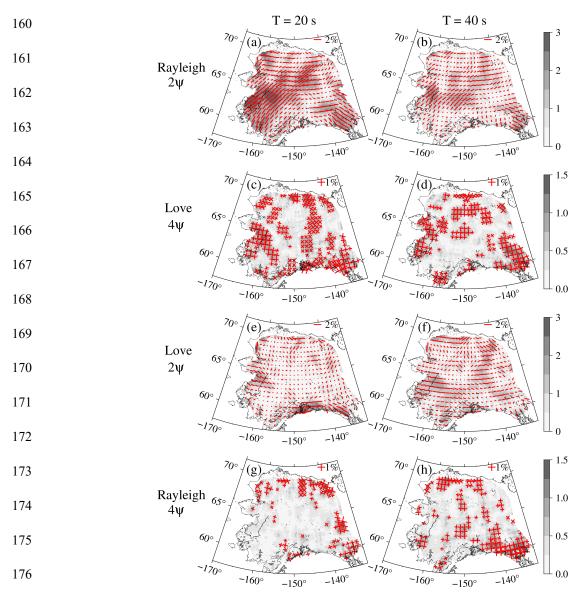
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- Our results for azimuthal anisotropy are summarized in **Figure 3** at two different periods, one
- that is mainly sensitive to the crust (20 s) and the other that is principally sensitive to the
- uppermost mantle (40 s). Expected anisotropy (Rayleigh wave  $2\psi$  and Love wave  $4\psi$ ) is
- presented in the first two rows and unexpected anisotropy (Love wave  $2\psi$  and Rayleigh wave
- 140  $4\psi$ ) appears in the bottom two rows. Results at other periods are also shown in the
- Supplementary Materials (Figure S6). Maps of the uncertainty in the isotropic phase speed, fast
- 142 axis orientations, amplitude, and the amplitude of each component of anisotropy normalized by
- its uncertainy are shown in the Supplementary Materials (Figures S9-S22). Here, by 'amplitude'
- we mean the peak-to-peak amplitude ( $A_2$  and  $A_4$  in equation (1)). For a general anisotropic
- medium with 21 independent elastic components, the fast axes of different signals (e.g., Rayleigh
- wave  $2\psi$ , Love wave  $2\psi$ , etc) can have any relationship because they are determined by different
- independent elastic parameters (X. Liu & Ritzwoller, 2025). However, in a tilted transversely
- isotropic (TTI) medium the fast axes for different signals may have a specific relationship. We
- focus on discussing observational results in light of these expectations for a TTI medium.

## 3.1 Expected anisotropy: Rayleigh wave $2\psi$ and Love wave $4\psi$

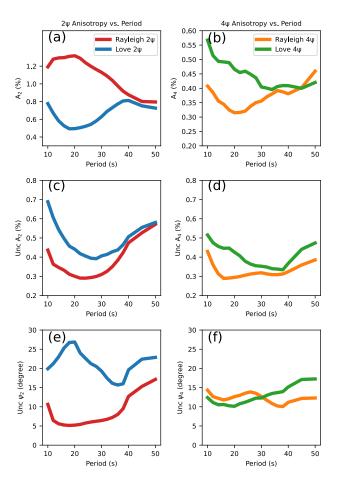
- "Expected anisotropy" dominates when Rayleigh-Love coupling is weak. The expected
- anisotropy for Rayleigh waves and Love waves closely mirrors the behavior of SV and SH
- waves, which display  $2\psi$  and  $4\psi$ , respectively, in the absence of SV-SH coupling (Backus,
- 154 1965). Rayleigh wave  $2\psi$  anisotropy has been observed in many studies but Love wave  $4\psi$  has
- presented a more significant observational challenge. Here, we first discuss these expected
- signals across Alaska, summarized in **Figure 3a-d** at two periods and **Figure S6** at other periods.
- We address questions #1 and #3 that motivate this study, notably whether Rayleigh wave  $2\psi$  and

Love wave  $4\psi$  anisotropy are observed across Alaska and if the fast axis orientations of these two components are related.



**Figure 3.** The amplitude and fast axis of Rayleigh (rows 1 and 4) and Love (rows 2 and 3) wave  $2\psi$  and  $4\psi$  azimuthal anisotropy at periods of 20 s (left column) and 40 s (right column). The red bars indicate the fast axis for  $2\psi$  or fast axes for  $4\psi$ , with length proportional to amplitude as shown in each panel. The background grey-shade is the amplitude of the specified component of anisotropy, with units of percent of isotropic phase speed at each location. To reduce clutter, red

crosses are shown for  $4\psi$  anisotropy only when the amplitude is larger than 0.5%, which are the major signals.



**Figure 4.** (a)-(b) Spatial average of the amplitude of Rayleigh and Love wave  $2\psi$  and  $4\psi$  compenents of anisotropy, computed within the blue rectangular box shown in **Figure 1** where amplitudes are most reliable. (c)-(f) Uncertainty in the amplitude and fast azimuth orientations of the  $2\psi$  and  $4\psi$  components also computed in the blue box but with an amplitude cutoff: the uncertainty is computed only where the amplitude exceeds the spatial average.

### 3.1.1 Rayleigh wave $2\psi$

After simultaneously resolving the  $2\psi$  and  $4\psi$  components for the Rayleigh wave, our Rayleigh wave  $2\psi$  results (**Fig. 3a,b**) remain very similar to those of Liu et al. (2022). The average amplitude of Rayleigh wave  $2\psi$  (**Figure 4a**) diminishes with period from about 1.3% near 20 s period to about 0.8% at 40s and longer periods. This reduction of amplitude with period is shown for point A in **Figures S1-S5** and it is also visually apparent in **Figure 3a,b**. In contrast, the average uncertainty (**Fig. 4c**) typically grows with period from about 0.3-0.4% at shorter periods

- 197 to about 0.5% at 40 s and above. The uncertainties, therefore, are well below the average
- amplitudes at all periods (Figure S13). 198
- The fast axis of Rayleigh wave  $2\psi$  mainly parallels major faults, which suggests that the 199
- potential cause of this strong anisotropy is fractures and cracks in the crust (e.g., Feng et al., 200
- 2020). This relatively simple fault-parallel pattern plays an important role in the comparison of 201
- Rayleigh wave to Love wave azimuthal anisotropy later in section 3.2. The uncertainty in the fast 202
- axis direction for Rayleigh wave  $2\psi$  increases with period as the amplitude decreases, with an 203
- average of about 5 degrees at 20 s period and 15 degrees at 50 s period. 204

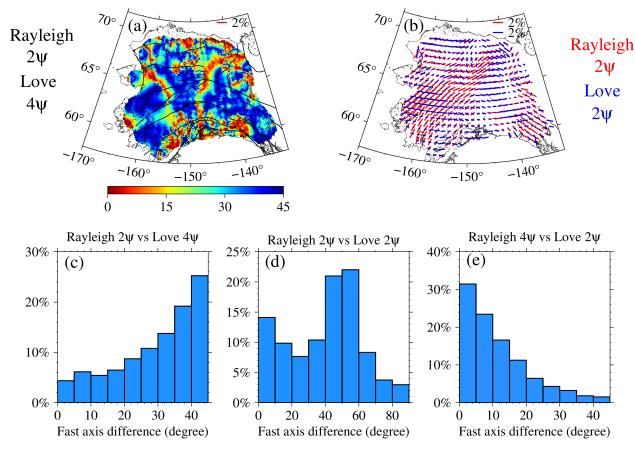
## 3.1.2 Love wave $4\psi$

- A principal novelty of this study is the clear observation of Love wave  $4\psi$  anisotropy over large 206
- parts of Alaska (Figure3c,d), for example Point A highlighted in Figure 2 as well as Figures 207
- S1-S5. Unlike  $2\psi$ , which has two fast directions (a single bar),  $4\psi$  exhibits four fast directions 208
- (two bars). As shown in **Figure 4a,b**, the amplitude of Love wave  $4\psi$  is smaller than the 209
- 210 Rayleigh wave  $2\psi$  at all periods. Like Rayleigh  $2\psi$ , it diminishes with period but more weakly,
- from an average of about 0.5% at shorter periods to 0.4% at longer periods. The smaller 211
- amplitude of the Love wave  $4\psi$ , particularly at longer periods, poses a challenge to its 212
- observation, in addition to the greater azimuthal resolution needed to observe it reliably. The 213
- average uncertainty (Fig. 4d) is relatively flat with period, averaging between 0.4-0.5%. The 214
- average amplitude of the Love wave  $4\psi$  lies closer to but above the uncertainty level on average. 215
- The uncertainty normalized amplitude across Alaska is in the supplementary material (Figure 216
- 217 **S16**).

- Largely due to limitations in azimuthal resolution, previous studies have not discussed the fast 218
- axis of Love wave  $4\psi$  or its comparison to Rayleigh wave  $2\psi$ . The uncertainty of the fast axis 219
- orientaitons for Love wave  $4\psi$  grows sightly with period from about 10 degrees to 15 degrees 220
- 221 (Figure 4f) as its average amplitude decreases (Figure 4b).
- Figure 5a presents the difference between the fast axes of Love wave  $4\psi$  (Figure 3c,d) at 20 s 222
- period and Rayleigh wave  $2\psi$  (Figure 3a,b) at 14 s period, and Figure 5c summarizes this 223
- difference across Alaska with a histogram. These periods are chosen so that the sensitivity 224
- 225 kernels with depth are relatively similar, both being confined to the crust. We find that the mode
- of the difference is about 45° (Figure 4c). Figure S7 presents another example histogram but for 226
- different periods (14 s for Rayleigh, 10 s for Love), which illustrates the fast axis differences 227
- accumulating near 45 degrees even more clearly. As discussed by X. Liu and Ritzwoller (2025), 228
- 229 observation of a near 45° fast axis difference indicates that the elastic tensor for a TI medium is
- tilted and the ellipticity parameter  $\eta_X < 1$  in the crust or at least in the upper crust.  $\eta_X =$ 230
- 4L/(A+C-2F), where A, C, L, and F are inherent Love moduli, and  $\eta_X \approx \eta_K$  of Kawakatsu 231
- (2016). Exceptions (red colors in Figure 5a) lie near the northern and southern boundaries or 232
- where the amplitude of  $4\psi$  is small. Locations where we do not observe the  $45^{\circ}$  difference is 233
- often associated with larger measurement uncertainty (e.g., Figure S14a). However, there are 234
- some regions where Rayleigh wave  $2\psi$  and Love wave  $4\psi$  are quasi-parallel to each other, both 235
- with large amplitudes, indicating that the ellipticity parameter  $\eta_X > 1$ . An example is Location A 236
- 237 in western Alaska, with data shown in Figures 2 and S1-S5. Further discussion of the ellipticity

parameter, such as its application in receiver function anisotropy (e.g., Schulte-Pelkum et al., 2020, 2023), is beyond the scope of this paper.





**Figure 5.** Comparison of fast axes observations. Histograms are computed where the amplitude of the  $2\psi$  component is greater than 0.5%, the amplitude of the  $4\psi$  component is greater than 0.3%, the fast axis uncertainty of the  $2\psi$  component is less than 20 degress, and the fast axis uncertainty of the  $4\psi$  component is less than 15 degress. (a),(c) Comparison is between Rayleigh wave  $2\psi$  at 14 s period and Love wave  $4\psi$  at 20 s period. (b),(d) Comparison is between Rayleigh wave  $2\psi$  at 20 s period and Love wave  $2\psi$  at 40 s period. (e) Comparison is between the Rayleigh wave  $4\psi$  at 45 s period and Love wave  $2\psi$  at 40 s period.

## 3.2 Unexpected anisotropy: Love wave $2\psi$ and Rayleigh wave $4\psi$

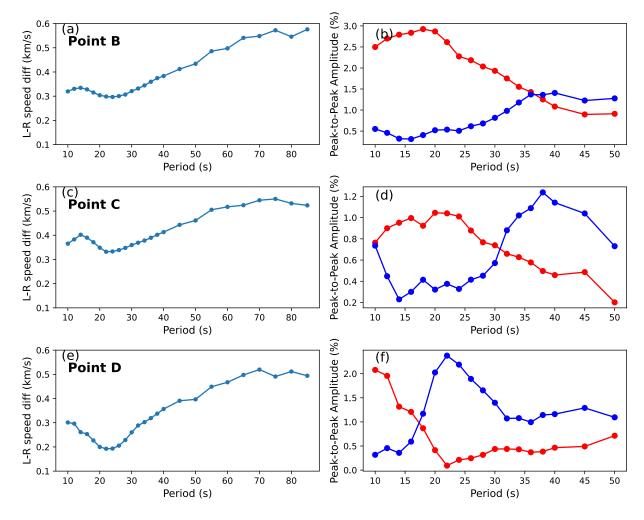
As shown in **Figures 2** and **3**, in addition to the expected anisotropy, at some places and certain periods we observe Love wave  $2\psi$  and Rayleigh wave  $4\psi$  anisotropy, which are unexpected without Rayleigh-Love coupling. Here, we discuss the observation of those unexpected signals across Alaska, which are more prominent at the mantle-sensitive longer periods but also observed at the crust-sensitive shorter periods (e.g. **Figure 6f**). We address questions #2, #3, and #4 that motivate this study, notably whether Rayleigh wave  $4\psi$  and Love wave  $2\psi$  anisotropy are observed across Alaska, if the fast axis orientations of Love wave  $2\psi$ 

- and Rayleigh wave  $2\psi$  and  $4\psi$  anisotropy are related, and whether the amplitudes of Rayleigh
- and Love wave anisotropy vary with period consistent with Rayleigh-Love coupling.

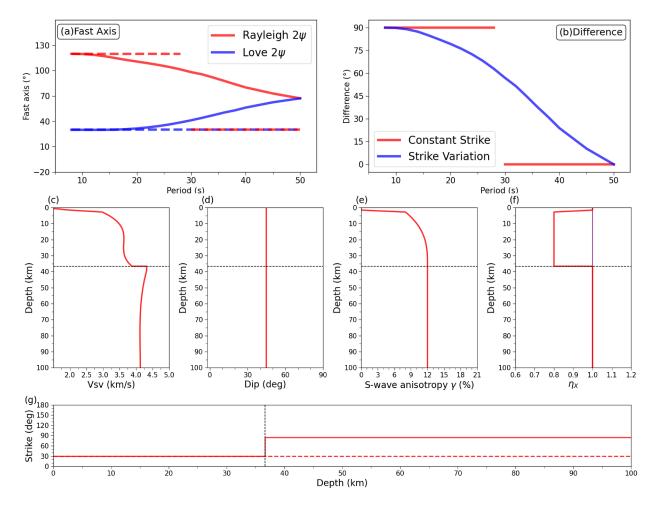
## 259 **3.2.1** Love wave $2\psi$

- The principal result of this study is the observation of Love wave  $2\psi$  anisotropy, depicted in
- Figure 3e,f and S6g-i. The amplitude of this signal on average grows with period from about
- 262 0.5% at 20 s period to 0.8% at 40 s period (Figure 4a). The uncertainty (Fig. 4c) varies with
- period, but averages between 0.4-0.6%, so the signal is typically larger than the uncertainty at
- 264 most points. The uncertainty normalized amplitude across Alaska is shown the supplementary
- 265 material (Figure S19).
- Figures 4a and 3a,b,e,f show that the amplitude of Love wave  $2\psi$  varies with period
- 267 approximately opposite from Rayleigh wave  $2\psi$ . A detailed local comparison of the amplitudes
- of Love and Rayleigh wave  $2\psi$  is shown in **Figure 6** for three points in Alaska (Points B, C, and
- D identified in Figure 1). These three points exemplify the spatially averaged statistics
- 270 presented in **Figure** 4a, that the Love wave  $2\psi$  amplitude increases and the Rayleigh wave  $2\psi$
- amplitude decreases with period (**Figure 6**b,d,f) which we infer to result of Rayleigh-Love
- coupling. For Points B and C, Rayleigh-Love coupling is mainly caused by anisotropy in the
- 273 mantle. For Point D, when the isotropic phase speed difference minimizes between 20 and 30 s
- period (**Figure 6**e), the amplitude of Rayleigh wave  $2\psi$  decreases rapidly while the amplitude of
- Love wave  $2\psi$  increases instead (**Figure 6**f). X. Liu and Ritzwoller (2025) discuss that this is the
- hallmark of Rayleigh-Love coupling, where the energy of the Rayleigh wave, which is strong at
- shorter periods, is transmitted to the Love wave through anisotropy coupling at the longer
- periods. The phase speeds of the Rayleigh and Love waves at Point D are similar enough
- between 20 and 30 s period (**Figure 6e**) that a small inherent anisotropy around  $4\% \sim 6\%$  in the
- crust can cause strong Rayleigh-Love coupling and the large observed amplitude of Love wave
- $2\psi$  (**Figure 6f**). Details about the inversion for the depth varying elastic-tensor at these points
- will be presented in a later contribution.
- These points are examples of common observations across Alaska (**Figure 4**a). For some regions
- in Alaska, the increase of Love wave  $2\psi$  amplitude is not accompanied with a decrease of
- Rayleigh wave  $2\psi$  amplitude and they may both increase with period. This may also result from

Rayleigh-Love coupling as explained in X. Liu & Ritzwoller (2025). Nevertheless, we find that on average their amplitudes are complementary across Alaska (**Figure** 4a).



**Figure 6.** Observational examples of the effect of Rayleigh-Love coupling at three points identified in Figure 1: Point B (first row), Point C (second row), and Point D (third row). (a),(c),(e) The phase speed difference between the Rayleigh wave and Love wave at periods of 10 s - 85 s. (b),(d),(f) The Rayleigh wave  $2\psi$  amplitude (red lines) and the Love wave  $2\psi$  amplitude (blue lines) from 10 s - 50 s period. The dots are our estimated quantities. The isotropic phase speed differences larger than 50 s are based on dispersion measurements from earthquakes taken from Liu et al. (2022).



**Figure 7.**Synthetic test of the effect of strike variation with depth on fast-axes. Two models, aspects of which are shown in (c)-(g), differ only in the strike angle of anisotropy. (a) Fast axes predictions (X. Liu & Ritzwoller, 2025) for Rayleigh  $2\psi$  (red line) and Love  $2\psi$  (blue line) using the two models: solid line (variable stike with depth) and dashed line (constant strike with depth). (b) The fast axis difference between Rayleigh  $2\psi$  and Love  $2\psi$  for the two models of strike variation with depth. (c-g) Aspects of the model used to produce the synthetic results: (c)  $Vsv = \sqrt{L/\rho}$ , (d) dip angle, (e) radial anisotropy  $\gamma = (N-L)/2L$ , (f) the ellipiticity parameter  $\eta_X$ , and (g) strike angle (solid line constant strike, dashed line variable strike), where L and N are the inherent Love shear moduli. Details about the definitions can be found in Xie et al. (2015) and X. Liu & Ritzwoller (2025).

As with all of the fast axis measurements, the Love wave  $2\psi$  fast axis uncertainty (e.g., **Figure S17**c) is strongly anti-correlated with its amplitude (e.g., **Figure 3f**); i.e., when the amplitude is high the fast axis uncertainty falls. The fast axis uncertainty for Love wave  $2\psi$  lies between 15 and 25 degrees, which is considerably larger than the uncertainty for the Rayleigh wave  $2\psi$ . This partially reflects the higher noise level on the T-T versus Z-Z components of ambient noise cross-correlations, but also that the Rayleigh  $2\psi$  has a larger amplitude, particularly at shorter periods. Love wave  $2\psi$  fast axis uncertainties are larger than those of Love wave  $4\psi$  because its

- azimuthal wavelength is longer by a factor of two (180 degrees versus 90 degrees), so relative
- uncertainties are about the same.
- Figure 5b overplots the Rayleigh and Love wave  $2\psi$  fast axes at 20 s and 40 s period,
- respectively. We choose 20 s rather than 40 s for comparison because the Rayleigh wave  $2\psi$
- amplitude is stronger at 20s period, but its fast axis directions change little between 20 s (**Figure**
- 3a) and 40 s (Figure 3b). In some places in Alaska, particularly in eastern Alaska as Figure 5b
- illustrates, the fast axes are approximately parallel to one another, but more commonly they
- differ by an angle between 40 and 60 degrees, which occurs across much of western Alaska,
- which is also reflected in the histogram shown in **Figure 5d**. The relationship between these fast
- directions is not simple, and we turn to theory to explain the observations.
- For very weak Rayleigh-Love coupling, Montagner and Nataf (1986) argued that the  $2\psi$  fast
- axes for Rayleigh and Love waves should be out of phase by about 90 degrees when they are
- sensitive to similar depths. Thus, absent strong Rayleigh-Love coupling, the Rayleigh wave  $2\psi$
- fast axes would be nearly perpendicular to the Love wave  $2\psi$  fast axes. With strong Rayleigh-
- Love coupling, however, X. Liu and Ritzwoller (2025) showed that when the strike of anisotropy
- is constant with depth, the Love wave  $2\psi$  fast axes should align with the strike of anisotropy
- whereas the Rayleigh  $2\psi$  fast axes could align with the strike or be perpendicular to it. Thus,
- angle difference between the Rayleigh and Love wave  $2\psi$  fast axes could be either 0 degrees
- 332 (parallel) or 90 degrees (perpendicular) if the strike of anisotropy is depth-invariant. In fact, we
- see neither the bimodal distribution (0 or 90 degrees) of fast axis differences predicted if
- Rayleigh-Love coupling is strong an strike is depth-invariant nor the single 90 degree difference
- predicted with weak Rayleigh-Love coupling (**Figure 5d**). We believe the reason for this is due
- to a significant variation in the strike of anisotropy with depth from the the crust to the mantle.
- To illustrate the effect of a strike variation with depth we use two synthetic TTI models (**Figure**
- 338 7c-g), one with a constrant stike angle with depth and another where the strike angle differs in
- the crust and mantle (Figure 7g). Other aspects of the models are the same, they have a depth-
- constant dip angle (45°, **Figure 7d**), an almost constant radial anisotropy (12%, **Figure 7e**), but
- different  $\eta_X$  in the crust and mantle (**Figure 7f**). Both have a strike angle of 30° in the crust, but
- one continues that strike angle in the mantle and the other has a strike angle of 85° in the mantle.
- (Figure 7g).  $\eta_X$  in the crust differs from 1 which makes the Rayleigh wave  $2\psi$  fast axis
- perpendicular to the strike in the crust as discussed by Xie et al. (2015).
- Figures 7a,b show examples of the fast axes orientations predicted by the theory of X. Liu &
- Ritzwoller (2025) with stong Rayleigh-Love coupling. With a constant strike angle with depth,
- 347 the Love wave  $2\psi$  fast axis is the same as the strike direction at all depths. At long periods (>30)
- s), the Rayleigh wave fast axis aligns with the Love wave fast axis, but at short periods it is
- perpendicular to it. Thus, with a constant strike angle with depth, the difference between the
- Rayleigh and Love wave  $2\psi$  fast axes bifurcates to be either 0 or 90 degrees (red line, **Figure**
- 7b). This is not what we observe, however (Figure 5d). Letting the strike angle vary from the
- 352 crust to mantle, produces a stike angle difference intermediate between 0 and 90 degrees (blue
- line, **Figure 7b**), similar to our observations. We conclude, therefore, that observed strike angle

- differences between Rayleigh wave  $2\psi$  and Love wave  $2\psi$  are evidence for strike variations 354
- with depth. 355

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- In conclusion, our observation of intermediate fast axes differences (40-60 degrees, Figure 5b,d) 356
- in western Alaska is diagnostic of a strike variation with depth. We refer to this as the fast axes 357
- being "quasi-perpendicular". On average, therefore, Rayleigh and Love wave  $2\psi$  fast axes are 358
- approximately quasi-perpendicular across western Alaska and they are approximately parallel 359
- across much of eastern Alaska. Thus, strike angles are more likely to vary only subtly with depth 360
- across eastern Alaska with stronger variation with depth in western Alaska. 361
- 362 Figure S8 presents another example histogram but for different periods (60 s for Rayleigh, 40 s
- for Love). This illustrates a stronger bifurcation of the fast axes differences near 0° and 90°, 363
- which suggests a more subtle strike variation with depth. 364

## 3.2.2 Rayleigh wave $4\psi$

Although Trampert & Woodhouse (2003) argued that Rayleigh wave  $4\psi$  should be observable, it has been largely overlooked in studies of anisotropy. We observe Rayleigh wave  $4\psi$  across substantial parts of Alaska at longer periods, as shown in Figure 3g,h, Figure S6l, and also in Figure 2 for an individual location. Rayleigh wave  $4\psi$  is not as strong or ubiquitous as the other components in Alaska, averaging between 0.3-0.45% in amplitude (Figure 4b). The uncertainty in the amplitude of Rayleigh wave  $4\psi$  averages between 0.3-0.4% (**Figure 4d**), which is smaller than the observed signal, on average, principally at the long periods. The uncertainty normalized amplitude across Alaska is in the supplementary material (Figure S22).

- 374 At shorter periods such as 20 s (Figure 3g), the amplitude of Rayleigh wave  $4\psi$  is negligible in central Alaska and largest near the periphery of the study region where uncertainties are largest 375
- 376 and reliability is suspect. As period increases (Figure 3h and 4b), the amplitude of Rayleigh
- wave  $4\psi$  becomes comparable to Love wave  $4\psi$  and at 50 s period it is a bit stronger, on 377
- 378 average. As illustrated in Figure 2, there are locations and periods where the azimuthal variation
- of the Rayleigh wave is actually dominated by  $4\psi$ . Figure 4b and also Figure 3c,d,g,h show 379
- that the amplitude of Rayleigh wave  $4\psi$  varies with period more or less opposite from Love 380
- wave  $4\psi$ . X. Liu and Ritzwoller (2025) say this is what is expected in the presence of Rayleigh-381
- Love coupling, where the amplitude of this component of the Love wave, which is stronger at 382
- shorter periods, is transmitted to the Rayleigh wave through anisotropy coupling at the longer 383
- periods. Ignoring Rayleigh wave  $4\psi$ , for example, can bias estimates of dip angle and ellipiticy 384
- parameters (X. Liu & Ritzwoller, 2025). 385
- Uncertainty in the fast axis direction for Rayleigh wave  $4\psi$  is about 10 degrees, which is similar 386
- to the uncertainty for Love wave  $4\psi$ . According to X. Liu & Ritzwoller (2025), the sensitivity 387
- kernels of Love wave  $2\psi$  and Rayleigh wave  $4\psi$  are basically the same as they are both mainly 388
- determined by the second term in X (equation (45) in X. Liu & Ritzwoller, 2025). Therefore, 389
- 390 their fast axes comparison is much more straightforward, compared to comparisons between

- Love wave  $2\psi$  and Rayleigh wave  $2\psi$ . We find that Rayleigh wave  $4\psi$  fast axis mainly align
- with those of Love wave  $2\psi$  (Figure 5e).
- Although this is beyond the scope of this paper, we note that inversion experiments indicate that
- strong Rayleigh wave  $4\psi$  at long periods may not be explainable with a TTI mantle, and these
- observations are more consistent with tilted orthorhombic elastic tensor in the mantle. A tilted
- orthorhombic medium in the mantle may further complicate the fast axes relationships compared
- with other signals.

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#### 4. Conclusions

Observations of Rayleigh wave and Love wave  $2\psi$  and  $4\psi$  azimuthal anisotropy are presented here across Alaska at periods ranging from 10 s to 50 s based on seismic ambient noise. We discuss the amplitude and fast axis orientations and address the four questions that motivated this study, with the following conclusions:

- (1) Both Rayleigh wave  $2\psi$  and Love wave  $4\psi$  are strong across Alaska with the average amplitude of both signals decreasing with period.
- (2) Love wave  $2\psi$  and Rayleigh wave  $4\psi$  anisotropy also are observed but with the average amplitudes growing with period. Love wave  $2\psi$  becomes similar in amplitude to Rayleigh wave  $2\psi$  at the longest periods of this study. Rayleigh wave  $4\psi$  is the weakest of the components studied, but is observable at the longer periods.
- (3) The fast axes of several components of anisotropy appear to be related. The mode of the distribution of the difference in fast axes between the Rayleigh wave  $2\psi$  and Love wave  $4\psi$  is 45 degrees and between the Love wave  $2\psi$  and Rayleigh wave  $4\psi$  is 0 degrees. The fast axis relationship between Rayleigh and Love wave  $2\psi$  is bimodal (0 and 45-60 degrees), where most differences are in the latter category, which we call quasi-perpendicular. Strike variations of the anisotropic fabric with depth can account for angle differences being substantially different than the expected 0 or 90 degrees for simple seismic models.
- (4) We interpret the observations of unexpected anisotopy and the complementary amplitude trends of Rayleigh and Love wave  $2\psi$  as well as Rayleigh and Love wave  $4\psi$  to result from Rayleigh-Love coupling becoming stronger at longer periods as the waves become sensitive to the mantle.
- We acknowledge that there are physical effects that are not accounted for in this study. For
- example, finite frequency effects can introduce some theoretical bias into our results such as
- apparent anisotropy in an isotropic heterogeneous structure (Lin & Ritzwoller, 2011) at longer
- periods. In addition, strong coupling between Rayleigh and Love waves will cause polarization
- anomalies that are not accurately accounted for by the tomography methods we apply (eikonal
- 426 tomography), particularly if the quasi-Rayleigh and quasi-Love waves are not well separated.
- Nevertheles, attributing all of the unexpected signals to measurement errors, noise, or theoretical
- bias is implausible, particularly given the systematics that result in the observations. This
- includes the comparisons between fast axes and the countervailing amplitude trends of Rayleigh

430 431	and Love wave components, which we attribute to Rayleigh-Love coupling. Further research about these topics would be worthwhile but is beyond the scope of this paper.
432	Although earthquake data have been used in many observational studies of azimuthal anisotropy
433	and in some cases in combination with ambient noise, we focus on the interpretation of ambient
434	noise data alone for the following reasons. First, although we find that earthquake data also show
435	strong anisotropy for all four-signals at long periods, especially the unexpected anisotropy, in
436	some regions the anisotropy inferred from earthquake data differs from that based on ambient
437	noise whereas in some regions they are very similar. We think a likely reason for this
438	discrepancy is the finite-frequency effects. Second, the azimuthal coverage from earthquakes is
439	limited. Third, we inspected the waveforms from earthquake data and these waveforms are often
440	very complex. These factors present challenges to observe azimuthal anisotropy reliably from
441	earthquake data, especially for the $4\psi$ components.
442	Observations of Rayleigh wave $2\psi$ anisotropy have been the primary data used to infer
443	information about anisotropy from surface waves. Observations of Love wave $2\psi$ and $4\psi$
444	anisotropy as well as Rayleigh wave $4\psi$ provide new information to improve the inference of the
445	elastic tensor in the crust and mantle in the future.
446	Acknowledgments
447	Open Research
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