

1 **The Incredible Lightness of Water Vapor**

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ABSTRACT

7 The molar mass of water vapor is significantly less than that of dry air. This
8 makes a moist parcel lighter than a dry parcel of the same temperature and
9 pressure. This effect is referred to as the vapor buoyancy effect and has of-
10 ten been overlooked in climate studies. We propose that this effect increases
11 Earth's outgoing longwave radiation (OLR) and stabilizes Earth's climate.
12 We illustrate this mechanism in an idealized tropical atmosphere, where there
13 is no horizontal buoyancy gradient in the free troposphere. To maintain the
14 uniform buoyancy distribution, temperature increases toward dry atmosphere
15 columns to compensate reduction of vapor buoyancy. The temperature differ-
16 ence between moist and dry columns would increase with climate warming
17 due to increasing atmospheric water vapor, leading to enhanced OLR and
18 thereby stabilizing Earth's climate. We estimate that this feedback strength
19 is about $O(0.2 \text{ W/m}^2/\text{K})$, which compares with cloud feedbacks and surface
20 albedo feedbacks in current climate.

21 **1. Introduction**

22 How fast would Earth's climate respond to increasing CO₂ (Manabe and Wetherald 1975; Flato
23 et al. 2013; Collins et al. 2013)? Why is tropical climate more stable than extratropical climate
24 (Holland and Bitz 2003; Polyakov et al. 2002; Pierrehumbert 1995)? What sets the inner edge
25 of the habitable zone of Earth-like planets (Yang and Abbot 2014; Pierrehumbert 2010)? Under-
26 standing and accurately estimating climate feedbacks are key to address these pressing questions.

27 The importance of water vapor seems to be widely recognized in the literature of climate feed-
28 backs (Manabe and Wetherald 1967; Ingersoll 1969; Held and Soden 2000; Flato et al. 2013).
29 Previous studies have focused on three basic effects of water vapor: E1) water vapor is a green-
30 house gas; E2) water vapor can condense to liquid water and release latent heat; E3) saturation
31 vapor pressure increases with temperature exponentially. The combination of E1 and E3 gives rise
32 to the water vapor feedback, the dominant positive climate feedback (Manabe and Wetherald 1967;
33 Held and Soden 2000; Flato et al. 2013). Increasing temperature leads to more water vapor, which
34 leads to an enhanced greenhouse effect, warming the planet further. The water vapor feedback
35 could even lead to a runaway greenhouse state when the atmosphere is sufficiently opaque to long-
36 wave radiation that the outgoing longwave radiation (OLR) is insensitive to surface temperature
37 (Ingersoll 1969). The combination of E2 and E3 gives rise to the (tropical) lapse rate feedback, a
38 negative climate feedback in the tropical atmosphere (Flato et al. 2013). Increasing temperature
39 leads to more water vapor, which leads to less steep lapse rate in the tropical atmosphere. This
40 effect increases upper troposphere temperature more than the lower troposphere, leading to higher
41 emission of outgoing longwave radiation (OLR), which cools the planet. At higher latitudes, tem-
42 perature lapse rate is no longer controlled by moist convection, so the lapse rate feedback is less
43 constrained. Both feedbacks are among the five most important climate feedbacks in the Intergov-

44 governmental Panel on Climate Change (IPCC) reports and have been extensively evaluated in general
45 circulation models (GCMs) (Flato et al. 2013).

46 However, the lightness of water vapor has been completely overlooked in the context of climate
47 feedbacks. The molar mass of water vapor is 18 g/mol, significantly lighter than that of dry air,
48 which is 29 g/mol. This makes a moist parcel lighter than a dry parcel of the same temperature
49 and pressure (Emanuel 1994). Here we refer to this as the vapor buoyancy effect, though it is also
50 referred to as the virtual effect (Yang 2018a,b).

51 We propose that the vapor buoyancy effect can increase Earth's OLR and helps stabilize Earth's
52 climate by regulating the atmosphere's thermal structure. Figure 1 shows temperature and virtual
53 temperature (buoyancy) fields in the moisture space from 2°S to 2°N using NASA AIRS data. In
54 the free troposphere ($p < 850$ hPa), buoyancy is horizontally uniform because of the small Coriolis
55 parameter and efficient gravity waves (Charney 1963; Bretherton and Smolarkiewicz 1989; Sobel
56 et al. 2001; Yang 2018a). However, temperature increases toward dry columns due to the vapor
57 buoyancy effect. Moving toward the dry columns, moisture and its associated vapor buoyancy
58 are reduced. To maintain uniform buoyancy, temperature has to increase. We propose that the
59 temperature tilt would increase with climate warming due to increasing atmospheric moisture,
60 leading to enhanced OLR over the dry area. This is a negative feedback and can stabilize Earth's
61 climate.

62 Previous studies have noticed that vapor buoyancy could make temperature increase toward
63 dry columns in the tropical atmosphere (Tompkins 2001; Bretherton and Smolarkiewicz 1989;
64 Bretherton et al. 2005; Yang 2018b,a). However, they have often considered this effect to be small
65 and negligible, simplifying the dynamics according to a weak temperature gradient approximation
66 (Sobel et al. 2001). These studies, therefore, did not consider that its radiative effect is significant,
67 which is the novelty of this study.

68 In Section 2, we explain our hypothesis in detail. We first illustrate how the vapor buoyancy ef-
69 fect increases Earth’s OLR (a negative radiative effect) and then explain why this effect strengthens
70 with climate warming. In Section 3, we derive a simple model for the radiative effect and feedback
71 strength of the vapor buoyancy effect. We then use the simple model to make order-of-magnitude
72 estimates for the radiative effect and feedback strength. In Section 4, we conclude and discuss
73 implications on the climate stability of Earth and other planets.

74 2. Hypothesis

75 We propose that the vapor buoyancy can increase OLR (a negative radiative effect) due to a clear-
76 sky effect, and that the radiative effect increases with climate warming. Figure 2 illustrates our
77 hypothesis by comparing OLR from two stand-alone atmospheres with overturning circulations:
78 one considers the vapor buoyancy effect (control), the other does not consider this effect. The
79 overturning circulation is analogous to the Walker Circulation or convective self-aggregation in
80 the tropics (Bretherton et al. 2005; Pritchard and Yang 2016; Yang and Ingersoll 2013, 2014).
81 The upwelling branch of the circulation is associated with deep convection and moist air, and the
82 downwelling branch is associated with clear sky and dry air. For illustrative purposes, we make a
83 few simplifications: S1) the two atmospheres are non-rotating; S2) the two atmospheres sit above
84 ocean surface with the same, uniform surface temperature; S3) the two atmospheres have the
85 same water vapor distribution. The first two simplifications are relevant to the tropical atmosphere
86 as the rotation effect and surface temperature gradient are both weak in the tropics. The third
87 simplification is often required when calculating the radiative effect.

88 Figure 2 shows that the control atmosphere emits more OLR than the no-vapor-buoyancy atmo-
89 sphere due to higher temperature in the dry area. OLR is primarily a function of temperature and
90 water vapor mixing ratio r . When r remains the same in the two atmospheres (S3), the OLR dif-

91 ference would come from temperature differences between the two atmospheres. The temperature
92 profiles of moist areas in the two atmospheres are set by convective plumes. Because these con-
93 vective plumes rise from the same surface temperature, the temperature profiles should be almost
94 identical in the two moist areas. Temperature profiles in the dry areas, however, differ significantly,
95 leading to differences in OLR. According to long-accepted results in geophysical fluid dynamics,
96 the horizontal buoyancy gradient is negligible in the free troposphere without rotation because
97 gravity waves can effectively smooth out buoyancy anomalies (Charney 1963; Sobel et al. 2001).
98 We refer to this effect as the weak buoyancy gradient (WBG) approximation (Yang 2018a). In
99 the control atmosphere, buoyancy is a function of both temperature and and water vapor mixing
100 ratio r due to the vapor buoyancy effect. The horizontal moisture gradient then leads to horizontal
101 temperature gradient: dry air is warmer than moist air. In the no-vapor-buoyancy atmosphere,
102 temperature is uniformly distributed in the free troposphere, as buoyancy is a function of tem-
103 perature only. The dry column of the control atmosphere, therefore, is warmer than that of the
104 no-vapor-buoyancy atmosphere, leading to enhanced OLR.

105 In warmer climates, the vapor buoyancy effect would become more significant due to increasing
106 water vapor. Therefore, we expect that the radiative effect due to the vapor buoyancy also increases
107 with climate warming. This is a negative climate feedback (Fig. 2b). The proposed mechanism
108 relies on ample atmospheric water vapor, so it would be most effective in stabilizing the tropical
109 climate.

110 We will construct a simple model of the proposed feedback mechanism. This will give an order-
111 of-magnitude estimate of the associated radiative effect and the rate at which it increases with
112 climate warming.

113 3. A simple model

114 We construct a simple model based on the schematic diagram (Fig. 2). Each atmosphere with
115 overturning circulations is represented by a dry column and a moist column (Pierrehumbert 1995).
116 Because the moist columns would have the same temperature profiles, the OLR difference pri-
117 marily comes from the dry columns, which we will focus on. Again, we aim to estimate the
118 ”radiative effect” due to the vapor buoyancy effect. Therefore, we assume that all basic dynamic
119 (*e.g.*, circulation and pressure) and thermodynamic features (*e.g.*, moisture) are the same in the
120 two atmospheres—one with the vapor buoyancy effect, and the other without it.

121 The goal of this simple model is to provide an order-of-magnitude understanding of our hypoth-
122 esis. Therefore, we employ a two-band radiative transfer model. The two-band model is more
123 realistic than a gray atmosphere model by allowing two absorption bands with distinct absorption
124 coefficients, leading to different emission levels. The two band model is, on the other hand, much
125 simpler than a real-gas radiative transfer model, so the results are easier to interpret.

126 *a. The two-band model*

127 We consider a plane-parallel atmosphere. Only the clear-sky longwave (IR) radiation is con-
128 sidered, and the IR opacity is mainly due to water vapor. Here we parameterize the water vapor
129 absorption spectrum by two broad bands that occupy roughly equal fractions of blackbody emis-
130 sion at Earth-like temperatures (Beucler and Cronin 2016): one with a strong absorption coefficient
131 (k_S) and the other with a weak absorption coefficient (k_W).

132 We first consider one absorption band with any given k . OLR is defined as

$$OLR^k \equiv F^\uparrow(p=0) - F^\downarrow(p=0); \quad (1)$$

133 where F^\uparrow and F^\downarrow are upward and downward longwave radiative fluxes. We know that $F^\downarrow(0) \approx 0$,
 134 so a primary focus is to solve for $F^\uparrow(0)$ in the gray atmosphere, which is given by

$$\frac{dF^\uparrow}{dt} = F^\uparrow - ST^4; \quad (2)$$

135 where T is temperature, t is optical depth, and S is the Stefan-Boltzmann constant. We integrate
 136 (2) and get

$$F^\uparrow(0) = e^{-t_s} F^\uparrow(t_s) + \int_0^{t_s} ST^4 \times e^{-t'} dt'; \quad (3)$$

137 The OLR is then given by

$$OLR^k = e^{-t_s} ST_s^4 + \int_0^{t_s} ST^4 \times e^{-t'} dt'; \quad (4)$$

138 where A_s represents the surface value of A , and we have used $F^\uparrow(t_s) = ST_s^4$. This equation shows
 139 that OLR has two components: one is the surface contribution, and the other is the atmosphere
 140 contribution.

141 We now use (4) to calculate the OLR difference between the two atmospheres, each containing
 142 one moist and one dry columns. We remind the readers that T_s , r and thereby t of the two atmo-
 143 spheres are identical, so the OLR difference primarily comes from dry columns, in which there is
 144 significant air temperature difference. The OLR difference of the dry column is given by

$$DOLR^k \equiv OLR_v^k - OLR_{nv}^k \approx \int_0^{t_s} 4ST_d^3 DT \times e^{-t'} dt' \approx \int_0^{t_s} 4ST_m^3 DT \times e^{-t'} dt'; \quad (5)$$

145 where OLR_v and OLR_{nv} represent OLR in the atmosphere with and without the vapor buoyancy
 146 effect. In the last equal sign, we assumed that $(T_d - T_m)/T_m \ll 1$. Because the strong and weak
 147 absorption bands occupy equal portions of the spectrum, the total OLR difference is given by

$$DOLR = 0.5 \times (DOLR^{ks} + DOLR^{kw}); \quad (6)$$

148 To compute $DOLR$, we need information of T_m , DT , t , and thereby r , which is the mixing ratio of
 149 water vapor.

150 *b. Temperature*

151 In the Earth’s tropical atmosphere, temperature profiles can be approximated by power-law re-
152 lations of pressure:

$$T = T_s \frac{p}{p_s}^{R_d G_M = g}; \quad (7)$$

153 where T_s is the surface temperature, p_s is surface pressure, R_d is the gas constant for dry air,
154 G_M is the moist adiabatic lapse rate, and g is gravity acceleration. This has been referred to as
155 the ”all-troposphere model” by Pierrehumbert (2010), as the lapse rate is entirely determined by
156 moist convection. Equation (7) fits the observed temperature profiles in the tropical troposphere,
157 but introduces significant biases in the stratosphere (Beucler and Cronin 2016). Earth’s OLR is
158 dominated by tropospheric contributions, which justifies the use of (7).

159 *c. Moisture*

160 The water vapor mixing ratio r is the ratio of the mass of water vapor to the mass of dry air and
161 is given by

$$r = RH \times r^*(T; p); \quad (8)$$

162 where RH is the relative humidity, and r^* is the saturation mixing ratio. For the moist column, we
163 assume that $r_m = r_m^*$ ($RH = 1$) at all vertical levels; For the dry column, we have $r = b \cdot r_m^*$, where
164 $0 < b < 1$. Here b is a more convenient parameter than the relative humidity of dry columns (RH_d).
165 This is because, at given T_s , the two dry columns are of different temperatures, so they would have
166 different RH values corresponding to same mixing ratio. In reality, b could have complicated
167 vertical structures, which requires multiple parameters to describe. For the purpose of illustrating
168 the proposed mechanism with minimal parameters, we take b as a constant at all vertical levels.
169 This simplification may affect the results quantitatively but will not affect the results qualitatively.

170 *d. Optical depth*

171 Outgoing longwave radiation is observed from space, so it would be convenient to define the
 172 optical depth t as an increasing function as pressure. We thus require that $t(p = 0) = 0$. We write
 173 the optical depth as

$$dt = k \cdot r \cdot \frac{dp}{g} \quad (9)$$

174 The optical depth would have different values for the two absorption bands: $dt = k^s \cdot r \cdot dp = g$ for
 175 the strong band, and $dt = k^w \cdot r \cdot dp = g$ for the weak band. In this model, we ignore the pressure-
 176 broadening effect and treat the absorption coefficients as constant: $k^s = 1.66 \times 0.1$ (m²/kg) and
 177 $k^w = 1.66 \times 0.02$ (m²/kg), where the factor of 1.66 is referred to as the diffusivity factor (Pier-
 178 rehumbert 2010). Here the absorption coefficients are consistent with previous modeling studies
 179 of similiar complexity (Ingersoll 1969; Pierrehumbert 2010; Beucler and Cronin 2016). We can
 180 integrate (9) to obtain the optical depth at an arbitrary pressure level

$$t(p) = \int_0^p k \cdot r \cdot \frac{dp'}{g} \quad (10)$$

181 At surface $p = p_s$, we then have $t_s = \int_0^{p_s} k \cdot r \cdot dp' = g$.

182 *e. The WBG approximation and DT*

183 Buoyancy is horizontally homogenized in the tropical free troposphere (Fig. 1). We refer to
 184 this constraint as the weak buoyancy gradient (WBG) approximation (Yang 2018a). This is an
 185 improvement of the weak temperature gradient (WTG) approximation, which neglects the vapor
 186 buoyancy effect (Charney 1963; Sobel et al. 2001). In a moist atmosphere, buoyancy is related to
 187 the virtual temperature, which is given by

$$T_v = T \frac{1 + r = e}{1 + r} \quad ; \quad (11)$$

188 where $e = M_v/M_d$, where M_v and M_d represent the molar mass of water vapor and dry air, respec-
 189 tively. In the free troposphere, uniform buoyancy requires the virtual temperature to be uniform
 190 across the moist and dry area:

$$T_m \frac{1 + r_m=e}{1 + r_m} = T_d \frac{1 + r_d=e}{1 + r_d} \quad ; \quad (12)$$

191 We substitute $T_d = T_m + DT_{WBG}$ into (12) and get

$$DT_{WBG} = T_m \frac{1 + r_m=e}{1 + r_m} - \frac{1 + r_d=e}{1 + r_d} \frac{1 + r_d}{1 + r_d=e} \quad ; \quad (13)$$

192 Equation (13) is derived without approximations about the amount of water vapor and the ampli-
 193 tude of DT . Although this form is quite accurate, we would like to simplify it by assuming water
 194 vapor is a trace gas: $r \ll 1$. This is a good assumption for the current climate and may still be
 195 good till surface temperature reaches 320 K, at which temperature $r^*(p_s) = 73$ g/kg. With this
 196 approximation, we get

$$DT_{WBG} = T_m \left(\frac{1}{e} - 1 \right) (r_m - r_d) \quad ; \quad (14)$$

197 This simplified equation clearly tells that DT depends on the contrast, not just absolute values, of
 198 mixing ratio and molar mass.

199 The above calculation is more accurate in the free troposphere, where gravity waves efficiently
 200 smooth out buoyancy anomalies. Although there is no such constraints in the boundary layer, we
 201 can assume that $DT = 0$ at the surface temperature because of the uniform sea surface temperature
 202 (SST). We, therefore, require DT equals DT_{WBG} in the free troposphere but smoothly decays to 0
 203 at surface:

$$DT = DT_{WBG} \times \left(1 - \frac{p}{p_s} \right)^n \quad ; \quad (15)$$

204 where n controls the decay rate with pressure. The $p=p_s$ term would decay faster (slower) with
 205 large (small) n , so different n could potentially result in different amplitudes and altitudes of the

206 maximum temperature difference. We, however, find that the values of $DOLR$ and its sensitivity
207 to surface temperature only change by 50% while we vary n over an order of magnitude, from 5
208 to 50. Therefore, we conclude the results are robust to the choice of n , and we take $n = 30$ in the
209 following calculation. Figure 3a shows DT profiles at different surface temperatures. We find that
210 the peak of DT is around 900 hPa, and that its peak is about 1.5 K at $T_s = 300$ and $b = 0.5$. We
211 also find that DT increases faster with T_s in drier columns, which would be used to explain the
212 sensitivity of $DOLR$ to T_s .

213 Equations (5-9, 15) form the complete set of this model. With proper parameter values, we can
214 estimate the magnitude of $DOLR$ and its change with surface temperature.

215 *f. Results*

216 Our calculation shows that the vapor buoyancy effect can significantly impact Earth's energy
217 balance and future climate changes. Figure 4a shows that $DOLR$ is of $O(4 \text{ W/m}^2)$ for a wide range
218 of parameter values. In the reference climate ($T_s = 300 \text{ K}$), $DOLR$ is about 2.5 W/m^2 with $b = 0.5$,
219 a similar magnitude to the radiative effect due to doubling CO_2 . According to (5), $DOLR$ would
220 increase with higher T , larger DT , or that the altitude of DT maximum becomes closer to the
221 emission level, where $t \sim O(1)$. We use this principle to understand the sensitivity of $DOLR$ to T_s
222 and b .

- 223 • $DOLR$ increases with T_s at given b . This is mainly because DT increase with warming, as
224 will be quantified in Figs. 4b & 4c.
- 225 • $DOLR$ is small at both moist and dry limits. In the moist limit ($b \rightarrow 1$), DT is small according
226 to (14). In the dry limit ($b \rightarrow 0$), although DT maximizes, $DOLR$ is dominated by surface
227 emission, insensitive to DT . The OLR difference, therefore, peaks at intermediate b values.

228 • The *DOLR* peak shifts toward smaller b in warmer climates. This is because, at high temper-
 229 atures, DT increases faster with warming in the small- b columns (Fig. 3a) and also because
 230 the large- b columns become increasingly opaque to IR emission (Fig. 3b-c).

231 Consistent with our hypothesis, *DOLR* increases with T_s , showing a negative climate feedback.
 232 To quantify the feedback strength, we define feedback parameters

$$l_t = \frac{dDOLR}{dT_s}; \quad (16)$$

$$l_{vb} = \frac{dDOLR}{dT_s} \Big|_{T;t}; \quad (17)$$

234 where l_t is the total sensitivity of *DOLR* to T_s , and l_{vb} is the vapor-buoyancy feedback parameter,
 235 which only concerns $dDT = dT_s$. Figure 4b shows that l_t is of $O(0.2 \text{ W/m}^2/\text{K})$ in the reference
 236 climate, which compares with the feedback strength due to clouds and surface albedo. The feed-
 237 back parameter keeps increasing with surface temperature and reaches about $1.4 \text{ W/m}^2/\text{K}$ at 320
 238 K, suggesting that the vapor buoyancy effect becomes increasingly important in future climates.

239 Figure 4c shows that l_{vb} is of similar magnitude to l_t , suggesting the vapor-buoyancy feedback
 240 dominates the entire *DOLR* sensitivity to T_s . We find that l_{vb} is small at the moist and dry limits.
 241 This is because $DT \rightarrow 0$ when $b \rightarrow 1$ at all surface temperatures, and *DOLR* is dominated by
 242 surface emission when $b \rightarrow 0$ at all surface temperatures, not feeling DT and its changes. In
 243 addition, we find that the peak of l_{vb} moves towards small- b columns with warming because DT
 244 increases faster with warming at small- b columns (Fig. 3a), and also because large- b columns
 245 become increasingly opaque at high temperature (Fig. 3b-c), insensitive to changes of DT that
 246 peaks in the lower troposphere.

247 The overall results do not depend on the assumed DT profiles in the boundary layer. Figure
 248 4d-f shows *DOLR*, l_t , and l_{vb} for the free troposphere ($p < 900 \text{ hPa}$). The free-troposphere results
 249 almost reproduce the full-column results, with amplitudes of 10 - 15% weaker than the full-column

250 calculation. This suggests that the vapor-buoyancy radiative effect and feedbacks occur primarily
251 in the free troposphere.

252 **4. Conclusion and discussion**

253 The conventional wisdom is that the vapor buoyancy effect is small, so its impact on temperature
254 is negligible in the free troposphere. However, using NASA AIRS observations, we have demon-
255 strated that the vapor buoyancy effect could lead to about 1.5 K horizontal temperature difference
256 in the lower troposphere, which has significant impact on Earth's radiative balance.

257 Based on the novel observation, this paper proposes that the vapor buoyancy effect can increase
258 Earth's OLR by increasing air temperature in the dry columns. We have developed a simple model
259 that computes the OLR difference between two atmospheres: one with the vapor buoyancy effect,
260 and the other without this effect. We show that the magnitude of this effect is of $O(1 \text{ W/m}^2)$
261 at $T_s = 300 \text{ K}$ and that it increases rapidly with climate warming due to exponential increase of
262 atmospheric water vapor, leading to a negative climate feedback (Fig. 2b). We further show
263 that the feedback strength l is of $O(0.2 \text{ W/m}^2/\text{K})$, the amplitude of which compares with major
264 climate feedbacks, including cloud feedbacks and surface albedo feedbacks. Therefore, faithful
265 representation of the vapor buoyancy effect in climate models is necessary for accurate estimates
266 of climate sensitivity and reliable predictions for future climate changes.

267 The vapor buoyancy effect may help explain why tropical climate has been more stable than
268 extratropical climate (Holland and Bitz 2003; Polyakov et al. 2002; Pierrehumbert 1995). The
269 strength of the vapor buoyancy feedback depends on water vapor contrast between moist and
270 dry columns, which in turn depends on water vapor abundance and thereby temperature in the
271 atmosphere. This effect, therefore, operates more efficiently in the tropics and less efficiently at
272 higher latitudes. This spatial pattern may explain why fluctuations of sea surface temperature in the

273 tropics are much smaller than that of higher latitudes in the past 100 million years (Pierrehumbert
274 1995).

275 The vapor buoyancy effect helps extend the inner edge of the habitable zone, in particular, for
276 tidally locked exoplanets. Tidally locked planets are often slowly rotating, so their free troposphere
277 could be in the WBG regime globally (Koll and Abbot 2016; Mills and Abbot 2013). These plan-
278 ets have one fixed diurnal hemisphere and one nocturnal hemisphere, corresponding to the moist
279 and dry columns of our model, respectively. When the tidally locked planets are approaching
280 the inner edge of the habitable zone, their surface temperature could be significantly higher than
281 Earth's tropical SST, providing an ideal environment for the vapor buoyancy feedback to work ef-
282 ficiently. However, previous studies have neglected the vapor buoyancy effect and assumed WTG
283 (Yang et al. 2013; Yang and Abbot 2014; Pierrehumbert 2010), which could lead to considerably
284 narrower habitable zones. Therefore, we suggest that the vapor buoyancy effect should be accu-
285 rately represented not only in GCMs but also in low-order models that are used to study climate
286 habitability.

287 To focus on order-of-magnitude understanding, we have inevitably introduced simplifications to
288 our model that only considers the clear-sky longwave radiation. An important one is that we use
289 the two-band radiative transfer model, lacking detailed representation of water vapor's absorption
290 spectrum. We have also assumed that b is uniform in altitude, whereas b often has complicated
291 vertical structures in the real atmosphere. However, a suite of cloud-resolving model (CRM) sim-
292 ulations has shown similar estimates of $DOLR$ and I . The CRM uses a comprehensive radiation
293 scheme and explicitly simulates atmospheric circulations and water vapor dynamics. The CRM
294 results have also shown that the vapor buoyancy effect does not affect the short-wave radiation
295 budget and that the clear-sky effect dominates the OLR response. The CRM results, therefore,
296 justify our simplifications and will be presented in a companion paper (Seidel and Yang 2018).

297 *Acknowledgments.*

298 **References**

- 299 Beucler, T., and T. W. Cronin, 2016: Moisture-radiative cooling instability. URL <http://doi.wiley.com/10.1002/2016MS000763>, 1620–1640 pp., doi:10.1002/2016MS000763.
- 300
- 301 Bretherton, C. S., P. N. Blossey, and M. Khairoutdinov, 2005: An Energy-Balance Analysis of
302 Deep Convective Self-Aggregation above Uniform SST. *Journal of the Atmospheric Sciences*,
303 **62 (12)**, 4273–4292, doi:10.1175/JAS3614.1.
- 304 Bretherton, C. S., and P. K. Smolarkiewicz, 1989: Gravity waves, compensating subsidence and
305 detrainment around cumulus clouds. *Journal of the Atmospheric Sciences*, **46 (6)**, 740–759,
306 doi:10.1175/1520-0469(1989)046<0740:GWCSAD>2.0.CO;2.
- 307 Charney, J. G., 1963: A Note on Large-Scale Motions in the Tropics. *Journal of the Atmo-*
308 *spheric Sciences*, **20 (6)**, 607–609, doi:10.1175/1520-0469(1963)020<0607:ANOLSM>2.0.CO;
309 2, URL [http://journals.ametsoc.org/doi/abs/10.1175/1520-0469\(1963\)020\(0607:ANOLSM\)2.0.CO;](http://journals.ametsoc.org/doi/abs/10.1175/1520-0469(1963)020(0607:ANOLSM)2.0.CO;2)
310 [10.1175/1520-0469\(1963\)020\(0607:ANOLSM\)2.0.CO;2](http://journals.ametsoc.org/doi/abs/10.1175/1520-0469(1963)020(0607:ANOLSM)2.0.CO;2).
- 311 Collins, M., and Coauthors, 2013: Long-term Climate Change: Projections, Commitments and
312 Irreversibility. In: *Climate Change 2013: The Physical Science Basis. Contribution of Working*
313 *Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change*
314 [Stocker, T.F., D. Qin, G.-K. Plattner, M. Tignor, S.K. Allen, J. Boschung, A. Nauels, Y. Xia, V.
315 Bex and P.M. Midgley (eds.)]. Cambridge University Press, Cambridge, United Kingdom and
316 New York, NY, USA.
- 317 Emanuel, K. A., 1994: *Atmospheric convection*. Oxford University press. 1994. pp. 580. isbn 0
318 19 5 6630 8.

319 Flato, G., and Coauthors, 2013: Evaluation of Climate Models. In: Climate Change 2013:
320 The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Re-
321 port of the Intergovernmental Panel on Climate Change [Stocker, T.F., D. Qin, G.-K. Plattner,
322 M. Tignor, S.K. Allen, J. Boschung, A. Nauels, Y. Xia, V. Bex and P.M. Midgley (eds.)].
323 Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA. doi:
324 10.1017/CBO9781107415324.020.

325 Held, I. M., and B. J. Soden, 2000: Water vapor feedback and global warming. *Annual Review*
326 *of Energy and the Environment*, **25** (1), 441–475, doi:10.1146/annurev.energy.25.1.441, URL
327 <https://doi.org/10.1146/annurev.energy.25.1.441>.

328 Holland, M. M., and C. M. Bitz, 2003: Polar amplification of climate change in coupled models.
329 *Climate Dynamics*, **21** (3), 221–232, doi:10.1007/s00382-003-0332-6, URL [https://doi.org/10.](https://doi.org/10.1007/s00382-003-0332-6)
330 [1007/s00382-003-0332-6](https://doi.org/10.1007/s00382-003-0332-6).

331 Ingersoll, A. P., 1969: The runaway greenhouse: A history of water on venus. *Journal of the*
332 *Atmospheric Sciences*, **26** (6), 1191–1198, doi:10.1175/1520-0469(1969)026<1191:TRGAHO>
333 2.0.CO;2, URL [https://doi.org/10.1175/1520-0469\(1969\)026<1191:TRGAHO>](https://doi.org/10.1175/1520-0469(1969)026<1191:TRGAHO>2.0.CO;2)
334 [2.0.CO;2](https://doi.org/10.1175/1520-0469(1969)026<1191:TRGAHO>2.0.CO;2), [https://doi.org/10.1175/1520-0469\(1969\)026<1191:TRGAHO>2.0.CO;2](https://doi.org/10.1175/1520-0469(1969)026<1191:TRGAHO>2.0.CO;2).

335 Koll, D. D. B., and D. S. Abbot, 2016: TEMPERATURE STRUCTURE AND ATMOSPHERIC
336 CIRCULATION OF DRY TIDALLY LOCKED ROCKY EXOPLANETS. *The Astrophys-*
337 *ical Journal*, **825** (2), 99, doi:10.3847/0004-637x/825/2/99, URL [https://doi.org/10.3847/](https://doi.org/10.3847/0004-637x/825/2/99)
338 [0004-637x/825/2/99](https://doi.org/10.3847/0004-637x/825/2/99).

339 Manabe, S., and R. T. Wetherald, 1967: Thermal equilibrium of the atmosphere with
340 a given distribution of relative humidity. *Journal of the Atmospheric Sciences*, **24** (3),

341 241–259, doi:10.1175/1520-0469(1967)024<0241:TEOTAW>2.0.CO;2, URL [https://doi.org/](https://doi.org/10.1175/1520-0469(1967)024<0241:TEOTAW>2.0.CO;2)
342 10.1175/1520-0469(1967)024<0241:TEOTAW>2.0.CO;2.

343 Manabe, S., and R. T. Wetherald, 1975: The effects of doubling the co₂ concentration on
344 the climate of a general circulation model. *Journal of the Atmospheric Sciences*, **32** (1), 3–
345 15, doi:10.1175/1520-0469(1975)032<0003:TEODTC>2.0.CO;2, URL [https://doi.org/10.1175/](https://doi.org/10.1175/1520-0469(1975)032<0003:TEODTC>2.0.CO;2)
346 1520-0469(1975)032<0003:TEODTC>2.0.CO;2.

347 Mills, S. M., and D. S. Abbot, 2013: UTILITY OF THE WEAK TEMPERATURE GRADIENT
348 APPROXIMATION FOR EARTH-LIKE TIDALLY LOCKED EXOPLANETS. *The Astrophys-*
349 *ical Journal*, **774** (2), L17, doi:10.1088/2041-8205/774/2/117, URL [https://doi.org/10.1088/](https://doi.org/10.1088/2041-8205/774/2/117)
350 2041-8205/774/2/117.

351 Pierrehumbert, R. T., 1995: Thermostats, radiator fins, and the local runaway greenhouse. *Journal*
352 *of the Atmospheric Sciences*, **52** (10), 1784–1806, doi:10.1175/1520-0469(1995)052<1784:
353 TRFATL>2.0.CO;2, URL [https://doi.org/10.1175/1520-0469\(1995\)052<1784:TRFATL>2.0.](https://doi.org/10.1175/1520-0469(1995)052<1784:TRFATL>2.0.CO;2)
354 CO;2, [https://doi.org/10.1175/1520-0469\(1995\)052<1784:TRFATL>2.0.CO;2](https://doi.org/10.1175/1520-0469(1995)052<1784:TRFATL>2.0.CO;2).

355 Pierrehumbert, R. T., 2010: A PALETTE OF CLIMATES FOR GLIESE 581g. *The Astro-*
356 *physical Journal*, **726** (1), L8, doi:10.1088/2041-8205/726/1/18, URL [https://doi.org/10.1088/](https://doi.org/10.1088/2041-8205/726/1/18)
357 2041-8205/726/1/18.

358 Pierrehumbert, R. T., 2010: *Principles of Planetary Climate*.

359 Polyakov, I. V., and Coauthors, 2002: Observationally based assessment of polar am-
360 plification of global warming. *Geophysical Research Letters*, **29** (18), 25–1–25–4,
361 doi:10.1029/2001GL011111, URL [https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/](https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2001GL011111)
362 2001GL011111, <https://agupubs.onlinelibrary.wiley.com/doi/pdf/10.1029/2001GL011111>.

363 Pritchard, M. S., and D. Yang, 2016: Response of the superparameterized Madden-Julian Oscil-
364 lation to extreme climate and basic state variation challenges a moisture mode view. *Journal*
365 *of Climate*, JCLI-D-15-0790.1, doi:10.1175/JCLI-D-15-0790.1, URL [http://journals.ametsoc.](http://journals.ametsoc.org/doi/abs/10.1175/JCLI-D-15-0790.1)
366 [org/doi/abs/10.1175/JCLI-D-15-0790.1](http://journals.ametsoc.org/doi/abs/10.1175/JCLI-D-15-0790.1).

367 Seidel, S., and D. Yang, 2018: The virtual effect of water vapor helps stabilize tropical climate.
368 *AGU Fall Meeting*, URL <https://agu.confex.com/agu/fm18/meetingapp.cgi/Paper/423217>.

369 Sobel, A. H., J. Nilsson, and L. M. Polvani, 2001: The Weak Temperature Gradient Approximation
370 and Balanced Tropical Moisture Waves*. *Journal of the Atmospheric Sciences*, **58** (23), 3650–
371 3665, doi:10.1175/1520-0469(2001)058<3650:TWTGAA>2.0.CO;2.

372 Tompkins, A. M., 2001: Organization of Tropical Convection in Low Vertical Wind Shears:
373 The Role of Water Vapor. *Journal of the Atmospheric Sciences*, **58** (6), 529–545, doi:
374 10.1175/1520-0469(2001)058<0529:OOTCIL>2.0.CO;2, URL [http://journals.ametsoc.org/doi/](http://journals.ametsoc.org/doi/abs/10.1175/1520-0469(2001)058{\%}3C0529:OOTCIL{\%}3E2.0.CO;2)
375 [abs/10.1175/1520-0469\(2001\)058{\%}3C0529:OOTCIL{\%}3E2.0.CO;2](http://journals.ametsoc.org/doi/abs/10.1175/1520-0469(2001)058{\%}3C0529:OOTCIL{\%}3E2.0.CO;2).

376 Yang, D., 2018a: Boundary layer diabatic processes, the virtual effect, and convective
377 self-aggregation. *Journal of Advances in Modeling Earth Systems*, **10** (9), 2163–2176,
378 doi:10.1029/2017MS001261, URL [https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/](https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2017MS001261)
379 [2017MS001261](https://agupubs.onlinelibrary.wiley.com/doi/pdf/10.1029/2017MS001261), <https://agupubs.onlinelibrary.wiley.com/doi/pdf/10.1029/2017MS001261>.

380 Yang, D., 2018b: Boundary layer height and buoyancy determine the horizontal scale of
381 convective self-aggregation. *Journal of the Atmospheric Sciences*, **75** (2), 469–478, doi:
382 10.1175/JAS-D-17-0150.1, URL <https://doi.org/10.1175/JAS-D-17-0150.1>, [https://doi.org/10.](https://doi.org/10.1175/JAS-D-17-0150.1)
383 [1175/JAS-D-17-0150.1](https://doi.org/10.1175/JAS-D-17-0150.1).

- 384 Yang, D., and A. P. Ingersoll, 2013: Triggered Convection, Gravity Waves, and the MJO: A
385 Shallow-Water Model. *Journal of the Atmospheric Sciences*, **70** (8), 2476–2486, doi:10.1175/
386 JAS-D-12-0255.1, URL <http://dx.doi.org/10.1175/JAS-D-12-0255.1>.
- 387 Yang, D., and A. P. Ingersoll, 2014: A theory of the MJO horizontal scale. *Geophysical Re-*
388 *search Letters*, 661–666, doi:10.1002/2013GL058542, URL [http://onlinelibrary.wiley.com/doi/](http://onlinelibrary.wiley.com/doi/10.1002/2013GL058542/epdf)
389 [10.1002/2013GL058542/epdf](http://onlinelibrary.wiley.com/doi/10.1002/2013GL058542/epdf).
- 390 Yang, J., and D. S. Abbot, 2014: A LOW-ORDER MODEL OF WATER VAPOR, CLOUDS,
391 AND THERMAL EMISSION FOR TIDALLY LOCKED TERRESTRIAL PLANETS. *The*
392 *Astrophysical Journal*, **784** (2), 155, doi:10.1088/0004-637x/784/2/155, URL [https://doi.org/](https://doi.org/10.1088/0004-637x/784/2/155)
393 [10.1088/0004-637x/784/2/155](https://doi.org/10.1088/0004-637x/784/2/155).
- 394 Yang, J., N. B. Cowan, and D. S. Abbot, 2013: STABILIZING CLOUD FEEDBACK DRAMAT-
395 ICALLY EXPANDS THE HABITABLE ZONE OF TIDALLY LOCKED PLANETS. *The As-*
396 *trophysical Journal*, **771** (2), L45, doi:10.1088/2041-8205/771/2/L45, URL [https://doi.org/10.](https://doi.org/10.1088/2041-8205/771/2/L45)
397 [10.1088/2041-8205/771/2/L45](https://doi.org/10.1088/2041-8205/771/2/L45).

398

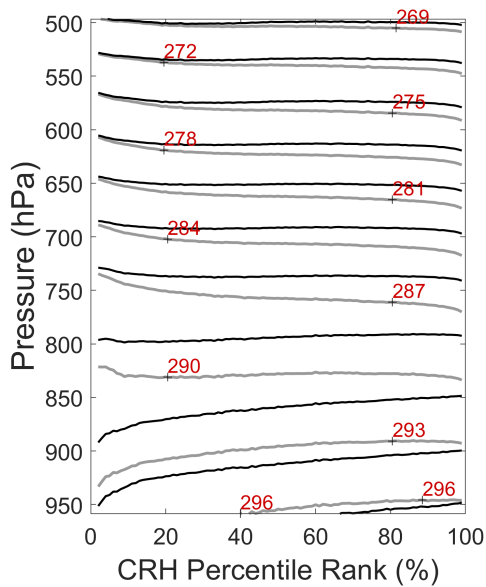
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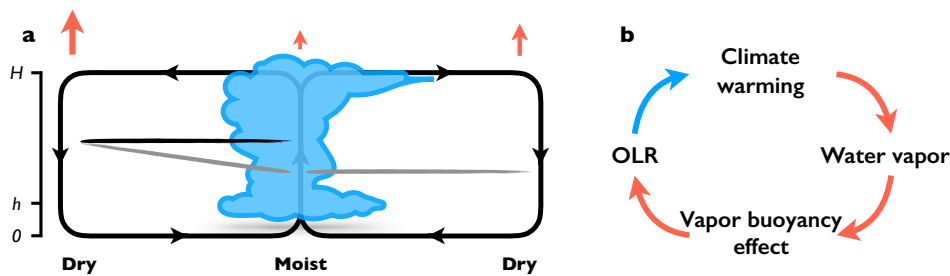
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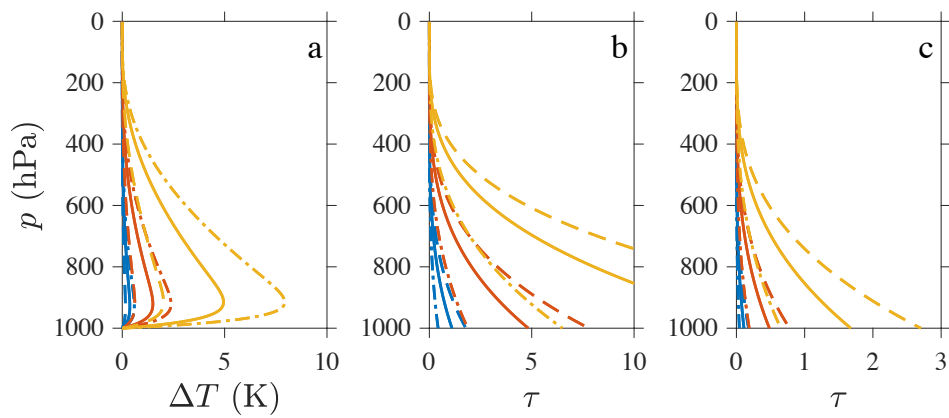
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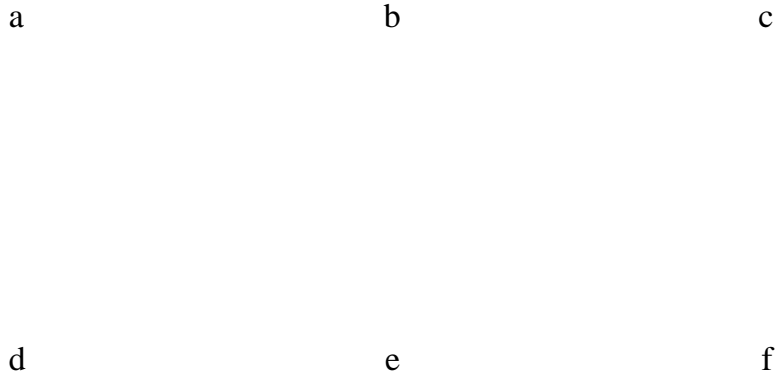
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