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# Moist Convection and Radiative Cooling: Dynamical Response and Scaling

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Moist convection is a fundamental process occurring in the Earth's atmosphere. It plays a central role in the weather and climate of the tropics where, to first order, the heating of the atmosphere by convection is in balance with the cooling of the atmosphere by the emission of radiation to outer space. In this study, we use a Cloud Resolving Model in Radiative-Convective Equilibrium with an imposed constant rate of radiative cooling and study the response of moist convection to varying this rate of radiative cooling. We recover the previously known result that in response to increasing radiative cooling, the area of convection expands rapidly while the intensity of convection does not change. We explore the robustness of this response under varying model parameters and find that this response is due to a combination of moist convective processes and changes in the boundary layer. We also propose a fundamental scaling of the non-dimensional cumulus mass flux in moist convection which is robust across models of different complexity. We aim to bridge the gap between highly idealised prototypes of moist convection such as "Rainy-Bénard convection" introduced by Vallis et al. (2019) and comprehensive cloud-resolving models.

## KEYWORDS

Moist Convection, Radiative Cooling

# 1 | INTRODUCTION

Convection is an overturning circulation of a fluid (Rayleigh 1916) driven by vertical differences in density where denser fluid falls vertically downward while lighter fluid is lifted vertically upward. In the atmosphere (Emanuel 1994), convective circulations are usually driven by local heating from the Earth's surface, which leads to the layer of air in contact with the surface to be lighter than the air above it, thus rising and bringing colder air from aloft to the surface through a compensating subsidence. Convection is a leading driver of heat and moisture transport in the Earth system and it is particularly important in the tropics, where deep, moist convection plays a dominant role in determining tropical weather and climate. Convection occurs either in isolated thunderstorms, as part of broader systems (Mesoscale Convection Systems, Monsoons) or in conjunction with other synoptic and planetary scale phenomena, such as Equatorial Waves, the Madden-Julien Oscillations, or the Hadley Cell (Houze Jr. 2004; Stevens 2005; Kiladis et al. 2009; Zhang 2005).

The Earth's atmosphere loses heat to outer space by (chiefly longwave) radiation (Manabe and Strickler 1964; Jeevanjee and Fueglistaler 2020). In the tropics, the leading order energy balance is between radiative cooling and the warming of the troposphere by convection, or radiative-convective equilibrium (RCE) (Tompkins and Craig 1998). RCE holds over large enough length and time scales in the tropics (Muller and O'Gorman 2011; Jakob et al. 2019) and is a key lens used to understand tropical dynamics. Studies of RCE use limited-domain cloud-resolving models (CRMs) where, without large-scale forcing in the steady-state, radiation and convection are in equilibrium. CRMs have proved a valuable tool in gaining insight and understanding into several aspects of moist convection and tropical dynamics (Wing and Emanuel 2014; Stauffer and Wing 2022), especially the changes in tropical climate in a global warming scenario characterised by higher surface temperatures (Muller et al. 2011; Singh and O'Gorman 2015). Behaviour observed in CRMs have instigated studies into realistic models and observations (Holloway et al. 2017; Wing et al. 2017). The utility of CRMs however goes well beyond mean-state tropical dynamics. CRMs can be used in non-RCE configurations with the boundary conditions and energetic and mass-balances configured to mimic real-world conditions and the influence from large-scales onto limited area models (Singh and Neogi 2022).

Convection warms the atmosphere by transporting heat upwards, mainly by the transport of water vapour (or latent heat) which condenses (and freezes) aloft in the atmosphere. The convective transport of latent heat occurs via rising cloud plumes in which the air is saturated with moisture. The dynamics of these plumes is set by complex, non-linear mutually interacting cloud processes involving both, the large-scale conditions as well as the microphysics of water condensates (Arakawa and Schubert 1974; Arakawa and Wu 2013). A commonly used measure for the strength of convection is the rate of upward transport of air within cloud-plumes, or the cloudy mass-flux (known henceforth as simply the mass-flux). Under RCE, the greater the radiative cooling, the greater the mass-flux. The mass-flux  $M_c$  at a given height can be written as

$$M_c = \rho \sigma w_c \quad (1)$$

where  $\rho$  is the density of dry air,  $\sigma_c$  is the area-fraction of the horizontal cross-section that is occupied by clouds and  $w_c$  is the typical vertical velocity within these clouds.

The scaling of the mass-flux with changes in radiative cooling has been previously studied in CRMs (Robe and Emanuel 1996) (see also similar simulations by Cohen and Craig (2006)), where it was found that while  $M_c$  increased strongly with the imposed rate of radiative cooling  $R$ , this increase occurred by an increase in the area of clouds (ie., increase in  $\sigma$ ) while the intensity of updrafts in the clouds (ie.,  $w_c$ ) remained nearly constant. This scaling has also been observed in other numerical simulations, for example in Shutts and Gray (1999) (see Figures 7, 8 and Table 1), and

Parodi and Emanuel (2009) (see Figure 8 showing updraft velocity for large changes in the radiative cooling). Further, the dynamic consequences of the expanding area of convection and constant vertical velocity in clouds in response to increase radiative cooling are reviewed in Yano and Plant (2012). More recently, the response of dry and moist convection to varying rates of bulk cooling was studied in idealised 2D Direct Numerical Simulations (Agasthya and Muller 2024; Agasthya et al. 2025). In Agasthya et al. (2025) (henceforth AMC25), using the Rainy-Bénard model of moist-convection (Vallis et al. 2019) the study found that the same scaling in response to radiative cooling holds even in highly idealised, 2D settings, establishing that this scaling is a fundamental feature of moist convection and not the consequence of parametrised sub-grid scale processes or not set by the microphysics of liquid water and ice.

In this study, we revisit the simulations of Robe and Emanuel (1996) (henceforth RE96) and study the scaling of moist convection in the light of several new findings around moist convection in the intervening time of three decades. In addition to changing the mass-flux, increased radiative cooling affects domain-mean temperatures, with more cooling leading to a colder domain. Here, we decouple the changes in the dynamics due to the changing temperature of the domain from the changes due to the altered circulation caused by varying radiative cooling. Further, we use the fundamental insights gained from idealised models to understand changes in cloudy area, arguing that fundamental constraints from convective dynamics and the changes in the boundary layer lie at the heart of the wide-spread convection seen in simulations with large radiative cooling. Finally, we identify that the average velocity in clouds is a fundamental velocity scale. Using this velocity scale, we show that in idealised moist convection, a simple power-law scaling exists between the non-dimensionalised mass-flux and the non-dimensionalised ratio of radiative cooling to condensation heating.

The rest of the article is organised as follows. § 2 details the cloud resolving model used for our RCE studies and the numerical experiments performed. § 3 outlines the main results and scientific insights gained from these simulations. In § 4 we summarise our work and point to potential future avenues of research.

## 2 | METHODOLOGY

We perform RCE simulations using the System for Atmospheric Modeling (SAM) (Khairoutdinov and Randall 2003) version 6.10.8. SAM uses anelastic momentum and scalar advection-diffusion equations with prognostic thermodynamic equations for liquid water/ice static energy, total precipitating water, and total non-precipitating water. Microphysical processes are parameterized using 1-moment microphysics while subgrid scale turbulence closure is parameterized using a Smagorinsky-type parameterization (as in Bretherton et al. 2005; Muller and Held 2012). Surface fluxes are parameterized using bulk formulas based on Monin-Obukhov similarity. The equations are solved on a 128 km × 128 km horizontally periodic square domain with horizontal grid-spacing of 1 km. The domain is 27 km high, with a sponge layer with Newtonian damping on all prognostic variables to absorb gravity-waves in the top 9 km. There are a total of 64 vertical levels, with 53 in the first 18 km including 9 levels in the lowest 1 km. The lowest atmospheric model level is at 37.5 m and the vertical resolution decreases with height to 400 m in the mid and upper troposphere. A constant radiative cooling rate  $-R \text{ K d}^{-1}$  is imposed up to a height of 10 km above which it is gradually relaxed to 0 at a height of 14 km. For temperature colder than 200 K, we applying a Newtonian damping to this temperature with a timescale of 2 days. This leads to a uniform cooling in most of the troposphere while maintaining stratospheric temperatures close to 200 K (similar to Pauluis and Garner (2006)).

The first set of 5 simulations are performed with an SST of 300 K and the magnitude of the imposed radiative cooling  $R$  varying from  $0.75 \text{ K d}^{-1}$  to  $7.2 \text{ K d}^{-1}$ . In-line with expectations and RE96, we find that the average air temperature in the domain decreases in response to a stronger cooling. Thus, any changes when  $R$  is increased could

$R$	Varying Air Temperature (VAT)				Tuned Air Temperature (CAT)			
	SST	$T_a$	SHF	LHF	SST	$T_a$	SHF	LHF
0.75	300	297.98	6.41	66.62	296.8	294.35	7.66	61.03
1.5	300	294.56	20.82	118.73	300	294.56	20.82	118.73
2.0	300	292.28	31.89	149.11	302	294.88	29.75	160.85
3.6	300	286.71	72.97	240.29	305.8	294.16	53.91	285.40
7.2	300	279.91	165.06	401.20	312.5	293.90	109.82	579.46

**TABLE 1** The imposed value of  $R$  ( $\text{K d}^{-1}$ ) and SST (K) for the VAT and CAT simulations. The various averaged quantities shown are respectively the lowest atmospheric level temperature  $T_a$  (K), surface Sensible Heat Flux SHF, and surface Latent Heat Flux LHF (both in  $\text{W m}^{-2}$ ).

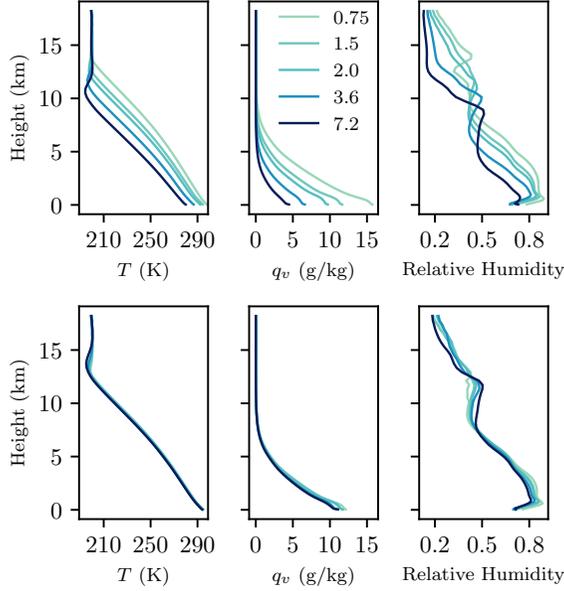
80 be due to changes in the circulation due to the direct effect of  $R$  or could be an indirect effect of the change in the  
81 air temperature. To isolate the former dynamic responses from the latter thermodynamic response, we perform an  
82 additional set of simulations where the SST is tuned by having a larger SST for simulations with larger  $R$ . This SST  
83 tuning ensures that the temperature of the lowest atmospheric level in the model  $T_a$  is within less than 1 K of that  
84 in the simulation with SST of 300K and  $R = 1.5 \text{ K d}^{-1}$ . This tuning leads to the average air temperature profile to be  
85 nearly identical across the simulations with different  $R$ . Henceforth, the first set of simulations with SST of 300K and  
86 varying  $R$  will be known as the varying air temperature, or VAT, simulations. The second set of simulations with SST  
87 tuned such that the air temperatures are identical will be known as constant air temperature, or CAT simulations. The  
88 reader must note that the simulation with SST = 300 K and  $R = 1.5 \text{ K d}^{-1}$  is common to VAT and CAT and is hence-  
89 forth referred to as control simulation (CTRL). All simulations are run until they reach a steady-state and all analysis is  
90 performed after that transient period using 50 days of steady-state dynamics.

## 91 | 3 | RESULTS

### 92 | 3.1 | Response of temperature, moisture and mass-flux

93 The imposed SST and various important simulation quantities are summarized in Table 1. For the VAT simulations,  
94  $T_a$  shows a sharp decrease with increasing  $R$  as the domain gets colder. The surface heat fluxes increase rapidly to  
95 balance the cooling in the domain, with the sensible heat showing a much larger relative increase than the latent heat.  
96 This can be attributed to the fact that sensible heat becomes more important as the domain becomes drier. As noted  
97 in §2 the CAT simulations have different SSTs but the resulting  $T_a$  are nearly the same, within  $< 1 \text{ K}$  of each other.  
98 For larger  $R$ , the CAT simulation domains have nearly identical temperature and moisture as CTRL. We also note that  
99 the surface fluxes (SHF + LHF) are stronger in the large  $R$  CAT simulations due to the increased convection depth in a  
100 warmer domain, leading to a higher stratosphere and thus, stronger vertically integrated radiative cooling that needs  
101 to be balanced by stronger incoming surface fluxes.

102 The time-averaged vertical profiles of temperature  $T$ , water-vapour mixing ratio  $q_v$  and relative humidity are  
103 shown in Figure 1. In the VAT simulations (top panels of Figure 1), for increasing  $R$  (darker shades of blue), the temper-  
104 ature in the domain decreases significantly, also leading to a decrease in  $q_v$  as well as large variations in the relative  
105 humidity profiles. We note in passing that the profiles shrink vertically with cooling, consistent with previous work  
106 (Singh and O’Gorman 2012). The profiles show consistency when plotted using temperature as a vertical coordinate  
107 (Jeevanjee 2022) (also see Appendix A), a theme that we will return to later.



**FIGURE 1** (From left to right) Horizontal and time-average profiles of Temperature, water vapour mixing ratio and relative humidity for the 5 values of  $R$  (values shown in legend with units  $\text{K d}^{-1}$ ) with VAT (top panels) and CAT (bottom panels).

108 For CAT simulations, we see that not only the lowest model level temperature but the temperature profiles of all  
 109 the simulation domains are very close to each other and are nearly indistinguishable from each other in the plotted  
 110 figure. Though temperature differences of the order of 2 K are present, this is a first indication that the temperature  
 111 profile, which is in turn set by convection, is a function of surface temperature and moisture, independent of the  
 112 radiative cooling.

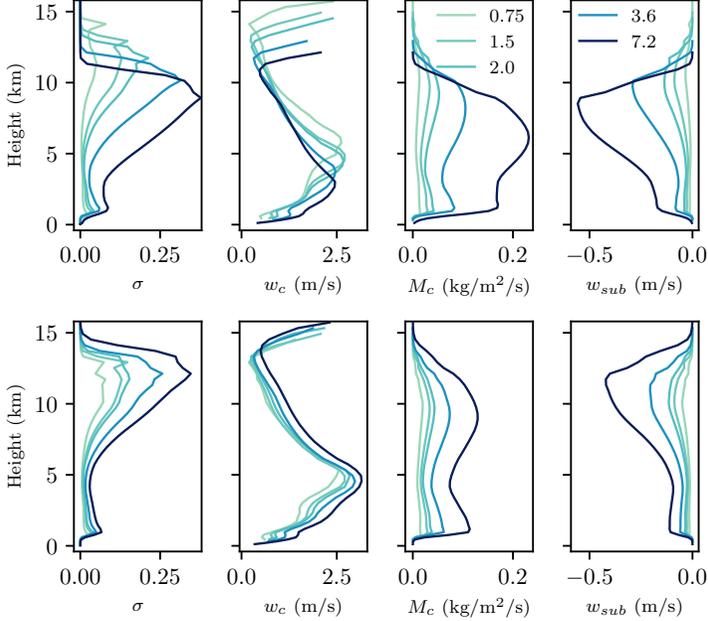
113 As discussed in RE96 and AMC25, greater radiative cooling leads to an increase in the magnitude of the average  
 114 subsiding vertical velocity  $w_{sub}$  outside clouds. This subsidence is radiatively driven and the subsidence adiabatic  
 115 warming plays an important role in balancing the imposed cooling. From the conservation of mass,  $w_{sub}$  is related to  
 116 the cloud mass-flux at a given height as

$$M_c = \bar{\rho} \sigma w_c = \bar{\rho} (1 - \sigma) |w_{sub}|, \quad (2)$$

117 where the line over  $\rho$  indicates that it is the anelastic density profile which is a function of height alone. Here  $w_{sub}$  is  
 118 the vertical velocity averaged only over regions that are not clouds while  $w_c$  and  $\sigma$  are the vertical velocity averaged  
 119 within clouds and the area fraction of clouds respectively. Away from clouds, assuming a balance between radiative  
 120 cooling and subsidence warming yields (e.g. [Robe and Emanuel \(1996\)](#))

$$w_{sub} = \frac{-R}{\frac{T}{\theta} \partial_z \theta} \equiv \frac{-R}{S} \quad (3)$$

121 where  $\theta$  is the potential temperature and  $S$  is known as the dry stability of the column, which is proportional to the  
 122 difference between the dry and moist adiabatic lapse rates (Bony et al., 2016; Jeevanjee, 2022). This suggests that, up  
 123 to changes in stability (which can be significant), the subsidence velocity must scale proportionally with the radiative  
 124 cooling.



**FIGURE 2** (From left to right) Time and horizontally averaged profiles of cloudy area fraction, average vertical velocity in clouds, cloudy mass flux and average vertical velocity outside clouds for (top panels) varying air temperature and (bottom panels) tuned air temperature simulations. A grid-point is considered to be cloudy if the mixing ratio of non-precipitating water (cloud water + cloud ice)  $q_n > 10^{-5}$  g/kg and  $w > 0$ .

125 Henceforth in our analysis, we define a grid-point to be “cloud” if it is rising ( $w > 0$ ) and has a non-precipitating  
 126 condensate mixing ratio  $q_n$  greater than  $10^{-5}$  kg/kg, a fairly standard definition of a cloud in the literature. Figure 2  
 127 shows the time-and-horizontally averaged profiles of  $\sigma$ ,  $w_c$ ,  $M_c$  and  $w_{sub}$ . In the top panels showing the averages of  
 128 the VAT simulations, we recover the result that while the mass-flux increases rapidly with stronger radiative cooling,  
 129  $w_c$  remains fairly constant, showing a mid-tropospheric maximum that is insensitive to  $R$ . The increase in the mass-  
 130 flux is driven by the large increase of  $\sigma$  – the mass-flux increases by having more clouds with the same intensity of  
 131 convection. On the flip-side,  $w_{sub}$  also shows a large increase with increasing  $R$ , closely mirroring the increase in  
 132 mass-flux, as expected from (3). Here,  $S$  decreases for simulations with larger  $R$  as the domain becomes drier and the  
 133 dynamics approach dry convection. It is important to note here that the convection becomes shallower for increasing  
 134  $R$  in the VAT simulations. This can be gauged either by observing the peaks of  $w_c$  and  $w_{sub}$  or by noticing that  $\sigma$   
 135 goes to 0 in the upper troposphere closer to  $z = 14$  km for  $R = 0.75 \text{ K d}^{-1}$  while this happens closer to  $z = 11$  km for  
 136  $R = 7.2 \text{ K d}^{-1}$ . This becomes important, while comparing convective quantities at a given height, something we will  
 137 come back to later.

The same profiles from the CAT simulations are shown in the lower panels of Figure 2. Here too, the increase in mass-flux with increasing  $R$  is pronounced and this increase is driven mainly by the increase in  $\sigma$ , an increase that is also seen in the magnitude of  $w_{sub}$ . However,  $w_c$  shows a small, monotonous increase at every height up to  $\sim 12$  km, and the highest value occurring at nearly the same height across the simulations. The maximum  $w_c$  shows a monotonic increase – however, the fractional increase in this peak is still small compared to the increase in  $R$  or  $\sigma$  – merely a  $\sim 25\%$  increase for a 9.6 times larger forcing (or 860% increase) and a 12.5 K warmer SST.

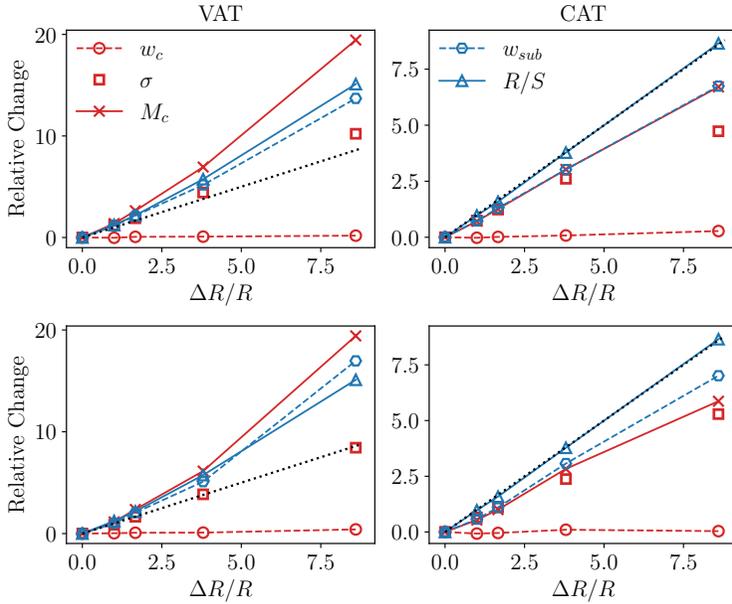
This is accompanied rather puzzlingly by a decrease of 80% and 18% in CAPE for VAT and CAT respectively. CAPE, short for convective available potential energy is a measure of the potential energy for convection in a given column of the atmosphere. It is defined as the positive part of the buoyancy of a moist-adiabatic parcel lifted from the surface and is known to be strongly associated with intense convective activity and thunderstorms (Johns and Doswell III, 1992). CAPE decreases in the VAT simulations because of a significant decrease in temperature and moisture available for convection, while in the CAT simulations, the decrease is due to small changes in the relative humidity and  $q_v$  in the domain (see lower middle and right panels of Figure 1). Thus, we see a decrease in CAPE but an overall increase in the average vertical velocity in clouds. Given that CAPE is usually associated with extreme events rather than average clouds, we also assess the extreme vertical velocity. The peak of the vertical profile of the 99.99-th %-ile  $w$  also shows a monotonic increase of 68% in CAT (not shown). For VAT, the increase from the simulation with the smallest  $R$  to the largest  $R$  is  $\sim 32\%$ , though this increase is not monotonic. Instead, the peak lies between 19 m/sec to 21.5 m/sec in all the simulations except the smallest  $R$ . It remains to be seen why despite a large increase in the surface forcing and a strengthening of the circulation with a strong subsidence flow,  $w_c$  increases by only a small amount, a change opposite in sign to the change in the moist instability as traditionally measured by CAPE.

### 3.2 | Relative changes in convective quantities

To understand the precise scaling of these quantities with the changes in radiative cooling, in the top panels of Figure 3 we plot their relative (or fractional) changes against the relative change in  $R$  for VAT (left) and CAT (right). The plots also have the black, dotted  $y = x$  straight-line for reference, as a linear relationship would lie on this curve. The relative changes of the quantities  $w_c$ ,  $\sigma$ ,  $M_c$ ,  $w_{sub}$  and  $R/S$  are plotted at a given temperature level rather than a chosen vertical height. While these quantities have been previously compared at a fixed height (for example a height of 6.7 km in RE96), we argue that the temperature level is a better way to ensure like-for-like comparisons (Jeevanjee 2022). This is discussed further in Appendix A.

The scaling for the VAT case are nearly identical to the scaling seen in Figure 3(a) of AMC25, with the mass-flux (solid curve, crosses) increasing faster than  $R$  (a super linear increase), the area of convection (square markers) increasing linearly with  $R$  and the velocity within clouds showing very little change (dashed curve, circular markers). Outside clouds, the subsidence velocity (solid curve, triangles) scales similar to the predicted  $R/S$  scaling where  $S$  is calculated from the mean vertical temperature profile. In colder domains,  $S$  is smaller, leading to a super-linear increase of  $R/S$  with  $R$ . Given that the changes are measured at the same temperature level, a small part of the increase in  $M_c$  can also be attributed to the change in density, where for the simulations with larger  $R$ , the 250 K level is lower down in the domain and thus the density of air is also larger.

In the CAT simulations, all the domains have the 250 K level at the same height and thus the changes are purely due to the changes in the circulation and the convection which are the result of the changing  $R$ . Here the mass-flux increases slightly slower than linearly, exactly with  $w_{sub}$ . The increase of  $R/S$  on the other hand is slightly faster than linear, which is due to small changes in the stability with changing relative humidity and the fact that the temperature profiles are not exactly identical.



**FIGURE 3** Relative change of various quantities in VAT (left) and CAT (right) simulations plotted against relative change in  $R$  linearly interpolated to the temperature level  $T = 250$  K within each simulation (note the change in  $y$ -axis range across panels). This corresponds to  $\sim 6.5$  km in CTRL and all CAT simulations, while it varies from 7.6 km to 3.8 km in VAT simulations. The relative changes plotted in the upper panels are average vertical velocity in clouds  $w_c$ , cloud area fraction  $\sigma$ , cloud mass flux  $M_c$ , average vertical velocity outside clouds  $w_{sub}$  and the radiative cooling  $R$  divided by the stability  $S$ . The lower panels show the same, but for convective regions (see main text for definition). All panels have the  $y = x$  black, dotted curve for reference.

179 While  $w_{sub}$  broadly scales with  $R/S$ , we must note here that the magnitude of the two quantities do not show a  
 180 very good match. As shown in Figure 9 in the Appendix, neither just a radiative balance nor a balance of radiative and  
 181 re- evaporative cooling combined with subsidence warming lead to a good match, with the greatest mismatch occurring  
 182 near the cloud anvil. This mismatch is briefly discussed in Appendix B. The fact that even the mid-tropospheric values  
 183 do not match well indicate that looking at vertical velocities purely outside clouds is not a good measure for purely  
 184 radiation driven subsidence. A cloud here is a point-wise metric requiring a threshold value of non-precipitating  
 185 condensate mixing ratio and rising motion ( $w > 0$ ). The mismatch is likely due to the large degree of turbulent vertical  
 186 velocity fluctuations and strong return flows near the clouds which are related to the cloudy dynamics rather than  
 187 subsidence in clear-sky regions far away from clouds.

188 Thus, we instead turn our attention to “convecting regions” – a vertical column is defined to be part of the con-  
 189 vecting region if the column-integrated cloud-water (CICW) is above a threshold of  $0.5 \text{ kg m}^{-2}$  (the results remain  
 190 unchanged for a broad range of thresholds from  $0.1$  to  $1 \text{ kg m}^{-2}$ ). The regions of high CICW are co-located with  
 191 regions of high column-integrated precipitable water, high surface precipitation and high mid-tropospheric vertical  
 192 velocity, indicating that these are regions of intense convective activity, even if momentarily they lack condensates at  
 193 some height. Figure 9 in the Appendix shows that the average subsidence outside these regions matches closely with  
 194 a pure radiative equilibrium. Thus, pure radiatively driven subsidence can be seen outside convecting regions rather  
 195 than outside clouds alone.

The lower panels of Figure 3 show the scaling of average vertical velocity and the mass-flux within convecting regions as well as the area fraction occupied by these regions at the same temperature level as the upper panels (250 K). In other words, we performed the same analysis as above using the column-integrated threshold to identify convective regions instead of clouds identified by a point-wise metric. All these three quantities (shown in red) scale nearly identically in the upper and lower panels for VAT as well as CAT. This crucial fact indicates that across simulations, the clouds have very similar structures and, for instance, the ratio between the cloud fraction and the convecting area fraction remain fixed at a given temperature level.

The picture that emerges is that of rising, saturated cloud-plumes, with the area occupied by them expanding to increase their mass-flux while the dynamics within them are fairly constant. These cloud-plumes carry with them a region of intense activity which are not radiatively driven, instead driven simply by the dynamics close to them and phase changes of water which doesn't participate in the core of the cloud. This "cloud baggage" also scales linearly with the cloud-plumes, so that when taken together, the cloud-plumes and their baggage form the convecting regions of the flow, responsible for clouds and precipitation. Outside these convecting regions, the dynamics is simply in balance with radiative cooling and directly feels the imprint of the varying  $R$ . The fact that the baggage is "well-behaved" is rather fortuitous, allowing the direct comparison of the vertical derivatives of  $R/S$  (or  $(R + H_e)/S$ ) with the vertical derivative of  $w_{sub}$  measured outside clouds (Bony et al., 2016; Jeevanjee, 2022).

The invariance of  $w_c$  across simulations taken together with the linear scaling between the properties of the cloud-plumes and the cloud baggage suggests that the properties of the clouds, such as the area occupied by individual clouds, do not change much. Instead their numbers simply increase. Previous studies which varied  $R$  in similar CRM set-ups do find that the increase in cloudy area fraction is due to an increase in the number of clouds while the distribution of the sizes of the cloud-cores do not vary (Craig and Cohen, 2006; Cohen and Craig, 2006).

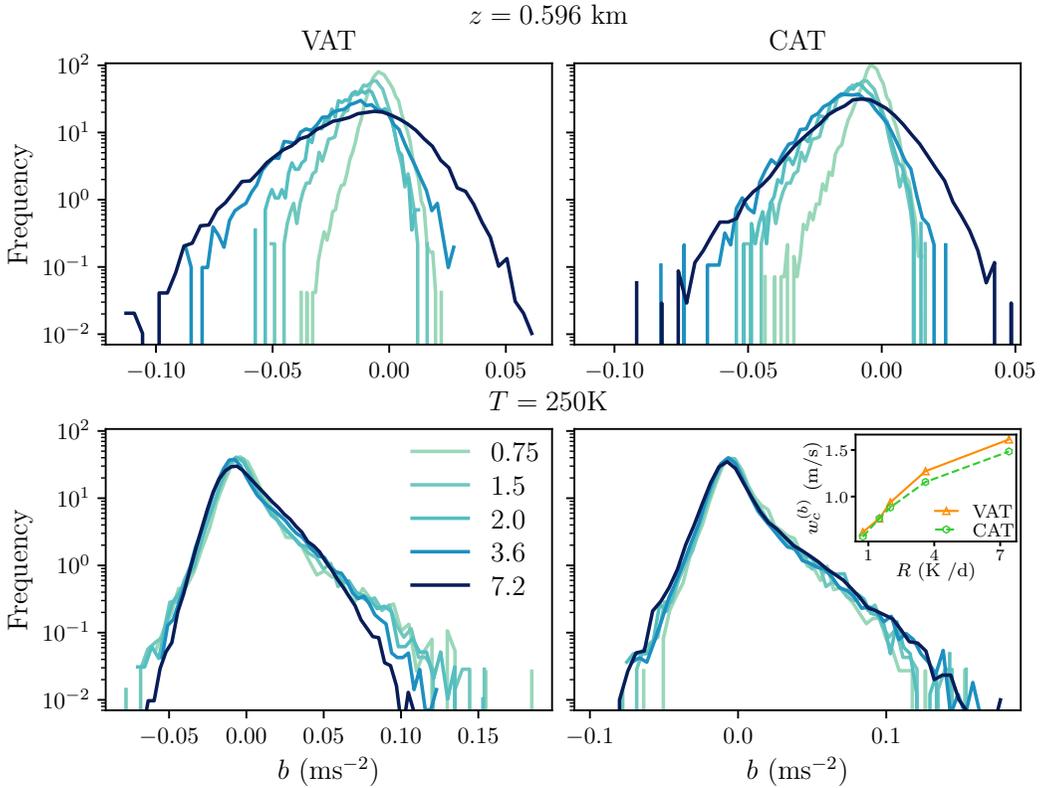
### 3.3 | Vertical Variation in Cloud Characteristics

To better understand the processes which set  $w$  in clouds, we assess the buoyancy within clouds. The lower panels of Figure 4 show the histogram of buoyancy at the 250 K level for the VAT and CAT simulation. We see here that the distribution of buoyancy does not show large differences, with even the tails of the distributions showing little difference in the CAT case. In the VAT case, the cases with larger  $R$  actually have smaller positive tails, consistent with the decrease in temperature and hence moisture and CAPE, leading to a smaller ability to create large positive buoyancies. This however is contrary to the observed small increase in  $w_c$  as well as extreme values of  $w$  with  $R$ .

The distribution of buoyancy at  $z \sim 600$  m (top panels) on the other hand shows large differences with varying  $R$ . This height corresponds to the first height at which the average cloud fraction is above 0.1 % in all simulations, ensuring that the histogram is reasonably smooth. It corresponds to the 6th model level, which is close to the theoretical Lifted Condensation Level (LCL) for all the simulations (between the 5th and 6th levels). It is below the lower-tropospheric peak of cloud fraction in all but one simulation ( $R = 0.75 \text{ K d}^{-1}$ ), so it represents a regime at or just below the cloud-base level, where the dynamics is strongly influenced by the boundary layer. At this level, the buoyancy histogram shows sharp differences, with the positive and negative anomalies being much larger for the simulations with large  $R$  in both sets of simulations. With increasing  $R$ , the distribution also becomes flatter and the tails are more pronounced. The inset to the top right panel shows the vertical velocity within clouds at the same height, which we denote  $w_c^{(b)}$ . Here, unlike its mid-tropospheric counterpart,  $w_c^{(b)}$  shows a noticeable increase with increasing  $R$ , showing a  $\sim 3$ -fold increase in value in both sets of simulations.

The above discussion, while holding many insights into the scaling of moist convection still does not address

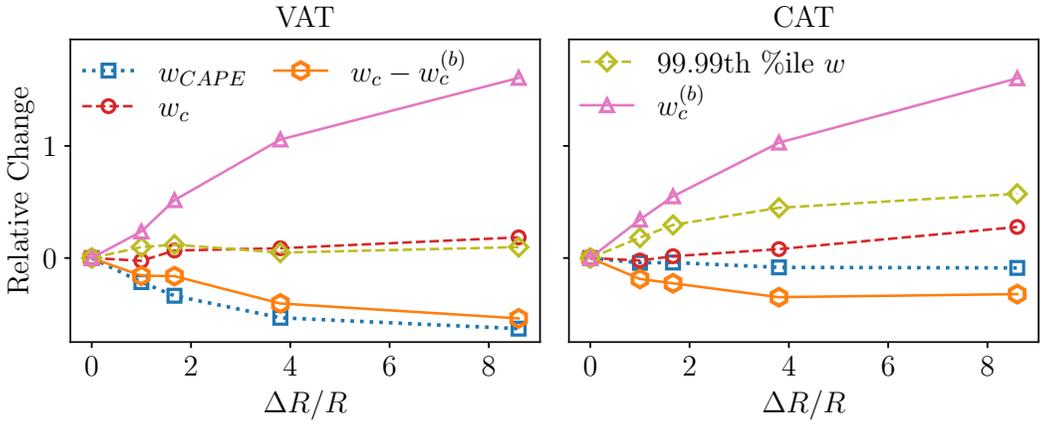
<sup>1</sup>We thank Prof. Robert Plant for suggesting this elegant terminology.



**FIGURE 4** Histogram of the buoyancy in clouds in the boundary layer ( $z = 596$  m - top panels) and temperature level closest to  $T = 250$  K (lower panels) for the simulations with different  $R$  for VAT (left) and CAT (right). Inset in lower right panel shows the boundary layer vertical velocity  $w_c^{(b)}$  at the same height.

236 the vexing central question – why does the area alone expand rapidly with increasing  $R$ ? Changing the radiative  
 237 cooling impacts a host of different flow characteristics, including the surface fluxes, the domain mean temperature,  
 238 the relative humidity and the stability. One set of quantities that remain remarkably constant across the simulations  
 239 are the vertical variations in the various cloud characteristics considered in the preceding paragraphs. In particular,  
 240 the vertical gradients in  $\sigma$ ,  $w_c$ ,  $M_c$  and  $w_{sub}$  plotted using temperature as a vertical coordinate follow almost exactly  
 241 the same curves above the boundary layer in the mid- and upper-troposphere with large differences closer to the  
 242 boundary. This is further discussed in Appendix A (see also Figure 3) and it shows that all these quantities have  
 243 the same vertical structure and a common vertical form function, independent of the forcing. Any inter-simulation  
 244 differences then must arise from differences which already exist at the top of the boundary layer.

245 Thus, instead of looking at the scaling of the velocity in clouds alone, in Figure 5 along with  $w_c$ , we plot various  
 246 other quantities related to the vertical velocity. The 99.99-th percentile of vertical velocity is shown to scale similarly  
 247 to  $w_c$ .  $w_{CAPE}$  is the prediction of  $w$  from the CAPE, that is the vertical integral of the buoyancy of a moist adiabatic  
 248 parcel lifted from the surface up to the given temperature level. As stated earlier, CAPE decreases with  $R$  in both  
 249 simulations. Finally, we show the scaling of  $w_c$  with the contribution of the top of boundary layer vertical velocity, or



**FIGURE 5** The relative change in various quantities plotted as a function of the relative change in  $R$  for varying air temperature and tuned air temperature simulations linearly interpolated to the  $T = 250$  K level. See main text for definitions.

250  $w_c^{(b)}$ , removed.  $w_c - w_c^{(b)}$  is thus a measure of the acceleration in the clouds above the boundary layer height, here  
 251 taken to be 596 m. We have already seen in the inset of figure 4 that this value responds quite strongly to  $R$ . Here  
 252 we see that the scaling of  $w_c - w_c^{(b)}$  (green curve) closely follows the scaling of the prediction from CAPE (blue curve)  
 253 in the VAT and CAT case. This scaling works similarly well for temperature levels below and close to the peak in the  
 254 vertical profile of  $w_c$ . Above this level, the value of  $w_c$  starts to decrease while the moist-adiabat is still positively  
 255 buoyant and it would be unphysical to continue to compare these curves. We note here that the peak in the extreme  
 256 cloud velocities occur far higher up in the domain, closer to the 230 K temperature level and higher.

257 This shows that the apparent insensitivity of  $w_c$  to CAPE is actually a boundary effect. Even as the boundary  
 258 becomes colder and drier, decreasing the moist instability and CAPE, the increasingly unstable boundary layer strongly  
 259 driven by the stronger surface fluxes leads to large variability in the boundary layer, creating strongly accelerated,  
 260 upward moving parcels at the cloud base itself. Above the cloud base, the on-average incremental upward acceleration  
 261 is related to the more traditional instability measure of CAPE. The increased boundary layer variability is seen not only  
 262 in the buoyancy distribution but also in the distributions of temperature, water vapour mixing ratio (and consequently,  
 263 the moist static energy). We note that generally, plume-based models for predicting vertical velocities do not take  
 264 into account large buoyancy or velocity anomalies arising within the boundary layer itself (Singh and O’Gorman, 2015).  
 265 Our results thus suggest, at least within the idealised settings used here, that boundary layer dynamics might play a  
 266 role in setting in-cloud velocities in addition to the acceleration from CAPE.

### 267 3.4 | A non-dimensional scaling for Moist Convection

268 In the appendix of AMC25, the authors suggested a non-dimensionalisation of the equations on the basis of setting  
 269 the rate of radiative cooling to unity. This was done by setting temperature scale  $\mathcal{T}$  and time-scale  $t_0$  such that

$$R = \mathcal{T} t_0^{-1}. \tag{4}$$

270 Through this relation, setting either a time-scale automatically sets the temperature scale and vice-versa. This  
 271 step is justified as in RCE, it is  $R$  that sets the dynamics of the entire system. As seen in this study, the dynamics also  
 272 strongly depend on the surface temperature, which determines the availability of heat and moisture in the boundary  
 273 layer. Increasing the SST for the same value of  $R$  increases  $w_c$ , decreases  $\sigma$  and  $M_c$ . The decrease of mass-flux with  
 274 warming has been noted and can be seen within references of [Jeevanjee \(2022\)](#). One way to interpret this decrease  
 275 is that a given rate of radiative cooling must be balanced by the transport of an equal amount of heat upward by  
 276 convection. This heat is either transported directly, as sensible heat, or as latent heat by the transport of moisture. A  
 277 warmer plume is also more moist, allowing the transport of more latent heat for the same mass of air, thus necessitating  
 278 fewer plumes to balance the same amount of cooling.

279 AMC25 suggested the dimensionless parameter given by

$$N = \frac{c_p R H}{q_0 U_0 L}, \quad (5)$$

280 where  $c_p$  is the specific heat-capacity of dry air,  $H$  is a vertical length scale,  $q_0$  is a water-vapour mixing ratio scale,  
 281  $U_0$  is a velocity scale and  $L$  is the latent heat of condensation of water.  $t_0$  then is given by  $H/U_0$ , which gives the  
 282 temperature scale from [\(4\)](#). In AMC25,  $U_0$  was given by the diffusive velocity scale [\(Vallis et al. 2019\)](#) and  $H$  was  
 283 simply the height of the domain. We notice here that the numerator of  $N$  is similar to a net cooling rate term for the  
 284 entire height while the denominator is similar to a rate of latent heating. It can be seen that using  $t_0 = H/U_0$  and  
 285  $R = \mathcal{T}/t_0$  gives simply

$$N = \frac{c_p \mathcal{T}}{L q_0}, \quad (6)$$

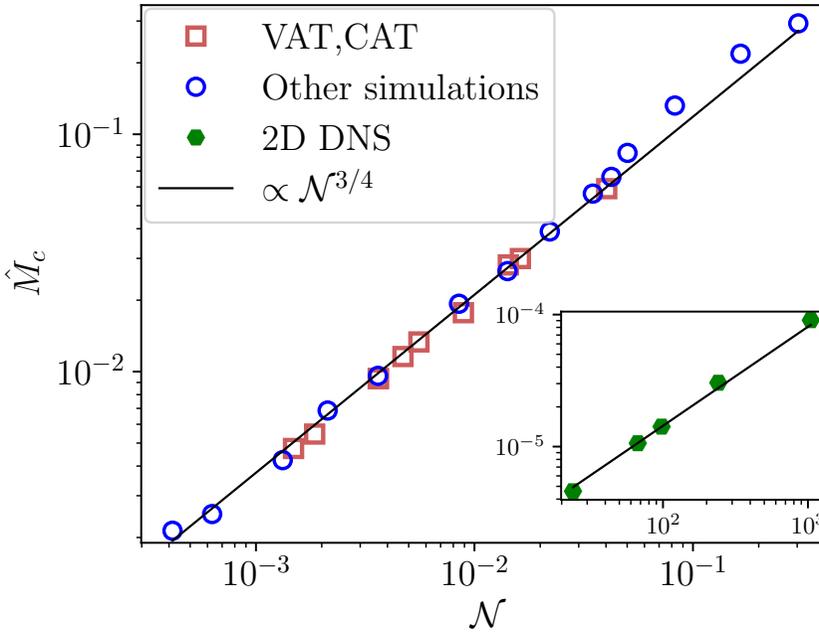
286 where  $\mathcal{T}$  and  $q_0$  are the appropriate temperature and water vapour mixing ratio scales.

287 We study the variation of  $N$  with the non-dimensionalised mass-flux, which we denote  $\hat{M}_c$ . We consider the  
 288 height of the lower tropospheric peak in  $M_c$  as the cloud-base height, which lies above the LCL. Taking the same  
 289 velocity scale  $U_0$  and a density scale  $\rho_0$  gives

$$\hat{M}_c = \frac{M_{c|cb}}{\rho_0 U_0}, \quad (7)$$

290 where  $M_{c|cb}$  is the cloud mass-flux at this cloud-base height. The mass-flux at cloud-base is known to be closely  
 291 related to the precipitation [\(Held and Soden 2006; Jeevanjee, 2022\)](#), which is in turn equal to the LHF (modulo a  
 292 constant related to  $\rho$ ,  $c_p$  and the latent heat of condensation of water). Figure [6](#) shows  $\hat{M}_c$  plotted against  $N$  in log-  
 293 log coordinates. This is plotted for the VAT and CAT simulations (red squares) as well as an additional set of simulations  
 294 (blue circles) where  $R$  and SST are varied widely using the same RCE set-up and details can be found in Appendix [C](#)

295 Figure [6](#) shows  $N$  and  $\hat{M}_c$  scale well with a simple  $3/4$  power law. A linear-fit performed for the logarithm of the  
 296 values yielded a slope of 0.783. In the inset of Figure [6](#) we show the log-log plot of the non-dimensionalised mass-  
 297 flux with the same non-dimensional number  $N$  calculated for the 2D direct numerical simulations detailed in AMC25,  
 298 discussed further in Appendix [C](#). Here too, the response of the mass-flux scales closely with the same power-law  
 299 when the imposed bulk cooling in the domain is varied by 1 order of magnitude. This indicates that the mass-flux in  
 300 moist convection, similar to the Nusselt number of Rayleigh-Bénard convection [\(Heslot et al. 1987; Grossmann and](#)  
 301 [Lohse 2000\)](#) and numerous other non-dimensionalised flux metrics for various other forms of convection [\(Klinger](#)  
 302 [and Marshall 1995; Yang et al. 2016\)](#), follows a scaling power-law which is a constant in the regimes explored here.



**FIGURE 6** Log-log plot of non-dimensional parameter  $\mathcal{N}$  versus the non-dimensionalised cloudy mass-flux  $\hat{M}_c$  for the VAT, CAT simulations (red squares) and other simulations carried out with varying values of  $R$  and SST with the same set-up (see Table 2). Inset shows the same plot for the DNS simulations from AMC25.

303 This scaling can be rationalised and interpreted by considering that within the set-up of a typical RCE model with  
 304 fixed radiative cooling as we have here, the dynamics is only a function of  $R$  and SST. When  $R$  increases, the difference  
 305 in temperature between the surface (SST) and the first atmospheric level in the model ( $T_a$ ) must increase so that the  
 306 surface fluxes, parametrised by bulk formulae as proportional to this difference  $\Delta T$ , also increase. The latent heat flux  
 307 is also proportional to the difference  $\Delta q$  between the saturation mixing ratio at SST,  $q^*(\text{SST})$ , and  $q_a$ . For a given SST,  
 308 increasing  $R$  leads to a smaller  $q_a$ , leading to a larger increase (relative to the increase in  $R$ ) in  $\mathcal{N}$ . This corresponds to  
 309 the VAT simulations described in this study. On the other hand, if  $q_a$  is to be kept fixed as  $R$  is increased (decreased),  
 310 then the SST must also be increased (decreased) accordingly. This corresponds to the CAT simulations described in  
 311 this study.

312 Our empirical scaling suggests that for fixed  $q_a$  achieved by tuning the SST,  $\hat{M}_c$  would scale roughly as  $\sim R^{3/4}$ ,  
 313 assuming small changes in  $U_0$  and  $H$ . In case  $R$  is fixed and  $q_a$  alone is varied by varying the SST, then  $\hat{M}_c$  would  
 314 scale as  $\sim q_a^{-3/4}$ . This latter scaling is the observed decrease in cumulus mass-flux for a warmer atmosphere. In fact,  
 315 under a global warming scenario with higher surface temperatures and increased atmospheric  $\text{CO}_2$  concentration,  $q_a$   
 316 is expected to increase strongly ( $\sim 7\%/K$ ) while the increase in total  $R$  in the troposphere is expected to be slower  
 317 (Held and Soden, 2006), leading to a decrease in  $\mathcal{N}$ , again assuming that changes in  $H$  and  $U_0$  are much smaller.

318 We make an informed guess that the scaling would break down at two different asymptotic regimes. Firstly, if  $R$   
 319 is held to be 0 but the SST is large enough, this would induce moist convection in the absence of the destabilisation by  
 320 radiative cooling. This is the situation for example in Rainy-Bénard convection (Vallis et al., 2019) and other systems  
 321 of simplified moist convection (Pauluis and Schumacher, 2010). Here,  $\hat{M}_c$  is finite while  $\mathcal{N}$  is 0. In the second scenario,

in a system with large  $R$  in conjunction with small SST, the system would approach dry convection and there would be no “clouds” or significant moist dynamics – thus the quantity of mass-flux in clouds would be ill-defined. The behaviour of radiatively cooled, purely dry convection has been studied in Berlingiero et al. (2012); Agasthya and Muller (2024). Finally, we note that recently an alternate non-dimensional quantity to characterise the static stability of moist-convection with radiative cooling has been proposed by Dritschel et al. (2025) using CAPE calculated from the steady-state temperature profile which depends on molecular diffusivity of air as well as the rate of radiative cooling.

## 4 | CONCLUSION AND DISCUSSION

In this study, we have considered the cloud-resolving model SAM in an RCE configuration with constant sea surface temperature and with the radiative cooling idealised as a constant, bulk cooling term with rate  $R \text{ K d}^{-1}$ . Our set-up and numerical experiments are very similar to previous work conducted in CRMs (Robe and Emanuel, 1996; Craig and Cohen 2006) and DNS (Agasthya et al., 2025). We systematically vary  $R$  and study the response of the domain and various moist-convective parameters in the simulations, in particular the cloud mass-flux, the area fraction of the domain and the vertical velocity in clouds. To decouple the direct impact of varying  $R$  on the domain from the indirect effect of the changing temperature, we conduct an additional set of simulations where the SST is changed from simulation to simulation to achieve a nearly constant temperature profile across simulations.

We study the scaling of these convective parameters as a function of the imposed radiative cooling and find that, consistent with previous studies, the increase in cloud mass-flux  $M_c$  (an increase required for energy balance) occurs by an increase in cloud area fraction  $\sigma$  while the vertical velocity in clouds  $w_c$  shows only small changes. This scaling occurs in both, simulations with the same surface temperature (VAT) and simulations with the same atmospheric temperature (CAT), showing that the impact of the decrease in temperature of the domain is not important in causing an increase in convective area.

Outside clouds, the dynamics is set directly by radiative cooling, with the magnitude of subsiding velocity  $w_{sub}$  that increases according to a theoretical balance between subsidence warming and radiative-cooling as in (3). However, it is pertinent to note that while this scaling was found to hold outside clouds,  $w_{sub}$  defined this way is not quantitatively representative of radiative balance as it is an order of magnitude larger than the prediction from radiative cooling. Instead, we find that the dynamics outside “convecting regions” are more akin to a pure radiative balance.

We further find that the various cloud characteristics are functions of temperature alone, independent of any changes in large-scale circulation. This constrains the buoyancy within clouds to not grow large enough to produce large vertical velocities even when the surface fluxes are very large. The changes we do observe in vertical velocity can be explained by a combination of larger variability in the boundary layer with increasing  $R$  followed by vertical acceleration that broadly scales with CAPE above the boundary layer. The large variability in the boundary layer can be seen in the vertical velocity in clouds, where for simulations with larger  $R$ , cloud parcels have already acquired a significant vertical velocity even before they are accelerated by CAPE. The extreme vertical velocities are also found to scale with  $R$  similar to the average in-cloud vertical velocities.

Finally, we use a non-dimensionalisation suggested previously in AMC25 to propose a scaling for moist convection which holds true for cloud resolving model simulations as well as direct numerical simulations. The non-dimensionalised mass-flux  $\hat{M}_c$  scales as  $\mathcal{N}^{3/4}$ , where  $\mathcal{N}$  is the non-dimensionalised ratio between the rate of radiative cooling  $R$  and the water-vapour mixing ratio at the surface  $q_a$ . The temperature scale is set by considering  $R$  to be of magnitude unity while the length and velocity scales are set to be the vertical extent of moist-convection in the

362 domain and the vertical velocity in clouds respectively. It remains to be seen to what extent the non-dimensional  
363 scaling discovered here is generally applicable to moist convection, particularly in the case where radiative cooling is  
364 not fixed externally but is represented realistically. Initial results from RCE simulations performed with fixed SST, fully  
365 interactive radiation and changing CO<sub>2</sub> by the authors indicate that the 3/4 power-law relationship also holds in this  
366 case, though the parameter space explored was fairly narrow.

367 A key aspect that needs further investigation is the scope and relevance of the current study. It remains to  
368 understand if the slow change in vertical velocity and strong response of cloud area to varying large-scale forcing  
369 studied and characterised here should be interpreted as a tropics-wide change in RCE or can also be seen over smaller  
370 time and length scales. The applicability of RCE simulation results to the Earth's atmosphere has been vigorously  
371 debated (Singh and O'Gorman, 2013; Seeley and Romps, 2015; Romps, 2021) and the results here are not immune  
372 from this debate. This gap could be bridged by assessing global climate model outputs, particularly storm-resolving  
373 models (Stevens et al., 2019) which resolve deep convection without parametrisations. Non-equilibrium studies of  
374 limited-domain models of moist convection are also a candidate to shed light further on this topic. An interesting  
375 question to ask is – under what model conditions could few clouds with large  $w_c$  be generated as a response to  
376 increasing  $R$ ?

377 In this study, we have taken a step in moving towards unifying studies on highly idealised prototypes of convection  
378 with more realistic models. This family of models ranges from classical Rayleigh-Bénard convection to realistic regional  
379 and global climate models, with various degrees of idealisations, simplifications and parametrisations in between.  
380 Fundamental studies of moist convection and general convection hold several insights into the behaviour of the earth's  
381 atmosphere, whether for dry convection, shallow moist convection or deep moist convection.

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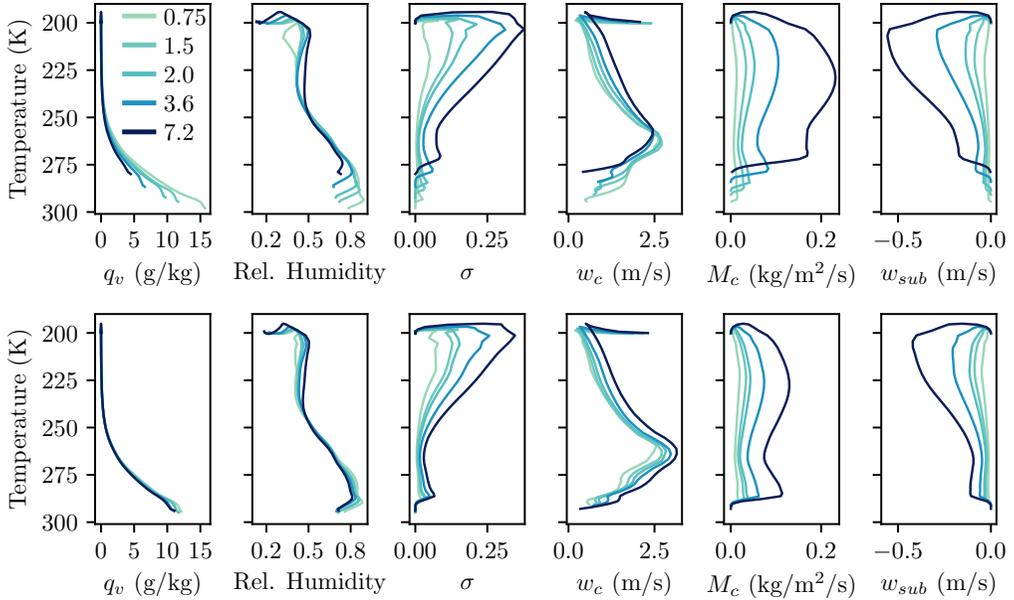
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## 392 conflict of interest

393 The authors report no conflict of interest.

## 394 Data Availability Statement

395 The data that support the findings of this study are available from the corresponding author, Lokhith Agasthya, upon  
396 reasonable request.



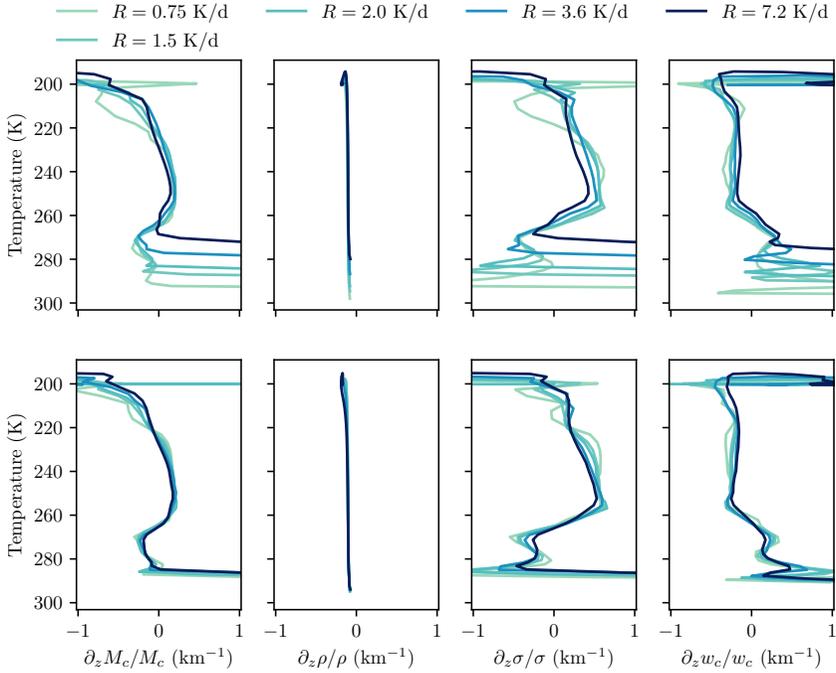
**FIGURE 7** (From left to right) Time and horizontally averaged profiles of water vapour mixing ratio  $q_v$ , relative humidity, cloud area fraction  $\sigma$ , vertical velocity in clouds  $w_c$ , cloud mass flux  $M_c$  and vertical velocity outside clouds  $w_{sub}$  for VAT simulations (top panels) and CAT simulations (bottom panels). The profiles are plotted against the averaged temperature profile, i.e., on isothermal coordinates.

## 397 A | ISOTHERMAL COORDINATES

398 The scaling of moist convective and other quantities are plotted at fixed temperature levels rather than a fixed height at  
 399 various points in the main text. Previous studies (Robe and Emanuel, 1996) compared  $w_c$ ,  $\sigma$  and  $M_c$  at the same height.  
 400 However, this captures different convective dynamics in different simulations. In Figure 7 we plot the profiles from  
 401 Figures 1 and 2 in isothermal coordinates, that is, using the average temperature profile as the vertical coordinate. We  
 402 note several features of interest. Firstly, despite very different conditions in the sub-cloud layer in the VAT simulations,  
 403 all the profiles of relative humidity are nearly invariant with temperature across simulations in the mid-troposphere.  
 404 The basic physics behind this RH-T invariance has been studied before (Roms, 2014).

405 The cloud-base can be discerned by looking at the lower tropospheric maximum of the cloud area fraction (which  
 406 is the same as the maximum in  $M_c$ ). For the VAT simulations, this cloud-base occurs at very different temperatures.  
 407 However, the mid-tropospheric peaks in  $w_c$  are much closer together, slightly shifting upward for larger  $R$ . The upper  
 408 peak in  $\sigma$  which closely corresponds to the minimum of  $w_{sub}$  is the cloud anvil and this too occurs at a roughly fixed  
 409 temperature, independent of the lower tropospheric temperature. This is however expected, given that we enforce  
 410 a fixed stratospheric temperature of 200 K. We choose a temperature level close to 250 K for our analysis as this is  
 411 close to the peak of  $w_c$  and is in a regime where the cloud area is increasing with height, influenced purely by in-cloud  
 412 processes.

413 In the CAT simulations, due to the temperature profiles being nearly identical, all the curves have the same shape



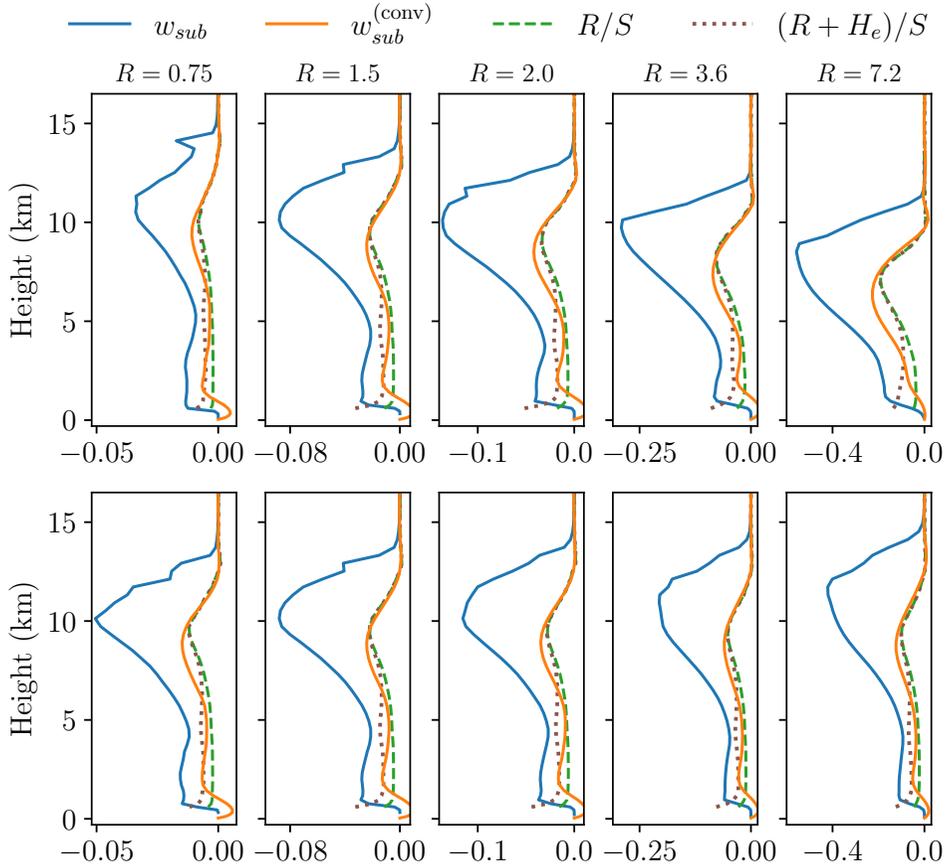
**FIGURE 8** Plots showing the fractional rate of vertical change for  $M_c$ ,  $\rho$ ,  $\sigma$ ,  $w_c$  for VAT (upper panels) and CAT (lower panels) plotted on isothermal coordinates.

414 in the vertical when plotted in isothermal coordinates as when they are plotted as a function of height.

415 In addition to the quantities themselves, in Figure 8 we study the vertical variation in the mass-flux and the three  
 416 quantities that make up the mass-flux. We find again in the mid-troposphere above the boundary layer, the vertical  
 417 derivative in these quantities are a function of temperature alone across simulations, as alluded to in § 3.3.

## 418 B | SUBSIDENCE VELOCITY

419 Figure 9 shows the average subsidence velocity  $w_{sub}$  outside clouds. It is seen here that there is a large mismatch  
 420 between this value (blue) and the prediction from a pure radiative balance (green, dashed curve). This match does not  
 421 improve greatly when the cooling due to the re-evaporation of falling rain is also included (dotted, brown curve) in the  
 422 prediction, as suggested by Jeevanjee (2022). The mismatch is especially large near the cloud anvil – here convergence  
 423 outside clouds is known to play a stronger role in producing subsidence velocities. However, we found that including  
 424 the horizontal convergence term still does not improve the prediction (not shown), indicating that non steady-state  
 425 forces also play an important role in the convective dynamics. A deeper investigation into this is beyond the scope of  
 426 the current study. Here we simply note that better quantitative agreement is obtained when we compute subsidence  
 427 velocity  $w_{sub}^{(conv)}$  outside convective regions defined with a CICW threshold (see § 3.2 for details).



**FIGURE 9** Time and horizontally averaged profiles of vertical velocity in  $\text{m s}^{-1}$  outside clouds (solid blue curve), outside convective regions (solid orange curve) compared with the prediction of pure radiative balance  $R/S$  (dashed green curve) and a radiation + reevaporation balance (dotted brown curve) in VAT simulations (top panels) and CAT simulations (bottom panels) for all the studied values of  $R$  in  $\text{K d}^{-1}$  as indicated by the plot titles. Note that the  $x$ -axes ranges vary for different  $R$ .

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## C | NON-DIMENSIONAL SCALING

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In addition to the VAT and CAT simulations, we performed RCE simulations with fixed radiative cooling  $R$  and constant SST across a wider range of parameters to understand the scaling of the non-dimensional parameters  $\mathcal{N}$  and  $\hat{M}_c$  introduced in §3.4. The values of  $R$  and SST chosen are listed in Table 2.

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For the DNS simulations of moist convection, the velocity scale was chosen as the usual diffusive velocity as given in AMC25, while the length-scale was simply the height of the domain. For  $q_0$ , the water vapour mixing ratio at  $z = 1$  was chosen while for  $\hat{M}_c$ , the mass-flux was assessed at  $z = 5$  in the simulation units of their study. For  $q_a$ ,  $z = 1$  corresponded a height above the diffusive boundary in all the simulations. The peak of  $M_c$  was close to  $z = 5$ , since the DNS simulations were performed assuming constant density (Boussinesq approximation) rather than decreasing

$R$ (Kd <sup>-1</sup> )	SST (K)
0.5	(295, 305, 310)
2	(295, 305, 310)
6	(295, 305, 310)
10	(290, 295, 305, 310)

**TABLE 2** List of parameters  $R$  and SST for which additional simulations with the same set-up were performed to obtain the points in Figure 6

437 with height.

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