A Method to Calculate the Centroid of Areas on a Sphere and its Application to Determining Geographic Centers

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June 6, 2025

Abstract

This study shows how a method of calculating the coordinates of the centroid of discrete points on a sphere can be generalized to determine the centroid of areas on a sphere by minimizing the surface integral of squared spherical distances. A quadratically convergent algorithm for calculating the coordinates of the centroid is presented. Equations are given to verify that the centroid so determined satisfies the condition that the surface integrals of the components of the associated distance vectors be equal to zero.

Evaluating a surface integral over an irregular geographic region is facilitated by using data from a digital elevation model to generate a digital image that, with appropriate editing, can accurately characterize a geographic region for the purpose of determining the location of its geographic center. A case study using the example of North America determines the location of the geographic center of the continent to be at 51°54' N 97°35' W.

1 Introduction

Peter Rogerson has given a comprehensive account of the history and methods of calculating geographic centers [1]. He cites a publication of the U.S. Geological Survey (USGS) that states: "There is no generally accepted definition of geographic center, and no completely satisfactory method for determining it." A commonly used method is based on projecting a region on a sphere onto a plane and then determining the center of the region on the plane based on Euclidean geometry. However the center so determined depends on the projection used which makes this method somewhat unsatisfactory.

Rogerson also cites a paper by Buss and Fillmore [2] who describe a projection-free method for determining the centroid of discrete points on the surface of a sphere based on minimizing the sum of squares of spherical distances. It will be shown how this method can be generalized to determine the centroid of areas on a sphere by replacing a discrete sum of squared distances with a surface integral. The centroid so determined satisfies the mathematical definition of a centroid which, in this context, specifies that the surface integrals of the components of the associated distance vectors be equal to zero.

2 The Centroid of Points on a Plane

The method of Buss and Fillmore for determining the centroid of points on a sphere is a generalization of the method of determining the centroid of points in a plane. Therefore the case of a plane will be considered as preliminary background. The $2 \times n$ matrix P contains the two Cartesian coordinates, x and y, of n points in a plane and the points have equal weight. The coordinates of the centroid, c, of the points are given by the average of the coordinates of the points:

$$c_i = \frac{1}{n} \sum_{k=1}^{n} P_{i,k} \quad (\text{for } i = 1, 2) \tag{2.1}$$

The sum of squares, Σd^2, of distances, d, from the points to the centroid is given by:

$$\Sigma d^2 = \sum_{k=1}^{n} d_k^2 = \sum_{k=1}^{n} \left[(P_{1,k} - c_1)^2 + (P_{2,k} - c_2)^2 \right] \tag{2.2}$$

The partial derivatives of Σd^2 with respect to the two components of c are given by:

$$\frac{\partial \Sigma d^2}{\partial c_i} = \sum_{k=1}^{n} -2\left(P_{i,k} - c_i\right) = 0 \quad (\text{for } i = 1, 2) \tag{2.3}$$

The sum of squares of distances is minimized at the point where the partial derivatives are zero and that point is the centroid given by Equation 2.1. The $2 \times n$ matrix, $d2$, of the vector components of the distances from the points to the centroid is given by:

$$d2_{i,k} = P_{i,k} - c_i \quad (\text{for } i = 1, 2 \text{ and } k = 1, \ldots, n) \tag{2.4}$$

For the purpose of this study the difference between two vectors, $d2$, will be referred to as the distance vector whose magnitude is the scalar distance, d. It follows from Equation 2.1 that:

$$\sum_{k=1}^{n} d2_{i,k} = 0 \quad (\text{for } i = 1, 2) \tag{2.5}$$

Although the values of the components of matrix $d2$ depend on the orientation of the coordinate system the value of the summation does not depend on the orientation. In physics the distance vectors $d2$ are considered as moment vectors. When the spatial distribution of discrete and masses are considered, the point where the components of the associated moment vectors sum to zero is the definition of center of mass, also referred to as the center of gravity.

Using a minimum sum of squares criterion to determine a centroid might appear to give excess influence to the locations of outlying points. This is not the case because Equation 2.1 giving the coordinates of the centroid is a simple average where all points have equal influence. The resolution of this apparent paradox lies in the fact that the partial derivatives of the sum of squared distances given by Equation 2.3 are linear, not quadratic, in the coordinates of the points.

3 The Arc Distance Between Two Points on a Unit Sphere

Geographic coordinates are commonly expressed as latitude and longitude. However for computational accuracy and efficiency Cartesian coordinates will be used here. The three Cartesian coordinates, x, y and z of two points on a unit sphere centered on the origin will be given by the unit vectors c and p. Two intermediate variables, a and b, are defined as:

$$a = (c_1 - p_1)^2 + (c_2 - p_2)^2 + (c_3 - p_3)^2 \tag{3.1}$$

$$b = (c_1 + p_1)^2 + (c_2 + p_2)^2 + (c_3 + p_3)^2 \tag{3.2}$$

The arc distance, d, between the two points c and p is given in radians by:

$$d = 2 \arctan\left(\sqrt{\frac{a}{b}}\right) \tag{3.3}$$

There are several mathematically equivalent ways of expressing this distance using Cartesian coordinates. For example d can be expressed as:

$$d = \arccos\left(c_1\, p_1 + c_2\, p_2 + c_3\, p_3\right) \tag{3.4}$$

But these mathematically equivalent expressions are not computationally equivalent due to the finite precision of floating-point operations. The second expression for d can result in a significant loss of precision if the two points on the sphere are very close to one another. On the other hand the first expression for d is much less susceptible to this problem [3].

As in Section 2, c will refer to the centroid of points on a unit sphere and p will refer one of those points. Newton's method of finding the roots of a system of equations will be used to calculate c so the partial derivatives of d with respect to the components of c will be needed. This is complicated by the fact that the three components of the unit vector c are not independent. The sum of squares of the components must always sum to one. This problem is solved by expressing the component c_3 in terms of the other two components when algebraically determining the partial derivatives of d with respect to c_1 and c_2:

$$c_3 = \pm\sqrt{1 - c_1^2 - c_2^2} \tag{3.5}$$

Under the conventional transformation from latitude and longitude to Cartesian coordinates a negative value of c would indicate a location in the Southern Hemisphere. The intermediate variables r and s are defined as:

$$r = \frac{2}{c_3\sqrt{a\,b}} \tag{3.6}$$

$$s_i = c_i\,p_3 - c_3\,p_i \quad (\text{for } i = 1, 2) \tag{3.7}$$

The partial derivatives of d with respect to c_1 and c_2 are components of the gradient vector g:

$$g_i = \frac{\partial d}{\partial c_i} = r\,s_i \quad (\text{for } i = 1, 2) \tag{3.8}$$

Another intermediate variable t is defined as:

$$t = \frac{4}{a\,b}\,(c_1\,p_1 + c_2\,p_2 + c_3\,p_3) \tag{3.9}$$

Next the partial derivatives of the gradient vector g with respect to c_1 and c_2 will form the components of the Hessian matrix h. Once again in algebraically determining these partial derivatives c_3 is expressed in terms of c_1 and c_2.

$$h_{i,i} = \frac{\partial g_i}{\partial c_i} = \frac{r}{c_3}\left[c_i\,p_i + c_3\,p_3 + s_i\left(\frac{c_i}{c_3} - t\,s_i\right)\right] \quad (\text{for } i = 1, 2) \tag{3.10}$$

$$h_{1,2} = h_{2,1} = \frac{\partial g_1}{\partial c_2} = \frac{\partial g_2}{\partial c_1} = \frac{r}{c_3}\left(\frac{c_1\,c_2\,p_3}{c_3} - t\,s_1\,s_2\right) \tag{3.11}$$

As with the expressions given above for the arc distance between two points on a sphere, there are mathematically equivalent expressions for the gradient and Hessian. The expressions given here were chosen to mitigate the potential loss of precision associated with floating-point operations.

4 The Procedure for Calculating the Centroid of Points on a Sphere

The equations of Section 3 pertain to the centroid, vector c, and one point, vector p, on a unit sphere. In this section n points on a unit sphere will be considered with the index k ranging from 1 to n. The index k will be applied to p to give a matrix of the coordinates of all n points, $P_{i,k}$. Similarly, the index k will be applied to the scalar distance, d (Equation 3.3), the gradient vector, g (Equation 3.8), and the hessian matrix, h (Equations 3.10 and 3.11), giving d_k, $g_{i,k}$ and $h_{i,j,k}$. Because c_3 is not independent of c_1 and c_2 the indices i and j will range from 1 to 2 as in Section 3 except for Equation 4.1. The sum of squares of spherical distances is minimized by using Newton's method to find the values of the centroid coordinates that make the components of the gradient of the sum of squared spherical distances equal to zero. The procedure is iterative and requires an initial estimate of these coordinates. A good estimate is given by first calculating the coordinates, q, of the centroid in 3D space and then projecting the coordinates back onto the spherical surface through normalization to give an initial estimate of the centroid coordinates::

$$q_i = \frac{1}{n}\sum_{k=1}^{n} P_{i,k} \quad (\text{for } i = 1, 3) \tag{4.1}$$

$$c_i \approx \frac{q_i}{|q|} \quad (\text{for } i = 1, 2) \tag{4.2}$$

Equation 3.8 gives the partial derivatives of the spherical distance between c and p with respect to two independent coordinates of c. To calculate the centroid the partial derivatives of the squares of the distances are determined by the chain rule. The gradient, G, of the sum of n squared distances, Σd^2, is given by:

$$G_i = \frac{\partial \Sigma d^2}{\partial c_i} = \frac{1}{n} \sum_{k=1}^{n} 2\, d_k\, g_{i,k} \quad (\text{for } i = 1, 2) \tag{4.3}$$

The Hessian matrix, H, associated with the gradient is also obtained by applying the chain rule:

$$H_{i,j} = \frac{\partial G_i}{\partial c_j} = \frac{1}{n} \sum_{k=1}^{n} 2\,(d_k\, h_{i,j,k} + g_{i.k}\, g_{j,k}) \quad (\text{for } i = 1, 2 \text{ and } j = 1, 2) \tag{4.4}$$

The values of c_1 and c_2 are now updated by solving the matrix equation $H \Delta c = -G$:

$$\Delta c_i = -\left(H^{-1} G\right)_i \quad (\text{for } i = 1, 2) \tag{4.5}$$

The components of the distances d_k gradient $g_{i.k}$ and Hessian $h_{i,j,k}$ are updated with Equations 3.3, 3.8, 3.10 and 3.11 and the next iteration starts with Equation 4.3. With each iteration the magnitude of the gradient vector G should approach zero with quadratic convergence. When the magnitude of the gradient is less than a reasonable threshold, say 10^{-9}, then no further iterations are required and the value of c_3 is calculated using Equation 3.5.

5 The Vector Components of a Spherical Distance

The vector components of the Euclidean distances from a set of points in a plane to the centroid of the points, given by Equation 2.4, are trivial to calculate. And the vector components of the distance from p to c are simply the negative of the vector components of the distance from c to p. The situation is very different with points on a sphere where Euclidean geometry does not apply. On a sphere the vector components of the spherical distance from p to c are not the negative of the vector components of the spherical distance from c to p. When considering the centroid of points p on a sphere it is the vector components of the spherical distances relative to the centroid c, i.e., from c to p, that are relevant.

A sphere can be considered to be a 2D surface embedded in a 3D space. When determining the vector components of the spherical distance, d, from c to p two equivalent methods may be used. The first method calculates a distance vector of three components, $d3$, in 3D space. The vector $u3$ is a unit vector giving the components of the direction of point p relative to point c. In the cross product notation of vector algebra $u3$ is given by:

$$u3 = \frac{c \times p}{|c \times p|} \times c \tag{5.1}$$

The distance vector, $d3$, is given by the product of the scalar distance d (Equation 3.3), and the unit direction vector $u3$:

$$d3 = d\, u3 \tag{5.2}$$

The second method calculates a distance vector of two components, $d2$, in 2D space. This is done by considering the tangent plane on the sphere at the centroid, c. A 2D Cartesian coordinate system on this tangent plane is parameterized by the unit vector z with three components. The final result doesn't depend on the orientation of this 2D coordinate system so the choice of the unit vector z is arbitrary. The direction vector $v2$ on the tangent plane gives components of the direction of point p relative to point c:

$$v2 = \begin{pmatrix} c \cdot (p \times z) \\ p \cdot z - (c \cdot p)\,(c \cdot z) \end{pmatrix} \tag{5.3}$$

where the vector algebra notation for the dot product is used.

The direction vector $v2$ is not a unit vector so it is normalized to give the unit direction vector $u2$:

$$u2 = \frac{v2}{|v2|} \tag{5.4}$$

The distance vector, $d2$, is given by the product of the scalar distance d and the unit direction vector $u2$:

$$d2 = d\,u2 \tag{5.5}$$

Now n points on a sphere will be considered. As before, the index k will be applied to the distance vectors $d2$ and $d3$. The $3 \times n$ matrix $d3$ contains the n distance vectors from the points, p, to the centroid of the points, c, using Equation 5.2. Similarly, the $2 \times n$ matrix $d2$ contains the distance vectors using Equation 5.5. When the centroid is determined by minimizing the sum of squared spherical distances Buss and Fillmore show that the following equations hold:

$$\sum_{k=1}^{n} d3_{i,k} = 0 \quad (\text{for } i = 1, 3) \tag{5.6}$$

$$\sum_{k=1}^{n} d2_{i,k} = 0 \quad (\text{for } i = 1, 2) \tag{5.7}$$

These equations applicable to distance vectors on a sphere have exactly the same form as Equation 2.5 applicable to distance vectors in a plane. The significance of this is that using a least-squares criterion to determine the centroid of points on a sphere is a rigorous generalization of using a least-squares criterion to determine the centroid of points in a plane. As with the case of a plane, the distance vectors $d2$ and $d3$ can be considered as moment vectors. Because their components sum to zero the centroid satisfies the physics definition of center of gravity.

The position taken by the USGS [4] regarding geographic centers is stated as: "There is no generally accepted definition of geographic center and no completely satisfactory method for determining it." The two parts of this statement are not independent. A satisfactory method of determining the geographic center would tend to influence the generally accepted definition. Indeed, the statement just quoted is immediately followed by "The geographic center of an area may be defined as the center of gravity of the surface, or that point on which the surface of an area would balance if it were a plane of uniform thickness." The method of Buss and Fillmore is the first step towards a satisfactory method that satisfies this definition. However, as Rogerson points out, this method is limited to the discrete domain. The second step is to generalize the method from the discrete domain to the continuous domain.

6 The Surface Integral of Squared Spherical Distances

As the name suggests, a surface integral is the integral of a function defined over a surface. It is a generalization of a line integral. The function under consideration here is the square of the spherical distance from a point on the surface of the sphere to a centroid to be determined. A surface integral requires that the surface be parameterized by a system of coordinates. In the case of of a sphere this can either be spherical or Cartesian coordinates. The surface is then projected onto a plane where a double integral can be evaluated. The double integral includes a term that accounts for the distortion of infinitesimal area elements when projected from the surface to the plane. Therefore the value of the surface integral does not depend on the projection used.

In the case of a sphere there are many projections available to choose from. The class of equal-area projections is particularly convenient because the area of elements in the plane is just a constant times the area of corresponding elements on the sphere and the location of a centroid does not depend on the value of this constant. For each point on the plane the inverse projection gives the coordinates of the corresponding point on the sphere. For two points on the plane the inverse projection is used to calculate the spherical distance between the points and, for a centroid determination, the square of this distance. So the purpose of the projection onto a plane is to define the limits of the surface integral in a convenient coordinate system. The surface integral can also be used to verify that the coordinates of the centroid satisfy the definition of a centroid that the surface integral of the components of the associated distance vectors be equal to zero.

An equal-area map projection of a particular geographic region on the globe will now be considered. The coordinates of any point on the map and the corresponding point on the globe can be determined. Therefore the squared spherical distance between any two points on the map can be determined. If the geographic region happens to be like the state of Colorado with regular boundaries then the corresponding double integral can be evaluated analytically. However most geographic regions have irregular boundaries and that presents a problem. The solution to this problem is to cover the region by a square grid of cells small enough to make numerical integration sufficiently accurate. Numerical integration returns the method of calculating the centroid from the continuous domain to the discrete domain where the method of Buss and Fillmore applies.

The accuracy of the method described here is limited by the fact that the Earth is not spherical. It is better represented by an ellipsoid. However the underlying principle of locating a geographic center by minimizing squared arc distances could be applied to an ellipsoid. But this would significantly complicate the mathematics of Section 3 and would likely result in only a modest improvement in the location of the geographic center.

7 Tools For Numerical Integration Over a Geographic Region

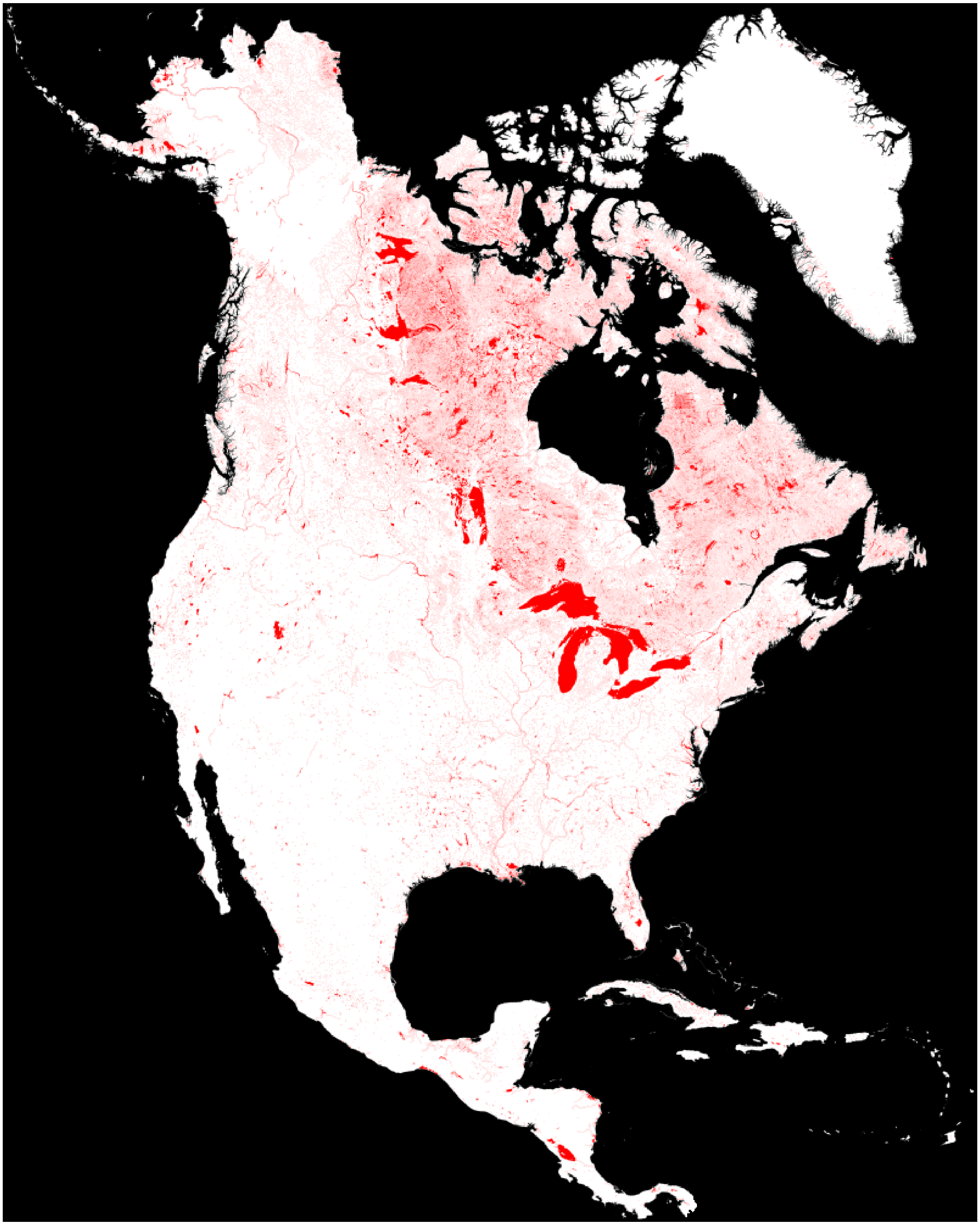
The task of setting up a square grid of small cells over a geographic region with irregular boundaries may seem to be difficult but a solution does present itself. A digital image of a geographic region is made up of pixels, i.e., a square grid of small cells. So a digital image of the region under consideration is generated using an equal-area projection. The digital image will include the boundaries of this region but it may also include extraneous regions. Digital image editing tools are then used to erase these extraneous regions from the image. The program to generate the digital image uses the specifications of a digital image file format. Windows bitmap format was used in this study. The program to calculate the geographic center uses the file specifications to access the geographic information encoded in the pixels of the edited image file. Each pixel corresponds to a cell in the square grid used for the numerical surface integration.

The borders between political regions are often defined by line segments facilitating accurate numerical integration. However coastlines can have a fractal nature that is not well represented by line segments. The solution to this problem is to use a digital elevation model (DEM) to distinguish land from sea under the assumption that offshore areas will have zero elevation. For the purpose of this study the term land will refer to regions that are not sea and so the term land will include lakes and rivers. The Shuttle Radar Topography Mission (SRTM) data provides a resolution of 90 meters between latitudes 60° N and 60° S with an elevation resolution of one meter. Land on coastlines has a minimum elevation of one meter making it possible to clearly distinguish land from sea. Data for polar regions is available from other sources such as ASTER GDEM.

Extraneous regions are the primary features of a digital image requiring editing. Other features that can require editing are major rivers flowing into the sea where the mouth of the river is at sea level. Because the elevation resolution of the SRTM data is one meter a river can have an elevation of zero far upstream from its mouth. For example, the Hudson River has an elevation of zero as far away as Albany over 200 km from the mouth of the river at New York Harbor. With an elevation of zero these river regions will be represented as sea in the digital image so editing of the image is required to represent the regions as land.

8 Case Study - The Geographic Center of North America

This azimuthal equal-area digital image of North America was generated using SRTM and other data. The projection is centered at 47°N 81°W. The vertical range along the 81°W meridian is 80°.



The most notable feature of this image is that extraneous regions of Russia, Iceland and South America have been erased using a standard digital image editing program. What is left is an image of North America that is based on the definition of the continent cited by authoritative sources [5], [6], [7], [8], [9], [10].

The procedure for generating a digital image of a geographic area begins with an azimuthal equal-area projection of the area onto an array of finely divided cells over which the numerical surface integration with be performed. DEM data is typically organized into tiles covering a limited area. The tiles needed to completely cover the geographic area are identified. The location of every data point in each tile is then projected onto the array and the corresponding cell is identified. For each cell two running counts are made, one for land and one for sea. If the corresponding elevation of the projected point is zero then the count for sea is incremented. Otherwise the count for land is incremented. At the end of this process the land fraction for each cell is calculated from the counts. Each cell in the array corresponds to a pixel in the image. The last step is to encode the land fraction in each cell as a shade of grey in the corresponding pixel. The numerical surface integration is a weighted sum over the cells with the weights being the land fraction for each cell.

This procedure is computationally intensive. For North America and the DEM data in 276 tiles of 5° by 5° 40 billion trigonometric evaluations are required. A more direct method would be to project from each cell in the plane to a single data point in the DEM file. The problem with this approach is that the single point in the data file is binary, land or sea. Therefore the value assigned to the cell is binary, zero or one, and not a land fraction. So with the direct method the high-resolution of the detailed coastline information in the DEM data would be degraded.

For this study a water mask for the DEM data was available making it possible to represent inland bodies of water by adding shades of red to the image palette. An 8-bit pixel format can represent 256 colours. Half of these can be assigned to a greyscale for coastal shorelines and the other half to shades of red for inland shorelines. The image below shows a closeup of the Long Island region taken from an image of the continent with 19200 by 24000 pixels. The SRTM data has the Hudson river at zero elevation in this region so editing of the image was required to change the river colour from black (sea) to red (land). The edited image file is read by the program for calculating the geographic center. The inverse of the projection used to create the image converts the planar image coordinates to spherical coordinates. And the pixel colour codes are converted back into land fractions for weighting each corresponding cell in the numerical surface integration.

A study was made to investigate the effect of varying the image resolution on the calculated coordinates of the geographic center of the North American continent. The vertical size of the image was varied from 1000 to 24000 pixels in increments of 500 pixels. Figures 1a and 1b give the latitude and longitude of the geographic center plotted against the number of pixels. The discrete nature of the images introduces a degree of random noise into the coordinates. Both coordinates vary only slightly from an asymptote for images in excess of 10000 pixels. This suggests that an image with the resolution of the SRTM data, equivalent to 96000 pixels, would yield similar results. The values of the two aymptotes give the coordinates of the geographic center of the North American continent to be 51° 54’ N 97° 35’ W.

Figure 1a - Latitude of the Geographic Center

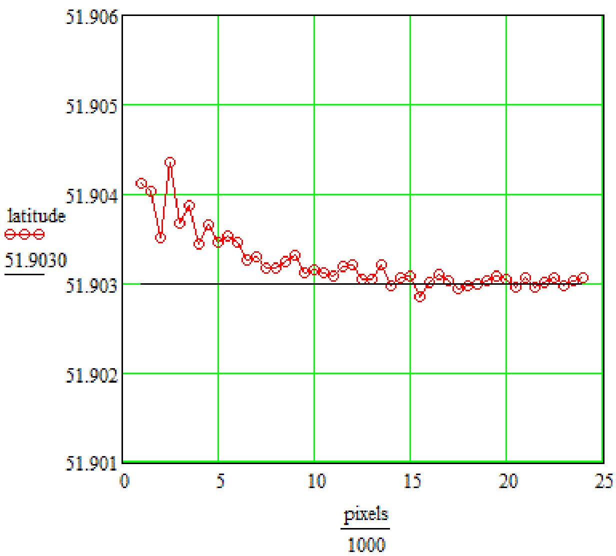
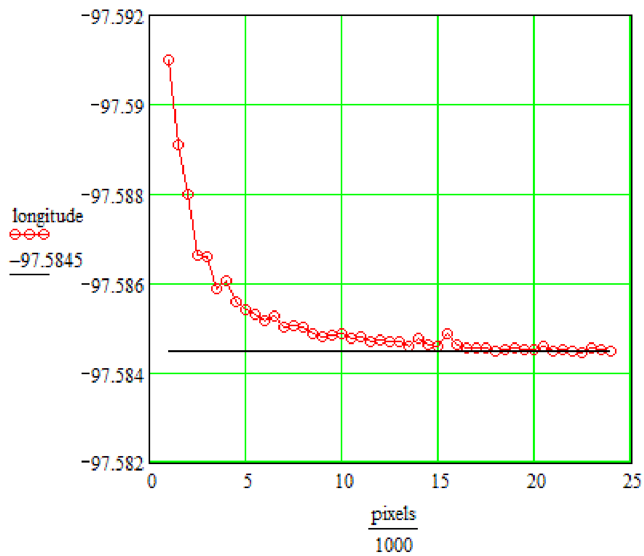
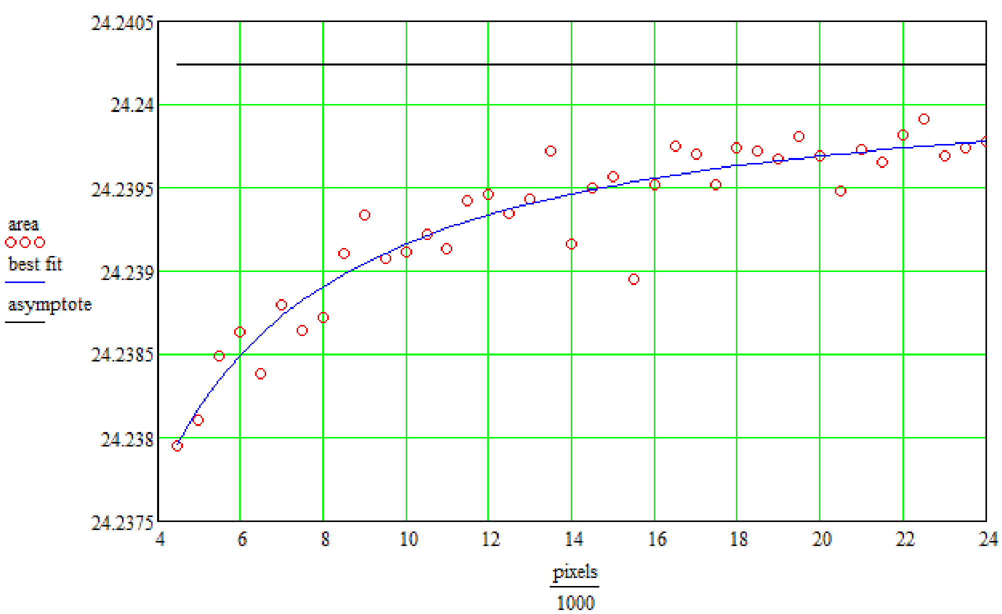


Figure 1b - Longitude of the Geographic Center



The surface integral can also be applied to determine the area of North America. With an equal-area projection the area of each pixel in the image is the same and is known. So the surface integral simply sums land pixels with coastline pixels having a fractional value. To convert the area from a unit sphere to the Earth a mean radius of 6371 km was used. Figure 2 shows how the area in millions of km^2 varies with image resolution expressed as the number of vertical pixels. The values of the areas converge linearly to an asymptote of 24.24 million km^2 with increasing resolution. This is in close agreement with the area cited by Encyclopedia Britannica of 24.23 million km^2 [5].

Figure 2 - Area of North America (km^2 / 10^6)



9 Conclusion

Buss and Fillmore showed how the standard method of determining the centroid coordinates of discrete points in a plane, equivalent to minimizing a sum of squared Euclidean distances, can be generalized to determine the centroid coordinates of discrete points on a sphere by minimizing a sum of squared spherical distances. This can be further generalized to determine the centroid coordinates of areas on a sphere by minimizing the surface integral of squared spherical distances. The minimization procedure is iterative but rapidly converges using a quadratically convergent algorithm. The centroid so determined satisfies the mathematical definition of a centroid, that the surface integrals of the components of the associated distance vectors be equal to zero.

Evaluating a surface integral over an irregular geographic region can be facilitated by using data from a digital elevation model to generate a digital image that, with appropriate editing, can accurately characterize a geographic region for the purpose of determining its geographic center. A case study using the example of North America determined the location of the geographic center of the continent to be at 51° 54' N 97° 35' W. This location is near the Kinonjeoshtegon First Nation community on the shore of Lake Winnipeg. It is a fitting coincidence that an Indigenous community resides at the heart of the North American continent.

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Contact

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Appendix - A Note on the Geographic Center of North America

Previous determinations of the geographic center of North America have placed it at two locations in the state of North Dakota. A plaque mounted on a stone monument in Rugby, N.D. in 1931 reads "Geographical Center of North America". A more recent plaque placed near Center, N.D. in 2017 reads "The Scientific Center of North America". Center, N.D. is 600 km from the location of the geographic center of the North American continent determined by this study, a discrepancy that merits an explanation.

In 1928 an employee of the USGS balanced a cutout of a profile of North America on a pin and determined the geographic center to be near Rugby, N.D. [11]. A depiction of this cutout indicating the location of Rugby, N.D. is shown below. The cutout could not physically accommodate offshore islands therefore they were excluded from the profile. Among these offshore islands were Greenland, the islands of the Arctic Archipelago, Newfoundland, and the islands of the Caribbean. These islands collectively comprise one sixth of the area of the North American continent. The USGS cutout of North America therefore fell far short of representing the entire continent. It should be noted that the inclusion of Greenland and the islands of the Caribbean as part of North America is cited by authoritative sources [5], [6], [7], [8], [9], [10].

Having set the precedent of excluding the offshore islands of North America in 1928, the USGS established the policy of excluding offshore islands in subsequent determinations of the geographic centers of individual states [4]. The 2016 study determining the geographic center of North America to be near Center, N.D. adhered to this policy [12]. This explains the discrepancy noted above. The policy of the USGS regarding the status of offshore islands originated with a cutout that was physically limited to representing just the mainland of North America. Computer based methods of determining geographic centers can accurately represent the entire continent. It is therefore recommended that the USGS policy be revised accordingly.

Photo by CBS News