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ORIGINAL PAPER

Limitations of M_w and M Scales: Compelling Evidence Advocating for the Das Magnitude Scale (M_{wg})—A Critical Review and Analysis

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Abstract Precise determination of earthquake size is crucial for various geoscientific and engineering applications. The Moment Magnitude (M_w) scale, introduced by Kanamori in 1977, was a significant advancement. Kanamori (1977) advocated use of M_w for large earthquakes (≥ 7.5). (Hanks and Kanamori in J. Geophys. Res. 84:2348–2350) later extended the M_w scale named as M scale by considering close coincidence of three equations. The use of the moment magnitude scale M scale for magnitudes below 7.5 is not appropriate, as Eq. (1) from Purcaru and Berckhemer (Purcaru and Berckhemer in Tectonophysics 49:189–198, 1978) was specifically derived for M_s values in the range of Ms $\lesssim$ 7.0. Furthermore, the M scale has not been validated globally for magnitudes below 7.5; its validation is limited to Southern California. Additionally, the M or M_w scale is based on surface waves and may not be applicable for all earthquake depths. Furthermore, Gutenberg and Richter (Gutenberg and Richter in Bull Seismol Soc Am 46:105–145, 1956) recommended using body waves, rather than surface waves, for the development of a magnitude scale as surface waves do not represent the earthquake source. To address these shortcomings, the Das Magnitude scale (M_{wg}) has been introduced in recent literature (Bulletin of Seismological Society of America, Das et al. (Das et al. in Bull Seism Soc Am 109:1542–1555, 2019); Natural Hazard, 2023), incorporating global data during 1976–2006 with 25,708 events with observed seismic moments (Mo) and body wave magnitudes (m_b), in line with the recommendations of Gutenberg and Richter (Gutenberg and Richter in Bull Seismol Soc Am 46:105–145, 1956). Recent seismological literature (Gasperini and Lolli, (Gasperini and Lolli in Bull Seismol Soc Am, 2024)) has inaccurately critiqued *Das* et al. (Das et al. in Bull Seism Soc Am 109:1542–1555, 2019), stating that the M scale is adequate and suggesting that certain foundational assumptions of the M_{wg} scale are inappropriate. However, our detailed analysis demonstrates that the M_{wg} scale is firmly grounded in robust scientific evidence and constructed on sound principles. It is important to note that all magnitude scales, including M_w, M, and Me, are developed using linear least squares methods. Therefore, if the fundamental assumptions underlying the Mw, M, and Me scales are considered valid, there is no justifiable reason to challenge the foundational assumptions of the M_{wg} scale. M_{wg} uses body-wave magnitude instead of surface waves and is applicable to all depths, making it more suitable for a wider range of earthquakes. M_{wg} is a better measure of energy release compared to M_w, providing a more accurate representation of earthquake strength. The M_{wg} scale demonstrates a closer correspondence with observed m_b and M_s values at a global level compared to the M scale. The average difference between observed m_b and M is -0.31 ± 0.30, whereas the difference between observed m_b and M_{wg} is significantly smaller at 0.008 ± 0.33 (Das et al. (Das et al. in Bull Seism Soc Am 109:1542–1555, 2019)). The uncertainty associated with the development of M_{wg} is limited due to its simplicity, while M_w or M involve a constant term and multiple substitutions, potentially introducing additional uncertainty. M_{wg} was developed and validated using global datasets, ensuring its applicability to a diverse range of seismic events. The M_{wg} scale significantly reduces the statistical differences with m_b and M_s compared to the larger discrepancies observed with M, offering a more reliable framework for

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understanding regional energy budgets. In this manuscript, we critically compare M_w, M, and M_{wg} scales. We revalidated the M scale using a comprehensive global dataset, finding that it significantly deviates from observed m_b (<5.5) and M_s (6–8) within their respective applicable ranges. Furthermore, revalidation of the M scale concludes that it is not appropriate to be below 7.5.

Keywords Seismic moment magnitude scale · Body-wave magnitude (mb) · Surface-wave magnitude · M_{wg} Scale · Seismic moment

Introduction

Accurate measurement of earthquake magnitudes is fundamental for all seismological and engineering studies. Over the years, several magnitude scales have been developed, each with its own methodology and scope. The Richter scale, introduced in 1935, was one of the first methods used to quantify earthquake size based on the amplitude of seismic waves. However, its applicability was limited to specific regions. Conventional magnitude scales, including m_b, M_s, and M_L, encounter saturation issues when the earthquake's rupture dimension exceeds the wavelength of the seismic waves used for magnitude determination. For large earthquakes, rupture dimensions often exceed the wavelengths of seismic waves commonly employed for magnitude estimation. This phenomenon, known as saturation, introduces systematic errors in assessing the energy released, hindering accurate magnitude determination. Traditional magnitude scales, such as M_L and m_b, based on short-period (1 Sec) seismic waves, typically saturate around magnitude 7.0 ([1], [2]). While the surface wave magnitude (Ms) extends the saturation limit to approximately 8.3, it remains inadequate for characterizing the largest earthquakes. To address these limitations, the moment magnitude scale (M_w) was introduced by Kanamori [3] in seismic literature to understand the earthquake size without saturation. As Kanamori [3] defined M_w scale for large earthquakes (≥7.5), therefore, Hanks and Kanamori [1] extended the M_w scale to measure lower, medium and large earthquakes and in this way M scale (1979) has been the standard for decades, offering a means to quantify the energy released during an earthquake. Despite its widespread use, the M scale is not without its shortcomings. Given the extensive use of the M scale as global scale in seismological and geophysical literature, it is crucial to assess its suitability for accurate evaluation and interpretation in terms of energy. The term M_w is commonly used by GCMT (Global Centroid Moment Tensor) for estimating moment magnitude. However, according to Hanks and Kanamori [1], the correct term should simply be M if one intends to measure earthquakes across all ranges (≥3), including small, medium, large, and very large events.

The inadequacy of surface waves in characterizing deeper earthquakes prompted Gutenberg and Richter [4] to propose a magnitude scale based on body waves. Subsequently, Kanamori [5] introduced a magnitude scale derived from seismic moment (Mo) for more accurate representation of seismic events for all depths. The complex nature of earthquakes precludes a singular metric for their quantification, necessitating the development of diverse magnitude scales, such as Me [6] and M_{wg}, [7–10] to comprehensively characterize seismic events. The M_{wg} scale aims to offer a more accurate representation of earthquake magnitudes by incorporating a broader frequency range and minimizing regional biases. Through a detailed comparison of the M_w, M and M_{wg} scales, including an analysis of their respective strengths and limitations, this study highlights their importance in seismology.

Background of M_w Scale

Kanamori's Introduction of the M_w Scale (1977)

In 1977, Kanamori introduced an earthquake magnitude scale that revolutionized the way seismic events are quantified and understood. This scale, commonly referred to as the moment magnitude scale (M_w), was designed to overcome the limitations of earlier magnitude scales such as the Richter scale (1935), body wave magnitude (m_b), surface wave magnitude (M_s), and local magnitude (M_L). The development of the M_w scale involved several key steps:

1. *Empirical Relationship:* Kanamori [3] utilized the empirical relationship between Gutenberg and Richter energy and magnitude:
   a. $\log E_s = 1.5\ Ms + 11.8$. This relationship was derived from the Gutenberg-Richter equation for seismic energy:
   b. $\log E = 5.8\ m + 2.4$. and the substitution $m = 2.5 + 0.63\ M$.

2. *Development of Moment Magnitude Scale M_w*: Kanamori [3] expressed Gutenberg-Richter energy equation in terms of seismic moment using the equation:
   a. $E/M_0 = (\Delta\sigma)/2\mu = 5\times 10^{-5}$, where σ represents earthquake stress drop and μ is the shear modulus. By substituting $E/M_0 = (\Delta\sigma)/2\mu = 5\times 10^{-5}$, into the Gutenberg-Richter energy equation, Kanamori derived: $\log M_0 = 1.5\ Ms + 16.1$. Kanamori [3] reversed the equation $\log M_0 = 1.5M_s + 16.1$ to

express surface wave magnitude (M_s) in terms of seismic moment (M_0) and termed this as the moment magnitude (M_w). The validation of the M_w scale was conducted by comparing it with observed surface wave magnitudes (M_s) (Table 2 of [3]). Kanamori asserted that the M_w scale could be seen as a natural extension of M_s, as outlined in Table 2 of Kanamori [3]. This comparison demonstrated that the M_w scale provided a consistent and reliable measure of earthquake magnitude that aligned well with observed M_s values earthquakes. M_w scale of Kanamori [3] was derived for large earthquakes (magnitude ≥ 7.5), addressing the saturation problem of earlier magnitude scales for such events. However, recognizing the need for a magnitude scale that could accurately measure smaller, medium, large and large earthquakes, Kanamori emphasized the importance of developing a scale appropriate for a broader range of seismic events. This led to the further refinement of the M_w scale in a later study by Hanks and Kanamori [1].

Hanks and Kanamori's Refinement (1979)

The moment magnitude scale (M_w) was further updated by Hanks and Kanamori in 1979 to extend the initial work by Kanamori [3]. To achieve this, Hanks and Kanamori [1] compared the Eqs. (4–6) from their study and found that these equations closely coincided. Based on this comparison, they proposed the extended moment magnitude scale M as 2/3log Mo-10.7. In the abstract of Hanks and Kanamori [1], they explicitly stated that the M scale is developed based on the close coincidence between their Eqs. (4–6) which lead them to define M scale as 2/3log (Mo)-10.7 and stated that M is uniformly valid for $3.0 \lesssim M_L \lesssim 7.0$, $5 \lesssim M_S \lesssim 7.5$, and $M_w \gtrsim 7.5$.

Furthermore the derivation of the M scale involved incorporating the ratio $E/M_0 = (\Delta\sigma)/2\mu = 5\times 10^{-5}$ into the Gutenberg-Richter energy magnitude equation as performed in Kanamori [3]:

$$\mathrm{Log}E = 1.5M_s + 11.8$$

where M_s represents the surface wave magnitude. Hanks and Kanamori [1] reversed the resultant equation and termed the M_s as M_w. Kanamori [3] clearly stated that the constant value $\Delta\sigma$ is strictly applicable for shallow and large earthquakes, and the share modulus μ ($3–6\times 10^{11}$) is strictly applicable for crustal earthquakes. Thus these limitations of M_w in the constant also applies in M scale of Hanks and Kanamori [1]. To enhance the accuracy of the M_w scale to lower and intermediate earthquakes, Hanks and Kanamori compared the equation $\log M_0 = 1.5Ms + 16.1$with the Eq. (1) by Purcaru and Berckhemer [11]. Additionally, they compared it with the California local equation within the range $3.0 \le M_L \le 7.0$. However, significant spatial variations in local magnitude were observed, indicating the need for further study (e.g., [6, 12, 13], Aditya et al. [14], Amit et al. [15], Amit, [16]). The validation of M scale [1] was conducted utilizing local and surface wave magnitudes of Southern Californian seismicity (Tables 1 and Table 2 of [1]).

Limitations and Recent Critiques

M scale is based on surface waves and hence not appropriate for deep and intermediate earthquakes. Recent investigations suggest that the M scale developed by Hanks and Kanamori [1] is not adequate on a global level, particularly for smaller and medium earthquakes. This discrepancy may arise from the limited data range used by Purcaru and Berckhemer [11], use of surface wave magnitude in equation Log Mo = 1.5 Ms + 16.1, use of constant value 5×10^{-5} in the Gutenberg Richter energy equation and the localized validation in Southern California. This localized approach raises questions about the universal applicability of the M scale across diverse tectonic settings. The details limitations we are discussing below.

Limitations of Purcaru and Berckhemer's Equation: Purcaru and Berckhemer [11] explicitly stated that their derived equation Log $M_0 = 1.5$ Ms + 16.1 is reliable exclusively below Ms $\lesssim$ 7.0, emphasizing its accuracy within the range $5.0 \le$ Ms $\lesssim$ 7.0. However, Hanks and Kanamori [1] asserted that Purcaru and Berckhemer [11] developed the equation Log $M_0 = 1.5$ Ms + 16.1 for the range $5.0 \le Ms \le 7.5$ (Page 2348 of [1], stating, "Which is remarkably coincident with the M0-Ms relationship empirically defined by Purcaru and Berckhemer [11] for 5 $\lesssim M_s \lesssim$ 7.5: Log$M_0 = 1.5$ Ms + 16.1 (± 0.1)" consequently, the M scale proves inadequate for measuring earthquake sizes below magnitude 7.5. However, M scale is applicable for large earthquakes (≥ 7.5).

Applicability of the Unsaturated M Scale: The unsaturated M scale holds global applicability for large earthquakes, as its foundation lies in equations such as log $E_s = 1.5M_s + 11.8$ [17] and LogMo = 1.5 Ms + 16.1. To extend support to smaller earthquakes within the M scale, Hanks and Kanamori [1] incorporated the relationship between Log M_o and M_L for Southern California [18], focusing on the magnitude range $3 \lesssim M_L \lesssim 7.0$. Consequently, the M scale can be deemed suitable exclusively for Southern California within the range 3 $\lesssim$ magnitude $\lesssim$ 7.0, raising concerns about its universal applicability. It's essential to recognize that different regions worldwide possess distinct tectonic environments and geological settings. Numerous studies have underscored notable spatial variations in local magnitude (e.g., [6, 12, 13]). Utilizing the M scale (2/3logMo-10.7) for smaller magnitude earthquakes on a global scale could potentially compromise earthquake hazard evaluations, early warning systems, and other seismic studies.

Validation of the M Scale: The development of a new method or scale requires a crucial step: validation. Typically, this involves the comparison of observed and estimated magnitudes, a standard practice in seismological literature ([1, 3], Ekström et al. [19]). Kanamori [3] validated the M_w scale (≥ 7.5) on a global level (see Table 1 of [3]), while Hanks and Kanamori [1] focused their validation specifically on the tectonic conditions in Southern California, as detailed in Tables 1 and 2 of their publication. However, the limited scope of validation for the M scale by Hanks and Kanamori [1], confined to Southern California, raises questions about its applicability beyond this specific region for magnitudes ≲7.5. The absence of validation across diverse global tectonic settings suggests potential limitations in the universal suitability of the M scale, especially when applied beyond the region for which it was derived and validated. An analysis by Das et al. [8] further highlights significant discrepancies between different magnitudes (m_b, M_s, M_e) and M scale on a global level (see Figs. 1, 2, and 3 of [8]). The differences between observed m_b and M_s values compared to M_{wg} and M are statistically significant, as evidenced by null hypothesis testing.

Applicability for Shallow Earthquakes: Since surface waves are inadequate for measuring the size of deeper earthquakes, Gutenberg and Richter [4] recommended creating a magnitude scale based on body waves. Likewise, Kanamori [5] introduced an alternative magnitude scale equation, defined in terms of Mo, for assessing deep earthquakes. Kanamori [3] clearly stated that the constant used in the development of the M_w scale is applicable only to shallow and large earthquakes, specifically within the context of crustal earthquakes. Kanamori [5] introduced a distinct equation tailored for measuring earthquake size in the case of intermediate and deeper earthquakes, acknowledging the limitation of the M or M_w scales primarily designed for shallow earthquakes. This differentiation highlights a fundamental inadequacy within the M_w or M scales.

In the evaluation of earthquake sizes across various depths — shallow, intermediate and deeper — two key equations come into play [1, 5]: Equation (i) for M = 2/3 Log Mo—10.7 (Based on Surface Wave, applicable for shallow depths) and Equation (ii) for m_w = Log Mo/2.4−10.1/2.4 (Based on Body waves, applicable for all depths). The existence of these distinct equations, each associated with specific depth ranges, emphasizes the nuanced nature of earthquake size assessment based on depth considerations. This delineation prompts the recognition of an inadequacy within the M scale, tailored predominantly for shallow seismic events. Consequently, there arises a compelling rationale for embracing a new earthquake scale that seamlessly captures seismic events across all depth ranges–shallow, intermediate, and deeper.

Derivation from Non-Direct Measurements: The formula for M scale (2/3logMo-10.7) was not derived from direct measurement of observed seismic moment. Instead, this scale was derived by substituting a constant term (Es/Mo = 5×10^{-5} = Δσ/2μ) into the Gutenberg energy equation Log $E_S = 1.5M_s + 11.8$. It is important to note that Δσ generally vary from a few bars to 125. The variability of Δσ is significant, and as a result, the assumption of constant stress drop cannot be made (Percaru and Berckhemer [11], [8, 9]). Consequently, depending on the constancy value, the M value for a given earthquake will change significantly. Furthermore, it is worth noting that the value of constancy (Es/Mo = 5×10^{-5}) suggested by Kanamori [3] is only applicable for shallow earthquakes.

Choy and Boatwright [6] used a different constant value (Es/Mo = 1.6×10^{-5}). Bormann and Di Giacomo [20] indicated that there can be significant variations in the constant value, ranging from ($-7\ominus_k-3$), while Kanamori used a value of θk = − 4.3. Therefore, there are significant limitations in the M_w or M scales due to the adoption of a fixed Es/Mo value of 5×10^{-5}.

Recent studies by Gasperini and Lolli [21] have claimed that the M scale is adequate for measuring earthquake magnitudes. However, they failed to address the limitations of the M scale as outlined in this study. Their assertion of adequacy is based on the introduction of a misprint, which lacks scientific evidence and is irrelevant to the broader discussion. Notably, Eq. (1) of Purcaru and Berckhemer [11] explicitly demonstrates that the Ms scale becomes unreliable for magnitudes exceeding 7.0.

Our comprehensive analysis, detailed in Appendix A, underscores the limitations of the M scale and demonstrates the strength of the Das scale (M_{wg}) in addressing these shortcomings. The claims by Gasperini and Lolli [21], which fail to adequately consider the well-documented constraints of the M scale, lack substantive evidence to counter our findings. By providing rigorous revalidation using global datasets and transparent methodologies, our study establishes a scientifically robust case for the adoption of the Das scale (M_{wg}) as a superior tool for earthquake magnitude assessment.

Development of Das Magnitude Scale

Kanamori [3] introduced the moment magnitude scale, M_w, to address the shortcomings of existing magnitude scales like M_L and M_s. This scale was validated for magnitudes ≥ 7.5 through comparisons with observed M_s values, as shown in Table 1 and Table 2 of Kanamori's study. Later, Hanks and Kanamori [1] extended the M_w scale to lower magnitudes using a similar approach. They validated the extended scale by comparing M estimates with observed M_s and M_L

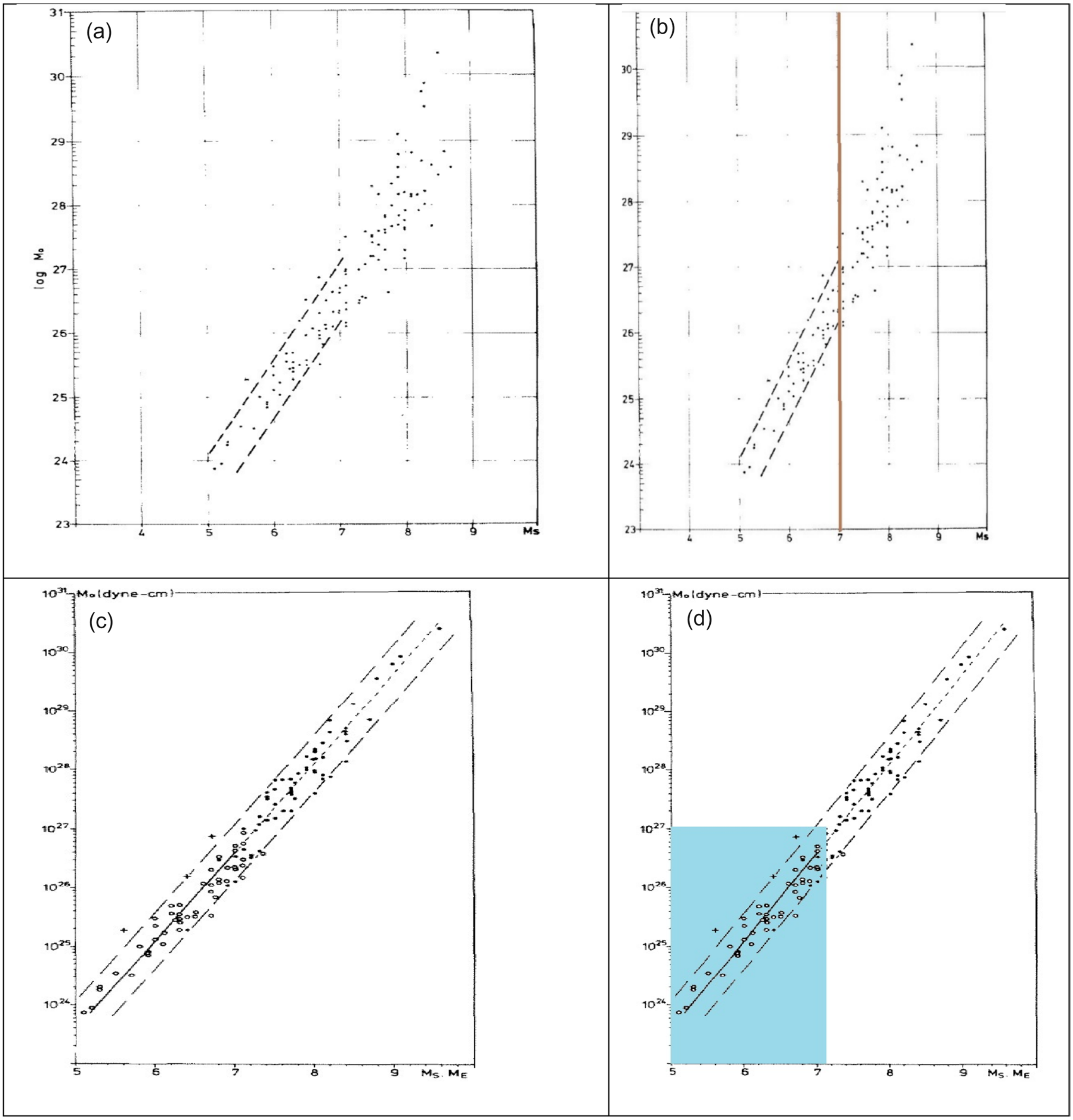

Fig. 1 We reproduce the plots in Figs. 1 and 4 of Percaru and Berckhemer [11]: **a** Original Fig. 1A of Percaru and Berckhemer [11], **b** Vertical brown line clearly shows that Log M_o and M_s shows linear relation, **c**: Original Fig. 4 of Percaru and Berckhemer [11]. **d**; Highlighted rectangle denotes the Log M_o and M_s data as shown in Fig. 4 of Percaru and Berckhemer [11]. Figures 1 and 4 of Percaru and Berckhemer [11] are reproduced with permission from @elsevier 1978

values for Southern California, as detailed in Tables 1 and 2 of their research. However, surface wave magnitudes (Ms) are not suitable for intermediate and deeper depth earthquakes, which is a critical limitation raised by Gutenberg and Richter [4]. They suggested using body waves to derive the magnitude scale. To address this, Kanamori [5] proposed a separate magnitude scale, $m_w = (\text{Log Mo} - 10.4)/2.4$, based on the linear equation Log Mo = 2.4 MB + 10.1, specifically for deeper earthquakes.

In summary, the current limitations and critiques of the M scale strongly warrant the development of a new magnitude scale. It is therefore inappropriate to claim that there is no room

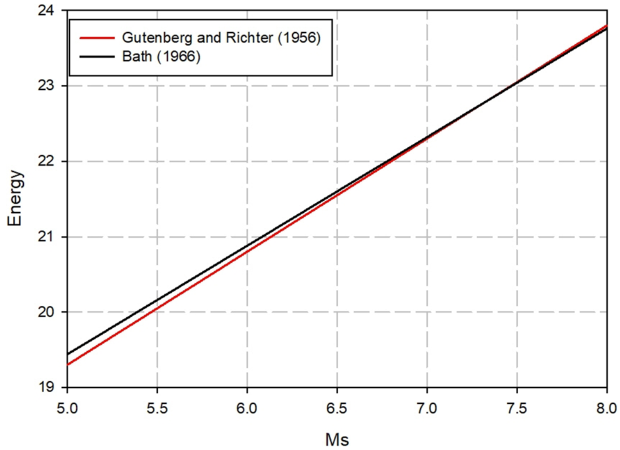

Fig. 2 Comparison of energy magnitude equations of Gutenberg and Richter [4] with Bath [23]. The figure illustrates the logarithmic relationship between earthquake magnitude and energy release as defined by Gutenberg and Richter's equation and Bath's equation

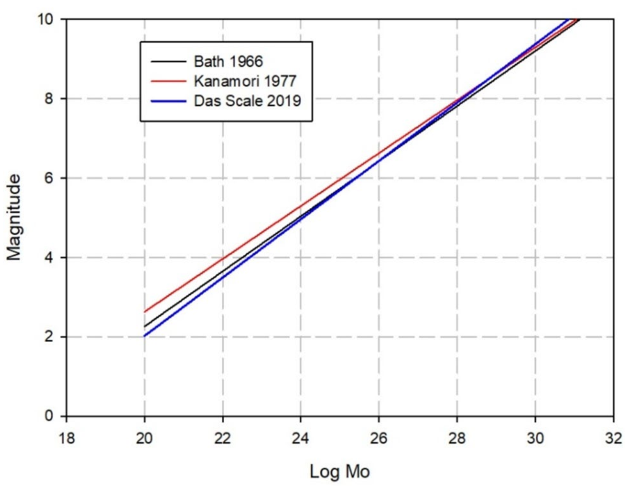

Fig. 3 Comparison of Kanamori Mw scale, Bath [23] $M_{w\text{-}bath}$, and Das Scale. The figure illustrates the logarithmic relationship between earthquake magnitude and energy release as defined by Gutenberg and Richter's equation and Bath's equation

for the development of a new earthquake magnitude scale, as recent literature suggests [21]. The development of M_L, M_s, m_b, Mw and M, further demonstrates the ongoing evolution and refinement of earthquake magnitude scales. Each iteration aimed to address limitations in the previous scale, reflecting the scientific community's commitment to a more accurate understanding of earthquakes. The very existence of the Choy and Boatwright [6] energy equation (M_e) serves as a testament to the need for further development. It emerged because the M_w and or M scale, while valuable, is not the ultimate answer. Therefore, ongoing research into a new earthquake magnitude scale based on body waves as suggested by Gutenberg and Richter [4] demonstrates a healthy scientific response to the complexities of measuring events.

Given the well-documented limitations of existing magnitude scales like M and M_w, there is a critical need for a scale that incorporates global tectonic factors and is based on body waves for more accurate earthquake measurement. This necessity has driven the development of the Das Magnitude Scale (M_{wg}) by Das et al. [8], offering a more comprehensive approach to seismic analysis.

Das et al. [8] adopted a meticulous approach to developing the M_{wg} scale. They began by assembling a vast instrumental dataset encompassing global seismicity. This dataset included 25,708 seismic moments directly observed from the Global Centroid Moment Tensor (CMT) database and m_b magnitudes from the International Seismological Centre (ISC) database. Notably, their study incorporated data for shallow, intermediate, and deeper earthquakes.

Recognizing the limitations of M_w or M scales, Das et al. [8] aimed to create the M_{wg} scale. This new scale aligns closely with existing magnitude scales within their respective applicable ranges. To ensure consistency with established techniques used by Hanks and Kanamori [1] and earlier works, Das et al. [8] derived a straightforward least-squares fitting relationship between seismic moment (M_0) and m_b magnitude ($\text{Log } M_0 = 1.36\ m_b - 17.24$) for the magnitude range of 3.5–7.0 and the timeframe of 1976–2006.

It is important to note that the linear relationship between seismic moment and any magnitude (Ms/mb/M_L) is fundamental in the development of any new magnitude scale. In the case of the M_w scale [3], this linear relationship ($\text{Log } M_o = 1.5\ M_s + 16.1$) was derived through several substitutions and use of constant terms. For development of the energy magnitude (Me), the linear relationship between $\log M_0$ and surface wave magnitude M_s was derived directly from the observed dataset, as performed by Das et al. [8].

In developing the M_{wg} scale, Das et al. [8] derived a linear relationship between $\log M_0$ and body wave magnitude (m_b) using observed seismic moment data, providing a more direct and accurate correlation based on actual measurements. This relationship, expressed as $\log M_0 = 1.36 m_b + 17.24$, offers an advantage over the M scale, which is based on the equation $\log M_0 = 1.5 M_s + 16.1$. Unlike the M scale, which was derived through several steps and involved constant terms ($E/Mo = 5 \times 10^{-5}$), Das et al.'s approach is more straightforward and precise. Das Scale was derived from simple calculation between Log Mo and m_b following the theoretical recommendation of Kanamori and Anderson [22]

The core principle behind Das et al.'s [8] approach lies in the relationship between m_b and M_0. If m_b is used in the equation, then M_0 should also be constrained. By treating M_0 independently, it can be placed on the left side of their equation, leading to the formula $M_{wg} = \text{Log } M_0$ /

1.36 – 12.68. This formulation provides a magnitude value that avoids saturation, a potential issue with other scales. Developing a magnitude scale (M_{wg}) based on body waves instead of surface waves has been a long-term demand initially raised by Gutenberg and Richter [4].

Furthermore, the correlation of seismic energy with the M_{wg} scale is explained through the well-established Gutenberg energy equation ($\text{Log } E_s = 1.5\ M_s + 11.8$). Finally, the validation of the M_{wg} scale follows the procedure outlined by Kanamori [3], ensuring adherence to established practices.

Through a comprehensive analysis, the advantages of the M_{wg} scale over the M_w and or M scales became evident:

(a) Hanks and Kanamori [1] applied Eq. (1) from Purcaru and Berckhemer (11) within the range $5.0 \lesssim M_s \lesssim 7.5$ for development of M scale. However, Purcaru and Berckhemer (11) specifically formulated their equation for the range $5.0 \le M_s \lesssim 7.0$, acknowledging the reliability of Ms only up to 7.0. This reveals a significant limitation of the M scale, which restricts its applicability to seismic events with M_s values ≤ 7.5. In contrast, the M_{wg} scale does not have this restriction.

(b) In contrast to M, which is limited to shallow earthquakes, the utilization of m_b in the M_{wg} scale allows for the measurement of earthquakes across all depths. This broader scope enhances the applicability and accuracy of the M_{wg} scale.

(c) In contrast to M_w or M which encounters limitations due to the assumption of a constant term ($E/Mo = 5 \times 10^{-5}$)in the Gutenberg Energy Equation ($\text{Log Es} = 1.5\text{Ms} + 11.8$), M_{wg} avoids such constraints. By utilizing 25,708 instrumentally recorded global seismic moments, M_{wg} overcomes issues related to constancy, ensuring a more accurate representation of seismic magnitudes while accommodating regional and tectonic variations.

(d) Unlike M, which primarily relies on surface wave magnitudes, M_{wg} is derived from observed M_o and m_b, capturing the initial cycles of P waves. This approach allows M_{wg} to establish a closer correlation with both low- and high-frequency spectra of seismic signals. Consequently, M_{wg} offers enhanced insights into seismic damage potential, making it invaluable for assessing the broader impact of earthquakes beyond their magnitude alone.

(e) In contrast to M, M_{wg} shows a remarkable alignment in applicable ranges with observed m_b and M_s values across all earthquake depths. Comparisons with M_E, M, and observed radiated energy consistently underscore M_{wg}'s superior agreement. Notably, 75% of estimated energy values using M_{wg} closely match observed radiated energy values, surpassing M. This heightened accuracy in energy magnitude representation significantly enhances the utility of the M_{wg} scale in earthquake size estimation.

(f) Enhanced Alignment: M_{wg} emerges as a superior magnitude scale compared to M, particularly in terms of alignment with old magnitude scales (M_L, Ms) and seismic event representation across various magnitude ranges. Figures 1, 2, and 3 of Das et al. [8] vividly illustrate the close alignment of M_{wg} with observed magnitude scales, contrasting with M. Notably, M_{wg} exhibits minimal deviation from observed old magnitude scales, statistically emphasizing its superior accuracy over M. The disparity between M_{wg} and observed magnitude scales is notably lower than that between M and the observed scales, with M tending to overestimate smaller and intermediate earthquakes. This discrepancy is evident in Fig. 3 and Table 2 of Das et al. [8], highlighting M_{wg}'s reliability over M, especially for smaller and medium earthquakes at the global level. In terms of seismic radiated energy E_s, M_{wg} proves more reliable than M, as depicted in Fig. 4 of Das et al. [8] and Fig. 1 of Das et al. [9]. Further analysis reveals significant disparities between observed m_b and estimated M, whereas M_{wg} demonstrates a considerably smaller average difference when compared to observed m_b. Similarly, M_{wg} showcases a closer alignment with observed M_s compared to M. The robust connection of M_{wg} with Me as addressed in Das et al. [8], further underscores its enhanced accuracy in representing seismic energy.

(g) The M_{wg} Scale: A More Accurate Measure of Earthquake Energy Release

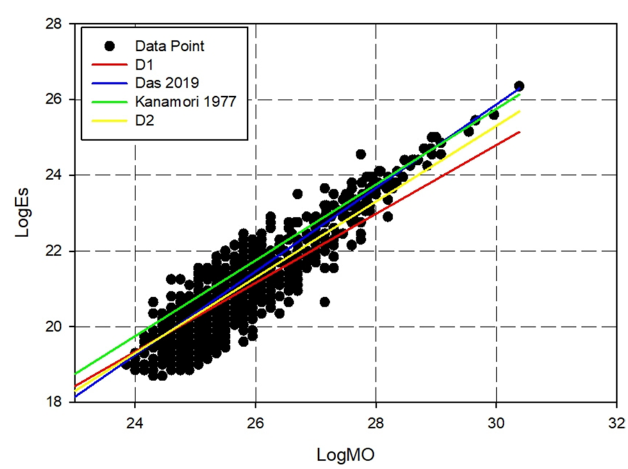

Fig. 4 Radiated Energy as a Function of Seismic Moment This figure depicts the radiated energy (Es) of a global earthquake dataset plotted against its seismic moment (Mo). The radiated energy values are estimated using both the Moment Magnitude (M) scale (represented by the Green solid line) and the proposed M_{wg} scale (represented by the blue solid line). Both estimations are derived from the Gutenberg-Richter energy equation: $\log(Es) = 1.5 * Ms + 11.8$

The most reliable method for understanding energy connection for a magnitude scale is to employ energy magnitude relations (e.g., Log E = 1.5 M_s + 11.8). One of the foundational energy magnitude relations is Gutenberg Energy magnitude relation Log E_s = 1.5 M_s + 11.8. However, Bath [23] derived an alternative equation relating energy and magnitude as:

$$\text{Log } E_s = 12.24 + 1.44M_s \quad (1)$$

Both equations exhibit nearly similar relationships (above M_s 6.5) between energy and magnitude (Fig. 2).

Above discussion indicates M_w and or M is a more precise energy representation for large and very large earthquakes (Fig. 3). However, Choy and Boatwright [6] demonstrated that M_w, in conjunction with the Gutenberg-Richter equation, overestimates radiated energy for magnitude values in the smaller and medium magnitude ranges.

For further understanding the M_w [3] scale's accuracy in representing earthquake energy release on a global scale for large earthquakes, we utilized radiated energy data from two sources. The first dataset, compiled by Choy and Boatwright [6] and another from Purcaru and Berckhemer [11]. The dataset of Purcaru and Berckhemer [11] encompasses a broader historical period, ranging from 1857 to 1977, and includes large earthquakes. Figure 4 indicates that for large and very large earthquakes, characterized by higher logM_0 values, the observed energy (log Es) closely matches the M_w scale (green line). In contrast, for smaller and medium-sized earthquakes, with lower and medium log M_0 values, the M_w scale tends to overestimate the actual energy released. This limitation of M_w scale is well-documented, as Choy and Boatwright [6] highlighted similar overestimations when using the Gutenberg Energy equation. Figure 4 demonstrates that the M_{wg} scale shows a stronger overall correlation between the observed energy (Log Es) and the M_{wg} scale (blue line) across all earthquake sizes, encompassing small, medium, and large events.

The overestimation of radiated energy seen with the M scale (in the range < 7.5) can be reduced by using the M_{wg} scale. Since the M_{wg} scale is derived from a more direct measure of the seismic moment (Mo), which closely reflects the actual energy released during an earthquake, it provides a stronger correlation with energy compared to the M_w scale. While the M_w scale is widely adopted, it tends to be less accurate for smaller and intermediate earthquakes because it depends on long-period surface waves (not directly connected to source).

Validity of the Moment Magnitude Scales (M, M_w, M_{wg}) in Global Tectonics and its Alignment with Existing Magnitude Scales (m_b, M_s)

One of the fundamental criteria in the development of magnitude scales is that it should align as closely as possible with the older magnitude scales in their applicable ranges. Different magnitude scales such as m_b, M_s, M_w, M and M_{wg} should closely coincide to ensure consistency in measuring the radiated energy from earthquakes. The close coincidence between M and previous magnitude scales (e.g., M_s, M_L) is crucial because it ensures continuity and consistency in the seismic size in terms of energy. Kanamori [3] compared M_w with global Ms (Tables 1, 2 of [3]). When Kanamori [3] founds M_w and M_s are closely related then he asserted M_w is a natural continuation of M_s. Hanks and Kanamori [1] also compared M with M_L and M_S (see Table 1 and Table 2 of [1]). Choy and Boatright [6] also compared M_e with M_w and M_s. The similarities between older magnitude scales (within their applicable range) and M demonstrate how closely these scales align with the measurement of seismic energy. For instance, using significantly different M_s and M values in energy calculations (Log Es = 1.5Ms + 11.8) can produce different energy estimates for the same earthquake. Therefore, when a new magnitude scale is developed, it is crucial that it closely aligns with existing older scales within their applicable ranges to ensure consistency and accuracy in seismic hazard assessments.

The body wave magnitude is effective up to magnitudes near 7.0 [1], and the surface wave magnitude is applicable up to around 8.3 [1]. However, a comprehensive scale is needed that can accurately reflect seismic activity across the entire spectrum of earthquake sizes, from smaller to medium and higher magnitudes. In deriving, M_w scale, Kanamori [3] compared M_w with the existing M_s scale using global data for range ≥ 7.5. This comparison was crucial to establish whether M_w could reliably replace M_s, providing a consistent measure of earthquake magnitude. Hanks and Kanamori [1] further validated the M scale by comparing it with the local magnitude scale (M_L) and Surface wave magnitude (M_s) using data from Southern California. These comparisons were essential to determine if M closely coincided with the traditional scales. Das et al. [8] statistically and numerically demonstrated how M_{wg} scale is closely connected with observed existing magnitude scales (m_b, M_s). The M_w or M, while effective for large earthquakes, encounters limitations when applied to smaller and medium events. Its applicability range is skewed towards higher magnitudes, hindering direct comparisons and accurate size estimation for smaller and medium earthquakes. To address this issue again, we revalidated the scale using global datasets from the period 1976–2014 obtained from the Global Centroid Moment Tensor (GCMT) and the International Seismological Centre

(ISC). In our analysis, we considered 39,792 m_b values from ISC and corresponding seismic moments from GCMT, as well as 21,256 Ms values from ISC and GCMT.

Our investigations clearly demonstrate (Fig. 5), as previously established by Das et al. [8], that **M** deviates significantly from $\mathbf{m_b}$ and $\mathbf{M_s}$ magnitudes within their applicable ranges. When comparing the alignment of amplitude-based scales like $\mathbf{m_b}$ and $\mathbf{M_s}$ with **M** and $\mathbf{M_{wg}}$, the $\mathbf{M_{wg}}$ scale shows statistically significant closeness to $\mathbf{m_b}$ and $\mathbf{M_s}$. As Fig. 5 shows, M aligns well with larger magnitudes (around 7.5 and above), which aligns with the range for which it was originally developed. However, for smaller and medium earthquakes, M deviates significantly from the observed trends of m_b and M_s data (refer to Figs. 5a, 2b).

All discussion reinforces the notion that the M scale is most appropriate for measuring large earthquakes exceeding 7.5, as initially suggested by Kanamori [3]. The M scale exhibits a tendency to significantly overestimate seismic moment values for earthquakes with surface wave magnitudes (M_s) below 6.5 and body wave magnitudes (m_b) across their applicable range, typically up to 7.0 [1]. However, the M_{wg} scale offers a potential solution by mitigating this overestimation for M_s values below 6.5 and m_b values up to 6.0.

To statistically assess the differences between M and M_{wg} estimates, a null hypothesis (Ho) is established, assuming both datasets share the same mean and variances. This hypothesis is rejected at confidence levels of 0.05 and 0.025 for m_b values between 4.5 and 5.5, and M_s values between 5.5 and 7.5 [8]. This rejection indicates statistically significant differences between M and M_{wg} estimates within these magnitude ranges. Consequently, the M_{wg} scale emerges as a single, uniformly valid magnitude scale applicable for earthquakes with magnitudes exceeding 4.5 [8].

Furthermore, the validation extends beyond m_b and M_s. To assess the similarity of M_{wg} and M with other observed magnitude scales, a globally distributed dataset of 1,361 energy magnitude (M_e) events from 1995 to 2007 was utilized [8]. A comparison of M_e, M, and M_{wg} revealed a noteworthy pattern: 53% of M_{wg} values were closer to M_e than M. This finding strengthens the case for M_{wg}, suggesting it aligns more closely with another independent measure of earthquake energy [8].

The strong correlation between M_{wg} and m_b, M_s, and Me indicates that M_{wg} is a more reliable measure than M. This is because M_{wg} closely aligns in the applicable ranges with scales ($m_b < 7.0$, $M_s < 8.3$) derived from direct amplitude measurements. While m_b and M_s are suitable for certain earthquake magnitudes, M_{wg} is particularly effective for large earthquakes and demonstrates a high correlation with m_b and Ms within their respective applicable ranges. Therefore, the close correspondence between M_{wg} and these other scales (in their applicable ranges) is critically important.

Hanks and Kanamori [1] derived the equation for M ($M = 2/3 * \text{Log Mo} — 10.7$) using the CGS (centimeter-gram-second) system, reporting results to one decimal

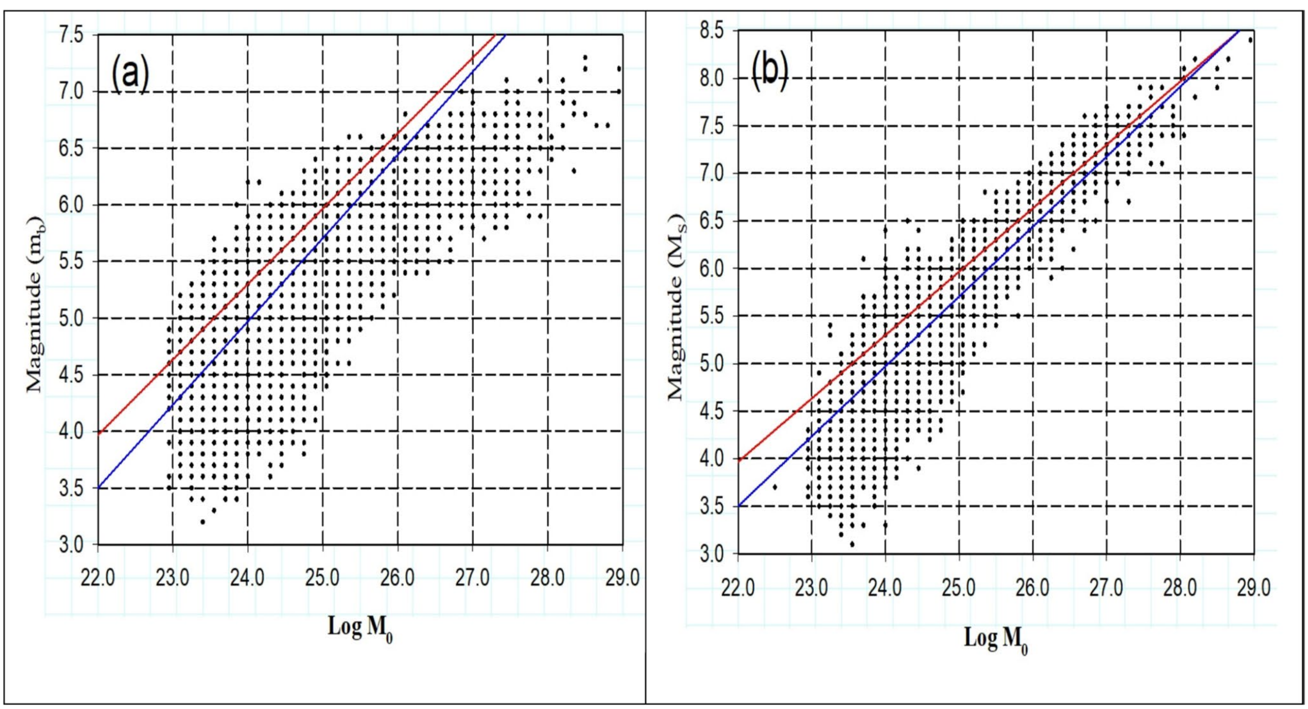

Fig. 5 Comparison of M scale (red line) and M_{wg} (blue line) with observed **a**: m_b and **b**: M_s

place. Kanamori [3] derived M_w by giving equation Log $W_0 = 1.5$ Ms + 16.1, where $W_0 = M_0/(2\times 10^2)$. The difference in the constant term between the two equations ($M_w = (LogW_0\text{-}16.1)/1.5$, M = 2/3LogMo-10.7) leads to a rounding error of up to 0.03. Notably, M = 2/3 Log Mo—10.7 is widely used and serves as a standard in global databases, including GCMT (Global Centroid Moment Tensor) and other regional seismic monitoring agencies. Thus, it is inaccurate to assert that Mw = $(LogW_0\text{-}16.1)/1.5$ and M = 2/3 Log Mo—10.7 yield identical estimates. This discrepancy arises from the constant term difference, and it is crucial to note that both forms of M will provide the same estimates only when using the alternate definition of M = 2/3 Log Mo—10.73333.The selection of the constant term in the M scale, specifically 10.7, is crucial for maintaining precision and consistency with the Eqs. (3,4,5,6, and 7) established by Hanks and Kanamori [1]. It is important to note that all the equations used in their work adhered to a consistent level of precision, which is why the constant term in the M scale was also set to 10.7 to match this precision.

To shed light on the discrepancies between M_e and M, we will scrutinize the equations formulated by Choy and Boatright [6] and the Gutenberg-Richter energy equation, both expressed within the CGS system.

$$\log E = 1.5\ M_s + 11.4 \text{ (with E in ergs)} \tag{2}$$

$$\log E = 1.5\ M_s + 11.8 \text{ (with E in ergs)} \tag{3}$$

Equation (2) can be redefined to derive a new magnitude expression, denoted as Me (Choy and Boatright, 2006):

$$M_e = \frac{\log E - 11.4}{1.5} \tag{4}$$

$$M_e = \frac{\log E - 11.4 + .4 - .4}{1.5} \tag{5}$$

$$M_e = M + 0.27 \tag{6}$$

Equation (6) shows that M and M_e are not equivalent, as they are derived from different physical principles. This distinction was emphasized by Choy and Boatright (2006) and Bormann and Giacomo [20]. Choy et al. [24] also pointed out that there is no inherent reason for Me and M to be the same. Since the M scale is defined based on the ratio $E_S/M_0 = 5\times 10^{-5}$, it differs from M_e. The disparity between M_e and M stems from their basis in distinct fundamental properties of seismic events.

A comprehensive analysis of 1385 worldwide energy data points revealed that 87% of the data deviated from the observed M values, underscoring the intrinsic dissimilarity between M and M_e. Moreover, 21% of observed M_e data deviates by 0.5 or more magnitude units from M. The maximum difference between M and M_e can reach up to 1.2 magnitude units. For instance, the earthquake on September 9, 2005, was reported with an M of 7.4, while the M_e was reported as 6.2.

The fundamental equation for the M_{wg} scale, as introduced by Das et al. [8], links seismic moment ($\log M_0$) with body wave magnitude (m_b), differing from the traditional relationship between $\log M_0$ and Surface wave magnitude (M_s) established by Kanamori [3]. The equation is as follows:

$$LogM_0 = 1.36m_b + 17.24 \tag{7}$$

Considering Eqs. (5) and (7), the relationship between energy magnitude (Me) and M_{wg} can be formulated in terms of the slowness parameter (⊖) as follows [20]:

$$M_e = \frac{1.36}{1.5}M_{wg} + \frac{5.44 + \ominus}{1.5} \tag{8}$$

From Eq. (8), it becomes evident that the M_{wg} scale is not identical to the energy magnitude (M_e) because both scales are based on different physical properties. This distinction emphasizes the importance of understanding and acknowledging the unique characteristics inherent in each seismic magnitude scale.

Upon scrutinizing 1385 observed radiated energy data, it is revealed that among these earthquakes, 730 events exhibited a closer alignment of M_{wg} with M_e compared to M. This implies that 53% of the data with M_{wg} demonstrated a closer agreement with M_e than M [8]. The same conclusion can be reached by using the dataset from Choy and Boatright [6] as demonstrated in Das et al. [8]. Hence, it is crucial to emphasize that M_{wg} exhibits a superior agreement in terms of energy magnitude (M_e).

Linearity Issue in Magnitude Scale Development

Most earthquake magnitude scales rely on a technique called linear least squares regression. This method assumes a straight-line relationship between two variables, such as seismic moment (Mo) and a specific magnitude (e.g., M_s, m_b, M_w, M etc.). Kanamori [3] utilized a linear least squares relationship between Log M_o and M_s to develop the M_w scale. Hanks and Kanamori [1] also employed the linear least square method between Log M_0 and M_s for the development of M scale. Linear least square relationship was the standard practice for developing relationships between Log Mo and magnitudes (e.g., Thatcher and Hanks [18], [1, 25]). Hanks and Kanamori [1] used linear least square relationship between Log M_o and M_L. As M_L and m_b are based on 1 s amplitudes, therefore, Hanks and Kanamori [1] considered the applicable range for M_L and m_b up to 7.0.

Similarly, the energy magnitude scale (M_e) was derived by Choy and Boatwright [6] using linear least squares regression. More than 90% of the seismic literature advocates the linear relation between Log Mo and magnitudes. However, a recent study by Gasperini and Lolli [21] has raised concerns about the validity of using linear regression between Log Mo and body wave magnitude (m_b). It is important to note that Gasperini and Lolli [21] overlooked the fact that most magnitude scales (e.g., M_w, M, M_e, M_{wg}) were based on linear least squares regression. While these authors (Gasperini and Lolli) conducted in their earlier studies (e.g., [25]) linear regression between M_w (= 2/3 logMo-10.7) and m_b, they raised questions about the linear regression performed by Das et al. [8] between Log M_o and m_b. Therefore, we will briefly explain this issue for the benefit of the reader. While deriving the M scale, Hanks and Kanamori [1] utilized a linear equation between Log M_o and M_s and in similar way Choy and Boatright [6] utilized linear least square equation between Log M_o and M_s for the range 5–8.6. Following the historical practices, Das et al. [8] utilized a linear equation between Log Mo and mb for the range 3.0–7.0. Hanks and Kanamori [1] advocated to use m_b up to 7.0 and M_s up to 8.3. Note that m_b and M_L are measured on 1 s and are considered applicable up to 7.0. Hanks and Kanamori [1] utilized the linear least square relation between Log M_o and M_L in the range 3.0–7.0. However, a recent study by Gasperini and Lolli [21] specifically targets the validity of linear least squares regression for M_{wg} scale, while overlooking its established use in developing other magnitude scales, such as those proposed by Hanks and Kanamori [1] and Choy and Boatright [6].

Furthermore, Gasperini and Lolli themselves utilized linear relationships between M_w and m_b (e.g., [25]), considering m_b values greater than 7.0. Since the M_w, M, and Me scales are fundamentally based on linear least squares, applying linear least squares in the development of the M_{wg} scale is appropriate. Additionally, Das et al. [8] clearly outlined the rationale for adopting linear least squares in the formulation of the M_{wg} scale.

To support nonlinear relationships between Log Mo and mb, Gasperini and Lolli [21] mischaracterized the findings of Kanamori and Anderson [22]. Therefore, we will clarify this issue with direct reference to Kanamori and Anderson's [22] original work. From theoretical considerations utilizing the viscoelastic model proposed by Haskell [26], Kanamori and Anderson [22] proposed three distinct slopes for the relationship between Log M_o and M_s (20 s). This conceptual framework applies across the entire magnitude spectrum, encompassing very small, small, medium, large, and very large earthquakes. Therefore, it is affirmed that employing a single linear relationship is not suitable for representing the complete magnitude range (−infinity to infinity, e.g., 0–8). According to Kanamori and Anderson [22], the expected scaling coefficient between the logarithm of scalar seismic moment (M_0) and M_s is approximately 1 for very small earthquakes, roughly 3/2 not for very large earthquakes (i.e., applicable for medium and large ones), and about 3 for the very large earthquakes (see abstract of Kanamori and Anderson [22] "*Except for very large earthquakes, the relation M s ~ (2/3) log M o ~ 2 log L is established by the data", "For very large earthquakes M s ~ (1/3) log M o ~ log L ~ (1/3) log E s. For very small earthquakes M s ~ log M o, ~ 3 log L ~ log E s.*"; page 1083 of Kanamori and Anderson [22], "*We have discussed only surface-wave magnitude but the same arguments hold for body-wave magnitude except that characteristic times are now to be compared with 1 s rather than 20 s. In this case, only very small earthquakes can be expected to scale as m b ~ log L 3 and more earthquakes can be expected to satisfy the mb ~ log L*:"). Kanamori and Anderson [22] further assert that similar arguments apply to the relationship between Log Mo and m_b (1 s), indicating that a single linear relation is not viable for the entire range of body wave magnitudes, encompassing very small, small, medium, large, and very large earthquakes.

In the context of Log Mo and m_b, with a slope of around 3 for very large earthquakes (> 8.0), around 3/2 not for very large earthquakes (i.e., medium and large earthquakes), and around 1.0 for very small earthquakes. The applicability of the linear relationship with a slope of 3 for very large earthquakes is not observed in m_b (1 s). Thus, justifying a linear relationship between Log M_o and m_b in the range of $3.5 \le m_b \le 7.0$ aligns with the theoretical model of Kanamori and Anderson [22], as magnitudes less than $m_b < 3$ can be considered very small earthquakes. It is, nevertheless, considered inappropriate to establish linear relationships across the entire magnitude range, covering very small, small, medium, large, and very large earthquakes, based on the considerations of Kanamori and Anderson [22].

Local magnitude M_L and teleseismic body wave magnitude m_b are determined from the amplitude of waves with a period of about 1.0 s. Thatcher and Hanks (1972) derived linear relationships between Log M_o and M_L for the magnitude range 3–7.0. Consequently, in deriving the moment magnitude (M) scale, Hanks and Kanamori [1] opted to utilize the relationship of Thatcher and Hanks (1972) between LogM_0 and M_L for magnitudes ranging from 3 to 7.0. Several authors (e.g., [25, 27]) also admitted that relationship between moment and m_b is also linearly related lower than the minimum current mb determination. A large number of studies (e.g., [28], Borman and Yadav [2], [25, 27],) have explored the linear regression between M and m_b. Bormann et al. [28] established a linear relationship between

M and m_b within the magnitude range 4.4–7.6. Borman and Yadav [2] also performed linear relationship between M and m_b within the magnitude range 4.4–7.6. Similarly, Scordilis [29] derived a linear relationship between M and m_b for the magnitude range $3.5 \le m_b \le 6.2$. Das et al. [30] also established a linear relationship within the magnitude range 2.9–6.5. Gasperini et al. [25] demonstrated a linear relationship between M and m_b greater than 7.0, as evident in their Figs. 1 and 2. The development of a linear relationship (in the range of medium and large, and small m_b earthquakes) between Log Mo and m_b aligns with the theoretical considerations of Kanamori and Anderson [22]. However, there is no physical reasoning why a non-linear relationship should exist between Log Mo and m_b in the medium, large and smaller m_b ranges.

Das et al.[8] established a linear relationship between Log M_o and m_b, grounded in the IASPEI [31] rule and the theoretical framework presented by Kanamori and Anderson [22], aligning with the findings of various researchers [1], Borman et al. [28], Wason et al. [32], [25], see page 932 of [25], "Almost all papers cited above used linear regression laws between Mw and mb", [9, 33, 34].

Critical Analysis of Gasperini and Lolli [21]

Recently, Gasperini and Lolli [21] claimed that the M scale is adequate, basing their argument on an alleged misprint in the caption of Table 1 in Purcaru and Berckhemer [11]. By introducing the misprint issue Gasperini and Lolli [21] distorted the original definition of ME of Purcaru and Berckhemer [11]. The misprint issue has been thoroughly discussed in the Appendix. It is scientifically and technically incorrect to assert that the M scale is adequate across all magnitudes and depths; this issue needs to be addressed to prevent bias in seismic energy budgeting. Gasperini and Lolli [21] questioned the validity of the basic assumptions of M_{wg} scale, specifically the application of linear least squares regression. However, as discussed above, we have demonstrated how linear least squares regression is indeed appropriate for developing magnitude scales. Note that linear least square regression was the core in building the M, M_w, M_e scales.

Gasperini and Lolli [21] argued that one could derive an infinite number of magnitude scales based on their chosen criteria, and they introduced hypothetical lines D1 (equation 15 of Gasperini and Lolli [21]) and D2 (equation 16 of Gasperini and Lolli [21]), claiming that these produce lower Absolute Average Deviation (AAD) and standard deviations compared to the M_{wg} scale. They attempted to imply that derivation of the M_{wg} scale is just one among countless possibilities. However, this argument is based on incorrect facts (e.g., use constant term −4.7 instead of correct value of −4.8) overlooks the critical limitations of the M and or M_w scale, particularly for measuring smaller and medium earthquakes, especially those below magnitude 7.5. Das and colleagues along with many researchers have demonstrated that the *M* scale is inadequate for these cases due to its inherent assumptions and constraints. The development of the M_{wg} scale specifically addressed these shortcomings, making it a more accurate and physically consistent alternative. Thus, the suggestion that an infinite number of magnitude scales can be defined dismisses the importance of addressing these known deficiencies and ignores the need for empirically validated and practically useful seismic measures.

The hypothetical lines D1 and D2, introduced by Gasperini and Lolli [21] in an attempt to invalidate the Das magnitude scale, were not derived following the standard procedures typically used for developing a magnitude scale. Specifically, the hypothetical line D2 exhibits a significant difference of 0.4 magnitude units (m.u.) across the higher magnitude range. This discrepancy arises from the use of a constant term of 4.8 instead of 4.3, as originally proposed by Kanamori [3]. The 0.4 m.u. difference for large and very large earthquakes ($\ge$7.5) undermines the close alignment with M_s data, which served as the basis for the development of the M_w scale by Kanamori [3].

D1 and D2 hypothetical lines did not adhere to the established methods used for scales such as M_w, M_e, and M_{wg}. There are key differences between the hypothetical lines and the original magnitude scales (i.e., M_w, M, M_e and M_{wg}). Kanamori's M_w scale, derived from surface wave magnitude, is primarily applicable to shallow earthquakes. Das's M_{wg} scale, on the other hand, utilizes body wave magnitudes, allowing it to function across the entire earthquake depth range. Additionally, the M scale was developed and validated for a specific region (Southern California), whereas the M_{wg} scale is formulated based on global data, offering a more comprehensive and consistent application. Importantly, the development of the M_{wg} scale addressed the limitations of the M scale. While energy comparison played a role in the M_{wg} scale's development, it wasn't the sole focus. Thus, hypothetical lines D1 and D2 don't meet the essential criteria for a new magnitude scale.

Furthermore, the evaluation criteria for D1 and D2 did not include any statistical tests, such as a null hypothesis, to determine their significance. Assessing their adequacy solely based on Absolute Average Deviation (AAD) and standard deviation does not provide a thorough comparison. In contrast, during the development of the M_{wg} scale, statistical significance was assessed using a null hypothesis to validate the results. The absence of rigorous statistical testing in the work of Gasperini and Lolli [21] undermines the validity of their arguments.

As illustrated in Fig. 4, the hypothetical lines D1 and D2 introduced by Gasperini and Lolli [21] significantly deviate in large magnitude ranges from the M and M_{wg} scales. While these lines align well with smaller and medium earthquakes, their divergence becomes pronounced for larger events. Although the hypothetical lines D1 and D2 appear reasonable in the small to medium magnitude range, they show significant deviations in the large and very large magnitude ranges. A fundamental criterion for a new magnitude scale is its alignment with existing scales (eg, m_b, M_s). Both D1 and D2 fail to meet this criterion, exhibiting significant deviations from observed M_s magnitudes (> 7.0).

Furthermore, the constancy issue inherent in Hanks and Kanamori's [1] scale persists in D1 and D2. However, it's important to recognize that similar arguments could be extended to other magnitude scales, such as M_w, M and M_e. If Kanamori [3] and Hanks and Kanamori [1] had used Bath's equation (Eq. 1) instead of Gutenberg Richter energy equation Log E = 1.5 M_s + 11.8, the formula for the moment magnitude (M_w) would have been:

$$M_{w-\text{Bath}} = \log \text{Mo}/1.44 - 11.625 \quad (9)$$

This illustrates that, just as with the M scale, it is theoretically possible to derive an infinite number of magnitude scales based on different mathematical approaches or assumptions. Therefore the same argument ("*In summary, our exercise teaches us that assumption and arguments leading to Mwg are scientifically unjustified because Mwg is only one of the infinite moment magnitude scales that could be defined and it is not even the best, based on the criteria proposed by its authors*", [21]) put forward by Gasperini and Lolli [21] can also be applied to the M_w and or M. The same argument can also be applied to Me [6] scale if one were to use Bath's equation instead of the Gutenberg Energy equation. Since Kanamori [5] stated that both Bath's equation and the Gutenberg-Richter energy equation are equivalent, it is feasible to use Bath's equation in the development of either the M_w or M_e scales. This illustrates that the choice of equations and assumptions in deriving magnitude scales can lead to various formulations, making the argument about the infinite possibilities of Gasperini and Lolli [21] equally valid for M_w, M, Me and Mwg scales. Given the limitations of the M_w and or M scales in accurately representing all magnitudes and depths, it is essential to address these issues and consider the development of a new, more comprehensive scale M_{wg}.

Comments on Gasperini and Lolli [21]

(i) Gasperini and Lolli [21] incorrectly used log10(Δσ/2μ) = −4.7, whereas the correct value, as derived from Choy and Boatwright [6], is −4.8, based on the equation Es = 1.6 × 10^{-5} Mo (pls see abstract of Choy and Boatright 1995). The implications of this error are discussed in points: a) *Inaccuracies in Eq. 12 of Gasperini and Lolli* [21]: The value − 4.7 used in Eq. (12) of Gasperini and Lolli [21] directly affects the computed relationship between M and M_E. Correcting this value to − 4.8 shifts the results and changes the interpretation of when M and M_E coincide. b) *Hypothetical Line D2 (equation 16 in G&L's Analysis) is incorrect:* The incorrect value for log10(Δσ/2μ) directly affects the derivation of Line D2 (Mwcb as per [21]), which was calculated based on the erroneous scaling relationship. As a result, this line does not accurately represent the intended relationship, leading to flawed conclusions. c) *Impact on Results and Discussion and Conclusion:* The miscalculation propagates through Gasperini and Lolli [21] analysis, affecting their interpretation of M_e and M. The incorrect scaling introduces bias, which undermines the validity of their arguments. Therefore, Table 1 and Fig. 1 of Gasperini and Lolli [21] are incorrect. d) *Incorrect comparisons of M_{wg} and M_{wcb}*: The incorrect value directly affects the computed M_{wb} and related comparisons with other magnitude scales. These comparisons, highlighted in the discussion and conclusions, are, therefore, invalid. Thus, main conclusions of the paper of Gasperini and Lolli [21] are incorrect and invalid.

(ii) Incorrect Representation of Eq. (1) of Purcaru and Berckhemer [11]

Gasperini and Lolli [21] incorrectly stated that Eq. (1) of Purcaru and Berckhemer [11] was developed in the range *5 ≤ Ms ≤ 7.5*. In their statement, Gasperini and Lolli [21] write, *"whereas* Purcaru and Berckhemer [11] *from earthquakes with 5 ≤ Ms ≤ 7.5 recorded all over the world obtained log10 M0 = 1.5Ms + 16.1."* However, Purcaru and Berckhemer [11] explicitly stated that their Eq. 1 is developed to earthquakes with magnitudes up to Ms ≲ 7.0 (Page 189 of Purcaru and Berckhemer [11] *"In the range of moderate to large earthquakes (Ms ≲7, M0 ≲ 10^27 dyn.cm), where Ms is a reliable measure, the relation between Log Mo and Ms is linear and average relation see* Fig. 4*: Log Mo = (16.1 ± 0.1) + 1.5 Ms—-(1) is* established to give the best fit with the observed data"*)*. Therefore, Gasperini and Lolli's reference to M_s ≤ 7.5 inaccurately extends the range specified by Purcaru and Berckhemer [11].

(iii) *Mischaracterization of* Purcaru and Berckhemer [11] *regarding a printing mistake:* Gasperini and Lolli [21] suggest the presence of a printing error in Purcaru and Berckhemer's work, without providing supporting references. This claim is unfounded and, if accepted, could distort the understanding of the ME scale as originally defined by Purcaru and Berckhemer [11]

(iv) *Misunderstanding the applicability of the Mw scale:* Gasperini and Lolli [21] fail to distinguish between the M scale and M_w scale, which differ by 0.03, and do not discuss the limitations of the Mw scale for global events below M_s 7.5. Kanamori [3] and Hanks and Kanamori [1] outlined the range of applicability for the M_w scale, with Kanamori [3] specifically noting that the M_w scale was designed as a natural continuation of the Ms scale for great earthquakes. However, the M_w scale is not suitable for smaller and medium earthquakes, as illustrated by evidence in Das et al. [8] and further explored in [35, 36], which demonstrate that the M_w scale overestimates smaller and medium earthquakes.

(v) *Inconsistency in the relationship between Log M_0 and mb:* While Gasperini and Lolli acknowledged a linear relationship between M_w and m_b in their earlier publications (e.g., [27]), they later suggest nonlinear relationships between Log M_O and mb for the Das et al. [8] model. However, we have demonstrated that a nonlinear relationship is not suitable for the 3.0 to 7.0 magnitude range, although nonlinear relations are appropriate for a broader range of magnitudes. We have provided a detailed justification for why a linear relationship is used in the specific range discussed.

(vi) *Criticism of m_b values up to 7.0:* Gasperini and Lolli [21] criticized the use of m_b values up to 7.0 in Das et al. [8], despite employing similar values in their own work (e.g., [27]). This inconsistency may confuse readers. To clarify, we have explained above the rationale behind adopting a linear relationship between Log M_O and mb for values up to 7.0, based on the foundational work of Kanamori and Anderson [22].

(vii) *Misinterpretation of* Kanamori and Anderson [22]: Gasperini and Lolli [21] claim that Kanamori and Anderson [22] noted the linearity of the relationship between Log M_O and mb only for magnitudes below 5.0. However, Kanamori and Anderson [22] discussed the relationship between Log M_O and Ms, not mb. They did not specify a range of mb ≤ 5.0, which Gasperini and Lolli incorrectly reference. This misinterpretation further misrepresents the work of Kanamori and Anderson [22].

Why the Old Standard M_w And or M Isn't Enough: The Need for M_{wg}

(1) *Improved Consistency of the M_{wg} Scale in Reducing Discrepancies with observed m_b and Ms:* The differences between the M scale and the other observed scales, m_b and M_s, are significantly greater (Null Hypothesis test was performed) compared to the differences between the M_{wg} scale and m_b and M_s (Figs. 1, 2, 3 of [8],Fig. 5, Fig. 6). The average difference between observed m_b and estimated M_{wg} is 0.008 ± 0.33. The average difference between observed mb and estimated M is − 0.31 ± 0.30. The average difference between observed M_s and estimated M is − 0. 43 ± 0.28, whereas the average difference between observed M_s and estimated M_{wg} is − 0.12 ± 0.26.

The M_{wg} scale's ability to reduce these discrepancies is crucial for achieving a more consistent representation of energy levels across different magnitude scales. Note that this parameter was critical when designing the M or M_w scale.

(2) *Closer Alignment with Seismic Energy Radiation:* It has been demonstrated [8, 9] that the M_{wg} scale provides a better correlation with seismic energy, making it the preferred choice for use.

(3) *Resolution of Technical Issues in the M Scale:* The technical problems associated with the M scale, arising

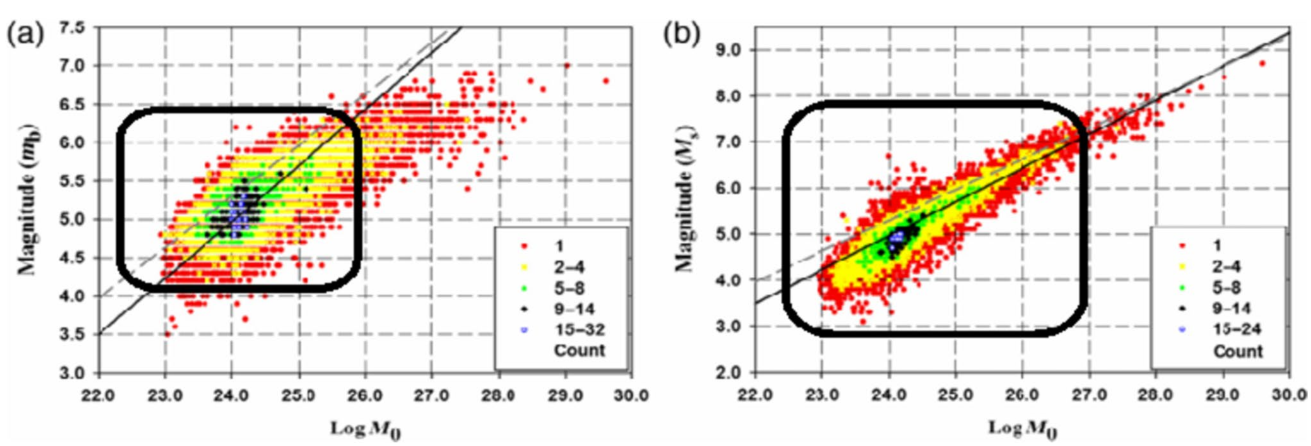

Fig. 6 Comparison between M_{wg} and M Scales [8]. **a** A statistically significant deviation is observed between m_b and M compared to M_{wg}, with this deviation being statistically significant. **b** A statistically strong deviation is observed between M_s and M compared to M_{wg}

from the misuse of Eq. 1 from Purcaru and Berckhemer [11] for magnitudes below 7.5, are effectively eliminated with the adoption of the M_{wg} scale.

(4) *Based on Global Tectonics:* The M_{wg} scale incorporates a global understanding of tectonic processes, making it more suitable for evaluating earthquakes across different tectonic settings.

(5) *Fulfills a Long-Standing Demand by* Gutenberg and Richter [4]: The M_{wg} scale addresses the recommendations of seismology pioneers Gutenberg and Richter [4], who emphasized the importance of developing magnitude scales based on direct measurements of body waves rather than surface waves ("*Determination of magnitude at large distances.—Theoretically, this should be based rather on body waves than on surface waves; unfortunately, readings of amplitude and period for body waves are less generally available*", page 142 of [4]). This approach ensures greater accuracy in magnitude determination, particularly for large-distance events, and aligns with their call for integrating body wave data into seismic analysis.

(6) *Addresses the limitations of the M_w Scale:* It overcomes certain shortcomings of the M_w and or M scales, such as inaccuracies in representing smaller and intermediate earthquakes, and provides a more accurate measure of seismic energy across various magnitude ranges.

(7) *Utilizes Instrumental Observations:* The M_{wg} scale is derived from directly observed seismic data, ensuring that it is based on actual measurements rather than relying on assumptions or indirect estimations. The M_{wg} scale aligns closely with the m_b and M_s scales within their respective applicable ranges. This is significant because amplitude-based scales, which directly measure seismic signal amplitudes, also show a strong correlation with M_{wg}, making it a more versatile and beneficial scale for assessing large earthquakes beyond the limitations of m_b and M_s.

These characteristics make the M_{wg} scale a robust and versatile tool for assessing earthquake magnitudes on a global level.

Conclusions

The earthquake magnitude scale plays a crucial role in earthquake science, making accurate knowledge of this scale essential for various seismological, geophysical, geoscientific and engineering applications. The introduction of the M_w scale by Kanamori [3] marked a significant advancement in seismology. However, the original scale was not suitable for earthquakes with magnitudes below 7.5. To address this limitation, Hanks and Kanamori [1] extended the scale to cover magnitudes as low as 3.0 by introducing the formula $M = 2/3\log Mo - 10.7$. M and M_w are distinct magnitude scales, differing by approximately 0.03 due to variations in their definitions and precision. The GCMT or other data agency should refrain from using the term M_w for magnitudes below 7.5, as Kanamori specifically defined M_w for earthquakes with magnitudes ≥ 7.5. Furthermore, the M scale, as defined by Hanks and Kanamori [1], is expressed as $M = 2/3\log Mo$ -10.7, and is intended for smaller, medium and large magnitudes. This distinction should be maintained to avoid confusion between the two scales. Despite its widespread use, the M scale has critical limitations and given that it was developed over 50 years ago, there have been substantial advancements in seismology and seismic instrumentation. These developments and along with critical limitations of M scale necessitate a thorough reevaluation of the M scale to ensure its continued accuracy and relevance.

This paper clearly demonstrates the limitations of the M and M_w scales for magnitudes below 7.5, with several reasons supporting this conclusion: 1) Kanamori [3] cautioned against using the M_w scale for earthquakes below this threshold, prompting the subsequent development of improvements by Hanks and Kanamori [1]. 2) Hanks and Kanamori [1] incorporated an equation $\log Mo = 1.5 Ms + 16.1$ from Percaru and Berckhemer (11) for surface wave magnitude (M_s), but this equation was incorrectly used for the range $5.0 \leq M_s \leq 7.5$. Importantly, Percaru and Berckhemer (11) themselves acknowledged that M_s reliability only extends up to 7.0. This inherent restriction in M_s translates to potential inaccuracies in the M scale for earthquakes exceeding this magnitude threshold. The inadequacy of the M scale for earthquakes below a magnitude of 7.5 is clearly evident from the instrumental datasets, as demonstrated in Figs. 5 and 6.

The key limitations of the M_w and M scales—namely, 1) inapplicability to deeper and intermediate-depth events, 2) lack of global validation (applicable for M scale), and 3) overestimation of energy for lower and intermediate-magnitude earthquakes—can be effectively addressed by using the M_{wg} scale. Both the M and M_{wg} scales are physics-based, as they are derived from Mo. The assumptions utilized in Das et al. [8] align with the principles established by Kanamori and Anderson [22], affirming their scientific validity.

Additionally, the M_{wg} scale aligns well with existing magnitude scales (m_b and M_s). The M scale relies on a constant value to relate moment magnitude and surface wave magnitude. This assumption, however, has been challenged since the work of Percaru and Berckhemer [11]. In contrast, the M_{wg} scale eliminates the need for a constant value altogether. Furthermore, the M_{wg} scale's reliance on body-wave magnitudes (m_b) holds promise for overcoming the limitations of the M_w scale, particularly for earthquakes at deeper and intermediate depths.

This investigation provides a detailed refutation of the claims made by Gasperini and Lolli [21], specifically their assertion that the M and M_w scales are adequate. Additionally, their more recent allegation that the basic assumption mainly using linear regression between Log Mo and m_b for development of M_{wg} scale are flawed has also been found to be incorrect. Linear regression is a standard method in the development of magnitude scales, including the M, M_w, and Me scales, and there is no valid reason to reject its application in the case of the M_{wg} scale. Moreover, the use of linear regression between log M_0 and m_b aligns with the theoretical framework established by Kanamori and Anderson [22]. The linear relationship between Log Mo and ML is a fundamental component of the M scale. If this relationship is correct, then a linear relationship between Log Mo and mb should also hold, as both M_L and m_b are based on 1-s seismic waves. Based on a thorough scientific analysis and critical evaluation, it is evident that the conclusions drawn by Gasperini and Lolli [21] are unfounded. This is primarily due to their use of an incorrect constant term, −4.7, as well as their misinterpretation of the works of Purcaru and Berckhemer [11] and Kanamori and Anderson [22].

We recommend that various international agencies, such as the ISC, NEIC, and GCMT, consider adopting the M_{wg} scale alongside existing magnitude scales to enhance the understanding of earthquake sources. It is important to note that the M_{wg} scale is based on body waves, fulfilling a longstanding demand originally emphasized by Gutenberg and Richter [4].

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Data availability Body- and surface-wave magnitudes of earthquakes for the entire globe from the International Seismological Centre (ISC, United Kingdom) database (http://www.isc.ac.uk/iscbulletin/ search/bulletin, last accessed August 2018), and the moment magnitudes from the Global Centroid Moment Tensor (CMT) database (http://www.globalcmt.org/CMTsearch.html, last accessed October 2018) during the period 1976–2014 have been compiled in this study.

Declaration

Conflict of interest The author acknowledges that there are no conflicts of interest recorded.

Appendix A

While criticizing Das et al. [8], Gasperini and Lolli [21] introduced an alleged misprint in the caption of Table 1 of Purcaru and Berckhmer [11] i.e., Ms = 7–7.5 without any reference or evidence. Note that this label 'Ms = 7–7.5' in the caption of Table 1 of Purcaru and Berckhmer [11] has been used in the development of ME scale by Purcaru and Berckhmer [11]. This label 'Ms = 7–7.5' has no connection in the development of M scale of Hanks and Kanamori [1]. When the original authors, Purcaru and Berckhmer [11], explicitly acknowledge in their article that they derived Eq. (1) for the magnitude range $\lesssim$ 7.0 due to the reliability of M_s up to 7.0 (*see page 189 of* Purcaru and Berckhemer [11], *"In the range of moderate to large earthquakes ($M_S \lesssim$ 7, Log M_O, < 10^27 dyn.cm), where M_S is a reliable measure, the relation between log M and M_s is linear and the average relation (see* Fig. 4*) ……is established to give the best fit with the observed data*") and the Eq. (1) of Purcaru and Berckhmer [11] has been used in the development of M scale [1] for magnitude range 5 ≤ Ms $\lesssim$ 7.5. Therefore, introducing alleged misprint in the caption of Table 1 of Purcaru and Berckhmer [11] i.e. 'Ms = 7–7.5' is irrelevant.

In the formulation of M scale, Hanks and Kanamori [1] leaned on the support provided by Eq. (1) from Purcaru and Berckhmer [11] to establish the global applicability of the M scale for magnitudes below 7.5 (Page 1223 of [1], "Which is remarkably coincident with M0 -Ms relationship empirically defined by Purcaru and Berckhmer [11] for 5 < = M_s = < 7.5"). However, a closer examination reveals that Purcaru and Berckhmer [11] originally defined the relationship between Log M_0 and M_s specifically for the magnitude range $\lesssim$ 7.0. Consequently, this reevaluation suggests that the M scale is inadequate for accurately representing seismic events below 7.5 on a global scale.

Gasperini and Lolli [21] argued that if Ms range is considered as ≤ 7–7.5 in the caption of Table 1 in Purcaru and Berckhemer [11], then the global applicability of the M scale could extend to earthquakes with magnitudes below 7.5. However, this argument is incorrect, as the validity of the M scale for magnitudes below 7.5 is solely linked to Eq. (1) in Purcaru and Berckhemer [11], which only applies up to a magnitude of 7 and has no connection to the caption of Table 1 of Purcaru and Berckhemer [11]. It is important to note that the magnitude range referenced (Ms = 7–7.5) in the caption of Table 1 pertains exclusively to the development of the ME scale and has no relevance to the M scale development.

Das et al. [8] argued that the Eq. (1) of Purcaru and Berckhemer [11] for the magnitude range 5.0 ≤ *Ms* 7.5 is not reliable due to the inconsistency of the defined magnitude range (moderate-to-large earthquakes defined as *Ms* ≤ 7.0 and *Ms* = 7–7.5).There is no importance of this inconsistency issue because Purcaru and Berckhemer [11] clearly explained that they derived their Eq. (1) for $M_s \lesssim 7.0$ (see page 189, [11]) and this equation is the key equation for M scale applicability for the range < 7.5).

Das et al. [8] identified two distinct ranges for moderate and large earthquakes in Purcaru and Berckhemer [11]: one for Ms ≤ 7 in the development of Eq. 1 of Purcaru and Berckhemer [11] and another for Ms = 7–7.5 in the development of ME scale, highlighting an inconsistency in the definition of moderate and large earthquake ranges. Gasperini and Lolli [21] attempted to resolve this inconsistency by suggesting a misprint in the range $M_s = 7–7.5$. However, even if one accepts the possibility of a misprint, the inconsistency remains because two separate magnitude ranges for moderate and large earthquakes persist. Furthermore, the reader should note that even with this misprint one cannot claim M scale is adequate below 7.5 because a) adequacy of M scale below 7.5 is connected only with Eq. (1) of Purcaru and Berckhemer [11] which is applicable to Ms ≲ 7.

Gasperini and Lolli [21] specifically focuses on labeling "Ms = 7–7.5" as a misprint, rather than stating that "Ms ≤ 7" may also be a misprint. This choice aligns with their defense of the applicability of the M scale in the range of 5.0 ≤ Ms ≤ 7.5. However, the Gasperini and Lolli [21] made a significant mistake by misinterpreting an alleged misprint of "Ms = 7–7.5" as:"Ms ≤ 7–7.5". This alleged misprint introduces a major error because in the caption of Table 1 header of Purcaru and Berckhemer [11] discusses moderate to large earthquakes and assigns the range as "Ms = 7–7.5" It is important to note that the range suggested by Gasperini and Lolli [21] using "Ms ≤ 7–7.5," cannot be considered as moderate to large earthquakes, as it would include magnitudes such as 1, 2, 3, 4, 5, 6, and 7.5. Magnitudes 1, 2, 3, 4, etc., are not classified as moderate earthquakes. Even with the inclusion of Ms ≤ 7–7.5 in the caption of Table 1 in Purcaru and Berckhmer [11], the essential condition of Eq. (1) of Purcaru and Berckhmer [11] remains unaltered.

In the caption of Table 1, Purcaru and Berckhemer [11] specified an M_s range of 7–7.5 to define the ME scale. However, Gasperini and Lolli [21] claimed that this range is a printing error, suggesting it should be Ms ≤ 7–7.5. This assertion by Gasperini and Lolli [21] is incorrect, as altering the Ms range would fundamentally change the definition of the ME scale as originally established by Purcaru and Berckhemer [11]. For justifying the arguments for M_S range in the caption of Table 1 of Purcaru and Berckhemer [11] they also have presented statements that lack clarity and supporting evidence. One of the statements is as quoted "The clear intention of Purcaru and Berckhemer [11] to indicate the range Ms < 7–7.5 is demonstrated by the parallel indication of the same range in terms of scalar moment as Mo ≤ 10^27". Gasperini and Lolli [21] claimed that the magnitude range $M_s \le 7.0–7.5$ is inferred from a parallel indication, specifically $M_0 \le$ 10^27 (dyn cm) in the caption of Table 1 of Purcaru and Berckhmer [11]. However, this assertion is considered incorrect, as the data in Table 2 of Purcaru and Berckhmer [11] indicates that even with $M_0 = 10^{27}$, M_s can reach as high as 8.4. The representation of the range is comprehensive, taking into account both M_s and M_0. Purcaru and Berckhmer [11] explicitly mentioned this when deriving their Eq. 1, stating the condition Ms ≲ 7.0 and $M0 \le$ 10^27(dyn.cm).

Figure 1 in Purcaru and Berckhmer [11] displays a plot of log M_0 and M_S in the range $M_S = 5—8.6$, incorporating a large set of data compiled from various literature sources, encompassing both ordinary and anomalous earthquakes (Fig. 1). However, they derived Eq. (1) specifically for Ms ≲ 7.0, acknowledging the reliability of M_s up to 7.0. Purcaru and Berckhmer [11] explicitly define the range for moderate to large earthquakes using $M_S \lesssim 7$ and $M_0, \le$ 10^27 (dyn.cm). From Fig. 1a of Percaru and Berckhemer [11], it clearly visible that Log $M_o = 1.5\ M_s + 16.1$ is derived using Ms ≲ 7.0 and Mo < = 10^27 (Fig. 1). The broken lines of Fig. 1a of Percaru and Berckhemer [11] clearly demonstrates that M_s data was considered up to < = 7.0 (Fig. 1). In Fig. 3 of Purcaru and Berckhmer [11], the relationship between M_0 and E_s (seismic energy) for moderate to large earthquakes is illustrated using crosses as symbols. Figure 4 of Purcaru and Berckhemer [11] clearly shows the relation between Log M_o and M_s for moderate to large earthquakes for Ms < 7 (Page 197 of Purcaru and Berckhemer [11], "The two broken lines indicate that M_O, corresponding to fixed M_S or M_E, varies within a factor of 6. Constant strain drop: Log M_O, = 16.1 + 1.5 M (full line)").

Gasperini and Lolli [21] claim that there are sentences on page 189 of Purcaru and Berckhemer [11] that indicate a magnitude range up to 7.5 ("the clear intention of Purcaru and Berckhemer [11] to indicate the range Ms ≤ 7.0–7.5 is demonstrated by the parallel indication of the same range in terms of scaler moment as Mo ≤ 10^{27} dyn.cm and even by other sentences of the articles e.g., at pages 189 and 195"). However, there is only one sentence on page 189 that mentions the range of "7.5" (page 189 of [11], "The scatter within the linear band in Fig. 1, for $M_s \le 7.5$, is therefore interpreted in terms of variable stress drop"). This sentence explains that the variation or spread of the data points within the linear band can be attributed to the concept of variable stress drop.

Based on above discussions, Fig. 1 and several statements (e.g., on page 189 "In the range of moderate to large earthquakes ($M_S \lesssim 7$, $M_0 \lesssim 10^{27}$ dyn.cm), where M_S

is a reliable measure, the relation between log M_0, and M_s is linear … is established", on page 190 "The two broken straight lines (the band) represent the linear trend of the relation log M_0-M_S for moderate to large earthquakes ($M_S \lesssim 7$ and/ or $M_0 \lesssim 10^{27}$ dyn.cm")) of Purcaru and Berckhemer [11], the conclusion can be drawn that Gasperini and Lolli [21] have inaccurately represented or mischaracterized the findings of Purcaru and Berckhemer [11].

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